Conjunctive Explanations: When Are Two Explanations Better than One?

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Abstract

When is it *explanatorily* better to adopt a conjunction of explanatory hypotheses as opposed to committing to only some of them? Although conjunctive explanations are inevitably less probable than less committed alternatives, we argue that the answer is not 'never'. This paper provides an account of the conditions under which explanatory considerations warrant a preference for less probable, conjunctive explanations. After setting out four formal conditions that must be met by such an account, we consider the shortcomings of several approaches. We develop an account that avoids these shortcomings and then defend it by applying it to a well-known example of explanatory reasoning in contemporary science.

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1 Introduction

When are two explanations better than one? That is, when is it *explanatorily* better to adopt the conjunction of individual explanatory hypotheses as opposed to choosing between them?[1] The options considered by this question are logically asymmetrical, the choice being between a "conjunctive explanation" $h_1 \wedge h_2$ and one of its conjuncts, say h_1. The single conjunct option h_1 is uncommitted with respect to h_2; it is the option of only committing with respect to h_1 and so is rightly thought of as equivalent to $h_1 \wedge (h_2 \vee \neg h_2)$. It is emphatically *not* the committed stance characterized by the conjunction $h_1 \wedge \neg h_2$, a stance which characterizes an alternative conjunctive explanation, on a logical par with $h_1 \wedge h_2$.

Accordingly, the original question can also be clarified as follows: when is it *explanatorily* better to opt for logically stronger positions? Logically stronger options, conjunctive explanations, commit to strictly more information about the world. There

[1]Here, we intentionally put aside the question of when it is *overall epistemically* right to favor a conjunction over one of its conjuncts. Our question pertains specifically to contexts of explanatory reasoning in which one is in search of the most explanatory conclusion. We leave open the possibility that the most explanatory conclusion might not be the overall epistemically best in some broader sense. Other related questions that are worth distinguishing from our own include: 1) When are two hypotheses better confirmed by the evidence than either individually (Atkinson et al., 2009)?; and 2) Under what conditions, and to what extent, do hypotheses compete with one another, relative to some body of evidence (Schupbach and Glass, 2017; Schupbach, 2019; Glass, 2019)?

are thus strictly more ways in which they might turn out false. That is, conjunctive explanations have an unavoidably smaller chance of being right. Nonetheless, the answer to our central question is not "never"; it can be explanatorily better to opt for necessarily less likely options.

On the surface, this may seem strange. But a moment's reflection makes it more plausible and even obvious. Adding more relevant information into an explanation can, of course, make for an overall better explanation. That Tweety is a penguin is a better explanation of her inability to fly than that Tweety is a bird.

Psychological studies have demonstrated that subjects often prefer conjunctive explanations of human actions suggesting "that a complete explanation is often one that includes multiple goals, or goals plus preconditions" (Leddo et al., 1984, p. 940; see also Abelson et al., 1987). Insofar as scientists, social scientists and historians appeal to multiple causes to explain a given explanandum, conjunctive explanations are also relevant. Explanations of this kind are particularly prevalent in the biological sciences. For example, in their defence of "integrative pluralism," Mitchell and Dietrich draw attention to work by Blaustein and Kiesecker (2002) providing "evidence that multiple factors in interaction are responsible for amphibian decline," but more generally they argue that "the nature of the complexity of the systems studied by biology in conjunction with a decomposition methodology necessitates a plurality of causal hypotheses that are not competing but compatible" (Mitchell and Dietrich, 2006, p. S77).

Consider a well-known example from science. Scientists evaluate several distinct hypotheses as potential explanations of the mass extinction event at the Cretaceous-Paleogene (K-Pg) boundary (responsible for the extinction of the dinosaurs). These include bolide impact, massive volcanic activity and flooding, climate change, and sea level regression. Until recently, some geologists have argued for bolide impact as a "smoking gun" explanation, sufficient to account for the variety of historical traces and evidence relevant to the event. Cited evidence includes, in particular, anomalously high levels of iridium in deep-sea limestones dating from the K-Pg boundary (Alvarez et al., 1980), the Chicxulub crater (Hildebrand et al, 1991), and "ejecta-rich deposits" in distribution patterns related to distance from the crater (Schulte et al., 2010).

More recent work has focused on the question of whether we don't explain more of this evidence at a deeper level by logically strengthening this position, opting for the impact hypothesis in conjunction with other of the above potential explanations (Archibald et al., 2010; Courtillot and Fluteau, 2010; Keller et al., 2010; Renne et al., 2015). For example, Archibald et al. suggest that the explanans which commits only to the impact hypothesis is explanatorily anemic when it comes to the evidence found in "countless studies of how vertebrates and other terrestrial and marine organisms fared at the end of the Cretaceous." In many recent publications on the topic, it is common to find scientists opting at least for an explanans which conjoins the impact hypothesis with volcanic flooding.

We will return to this example below in order to test the adequacy of our proposed account. For now, the case serves to motivate our study by exemplifying the phenomenon of conjunctive explanation. Sometimes explanatory considerations seem to warrant a preference for saying strictly more about the world, in spite of the resulting price of being more likely wrong.

This paper seeks to provide a principled account of conjunctive explanation; we seek an explication of the logical-epistemic conditions under which it is explanatorily better to opt for strictly stronger, less probable explanations. Section 2 introduces and motivates a set of desiderata for any such explication. Section 3 then applies these desiderata in order to highlight the shortcomings of some plausible, candidate explications. These criticisms are illuminating, providing positive direction for a more successful account of the conditions under which conjunctive explanations might be preferable. Such an account is developed and defended in the same section. Section 4 concludes by applying our account back to the K-Pg example. It is argued that our preferred explication appropriately captures exactly the right considerations at work in this case study.

