London School of Economics and Political Science

Doctoral Thesis

Essays in Banking

Author:
Arthur Taburet

Supervisor:
Dr. Alessandro Gavazza,
Dr. Martin Oehmke,
Dr Daniel Paravisini
and Dr. Ricardo Reis

June 1, 2023

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work, other than where I have clearly indicated that it is the work of others. The copyright of this thesis rests with the author. Quotation from it is permitted, provided that full acknowledgement is made. This thesis may not be reproduced without my prior written consent. I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of approximately 35,000 words.

Statement of Conjoint Work

I certify that Chapter 1 of this thesis is co-authored with Alberto Polo and Quynh-Anh Vo. I contributed 90% of the work in Chapter 1.

Acknowledgements

I am deeply grateful to my advisors Alessandro Gavazza, Martin Oehmke, Daniel Paravisini and Ricardo Reis for their patient guidance, constant encouragement and invaluable advice. I also thank Ulf Axelson, Cynthia Balloch, Matteo Benetton, Jamie Coen, Vicente Cuñat, Kim Fe Cramer, Dirk Jenter, Christian Julliard, Walker Ray, Amirabas Salarkia, Silvia Sarpietro, Huan Tang, Liliana Varela, Dimitri Vayanos, seminar participants at the London School of Economics, HEC Paris and the Bank of England for their valuable comments and suggestions. The first chapter uses data from the Financial Conduct Authority and the Bank of England that has been provided under a data-access agreement for external research purposes. The views expressed in this paper are those of the authors and do not represent those of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Authority Board views of the Financial Conduct Authority. All errors are my own.

Abstract

This thesis consists of three essays on household finance and banking.

The first chapter, co-authored with Alberto Polo and Quynh-Anh Vo from the bank of England, examines the role of menus of contracts in the UK mortgage market. Using data from the UK mortgage market and a structural model of screening with endogenous menus, I quantify the impact of asymmetric information on equilibrium contracts and welfare. I show that when lenders screen borrowers using a menu, they generate a contractual externality by making the composition of their competitors' borrowers worse. Counterfactual simulations of a social planner problem show that, because of the externality, there is too much screening along the loan-to-value dimension. The deadweight loss, expressed in borrower utility, is equivalent to an interest rate increase of 30-60 basis points (a 15-30 percent increase) on all loans.

The second chapter theoretically analyses the interaction between competition and adverse selection in markets where menus are used. Using a discrete choice approach to model competition, I characterise the unique pure strategy Nash equilibrium in a contract theory model with adverse selection and imperfect competition. I highlight a novel contractual externality leading to a welfare trade-off between competition and adverse selection. Lowering competition lowers concerns of losing market shares; this can improve welfare by giving lenders more flexibility on how to use contract terms and prices to sort borrowers efficiently. It also lowers lenders' incentives to implement socially inefficient strategies that rely on taking advantage of their competitors' menus to attract only low-cost borrowers (cream-skimming). However, low competition also allows lenders to apply high mark-ups, reducing borrowers' utility. When the externality is high, lowering competition leads to a Pareto improvement.

The third chapter theoretically studies the impact of designing lender-specific capital regulation regimes. To that end, I build a novel model of banking in which setting individualized capital requirements allows to better deal with each lender's excessive lending behaviour. However, creating different capital requirements also increases lenders' fixed cost of understanding and interpreting the regulation. Changes in the fixed cost endogenously affect the market structure and bank interest rate markups. Those changes feed back into lenders' incentives to over-lend. Due to this general equilibrium effect, I show that increasing capital requirement heterogeneity can increase the friction it was designed to reduce. Motivated by this theoretical result, I develop a sufficient statistic approach to empirically assess the impact of capital requirement complexity on welfare.

Contents

List of Figures

List of Tables

Chapter 1

Screening using a menu of contracts: a structural model for lending markets

1.1 Introduction

Menus of contracts are widely used in financial markets. For instance, mortgage borrowers often have the choice between fixed or flexible interest rates, high or low loan-to-value (LTV) ratios, and different combinations of interest rates and fees. A leading explanation is that lenders offer menus to make borrowers reveal their private information through their choices (i.e., screening). For example, if high LTV contracts are more valuable to high-default borrowers, lenders can make them self-select into a high-interest rate high LTV contract. However, in that case, low-default borrowers get a lower LTV than high-default borrowers, which is not necessarily what would happen in the first best. By screening borrowers, lenders can thus restore perfect information pricing, but this may come at the cost of distortions in other contract terms.

The theoretical literature has highlighted that pooling contracts cannot be offered in competitive markets even when pooling is a Pareto improvement over screening (see for instance Rothschild and Stiglitz (1976) and Lester et al. (2019)). This is because lenders could take advantage of their competitors' pooling contracts attracting safer and more profitable borrowers without having to distort their contract terms — such as LTV — too much.[1] Yet, how large this issue is in practice, and more generally, how adverse selection impacts contract terms and welfare, is still an open question (see Einav, Finkelstein, and Mahoney (2021) for a literature review). Quantifying the impact of adverse selection on contract terms requires determining what contracts would be offered if there were no adverse selection ("first best") or the ones offered by a social planner who internalizes that deviating from pooling may be inefficient ("second best"). This is, however, challenging as those situations are not observed in the data.

In this paper, we quantify the impact of asymmetric information on contract terms and welfare using the first structural model of screening for default probabilities. We use our structural model to simulate the contracts that would be offered in the first and second-best cases. By comparing the simulated contracts to the ones in the data, we assess the extent to which contracts are distorted and quantify the welfare loss. To flexibly capture screening incentives, we develop a supply and demand model with imperfect competition and allow borrowers to have private information about their default probabilities and their preferences over each contract characteristic. We identify and estimate the model parameters using administrative data on lenders' menus,

1. In Rothschild and Stiglitz (1976) the pure strategy equilibrium does not exist in that case because there is also a profitable pooling deviation when all lenders screen. Papers such as Lester et al. (2019) characterize the mixed strategy equilibrium and show that lenders cannot cross-subsidize — and thus pool — when competition is high enough.

borrowers' contract choices and defaults in the United Kingdom (UK) mortgage market for first-time buyers from 2015 to 2019.

A key challenge when identifying screening incentives comes from the fact that the difference in the default probability of borrowers choosing different contracts can be due to the causal impact of contract terms (i.e., burden of payment or moral hazard) rather than borrowers' type (i.e., adverse selection). Indeed, even identical borrowers, once they choose contracts with different interest rates, can have different default probabilities simply because they have to pay a different amount each month. We propose a novel approach to disentangle moral hazard from adverse selection. We use the fact that borrowers' choices of contract (i.e., adverse selection) are a function of the interest rate spread between products in the menu, while how hard it is to repay a loan (i.e., burden of payment or moral hazard) only depends on the interest rate of the contract chosen. As a result, exogenous variation in the interest rate of high LTV loans can be used to identify adverse selection as it changes the type of borrowers that chose low LTV loans while keeping the burden of payment channel constant. We show how to implement this idea formally within a structural model using an instrumental variable approach based on contract-specific capital requirements.

We deliver three new empirical results. First, we find that the LTV ratio is used together with interest rates to screen borrowers along their default probability. Lenders set their LTV pricing schedule such that high-default borrowers chose a higher LTV-higher interest rate contract relative to low-default borrowers. Screening works because high-default borrowers — who also tend to be less price elastic[2] — have a higher "willingness to pay" for LTV. That is high default borrowers are more reluctant to provide a higher down payment for each pound they borrow (i.e., they have a higher marginal rate of substitution of interest rate for LTV). We also find that other contract characteristics (fees and the type of rate) are used to screen as well.

Second, using counterfactual simulations, we show, that maintaining incentives to self-select requires distorting contract terms away from their perfect information value. In the data 50 percent of borrowers (those with a lower default probability) choose contracts with an LTV between 70 and 85 percent. However, under perfect information, those borrowers — as well as most other borrowers — would have obtained an LTV above 85 percent and bought a bigger house. Thus, contracts with an LTV between 70 and 85 percent are introduced primarily to screen borrowers rather than to cater to their preferences. We also find that because of screening, the interest rate on 95 percent LTV loans is lower by 70 basis points (bps) relative to what those borrowers would have gotten under perfect information.

2. The correlation between default and price elasticity is consistent with risky borrowers internalizing the probability that their application is rejected and thus behaving as if they had higher search costs (see Agarwal et al. (2020) for empirical evidence).

Finally, by comparing the menu in the data to the one offered by an informationally constrained social planner, we isolate the effect of the contractual externality and show that there is excessive screening. If a lender were to offer the social planner's pooling contracts, its competitors could take advantage of it by introducing a contract with a lower rate and an LTV just below the ones currently offered, thereby attracting a high proportion of low-cost-high price elasticity borrowers. This deviation from pooling is inefficient as lenders do not internalize that how low the LTV must be is a function of their competitors' contracts. A lower bound of the deadweight loss generated by this externality is equivalent to the utility loss caused by a 30 bps interest rate increase on all loans.[3]

Our findings imply that, as maximum leverage is the dominant screening device in this market, regulations affecting LTV can have an unintended effect on banks' incentives and ability to screen borrowers. Common examples of such policies are bans on high LTV products, high LTV mortgage guarantee schemes, or LTV-specific capital requirements (see the IMF's Global Macroprudential Policy Instruments database). To analyse these unintended effects, we provide a policy simulation of a ban on high LTV mortgages. We find that the cost of that policy is equivalent to a 30 bps increase in interest rates and that the cost is underestimated by a factor of three when not considering screening. The rationale is that the high LTV ban will move high default-low price elastic borrowers to lower LTV, and screening incentives push the safer borrowers into new and cheaper products but with characteristics that do not match those of perfect information products.

Overall, our results show that screening is an important force in the UK mortgage market and that the associated contractual externality is costly. This suggests there is room for Pareto improving policy interventions. As shown in the theoretical companion paper Taburet (2022), lowering competition, increasing the capital requirement on low LTV in a low-competition environment, or banning the use of lower LTV products could reduce the impact of the contractual externality by preventing cream-skimming deviations to occur.

We deliver our empirical results thanks to two methodological innovations. One is related to how to formally implement our novel identification strategy for screening within a structural model, the other is related to how to develop and solve the structural model of screening.

Our identification strategy relies on recovering the correlations between borrowers' preferences and default probabilities. This is a key statistic for screening as it tells whether borrowers that are more reluctant to, let us say, borrow at low LTV are also more likely to default. Formally, we assume that borrowers' utility is linear in

3. Considering an average loan size of £200,000 and a 25-year maturity, this corresponds to a £25 monthly increase in borrowing expenses for all borrowers. In practice, this cost is borne by a third of borrowers and is thus equivalent to a £75 monthly increase.

contract terms and decompose the choice of contract into two equations: a product choice and a quantity choice (discrete-continuous demand system). We employ a two-step approach. First, we use a revealed preference in the spirit of Nevo (2001) to recover moments of the distribution of borrowers' ex-ante unobservable preferences from contract product choice (interest rate type, LTV, lender and fees) and loan size choice data. Our identification leverages the idea that if a borrower chooses a contract with a maximum LTV of 90 percent while they had access to a contract with a maximum LTV of 95 percent for an interest rate increase of, say, 100 bps, it must be that their willingness to pay for this LTV increase is lower than 100 bps. In the second step, we build a measure of the average preference of borrowers choosing a given contract and compare the default of groups of borrowers that chose the same contract but have different average preferences. By comparing borrowers that chose the same contract, we fix moral hazard and burden of payment. We show that the variation in our preference measure comes from changes in the spread between contract prices or new contract introduction or withdrawal. Our measure of contract-level average preferences can thus be instrumented using a weighted average of lenders' product-specific cost shifters. The instruments affect the spread between interest rates and, thus, the average type of borrower choosing a contract. We then use the demand and default parameters together with formulas derived from the lenders' profit maximization problem to back out the marginal costs of originating mortgage products and the fixed cost of changing menus.

The challenges of our identification strategy are the following. In the first step, our measures of borrowers' preferences need to be correctly identified. In the second step, variations in the average preferences of borrowers choosing a given contract need to be uncorrelated with borrower characteristics (e.g., soft information) or economic conditions unobserved by the econometrician.

The endogeneity concerns in the first step are addressed using standard approaches. We mitigate the endogeneity problem from omitted product characteristics by instrumenting for interest rates using predetermined risk weight as a cost shifter as in Benetton (2018). We deal with unobserved rejection of mortgage applications based on soft information using a consideration set approach (see Crawford, Griffith, Iaria, et al. (2021)). We deal with the selection on unobservables in the loan size regression by allowing for the product choice and loan size parameters to be correlated similarly to K. Train (1986). Finally, we use product fixed effects in our specification of preferences over contract characteristics to recover the component of preferences that is a function of expected default but is not contaminated by moral hazard or burden of payment incentives. This component is used as an independent variable in the second step.

To address the endogeneity concern in the second step, we instrument borrower average LTV preference using a weighted measure of product-specific risk weights and minimum capital requirements as a product-specific cost shifter. Risk weights are pre-determined and vary over time across lenders and mortgages with different maximum LTVs. Minimum capital requirements vary over time and across lenders. Both have been extensively used as an instrument for interest rates (e.g., Aiyar et al. (2014), Benetton (2018), Robles-Garcia (2019)). Our instrument is relevant as it affects the spread between interest rates and, thus the type of borrower choosing a given contract. We control for unobserved characteristics that are common among products (lender shocks) and those that are common across lenders (market shocks). Thus, given the absence of individual-based pricing in the UK (see Benetton (2018)), the exclusion restriction requires that our cost shifter is not correlated with economic shocks affecting borrower types differently, and with acceptance and rejection rules based on characteristics unobserved by the econometrician only. It is plausible that the endogeneity from mortgage application rejections based on soft information is not fully addressed, as lenders can update their acceptance and rejection criteria following a product cost shock. In that case, our results should be interpreted as a lower bound on adverse selection as lenders are likely to become stricter to mitigate the increase in the cost of lending.

The second methodological contribution of the paper comes from using methods and results from the theoretical literature on screening to solve for the equilibrium contracts in the counterfactual simulations. The simulations are needed to provide an economic interpretation to our model parameters — such as the correlation between preferences and default. The key challenges are that (i) equilibria in selection markets are difficult to characterize and are often fraught with nonexistence, and (ii) the computational burden associated with structural model simulations is high when more than one product characteristic is endogenised.

Our approach to solve for the equilibrium contracts is based on three innovations. First, we simplify the analysis of contract distortions using the perfect information case as a benchmark in our first counterfactual exercise. This framework eliminates both the existence and computational burden concerns. We further construct a model-based and analytically tractable decomposition of the equilibrium interest rate observed in the data into a perfect information competitive interest rate, a perfect information-imperfect competition markup, and an asymmetric information discount or premium.

Second, as in the companion paper Taburet (2022), we show that the contractual externality can be measured by setting the social planner problem similarly to a monopoly screening model. Monopoly models are convenient as they do not feature the non-existence result of Rothschild and Stiglitz (1976). Formally, we consider the hypothetical case in which each lender becomes a monopolist and borrowers' outside option is their utility in the competitive equilibrium. The outside option constraint is made to focus on Pareto improvements. The monopoly assumption keeps the asymmetric information but eliminates the externality by preventing borrowers from moving from one bank to another. This formulation of the social planner problem is convenient as it does not feature the non-existence of equilibrium (see, for instance, Taburet (2022)). It also allows us to focus exclusively on the screening externality by preventing an increase in welfare generated by a better allocation of borrowers to cheaper banks.

Finally, as in Wollmann (2018), we discretise the product choice set to reduce the computational burden of the counterfactual simulations for our policy analysis counterfactual. We innovate by introducing random fixed costs of designing a new menu. The latter assumption makes the estimation tractable and also disciplines the counterfactual simulations of LTV bans by assigning a probability to each potential market outcome.[4]

The rest of the paper is structured as follows. In section we provide a literature review. In section 2.3, we describe the institutional features of the UK mortgage market, outline the data used, and conduct a descriptive analysis to motivate the modelling assumptions. In section 2.4, we present the structural model. Section 2.6 discusses the identification strategy and estimation procedure. In section 1.6, we analyse the estimation results and the counterfactual experiment outcomes.

1.2 Literature Review

This paper contributes to the empirical literature on price discrimination, the literature on adverse selection and the empirical literature on credit markets.

The recent empirical literature on price discrimination is mainly structural and is theoretically grounded in the seminal monopoly model of Mussa and Rosen (1978).[5] As such, the empirical literature studies how product prices and product distortions (generally along one dimension such as quality) react to changes in the economic environment. Recent examples include Crawford, Shcherbakov, and Shum (2019), which uses a demand and supply structural model with endogenous quality and price to study quality and price distortions in the cable television market, and Benetton, Gavazza, and Surico (2021), which examines the impact of a funding policy in the mortgage market using a structural model with endogenous product fees and rate. Our paper is closely related to Wollmann (2018), which analyses the impact of mergers

4. This approach also helps in the estimation of the fixed cost as it allows using the sufficient set approach (Crawford, Griffith, Iaria, et al. (2021)) to avoid having to compute all potential combinations of product introduction and withdrawal.

5. See Busse and Rysman (2005) for a reduced-form empirical analysis.

using a model of product entry and exit for the car market. To the best of our knowledge, Wollmann (2018) is one of the first to propose a supply and demand structural model that endogenizes more than two variables for product characteristics. Our paper builds on the numerical method developed in Wollmann (2018)[6] to solve for endogenous contracts and extend it to an empirical model of banking with adverse selection.

The study of the impact of adverse selection on market outcomes is well established in the literature. On the theory side, the seminal references are Akerlof (1978) for a model with single-product firms and endogenous prices and Rothschild and Stiglitz (1976) in which firms design and offer menus. Akerlof (1978) shows that adverse selection can lead to a market breakdown. Rothschild and Stiglitz (1976) document that a (pure strategy) equilibrium may not exist in the perfect competition setting. To overcome the non-existence result, the literature has developed equilibrium concept refinements, such as that in Riley (1979), Bisin and Gottardi (2006) or Wilson (1980).[7] Alternatively, the literature introduces modelling changes to be able to solve for an equilibrium. For instance, Guerrieri, Shimer, and Wright (2010) assume that the principal can match one borrower at most. Finally, allowing banks to play mixed strategies can resolve the non-existence problem (see Dasgupta and Maskin (1986) for a proof that a mixed strategy equilibrium exists). However, solving for a mixed strategy equilibrium is computationally demanding (see Lester et al. (2019) or Farinha Luz (2017)), and as such, the properties of screening models are still understudied. Our model is based on a companion paper Taburet (2022) that analyses the properties of a screening model (existence and uniqueness of the equilibrium) that features the logit demand form used in this paper. In this paper, we adapt the theoretical literature modelling to bring a screening model to the data to measure the impact of the market inefficiencies studied in the theoretical literature.

A large empirical literature tests whether or not adverse selection and screening occur in practice. They do so using reduced-form approaches. The most common identification strategy in the screening literature is to directly compare the default of observationally equivalent borrowers that chose different contracts (Chiappori and Salanie (2000)). However, this correlation can be due to the causal effect of contact terms (moral hazard or burden of payment) rather than borrower unobservable attributes (adverse selection). To mitigate this concern, Hertzberg, Liberman, and Paravisini (2018) exploits a natural experiment to compare the default of borrowers that chose the same contract before and after a new product was introduced in the menu. Our approach has three advantages relative to the existing literature. First, it

6. As discussed more in depth in the estimation section, Wollmann (2018) relies on discretizing products and iterating on firms' best-response function.

7. The two first equilibrium concepts restore the existence of the screening equilibrium, while the third one restores the pooling equilibrium.

relies on weaker identifying assumptions than the positive correlation test literature (Chiappori and Salanie (2000)). Second, it uses variation in interest rates and is thus applicable to a wide variety of setups. In contrast, the literature such as Hertzberg, Liberman, and Paravisini (2018) relies on the analysis of lenders that just started using menus or the use of experimental data (Karlan and Zinman (2009)). Finally, it is implemented within a structural model, which allows answering a wider range of questions by doing counterfactual simulations.

Another empirical method to test adverse selection has been developed in Einav, Finkelstein, and Cullen (2010) and is called a cost curve test. It has been mainly used in the public economics literature as it allows using the cost curve estimates as part of a sufficient statistic for welfare. This test consists of using data on expected costs and prices and identifying whether the cost is upward-sloping (adverse selection) or downward-sloping (advantageous selection) with respect to prices. The main identifying assumptions are that the marginal cost curve is monotone and that the change in prices used for the identification affects product choices but not moral hazard.[8] Landais et al. (2021) and DeFusco, Tang, and Yannelis (2022) are recent examples of empirical applications of this method to labour market insurance and consumer credit, respectively. The sufficient statistics literature requires detailed data on the cost of products and has focused on situations in which menu offers are fixed and thus do not endogenize the product offering. Given those considerations, we use a structural approach instead. However, our paper shares with this literature the idea of relying on revealed preferences in the demand estimation instead of modelling the underlying process that is driving choices. In this way, our results are robust to the underlying model that drives borrowers' expectations, for instance.

Similar to the sufficient statistics literature, structural frameworks in the adverse selection literature are based on theoretical models such as Akerlof (1978). They focus on a situation in which the product is fixed but prices are not (see Einav, Finkelstein, and Mahoney (2021) for a recent literature review). As a result, this literature mainly studies the effect of adverse selection on prices, quantities and welfare but has neglected its impact on product offerings. A recent example of this literature is Crawford, Pavanini, and Schivardi (2018). The paper studies the interaction between competition and adverse selection in the business lending market. A notable exception is Handel, Hendel, and Whinston (2015), which uses a perfect competition structural model based on Rothschild and Stiglitz (1976) to study health insurance policies. The model allows for menus to be composed of at most two products with their coverage exogenously fixed to 90 and 60 percent actuarial values. Because equilibria in competitive markets are difficult to characterize and are often fraught with

8. This assumption is harder to satisfy in credit markets — in which the cost variable is default — relative to insurance markets — in which the relevant cost is the number of claims made. Indeed, in credit markets, default is directly affected by prices as a result of the burden of payment.

nonexistence, Handel, Hendel, and Whinston (2015) uses Riley (1979) equilibrium concepts, which forces the screening equilibrium to happen. Our approach allows us to avoid the use of equilibrium concept refinements, relax the perfect competition assumption and endogenize both the contract terms and the menu size. By doing so, we fit the credit market structure and more flexibly capture incentives to pool (or, more generally, to cross-subsidize) or screen borrowers. The competition assumption is important given recent theoretical and empirical papers[9] have shown that the effects of asymmetric information on prices and contract terms — via, for instance, the ability to pool borrowers — depends on the market structure itself.

This paper is also related to the literature analysing consumers' and lenders' behaviours in retail financial markets. This topic has been an important one in economics in recent years. Several papers have focused on the demand side and documented limited search, mistakes, and inertia (Coen, Kashyap, and Rostom (2021), Agarwal, Ben-David, and Yao (2017), Andersen et al. (2020)). Other papers have taken a more structural approach to look at how lenders may gain from borrowers' choice frictions (Buchak et al. (2018)) or at the effects that, for example, capital or broker regulations have had on market outcomes and welfare (Benetton (2018), Robles-Garcia (2019)). Our paper contributes to this literature by studying screening in the context of credit markets. It builds on the framework in Benetton (2018) and Crawford, Pavanini, and Schivardi (2018) and further includes unobserved borrowers' preferences over contract characteristics, adverse or positive selection and endogenous menus of contracts.

1.3 Institutional Setting, Data, and Motivating Evidence

This section describes the key institutional features of the market and the data used in this paper. It then provides suggestive evidence that screening is an important driver of the UK mortgage market contracts offering.

1.3.1 Institutional setting

Market features: While mortgage markets are important credit markets in most countries, their institutional features vary (Campbell (2013)). The UK mortgage market differs from other mortgage markets — such as that in the US, for instance — along three dimensions.

First, lenders do not offer long-term fixed rate contracts in the UK market. Instead, borrowers can fix the interest rate for a given number of years (typically two, three, or five). After that period, the "teaser rate" is reset to a generally significantly higher and flexible "follow on rate". Coupled with the fact that contracts feature

9. For instance, Lester et al. (2019) and Crawford, Pavanini, and Schivardi (2018).

high early repayment charges — which typically account for 5 or 10 percent of the outstanding loan — refinancing around the time when the teaser rate period ends is very frequent in this market (Cloyne et al. (2019)).

Second, the interest rate of a contract advertised by a given bank on its website or other platforms is the one paid by every borrower choosing that contract. This is because minimal negotiation takes place between borrowers and lenders, and banks do not practice individual-based pricing.[10] However, while pricing is independent of borrowers' characteristics, banks may reject loan applications based on individual characteristics. This approach is common in other markets (credit cards, hedge funds) or online platforms.[11]

Finally, the UK mortgage market is very concentrated. The "big six" lenders account for approximately 75% of mortgage origination. The number of active banks is stable over time, even during times of financial disturbance such as during the COVID-19 pandemic.

Loan contracts: As illustrated in Figure A.1.1, a borrower who is willing to take on a mortgage from a particular bank in the UK can choose from a menu of standardized loan contracts.

The pricing of those contracts is primarily based on product characteristics such as lender name, rate type, maximum LTV and fees. Indeed, using a linear regression of rate on product characteristics, we show — consistent with other papers on the UK mortgage market (Benetton (2018), Robles-Garcia (2019)) — that 90 percent of the price variation is explained by interacting time dummies with lender dummies, rate type, maximum LTV and fees dummies. The remaining variation is independent of the characteristics of the borrowers choosing the contract.

Conditional on those product characteristics, borrowers can borrow as much as they want and can freely choose the maturity within some bounds without any impact on the interest rate.

While the contract pricing is independent of borrowers' characteristics, a bank can choose to reject a borrower's loan application based on their observable characteristics (e.g., income, age, credit score). As we do not observe loan applications or the criteria used by banks, we will build our empirical strategy considering this limitation.

10. The search platform Moneyfacts reports: "A personal Annual Percentage Rate is what you will pay. For a mortgage this will be the same as the advertised APR, as with a mortgage you can either have it or you can't. If you can have the mortgage, the rate doesn't change depending on your credit score, which it may do with a credit card or a loan." See Leanne Macardle, "What is an APR?" Moneyfacts, https://moneyfacts.co.uk/guides/credit-cards/ what-is-an-apr240211/.

11. This can be rationalized by the fixed cost of negotiation being high compared to the size of loans in the consumer market compared to the firm market.

Based on those facts, we thus define as a loan contract the object (L, X, r) where L is the loan size, X is a vector containing other contract characteristics (lender dummy, maturity, rate type, maximum LTV and fees), and r is the interest rate on the loan.

Following the vocabulary in the industrial organisation literature, we also refer to the vector of characteristics (X) as a product, r as the product's price, and L as the quantity of that product being bought.[12]

1.3.2 Data

We use the Product Sales Database 001 (hereafter, PSD 001). The data are collected quarterly by the Financial Conduct Authority (FCA) and contain contract-level information about households' mortgage choices and detailed information on mortgage origination characteristics for the universe of residential mortgages in the UK. The dataset is available to restricted members of staff and associated researchers at the FCA or the Bank of England.

