Imperfections in heralding three-photon GHZ states for quantum computing applications

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Abstract: We compute the effect of imperfections, specifically partial photon indistinguishability, on a linear-optical scheme that heralds three-photon Greenberger-Horne-Zeilinger (GHZ) states for use in a recently demonstrated scheme for fusion-based quantum computation.

1. Introduction

Fusion-based quantum computation (FBQC) is a promising model for realizing universal photonics-based quantum computing [1]. The large cluster states required for FBQC are formed by fusing many smaller entangled resource states, such as 3-qubit Greenberger-Horne-Zeilinger (GHZ) states [2]. One way of generating these resource states is by using linear optical circuits to interfere single photon states. A given partial measurement of the multiphoton output state heralds an entangled multiphoton resource state in the remaining outputs. These schemes utilize the symmetrization requirement of the overall wavefunction inherent in bosons to transform a set of product states onto a joint entangled state. This bosonic nature manifests most strongly when the photons are maximally indistinguishable, a requirement which is never met experimentally in practice.

Recently, Chen *et al.* reported on the first experimental demonstration of a dual-rail encoded 3-qubit GHZ state [3]. Their work is based on the photonic implementation of the six-photon 10-mode (6P10M) heralding scheme proposed by Gubarev *et al.* [4]. The realized heralded three-photon GHZ state fidelity is $F = 0.573 \pm 0.024$ after post-selection within the computational subspace.

Here, we numerically reconstruct the heralded three-photon GHZ state within the computational subspace for imperfect single photon states. The interference of partially distinguishable photons according to the unitary transformation of the 6P10M heralding scheme is simulated using the formalism of [5] as cast in the form of [6]. Although the reported fidelity by Chen *et al.* cannot be fully attributed to imperfect photon partial distinguishability, we demonstrate that partial distinguishability eventually limits the maximum achievable fidelity in absence of any other type of imperfections.

2. Results

We estimate the partial distinguishability of the single photon states used in the work of Chen *et al.* based on the reported Hong-Ou-Mandel interference visibilities V_{HOM} and second-order time correlation function $g^{(2)}(0)$ [7]. For simplicity, we model all photons as having equal internal wave function overlap $x = \langle\psi|\phi\rangle$. We estimate this quantity as $x = \sqrt{\frac{V_{HOM}+g^{(2)}(0)}{1-g^{(2)}(0)}}$ (or $x = \sqrt{V_{HOM} + g^{(2)}(0)}$) , based on the assumption of the multiphoton contribution to consist of fully distinguishable (or indistinguishable) noise photons [8]. We estimate for fully (in)distinguishable noise photons a partial distinguishability of $x = 0.959 \pm 0.006$ ($x = 0.947 \pm 0.006$).

With this information, and the linear-optical transformation required for the GHZ heralding scheme, we can compute the counting statistics behind this interferometer. Fig.1(a) shows the expectation value for each state assuming distinguishable noise photons when post-selecting on the computational subspace. We calculate the population term $P = |000\rangle\langle000| + |111\rangle\langle111|$ to be $P = 0.941 \pm 0.009$ ($P = 0.923 \pm 0.009$) for (in)distinguishable noise photons. Similarly, the expectation values for the coherence observables ${M_\theta}^{\otimes 3}$ defined in Ref. [3] assuming distinguishable noise photons are presented in Fig.1(b). Here, we calculate the coherence term $C = (1/3)\sum_{k=0}^{2}(-1)^k\langle {M_{k\pi/3}}^{\otimes 3}\rangle$ to be $C = 0.838 \pm 0.022$ ($C = 0.791 \pm 0.021$) for (in)distinguishable noise photons. The fidelity $F = (P+C)/2$ is $F = 0.890 \pm 0.015$ ($F = 0.857 \pm 0.015$) for (in)distinguishable noise photons. For $x = 0.99$, we calculate a fidelity $F = 0.972$ and for perfectly indistinguishable photons ($x = 1$) the fidelity is $F = 1$.

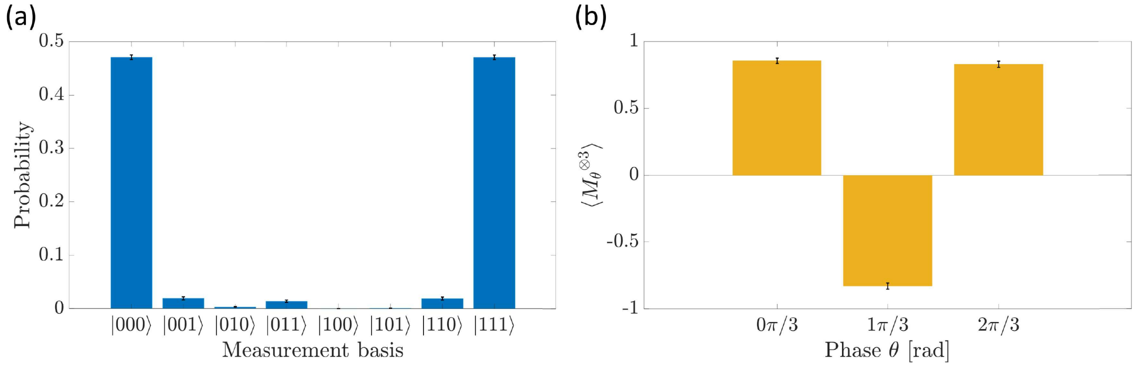

Fig. 1. Numerical results of the observables for the reconstructed heralded three-photon GHZ state within qubit space assuming distinguishable noise photons. (a) Occupancy probability of the computational basis states. Population observable $P = 0.941 \pm 0.009$. (b) Expectation values of the coherence observables. Coherence observable $C = 0.838 \pm 0.022$. Error bars represent deviation due to error in the estimated partial distinguishability $x = 0.959 \pm 0.006$. The underlying calculations only take into account imperfections caused by partial distinguishability.

3. Discussion

The reported fidelity by Chen *et al.* is lower than both expected fidelities calculated for partially distinguishable photons in absence of any other type of imperfection. We attribute this difference to the actual presence of other imperfections, such as limited gate fidelities. The quantum photonic processor used in the experiment has a typical average gate fidelity of ~ 0.9, resulting in an imperfect implementation of the unitary transformation of the heralding scheme [9]. Nevertheless, assuming that all imperfections accumulate infidelity, our simulation shows that partial distinguishability limits the maximum achievable fidelity to $F \approx 0.9$ even if all other imperfections are suppressed.

In conclusion, we demonstrate that the heralded three-photon GHZ state fidelity is limited by imperfect photons. Partial distinguishability reduces the fidelity in absence of other type of imperfections. Our work contributes to the understanding of implications of single photon state imperfections in linear optical quantum computing and highlights the need for indistinguishable photon distillation methods [10].

References

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