2 Desiderata for Conjunctive Explanations

We seek an account of the conditions under which a conjunctive explanation is *explanatorily better* than its logically weaker components. Our general strategy is to identify a measure of explanatory goodness, $\mathcal{E}$, and use it as follows:

Conjunctive Explanation based on explanatory goodness, $\mathcal{E}$. Two hypotheses are explanatorily better together if their conjunction $h_1 \wedge h_2$ has more explanatory goodness with respect to explanandum e according to measure $\mathcal{E}$ than either conjunct individually: $\mathcal{E}(e, h_1 \wedge h_2) > \mathcal{E}(e, h_1[h_2])$.

Naturally then, we must explicate exactly what notion of "explanatory goodness" is at work here. There are as many distinguishable notions of explanatory goodness as there are dimensions along which we evaluate explanations (Schupbach, 2017). In different contexts, explanations are preferred for being simpler, more unifying, having broader scope and/or more power, etc., or for having any combination of these putative virtues. But what particular notion(s) of explanatory goodness captures the virtue at work when we prefer conjunctive explanations as "explanatorily better"?

To address this question, we set out a number of minimally necessary conditions such an account should satisfy. Our goal is to identify criteria relevant to the explanatory goodness of a conjunction of hypotheses, $h_1 \wedge h_2$, and then to obtain a measure that can enable us to specify when such a conjunctive explanation provides a better explanation than either conjunct according to our strategy above. Most obviously, we should require the following:

Possibility. Conjunctions of explanatory hypotheses may be explanatorily better with respect to some explanandum than their corresponding component hypotheses.

That is, the account should allow for the possibility that conjunctive explanations sometimes be better than logically weaker alternatives. This requirement is in line with our argument above, that logically stronger hypotheses can sometimes be explanatorily superior to weaker alternatives.

Two further natural requirements that we shall have to make more precise below are as follows:

Power. Conjunctive explanations may be preferred because they more powerfully account for given evidence than any weaker position.

Scope. Conjunctive explanations may be preferred because they account for a wider array of evidence than any weaker position.

These desiderata point to two closely related reasons that may lead a reasoner to opt for a conjunctive explanation, and it clarifies that the account we aim for should reflect both types of consideration. In cases like the K-Pg example above, reasoners might endorse a logically stronger explanatory stance because such an explanans has greater power over some given evidence; i.e., one can account more convincingly for some body of evidence by doing so. But reasoners may also endorse the conjunctive position because doing so allows one to account for a broader swath of evidence. The relevant notion of explanatory goodness should thus incorporate the virtue of scope; the more evidence an explanation is able to account for, ceteris paribus, the better the explanation.

Note that the general strategy we outlined at the start of this section concentrated on whether a conjunctive explanation provides a better explanation for a single explanandum e. However, **Scope** concentrates on a wider array of evidence and so may not seem relevant to our account.[2] Nevertheless, it is still indirectly relevant since it provides one way in which explanatory goodness can be enhanced. In fact, as we shall see, this can be formulated as a criterion for a measure of explanatory goodness for explanations in general, whether conjunctive or not. Once an appropriate measure is identified that satisfies this criterion (along with the others), it can then be applied to conjunctive explanations.

There is, of course, an inevitable cost that comes by accepting conjunctive

[2]We thank a reviewer for raising this point.

explanations, and the following desiderata requires that our account also be sensitive to this cost:

Complexity. Logically stronger explanatory hypotheses by necessity convey strictly more information than weaker alternatives, making them necessarily less likely to be true. This cost in informational complexity can sometimes outweigh improvements in explanatory power or scope.

We now use these four informal desiderata to help motivate formal criteria for a measure of explanatory goodness that will be appropriate for conjunctive explanations.

It is straightforward to express the **Possibility** criterion formally in terms of a measure of explanatory goodness $\mathcal{E}$ as follows:

C1. For two distinct explanatory hypotheses, h_1 and h_2, for explanandum e, it is not necessarily the case that the explanatory goodness of $h_1 \wedge h_2$ is less than or equal to that of each conjunct. Formally, it is possible that $\mathcal{E}(e, h_1 \wedge h_2) > \mathcal{E}(e, h_1[h_2])$.

The other informal criteria are not so easily translated into corresponding formal criteria. The reason for this is that there is a balance to be struck between **Power** and **Scope** on the one hand and **Complexity** on the other. Consider **Power** for example. As already noted, there is a cost in terms of complexity associated with a conjunctive explanation, so it is not immediately obvious how to state the condition under which conjunctive explanations are to be preferred because of their greater power. However, we can state a related condition that is relevant for the measure of explanatory goodness even though it does not specifically mention conjunctive explanations. Essentially, the idea is that the greater the power, ceteris paribus, the better the explanation. For the ceteris paribus condition, we need to keep the complexity (or information content) of the hypotheses fixed and in standard approaches to information content this can be done by setting the probabilities of the hypotheses to be equal. How can we express the idea that one hypothesis has greater power than another? As will become clear when we consider measures of explanatory power, the extent to which the explanandum is rendered more probable by a hypothesis is closely related to its power. This suggests the following criterion:[3,4]

C2. For two distinct explanatory hypotheses, h_1 and h_2, for explanandum e, such that $P(h_1) = P(h_2)$, then $\mathcal{E}(e, h_1) > \mathcal{E}(e, h_2)$ if $P(e|h_1) > P(e|h_2)$.