We merge the data with PSD 007 containing the credit events on mortgages. We use arrears as a measure of default, which is defined as being 90 or more days delinquent on monthly payments.

In this paper, we focus on the years 2015 to the end of 2022. During this period, we observe for each mortgage origination details on the loan (interest rate, loan amount, initial fixed period, maturity, lender, fees), the borrower (income, age), and the property (value, location). The estimation is done excluding the COVID-19 period as the policies implemented during that time may confound the identification. However, we provide stylized facts about default and product offers during that period. We focus on the first-time buyer market to abstract from preexisting lending relationships between lender and borrower.

The structural estimation is done using 2018 data (See table B.1 in Appendix B for the data summary statistics). For that year, we observe 847,000 first-time buyer contracts, of which almost 90% are mortgages with initial fixed periods of two, three, or five years. The average interest rate is 2.5 percentage points, and the average origination fee is £503. The average loan is almost £165,000 with an LTV of 80 %, a loan-to-income of 4.6, and an average maturity of 29 years. Borrowers are, on average, 31 years old and have an annual income of £36,000.

We supplement the data with a survey on credit events during the COVID-19 period and 2015–2018 surveys on capital requirements policies at the bank-product level .

12. This vocabulary is relevant here as the vast majority of the contract price does not depend on the loan size.

1.3.3 Motivating evidence

This section discusses descriptive patterns about banks' menus. We also provide suggestive evidence that screening is feasible in this market as borrowers' (observable) characteristics are correlated with contract choices and default.

Variation in product offering: As shown in Figure A.1.2, the number of products varies over time and across market participants. In particular, first-time buyers shopping for 90% LTV contracts faced on average two different options at each bank in 2010, six options before the COVID-19 crisis and only one or no options during the peak of the COVID-19 period. Menu sizes are larger at 75% LTV. Indeed, the average menu contains 6 alternative contracts at 75% LTV in 2010 and during the COVID-19 period but 16 in 2017. Typically, in 2017 the average bank offers at 75% LTV the option of fixing the rate for 0, 2, 3 or 5 years and proposes three levels of fees (0, 750, 1500). A higher level of fee is associated with a lower rate. Considering all combinations of fixed rates and fees for all LTV levels offered starting from 60% LTV (i.e., 60, 65, ..., 90, 95), we find that, on average, only 40 percent of those products are offered by the average bank.

This empirical result motivates the fact that the number of products needs to be endogenized in the model.

Sorting on observables: As suggestive evidence that borrowers with different characteristics tend to select different products, we regress borrowers' observable characteristics on LTV dummies (see table B.2).

We document that — compared to borrowers choosing 75% LTV contracts — borrowers choosing 95% LTV contracts are on average 1.5 years younger, earn 7,400 net pounds less a year, and are 20 percent more likely to be part of a couple.

This correlation between LTV and borrowers' characteristics can be the result of borrowers' self-selection or the fact that banks may decline the loan applications of riskier borrowers for a high LTV loan. As banks generally offer high LTV loans only to safer borrowers, it is likely that the income and age gap between high and low LTV loans would be higher absent banks' rejection behaviour. Making borrowers self-select (on observable characteristics) using LTV is thus feasible.

Sorting on default: As suggestive evidence that borrowers that choose different products have different default behaviour, we regress default on borrower and contract characteristics (see table B.3):

$$Default_i = \beta X_i + \epsilon_i \tag{1.1}$$

$Default_i$ is equal to 1 if borrower i has been in arrears by the end of 2019, andX_i includes borrower i's contract terms (lender, LTV, rate, fees, teaser period, mortgage term) and borrower i's characteristics (age, income, location of the house, number of applicants, time at which the contract has been taken).

We document that 1.2 percent of the loans originated in 2018 had defaulted by 2020. The default probability on 85-95% LTV loans is 1.4 percent, while the average for 75-85% LTV loans is 0.8 percent. We excluded the COVID-19 period as a payment deferral (mortgage holiday) policy was implemented to help borrowers facing financial difficulties.

Using a baseline default of 1.2%, the regression of default on product and borrowers' characteristics implies that a 100 bps increase in rate is associated with a 50 percent increase in default probability; the default probability of a 5-year fixed rate contract is 40 percent lower than that of flexible rate contracts; the default probability of a zero fee contract is 30 percent lower than contracts with fees of 1,000; and borrowers whose income is one standard deviation lower (16,000) are 16 percent more likely to default.

In Figure A.1.4, we plot the share or mortgages that are in arrears as a function of the LTV at origination. The average arrears is 1.2 percent. Loans with an LTV bellow 75 percent are twice as less likely to be in arrears than loan with an LTV above 90 percent. This can for instance be due to the causal impact of borrowers type (i.e., adverse selection) or to the causal impact of contract terms (i.e., moral hazard or burden of payment).

As a complementary study, we use a proprietary survey from the bank of England made during the COVID-19 period. According to the survey, 25 percent of borrowers asked for a mortgage holiday. As illustrated by Figure A.1.4, the amount of loans benefiting from the policy was 60 percent higher than the average for high LTV loans and 30 percent higher for small banks. Of these loans, 6 percent of those originated before the 2008 financial crisis had defaulted by 2020. Those two facts illustrate that, while the baseline default probabilities may be small in normal times, they become large during an economic crisis. For this reason, the default probability estimated in this regression and in the structural model may not reflect banks' actual expected default probabilities. Consequently, we do a sensitivity analysis based on default probabilities in the structural model.

Those results, together with the one on borrowers' choice of contract — and given that pricing is independent of borrowers' income — provides suggestive evidence of adverse selection along the income dimension. Indeed, we documented that low-income borrowers are more likely to choose high LTV contracts and are more likely to default.

Need for a structural model: To further understand the impact of screening on equilibrium quantities, one needs to compare the observed equilibrium contracts' terms to a counterfactual in which there is no private information. Given the difficulty of finding the right counterfactual in the data, we build a structural framework to rely on simulations instead. The following sections discuss the model assumptions and our identification strategy. Our modeling approach and identification strategy also allow us to look at selection on unobservable borrowers' characteristics, take care of the bias generated by the rejection of mortgage applications, and disentangle moral hazard or burden of payment from adverse selection in the default regression.

1.4 Model Setup

For each month t, we read the data through the lens of the model of supply and demand described in this section. To simplify the notation, we drop the index t on the variables except when necessary.

Sections 1.4.1 and 1.4.2 provide a general overview of the setup and the equations that will be used to both identify the model parameters and solve for the counterfactual simulations.

An interested reader can look at Appendix C.1 for an in-depth discussion about the modelling assumptions. Appendix C.9 provides an analysis of the model product introduction incentives and the contractual externality.

1.4.1 General Considerations

We consider a T-period model ($T \in \mathbb{N},\ T > 1$) with two groups of agents: borrowers and lenders. We also refer to the second group as banks. There are n potential borrowers ($n \in \mathbb{N},\ n \geqslant 1$) indexed by i. There is a finite number of banks indexed by b. We denote B as the set of banks.

Definition of contracts and products: Banks offer a menu of contracts. Based on the UK institutional features, we define as a loan contract the object (L, X, r) where L is the loan size, X is a vector containing other contract characteristics (lender dummy, rate type, maximum LTV and fees) and r is the interest rate on the loan. Following the IO literature vocabulary, we also refer to the vector of characteristics (X) as a product, r as the product price, and L as the quantity of that product being bought. We index a product by the subscript c. We denote P_{ib} as the set of products (c) available to borrower i at bank b.[13] We denote by $M_{ib} := \{(X_{cb}, r_{cb})\}_{c \in P_{ib}}$ the menu of products offered to borrower i at bank b. We drop the b or i index in M and P to refer to the market menu ($M_i := \cup_b M_{ib}$ and $P_i := \cup_b P_{ib}$) or the bank menu ($M_b := \cup_i M_{ib}$ and $P_b := \cup_i P_{ib}$). $C_b := \mathbf{card}(P_b)$ is the number of products sold by

13. Each combination of product characteristics (X) is a one-to-one mapping to a natural number.

bank b.

Key features: The following considerations formally summarize the key features of the UK mortgage market as discussed in section 2.3:

(i) Each bank b posts a menu of products M_b that is visible to everyone.

(ii) Bank b may reject borrower i's mortgage application, so the menu available to borrower i at bank b M_{ib} may be smaller than M_b.

(iii) Each bank b offers a finite number of products ($C_b := \mathbf{card}(P_b)$).

(iv) For each product $c \in P_b$, there exists a contract (L, X_c, r_c) for any loan amount $L \in [a, b] \subset \mathbb{R}^+$ (see Figures A.1.1 and A.1.3).

(v) The pricing (r) of contract (L, X, r) depends on product characteristics (X) and not on the loan size L or maturity (see Robles-Garcia (2019), Benetton (2018) or Benetton, Gavazza, and Surico (2021)).

Features (iv) and (v) justify our definition of a product as a bundle of characteristics (X). While facts (i)–(v) can all arise endogenously from an optimal contract design (see the appendix in Taburet (2022)), in this paper, we take facts (iv) and (v) as given.

Timing: At the beginning of period t, each borrower decides whether or not to enter the credit market. Conditional on participation, a borrower chooses one loan contract from one lender.

Loan c matures in m_c periods with $t < t + m_c \leqslant T + t$. A borrower may default on his loans.

Heterogeneity: Borrowers have heterogeneous characteristics (age, income, savings, risk aversion), which translates into borrowers having different preferences over the characteristics of loan contracts and banks. As a result, each lender may have market power over borrowers. Borrowers also have heterogeneous default probabilities.

Information structure: There is asymmetric information in the economy: lenders do not perfectly observe borrowers' types (i.e., their preference, some of their characteristics and default probabilities). Whenever profitable and feasible, they use a menu of contracts to make borrowers self-select.

1.4.2 Overview of the Model

Our model is based on the following demand and supply maximization problems. Borrowers choose the bank and contract among its individual specific set that maximizes its indirect utility. Lenders maximize their expected profits. Lenders do not perfectly observe borrowers' characteristics but know the characteristics' distribution. Formally, for each period t we have:

Borrower i: contract c and lender b choice

$$(c_i, b_i) = \text{argmax}_{\{b \in B_i, c \in P_{ib}\}} \{ \underbrace{V_i(\overbrace{X_{cb}, r_{cb}}^{\text{contract terms and price}}, \overbrace{L_i(X_{cb}, r_{cb}, d_i(X_{cb}, r_{cb}))}^{\text{Loan demand}}, \overbrace{d_i(X_{cb}, r_{cb})}^{\text{Default probability}})}_{\text{Indirect utility}} \} \tag{1.2}$$

Lender b: menu offering M

$$M_b \subset \text{argmax}_{\{M, C_b, P_{ib}\}} \; E[\sum_{i,c} 1_{\{(c_i,b_i)=(c,b)\}} \underbrace{NPV(r_{cb}, d_{icb}, \overbrace{mc_{cb}}^{\text{marginal cost of lending}})}_{\text{Expected NPV if i chooses cb}}] - \underbrace{\frac{F(M, M_{bt-1})}{\beta^F} + \beta^F e_{Mb}^F}_{\text{Fixed cost of changing menu}} \tag{1.3}$$

Menus have the form $M_b = \{(X_{cb}, r_{cb})\}_{c \in [\![1, C_b]\!]}$, with C_b being the number of contracts in the menu. P_{ib} is the subset of the menu M_b available to borrower i at bank b. B_i is the subset of banks that are considered by borrower i.

M_{bt-1} is the menu offered by bank b in the previous period.

$\beta^F e_{Mb}^F$ is a random variable modelling product-specific introduction or withdrawal fixed costs.

The expectation in equation 1.3 is conditional on the lender information set. The information set contains contract terms and prices, and observable borrower characteristics. Lenders know that borrowers behave according to problem 1.2.

1.4.3 Discussion about the model's assumptions

Any model simplifies the reality of focusing on a given economic phenomenon. In our structural model, we use consider borrowers' participation in the mortgage market as given. On the supply side, we do not endogenize the house price upon default and do not model dynamic considerations in order to be able to model screening incentives in more detail. The counterfactual simulations thus consider that those elements — as well as unobserved product characteristics (captured by product-lender fixed effects) — remain constant.

Demand

Savings: As we do not observe savings, we cannot explicitly model the constraints on the level of down payment (d) a borrower can provide. We address this issue by modelling borrowers' choice of both LTV and the loan size and relying on a revealed preference approach to recover the demand parameters. Indeed, using the definition of LTV, we get: $LTV := \frac{L}{d+L} \Leftrightarrow d = L \cdot \frac{1-LTV}{LTV}$. In the situation in which a borrower is constrained by their savings (s_i) when selecting their level of down payment, their loan demand function is: $L_i(LTV) = s_i \frac{LTV}{1-LTV}$. Where s_i is a parameter to be recovered using choice data. Our specification of the demand allows capturing this situation.

Rejection of mortgage application: In borrowers' maximisation problem (1.2), we allow for the menu available to each borrower (P_{ib}) to be different as a result of rejections of borrowers' applications for a particular contract. The modelling of the choice of product is general enough to encompass the case in which borrowers have or do not have perfect knowledge of which applications would be successful and which would not. We favour the perfect information case interpretation as this case can be justified by the heavy use of brokers in this market. The imperfect information case is discussed in Appendix (C.4).

Borrowers' participation in the mortgage market: As shown in Andersen et al. (2021) and Benetton, Gavazza, and Surico (2021), borrowers' entry decision in the mortgage market is very inelastic to loan prices and characteristics.[14] Furthermore, Robles-Garcia (2019) and Benetton (2018) show that the level of competition is high in the UK mortgage market, making it unlikely that banks will be able to extract the full surplus from borrowers. This motivates the assumption of taking borrowers' participation as given and the use of a static demand model.

In appendix C.7, we derive a nested logit version of the model in which borrowers actively choose to participate or not participate in the mortgage market. This extension yields a closed-form formula for the expected utility of participating in the market V_i, which can then be estimated. This modelling is convenient as it makes the logit coefficient independent of the assumptions on the set of potential mortgage buyers that did not enter the market.

Supply

In this section, we discuss how our assumptions affect the interpretation of the supply parameters.

Collateral: Our NPV parametrization is derived in Appendix D.3 from a model in which banks do not recover anything following borrowers' default. This assumption

14. They estimate the entry decision in regular time, as opposed to a financial crisis. But it seems that even during the COVID-19 crisis, the number of borrowers did not drop on average.

does not affect the demand estimation as we do not explicitly model the cost of default and instead rely on a revealed preference approach. However, it affects the interpretation of the marginal cost of lending parameter that is recovered in the estimation section. To provide intuition for how to interpret the results given our assumption about collateral, let us introduce the following notation. Upon default, the mortgage originator can seize the lender's house and get $min\{\delta \cdot \frac{L}{LTV}, rL\}$. L is the loan size, r the interest rate, $\frac{L}{LTV}$ is the house value at the origination date, and δ is the ratio of the house price upon default over the one at origination. Default happens with probability d. If δ is not equal to zero, the estimated marginal cost we recover will capture the average loss given default conditional on LTV $E[mc - min\{\delta \cdot \frac{1}{LTV}, r\}d|LTV]$. Given our identification strategy, we cannot identify δ and mc separately. However, we discuss how one could do so using an integrating over approach in Appendix C.13.1.

Finally, although the use of collateral has been taken as given rather than derived from a first principle, conditions for collateralized debt to be the optimal contract is in Appendix *D*.2.1.

Static model of supply: The supply model used in this paper is static, as each period lenders maximize the expected profits generated by current lending activities only. This consideration is justified by the demand also being static. Static demand is heavily used in the literature and is a good approximation for mortgage markets as recent studies show that borrowers' entry and exit decisions — and thus their decisions on when to borrow — are almost never affected by mortgage prices and product offerings (Andersen et al. (2021) and Benetton, Gavazza, and Surico (2021)). However, the use of the fixed cost function in the lenders' problem creates a dynamic relationship between current and past maximization problems and makes the use of a dynamic model natural.

The static supply approach can nonetheless be justified by the following considerations. First, our static modeling can be written as the hurdle rate approach, which is a good approximation of firms' product-offering decisions according to recent surveys (see Wollmann (2018)). The hurdle rate approach assumes that firms choose to offer a set of products such that, for any other feasible set, the expected ratio of the added profits to added sunk costs does not exceed a set number (the hurdle rate).

Second, the only parameter affected by a dynamic modeling approach is the fixed cost function, which is not an object of interest of our analysis. Indeed, the marginal costs are not affected as they are identified from a model optimality condition that depends on the number of products being fixed. The counterfactual experiment is not affected by the use of the static model as long as the relationship between current and expected profits in the counterfactual experiment remains the same as in the data. The static estimation affects the economic interpretation of the size of the fixed cost. As a complementary approach, we show in Appendix C.13.3 how methods used in the dynamic demand estimation literature could be used in a dynamic version of our model to estimate the supply parameters. However, the dynamic estimation increases the computational burden of counterfactual experiments to the point where the counterfactual model would not be solvable with the current methods available.

1.4.4 Overview of the methodology

We parameterize the indirect utility (V), the loan demand (L) and the default probability (d) functions of the problem (1.3). In the identification exercise, we recover the parameter value using choice and default data.

In our parametrization, we acknowledge that the parameters of the indirect utility, the loan demand and the default probability are correlated as they derive from the same maximization problem. In particular, we allow for the utility derived from a given contract to be a function of borrower default probability. We provide a summary of our methodology in the following paragraphs. An in-depth discussion of the indirect utility (V) and loan demand (L) assumptions and their derivation is provided in Appendix C.1.1. We discuss the default probability functional form in Appendix C.1.2.

Preferences: Following the tradition in the IO literature, we use a hedonic demand system a la Lancaster (1966). That is, the utility derived from a contract at a given bank reflects the sum of the characteristics (e.g., LTV, loan amount, branch network) of that contract-bank.

Given the UK mortgage structure discussed in the previous section, we decompose the choice of contract into two equations: a product choice and a quantity choice. The choice of product-bank is based on a logit model (discrete choice). The choice of the borrowing amount is continuous and is modelled by a linear regression (continuous choice). This modelling is called a discrete-continuous demand system in the IO literature (see, for instance, Hanemann (1984) or K. Train (1986) for a discussion of the micro-foundation of that class of model).

Instead of specifying the demand system in a reduced-form way (as in Crawford, Pavanini, and Schivardi (2018) for instance) or fully specifying the maximization problem (see appendix C.5.1), we instead use a method developed in the discrete-continuous demand system literature. That is, we first parametrize the indirect utility function of borrowers at the optimal loan size for a given choice of product. We then derive the loan demand and product choice using optimality conditions: the borrower chooses the product c that maximizes its indirect utility, and the optimal loan size functional form is derived using Roys' identity.

In the following paragraph, we discuss the parametrization in light of the discrete-continuous literature. For those that are sceptical about the discrete-continuous approach, the same functional form used in this paper can be derived using a reduced form approach assuming that the default function, the log loan demand (1.3) are linear in their arguments, that borrowers' utility function in linear in contract terms and assuming that how borrower value contract terms is a linear function of the loan demand and default (see equation (1.5) bellow and the associated discussion).

In addition to being theoretically grounded, the approach taken in this paper has two advantages. First, it allows for mitigating the selection bias in the estimation (discussed in section 2.6 and in Table B.8). Indeed, since the choice of loan amount and contract derives from the same utility maximization problem, the demand system's coefficients should be correlated. As shown in (K. Train (1986)), this correlation can create selection bias in the quantity regression. This happen if, for instance, someone with a high unobserved propensity to borrow may also compare products more intensively (i.e., a higher unobserved price elasticity). Comparing the average loan size of a similar contract price differently thus captures the direct effect of the rate on loan demand but also the fact that borrowers with a high propensity to borrow tend to choose cheaper contracts. In our empirical exercise, we show that this method doubles the size of some product characteristic coefficients (Table B.8).

Second, this semi-reduced-form approach avoids having to explicitly model the underlying heterogeneity and actions in the structural model. As a result, the estimation relies on revealed preferences only and is robust to the underlying model driving borrowers' expectations or the borrower being credit-constrained due to an income multiplied for instance. This approach is traditional in the empirical literature on adverse selection (see, Chetty and Finkelstein (2013), Landais et al. (2020)).

We innovate with respect to the literature by generalizing the utility functional form used in K. Train (1986). This is done for two reasons (see Appendix C.1.1 for the derivations and the equation below for the indirect utility). First, the traditional functional form may not be adapted to financial markets. Indeed, applied directly, it implicitly implies that the bigger the loan size, the more utility one derives from the loan product. This assumption might not be true in the lending market as a high LTV makes the borrower put less of his own money into the house. It then forces the borrower to borrow more to buy the same house, and the costs in terms of reduced consumption in the future may be too high.

The second reason for the departure from K. Train (1986) is purely technical. As shown in the next paragraph, our assumption allows a classic linear logit model and linear loan size demand functions with correlated parameters. This simplifies the estimation through reduced computation time and also simplifies the counterfactual analysis via both reduced computation time and the uniqueness of the interest rate equilibrium. As mentioned in Wollmann (2018) and Einav, Finkelstein, and Mahoney (2021), those technical limitations are central issues when it comes to the counterfactual estimation.

Key parametrization: One of the key parametrization is the indirect utility ($V_i(c,b)$) and the expected default probability given the borrower information set ($E[d_{icb}|\mathcal{I}_i^B]$):

$$V_i(c,b) = \beta_{icb}X_{cb} + \alpha_{icb}r_{cb} + \overbrace{\xi_{cb}}^{\text{unobserved contract characteristics}} + \overbrace{\mu D_i + \epsilon_i}^{\text{borrower's characteristics}} + \varepsilon_{icb};\ \varepsilon_{icb} \sim EV \tag{1.4}$$

$$With:\ (\beta_{icb}, \alpha_{icb}) := \underbrace{f_{cb}(X_{cb}, r_{cb}) + \mu D_i + \beta_i^P}_{Contains\ E[d_{icb}|\mathcal{I}_i^B]};\ \beta_i^P \sim \mathcal{N}(0, \sigma_\alpha^2) \tag{1.5}$$

$$and\ E[d_{icb}|\mathcal{I}_i^B] := \beta^{di}(X_{cb}, r_{cb}) + \nu^{di} D_i + \underbrace{PI_i^d + e_i^{Ed}}_{\text{borrower's private information about default probability}} \tag{1.6}$$

The main objects of interest for screening in equations (1.5) and (1.6) are the correlations between default probabilities and borrowers' characteristics (D_i), and the correlations between borrowers' unobserved preferences heterogeneity (β_i^P) and the private information about borrower baseline default probability (PI_i). This is relevant because when preferences ($\beta_{icb}, \alpha_{icb}$) are heterogeneous, banks can influence the average characteristic (D) and preference (β^P) of borrowers choosing a given product[15] by changing their contract menus. For instance, high default borrowers find it relatively more costly to provide a high level of down payments for each additional unit they borrow, then low LTV contracts attract unobservably safer borrowers and can be offered at a lower price. An in-depth discussion of the model parameters is provided in section 2.6 and in Appendix C.1.

Once the demand and supply parameters estimated, we then parametrize the net present value of lending (NPV) and the fixed cost (F) functions. We use the model optimality conditions together with data on menus offered and estimated demand parameters to recover the supply parameters. The supply model assumptions are discussed in section C.1.3.

The identification and estimation of the model parameters are discussed in section 2.6.

In the counterfactual simulations, we change fundamental parameters, such as the information set of lenders and use the maximization problem, to recover the new

15. Given that banks do not offer a different price based on D_i in the UK, observable characteristics also drive the menu design.

equilibrium. The counterfactual section 1.7 presents the main empirical results and also provides a graphical illustration of the main mechanism at play. A more general analysis is provided in Appendices C.9, C.10 and C.11.

1.5 Identification and Estimation

This section discusses the identification and estimation of the model parameters defined in section 2.4. Recall that D_i is the vector of borrower i's observable characteristics, Γ_{icb} is a vector of parameters driving how borrower i values the characteristics and the price of contract c at bank b, and M_i is the set of contract menus offered to borrower i. The parameters to be identified are: (i) the moments of the distribution of the product and loan demand elasticity conditional on borrowers' observable characteristics ($E[\Gamma_{icb}|D_i, M_i, i\ choose\ cb]$, $V[\Gamma_{icb}|D_i, M_i, i\ choose\ cb]$); (ii) how borrowers' default probabilities vary with contract terms (β^d), with observable borrowers' characteristics (ν^d), and with borrowers' demand elasticities (ρ^d); and (iii) the lender-product-specific unobservable marginal costs of lending (mc_{cb}), and the lender-specific fixed costs of introducing or withdrawing a new type of contract in their menu (F_b).

We collect all the parameters into the vector $\Theta := (\Theta^D, \Theta^d, \Theta^S)$ where $\Theta^D := (\Theta^P, \Theta^L)$ denotes the demand parameters related to the product demand (Θ^P) and the loan demand (Θ^L). Θ^d contains the default parameters (β^d, ν^d, ρ^d) and Θ^S the supply ones (mc, F). The elements of Θ^P and Θ^L are defined in the relevant sections. Each following section — demand (section 1.5.2), default (section 1.5.3), and supply (section 1.5.4) — focuses on the identification and estimation of their respective Θ element.

1.5.1 Econometrician information set and parametric assumptions

Econometrician information set: The econometrician observes each borrower i's choice of contract (c_i, b_i), their characteristics (D_i), the amount borrowed $(L_{ic_ib_i})$, the set of banks operating in the market (B) and the price and characteristics of each product c offered by each bank b if borrower i were to choose it $M := (X_{cb}, r_{cb})_{cb \in P}$.[16] The econometrician also observes whether borrower i defaulted on their mortgage contract before 2020 $(d_{ic_ib_i})$ and the origination date of the loan (t_i).

Some of the product and borrower characteristics are observed by banks but not by the econometrician. When necessary, we denote the observable characteristics with a superscript o and the unobservable characteristics (by the econometrician only) by the superscript u.

16. There is no i index as the pricing is independent of i in the UK mortgage market.

The econometrician does not perfectly observe the subset of menus available to each borrower i ($M_i \subset M$). In particular, borrowers' loan applications and banks' rejections of the applications are not observed.

The econometrician information set is defined as $\mathcal{I}^E := \{M, (D_i, SP_{ib}, c_i, b_i, L_{ic_ib_i}, d_{ic_ib_i})_i\}$ where SP_{ib} is the subset of the (indexes) contract available to each borrower i. We discuss how SP_{ib} is constructed and how it affects the identification in the relevant section. For convenience, we introduce the econometrician information set prior to the borrower's choice of contract: $\mathcal{I}^E_P := \{M, (D_i, SP_{ib})_i\}$.

1.5.2 Step 1: Demand

In this first step, we use contract choice data to identify and estimate borrowers' heterogeneous demand elasticities. We thus capture banks' ability to screen borrowers along their outside option. For instance, lenders can benefit from screening if borrowers that tend to choose high LTV contracts also tend to compare less intensively products across banks and are thus less price elastic.

The demand parameters (Θ^P, Θ^L) are identified and estimated using the cross section for a given month. We first provide an overview of the identification and estimation process and challenges before explaining them in detail.

Identification of the product choice parameters

In this section, we describe the identification strategy and the estimation approach for the parameters driving borrowers' choice of product (i.e., the choice of a combination of LTV, maturity, fixed rate, fees) and bank. Borrowers' choice is based on a mixed logit model (see Nevo (2001) for a supply and demand approach based on a mixed logit demand model in the cereal industry).

Product choice equation: Given the parametric assumptions on the indirect utility, the loan demand and the default regression discussed in depth in Appendix C.1, the probability of borrower i choosing contract c at bank b can be written as the following (mixed) logit model. It allows identifying the indirect utility parameters

$(V_i(c,b))$.

$$\begin{aligned}
&\Pr(\text{i chooses cb}\,|\mathcal{I}_P^E,\Theta^P,\beta_i^P) && (1.7)\\
:=&\Pr(cb\in \text{argmax}_{\{y\in SB,\ x\in SP_{iy}\}}\{\underbrace{\overbrace{\beta_i X_{xy}-\alpha_i r_{xy}+\xi_{xy}}^{\tilde{u}_i(x,y)}+\epsilon_{ixy}}_{V_i(c,b)}\}|\mathcal{I}_P^E,\Theta^P,\beta_i^P)\\
=&\frac{\exp(\tilde{u}_i(c,b))}{\sum_{y\in B}\sum_{x\in SP_{iy}}\exp(\tilde{u}_i(x,y))},\ \text{when } \epsilon_{icb} \text{ iid and EV distributed}
\end{aligned}$$

$$\begin{aligned}
&\textit{with } \beta_i := \beta+\nu_X^P D_i+\beta_i^X && (1.8)\\
&\alpha_i := \alpha+\nu_r^P D_i+\alpha_i^r && (1.9)\\
&\beta_i^P := (\beta_i^X,\alpha_i^r)' \sim \mathcal{N}(0,\Omega^P) && (1.10)
\end{aligned}$$

where the product demand parameters are denoted $\Theta^P := (\beta,\alpha,\nu_X^P,\nu_r^P,\Omega^P,(\xi_{cb})_{cb})$.

β_i and α_i drive how borrower i values product characteristics and prices. We loosely refer to them as borrowers' preferences. β_i and α_i refer to the part of the valuation that is not a function of contract terms (i.e., borrower i values product characteristics and prices according to $\beta_{icb} := \beta_i + f_{cb}(X_{cb}, r_{cb})$). The elements that do depend on contract terms (i.e., $f_{cb}(X_{cb}, r_{cb})$) are absorbed by the bank-product fixed effect ξ_{cb}.

As acknowledged by the notation of equation (1.3) and discussed in section C.1.2, β_i and α_i are potentially a function of borrowers' default probability, the cost of defaulting — which depends on the loan being recourse or not — or how much the borrower values housing relative to consumption and how much savings the borrower has. For instance, high default may be less sensitive to the face value of the debt if borrowers expect that they will not have to repay it fully upon default (for example, if the loan is non-recourse).

$\nu^P := (\nu_X^P, \nu_r^P)$ are parameters capturing observable heterogeneity in borrowers' preferences.

β_i^P is a random coefficient modeling unobserved heterogeneity in borrowers' preference. It is a key parameter for screeningas it potentially contains information about borrower i's unobserved baseline default probability. β^P also contains borrowers' characteristics that are unobservable by the econometrician but observable by banks.[17]

The ratio $\frac{\beta_i}{\alpha_i}$ represents borrower i's willingness to pay for a characteristic. Indeed, if a bank proposes a new high LTV contract, borrower i would be happy to take it (i.e., its utility would increase by taking the contract) as long as the price increase is

17. If the model is misspecified, this term includes the misspecification error terms as well.

below the borrower's willingness to pay. Formally, borrowers accept the new contracts if $U(L(LTV_2); r_2, LTV_2) \geqslant U(L(LTV_1); r_1, LTV_1) \iff \frac{\beta_i}{\alpha_i}(LTV_2 - LTV_1) > (r_2 - r_1)$.

ξ_{cb} is a product bank fixed effect. As discussed in Appendix C.1.1, it captures the part of the average indirect utility that comes from unobserved (by the econometrician) contract characteristics.

ε_{icb} is the demand shock. As discussed in Appendix C.1.1, it contains borrower i's deviations from the average borrowers' valuation of unobserved product characteristics and bank shocks. We assume that $E[\sigma_i^{-1}\varepsilon_{icb}|X_{cb}, r_{cb}, \beta_i, \alpha_i] = 0$ so that $\sigma_i^{-1}\varepsilon_{icb}$ represents the part of borrowers' demand that cannot be screened by banks when they use product characteristics (X_c, r_c) only (cf. proposition 1). The potential identification threat caused by this assumption is discussed in the following paragraphs.

SP_{ib} is the subset of (indexes of) products available to each borrower at bank b. We describe how it is constructed in the identification challenges paragraphs. We denote it SP_{ib} to distinguish it from the actual one, P_{ib}, used by banks.

The distribution of the interest rate coefficient (α_i) — or its moments, in our case — can be identified from banks offering the same product at different interest rates. The coefficients in front of product characteristics are identified from the pricing schedule of banks along the relevant dimension (max LTV, fixed rate, maturity, fees).

In the following paragraphs, we discuss — first informally, then formally — how variables that are unobservable by the econometrician such as borrowers' characteristics, product characteristics and preference heterogeneity for those characteristics and rejection of mortgage application may challenge our identification and how we address them. The identification is formally discussed in Appendix C.11.1 (for a linearized version of the model) or in Fox et al. (2012) (for a standard mixed logit model).

Overview of the methodology:

Figure 1.1 provides a visual representation of the mixed logit identification strategy and its challenges. For simplicity of the exposition, we set demand shocks (ϵ_{icb}) to zero, consider only one bank b, and plot on the (LTV, interest rate) plane the following objects: borrower i's indifference curves and bank b's pricing schedule for LTV, taking other contract characteristics as constant.

The object of interest is the slope of the pricing schedule curve at borrower i's optimal contract choice. The slope provides information about borrower i's willingness to pay for LTV (i.e., $\frac{\beta_i}{\alpha_i}$). For instance, absent the demand shock ϵ_{icb}, and under a continuous and convex pricing schedule,[18] the slope is exactly equal to $\frac{\beta_i}{\alpha_i}$.[19] The

18. In our setup, the pricing schedule is convex.

19. A maximization problem of the utility $\tilde{u}_i(c, b)$ with a continuum of product yields that the willingness to pay is equal to the slope of the pricing schedule at optimum (for an interior solution and under convexity of the pricing schedule for all valuable product characteristic X).

intuition is that if a borrower chooses, for instance, a 85% LTV loan while he had access to a 90% LTV loan for an interest rate increase of 100 bps, it must be that his willingness to pay for a 5 percent LTV increase is below 100 basis points.

Given that we observe the pricing schedule for each bank and product characteristic, as well as each borrower choice, we can recover or bound the distribution of willingness to pay ($\frac{\beta_i}{\alpha_i}$) for each contract term from the slope of the pricing schedule of banks at borrowers' optimal contract choice. Similarly, we can recover the level of product demand elasticity (α_i) from banks selling similar products at different prices.

The simplifying assumptions made in this overview about the demand shock ϵ_{icb} being equal to zero — that is, the fact that there is a continuum of products and only one bank — does not affect our identification strategy. The demand shock distribution is fixed and independent of borrowers' preferences, so a deconvolution argument allows to back out the parameters $\frac{\beta_i}{\alpha_i}$ from their contract choice. Dealing with discrete choice requires doing an interpolation. For instance, one can use a linear interpolation of the pricing curve and use the left and right derivatives as bounds. The logit model is a particular way to construct the pricing schedule and do the interpolation when there are multiple product characteristics and lenders as well as a discrete number of products.

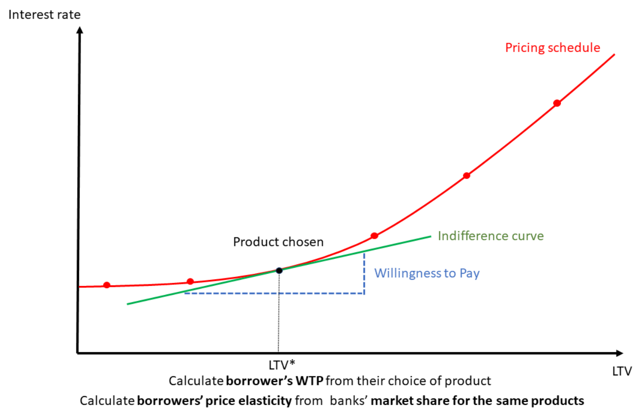


FIGURE 1.1: Revealed preference approach

The product choice identification presents three main identification challenges. The first two come from (i) the unobserved contracts' characteristics (for instance, marketing expenses) that are potentially correlated with rates or (ii) the correlation between unobserved preferences for observed and non-observed contract characteristics (for instance, characteristics arising from both of them depending on default probabilities). In Figure 1.1, this would mean that the pricing schedule slope is too flat or too steep as other contract terms move with LTV. The third one is caused by the (unobserved) rejection of borrowers' applications based on unobserved variables by the econometrician but observable by the bank (referred to as consideration set bias). In Figure 1.1, this means that each point in the pricing schedule slope is not actually available to the borrower. This biases the result if the econometrician wrongly includes in a borrower's choice set a contract with attractive features—for instance, a high LTV—that was in practice not available to the borrower. In our example, this inclusion tends to downwardly bias the willingness to pay estimates for LTV to rationalize that the borrower did not choose the high LTV contract.

To deal with the unobserved product characteristics and rejection thresholds, we use an instrumental variable approach together with bank and product fixed effects. As in Benetton (2018) or Robles-Garcia (2019), product-specific capital requirements are used as cost shifters. The consideration set bias is dealt with using a sufficient set approach, as in Crawford, Griffith, Iaria, et al. (2016). This approach shows that taking a subset of the menu for which banks' rejection is independent on variables unobserved by the econometrician restores the consistency of the estimates. The choice of subset is subject to the econometrician's judgment. Since a failure of the sufficient set correction would lead to a downward bias of the WTP LTV estimates, our main results about the LTV distortion level and the cost of those distortions should be interpreted with caution as a lower bound to the true effect.

Identification challenges: Let us now formally discuss the identification challenges. The main parameters of interest in $\Theta^P := (\beta, \alpha, \nu^P, \Omega^P, (\xi_{cb})_{cb})$ are the mean coefficients (β, α), the observable heterogeneity coefficient (ν^P) and the variance of the unobservable heterogeneity component (Ω^P). The coefficients (ν^P) do not need to have a causal interpretation as we are interested in how lenders can use them as a proxy for borrowers' demand elasticity.

Challenge (i): to make the identification threat concerning the mean coefficient (β, α) salient, let us rewrite as in Nevo (2001), the logit model using the bank-product fixed effect (δ_{cb}).[20] As shown in Fox et al. (2012), the parameter δ_{cb} can be identified from product choice data and then regressed on (X,r) to recover the average coefficients. In that second step, ξ_{cb} are residual terms and can be interpreted as unobservable product characteristics. By definition, δ_{cb} is equal to

$$\delta_{cb} := E_i[\tilde{u}_i(c,b)|\mathcal{I}_P^E] = \underbrace{\beta X_{cb}^o - \alpha r_{cb}}_{Average\ effect} + \underbrace{\xi_{cb}}_{Contains:\ \beta^u X_{cb}^u}$$

The potential identification threat thus comes from unobserved product characteristics (X_{cb}^u) that are not captured by the bank dummies and correlated with observed product characteristics' levels and interest rates (X_{cb}^o, r_{cb}). The correlation can be a result of, for instance, banks promoting a particular product via higher broker commissions (X_{cb}^u) and passing through the marketing expense to the product

20. Using the bank-product fixed effect also limits the threat that the heterogeneous component (e_i) contains adverse selection information.

interest rate (r_{cb}). This creates a positive correlation between the interest rate of the product and the unobserved product promotion, which would upwardly bias the estimates of the interest rate coefficient ($-\alpha$). To mitigate this concern, we use the bank fixed effect and instrument rates with a cost shifter, as in Benetton (2018) and Robles-Garcia (2019). In particular, we exploit the variation in risk-weighted capital requirements both across lenders and across LTV levels within lender (henceforth denoted as Z). Our empirical strategy thus controls for differences across lenders that are common among products (lender shocks) as well as differences across products that are common across lenders (market shocks).

Challenge (ii): let us now look at the identification threat concerning the variance of the random coefficients (Ω^P). To make the identification threat clear, let us explain why we impose the assumption about ε_{icb} being independent and identically EV distributed in equation (1.7). A more theoretically founded assumption on the error term would be as follows:

$$\varepsilon_{icb} := \beta_i^{Pu} X_{cb}^u + \tilde{\sigma}_i \tilde{\varepsilon}_{icb}, \text{ with } \tilde{\varepsilon}_{icb} \text{ iid and EV distributed} \tag{1.11}$$

Let us further consider that the unobserved heterogeneity associated with unobservable product characteristics (β_i^{Pu}) is correlated with the one associated with observable product characteristics (β_i^{Po}). Formally, we assume that, conditional on (e_i^o, X^o, r^o), $\tilde{\varepsilon}_{icb}$ follows an extreme value distribution with mean $\rho^e \beta_i^{Po} X_{cb}^u$,[21] and draws are independent across borrowers and products. With that assumption, one can, without loss of generality, normalize $\tilde{\sigma}_i$ to 1 for each borrower (in that case, α_i will play the same role as $\tilde{\sigma}_i$). The probability of seeing borrower i choose contract c at bank b is thus

$$Pr(i\ choose\ cb | \mathcal{I}_P^E, B, P_i) = \int \frac{exp(\tilde{u}_i(c,b) + \overbrace{\rho^e \beta^{Po} X_{cb}^u}^{Correlated\ preferences\ bias})}{\sum_{y \in B} \sum_{x \in P_{iy}} exp(u_i(x,y))} dF_{\beta^{Po}}(\beta^{Po}; \Theta^P)$$

The variance that is identified for the x^{th} element of X_{cb}^o (denoted x) is thus

$$V_x := V[\beta_i^{Po} | \mathcal{I}_P^E, B, P_i] + \underbrace{V[\rho^e \beta_i^{Po} \frac{X_{cb}^u}{x} | \mathcal{I}_P^E, B, P_i]}_{Correlated\ preferences\ bias} \tag{1.12}$$

The bias thus comes from a correlation between preferences for observable and

21. This functional form $\rho^e \beta_i^{Po} X_{cb}^u$ arises when error terms ($\beta_i^{Pu}, \beta_i^{Po}$) are jointly normally distributed. The variance assumption does not matter as it scales the value of the preferences parameters by the same amount.

unobservable product characteristics (e_i^o and ε_{icb}). The correlation exists if, for instance, more price-elastic borrowers are also pickier about better customer service quality (an unobserved characteristic). The possibility of correlated preferences has not been the focus of the literature. Following the IO literature, we assume that $\rho^e = 0$ in the main model and thus go back to the assumption written in equation (1.7). To verify this assumption's validity, we check whether the random coefficients associated with the observable product characteristics are correlated. We find that there is no correlation conditional on observable borrower characteristics. We provide in Appendix C.11.1 an alternative new methodology to mitigate this concern in a linearized version of the model.

Challenge (iii): the last concern comes from loan application rejections being unobservable. This is an issue to the extent that the econometrician cannot reconstruct the true choice set of borrowers from the choice of observationally equivalent borrowers. This happens if banks reject borrowers' applications based on information that is observable by banks but not by the econometrician (for instance, soft information or credit history). Following Crawford, Griffith, Iaria, et al. (2021), we show in Appendix C.12 that the bias results from including in the menu a contract that borrowers would have chosen if it had been offered to them.[22] For instance, wrongly including a cheap high LTV contract in a menu tends to downwardly bias the willingness to pay for LTV estimates to rationalize that borrowers did not choose the contract.

As shown in the consideration set literature (see, for instance, Crawford, Griffith, Iaria, et al. (2021)), this issue can be dealt with by using a subset (denoted SP) of the contract menus truly available to borrowers. We provide in Appendix C.12 a sketch of the proof, including the case with random coefficients. The random coefficient inclusion requires the additional assumption that the random coefficient draws should not affect SP. Formally, the bias can be written as

$$Pr(i\ choose\ cb|\mathcal{I}_P^E, \rho^e = 0) = \int \frac{exp(\tilde{u}_i(c,b) - \overbrace{ln(\pi_i)}^{unobserved\ rejection\ bias})}{\sum_{x \in SB} \sum_{c \in SP_x} exp(\tilde{u}_i(c,x))} dF_{e^o}(e;\theta)$$
$$with:\ ln(\pi) := ln(\frac{\sum_{x \in SB \cap B} \sum_{c \in SP_x \cap P_{xi}} exp(\tilde{u}_i(c,b))}{\sum_{x \in B} \sum_{c \cap P_{xi}} exp(\tilde{u}_i(c,b))})$$

as $ln(\pi)$ is equal to 0 when the subset $SB \subset B,\ SP_{xi} \subset P_{xi}$.

We construct individual subsets of menus in the following way. First, we divide households into time periods (months) and geographical regions. We assume households in each group can access all products sold by banks during that period but not

22. This is slightly more general than loan rejections as it includes expected loan rejections as well.

those sold in other periods. The time restriction accounts for the entry and exit of products. The geographical restriction mostly affects building societies and smaller banks because they often have limited coverage across regions.

We then impose additional restrictions. For each product, we select the oldest households. We then assume a household will not qualify for that product if it is older than the cutoff value. This restriction is based on a commonly used affordability criterion.

We also restrict households that received a product from the biggest eight banks to the menus offered by those banks. This restriction captures the fact that some borrowers may not consider smaller banks when shopping. Similarly, we consider that borrowers that received a product from fringe lenders are restricted to menus offered by those lenders. This is rationalized by large lenders being stricter in terms of compliance resulting from regulatory oversight. As a result, some some borrowers — for example, self-employed workers — may only be able to borrow from fringe banks specialized in lending to them. We also restrict the product maximum LTV of each contract belonging to a borrower menu to a maximum LTV category just above and below the chosen product. This limits the concern over a borrower not having enough of a down payment to select another product category and mitigates the threat of rejection based on borrower characteristics that are not observable by the econometrician.

Moments: Denoting the parameter to be estimated using the logit model $\theta := (\delta_{cb}, \nu^P, \Omega^P)$, the product demand parameters ($\Theta^P := (\beta, \alpha, \nu^P, \Omega^P, (\xi_{cb})_{cb})$) can be identified and estimated using the following identification conditions:

$$E[1_{\{i \ choose \ cb\}}|\mathcal{I}^E, \theta, \rho^e = 0] = \int \frac{exp(\tilde{u}_i(c,b))}{\sum_{y\in SB}\sum_{x\in SP_y} exp(u_i(x,y))} dF_{\beta^P}(\beta^P;\theta) \tag{1.13}$$

$$E[\underbrace{\delta_{cb} - (\beta X_{cb} - \alpha r_{cb})}_{\xi_{cb}}|\mathcal{I}^E, \theta, Z] = 0 \tag{1.14}$$

The first moment identifies θ, and the second moment allows us to identify the average preferences (β, α) and the unobservable contract characteristics ξ_{cb}. Z is the instrument used for interest rates.

Estimation procedure: The logit model is estimated using a two-step approach, as in K. E. Train (2009). The parameters of equation (1.13) are estimated using the simulated maximum likelihood procedure, as in Berry, Levinsohn, and Pakes (1995) or Nevo (2001). Based on the moment condition (1.14), we use an instrumental variable approach to estimate the interest rate and the product mean coefficients (β, α).

We replace δ_{cb} by the estimated product-bank fixed effect $\hat{\delta}_{cb}$ taken from the first step. The standard errors are calculated using a bootstrap method.

Identification of loan amount choice parameters

In this section, we describe the identification strategy and the estimation approach of the loan amount choice. The borrowers' choice is based on a random coefficient linear model.

Loan choice equation:

$$ln(L_{icb}) = \tilde{\beta}_i X_{icb} - \tilde{\alpha}_i r_{icb} + \nu^L D_i + e^L_{icb} \tag{1.15}$$

$$with\ \tilde{\beta}_i = \tilde{\beta} + \tilde{\nu}^L_X D_i + \tilde{\beta}^X_i, \tag{1.16}$$

$$\tilde{\alpha}_i = \tilde{\alpha} + \tilde{\nu}^L_r D_i + \tilde{\beta}^r_i, \tag{1.17}$$

$$\tilde{\beta}^L_i := (\tilde{\beta}^X_i, \tilde{\beta}^r_i)' \sim \mathcal{N}(0, \Omega^L) \tag{1.18}$$

$\tilde{\beta}_i$ and $\tilde{\alpha}_i$ parameterize the loan demand heterogeneity with respect to product characteristics and prices. Those parameters are a function of observable and unobservable heterogeneity (respectively, $\nu^L_X D_i$ and $\tilde{\beta}^L_i$).

$(\tilde{\beta}^L_i)$ are random coefficients that are correlated with the product choice coefficients (β^P_i) to capture the fact that the demand and product choice derive from the same maximization problem (1.2). For the reasons discussed in C.1.1, we consider that $(\beta^P_i, \tilde{\beta}^L_i)$ follows a joint normal distribution with mean zero to address the potential selection bias that can arise in the loan choice equation estimation. It follows that $E[\tilde{\beta}^L|\beta^P_i] = \Sigma_{\tilde{\beta}^L} \cdot \beta^P_i$, where $\Sigma_{\tilde{\beta}^L}$ captures the correlation between $\tilde{\beta}^L$ and β^P.[23]

ν^L parameterizes how the loan demand for a given contract varies with borrowers' observable characteristics.

As in the product choice equation, the error term e^L_{icb} contains unobserved product characteristics and deviations from the average loan demand coefficients.

Conditional on the choice of product coefficient Θ^P being identified, we can identify $\Theta^L := (\tilde{\beta}, \tilde{\nu}^X, \tilde{\alpha}, \nu^r, \Sigma, \nu^L)$ using loan size data. The correlation coefficient Σ is identified from variation in incentives to choose a given product (for instance, changes in the interest rate spread between similar product categories or borrowers facing different menus). We discuss the identification assumptions of the correlation coefficients more in detail in the default probability sections as they are the centre of the default regression analysis. For a given correlation coefficient Σ, the contract and borrower characteristics coefficients are identified by comparing the average loan

23. $\Sigma_{\tilde{\beta}^L}$ is the product between the covariance matrix and the inverse of the variance of the conditioning variable.

size of observationally equivalent borrowers that choose contracts that are similar in all but one dimension — for instance, interest rates. In this section, we focus on how allowing for correlated coefficients (via Σ) mitigates the selection bias caused by the simultaneous decision of loan size and product choice. The endogeneity of interest rates is dealt with using the same instrumental variable approach as in the product choice estimation.

Identification challenges: Let us now formally discuss the identification challenges. The loan choice regression features a selection bias problem. It happens when, for instance, borrowers with a high unobserved propensity to borrow tend to compare products more intensively and thus choose cheaper contracts.

This issue is dealt with by explicitly modeling how the selection occurs. Traditionally, this is done using the structural discrete-continuous approach, as in K. Train (1986). This approach is developed for the logit model instead of the mixed logit model (i.e., the random coefficient logit), and we impose the restriction that the product and quantity parameters are the same ($\beta = \tilde{\beta}$). As discussed in section 2.3, we modify the indirect utility functional form used in K. Train (1986) to adapt it to financial markets. In our setup, the selection bias is thus captured by the correlation between the random components of the loan demand equation (i.e., $\tilde{\beta}_i^L$) and those of the product choice equation (β_i^P).

Formally, given the model specification and using the fact that the random coefficient variables ($\beta_i^P, \tilde{\beta}_i^L$) follow a multidimensional normal distribution?, as well as the assumptions that the demand shock (ε_{icb}) does not give any information about the loan size demand, the loan size equation (C.3) can be rewritten as

$$\begin{aligned} E[ln(L_{icb})|\mathcal{I}_P^E, i\ choose\ cb] = \ & (\tilde{\beta} + \nu_X^L D_i)' X_{icb} - (\tilde{\beta} + \nu_r^L D_i) r_{icb} + \nu D_i \\ & + E[e_i^L|\mathcal{I}_P^E, i\ choose\ cb] \\ & + \underbrace{E[\overbrace{\Sigma_{\tilde{\beta}_i^X} \cdot \beta_i^P \cdot X_{icb} + \Sigma_{\tilde{\beta}_i^r} \cdot \beta_i^P \cdot r_i + \Sigma_{e^L} \cdot \beta_i^P}^{sb_{icb}} |\mathcal{I}_P^E, i\ choose\ cb]}_{Selection\ bias} \end{aligned} \tag{1.19}$$

The above expression formalizes the idea that not controlling for borrowers' preferences (β_i^P) can lead to a bias when borrowers that tend to choose a particular type of contract consistently have a higher or lower than average demand elasticity (i.e., when $E[\beta_i^P|M_i, D_i, i\ choose\ cb] \neq 0$ and $\Sigma \neq 0$).

Σ is identified by variation in the average unobserved heterogeneity conditional on product choice (i.e., variation in $E[\beta_i^P|\mathcal{I}_P^E, i\ choose\ cb]$) holding contract c at bank b terms constant. $E[\beta_i^P|\mathcal{I}_P^E, i\ choose\ cb]$ varies with the product characteristics or interest rate spread between product c at bank b and the products of its competitors.[24] Σ is thus identified using variation in contract terms and prices other than contract c at bank b, keeping the terms and prices of contract c at bank b constant.[25]

For a given Σ, the average coefficients (β, α, ν) are identified by comparing the loan size of observationally equivalent borrowers borrowing using contracts that vary in X and r.

As in the product choice, one might worry that the unobserved product characteristics are being correlated with interest rates. We thus use the same cost shifter to identify the rate coefficient.

Moments: The loan size demand model can be identified and estimated using the following identification assumptions:

$$E[\underbrace{L_{icb} - (\tilde{\beta} + \nu_X^L D_i)X_{icb} - (\tilde{\beta} + \nu_r^L D_i)r_{icb} - \nu D_i - sb_{icb}}_{e^L} | \mathcal{I}_P^E, L_{icb}, Z, i \ choose \ cb] = 0 \tag{1.20}$$

where Z is the instrument used for interest rates and sb_{icb} is the selection bias correction term defined in equation (1.20).

Estimation procedure: Given consistent estimates for Θ^P — taken from the product demand estimation and denoted $\hat{\Theta}^P$ — we construct a consistent estimate of $E[\beta_i^P | \mathcal{I}_P^E, i \ choose \ cb])$ by using Bayes' rule and the estimated preferences coefficients of equation (1.13) to get

$$\hat{E}[\beta_i^P | \mathcal{I}_P^E, i \ choose \ cb] = \int \beta^P \frac{Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \hat{\Theta}^P, \beta_i^P)}{Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \hat{\Theta}^P)} dF(\beta^P; \hat{\Omega}^P) \tag{1.21}$$

$Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \Theta^P, \beta_i^P)$ is defined in equation (1.7). $Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \hat{\Theta}^P)$ is given by integrating $Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \hat{\Theta}^P, \beta_i^P)$ over β^P using the cumulative distribution function $F(\beta^P; \hat{\Theta}^P)$.