Essentially, **Scope** requires that the explanatory goodness of a given conjunctive explanation can be greater in some cases for multiple pieces of evidence than it is for just one of them. In fact, there is no reason to restrict this criterion to conjunctive explanations, so we can formulate it more generally in terms of an explanatory hypothesis h. Of course, if it does not account for a piece of evidence very well, that may not enhance explanatory goodness, but we can propose the following condition

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[3]We will assume throughout that the relevant probabilities are defined and in particular that $0 < P(e), P(h_1), P(h_2), P(e \wedge h_1), P(e \wedge h_2), P(h_1 \wedge h_2) < 1$.

[4]C2 is crucially different from a straightforward "one-likelihood inequality" condition (Roche and Sober, 2023), which takes $P(e|h_1) > P(e|h_2)$ to be a sufficient condition for $\mathcal{E}(e, h_1) > \mathcal{E}(e, h_2)$, leaving out C2's additional stipulation that $P(h_1) = P(h_2)$. As we will argue in the discussion of measures of explanatory power below, such a condition spells trouble in attempting to explicate the notion of explanatory goodness at work in conjunctive explanations.

for a specific case in which it is clear that a hypothesis does have greater explanatory goodness if it explains more evidence:

C3. Suppose that e_1 and e_2 are two distinct and logically independent explananda. If an explanatory hypothesis h entails both e_1 and e_2, $h \vDash e_1$, $h \vDash e_2$, then $\mathcal{E}(e_1 \wedge e_2, h) > \mathcal{E}(e_1, h)$.

Finally, for **Complexity** we can ensure that the scope is kept fixed by keeping the explanandum fixed as e and ensure the power is fixed by keeping the probability of e given the hypotheses fixed. The next condition then needs to capture the idea that in such a case a hypothesis that is more complex cannot increase the explanatory goodness. We do this by comparing the hypothesis h_1 with the conjunction $h_1 \wedge h_2$ since $P(h_1 \wedge h_2) \leq P(h_1)$, and hence the informational complexity of $h_1 \wedge h_2$ is at least as great as that of h_1. In fact, our fourth condition is just a statement that ensures irrelevant conjunctions do not result in better explanations:

C4. Suppose that hypothesis h_2 is probabilistically independent of hypothesis h_1, explanandum e and their conjunction.[5] Then $\mathcal{E}(e, h_1 \wedge h_2) < \mathcal{E}(e, h_1)$.

To illustrate, let e be a description of the bending of light from a distant source by the sun and h_1 an explanation in terms of Einstein's theory of general relativity. Let h_2 be the hypothesis that Covid-19 was the result of a lab leak. A plausible measure of explanatory goodness in the sense intended here should show that this conjunction provides a worse explanation since it is more complex and does not provide a better

[5]In which case, $P(e|h_1 \wedge h_2) = \dfrac{P(e \wedge h_1 \wedge h_2)}{P(h_1 \wedge h_2)} = \dfrac{P(e \wedge h_1)P(h_2)}{P(h_1)P(h_2)} = P(e|h_1)$.

account of the explanandum. As such, C4 can be seen as an instance of Ockham's razor.

3 Accounting for Conjunctive Explanations

The **Power** criterion suggests an obvious place to turn in attempting to explicate the notion of explanatory goodness at work in conjunctive explanations, namely, to the formal literature explicating and measuring explanatory power (Sprenger and Hartmann, 2019, Variation 7). In the remainder of this section, we show that a particular trajectory through this expanding literature does indeed provide us with some promising directions for developing a working account. A way forward presents itself if we carefully attend to the unique shortcomings—apropos the above desiderata—of some candidate accounts.

Schupbach and Sprenger's (2011) account provides a fruitful launching point. This account is meant to capture "one familiar and epistemically compelling sense of explanatory power that is common to human reasoning." The salient notion of *power* "has to do with a hypothesis's ability to decrease the degree to which we find the explanandum surprising (i.e., its ability to increase the degree to which we expect the explanandum)." As one could argue by way of the K-Pg example above, one reason why one might be willing to commit to a logically stronger explanation is if doing so purchases greater explanatory power in this sense; i.e., if by doing so, the explanandum becomes much less surprising.

Schupbach and Sprenger argue that the following measure provides a uniquely best explication of their informal notion of power:

$$\mathcal{E}_{SS}(e,h) = \frac{P(h|e) - P(h|\neg e)}{P(h|e) + P(h|\neg e)}.$$

However, even if $\mathcal{E}_{SS}$ adequately captures the notion of power that Schupbach and Sprenger have in mind, the following observation reveals that it does not provide an explication of explanatory goodness appropriate for an account of conjunctive explanations:

Proposition 1. *$\mathcal{E}_{SS}$ satisfies criteria C1 and C2, but not C3 or C4.*

Since it does not satisfy C3, $\mathcal{E}_{SS}$ fails to reflect the virtue of scope in important contexts. Consider the scenario specified in C3 where an explanatory hypothesis h entails two distinct (and logically independent) explananda individually: $h \vDash e_1$, $h \vDash e_2$. For example, let e_1 be a description of Earth's planetary orbit, e_2 a description of the perihelion of Mercury and h an explanation in terms of Einstein's theory of general relativity along with relevant auxiliary assumptions. In this case, h has equal (and maximal) degree of explanatory power (à la Schupbach and Sprenger) over e_1, e_2, *and* $e_1 \wedge e_2$; i.e., $\mathcal{E}_{SS}(e_1, h) = \mathcal{E}_{SS}(e_2, h) = \mathcal{E}_{SS}(e_1 \wedge e_2, h)$. In terms of $\mathcal{E}_{SS}$, no explanatory power is gained here by accounting for $e_1 \wedge e_2$ as opposed to accounting only for e_1 (or e_2). What is needed in order to account more fully for the potential virtues of conjunctive explanations is a measure like Schupbach and Sprenger's, but which more adequately reflects the virtue of scope.[6]

[6]Lange (2022, p. 260) offers a similar criticism of Schupbach and Sprenger's measure in contexts where deductive entailment relations imply that this measure takes its

This is exactly what seems to be provided by the following alternative measure of explanatory power, $\mathcal{E}_{GM}$, defended by Good (1960)[7] and McGrew (2003):

$$\mathcal{E}_{GM}(e,h) = \frac{P(e|h)}{P(e)}$$

Like Schupbach and Sprenger's measure, $\mathcal{E}_{GM}$ is straightforwardly interpreted as measuring "a hypothesis's ability to decrease the degree to which we find the explanandum surprising." But it has the advantage of satisfying C3 and hence more adequately incorporating the virtue of scope.

Nonetheless, $\mathcal{E}_{GM}$ also does not provide an adequate account of explanatory goodness appropriate for conjunctive explanations:

Proposition 2. *$\mathcal{E}_{GM}$ satisfies criteria C1, C2 and C3, but not C4.*

Like $\mathcal{E}_{SS}$, $\mathcal{E}_{GM}$ fails to satisfy C4 and so does not adequately reflect the cost of informational complexity in the context of conjunctive explanations. The following result highlights the close relationship between both of these measures and likelihoods:[8]

Proposition 3. *Let $\mathcal{E}$ represent either $\mathcal{E}_{SS}$ or $\mathcal{E}_{GM}$. $\mathcal{E}(e,h_1) \gtreqless \mathcal{E}(e,h_2)$ if and only if $P(e|h_1) \gtreqless P(e|h_2)$.*

In fact, this result applies to virtually all proposed measures of explanatory power (offered as explications of the degree to which *h* alleviates our surprise in *e*). This

maximum value across hypotheses that are intuitively unequal in their "explanatory loveliness."

[7] Good's measure is technically the log-normalized version of $\mathcal{E}_{GM}$.

[8] See Glass (2023a) for further discussion of this result.

includes the measures proposed by Popper (1959) and Crupi and Tentori (2012). Since it follows immediately that they fail to satisfy C4, they are therefore unsatisfactory for present purposes.[9]

Recall that the motivation for **Complexity** is the observation that there is a cost in informational complexity when we favor a logically stronger position; after all, there are strictly more ways that such a position could be wrong. The problem with the measures considered so far is that they ignore this cost in complexity altogether. According to accounts of conjunctive explanation based on these measures, any benefit in accounting for e (no matter how slight) is worth any cost that comes by complicating our explanatory stance (no matter how great, short of inconsistency). For this reason, this account too is not capturing the salient notion of explanatory goodness at work in conjunctive explanations. What is needed (per **Power**, **Scope** and **Complexity**) is an explication of explanatory goodness that combines the scope-incorporating notion of explanatory power with a penalty for informational complexity.

Interestingly, Bayes's Theorem can be thought of as combining exactly these considerations. For a particular h and e, the $\mathcal{E}_{GM}$ measure captures a scope-incorporating notion of explanatory power. And as already emphasized, strictly increasing informational complexity (increasing logical strength) corresponds

[9]Roche and Sober (2023) similarly criticize "purely probabilistic measures of explanatory power" for satisfying their "one-likelihood inequality" condition (a special case of the above **Proposition 3**). We agree with them in this paper at least insofar as we believe this to be a problem when it comes to using such measures for explicating the virtue of *conjunctive* explanations.

to a strictly decreasing probability. Thus, a hypothesis's probability provides a straightforward penalty factor for informational complexity. For example, for logically independent h_1 and h_2, $h_1 \wedge h_2$ is strictly more informationally complex than either hypothesis individually, and so it should be penalized relative to these weaker options; this can be achieved simply by weighting a hypothesis's explanatory goodness by it's probability—since $P(h_1 \wedge h_2) < P(h_1[h_2])$. Bayes's Theorem does precisely this:

$$P(h|e) = \frac{P(e|h)}{P(e)} \times P(h) = \mathcal{E}_{GM}(h, e) \times P(h).$$

Does this mean that we could use the posterior probability of a hypothesis as suitable measure of explanatory goodness? While posterior probability is not generally considered as a measure of explanatory goodness, its weakness is particularly obvious in the context of conjunctive explanations. Such an account would fail to satisfy C1 and hence **Possibility**, since necessarily $P(h_1 \wedge h_2|e) \leq P(h_1|e)$. Whereas conjunctive explanations would be altogether too easy to come by based on $\mathcal{E}_{GM}$, they would be too difficult to come by (impossible in fact!) if we were to use posterior probability.

The scope-incorporating, explanatory power account based on $\mathcal{E}_{GM}$ ignores complexity altogether. By contrast, an approach based on posterior probability places extreme weight on considerations of complexity such that conjunctive explanations are effectively banned from the start. These problems are instructive. What we can now see is that the salient notion of explanatory goodness, the one at work when we favor conjunctive explanations, should strike a balance between (scope-incorporating) explanatory power and complexity. This can be achieved by finding a principled middling position between the problematically extreme accounts based on $\mathcal{E}_{GM}$ and posterior probability.