24. $E[\beta_i^P | \mathcal{I}_P^E, i \ \ choose \ \ c] = \int \beta^P \frac{Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \Theta^P, \beta_i^P)}{Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \Theta^P)} dF(\beta^P; \Omega^P)$ and $Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \Theta^P, \beta_i^P)$, given by equation (1.7), depends on the spread between contracts only $(\frac{exp(\beta X_c - \alpha r_c)}{\sum_x exp(\beta X_x - \alpha r_x)} = \frac{1}{\sum_x exp((\beta X_x - \alpha r_x) - (\beta X_c - \alpha r_c))})$. $Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \Theta^P)$ is given by integrating $Prob(\text{i chooses cb} \,| \mathcal{I}_P^E, \Theta^P, \beta_i^P)$ over β^P.

25. Let us consider the menus M and $\tilde{M}$ where $\tilde{M}$ is composed of the same contract as M except that all product prices, save for the one indexed by cb, increase by a given positive amount. To simplify the notation, let us assume without loss of generality that $\Sigma_{\tilde{\beta}_i^r} = \Sigma_{e^L} = 0$ In that case we have $E_i[ln(L_{icb})|M, D, i \ choose \ cb] - E_i[ln(L_{icb})|\tilde{M}, D, i \ choose \ cb] = \Sigma_{\tilde{\beta}_i^X} \cdot E[\beta_i^P \cdot |M, D, i \ choose \ cb] X_{icb} - \Sigma_{\tilde{\beta}_i^X} \cdot E[\beta_i^P |\tilde{M}, D, i \ choose \ cb] \cdot X_{icb}$. As $E[\beta_i^P \cdot |M, D, i \ choose \ cb] \neq E[\beta_i^P \cdot |\tilde{M}, D, i \ choose \ cb]$, we can identify $\Sigma_{\tilde{\beta}_i^X}$.

We then use an instrumental variable approach to estimate the loan demand coefficients Ω^L based on the moment condition (1.20).

The joint estimation of the product and loan demand is computationally demanding as it would require iterating on the estimate of $E[\beta_i^P|\mathcal{I}_P^E, i\ choose\ cb]$ for each θ^P. For this reason, we estimate the product and loan demand separately and calculate the standard errors using a bootstrap method.

1.5.3 Step 2: Default probabilities

This section discusses the identification and estimation procedure of the default parameters (Θ^d). We present the econometric model, highlight the identification challenges, and discuss the estimation procedure. The default parameters are identified and estimated using the cross-sectional variation and the variation in the month of the mortgage origination.

Borrowers' default equation: From the micro-foundations presented in Appendix (C.5), How borrowers value contract terms $\Gamma_{icb} := (\alpha_{icb}, \beta_{icb}, \tilde{\alpha}_{icb}, \tilde{\beta}_{icb})$ might be a function of the default probabilities. Indeed, risky borrowers might be less sensitive to prices if they expect that they won't be forced to repay the full face value of the loan upon default. In that case, α_i would be a decreasing function of default. Alternatively, instead of being a default function directly, "price elasticity" α_i and default probability d_i might be influenced by the same fundamental parameter. For instance, a borrower with greater financial sophistication might find it less time-consuming to compare products and thus may end up with a cheaper product. The same borrowers might be more likely to make better financial decisions in general and thus may have a lower baseline default rate. For those reasons, we model default probabilities[26] (d_{icb}) and preferences Γ_{icb} the following way. Using the superscript o to denote the variable that is observed by both the econometrician and banks and the superscript u for the variable observable by banks only, the default model (d_{icb}) for borrower i choosing contract c at bank b has a default probability is:

26. The logic behind our approach is as follows. The default probability is a function a of monthly repayment, the cost of defaulting and losing the house, the borrower's future income profile and the borrower's propensity to save. The loan size is an endogenous variable, so we replace it by its function defined in C.3. We linearize the expression around the contract and borrowers' characteristics. Then, we explicitly acknowledge that the choice of contract and loan size depends on default in equation (C.8).

Default probability from borrower i point of view:

$$d_{icb} = \delta t_i + \beta^d (X_{cb}, r_{cb}^{t_i})' + \nu^d D_i + \rho \overbrace{PI_i^d + \tilde{e}_{icb}^d}^{borrower's\ private\ information} \quad (1.22)$$

Default probability from the econometrician's point of view:

$$d_{icb} = \delta t_i + \beta^d (X_{cb}^o, r_{cb}^{t_i})' + \nu^d D_i^o + \underbrace{\rho^d \hat{\beta}_{icb}^P}_{contains:\ PI^d\ and\ D_i^u} + \underbrace{e_{icb}^d}_{contains:\ \beta^{ud} X_i^u} \quad (1.23)$$

d_{icb} is equal to 1 if borrower i has been in arrears by the end of 2019. t_i is the origination date of the mortgage acquired by borrower i, δ is the parameter associated with t. β^d captures the causal impact of the contract terms $(X_{cb}, r_{cb}^{t_i})$ on default probabilities due to moral hazard or burden of payment. $r_{cb}^{t_i}$ is subindexed by the origination date as the pricing of product c at bank b may vary with the origination date.

ν^d parametrizes how the baseline default probability varies with observable borrower characteristics (D_i), ρ^d parametrizes how the baseline default probability varies with unobservable borrower characteristics.

e_{ic}^d represents, for instance, the characteristics of borrower i (observable by the lender) that influence default but are not considered by borrowers when they make their loan decision (i.e., they do not enter Γ_i and cannot be recovered by banks).

$\hat{\beta}_{icb}^P := \hat{E}[\beta_i^P | \mathcal{I}_P^E, i\ choose\ cb] = \hat{E}[\rho PI_i + \nu^P D_i^u | \mathcal{I}_P^E, i\ choose\ cb]$ is the estimated average borrower preference given the menus offered by lenders and the choice of contract of borrower i, and β_i^P is defined in section 1.5.2.

Conditional on the choice of product coefficients Θ^P being identified, we can get an estimate of $\hat{\beta}_{icb}^P$ using the procedure defined in section (1.5.2). We can then identify $\Theta^d := (\beta^d, \delta^t, \nu^d, \rho^d)$ using default data. Similar to the selection bias terms in equation (1.20), the correlation vector ρ^d is identified using variation in incentives to choose a given product (see footnotes 24 and 25 for a sketch of the proof). Incentives to choose a product vary with, for instance, changes in the interest rate spread over time between similar product categories.[27] The mortgage origination coefficient is identified from the month in which menus do not change. Given ρ^d, the contract and borrower characteristics coefficients are identified by comparing the average default

27. Indeed, $E[\beta_i^P | \mathcal{I}_P^E, i\ \ choose\ \ c] = \int \beta^P \frac{Prob(\text{i chooses cb} | \mathcal{I}_P^E, \Theta^P, \beta_i^P)}{Prob(\text{i chooses cb} | \mathcal{I}_P^E, \Theta^P)} dF(\beta^P; \Omega^P)$ and $Prob(\text{i chooses cb} | \mathcal{I}_P^E, \Theta^P, \beta_i^P)$, given by equation (1.7), depends on the spread between contracts only $(\frac{exp(\beta X_c - \alpha r_c)}{\sum_x exp(\beta X_x - \alpha r_x)} = \frac{1}{\sum_x exp((\beta X_x - \alpha r_x) - (\beta X_c - \alpha r_c))})$. $Prob(\text{i chooses cb} | \mathcal{I}_P^E, \Theta^P)$ is given by integrating $Prob(\text{i chooses cb} | \mathcal{I}_P^E, \Theta^P, \beta_i^P)$ over β^P.

of observationally equivalent borrowers that choose contracts that are similar in all but one dimension — for instance, interest rates.[28]

In the following paragraphs, we focus on how banks' rejection of loan applications can challenge the identification of ρ^d.

Overview of the methodology: Throughout the example in this section, we consider that the effect of the origination date on default (δ) is known as it can be recovered from periods in which menus did not change. Without loss of generality, we set it to zero in this methodology overview.

The ρ^d parameters are identified by comparing groups of ex ante observationally equivalent borrowers that choose the same product at the same price but at a time when incentives to choose the contract are different. We fix contract terms and price controls for the impact of moral hazard or burden of payment on default (captured by $\beta^d(X^o_{cb}, r^{t_i}_{cb})'$). The variation in incentives to choose the contract affects the level of adverse selection in each group (captured by the average preference parameter $\hat{\beta}_{icb}$).

Figure 1.2 provides a visual representation of the identification strategy. We consider a simple case in which only two products are offered in the mortgage market and all products are offered by the same bank. We drop the bank index in the notation and index the contract by $c \in \{1, 2\}$. Contracts are identical in all but one dimension: contract 1 has a higher maximum LTV. We have two groups of observationally equivalent borrowers (i.e., D^o_i is constant across i). Each borrower group makes its contract decision in a different period ($t_i = 1$ for the first group and $t_i = 2$ for the second). We assume that the price of contract 1 varies with the origination date t_i but the price of contract 2 does not.

Based on the origination date and the contract chosen, borrowers are categorized into four subgroups. We index each subgroup by $g \in \{1, 2, 3, 4\}$. We observe the average default of each subgroup (d_g), and we can estimate the average preference ($\hat{\beta}_g$) of each subgroup using the method presented in section 1.5.2.[29] For simplicity of the exposition, we consider the case in which the distribution of the borrower's unobserved preference (β^P_i) is constant across time periods. In figure (1.2), we represent borrower i's preferences (β^P_i) by the colour of the borrower's avatar.

In our example, ρ^d is identified by comparing borrowers that choose contract 2 in period 1 (group g=2) and period 2 (group g=4). In figure 1.2, we illustrate why the average preference groups 2 and 4 are different. The borrowers in red have a higher

28. Alternatively, given δ, the contract coefficients can be recovered from an increase in the interest rate of all contracts. Variations in the interest rate r_{cb} — while keeping the interest rate spreads constant — keep the incentives to choose a given contract unchanged ($E[\beta^P_i | \mathcal{I}^E_P, i \textit{ choose } c]$ does not vary) but change the burden of payment of the borrower ($\beta^d r_{cb}$).

29. In this example with two products only, we can identify separately for each period the mean and the variance of the random variable $\beta := \frac{\beta_{LTV}}{\alpha}$ where β_{LTV} is the preference over the max LTV of the contract. See, for instance, C.11.1 for a proof using the linearized logit.

willingness to pay (WTP) for LTV compared to the borrowers in green but have a lower WTP relative to the borrowers in black. As a result, in period 1, when the interest spread between products is low, the borrowers in red and black choose the contract with a high LTV (contract 1), but the borrowers in green choose contract 2. However, when the price of contract 1 becomes too high relative to the price of contract 2, the borrowers in red choose contract 2. This switching changes the average preference of borrowers choosing each contract ($\hat{\beta}^P_g$) as the borrowers in red have a higher WTP for LTV relative to borrowers in green but have a lower WTP relative to the borrowers in black. We can thus recover ρ^p as $\hat{\rho}^d = \frac{d_4 - d_2}{\hat{\beta}^P_4 - \hat{\beta}^P_2}$.[30]

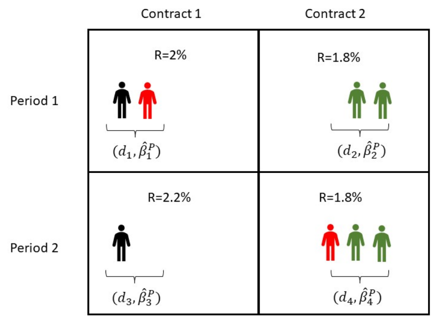


FIGURE 1.2: Identification strategy $d_4 - d_2 = \rho^d(\hat{\beta}^P_4 - \hat{\beta}^P_2)$

Two identification challenges are associated with the default model. The first one (i) comes from the fact that borrowers' valuation of a contract characteristic is itself a function of default. Not considering it in our specification, the identification and estimation of $\hat{\beta}^P_{icb} := \hat{E}[\beta^P_i | \mathcal{I}^E_P, i \text{ } choose \text{ } cb]$ would bias our default estimates as the term $\hat{\beta}^P_{icb}$ would then include information about both moral hazard (i.e., how default probabilities vary with contract terms) and adverse selection (i.e., the private information component of PI_i). The second threat (ii) to identification comes from the possibility that banks, relative to the econometrician, observe a larger set of borrower characteristics and may design acceptance and rejection rules based on those variables. This informational issue can bias the ρ^d estimates if banks tend to change acceptance and rejection rules along with contract terms.

Figure 1.3 gives a visual representation of the threat coming from acceptance and rejection rules. In Figure 1.3 the borrower in red would like to borrow via contract 2 in period 1, but his application is rejected. In period 2, the acceptance threshold changes at the same time as the price of contract 1. The borrower in red is accepted into contract 2, but this is the result of the acceptance threshold rather than the

30. We assume that incentives to choose a given contract vary as a result of a change in the spread between rates but that a product introduction or exclusion would yield the same outcome.

price change. The value $d_4 - d_2 = \rho^d(\hat{\beta}_4^P - \hat{\beta}_2^P)$ thus wrongly attributes the default to the screening behaviour of banks rather than to the rejection policy. This is shown formally in equation (1.25) in the identification challenges paragraphs.

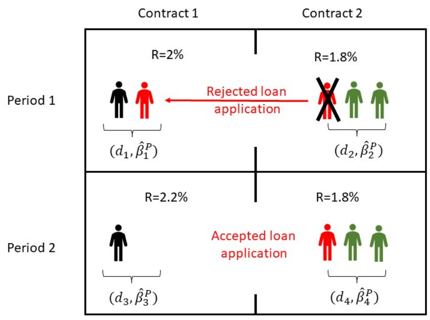


FIGURE 1.3: Acceptance and rejection identification treat

We deal with potential issues (i) and (ii) the following way. As concern challenge (i), we use a parametrization of the borrower's valuation of contract characteristics in section 1.5.2 so that $\hat{\beta}_{icb}^P$ do not contain the part of the valuation that depends on the moral hazard or burden of payment channels. We deal with challenge (ii) by controlling for observable borrower characteristics, using product and lender fixed effects and an instrumental variable (IV) approach for $\hat{\beta}_{icb}^P$. We instrument $\hat{\beta}_{icb}^P$ using variations in product- and lender-specific capital requirements. Capital requirements vary at the lender and LTV level. They have been used in many papers such as Aiyar et al. (2014), Benetton (2018) and Robles-Garcia (2019) as an instrument for interest rates. Capital requirement decisions can be seen as orthogonal to credit risks because they are predetermined and are often based in the UK on procedural risk such as IT systems and organizational structures (see Aiyar et al. (2014) and Bridges et al. (2014) for evidence). As we want to instrument for exogenous changes in the interest rate spread between contracts, we build a measure of the spread between capital requirements.

Identification challenges: Let us now formally discuss the identification challenges. As in the case of product choice, one might worry that the unobserved product characteristics are being correlated with interest rates. We thus use the same cost shifter to identify the rate coefficient.

The coefficients (ν^d) do not need to have a causal interpretation as we are interested in how lenders can use them as a proxy for default rather than the causal effect of those variables.

The coefficient (ρ^d) associated with the unobserved willingness to pay $(\hat{\beta}_i^P)$ must, however, only be related to screening. As discussed in the above paragraph, two potential identification challenges are associated with ρ^d. The first is related to (i) disentangling moral hazard from adverse selection. The second is related to (ii) the rejection of loan applications based on unobserved (to the econometrician) borrower characteristics.

Concerning point (i), because of the specification of preferences β_i^P and the use of bank-product fixed effects (ξ_{cb}), β_i^P is uncorrelated with observable and unobservable contract characteristics (X_{cb}^o, X_{cb}^u). As a result, the coefficient $\hat{\beta}_{icb}^P$ contains no information about moral hazard or burden of payment.

Concerning point (ii), the issue comes from the fact that our measure of private information contains information about borrower characteristics that are observed by lenders but not by the econometrician ($\hat{E}[PI_i + \nu^P D_i^u | \mathcal{I}_P^E, i\ choose\ cb]$). This is an issue to the extent that the distribution of $\hat{E}[\nu^P D_i^u | \mathcal{I}_P^E, i\ choose\ cb]$ can be controlled both with the menu design and by acceptance and rejection rules. To formalize the discussion, let us introduce the cutoff for the rejection rule used by bank b for contract c. We denote it ($\bar{D}_{cb}^u$) and assume that borrower characteristic D_i^u must be above $\bar{D}_{cb}^u$ for the borrower to be accepted into a given contract. Our private information measure can thus be written as

$$\hat{E}_i[\beta_i^P | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u] = \hat{E}_i[PI_i | \mathcal{I}_P^E, i\ choose\ cb] + \nu^P \hat{E}_i[D_i^u | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u] \tag{1.24}$$

$$\approx \underbrace{P\hat{I}_{icb} + \nu^P \hat{D}^u(M)}_{Vary\ with\ menus\ only} + \underbrace{\nu^P \hat{D}^u(\bar{D}_{cb}^u)}_{Vary\ with\ rejection\ rules\ only} \tag{1.25}$$

where $\hat{D}^u(M) + \hat{D}^u(\bar{D}_{cb}^u)$ is a linear approximation of the function $D(M, \bar{D}_{cb}^u) := \hat{E}_i[D_i^u | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u]$ around an arbitrary cutoff rule and menus.

Equation (1.25) illustrates that the use of $\hat{E}_i[\beta_i^P | \mathcal{I}_P^E, i\ choose\ cb]$ in regression (1.23) creates an endogeneity issue when banks change their screening behavior using contracts (for instance, by changing the spread RS) together with their acceptance and rejection rule ($\bar{D}_1^u$).[31]

To limit this concern, we use bank fixed effects, control for the mortgage origination date, and use a new instrument for our measure $\hat{E}_i[\beta_i^P | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u]$. We instrument $\hat{E}_i[\beta_i^P | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u]$ using changes in the spread between capital requirements. Such changes affect the spread between interest rates and, thus, as

31. Changes in acceptance and rejection rules only are irrelevant as $\hat{E}_i[\beta_i^P | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u]$ varies with banks' pricing schedules. This point is not reflected by the linear approximation of $\hat{E}_i[D_i^u | \mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u]$.

illustrated in figure 1.2, the type of borrower choosing a given contract. As capital requirements vary across lenders and mortgages with different maximum LTVs, our empirical strategy thus controls for differences across acceptance and rejection rules that are common among products (lender shocks) and differences across products that are common across lenders (market shocks).[32] As shown in Benetton (2018), capital requirements levels are exogenous and correlated to rates. Consequently, the capital requirements spread is thus exogenous as well and correlated with $\hat{E}_i[\beta_i^P|\mathcal{I}_P^E, i\ choose\ cb, \bar{D}_{cb}^u]$.

This is a variation in the incentives to choose a contract that is plausibly uncorrelated with the bank's acceptance and rejection rule. Formally, denoting Z as the instrument, and using the linear approximation in equation (1.25), the identification assumption is thus

$$E[\underbrace{d_{icb} - \delta t_i - \beta^d (X_{cb}^o, r_{cb}^{t_i})' - \nu^d D_i^o - P\hat{I}_{icb} + \nu^P \hat{D}^u(M)}_{e_{icb}^d} |\mathcal{I}_P^E, i\ choose\ cb, Z] = 0 \tag{1.26}$$

Estimation procedure: Given a consistent estimate for Θ^P — taken from the product demand estimation — we construct a consistent estimate for $E[\beta_i^P|\mathcal{I}_P^E, i\ choose\ cb])$ using the procedure defined in section 1.5.2.

We then use an instrumental variable approach to estimate the loan demand coefficients Θ^d, based on the moment condition (1.26).

As in section 1.5.2, the joint estimation of the demand and default parameters is computationally demanding as it would require iterating on the estimate $E[\beta_i^P|\mathcal{I}_P^E, i\ choose\ cb]$ for each θ^P. For this reason, we estimate the product and loan demand parameters separately and calculate the standard errors using a bootstrap method.

1.5.4 Step 3: Supply

In this section, we describe the identification and the estimation approach for the supply parameters (Θ^S): the marginal costs of lending and the fixed cost of designing a new product.

Conditional on the demand and default parameters being identified and estimated, the supply parameters are identified and estimated using the cross-sectional variation.

32. Alternatively, the IV approach could exploit the timing of a bank-specific internal rate-based approval as an exogenous variation in the interest rate spread between products. However, the internal rate-based model mostly happens around 2010, period in which the PSD data feature less information about contract characteristics.

Parametric assumption NPV: NPV_{icb} is the net present value of lending to borrower i via contract c at bank b. The derivation of the formula is in Appendix (D.3). As in Benetton (2018) and Crawford, Pavanini, and Schivardi (2018), we consider that banks are risk neutral and that all borrowers refinance at the end of the teaser rate period, and we approximate the NPV by:

$$NPV_{icb} := L_i(c, b) \cdot [(1 - d_{icb})r_{cb} - mc_{cb}]F_{cb} \tag{1.27}$$

where $L_i(c, b)$ is borrower i's loan demand conditional on choosing contract c at bank b (defined in equation C.3), d is the default probability (defined in equation C.7), r is the interest rate, F is the fixed rate period and mc_{cb} is the marginal cost of lending.

For convenience, we denote bank b's gross margin when menu M is offered in the market as:

$$\hat{\Pi}_b(M) := \sum_{i=1}^{n} \sum_{c \in P_{ib}} \hat{E}_\Theta[\overbrace{\Pr(\text{i chooses cb} \,|M, (D_i)_i, \Theta, \beta_i^P) L_{icb}}^{\phi_{ic}}[(1 - d_{icb})r_{cb} - mc_{cb}]|M, (D_i)_i] \tag{1.28}$$

To simplify the notation, we drop the borrower-specific menus (P_{ib}) in the gross margin function notation.[33] The gross margin is composed of the probability of borrower i choosing contract c (denoted $\Pr(\text{i chooses cb} \,|M, (D_i)_i, \Theta, \beta_i^P)$), multiplied by the present value of lending to that borrower (denoted $L_{icb} \cdot [(1 - d_{icb})r_{cb} - mc_{cb}]$).

Model-implied marginal costs: Given the demand and default parameters and the observed bank contract menus, we recover the model-implied marginal costs for each bank b and contract c ($(\hat{mc}_{cb})_{cb}$) by solving the system of equations derived from banks' first-order conditions with respect to prices. This approach is traditional in the IO literature (see, for instance, Berry, Levinsohn, and Pakes (1995)). It yields the following expression:

$$\hat{MC}_b := (\hat{mc}_{1b}, ..., \hat{mc}_{C_b b})' = B_b^{-1} A_b, \ \forall b \tag{1.29}$$

where A_b is a column vector composed of elements $(a_c^b)_c$, $a_c^b := \sum_{i=1}^n E_{\Theta^D, \Theta^d}[\sum_{x=1}^C \partial_{r_{cb}}(\phi_{ixb})(1 - d_{ixb})r_{xb} + \phi_{i1b}\partial_{r_{cb}}((1 - d_{icb})r_{cb})|\mathcal{I}_P^E]$ is the impact of a marginal increase in the interest rate of contract c on bank b revenues, $\phi_{icb} := Prob(\text{i chooses cb} \,|\mathcal{I}_P^E, \Theta^D, \Theta^d, \beta_i^P) \cdot L_{icb}$ is the probability of borrower i choosing contract c at bank b multiplied by the loan demand of borrower i if they choose contract c at bank b (defined in equations (1.14) and (1.15)), and d_{icb} is the default probability of borrower i if they were to choose

33. A more precise notation would replace M by $(M, (P_{ib})_i)$ in the gross margin definition.

contract c at bank b. It is defined in equation (1.23). Given the demand and default parameters and the observed bank menus, a_c^b is known.

B^b is a matrix of the size of bank b's menu. It is composed of the elements $B_{xy}^b := \sum_{i=1}^n E_{\Theta^D,\Theta^d}[\partial_{r_{xb}}(\phi_{iyb})|\mathcal{I}^E]$. The scalar $(B_{c1}^b, ..., B_{cC}^b) \cdot \hat{MC}_b$ is the impact of changes in the rate of contract c on bank b costs. Given the demand and default parameters and the observed bank menus, B_{xy}^b is known.

Equation (1.29), can thus be interpreted in the following way. Given the estimated level of demand and default elasticities, the banking model implies that — given competitors' menus — lender b should apply a certain markup level for each contract c. The model-implied optimal markup is a function of estimated or observable objects. It allows us to recover the marginal costs by scaling down the observed contract c interest rate.

Fixed cost equation: As discussed in section 1.4.2, the following equations are derived from the model-implied best-response function of bank b. Using hat superscripts to denote the mathematical objects that are known given a value of the demand, the default parameters (Θ^D, Θ^d) and the marginal costs $(\hat{mc})$, we have

$$\Pr(M_b|M_{-b},\Theta) = \Pr(M_b \in \text{argmax}_{m\in\mathcal{F},P_{ib}}\{\overbrace{\hat{\Pi}_b((m,M_{-b})) - \frac{F_b(m)}{\beta^F} + \beta^F e_m^F}^{Profits}\}|M_{-b},\Theta) \tag{1.30}$$

$$= \frac{\exp(\beta^F \hat{\Pi}_b(M) - F_b(M_b))}{\sum_{m\in\mathcal{F}} \exp(\beta^F \hat{\Pi}_b((m,M_{-b})) - F_b(m))}, \text{ when } e_m^F \text{ iid and EV distributed} \tag{1.31}$$

We use the notation M_{-b}, to refer to the menus of contracts offered by banks other than bank b (i.e., $M_{-b} := (M_x)_{x\in B\setminus\{b\}}$) where

$F_b(M_b)$ is the cost of designing menu M_b. The fixed costs are needed to rationalize the fact that banks do not offer a continuum of products despite the large heterogeneity in preferences. Formally, we consider that only changes in product characteristics are costly, so $F_b = \sum_{c\in P_{bt}} \theta' X_{cb}[\underbrace{\mathbf{1}_{c\in P_{bt}, c\notin P_{bt-1}}}_{Inclusion} + \underbrace{\lambda \mathbf{1}_{c\in P_{bt-1}, c\notin P_{bt}}}_{Exclusion}]$, where $\theta' X$ is the cost of introducing a new contract with characteristics X, λ is a scaling parameter that captures the cost or benefits of withdrawing a contract from the menu, and $\mathcal{F}$ is the set of potential menus a bank can offer.

Equation (1.31) has a logit form. However, the denominator contains simulated dependent variables (i.e., the gross margins $\hat{\Pi}(m, M_{-b})$ for all possible bank b M menus) rather than observed ones.

The product introduction game: Since $\hat{\Pi}(m, M_{-b})$ are simulated, we need to take a stance on whether (i) banks are playing a two-stage game in which they first select product design and then choose rates to clear markets or (ii) interest rates equilibrate simultaneously with other products' characteristics.

In the context of our model, this translates into two considerations: (i) the interest rates of other banks' menus (M_{-b}) react to bank b's menu offering (m) when we calculate the function ($\hat{\Pi}(m, M_{-b})$), or (ii) the interest rates of other contracts do not react with m.

Using a two-stage game timing may be more compelling as it captures the fact that banks' interest rates change more frequently than products' characteristics.

The two-stage game timing is as follows. In the first phase, banks choose the number of products as well as their characteristics (LTV, fixed rate period, fees). We consider that banks also fix their acceptance and rejection rules in that stage. Banks pay fixed costs when introducing or withdrawing a product, not when changing acceptance and rejection rules.

In a second step, banks compete on rates given their product offering. Given the logit form assumption, the equilibrium prices can be calculated using a fixed point approach similar to (Morrow and Skerlos (2011)). Given our model assumptions, there are unique equilibrium profits as a function of a product being offered.

The timing assumption does not affect the estimation of the marginal costs. Contrary to the marginal costs estimation, the timing assumption will matter for the fixed cost as it affects the NPV of product introduction or withdrawal. As the fixed cost is not at the center of our analysis, this is not an issue. Furthermore, our results on the product and interest rate distortions are robust to the timing assumption.

Identification: The fixed costs parameters (θ, λ) are identified by the model optimality condition. Given that the gross margin function is increasing and concave in the number of products, the fixed cost is identified by the fact that, given the menus offered in the data, any additional revenue created by the product introduction is lower than the fixed cost of introducing the said product. Using this condition for all banks, we can point-identify the fixed cost parameters using a standard logit model argument. Similarly, the fact that any product withdrawal is not optimal allows us to identify (λ).

Identification challenges: As in the demand estimation, two potential identification issues arise with the product choice equation. The first one is the omitted variable bias. This can happen if, for instance, high LTV products are often associated with higher marketing expenses. This would tend to upwardly bias the cost of high LTV products. The use of the residual from the loan demand regression to control for unobserved product characteristics can be use to mitigate this in the spirit of the control function approach. The second type of issue is the consideration set bias. This bias would occur if, for instance, a highly profitable product is not being offered because of regulations that constrain banks' product offerings or that are wrongly not considered by the banks. To mitigate that issue, we do the estimation only at product introduction and product exclusion periods and calculate counterfactual profits in the equation using the menu from the previous period. As a robustness check, we also do the estimation considering as a set of potential products the combinations of the most common values for the characteristics of the existing products in the market.

As discussed in section C.1.3, we consider a static problem and thus use the gross margin Π in lieu of a more general value function $V_{bt}(M')$. We show in Appendix C.13.3 how the parameters could be estimated using a dynamic approach.

1.6 Estimation Results

This section presents the estimation results for the demand, default and supply parameters. Then, we look at how the coefficient heterogeneity shapes the equilibrium contract terms and prices using the model. In particular, we provide a measure of the price and product distortions (section 1.7.1).

1.6.1 Demand results

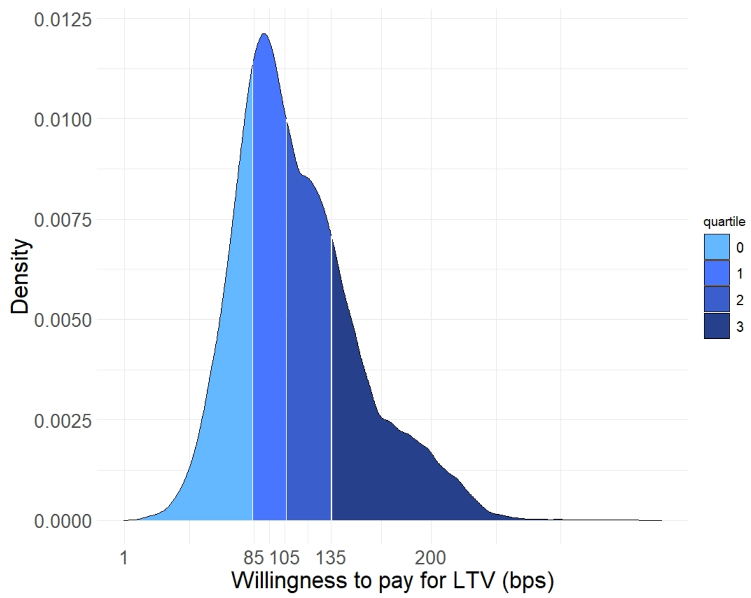


FIGURE 1.4: Distribution of WTP for LTV for the full population

Discrete choice: The average point estimate of the coefficient on interest rates across all income and region groups is significant and equal to -1.9. This implies

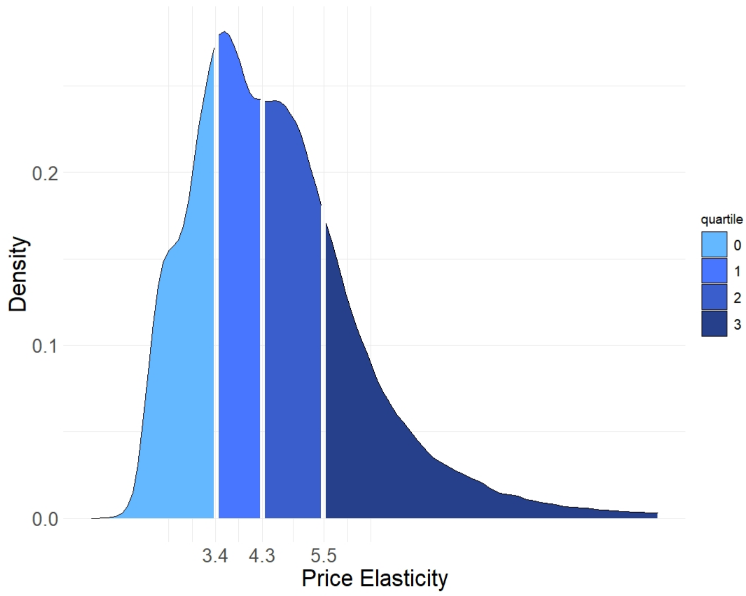


FIGURE 1.5: Distribution of price elasticity for the discrete choice regression for the full population

that borrowers dislike more expensive mortgages. There is substantial heterogeneity, mainly based on income (see Table B.5).[34] Indeed, people shopping for 70%-85 % LTV loans on the first, second and third quartile on the income distribution have an interest rate coefficient of, respectively, -2.3, -1.9 and -1.5 on average (see Table B.6). This result implies that borrowers with higher income are more sensitive to rates. It can be rationalized by, for instance, search costs as in Agarwal et al. (2020). Borrowers with higher income are more likely to be accepted into any loan contract and thus have more incentives to search intensively. The correlation between income and price elasticity can also be related to the fact that income could be a proxy for other variables such as financial sophistication. Alternatively, this correlation can be rationalized by the direct effect of default probabilities: borrowers who are more likely to default are also less likely to repay the full face value of the debt and thus end up being less price elastic. As shown in the motivating evidence and in the next section, default is indeed correlated with income.

The corresponding average own-product demand elasticity is equal to 2.6, 3.6 and 5.1 for the average borrower — borrowers in the first, second and third quartile of the income distribution of 70-85% LTV shoppers (see Table B.7). Those results imply that, on average, a 1% increase in the interest rate decreases the market share of the mortgage by 3.6% for 70-85% LTV shoppers. Looking at the market share of low-income borrowers only (the first quartile of the distribution), we see that a 1% increase in the interest rate decreases the market share by 2.6%. Figure **??** represents

34. The other source of heterogeneity coming from the observable heterogeneity and the random coefficients term are non-significant (statistically and economically).

the distribution of WTP for LTV for the whole borrower population.

These results imply that average borrowers like high LTV loans. The average coefficient is 0.17. Contrary to the interest rate case, the heterogeneity comes from the random coefficient term. This coefficient is significant at the 0.1 level[35]. Indeed, the first quartile of the distribution is 0.13 while the third quartile is 0.21. However, when considering only the observable heterogeneity, we find that the lower quartile of the distribution has an average of 0.16 and the third quartile's average is 0.18. One interpretation for the positive coefficient results is that borrowers do not like to make down payments as they may be credit constrained. Combining the two coefficients' estimates, we find that 70-85% LTV shoppers in the first, second and third quantile are, respectively, willing to pay $(\frac{\beta}{\alpha})$ up to 7, 10 and 14 bps for a 1 percent LTV increase. Figure 1.5 represents the distribution of WTP for LTV for the whole population.

We also find substantial heterogeneity for the teaser rate parameter. The heterogeneity comes from the random coefficient term rather than income and is significant. This is the only parameter for which there is a sign change. Fixing rates for a longer period provides a hedge against interest rate increases when borrowers refinance their loan. The interest rate risk, and thus the benefit of fixing rates, can be a result of future changes in borrowers' credit risk or variation in lenders' cost of lending. Consequently, the teaser rate coefficients can be rationalized by borrowers having different degrees of risk aversion or expectations about the future economic path. This implies that some borrowers prefer a fixed rate while others prefer a flexible rate. Borrowers in the first, second and third quantile have a coefficient of -0.4, 0.1 and 0.9. Those coefficients imply a willingness to pay of -30, 8 and 50 bps for a one-year increase in the teaser rate.

The average borrower dislikes fees. There is no observable and unobservable heterogeneity for that coefficient given the other coefficient heterogeneity. Borrowers have an average coefficient of $-7 \cdot 10^{-4}$. Those coefficients imply a willingness to pay of 32, 43 and 60 bps for a 1,000-pound decrease in fees.

Loan demand: The loan coefficients are all significant and reported in Table (B.8). The use of a model allowing for a correlation between the product choice and loan borrowed parameters allow us to correct the selection bias mentioned in the identification section. Comparing the models with and without the correlation term, we find that the LTV and the fixed rate parameters are the most affected. We find that high LTV increases the amount borrowed by 7.6 percent in the non-correlated case and by 15 percent in the correlated model. For the teaser rate, we find that increasing the teaser rate by 1 year decreases the amount borrowed by 0.1

35. The income interaction term is not significant and has almost no impact on the parameter

percent under the non-correlated model and by 0.8 percent in the correlated model. We further document that borrowers with a high unobserved preference for LTV or a fixed rate also have a higher propensity to borrow. Indeed, borrowers with an unobserved preference for a fixed rate that is one standard deviation higher borrow, on average, 20 percent more. Borrowers with a unobserved preference for an LTV that is one standard deviation higher borrow, on average, 1.3 percent more. If those borrowers are also profitable, this creates incentives for banks to create a menu to extract more surplus from them.

1.6.2 Default results

For a given level of income and other observable characteristics, borrowers that have an unobserved propensity to choose high LTV products (high $\hat{e}_{LTV}$) that are one standard deviation above the average of the $\hat{e}_{LTV}$ distribution also have a baseline default probability that is twice as low relative to the average borrower (assuming the average is 1.2%). As high LTV loans are more expensive, this effect goes in the other direction relative to the income effect. Indeed, low-income borrowers are more likely to default and are also more likely to choose a high LTV loan. The positive selection along the $\hat{e}_{LTV}$ dimension can be the result of borrowers that are less likely to default are more likely to stay in the house they bought and are thus more willing to take a larger loan for a fixed level of down payment.

As mentioned in the demand section, longer teaser rates hedge borrowers against changes in interest rates. Variation in future rates can be a result of, for instance, general economic conditions or borrower-specific credit risk changes. Borrowers preferring higher teaser rates are thus likely to be more risk averse or see their credit score decrease (and thus their refinancing rate goes up). Those two channels imply opposite predictions regarding adverse or advantageous selection. Indeed, theoretically, borrowers who are highly risk averse are less likely to default. In contrast, private information about a credit risk interpretation will likely lead to adverse selection along the teaser rate dimension. Indeed, borrowers with private information about their credit risk being likely to go up over time are more likely to fix their contract terms. Those borrowers are also more likely to default.

Our estimates imply mild positive selection along the teaser rate dimension. Indeed, borrowers who are one standard deviation above the mean are 2 percent less likely to default. The results suggest that the risk aversion channel dominates. This interpretation is also consistent with the loan regression results showing that those customers tend to borrow more. Indeed, those borrowers are less likely to lose their house and thus benefit more from each extra unit of house bought. However, the fact that the teaser rate coefficients are low may be a result of both channels being present.

1.6.3 Marginal costs and fixed cost results

Marginal costs: We find that the average marginal cost is 220 bps. Scaled up by a default probability between 0 and 5 percent, this implies an average fair price of between 220 and 231 bps. The marginal costs are increasing in LTV in a convex fashion. While the average marginal cost increases by 10 bps between 70 and 80% LTV loans, it increases by 110 bps between 90 and 95% LTV loans. Longer teaser rate products are more expensive to produce. One year longer costs 4 bps at a low level but 14 bps per year above the fifth one. Finally, higher fee products are associated with lower marginal costs. A 500 fee increase is associated with a marginal costs decrease of 10 bps starting from a zero fee product. This decrease is even bigger for higher fee products.

Fixed costs: we find that the average fixed costs of introducing a new product are about (£ 16 M) per product or 2% of current profits. Around 30% of the fixed cost is recovered after the withdrawal of an existing product. Those numbers are comparable to Wollmann (2018), who analyses the car industry. The estimates are the ones implied by the model to justify that banks offer a discrete number of products. The sunk cost includes monetary costs such as marketing expenses, updates of the menu on all lending platforms, and changes in risk weights calculations. They also include non-monetary costs such as within-firm managing frictions. Their large magnitude suggests that analysing their drivers is an essential force of the lending market and should thus be included in theoretical models or analysed empirically in future work.

1.7 Counterfactual Analysis

In section 1.7.1, we use simulations to provide a measure of product distortions relative to the perfect information benchmark. We show that contract characteristics are distorted compared to the first best. We provide a decomposition of the interest rate into three components: a perfect information perfect competition price, a perfect information markup, and an asymmetric information discount or premium. Those components are functions of the model parameters and the data and do not require simulations.

In section 1.7.2, we calculate the cost of the screening externality (see Appendix C.9 for intuition or Taburet (2022) for an in-depth theoretical analysis).

In section 1.7.3, we simulate the impact of a ban on high LTV contracts.

1.7.1 Product and interest rate distortions

Graphical intuition from Rothschild and Stiglitz (1976)

Let us start by providing intuition on how borrowers' private information about default probabilities and preferences can distort product characteristics and interest rates.

To simplify the analysis and make our model similar to Rothschild and Stiglitz (1976) (see Appendix C.11 for a formal description of the assumptions), we consider a perfectly competitive world with two borrower types in which lenders are all identical.

In Figure 1.6, we plot on the LTV-interest rate plane the perfect information contracts (c_1, c_2), borrowers' indifference curves, and the break-even rates. We focus on the case in which the perfect information contracts are not incentive compatible: the high-default borrower would prefer the low-rate contract (c_2) designed for the low-default borrower. As in our empirical application, the break-even rate (i.e., the cost of lending) is increasing in LTV. This can be rationalized by the cost given default being an increasing function of leverage.

In Figure 1.7, we illustrate how lenders can lower the LTV of the low WTP borrower to maintain borrowers' incentives to self-select.

In Figure 1.8, we illustrate how lenders can also cross-subsidize borrowers to maintain incentives to self-select.

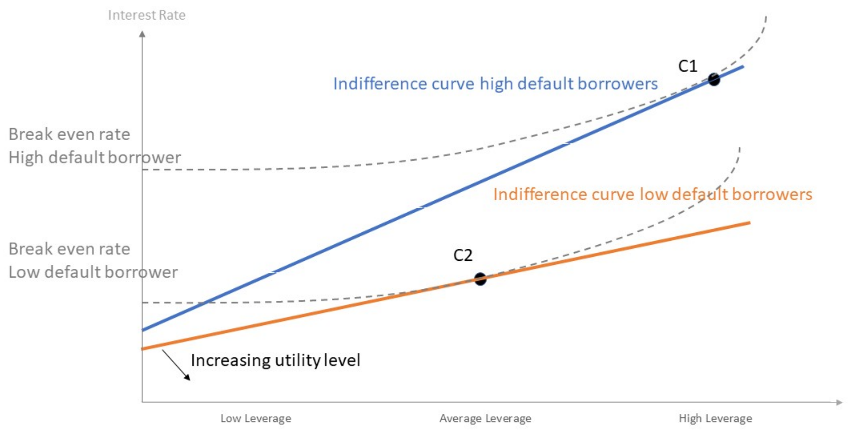


FIGURE 1.6: The perfect information, perfect competition contracts (c_1, c_2) are not incentive compatible. The high default borrower prefers c_2 to c_1.

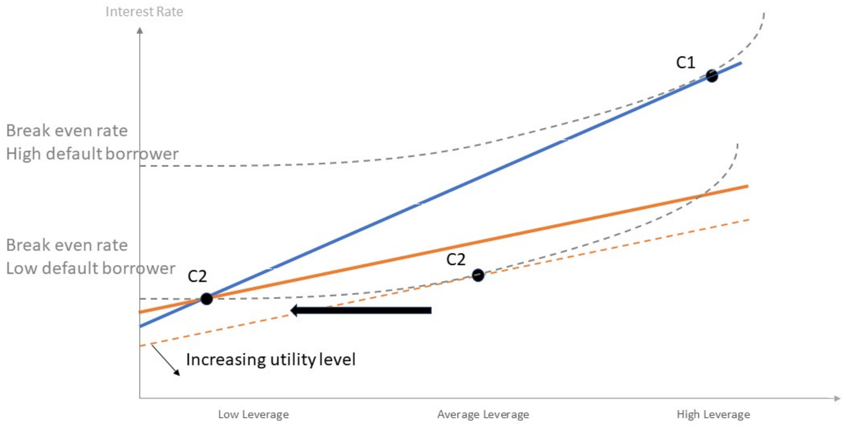


FIGURE 1.7: The perfect information, perfect competition contracts are not incentive compatible. Solution (i): Leverage distortions.

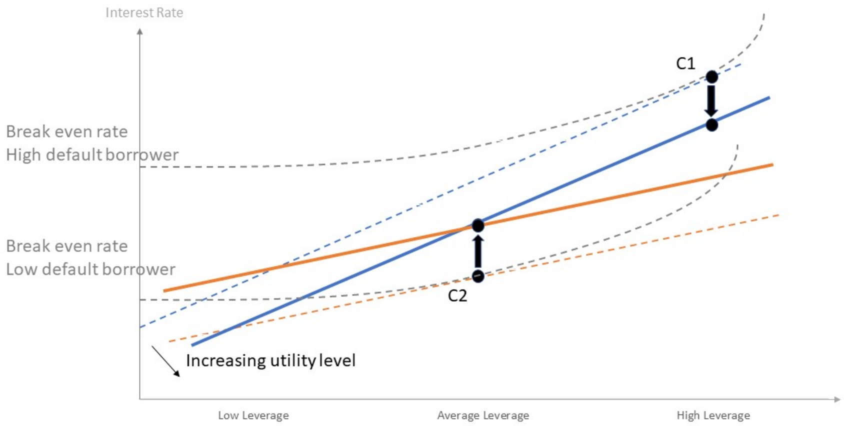


FIGURE 1.8: The perfect information, perfect competition contracts are not incentive compatible. Solution (ii): Interest rate distortions.

Product distortions: conceptual framework

In our model, product characteristics are distorted as a result of two frictions: market power and imperfect information about borrowers' preferences or default probabilities. Solving for the counterfactuals in which the degree of competition or the level of information arbitrarily changes is, however, too computationally demanding (see Einav, Finkelstein, and Mahoney (2021) for a survey). Consequently, we assess the amount of product distortions by comparing the contract in the data to the perfect information benchmark. See Appendix C.14 for a formal analysis of the benchmark.

The perfect information benchmark is based on our structural model with the assumption that lenders can observe borrowers' preferences and demand shocks. We ignore the fixed costs of designing a product (F) and allow contract characteristics to be continuous instead of discrete. Neglecting the fixed cost and allowing for a continuum of products makes the problem tractable. Abstracting from the fixed cost is not an issue for our exercise as it does not change the underlying economic mechanisms under perfect information: for each borrower, lenders design the contract that maximizes the surplus generated by the trade and then use the interest rate to split the surplus between lenders and borrowers. How the surplus is split is driven by the constraint that lender b needs to offer borrower i a contract that provides them at least a certain utility level (denoted $\bar{u}_i$) for hem to accept the contract. The utility level can be set arbitrarily or estimated in the data and captures the degree of competition. For instance, the situation in which the promised utility level is such that the bank breaks even on borrowers represents the perfect competition case.

Under perfect competition, the model implies that it is optimal to increase the contract LTV when the increase in borrower i utility generated by a higher LTV ($\frac{\beta_i^{LTV}}{\alpha_i}$) is greater than the lender marginal cost of increasing the contract LTV ($\frac{mc}{1-d_{ic}}$). The cost is the marginal cost scalled up by the survival probability $(1-d_i)$. Formally:

$$\underbrace{\frac{\beta_i^{LTV}}{\alpha_i}}_{Willingness\ to\ pay} > \underbrace{\partial_{LTV}(\frac{mc}{1-d_{ic}})}_{cost\ of\ increasing\ LTV} \tag{1.32}$$

Product distortions: results

Our results imply that maintaining borrowers' incentives to self-select requires distorting contract terms away from their perfect information value. Because high default-low price elastic borrowers have a high willingness to pay for LTV, low default-high price elastic borrowers get a lower LTV, and thus a lower house size, under imperfect information.

Using equation (1.32), we find that more than 90 percent of borrowers shopping between 70 and 95% LTV would get a 85-95% LTV product under perfect information-perfect competition (see figure 1.9 or table B.12). This finding suggests that products below 85% LTV are introduced to screen rather than to cater to borrowers' heterogeneous preferences. We exclude borrowers shopping below 65% LTV as they constitute less than 10 per cent of the loans originated, and the data quality is lower for that sub-sample.[36]

Our results are robust to the use of models with observable heterogeneity and observable heterogeneity and estimating the coefficient separately for each sufficient

36. including them would imply that LTV between 50 and 75 would be introduced but would account for less than 5 percent of the market shares.

set.[37] As discussed in the Appendix (1.32), the amount of product distortion relative to the perfect information situation is accentuated when moving away from perfect competition. Finally, the result is robust to changing the fact that a higher LTV decreases default. Indeed, one may be worried that this sign results from banks selecting good borrowers into high LTV loans based on soft information not observable by the econometrician. However, the LTV coefficient of the default regression would need to be positive and one hundred times larger in absolute value to imply that 10% of borrowers get offered lower than 90% LTV products. Given the standard errors of 2.810^{-6} and the average coefficient of $-3.9 \cdot 10^{-5}$ on the LTV coefficient, this situation is not likely.

As summarized in table B.12 in the appendix, we find that the product distortions when it comes to fees and teaser rates are milder. Indeed, the model implies that more products should be offered. In particular, higher fee products (more than £1500), and longer teaser rate periods (longer than 7 years). The share of the population that would like to get them is low (below 20 percent of the 80+ LTV borrowers). In addition, this result highly depends on how the marginal costs of lending vary with fees and teaser period. As the marginal costs are estimated for products with fees ranging from 0 to 1500 and teaser rate from 0 to 7, the product introduction results are highly dependent on our extrapolation of the marginal cost function. We find that the distribution of borrowers would shift towards lower-fee products and more flexible rate contracts. This is the result of interest rate distortions. Those distortions are analyzed in the next section.

Interest rate distortions: conceptual framework

Using the first order condition of the structural model (E.1) with respect to interest rates, we decompose the interest rates into a fair price, a perfect information imperfect competition "mark up"[38] and an asymmetric information discount or premium. The asymmetric information premium or discount refers to the increase or decrease in interest rates relative to the perfect information benchmark. Banks use it in order to maintain borrowers' incentives to self-select. For instance, if banks know that high LTV products are chosen by borrowers that are, on average less price elastic, they could potentially set a higher markup for these products. However, how high this markup can be is limited by how high the markups on other products that are close substitutes are (for instance, lower LTV products that are designed for highly

37. As the unobservable heterogeneity uses a normal random variable, there is always a mass of borrowers with a very low WTP for any characteristics. However, the borrowers that will choose lower than 90% LTV in the heterogeneity case account for less than 5 per cent of the population

38. The theoretical literature usually refers to the markup as the output price divided by the marginal cost. We instead define the markup as the pricing above the marginal costs. The empirical IO literature sometimes uses the same terminology (Crawford, Pavanini, and Schivardi (2018)).

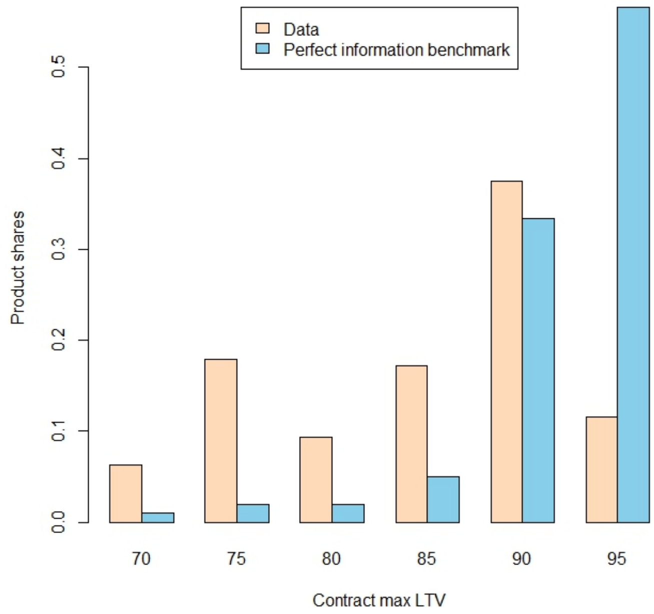


FIGURE 1.9: Product shares, data vs perfect information- perfect competition benchmark

price-elastic borrowers). As illustrated by figure 1.7, borrowers' heterogeneity creates incentives to decrease the price of the high LTV product and increase the one on the high LTV products relative to the perfect information situation.

Formally, the decomposition is:

$$r_c = \{\overbrace{\frac{mc}{1-E[d|bc]}}^{Fair\ price} + \overbrace{\frac{E[\Phi_c]}{E[-\Phi'_c]}(\frac{1-E[d|bc]+\beta^d_r r}{1-E[d|bc]})}^{PI\ mark-up} + \overbrace{\sum_{j\neq c}\frac{E[\Phi'_j]}{E[-\Phi'_c]}\frac{\tilde{\pi}_c}{1-E[d|bc]}}^{AI\ discount/premium}\}\frac{1-E[d|bc]}{1-E[d'|bc]} \tag{1.33}$$

Where $\Phi_c := \sum_i \frac{exp(u_{ic})}{\sum_x exp(u_{ix})} L_{ic}$ is the expected amount lent, $\tilde{\pi}_c := (1-E[d|cb])r_c - mc_c$ is the expected profit on each loan unit given that the borrower choose the contract c at bank b.

The first term $\frac{mc}{1-E[d|bc]}$ is the pricing at which banks break even given the expected default probability of borrowers choosing the contract c at bank b ($E[d|bc] := \beta^d X_c + \alpha^d r_c + \rho\frac{E[\Phi_c \beta_i]}{E[\Phi_c]}$). It is the marginal cost scaled up by the survival probability.

The second term is $\frac{E[\Phi_j]}{E[-\Phi'_j]}(\frac{1-E[d|bc]+\beta^d_r r}{1-E[d|bc]})$ is the pricing set by banks above the fair price if they could observe the average default probability of the type of borrowers choosing each contracts ($E[d|bc]\forall cb$). $\frac{E[\Phi_j]}{E[-\Phi'_j]}$ is the impact of borrowers product elasticity (i.e., competition). $\frac{(\beta^d_r r}{1-E[d|bc]})$ accounts for the burden of payment: when increasing r, borrowers are more likely to default ($\beta^d < 0$), this creates incentives to lower the mark-up.