Interestingly, I. J. Good (1968) discusses the same shortcoming of his own $\mathcal{E}_{GM}$ measure that we highlighted above, emphasizing the fact that this measure ignores considerations of informational complexity. For this reason, he labels $\mathcal{E}_{GM}$ his "weak" measure of explanatory power. Good proceeds to distinguish this measure from a family of "strong" measures that penalize for complexity. Good's strong measures are characterized by adding a γ-weighting parameter to the probability of a hypothesis in a log-normalized version of Bayes's Theorem (where $0 < \gamma < 1$):

$$\begin{aligned}\mathcal{E}_G(e,h) &= \log\left(\frac{P(e|h)\cdot P(h)^{\gamma}}{P(e)}\right)\\ &= \log\left(\frac{P(e|h)}{P(e)}\right) + \gamma \log P(h). \end{aligned} \tag{1}$$

Note, on the one hand, that if $\gamma = 0$, then the resulting measure ends up being Good's weak measure $\mathcal{E}_{GM}$ again, with the result that complexity is given no weight. On the other hand, if $\gamma = 1$, then the resulting measure is effectively Bayes's Theorem, with the aforementioned result that complexity is given implausibly extreme weight in potential conjunctive explanation scenarios.[10] Since $0 < \gamma < 1$, this family of measures provides any number of more plausible explications of the notion of explanatory goodness driving judgments about conjunctive explanations. For the rest of this paper, we follow Good's suggestion of setting $\gamma = 1/2$ (see also Glass

[10]Setting $\gamma = 1$ also has the effect of simplistically equating the conceptually distinct notions of "explanatory loveliness and likeliness," a move criticized convincingly by Lipton (2004).

(2023b) for a defence of this parameter setting). Selecting a different value of γ would not fundamentally change the picture, but would result in a different balance between (scope-incorporating) explanatory power and complexity.[11]

We can now state the following result:

Proposition 4. *$\mathcal{E}_G$ satisfies criteria C1, C2, C3 and C4.*

Hence, $\mathcal{E}_G$ provides a more appropriate measure for expounding the formal epistemology of conjunctive explanations.[12] Following the approach set out at the start of section 2, we can now state the following account of conjunctive explanation:

Conjunctive Explanation based on $\mathcal{E}_G$. Two hypotheses are explanatorily better together if their conjunction $h_1 \wedge h_2$ has more explanatory goodness with respect to explanandum e according to measure $\mathcal{E}_G$ than does either conjunct individually: $\mathcal{E}_G(e, h_1 \wedge h_2) > \mathcal{E}_G(e, h_1[h_2])$.

Letting h_1 be the explanatory hypothesis with the greater individual degree of explanatory goodness with respect to e, i.e. $\mathcal{E}_G(e, h_1) \geq \mathcal{E}_G(e, h_2)$, this account

[11] In particular, proposition 4 would still hold provided $0 < \gamma < 1$.

[12] Does this mean that Good's measure provides a uniquely best overall measure of explanatory goodness? Insofar as it satisfies C4, it explicates a different concept from the other measures and in so doing captures an important feature of explanatory goodness. However, a much more detailed argument would be needed to defend Good's measure as the best measure of explanatory goodness overall, and so we take no position on that question here. For present purposes, we only wish to claim that it provides an adequate account of conjunctive explanation. For a more general defence of Good's measure, see Glass (2023a).

requires the following inequality for a conjunctive explanation to be favored over both of its component conjuncts (i.e., $\mathcal{E}_G(e, h_1 \wedge h_2) > \mathcal{E}_G(e, h_1)$ is equivalent to the following condition):

$$\log\left(\frac{P(e|h_1 \wedge h_2)}{P(e|h_1)}\right) > \log\left(\frac{1}{P(h_2|h_1 \wedge e)}\right). \tag{2}$$

We can express this result in information-theoretic terms. As Good (1968, p. 126) observes, citing the work of Bar-Hillel and Carnap (1953) and in accordance with our comments above, the informational complexity of h can be measured as a function of its prior, by $\mathrm{Inf}(h) = -\log P(h)$. Informational complexity can also of course be measured conditionally, such that, for example, $\mathrm{Inf}(h|e) = -\log P(h|e)$. Moreover, the amount of information concerning e provided by h can be quantified as $\mathrm{Inf}(e, h) = \log[P(e|h)/P(e)]$, which is just the logarithm of $\mathcal{E}_{GM}$. Note that this can also be expressed as $\mathrm{Inf}(e, h) = \mathrm{Inf}(e) - \mathrm{Inf}(e|h)$, which represents the reduction in informational complexity of e brought about h. As Glass (2023b) shows, $\mathrm{Inf}(e, h)$ and $\mathrm{Inf}(h|e)$ may plausibly be thought of as quantifying *explanatory gain* and *explanatory cost* respectively. Good's measure (with $\gamma = 1/2$) can then be expressed in terms of the difference between explanatory gain and cost as follows:

$$\mathcal{E}_G(e, h) = 1/2 \times \left[\mathrm{Inf}(e, h) - \mathrm{Inf}(h|e)\right]. \tag{3}$$

We can think of h as providing a good explanation of e to some extent if $\mathcal{E}_G(e, h) > 0$, or equivalently, if the explanatory gain is greater than the explanatory cost.[13]

[13]In fact, this approach to explanatory goodness provides a justification for setting

Based on these considerations, the inequality in (2) can be expressed in information-theoretic terms as:[14]

$$\mathrm{Inf}(e, h_2|h_1) > \mathrm{Inf}(h_2|h_1 \wedge e). \tag{4}$$

Suppose h_1 provides an explanation for e, and we wish to know whether it would be an explanatory improvement to commit to the logically stronger position described by the conjunctive explanation, $h_1 \wedge h_2$. Expression (4) asks us to consider whether, given h_1, the additional reduction in informational complexity of e brought about by h_2 is greater than the complexity arising from the introduction of h_2 in the context of h_1 and e. Informally, one should opt for the conjunctive explanation if doing so results in an explanatory gain (in terms of reduced informational complexity in the explanandum) that is worth the explanatory cost (of accepting an explanatory stance with overall greater informational complexity).