The last term $\sum_{j\neq c}\frac{E[\Phi'_j]}{E[-\Phi'_c]}\frac{\tilde{\pi}_j}{1-E[d|bc]}$ is the equivalent of the information rent in the textbook principal agent model.

The ratio $\frac{1-E[d|bc]}{1-E[d'|bc]}$ in which $E[d'|bc] := \beta^d X_c + \alpha^d r_c + \rho\frac{E[\Phi'_c\beta_i]}{E[\Phi'_c]}$ is scale up the three terms by taking into account the fact that changes in r impact the type of borrowers choosing a given contract.[39]

Interest rate distortions: results

The results on the interest rate decomposition are summarized in table B.15, table B.14 and figure 1.10. Doing this decomposition, we find that the average fair price is 231 bps, the markup is about 116 bps while the average information rent is -70 bps for high LTV loans (above 80). For loans with LTV between 70 and 80, the average fair price is 202 bps, the markup is about 60 bps while the average information rent is -30. These difference across LTV are mainly due to the fact that lower LTV loans are chosen by borrowers that are more price elastic on average. As a result banks have less able to apply large interest rate or large information rents. The impact of default is mild when explaining the interest rate level. For instance the difference between the effective marginal cost and the marginal cost is on average less than 5 bps (and less than 10 bps when we scale up all default probabilities by 5 to take into account that the estimated default probabilities may underestimate banks true default expectations). However, as mentioned in section C.11 even mild difference is default can lead to big product distortions when the screening device is not very effective.

Looking at the differences in the average information rent between different products, we find that high LTV products (95% LTV) earn low information rents (5 bps) compared to 75% LTV products. This is due to the fact that high LTV products are also more expensive to produce, implying that the information rent need not be large. This result is also consistent with the fact that banks maintain incentives to self-select by distorting the LTV rather than rates. Contrarily, we find that lower fee contracts and longer rate contracts get a substantial information rent. This can be explained by the fact that high fees products are chosen by more price elastic borrowers. Under perfect information those borrowers would thus get a lower markup (see mark up columns in tables B.15 and B.14). To be able to extract more surplus from other borrowers, banks make high fee product relatively more expensive than what they should be. This is consistent with the product distortion and the shift in the low fee products category observed under perfect information: banks increase rates

39. When the number of product in the market is large and the loan rate elasticity is low ($\tilde{\beta}_r$ low), $E[d|bc]$ and $E[d'|bc]$ are relatively close to each other. Indeed, $\Phi' \approx \Phi(\tilde{\beta}_r + 1)\Phi \approx \Phi$.

in low fee products to extract more surplus from the low price elastic borrowers, as a result more price elastic borrowers are pushed to high fees products when they exist. This creates incentives introduce more high fees products relative to the first best in order to implement the screening.

Longer teaser rate products are more expensive to produce. They are chosen by less price elastic borrowers. Under perfect information those borrowers would get a higher markup. Those products also benefits from an information rent.

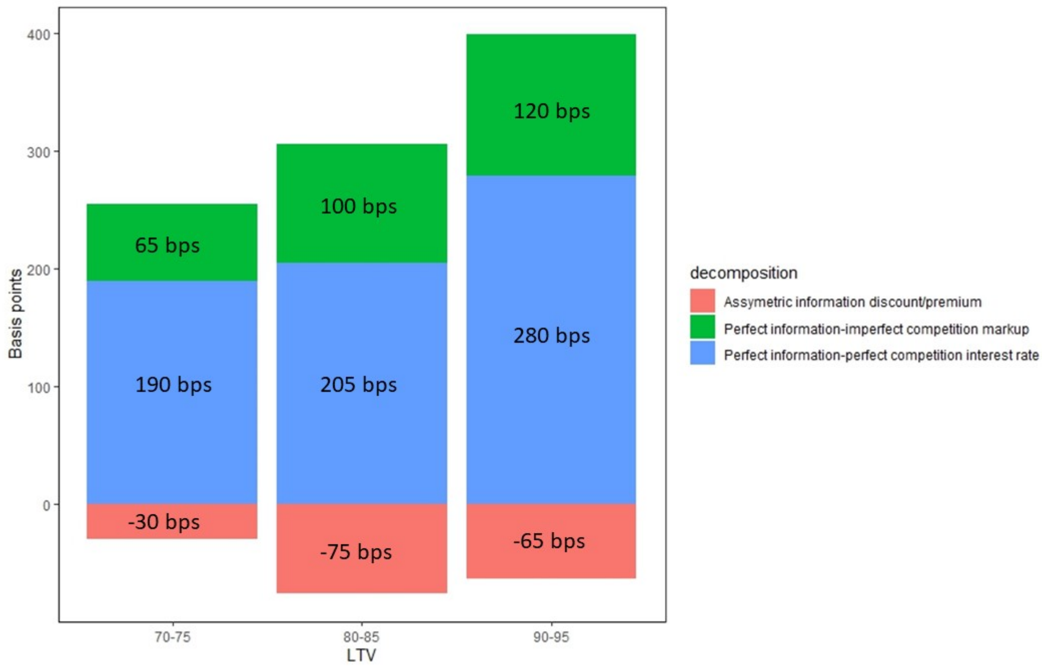


FIGURE 1.10: Interest rate decomposition by LTV

Summary of the results and economic interpretation

Our estimates imply that, in the perfect information case, borrowers in the first and last willingness to pay quartile of the LTV distribution would get contracts with similar LTVs — respectively, 85 and 95 — and get charged different prices because of their heterogeneous price elasticity and default probability. As a result, a menu composed of perfect information contracts cannot be offered under imperfect information as high default-low price elastic borrowers would be tempted to choose the lower rate contracts. This creates incentives to decrease the interest rate on high LTV contracts (i.e., an asymmetric information discount, also called information rent in monopoly models) and increase the interest rate on low LTV contracts (i.e., an asymmetric information premium) relative to the perfect information case. As a complementary incentive, lenders also introduce LTV contracts that are lower than 85. As high default-low price elastic borrowers are more reluctant to provide higher down payments for each loan unit, low LTV contracts attract unobservably safer borrowers and can be offered at a lower price.

Those results imply that welfare is lower relative to the perfect information-perfect competition case. The overall loss in borrowers' utility in the current data is equivalent to the loss in utility following a 100 basis point interest rate increase on all loans.

The perfect information-imperfect competition case is not a natural benchmark to study welfare given that asymmetric information and imperfect competition interact. Removing one friction can thus increase the other. For instance, by removing asymmetric information, lenders are able to set a higher interest rate (70 bps) to high LTV contracts without the fear of borrowers substituting to a lower LTV contract designed to attract safer borrowers.

Reducing the level of asymmetric information, or allowing lenders to price borrowers on all observable characteristics such as ethnicity, gender, disability, or religious beliefs may not be feasible or desirable. As a result, it is also relevant to look at how far the product offered are from the second best (i.e., the menus offered by an informationally constrained social planner). This is the purpose of the following section.

1.7.2 Screening externality

Graphical intuition from Rothschild and Stiglitz (1976)

We use the same stylized model as in the previous section to provide graphical intuition on how we measure the screening externality cost. In Figure (1.11), we start from a set of contracts (c_1, c_2) observed in the data. From our estimation, we are able to back out borrowers' indifference curves and lenders' cost of lending to each borrower. We then check if the lenders can offer menu that would be a Pareto improvement over the existing menus (c_1', c_2'). Figure (1.13) illustrate why the socially optimal menu (c_1', c_2') cannot be offered in equilibrium: a competitor could then offer a menu in the cream-skimming region and make profits by attracting the safer borrowers. The results hold under imperfect competition, the deviation relive on creating a menu that will attract a large proportion of the most profitable borrowers.

Quantitative analysis

We define social welfare as the sum of firms' profits plus the sum of borrowers' utility expressed in monetary term. We measure the cost of the screening externality by comparing the utilitarian social welfare level implied by our structural model to one achievable in a benchmark in which the contractual externality is internalized.

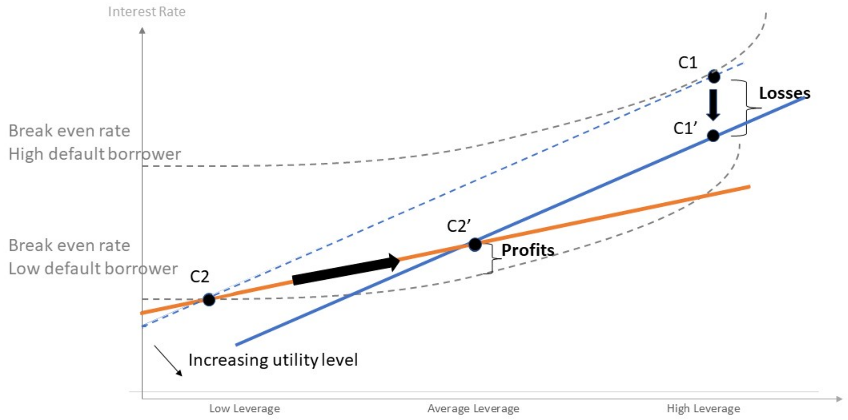


FIGURE 1.11: Cross-subsidization is a Pareto improvement when the number of high default borrower is low.

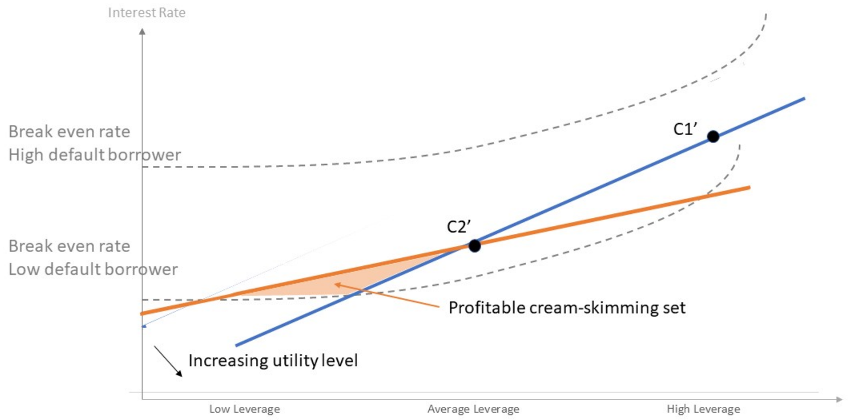


FIGURE 1.12: Cross-subsidization is not possible because a competitor can take advantage of the cross-subsidy to attract the most profitable borrowers (cream-skimming).

We use the following benchmark. We consider the hypothetical case in which each lender becomes a monopolist and borrowers' outside option is their utility in the competitive equilibrium (i.e., the structural model implied utility). The monopoly assumption gets rid of the externality by preventing borrowers from moving from one bank to another. It also allows us to focus exclusively on the screening externality by preventing increase in welfare generated by a better allocation of borrowers to cheaper banks. The outside option assumption is made to focus on Pareto improvements. A the focus on Pareto improvements, our measure can be interpreted as a lower bound on the screening externality cost.

Formally, lender problem is defined as:

$$max_{C_{bt}, M_{bt} \in \mathcal{F}^{C_{bt}}, P_{ibt}} \sum_i n_i \sum_{c=1}^{C} \overbrace{\Pr(\text{i chooses c} \,|\, \text{i chooses b})}^{IC} NPV_{ic} - F(M_{bt}, M_{bt-1}) \tag{1.34}$$

$$s.t. \ \forall i \ E[\max_c u_{ic} + \epsilon] \geqslant E[\max_c \bar{u}_i + \epsilon] \ (PC)$$

$\Pr(\text{i chooses c} \,|\, \text{i chooses b}) := \frac{exp(u_{ic})}{\sum_{x \in [\![1,C]\!]} exp(u_{ix})}$ captures how borrowers i make their choice of contract when having only access to bank b contracts. We use this demand instead of the one used in the structural model ($\frac{exp(u_{ic})}{\sum_{x \in B} exp(u_{ix})}$) to shut down the intensive margin (i.e., competition) channel.

$\Pr(\text{i chooses b}) := \frac{\sum_{x \in [\![1;C]\!]} exp(u_{ix})}{\sum_{x \in [\![1;C]\!]} exp(u_{ix})}$

$E[max_c u_{ic} + \epsilon] \geqslant E[max_c \bar{u}_i + \epsilon] \iff \sum_{c=1}^{C} exp(u_{ic}) \geqslant C exp(\bar{u}_i) \iff E_c[exp(u_{ic})] \geqslant exp(\bar{u}_i)$ states that borrower i expected utility should be at least as big as what they got under the competitive equilibrium if they chose bank b.

NPV_{ic} is the net present value of lending to borrower i via contract c. It is formally defined in section C.1.3 as the amount then to borrower i multiplied by the expected revenues generated by each lending unit minus the cost of lending each unit via contract c.

Summary of the results and economic interpretation

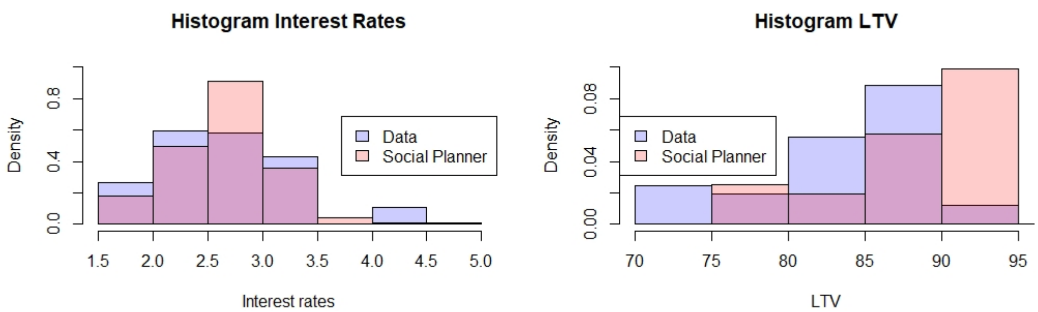


FIGURE 1.13: Data and social planner simulation distribution of the equilibrium interest rate and LTV distribution.

As illustrated by figures 1.13, the counterfactual simulation shows that the social planner could do a Pareto improvement by pooling more borrowers at higher LTV. Low-default borrowers are better off because they can buy a larger house. High-default borrowers benefit from being pooled by getting a lower interest rate. Lenders are also better off because lower LTV distortions imply that the surplus generated by the lending activity is larger, and they are thus able to extract more surplus and increase their profits.

We find that despite the low spread between defaults, the cost of the screening externality is quite large. The deadweight loss associated with the externality is equivalent to the loss in borrowers' utility following a 32 bps increase in interest rates for all contracts. As discussed in section C.9, even with low spread between baseline default probabilities, the cost of the externality can still be large as long as WTP are relatively flat.

This finding suggests there is room for Pareto improving policy interventions. As shown in the theoretical companion paper Taburet (2022), lowering competition, increasing the capital requirement on low LTV in a low-competition environment, or banning the use of lower LTV products could reduce the impact of the contractual externality by preventing cream-skimming deviations to occur. However, our model focuses on asymmetric information distortions and does not explicitly model other frictions. For instance, deposit insurance could lead banks to underestimate the risk of lending via higher LTV. This friction would then lead to too much leverage in the mortgage market instead of too little leverage. As a result, a policy Policy interventions should consider both frictions before implementing a low LTV ban.

In the following section, we take as given the policy implemented in the market and focus on quantifying its unintended effect when lender screen.

1.7.3 Ban on High LTV products

Limits on LTV are becoming increasingly popular. Indeed, according to the IMF's Global Macroprudential Policy Instruments (GMPI) database, 47 countries have introduced limits on LTVs. While those policies are used as part of the macroprudential policy toolkit, LTV limits also have an effect on the market equilibrium by restricting banks' ability to screen using LTV.

Indeed, by doing so, borrowers shopping at high LTV will be forced to move to lower LTV loans. Banks thus have to pool borrowers with different price elasticities and default probabilities or introduce new products in order to sort borrowers. To assess the impact of those policies, we solve for the situation in which the banks cannot change their menu offers and the situation in which the product offering is endogenous.

Solving the model: Given the difficulties of solving for more than one endogenous characteristic using the first order condition approach (Einav, Finkelstein, and Mahoney (2021)), the numerical exercise is based on discretizing products' characteristics and using a contraction mapping to solve for rates using the interest rate first-order conditions for a given menu offering. Instead of looking at all the possible menu offering combinations, which would be too computationally demanding,footnoteIndeed, even restricting ourselves to 10 potential products of 6 banks, the potential equilibriums to compute are greater than 10^6. , we use an algorithm proposed by Lee and Pakes (2009). The idea is to start from a given equilibrium, change a fundamental parameter and allow a first bank to optimally choose which products to enter or exit, taking other banks' offers as given and knowing what the interest rate equilibrium will be.[40] We compute the new equilibrium prices using a classic contraction mapping. Then, we allow a second bank to best respond to the new equilibrium. The program cycles through the banks, continually updating the offerings until an entire cycle is complete and no firm wishes to deviate.

Fixed products scenario: The average rate for 80-90 products increases from 244 bps to 255 bps. Using the interest rate decomposition we find that the average markup for 80-90 products goes from 33 bps to 48 bps. This is because borrowers who previously shopped at 95% LTV are, on average, less price elastic and more likely to default. After the LTV ban, they substitute for a lower LTV. The average price elasticity and default probability of borrowers shopping at lower thus increase leading to a price increase. The average information rent decreases from 66 bps to 58 bps implying either that banks pool more borrowers or that the incentive compatibility constraints are easier to maintain. Using the structural model, we find that the average cost of the LTV ban is equivalent to a 10 bps interest rate increase for all borrowers.

Endogenous products scenario:

Allowing for product entry increases the average price from 244 bps to 283 bps and expands the choice set. This is a 30 bps increase relative to the fixed product scenario. While allowing for endogenous products could have disciplined prices by increasing competition in market segments with high markups, we find that the opposite result holds because that endogenous products allows banks to extract more surplus from high WTP borrowers. In particular, we find that the products introduced by banks following the high LTV ban are the ones that are more likely to be chosen by the new borrowers that are less price elastic: 90% LTV products, low fees, and longer teaser rates. The number of products increases for two reasons. The first reason is that the number of borrowers shopping at a given LTV range increases, and the price elasticity decreases. As a result, the expected profit for any given product increases due to the market size and the markup effect; thus, it is more likely that the fixed cost becomes lower than the potential profits. This product introduction effect lowers mark-ups. However, as discussed in Tirole (1988) and in Appendix **??**, the existence of fixed costs

40. We could also consider that other banks do not change their rate

can lead to too much product being offered. This happens because lenders do not internalize the business stealing effect (cannibalization) of their product introduction on competitors. As a result, competitors tend to offer too many products. Including product introduction and exclusion thus also allows for this effect to be present.

The second effect comes from incentives to screen borrowers. As the preference heterogeneity of borrowers shopping at lower LTV increases, banks have incentives to increase the number of products to screen borrowers. As discussed in section C.9, because of the screening externality, banks may create too many products (i.e., screen borrowers) even when the social planner would not do so.

The overall effect of product introduction on welfare in thus theoretically ambiguous. Using the structural model, we find that, compared to the situation without the ban, welfare decreases by 30 bps. This result implies that product introduction is, in our case, detrimental to borrowers' welfare as it allows banks to extract more surplus from high WTP borrowers and pushes other borrowers towards products with distorted characteristics. Not considering product introduction thus underestimates the negative impact of an LTV ban by a factor of three.

1.8 Conclusion

The main contribution of this paper is to provide the first analysis of product and price distortions in the context of credit markets in which menus of contracts are used. We do so by developing the first structural model of screening with endogenous menus of contrists.

To identify and estimate the model, we make several technical contributions. First, we develop a new identification strategy to test whether screening for default probability is possible. Along the way, we discuss how to adapt classic structural models to the banking market. Those changes are guided by the fact that financial markets are not a classic IO market in many regards. For instance, contrary to a traditional IO market, the quantity (loan size) of products being sold to a given borrower may be limited by sellers, sellers may not accept to sell borrowers some products (rejection of loan applications), and the market is likely to feature adverse or positive selection. The second contribution is to propose a new set of tools to analyse the impact of screening on product and price distortions. Instead of using the classic counterfactual analysis — for which the technical properties (equilibrium uniqueness) have not been fully analysed by the literature in the context of multiple endogenous variables — we propose a new complementary approach. We first use perfect information, well-behaved model, as a benchmark to analyze product distortions. Second, we use a "sufficient statistic approach" to decompose the equilibrium interest rates into a fair price, a perfect information markup and an asymmetric information premium or discount. Finally, we propose a social planner benchmark to deliver a measure of the cost of coordination problems related to screening. The third contribution is to estimate the impact of policies affecting incentives to screen using the classic structural approach and discuss why their impact on contract terms is theoretically ambiguous.

In addition, our paper touches on several topics that we think are exciting avenues for future research. First, although not at the centre of our analysis, we document that the banking market features a large fixed cost of introducing products (30 million pounds). That results is comparable to the one of Wollmann (2018) for the car industry. Given that introducing a new product in credit markets does not require — contrary to the car industry — any new machine or raw material expenses, that result may imply large managerial frictions or collusion between banks. However, given the static nature of our supply model, our estimated fixed cost should not be taken at face value. We believe using a dynamic approach like the one explored in Appendix (C.13.3) instead of the static one used in this paper could help provide better estimates of those fixed costs. In turn, this would help in designing better models and policies in credit markets. Second, although acceptance and rejections are important drivers of the market equilibrium, those thresholds are unobserved in most data sets. We deal with this limitation by using a sufficient set approach in this paper, but, we believe that using a structural approach to back out those rules is also an interesting avenue for research.[41] To that end, we propose in Appendix (C.13.1), a methodology to recover the acceptance and rejection using an integrating over approach. This methodology would also allow relaxing the assumptions about lender risk neutrality and the functional form of the present value of lending.

41. We do not compute it in our estimation due to the computational burden.

Chapter 2

Screening using a menu of contracts in imperfectly competitive and adversely selected markets

2.1 Introduction

Many markets in which firms screen their customers using a menu of contracts feature some degree of imperfect competition *and* adverse selection. Examples includes the insurance market (Einav, Finkelstein, and Tebaldi (2019)), the mortgage market (Benetton (2018), Polo, Taburet, and Vo (2022)) and the market for credit cards (Nelson (2020)). Most of the literature analyses those two frictions in isolation or in the context of fixed contract terms. Yet, there is growing concern that these assumptions lead to welfare decreasing policy recommendations or limited understanding of market outcomes (Lester et al. (2019)). Indeed, as there usually is a significant interplay between market imperfections, policies targeting one imperfection may increase another (Handel, Kolstad, and Spinnewijn (2019)).

How imperfect competition and adverse selection interact when firms use menus to learn about their customers' private information is still an open question. The shortage of theoretical analysis is due to technical difficulties related to solving adverse selection models with multiple principals.[1] Given these limitations, the vast majority of the theoretical literature has either used monopolistic market structures or assumed perfect competition (see Lester et al. (2019) for a review). On the one hand, analyzing a market assuming monopoly can be problematic as they do not reflect the most common market structure, abstract from the interaction between firms, and are not suited for welfare analysis. On the other hand, the analysis of perfect competition models vastly relies on equilibrium concept refinements that exogenously fix contract terms.[2] Consequently, not much emphasis has been put on the predictions under perfect competition as they highly depend on the equilibrium concept used.

In this paper, I develop a screening model covering general imperfect competition cases. The framework benefits from the equilibrium properties of monopoly models: it delivers — without any equilibrium refinement — a unique and closed-form pure strategy equilibrium as long as the demand elasticities are lower than a threshold. For simplicity of the exposition, I present my framework as a model of credit markets in which borrowers have private information about their default probability, but the model components can be relabelled to capture other cases, such as the insurance market or a trade situation with different quality goods.

1. Those frameworks have to be solved using mixed strategies as long as some degree of competition is allowed (Dasgupta and Maskin (1986)) to overcome the Rothschild and Stiglitz (1976) results on the nonexistence of a competitive equilibrium. However, characterizing the mixed strategy equilibrium is computationally demanding in screening models.

2. For instance Wilson (1980) forces the principals to pool agents in one contract, Riley (1979) forces principals to screen agents using a menu. Another way to overcome this issue has been to change the modelling. For instance, Guerrieri, Shimer, and Wright (2010) assumes that the principal can match at most one borrower. Solving the model using mixed strategies has been done recently in, for instance, Lester et al. (2019).

I use my framework to highlight a novel contractual externality. In my setup, lenders can use menus of contracts to screen borrowers on their private information. Screening allows lenders to (partially) restore perfect information pricing at the cost of contract terms distortions relative to the perfect information case. Yet, due to the contractual externality, the socially optimal menus may not be offered. The friction emerges because lenders do not internalize how their screening strategies change the types of borrowers selecting competitors' products — and thus the cost of lending via those products. To illustrate this point, let us consider a perfectly competitive market in which screening is achieved by making high-default borrowers self-select a high-rate contract because lower-interest-rate contracts contain features that are relatively more costly to them, such a low maximum loan size. When the loan size distortions needed to screen borrowers are high, pooling at least some types of borrowers is a Pareto improvement over screening. This happens because, under pooling contracts, high-default borrowers get a lower rate and low-default borrowers are less credit constrained. Yet, if a lender prices their customers using the average default probability (pooling), a competitor can take advantage of the lender pooling strategy to introduce contracts that will steal only the low-cost customers (cream skimming) by offering a low rate-low credit constraint contract.[3]

I then provide an analysis of the impact of the externality on contract terms. In particular, I analyze the following welfare trade-off between competition and adverse selection. A low competition level allows lenders to apply high mark-ups, which reduces borrowers' utilities. Nevertheless, a low competition level can also improve welfare via two channels. First, by limiting concerns of losing market shares, it gives lenders more flexibility in using contract terms and prices to sort their customers. Second, it lowers lenders' incentives to implement socially inefficient strategies that rely on taking advantage of their competitors' menus to attract only the most profitable borrowers. I show that decreasing competition leads to a Pareto improvement when the externality is high.

My setup builds on a textbook screening model. Borrowers choose the contract and lender that maximize their utility. In the baseline model version, a contract is composed of a loan size and an interest rate. Borrowers value loan amounts positively and have quasi-linear indirect utilities in interest rates. Borrowers are the same across the following dimensions: savings and their outside option when not borrowing. They have private information about their willingness to pay (WTP) for each loan unit and

3. While Rothschild and Stiglitz (1976) shows the profitability of cream skimming deviations, it does not analyse its impact on welfare as the deviations are profitable in the region where the equilibrium contracts cannot be computed in pure strategy. Most papers, like Rothschild and Stiglitz (1976), focus on pure strategy equilibrium. Papers that characterized the mixed strategy equilibrium such as Lester et al. (2019) have not focused yet on the externality.

their default probabilities. For screening to be possible, I consider that borrowers have heterogeneous WTP and that their WTP is positively correlated with their default probabilities. Screening can thus be achieved by offering a menu featuring: a low-interest rate but small loan size contract, and a larger loan contract with each extra lending unit being priced above the lowest WTP for loan size. That way, the small loan size attracts unobservably safer borrowers. In order for screening to be costly, I consider that both borrowers want to borrow the same amount in the first best. The low default probability borrower is thus credit constrained as a result of screening.[4] In an extension, I allow for borrowers' price elasticity to be heterogeneous correlated with WTP and unobserved by lenders.

The main technical innovation of the paper is to use a discrete choice approach (McFadden (1981)) to model competition (i.e., borrowers' product elasticity) within an otherwise standard principal-agent model. Following the discrete choice literature, I assume that part of borrowers' utility contains a borrower-bank-specific random shock following a continuous probability distribution and entering the utility additively. This shock can, for instance, be interpreted as borrowers living at a different distance from the closest bank branch (Hoteling (1929)) or as borrowers being imperfectly informed about the contracts offered by each bank (Varian (1980)). I consider the situation in which the random shock is uncorrelated to borrowers' preferences for contract characteristics so that banks cannot use their menus to sort borrowers on their random shock realization. When the shocks follow an extreme value distribution, its variance parameterizes the product demand elasticity. In the limit case in which the variance of the shock tends to infinity, each lender behaves like a monopolist. When the variance tends to zero, borrowers' demand elasticity becomes infinite, as in the perfect competition case.

The discrete choice modelling makes our framework suitable for empirical works. First, our demand function is closely related to logit models and can thus be estimated using traditional empirical tools.[5] This is done in the companion paper Polo, Taburet, and Vo (2022). Second, the discrete choice approach yields a continuous demand function — in contrast to other approaches to model competition (for instance Burdett and Judd (1983)). This property allows decomposing the effect of the frictions on interest rates as the sum of the pure adverse selection effect, a pure competition effect, and an interaction term. The formula components can be estimated using standard industrial organisation tools to decompose the different channels at play in an empirical setup. Finally, the tractability of the discrete choice approach also

4. The product distortions relative to the first best concern loan size here but could be relabelled maturity, fixed-rate period, collateral or any other contract term.

5. This allows, for instance, to deal with unobserved product characteristics using Berry, Levinsohn, and Pakes (1995) method developed for logit models.

facilitates the analysis of the impact of the functional form assumptions on the predictions of the model. This allows checking if a particular functional form assumption is adapted to a given empirical application.[6]

The discrete choice approach facilitates the analysis of adverse selection and competition by restoring the ability to solve the model in pure strategy. This feature allows studying the contractual externality as it is only present for the set of parameters for which the pure strategy equilibrium does not exist in competitive models such as Rothschild and Stiglitz (1976).[7]

The reason a pure strategy equilibrium does not exist in perfect competition models used in the literature is the following. When competition is high enough, offering one contract and pricing it using the average borrower default probability (i.e. pooling) is not sustainable: each lender has incentives to take advantage of its competitors' offers to attract the most profitable borrowers only. This forces lenders to screen even when both borrowers would benefit from being pooled. However, if pooling is a Pareto improvement, offering screening contracts is also not sustainable: a competitor can make profits by offering a pooling contract with a small markup over the break-even rate. Those deviations from screening exist if the number of high-default borrowers in the market is low enough so that low-default borrowers are better off being pooled.

Instead, in my model, deviations from screening are not profitable when the demand elasticities are low enough as they attract too many high default borrowers to be profitable. This derives from the fact that deviations do not attract the full market size. Consequently, the relative number of high versus low default borrowers attracted by pooling deviations not only depends on the relative market sizes but also on how attractive the deviation from pooling is to each borrower type. Given that pooling deviations requires increasing a contract loan size — which is relatively more valuable to the high-default borrower—, the proportion of high-default borrowers it attracts is higher.[8] This makes pooling deviations not profitable and restores the existence

6. For instance, due to the utility of loan size and interest rate being additively separable, absent asymmetric information, imperfect competition does not distort loan size away from its first best value. The reason is that, under perfect information, banks set the contract characteristics that maximise the lending surplus and use interest rates to split the surplus between borrowers and lenders. This separability assumption may not be valid when the loan is used to buy a good, such as housing, that complements consumption.

7. Furthermore characterizing the mixed strategy has only been done recently (Lester et al. (2019)) in competitive models. Those papers do not focus on the externality

8. Formally, denoting the utilities $u(L, R; WTP) = WTP \cdot L - R$, we have that the high WTP borrower (denoted WTP_1) derives more utility out of the same contract than the low WTP borrower (denoted WTP_2): $u(L, R; WTP_1) = u(L, R; WTP_2) + (WTP_1 - WTP_2)L$. The revelation principle imposes that it is optimal for the high WTP to be indiferent between his contract and the contract chosen by the other borrower $u(L_2, R_2; WTP_2) = u(L_1, R_1; WTP_1)$. Thus, any increase in the loan size of the contract designed for the low WTP borrower as in the pooling deviation increases the utility of the high WTP relatively more.

of a pure strategy equilibrium.[9] When markets are sufficiently imperfectly competitive, there exists a unique pure strategy equilibrium in which the spread between contract terms and prices is such that any pooling or screening deviations cannot attract enough low default borrowers for it to be profitable.

The model implies that lowering competition decreases the contract terms distortions relative to the first best as long as borrowers' participation constraints are not binding. Due to the threat of losing customers to a competitor, a high level of competition forces banks to screen borrowers to (partially) restore perfect information pricing. This implies a high credit constraint. When borrowers' product demand elasticities decrease in the same proportion, the level of credit constraint decreases.[10] The rationale behind that result is the following. When competition decreases, banks can increase interest rates on both contracts without losing too much market share. However, lenders benefit from applying a higher markup —relative to the one they would apply under perfect information — for the contracts designed to low default borrowers. In doing so, they relax the high-default borrowers' incentives to choose the contract designed for low-default borrowers. As a result, lenders can reduce the credit constraint on the contract designed for low-default borrowers, thereby increasing their profits by lending more. The prediction that a low level of competition always leads to a low level of credit constraint does not hold in existing monopoly models — which predict that both situations can arise depending on the parameters.[11]

Given the previous paragraph's discussion, a decrease in competition increases welfare[12] when the participation constraints are not binding. When the contractual externality level is high, lowering competition is also a Pareto improvement as it limits incentives to implement ex-post inefficient deviation from pooling. The model allows analysing when changes in competition lead to a Pareto improvement. Under adverse selection, a lower level of competition can be beneficial to low-default borrowers as it lowers their credit constraint at the cost of a higher markup. It is also beneficial to high-default borrowers as competition can lower the interest rate of their contracts. The decrease in interest rate is due to the fact that the pure competition effect can

9. When competition is large enough, as the problem becomes convex: the deviation starts attracting bad borrowers in high proportion, however, once most of the bad borrowers have been attracted the good borrowers become easier to attract relative to the bad borrowers.

10. However, the loan size distortion relative to the first best can increase due to the pure competition effect only. Indeed, high markups can lead to a lower amount of quantity sold compared to the first best. As discussed in footnote 6, in my model, due to the quasi-linearity assumption of rates, the effect of pure competition on loan size if absent.

11. The reason is that low default borrowers are at their participation constraint in monopoly models, and thus, the bank cannot increase the interest rate of the low loan size contract without losing all its customers. In my model, when competition is so low enough that borrowers' outside option becomes not borrowing instead of going to a competitor bank, the predictions then become similar to the monopoly case.

12. The amount of credit rationing is the utilitarian welfare measure in our model.

be compensated by an information rent (i.e. interest rate decrease). The information rent prevents the high-default borrower from pretending to be a low-default one.

Finally, motivated by policies implemented in screening markets, I analyze the effect of product-specific lending costs and preferences on contract terms, prices and welfare. Common real-world examples of such shocks are product-specific capital requirements in lending markets or product-specific government subsidies in insurance markets.[13] I show that there is an interplay between market imperfections, and that policies targeting one imperfection may increase another. In particular, when competition is low, a decrease in the cost of creating contracts designed for borrowers with high default probability leads to worse contract terms and higher prices for all borrowers. The reason is that this incentivizes lenders to extract more surplus from high-default borrowers. Increasing the rate of the high default contract makes the adverse selection stronger as taking the low default contract price and terms as fixed, the high default borrowers have more incentives to pretend to select the low default contract. This negatively impacts the low-default borrowers' contract terms and prices. The opposite effect obtains in competitive markets or when the cost decrease affects contracts designed to be chosen by low default probability borrowers. When competition is low enough, the lower rate is passed through the high default contract, this makes the adverse selection weaker and thus benefits the low-default borrowers. Analyzing the impact of those policies thus requires measuring the degree of competition and how lenders implement their screening mechanism in practice.

In the rest of the paper, we first describe the model setup in section 2.3. Then, in section 2.4, I provide intuition about the incentives to screen, I prove the existence and uniqueness of the equilibrium and solve for the model in closed form. Finally, in section 2.6, I analyse the theoretical effects of changes in the fundamental parameters of the model and discuss the potential implications of various widely used policy interventions.

2.2 Literature Review

This paper is related to the literature studying both adverse selection and competition, and to the literature studying the role of contract terms and prices as a screening device.

There is a growing literature on the interaction between adverse selection and competition. Papers have focused on the situation in which lenders can only choose interest rates (see Crawford, Pavanini, and Schivardi (2018) for a structural empirical framework), or the case in which lenders can pay a fixed cost to learn about borrowers

13. See for instance the 2014 BIS report "Basel capital framework national discretions" for the lending market and Einav, Finkelstein, and Tebaldi (2019) for the insurance market

type (Yannelis and Zhang (2021)). My paper differs from those two by looking at a situation in which lenders can screen their customers by designing their menu of contracts.

Our paper contributes to the literature on adverse selection and on the role of contract terms and prices as screening device. The vast majority of the literature analyses model using perfect competition (Rothschild and Stiglitz (1976)) or monopoly (à la Stiglitz (1977)). Recent examples are Farinha Luz (2017) for a characterization of the mixed strategy contract terms under perfect competition and Guerrieri, Shimer, and Wright (2010) for a pure strategy characterization based on the assumption that the principal can match with at most one agent. The closest paper to our is Lester et al. (2019). It uses a search model à la Burdett and Judd (1983) to model imperfect competition in and otherwise standard screening model in a goods of different quality (lemon market). We use a different modelling approach based on a discrete choice (à la McFadden (1981)) and focus on the lending market. Our discrete choice approach allows to solve our model in pure strategy and makes our model closely related to empirical industrial organisation models (see Polo, Taburet, and Vo (2022) for an empirical application), while Lester et al. (2019) rely on a mixed strategy characterization. The reason their model have to be solved in pure strategy comes from their modeling of demand. By modeling borrowers as some being infinitely price elastic while other being completely inelastic, Lester et al. (2019) end is a situation similar to Rothschild and Stiglitz (1976) in which both pooling deviations attract the same proportion of good and bad borrowers as in the full market. As discussed in the introduction, this creates in some situation incentives to deviation from both pooling and screening prevents the model to be solved using pure strategies. Our paper complement Lester et al. (2019) analysis on the interaction of competition and adverse selection trade-off by highlighting the screening externality.

2.3 Model Set-up

2.3.1 General considerations

I consider a 2-period model with two groups of agents: borrowers and lenders. I also refer to the second group as banks. There is a finite number of banks $B > 1$ indexed by $b \in 1, B$. Banks offer a menu of contracts. I index a contract by c.

Timing: At the beginning of the first period, each borrower makes a decision to enter or not the credit market. There is no entry cost, borrowers choose to participate if the utility they get from borrowing is higher than the one of not borrowing. Conditional on participation, a borrower chooses one loan contract from one lender.

Loans mature in the second period. Borrowers may default on their loans.

Heterogeneity: Borrowers have heterogeneous characteristics (age, income, risk aversion..). This translate into borrowers having different preferences over loan contracts and banks' characteristics. Borrowers also have heterogeneous default probabilities. There are two types of borrowers indexed by $i \in \{G, B\}$. The number of type i borrowers is denoted n_i.

Information structure: There is asymmetric information in the economy: lenders do not perfectly observe borrowers' type (i.e. their preference and their default probabilities).

Lenders are using the revelation principle: whenever it is profitable and feasible, they use a menu of contracts to make borrowers self-select.

2.3.2 Borrowers

In this section, I model borrowers' decision to participate in the credit market as well as their choice of loan contract and repayment behaviour. I then provide a micro-foundation of the demand system.

Information structure: All parameters defined in this section are part of borrower i information set at the time they make their choice of contract and bank.

Choice of contract and bank

A loan contracts is composed of the loan size L and the amount the borrowers promised to pay at maturity R. This is extended in the appendix to any finite number of contract characteristics X as long as they enter linearly in the utility.

Utility: The utility of type i borrowers when borrowing an amount L_c in period 1 via contract c requiring a repayment of (R_c) at maturity is specified as:

$$u_i(L_c, R_c) := \beta_i F(L_c) - R_c, \ \beta_i > 0 \tag{2.1}$$

$\beta_i F'(\cdot)$ is the borrowers' willingness to pay for an extra unit of lending.

Without loss of generality, I assume that $\beta_G < \beta_B$. Since the interest rate preference parameter is normalize to one, a high β captures that the borrower derive a high utility level from housing, or less disutility from having a high interest rate. I provide in the next section (section 2.3.2) a micro-foundation for this utility form. Heterogeneous β coefficient imply that lenders can screen using loan size L and the face value of the debt R. Screening is achieved by offering a contract whose pricing for an extra loan unit is in between the two borrowers' willingness to pay. That way, the extra loan units will be purchased only by the high WTP borrowers only.

In an extension, I include down-payment, collateral and loan size to endogenize the contract Loan-to-Value. This is done as many policy regulations (Capital requirements, government guarantee schemes) are based on Loan-to-Values.

Choice of bank b contract c: Borrower i chose the bank that offers him the best deal:

$$max_{cb}\{u_i(L_c, R_c) + \sigma^{-1}\epsilon_{ib}\}, \ \epsilon_{ib} \ iid, \ EV \tag{2.2}$$

$\sigma^{-1}\epsilon_{ib}$ is the main departure from the classic principal-agent model. σ drives the product elasticity (competition) and can be interpreted as the distance between the borrower and the closest bank branch as in, for instance, Hoteling (1929). When σ tends to infinity, borrowers only care about the contract features offered by the banks (i.e perfect competition). When σ^{-1} tends to 0, each bank behaves like a monopolist with their borrowers.[14]

In the Appendix (D.2), the model is also solved with a different functional form for ϵ_{ib} that yields a CES type of demand function instead of a logit one.

Participation constraint (PC): Borrower i accept a contract if it provides them a higher level of utility than the one they would get if they do not take any loan. Formally, borrower i accepts the loan if:

$$u_i(L_c, R_c) \geqslant \bar{V}_i = \bar{V} \in \mathbb{R}^+ \tag{2.3}$$

The fact that ϵ_{ib} is not present here is in favour of the interpretation of ϵ_{ib} being a sunk cost that has to be paid in order to go to the bank. Using a nested logit approach, one could write a similar condition that would model the entry decision in the borrowing market.

Survival probability: Borrowers have heterogeneous and exogenous survival probabilities (θ_i).

$$\theta_i = 1 - \rho\beta_i, \ \rho > 0 \ (Adverse \ selection) \tag{2.4}$$

I consider the case in which the market is adversely selected. That is borrower with a high willingness to pay $\beta_B > \beta_G$ are more likely to default $\theta_B < \theta_G$). In the

14. when $(\varepsilon_{i,j})_j$ are not all equal

following micro-foundation, I justify this assumption by the fact that high default borrowers are more likely to have a high willingness to pay for loan as they do not expect to repay the full face value of the debt.

Possible micro-foundation borrowers' indirect utility

This section micro-found the assumption made about adverse selection ($\rho > 0$) using first principles. As I will look at mortgage policies in the last section of the paper, I use a mortgage micro-foundation. Alternatively, one can assume that the lending is used for consumption in the first period and get rid of the housing modeling.

Borrowers do not have any income in period 1. They can get a loan (L_c) to invest in a house of size (H_1) yielding the utility $F(H_1)$ in each period as long as they do not sell it. They can also consume (C_1), from which they derive utility ($u(C_1)$). The function F and u are increasing and concave with $F(0) = u(0) = 0$. Borrowers discount period 2 utilities with the discount factor δ. In the second period, borrowers' income is either W with probability (θ) or 0 with probability ($1-\theta$). Borrowers use their income W to consume (C_2) and to repay the loan (R_c). When their income is equal to zero, they fire sell the house and get (γH) to repay for the loan R_c.

$$max_{\{C\}}\ u(C_1) + F(H_1) + \delta\theta \overbrace{[u(C_2) + F(H_1)]}^{utility\ when\ not\ defaulting} + \delta(1-\theta)\overbrace{[u(max\{\gamma H_1 - R, 0\})]}^{utility\ when\ defaulting} \tag{2.5}$$

$$s.t.\ C_1 + H_1 = L_c \tag{2.6}$$

$$C_2 = W - R_c \tag{2.7}$$

Assuming for simplicity of the notation that fire selling is costly ($\gamma = 0$)[15] and that borrowers prefer to invest in the house rather than consuming in the first period (i.e. $u'(0) < [1 + \beta(1-d)F'(\bar{L})]$, with $\bar{L}$ being the maximum loan size available), but that they prefer consuming rather than getting a house in the second period (i.e. $u'(W) > F'(0)$) the indirect utility can be written:

$$\overbrace{[\frac{1}{\delta\theta} + 1]}^{\beta} F(L_c) - u(W - R_c) \tag{2.8}$$

15. In that particular case, borrowers would be better off keeping the house (and lender would be no worse off). Selling the house upon default can be however rationalized by the use of house as collateral in order to prevent borrower to fill for default even when income is equal to W (see appendix D.2.1).

Without loss of generality, I normalized the utility parameter to one in the above expression. The expression implies that high default borrowers are more likely to have a high willingness to pay for loan as they do not expect to repay the full face value of the debt (captured by $\frac{1}{\delta\theta}$). When the utility of consumption is linear, the same indirect utility as in 2.1 is obtained. This assumption is consistent with Hertzberg, Liberman, and Paravisini (2018) empirical findings that the self-selection in the consumer lending market seems to be driven by private information on the income process rather than risk aversion.

The model can be extended by allowing borrowers to have income (A) in the first period at contract to allow for down-payments (D). The utility thus become:

$$[\frac{1}{\delta\theta} + 1]F(L_c + D_c) + \frac{1}{\delta\theta}u(A - D_c) - u(W - R_c) \tag{2.9}$$

In that case, we get that high default borrowers are less willing to put their own wealth into their house. This justifies the fact that, under screening, high LTV loans will be selected by borrowers with unobservably high default probabilities.

2.3.3 Lenders

Lenders do not observe borrowers' type (β_i) but they know its distribution and its correlation with survival probabilities. They use a menu of contracts to make borrowers self-select on their private information. They use the revelation principle: they maximize profits subject to incentive compatibility constraints. I assume that ϵ_{ib} is independent of contract characteristics so that banks cannot screen borrowers on their ϵ_{ib} draw.

The model is thus the classic textbook principal-agent model but with the possibility for borrowers to move to other banks. Each bank maximization problem can be written:

$$max_{\{(L_{ib},R_{ib})\in\mathcal{F}\}} E_\epsilon[\sum_{i\in\{G,B\}} \overbrace{\underbrace{n_i}_{market\ size} \cdot \underbrace{1_{u_i(L_{ib},R_{ib})\geqslant \bar{V}_{ib}(\epsilon)}}_{PC^1} \cdot \underbrace{1_{u_i(L_{ib},R_{ib})\geqslant \bar{V}}}_{PC^2}}^{Demand} \underbrace{PV_i}_{Expected\ profit\ on\ loan} |\beta_i] \tag{2.10}$$

$$s.t.\ (IC_i) : u_i(L_{ib}, R_{ib}) \geqslant u_i(L_{jb}, R_{jb})\ \forall i, j \in \{G, B\} \tag{2.11}$$

Where PV_{ib} is the present value of lending to borrower i via a contract i at bank b. Following the micro-foundation presented in section 2.3.2, the present value can be written $PV_{ib} := \theta_i R_{ib} + (1 - \theta_i)min\{\gamma H, R_{ib}\} - mcL_{ib}$ where mc is the marginal cost of lending via contract c. To focus on risk discrimination, I consider the case in which the price of house upon default $\gamma = 0$ is low enough so that the collateral is not enough to repay for the face value of the debt.

The first constraint (PC^1) states that, a borrower comes to bank b if bank b contract delivers a higher utility level than its competitors. Formally, the utility of going to another bank is $\bar{V_{ib}}(\epsilon) := max_{\tilde{b}}\{u_i(L_{i\tilde{b}}, R_{i\tilde{b}}) + \sigma_i\epsilon_{i\tilde{b}} - \sigma_i\epsilon_{ib}\}$.

The second constraint (PC^2) states that, given that a borrower comes to bank b, they accept the contract if it provides a higher utility than not borrowing.

The incentive compatibility constraint (C.41) makes sure that borrower i chooses the contract designed for them. As the random term (ϵ_{ib}) is the same for all the product of the same bank, lenders cannot screen borrowers on their (ϵ_{ib}) parameter. Indeed those two terms cancels each other. Under the assumption that ϵ_{ib} is extreme value distributed, the probability that borrower i chooses bank b is:

$$N_i^b(u_i(L_{ib}, R_{ib})) := E_\epsilon[1_{u_i(L_{ib},R_{ib})\geqslant\bar{V}_{ib}(\epsilon)}|\beta_i] = \frac{exp(\sigma_i u_i(L_{ib}, R_{ib}))}{\sum_{x\in B_i} exp(\sigma_i u_i(L_{ix}, R_{ix}))} \tag{2.12}$$

B_i is the set of banks available to borrower i.

For the problem (2.10) to be well defined when the function F is linear, I assume that contracts' characteristics are bounded: $\mathcal{F} := \{(L_c, R_c) : L_c \in [0, \bar{H}],\ R \geqslant 0\}$ with $\bar{H} > 0$. $\bar{H}$ reflects maximum house size availability or the fact that the house utility function F has a kink. Alternatively, I can write this constraint as a maximum Loan-to-Value constraint to model existing regulations. I use a constraint on L rather than a constraint on borrowers second period income (and thus R) as this allow the house size to be fixed in the first best and independent of competition in order to focus on the effect on rates. R needs to be positive, I do not put any upper bound as competition or the participation constraint will naturally impose a bound (i.e. I assume that W in the second period is high enough).

Assumptions A1: We assume that F is linear on $[0, \bar{H}]$ and equal to $F(L) = L$. We consider that both market segment have positive NPV $\beta_i - \frac{mc}{\theta_i} > 0\ \forall i$. This assumption is made to make the screening problem interesting. If only the low WTP borrower had positive NPV, then the model will be similar to Akerlof (1978), in which only one contract is offered. Similarly, if only the high WTP borrowers have a positive NPV, then banks will easily exclude the low WTP borrowers.

2.4 Optimal Menu Design

In this section, I analyze how each lender set contract terms under under perfect and then under imperfect information.

2.4.1 Contracts when β_i observable

Before focusing on the meaning and the impact of the of the incentive compatibility (IC) constraints, let us first solve the problem without it. That is, let us focus on the case in which borrowers' type is observable.

From the problem (2.10) without (IC_B):

Proposition 1: Banks' incentives to screen under perfect information *Each bank uses product characteristics (L) to maximize the surplus of lending ($S := (\beta_i - \frac{mc}{\theta_i})$), then uses the interest rates to split the surplus between itself and the borrowers.*

Formally, from the first order conditions of the problem (2.10) without (IC_B) and dropping the b index for clarity of the notation:

Characteristics:

$$L_i^{PI} := \bar{H} \tag{2.13}$$

Pricing:

$$r_i^{PI}(L_i^{PI}) := \begin{cases} \overbrace{\frac{mc_i L_i^{PI}}{\theta_i}}^{fair\ price} + \overbrace{\frac{N_i}{\partial_R N_i}}^{\text{“}markup\text{”}} & \text{if } u_i \geqslant \bar{V}_i \\ \beta_i L_i^{PI} - \bar{V}_i & \text{Otherwise} \end{cases} \tag{2.14}$$

Utility:

$$u_i = \begin{cases} \overbrace{S_i(L_i^{PI})}^{Lending\ surplus} - \overbrace{\frac{N_i}{\partial_R N_i}}^{\text{“}markup\text{”}} & \text{if } u_i \geqslant \bar{V}_i \\ \bar{V}_i & \text{Otherwise} \end{cases} \tag{2.15}$$

with $S_i(L) := (\beta_i - \frac{mc}{\theta_i})L$ being the surplus generated by the lending activity.

Equation (2.13) states that the optimal contract allows borrowers to get the biggest house possible $\bar{H}$. This is because we assumed that lending generates positive NPV ($\beta_i - \frac{mc}{\theta_i} > 0\ \forall i$).

The upper right-hand side of the equation (2.14) reflect the case in which competition is high enough so that the participation constraint is not binding. This equation captures the classic extensive and intensive margin channels driving the interest rate level. The first term ($\frac{mcL_i^{PI}}{\theta_i}$) is the fair price. That is the price at which banks would break even. The second term is a "markup" term[16]. It is equal to the inverse price elasticity multiplied by R ($\frac{N_i}{\partial_R N_i \cdot} = \frac{1}{\sigma_i(1-N_i)}$). The numerator of the "markup" captures the intensive margin: by increasing R, banks earn more on each borrower. The denominator ($\partial_R N_i$) captures the impact of the extensive margin on pricing: by increasing R the bank loses customers. In equilibrium, the right-hand side part of the equation will not depend on rates as the mark-up $\frac{N_i}{\partial_R N_i}$ will simplifies to $\frac{1}{\sigma_i(1-B^{-1})}$.[17]

Due to the utility of loan size and interest rate being additively separable, absent asymmetric information, imperfect competition does not distort loan size away from its first best value. The reason is that, under perfect information, banks set the contract characteristics that maximise the lending surplus and use interest rates to split the surplus between borrowers and lenders. This separability assumption may not be valid when the loan is used to buy a good, that complements consumption. We will not analyze this case in our model as we focus on the distortions related to the contractual externality only.

2.4.2 Contracts when β_i unobservable

Now let us focus on the incentive compatibility constraints and how it impacts banks' menu offering.

As shown in the previous section, given assumption A2 stated in the previous section: (i) borrowers would get offered the same product in the first best, but (ii) banks would like to price them differently due to their different default probability or price elasticity. Said otherwise, the first best contracts are not incentive compatible as borrowers will always choose the cheaper product.

As a result, banks have to distort the first best contracts to maintain borrowers' incentives to self-select. Formally, they do so using a system of incentive compatibility constraints (IC).

Simplifying the maximization problem

As in the textbook principal-agent model, the system of IC can be simplified. Formally, the simplifications are summarized in the following Lemma 1, 2 and 3.

16. The theoretical literature usually refers to the markup as the output price divided by the marginal cost. I instead define the mark-up as the pricing above the marginal costs. The empirical IO literature sometimes uses the same terminology (Crawford, Pavanini, and Schivardi (2018)).