The following result follows trivially from expression (2), but it is worth stating since it distinguishes the proposed account of conjunctive explanation based on $\mathcal{E}_G$ from those based on $\mathcal{E}_{SS}$ and $\mathcal{E}_{GM}$.

Proposition 5. *A necessary but not sufficient condition for a conjunctive explanation $h_1 \wedge h_2$ to provide a better explanation of explanandum e than h_1 does is: $P(e|h_1 \wedge h_2) > P(e|h_1)$.*

$\gamma = 1/2$ (Glass, 2023b).

[14]In the more general case where γ is not set to a value of $1/2$, the condition for conjunctive explanation, corresponding to expression (2), would become $\log\left(\frac{P(e|h_1 \wedge h_2)}{P(e|h_1)}\right) > \frac{\gamma}{1-\gamma}\log\left(\frac{1}{P(h_2|h_1 \wedge e)}\right)$ and hence (4) would be replaced by $\mathrm{Inf}(e, h_2|h_1) > \frac{\gamma}{1-\gamma}\mathrm{Inf}(h_2|h_1 \wedge e)$.

For accounts based on $\mathcal{E}_{SS}$ and $\mathcal{E}_{GM}$, this condition would be sufficient as well as necessary. This result reveals that explanatory power is still important in the proposed account. However, it is no longer sufficient since even though the inclusion of h_2 raises the probability of e, this may not be sufficient to outweigh the additional informational complexity introduced by h_2.

The follow result identifies certain limiting cases relating the explanatory goodness of a conjunctive explanation to that of a single component conjunct.

Proposition 6. *As an explanation of e, the conjunctive explanation $h_1 \wedge h_2$:*

(i) must be exactly as good an explanation as h_1 if h_1 along with background knowledge probabilistically entails h_2; formally, $\mathcal{E}_G(e, h_1 \wedge h_2) = \mathcal{E}_G(e, h_1)$ if $P(h_2|h_1) = 1$;[15]

(ii) cannot provide a better explanation than h_1 if h_1 along with background knowledge probabilistically entails e; formally, $\mathcal{E}_G(e, h_1 \wedge h_2) \leq \mathcal{E}_G(e, h_1)$ if $P(e|h_1) = 1$;

(iii) provides a better explanation than h_1 if e (or $e \wedge h_1$) along with background knowledge probabilistically entails h_2, but h_1 along with background knowledge does not probabilistically entail h_2 or e; formally, $\mathcal{E}_G(e, h_1 \wedge h_2) > \mathcal{E}_G(e, h_1)$ if $P(h_2|e) = 1$ or $P(h_2|e \wedge h_1) = 1$, $P(h_2|h_1) < 1$ and $P(e|h_1) < 1$.

Each of these plausible results lends credibility to the account. For result (i), the fact that the information provided by h_2 is effectively already included in h_1 means that there is no semantic difference between the conjunctive explanation and the component explanation h_1; hence there is no consequent difference in their

[15]We have excluded background knowledge from the specified probabilities for simplicity.

explanatory values with respect to *any* e. The fact that h_1 probabilistically entails e in result (ii) means that there is no scope for h_2 to improve the explanation by increasing the probability of e and so again it does not result in a better explanation. Finally, with result (iii), the inclusion of h_2 increases the probability of e at least to some extent and, crucially, since $P(h_2|e)$ or $P(h_2|e \wedge h_1)$ equals one there is no explanatory cost in informational complexity associated with the introduction of h_2.

To clarify further this last, more complicated result, consider a simple example. Suppose Smith died as a result of anaphylactic shock (e). The conjunctive explanation that Jones spiked Smith's drink with peanut oil and Smith had a nut allergy ($h_1 \wedge h_2$) would be a far better explanation than the simpler individual hypothesis that Jones spiked Smith's drink with peanut oil (h_1). This is due to the fact that e is made much more probable by conjoining the hypothesis that Smith had a nut allergy (h_2) to h_1 than it is by merely committing to h_1, regardless of whether or not h_2; moreover, there is no additional cost associated with h_2 since it is probabilistically entailed by $e \wedge h_1$.[16]

[16] Strictly speaking, depending upon the background knowledge, h_2 might of course not be entailed by $e \wedge h_1$ in this example. However, the story could be enriched—e.g., by ruling out other possible allergens in the spiked drink—in order for such background knowledge to force the probabilistic entailment.

4 Conjunctive Explanation and the K-Pg Extinction Event

In this closing section, we argue that our account captures the salient factors that ought to be considered (and balanced) in important real-life cases. We argue this by returning to the K-Pg extinction example.

Recall that scientists, in a number of different publications, have recently argued for the possibility that we can account for more of the available evidence by logically strengthening our explanans, conjoining the already favored impact hypothesis with other explanatory hypotheses. Archibald et al. (2010, p. 973), for example, write:

> A simplistic extinction scenario has not stood up to the countless studies of how vertebrates and other terrestrial and marine organisms fared at the end of the Cretaceous. Patterns of extinction and survival were varied, pointing to multiple causes at this time—including impact, marine regression, volcanic activity, and changes in global and regional climatic patterns.

The argued upshot is that multiple causal-explanatory hypotheses are going to be needed to account for the complex evidence pertaining to this event. And the difference is logically captured with a simple appeal to the (scope-incorporating) notion of explanatory power. Letting e represent the complex conjunction of evidence, and focusing for simplicity on the bolide impact h_1 and volcanic flooding h_2

hypotheses, we may explicate the conclusion for which these scientists are arguing as:

$$\frac{P(e|h_1 \wedge h_2)}{P(e)} \gg \frac{P(e|h_1)}{P(e)}.$$

Of course, as we've argued, more needs to be said at this point; after all, if this were the sole desideratum for deciding whether to strengthen one's explanans, there would seem to be no end to the additional h_i's we could introduce that would still further boost e's likelihood (conditional on the hypotheses so far accepted). One wants to know that the cost in terms of complexity introduced by logically strengthening the position is worth the resulting increased explanatory gain over the evidence.

It is thus reassuring to find scientists arguing for exactly this further point. For example, Renne et al. (2015) write:

> Bolide impact and flood volcanism compete as leading candidates for the cause of terminal-Cretaceous mass extinctions. High-precision $^{40}\text{Ar}/^{39}\text{Ar}$ data indicate that these two mechanisms may be genetically related, and neither can be considered in isolation. The existing Deccan Traps magmatic system underwent a state shift approximately coincident with the Chicxulub impact and the K-Pg mass extinctions [...] Initiation of this new regime occurred within $\sim 50,000$ years of the impact, which is consistent with transient effects of impact-induced seismic energy.

The point that Renne et al. are arguing is distinct from the previous point. Their point in offering the high-precision $^{40}\text{Ar}/^{39}\text{Ar}$ data is not that this evidence can better (or only) be accounted for by the conjunction of distinct hypotheses. Rather, this data "indicate that [bolide impact and flood volcanism] may be genetically related," and so their article makes the case for thinking that h_1 and h_2 are themselves causally tied together. That is, they provide reason to think that a bolide impact would kick off massive volcanic activity. To the extent that they establish this point, volcanic flooding is much more likely to occur on the heels of a bolide impact (conditional or not on e), and so $P(h_2|h_1 \wedge e) \gg P(h_2|e)$.[17] And this inequality directly bears on the informational complexity added to an explanation by h_2, assuming that the explanans already contains h_1 (conditional or not on e), for it shows that the complexity penalty $\text{Inf}(h_2|h_1 \wedge e)$ is unlikely to be substantial.

Our account clarifies that a conjunctive explanation would be superior in this case if

$$\text{Inf}(e, h_2|h_1) = \log\left(\frac{P(e|h_1 \wedge h_2)}{P(e|h_1)}\right) > \log\left(\frac{1}{P(h_2|h_1 \wedge e)}\right) = \text{Inf}(h_2|h_1 \wedge e).$$

Of course, our purpose in discussing this work is not to argue that this inequality is satisfied in this example. We leave it to the scientists to figure that out. But the point *is* that this formal account nicely explicates what the scientists are trying to figure out. Our account captures the relevance for this question of exactly those considerations that the scientists are raising. Those arguing that the conjunctive explanation is better in this case are putting forward two distinct arguments. The first

[17] Or at least that this is the case if $P(h_2|e)$ is not too high. Note also that $P(h_2|h_1 \wedge e) = \frac{P(e|h_1 \wedge h_2)}{P(e|h_1)} P(h_2|h_1)$. We have already seen that there are good reasons for thinking that $\frac{P(e|h_1 \wedge h_2)}{P(e|h_1)}$ is high. Renne et al. provide reasons for thinking that $P(h_2|h_1)$ is substantially greater than it would be under the assumption that h_1 and h_2 are independent.

is probabilistically explicable as the argument that the left hand term of the above inequality is substantial; the second can be explicated as the argument that the right hand term is not too high. To the extent that these two arguments are compelling, our account agrees that this would be a case where the logically stronger, conjunctive explanation is superior.

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Appendix

As noted earlier, we assume that the relevant probabilities are defined and that $0 < P(e), P(h_1), P(h_2), P(e \wedge h_1), P(e \wedge h_2), P(h_1 \wedge h_2) < 1$.

A.1 Proof of Proposition 1

C1. From proposition 3, we know that $\mathcal{E}_{SS}(e, h_1 \wedge h_2) > \mathcal{E}_{SS}(e, h_1)$ iff $P(e|h_1 \wedge h_2) > P(e|h_1)$. Since this condition can be satisfied, $\mathcal{E}_{SS}$ satisfies C1.

C2. This condition follows trivially from proposition 3.

C3. Since h entails e_1, it follows that $P(h|\neg e_1) = 0$ and so $\mathcal{E}_{SS}(e_1, h)$ takes on its maximum value of 1. However, this is also true for $\mathcal{E}_{SS}(e_1 \wedge e_2, h)$ since h also entails e_2. Hence, $\mathcal{E}_{SS}$ fails to satisfy C3.

C4. By assumption $P(e|h_1 \wedge h_2) = P(e|h_1)$. From proposition 3, it follows that $\mathcal{E}_{SS}(e, h_1 \wedge h_2) = \mathcal{E}_{SS}(e, h_1)$. Hence, $\mathcal{E}_{SS}$ fails to satisfy C4.

A.2 Proof of Proposition 2

C1. As for proposition 1.

C2. As for proposition 1.

C3.

$$\begin{aligned}
\mathcal{E}_{GM}(e_1 \wedge e_2, h) &= \log\left[\frac{P(e_1 \wedge e_2|h)}{P(e_1 \wedge e_2)}\right] \\
&= \log\left[\frac{P(e_2|h \wedge e_1)}{P(e_2|e_1)}\right] + \log\left[\frac{P(e_1|h)}{P(e_1)}\right] \\
&= \log\left[\frac{1}{P(e_2|e_1)}\right] + \log\left[\frac{1}{P(e_1)}\right] \text{(since } h \text{ entails } e_1 \text{ and } e_2\text{)} \\
&> \log\left[\frac{1}{P(e_1)}\right] \\
&= \log\left[\frac{P(e_1|h)}{P(e_1)}\right] = \mathcal{E}_{GM}(e, h).
\end{aligned}$$

Hence, $\mathcal{E}_{GM}$ satisfies C3.

C4. As for proposition 1.

A.3 Proof of Proposition 3

Consider the measure $\mathcal{E}_{SS}$ (the proof is trivial for $\mathcal{E}_{GM}$). Then $\mathcal{E}_{SS}(e,h_1) \gtreqless \mathcal{E}_{SS}(e,h_2)$ iff

$$
\begin{aligned}
& \frac{P(h_1|e) - P(h_1|\neg e)}{P(h_1|e) + P(h_1|\neg e)} \gtreqless \frac{P(h_2|e) - P(h_2|\neg e)}{P(h_2|e) + P(h_2|\neg e)} \\
\text{iff} \quad & P(h_1|e)P(h_2|\neg e) \gtreqless P(h_2|e)P(h_1|\neg e) \\
\text{iff} \quad & \frac{P(e|h_1)}{P(e)}P(h_1)\frac{P(\neg e|h_2)}{P(\neg e)}P(h_2) \gtreqless \frac{P(e|h_2)}{P(e)}P(h_2)\frac{P(\neg e|h_1)}{P(\neg e)}P(h_1) \\
\text{iff} \quad & P(e|h_1) \gtreqless P(e|h_2).
\end{aligned}
$$

A.4 Proof of Proposition 4

C1. It is easy to show that $\mathcal{E}_G(e, h_1 \wedge h_2) > \mathcal{E}(e, h_1)$ iff ${}^{P(e|h_1 \wedge h_2)}/_{P(e|h_1)} > {}^{1}/_{P(h_2|h_1)^{1/2}}$. To see that this inequality can be satisfied in some cases, let $P(e|h_1 \wedge h_2) = 1$. It will then be satisfied provided $0 < P(e|h_1)^2 < P(h_2|h_1)$.

C2. This follows trivially from the definition of $\mathcal{E}_G$.

C3. $\mathcal{E}_G(e_1 \wedge e_2, h)$ will be greater than $\mathcal{E}_G(e_1, h)$ provided $\mathcal{E}_{GM}(e_1 \wedge e_2, h) > \mathcal{E}_{GM}(e_1, h)$ and so the result follows from proposition 2.

C4. Since it is assumed that $P(h_2) < 1$ and by hypothesis h_2 is independent of h_1, it follows that $P(h_2|h_1) < 1$.

$$\begin{aligned}
\mathcal{E}_G(e, h_1 \wedge h_2) &= \log \left[\frac{P(e|h_1 \wedge h_2)}{P(e)} P(h_1 \wedge h_2)^{1/2} \right] \\
&= \log \left[\frac{P(e|h_1)}{P(e)} P(h_2|h_1)^{1/2} P(h_1)^{1/2} \right] \\
&\quad \text{(since } h_2 \text{ is independent of } h_1 \text{ and } e \wedge h_1\text{)} \\
&< \log \left[\frac{P(e|h_1)}{P(e)} P(h_1)^{1/2} \right] \text{ (since } P(h_2|h_1) < 1\text{)} \\
&= \mathcal{E}_G(e, h_1).
\end{aligned}$$

A.5 Proof of Proposition 6

(i) By assumption $P(h_2|h_1) = 1$ and hence $P(h_1 \wedge h_2) = P(h_1)$. Also, since $P(e|h_1) = P(e|h_1 \wedge h_2)P(h_2|h_1) + P(e|h_1 \wedge \neg h_2)P(\neg h_2|h_1)$ and given that $P(h_2|h_1) = 1$, it follows that $P(e|h_1) = P(e|h_1 \wedge h_2)$. It thus follows from equation (1) that $\mathcal{E}_G(e, h_1 \wedge h_2) = \mathcal{E}_G(e, h_1)$.

(ii) If h_1 entails e so that $P(e|h_1) = 1$, then $P(e|h_1 \wedge h_2)$ cannot be greater than $P(e|h_1)$ and so by proposition 5 the conjunctive explanation cannot be better than h_1.

(iii) Recall that the conjunctive explanation will be better than h_1 provided the condition in expression (2) is met, i.e. $\log\left(\frac{P(e|h_1 \wedge h_2)}{P(e|h_1)}\right) > \log\left(\frac{1}{P(h_2|h_1 \wedge e)}\right)$. This can equivalently be expressed as $\log\left(\frac{P(h_2|h_1 \wedge e)}{P(h_2|h_1)}\right) > \log\left(\frac{1}{P(h_2|h_1 \wedge e)}\right)$. If e or $e \wedge h_1$ entails h_2, then this becomes $\log\left(\frac{1}{P(h_2|h_1)}\right) > 0$, which is satisfied provided $P(h_2|h_1) < 1$. This also requires that $P(e|h_1) < 1$ since if $P(e|h_1) = 1$ and

$P(h_2|h_1 \wedge e) = 1$, then $P(h_2|h_1) = 1$.

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