17. Indeed, under the logit formulation the r terms cancels each other to become $\frac{1}{\sigma_i(1-n_i N_i)}$. The parameter σ drives the product elasticity. When sigma is high, a lot of the surplus is given back to the borrower. $(1-n_i N_i)$ captures the fact that the price elasticity under a logit depends on the number of competitors. In a symmetric equilibrium, this term will be equal to the relative number of borrowers of type i over the total number of banks.

Lemma 1: At least one IC is biding. *Under the assumption that* $\sigma_B < \sigma_G$, (IC_B) *is always binding.*

Proof: Appendix D.4.

$\sigma_B < \sigma_G$ makes sure that the high WTP borrower is always the one that benefits from pretending to be the other type. This is because their perfect information interest rate is always higher due to their higher default probability and lower price elasticity. This assumption makes the problem easier to present as one does not have to track the IC constraint that is binding. This assumption can be relaxed.

Using Lemma 1, we know that (IC_B) will always be binding in problem (2.10). (IC_B) states that borrower B (i.e. the one that wants to pretend to be of the other type) must be indifferent between his contract and the contract chosen by borrower G. It can be written:

$$(IC_B) : R_B - R_G = \tilde{\beta}_B(L_B - L_G) \tag{2.16}$$

(IC_G) can be written as a monotoniticy constraint:

$$(IC_G) : L_B \geqslant L_G \tag{2.17}$$

Equation (2.16) and (2.17) implies that high default will self-select a interest rate contract because they find the loan size increase $L_B - L_G$ cheap enough. Equation (2.16) states that each loan units above L_G must be priced at he maximum willingness to pay ($\beta_B > \beta_G$) so that the G borrower prefer the smaller loan size contract.

Solving for the problem 2.10 using Lemma 1, we get:

Lemma 2: No distortion at the top. *The product characteristics of borrowers B are equal to their first best value.*

Proof: Formally, this is shown by solving for the maximization problem in its promised utility form and noticing that (IC_B) does not depend on L_B nor D_B. Intuitively, the self-selection problem comes from the fact that high default (high WTP) borrowers want to pretend to be low default (low WTP) borrowers to get a cheaper loan. If the bank offers a contract with distorted product characteristics ($L_B < \bar{L}$), it means that the surplus is not maximized (as $\beta_B - \frac{mc}{\theta_B}$). There is thus a Pareto improvement that leads to either more profits in the B market segment or relaxes the IC constrain. Indeed, this is the case if the bank increase L_B and changes the rate of the B contracts in between the marginal cost of L_B and the willingness to pay of the B borrower.

Lemma 3: Banks' trade-off and product distortions. *(i) Only the loan size of the contract designed for borrower G is distorted. The distortion is proportional to the interest rate spread:*

$$(IC_B) : \Delta L_G^{II} := L^{PI} - L_G^{II} = \frac{R_B^{II} - R_G^{II}}{\beta_B^c} \in [0, \bar{H}] \tag{2.18}$$

(ii) Relative to the first best, the interest rate of the B borrower is lower, the one of the G borrower is higher. The pricing have the form:

$$R_i^{II} := \begin{cases} \overbrace{\frac{mcL_i^{II}}{\theta_i}}^{fair\ price} + \overbrace{\frac{N_i}{\partial_R N_i}}^{\text{``}mark\ up\text{''}} + \overbrace{AI_i}^{Asymmetric\ information\ discount/premium} & \text{if } \ u_i \geqslant \bar{V}_i \\ \tilde{\beta}_i L_i^{II} - \bar{V}_i & \text{Otherwise} \end{cases} \tag{2.19}$$

with D_B^I and L_B^I and the non distorted characteristic of borrower G are equal to their perfect information values defined in section 2.4.1. The distorted characteristic of borrower G (found using lemma 3) is given by equation (2.18).

Proof: (i) Use lemma 2 and Lemma 1. (ii) solve for the problem using the Lagrangian.

Equation (2.19) states that the price can be written as in the perfect information case (fair price and mark up) with an extra additive term. I call this extra term the asymmetric information discount or premium (AI). The AI term enters positively for the G borrower (i.e. premium) and negatively for the B borrowers (i.e. discount). Absent AI, the spread between rates would be higher, implying that banks would have to distort the L_G characteristic more intensively. AI thus provides information about the incentives to screen: setting high AI allows to lower the spread between rates and thus lower the product distortion on the G market segment. In the extreme case, high AI implies that banks offer just one (pooling) contract.

The modelling of imperfect competition yields a new expression for the loan pricing. Indeed, under perfect competition, only the fair price $\frac{mcL_i^I}{\theta_i}$ would be present. Under monopoly, the participation constraint will be binding for one borrower, and the price of the other borrower would be (partially) driven by this outside option.

The extra two terms $\overbrace{\frac{N_i}{\partial_R N_i}}^{\text{"mark up"}} + \overbrace{AI_i}^{Asymmetric\ information\ discount}$ are thus specific of the imperfect competition case. They allow studying how different changes in the economic environment affect incentives to screen.

Banks' trade-off: Using Lemma 3, the perfect information solution and the optimization problem (2.10), we can see that the incentive compatibility constraint creates the following trade-off. If a bank wants to extract a lot of surplus from market B (i.e. set a high rate $R_B > R_G$) as in the perfect information contract, it has to distort the market G (i.e. lower L_G or higher R_G). If the bank wants to distort less the market G, it has to provide an information rent to market G (i.e. set a lower rate $R_B \leqslant R_G$).

Solving Lenders' Problem

Making use of Lemmas 1-3 of the previous section, I solve for the optimal contract terms of lender problem (2.10).

Proposition 2: Banks' incentives to screen *Banks have incentives to screen when the relative profits in the G market segment are low, when screening is costly and when price elasticities are high. Looking at an interior solution for L_G^{II} (i.e. $L_G^{II} \in [0, \bar{H}]$ and $u_i \geqslant \bar{V}$, the level of product distortion is:*

$$L_G^{II} = \underbrace{\Theta \bar{H}}_{No\ markup\ no\ AI} + \frac{1}{MC}[\underbrace{\frac{N_G}{\partial_R N_G} - \frac{N_B}{\partial_R N_B}}_{price\ discrimination\ incentives} + \underbrace{AI_B - AI_G}_{risk\ discrimination\ incentives}] \tag{2.20}$$

Where

$$0 < \Theta := \frac{MB}{MC} \leqslant 1, \quad \overbrace{MC}^{IC\ cost\ increase\ in\ L} := \beta_B - \frac{mc}{\theta_G}, \quad \overbrace{MB}^{Benefits\ increase\ in\ L} := \beta_B - \frac{mc}{\theta_B} \tag{2.21}$$

$$Asymmetric\ information\ discount\ given\ to\ type\ B:$$
$$AI_G := \frac{N_G}{-\partial_{R_B} N_B} \Theta \frac{\theta_G}{\theta_G} \frac{n_G}{n_B} > 0 \tag{2.22}$$

$$\text{Asymmetric information premium paid by type } G:$$
$$-AI_G := \frac{N_G}{-\partial_{R_G} N_G}\Theta > 0 \tag{2.23}$$

We will show in the next section (section 2.5) that the conditions for the interior solution (*i.e.* $L_G^{II} \in [0, \bar{H}]$ *and* $u_i \geqslant \bar{V}$) are satisfied when competition is high enough. The general case with multiple characteristics is solved in the in Appendix (D.4).[18] The channels at play, however, remain the same. We can thus focus on the interior solution only to gain intuition about incentives to screen.

The asymmetric Information terms (AI) represent the interest rate distortions relative to the perfect information pricing. To understand incentives to screen, we can thus focus on those terms only.

The size of the asymmetric information discount (AI_B) captures the incentives to pool or screen borrowers using the rate on the B contract. Indeed, providing an information rent ($AI_B \uparrow$) to market B allows to relax the product distortion in market G (and thus be closer to the pooling contract). Doing so, banks make less profits on the B market segment but increase it on the G market segment. Banks thus have incentive to provide and information rent if potential profits in market G are higher. This is the case when the relative market size $\frac{n_G}{n_B}$ is high or if the probability of the loan being repaid is the B market segment is higher ($\frac{\theta_G}{\theta_B}$ high). In addition, the benefit of providing an information rent (AI_B) is higher when the screening device is effective. That is, when the marginal surplus generated by market B for each extra loan unit (MB) is high, and the information rent allows to increase loan size a lot (i.e. $(MC)^{-1}$ high). The term $\frac{1}{\sigma_B(1-N_B)}\frac{N_G}{N_B} = \frac{N_G}{\partial_{R_B} N_B}$ is a feedback effect. It captures the fact that providing an information rent while maintaining the utility of the other borrower constant increase the relative market share of market B relative to market G. As the size of market B increases, the incentives to provide an information rent decreases. This feedback effect is stronger when the price elasticity is high.

The value of the information rents (AI_G) illustrate the different the incentives to pool or screen borrowers by distorting the rate on the G market segment. Its purpose is also to relax the incentives compatibility constraint in order to lower the distortion in characteristic X. Increasing the rate by one unit allows to increase X by $\frac{1}{MC}$ while maintaining incentives to self-select and to thus generate and extra surplus of MB. However this come at the cost of loosing customers. The lost in customers is

18. When the participation constraint of borrowers i is binding the element $\frac{N_i}{\partial_R N_i}$ and AI_i will be replaced by a function of the participation constraint. When the product distortion is equal to zero, the AI have a value so that the pricing equation 2.19 is equal to the break-even pooling condition plus a weighted average markup.

captured by the parameter driving the price elasticity $\frac{n_G N_G}{n_a \partial_{R_G} N_G} = \frac{1}{\sigma_G(1-N_G)}$. Again, this feedback effect is stronger when the price elasticity is high.

2.5 Equilibrium

In this section, I solve for the equilibrium contracts. In contracts with other screening models, my model features a unique pure strategy equilibrium exists. This " existence result" is important as it allows the analysis of the impact of screening when banks interact with each other without the use of equilibrium refinements. In that context, I show that there is a screening externality.

2.5.1 Existence and Uniqueness

As shown in Rothschild and Stiglitz (1976), an equilibrium may not exist in the perfect competition setting. This result rests on the existence of profitable pooling deviations that prevent banks to screen borrowers. Those deviations depend, among other things, on the relative number of types of agents. To overcome this one can use another equilibrium concept as in for instance Riley (1979), Bisin and Gottardi (2006) or Wilson (1980). The two first equilibrium concepts restore the existence of the screening equilibrium, while the third one restores the pooling equilibrium. Another way to overcome this issue has been to change the modelling. For instance, Guerrieri, Shimer, and Wright (2010) assumes that the principal can match at most one borrower. Finally, allowing banks to play mixed strategies can solve the non-existence problem (see Dasgupta and Maskin (1986) for a proof that a mixed strategy equilibrium exists, and Lester et al. (2019) or Farinha Luz (2017) for a numerical solution to those mixed strategies).

In the model presented in this paper, equilibrium exists in the pure strategy setting with general utility functions. The logit demand system leads to product elasticities and equilibrium promised utilities that prevent the pooling deviations from existing as long as the product elasticity is low enough. The reason is that any pooling deviation attracts relatively more costly borrowers so that banks cannot break even. Said otherwise, the equilibrium under imperfect competition features just enough of cross-subsidy so that any pooling deviations is not be profitable as they attract a relative number of types of agents so that the existence condition in Rothschild and Stiglitz (1976) is satisfied.

Proposition 3: Uniqueness and existence of the equilibrium
Existence: *There exist a pure strategy equilibrium as long as the product elasticities are low enough (see Appendix D.5 for the conditions). The equilibrium is symmetric*

and is given by replacing N_i by $\frac{1}{B}$ and $\frac{N_i}{\partial_{r_B} N_i}$ by $\frac{1}{\sigma(1-\frac{1}{B})}$ in the formulas (2.19) and (2.20). B being the number of banks.
Uniqueness: *In the spirit of Dixit and Stiglitz (1977), considering the limit case in which the number of banks is high enough so that the $\partial_R N_{ibi} = \sigma_i N_{ibi}(1 - N_{ibi}) \approx \sigma_i N_{ibi}$. The equilibrium is unique.*

Proof: See appendix D.5.

The conditions on the level of price elasticity for the equilibrium to exist depends on the strength of the adverse selection. This drives how profitable the cream-skimming deviations are.

2.6 Analysis of the Equilibrium Contracts

In this section, I do a positive and normative analysis of the equilibrium contracts. For simplicity, as they do not bring anything to the policy analysis, we assume that the price elasticity of both borrowers are the same ($\sigma := \sigma_G(1 - \frac{n_G}{B}) = \sigma_B(1 - \frac{n_B}{B})$).

2.6.1 Positive Analysis of the Equilibrium Contracts

In Figure 2.1, I summarize the various types of equilibrium and the regions in which they occur. To focus on risk discrimination, the figure is plotted for the case in which $\frac{\sigma_B}{\sigma_G} = 1$ and varies the level of product elasticity (σ) while keeping the ratio of product elasticities constant. I discuss how changes in this assumption affect the shape of the graph in the following paragraphs.

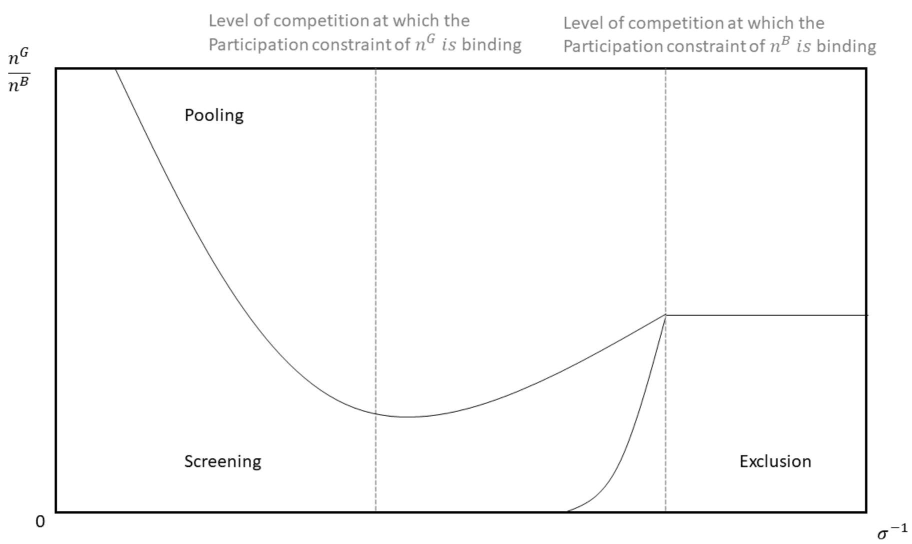


FIGURE 2.1: Equilibrium regions

I lay out the general results and explain the mechanisms behind them in the following paragraphs.

Result 1: Banks screen when competition is high enough*. Screening happens even in cases both borrowers type would prefer to be pooled.*

To explain why pooling is not feasible, let us consider a close to perfect competition situation in which banks pool borrows and break even on the lending contract. This situation implies that banks make losses in the G market segment and profits in the B market segment. This creates incentives for a competitor to offer a new contracts (L_G) with lower X and price lower price by any amount in between $[(\bar{H}-L_G)\beta_G,(\bar{H}-L_G)\beta_B]$ to attract the G borrower only. The contract is attractive to borrower G (with lower WTP for X) only as they find the decrease in interest rate more valuable than the cost of having a lower characteristic L while its the contrary for the other borrower type. This contract allows the competitor bank to make positive profits. As shown in Rothschild and Stiglitz (1976), under perfect competition, the screening equilibrium may not be possible if low default borrowers are numerous enough and both borrowers prefer to be pooled. In that case, there is a profitable pooling deviation. Indeed, the bank will attract all borrowers and make profits by offering the first-best contract characteristics and pricing just above the break-even cost. This situation does not happen in my model as long as the price elasticity is not infinite. Indeed, in that case, if banks are in a screening equilibrium, the pooling deviation attracts relatively more bad borrowers making it impossible to break even with it.

The screening deviation mentioned above implies that there is a screening externality. I analyze this externality in the net section.

Result 2: Banks tend to pool when competition decreases as long as the participation constraint are not binding*. When $\sigma_B \leqslant \sigma_G$, Banks screen when competition is high enough ($\sigma \in [\bar{\sigma},\infty)$). When the participation constraints are not binding, the switching point ($\bar{\sigma}$) is defined by:*

$$\bar{\sigma} := \frac{\left(\overbrace{\frac{(1-\Theta)\bar{L}}{\Theta}}^{\text{Level of distortion under PC}}\right)^{-1} \overbrace{\left[\tilde{\delta}\left(1+\frac{n^B}{n^G}\frac{\theta^B}{\theta^G}\right)\right]^{-1}}^{\text{Relative surplus measure}}}{\left(\overbrace{MC}^{\text{“Cost” of screening}}\right)^{-1}}$$

δ: Product distortion under perfect competition (defined in Proposition 2)
$\tilde{\delta} := \frac{\beta_G - \frac{mc}{\theta_G}}{\beta_B - \frac{mc}{\theta_G}} < 1$: Relative surplus measure

The intuition for why banks tend to pool when competition decreases comes from the fact that the product distortion is proportional to the spread between contracts (see. Lemma 3 equation (2.18)). As competition decreases, banks can increase rates by approximately $\frac{1}{\sigma_i}$. However, they increase the rate less strongly for high WTP borrowers as they want to provide an information rent to maintain borrowers incentives to self select. Thus as competition increases, the spread between interest rate decreases as long as $\frac{1}{\sigma_G} \geqslant \frac{1}{\sigma_B}$ or $\frac{1}{\sigma_G} < \frac{1}{\sigma_B}$ and the incentive to provide an information rent dominates.[19]

As shown in Proposition 2, having $\frac{\sigma_B}{\sigma_G} > 1$ does not change the figure shape. Indeed, in that case, the price discrimination incentives goes in the same direction as the risk discrimination ones. as it creates incentives to pool as the G market segment become more profitable since they are relatively more less price elastic. Contrarily, $\frac{\sigma_B}{\sigma_G} > 1$ creates incentives to screen as competition increases (keeping the ratio of product elasticities constant).

Result 3: Depending on the level of competition, changes in adverse selection ($\frac{\theta_G}{\theta_B}$) can lead to more pooling or more screening.

The result derives from Proposition 2 and proposition 2. Intuitively, under a low level of competition, the decrease in the default probability of the B borrowers ($\uparrow \frac{\theta_G}{\theta_B}$) increases the potential profits in this market. As shown in Proposition 2, this creates incentives to distort the market segment G to enable extracting more surplus from contracts designed for market B. Under a high level of competition, however, the decrease in the default probability must be passed through to the interest rate of the contract designed for the B market segment. This, in turn, relaxes the distortion in the G market segments as the high default borrowers (B) then have fewer incentives to choose low loan size contracts.

When adverse selection decreases, the equilibrium region moves from the dotted lines to the solid lines in figure 2.4.

Normative Analysis: Screening Externality and Pareto Improvement

In this subsection, I analyze the screening externality. I show that different policy interventions, such as a decrease in competition, can mitigate the externality.

Screening externality: The fact that banks must screen when competition is high enough may be striking as, in some cases, both borrowers would prefer to

19. Formally, as long as: $[\underbrace{\frac{1}{\sigma_G} - \frac{1}{\sigma_B}}_{price\ discrimination\ incentives (\lesseqgtr 0)} + \underbrace{AI_B - AI_G}_{risk\ discrimination\ incentives (>0)}] > 0$ with AI formulas given by using Proposition 2 and 3.

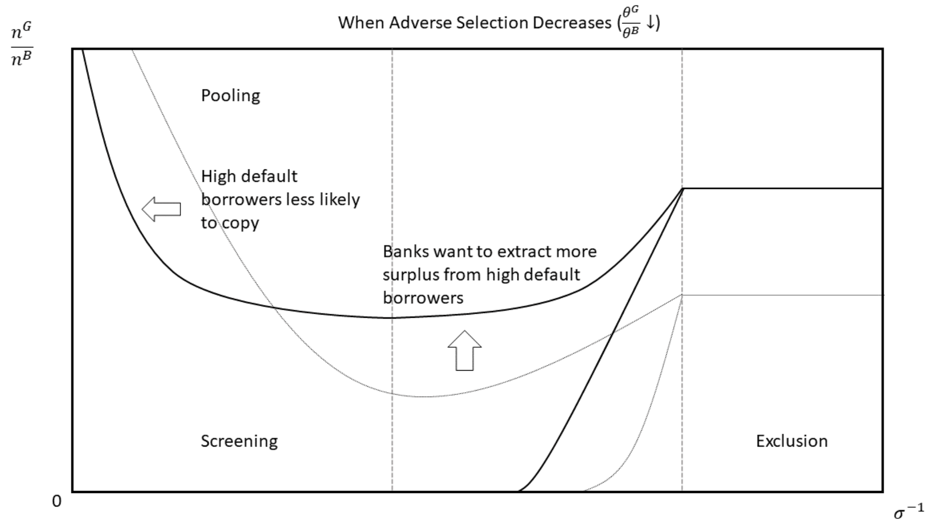


FIGURE 2.2: Equilibrium regions when adverse selection decreases

be pooled. This result is due to a screening externality: when banks design their contracts, they do not internalize that their screening behaviour affects other banks' screening ability. This externality is absent from monopoly models as, by definition, they abstract from the interaction between the actors creating menus. It has not been studied in perfect competition models due to the reliance on equilibrium refinements.

To fix ideas on the mechanisms driving the screening externality, let us consider a situation with two types of borrowers where banks price close to marginal costs. One borrower type has a higher default probability, and both prefer not to put down payments (i.e. low X in my model). Let us further assume that screening is possible: putting more down payment is relatively more costly to the high default borrower. Let us further assume that using collateral is inefficient in the first best: borrowers value the down-payment more than banks. In that set-up, both borrowers type may prefer to be pooled (i.e. getting the same contract). Indeed, the high default borrower is better of being pooled as he gets a lower rate. The low default borrower might be better off as well. This happens when the cost of being pooled (i.e. getting a higher rate) is lower than the costs of being screened (i.e. having to put more down payments). However, the pooling equilibrium may not be feasible due to so-called cream-skimming deviations. Indeed, because both borrowers pay the same price, banks make more profits on the low default borrowers. This creates incentives for a competitor bank to offer a low rate, positive but low down payment loan that will attract the highly profitable borrowers only. The key intuition is that the cost of screening (i.e. the amount of down payment required to screen borrowers) is low if the other banks keep offering the same pooling contract at the same rate. In other words, given that other banks pool borrowers, high default borrowers derive a lot of utility from the pooling contract, and thus the amount of collateral required for screening need not be large. However, as the competitor introduce the screening contracts, the low default borrowers start choosing the low X loans. Thus, banks that do not screen have to increase the rate on the pooling contract. In turn, this forces the bank that screen to ask for even more down payment. At equilibrium, each borrower chooses a different contract, and there is too much screening in the economy. Under perfect competition, the pure strategy screening equilibrium may not be feasible (Rothschild and Stiglitz (1976)) as a competitor can offer a pooling contract that is preferred by both market segments if the number of high default borrowers is low enough. In my model, this does not happen as long as the price elasticity is not infinite as the separating equilibrium feature enough cross-subsidization between borrowers so that pooling deviation will attract relatively more high default borrowers and end us being not desirable by banks.

Measuring the screening externality losses: To get a lower bound on the screening externality impact, one can look at a monopolist framework subject to the constraint of providing at least as much utility to borrowers as in the one given in a competitive market with price elasticity σ.

Let us denote those set of utility as (u_B^*, u_G^*) and use the vector $X := (L, D)$, the problem is thus:

$$maL_{\{(X^G, L_B) \in [0,\bar{L}]^2\}} \sum_i \overbrace{n_i}^{Market\ size} \overbrace{[(\beta_i - \frac{mc}{\theta_i})L_i - u_i^*]}^{profits\ on\ type\ i\ market\ segment}$$
$$s.t.\ (IC_B) : u_B^* = u_G^* + (\beta_B - \beta_G)L_G$$

Proposition 4: *Banks should offer and information rent and pool borrowers together iif:*

$$\frac{\beta_G - \frac{mc}{\theta_G}}{(\beta_B - \beta_G)} \frac{\theta_G}{\theta_B} \frac{n_G}{n_B} > 1 \tag{2.24}$$

However, except under monopoly, the competitive equilibrium switching point is different than the social planner one (2.24). For instance, according to Result 2, when both participation constraint are not binding, banks pool borrowers iif: $\sigma \in [0, \bar{\sigma}]$

The left hand side $\frac{\beta_B - \frac{mc_f}{\theta_B}}{(\beta_B - \beta_G)} \frac{\theta_B}{\theta_G} \frac{n_G}{n_B}$ is also the rate at which competition increases the information rent (ef. equation 2.38). It is increasing in the relative profit made in the G market segment relative to the B one $(\frac{\theta_G}{\theta_B} \frac{n_G}{n_B})$ and the benefits of relaxing the contracts $(\beta_B - \frac{mc_f}{\theta_B})$. It is also decreasing in the information rent costs $(\beta_B - \beta_G)$.

The right hand side is the rate at which competition increase the interest rate in the B market segment.

The gain welfare gains are:

$$\Delta W := n_G\alpha_G\Delta u_G + n_B\alpha_B\Delta u_B + \alpha_F\Delta\Pi \tag{2.25}$$

$$= n_B\alpha_B\Delta u_B + - \underbrace{n_B\theta_B(\beta_B - \beta_G)'\Delta_G}_{profit\ decreases\ default\ utility} + \underbrace{n_G\theta_G(\beta_G - \frac{mc}{\theta_G})'\Delta_G}_{profit\ increases} > 0\ \forall(\alpha_G, \alpha_B, \alpha_F) > 0 \tag{2.26}$$

α_i is a weight that normalize the utilities into the same unit. For instance, if the loan is not recourse and borrowers are risk neutral then the weights are θ_G,θ_B and 1. It that case everything is expressed in monetary units. Δu_i is the increase in utility of borrower of type i Δ_G is the difference between $\bar{L}$ and the banking equilibrium.

Back of the envelope calculation: Considering only two types of borrowers and using the estimates similar ofRobles-Garcia (2019) for the X and price elasticity, I get:

The perfect competition contracts would be 95 and 87 X. The Information rent allows to get the imperfect competition contracts at 95 and 92 X. T The equilibrium contracts rates are 5.5 at 95 X and 4.8 at 90 X. The information rent are 0.26 for B and 0.25 for G. There is a screening externality, the monetary equivalent cost of the imperfection is costs 10 basis points per contract. If banks could get this dead-weight loss it would corresponds to a 30 percents increase in profits. When the surplus is gave back to borrowers this would lead to 120 £ payments reduction per year on a 30 year mortgage of 180,000£. When the default spread is 0.1 instead of 0.05, the cost rises to 50 basis points (600£ per year).

Effect of competition, adverse selection, relative profits in each market segment on the screening externality: We can look at how changes in the economic environment affects welfare. I define the set of changes leading to an increase of utility to both borrower type (Pareto set).

Has hinted by Proposition 3, decreasing competition can welfare increasing as long as some borrowers are credit constrained. Indeed, decreasing competition can restore banks ability to pool borrowers, low default probability borrowers are happy to be pooling if the cost of screening is too high (i.e. high credit constraint). High default borrowers are happy to be pooled since they a lower rate. This is shown formally in the following result:

Results 4 *The impact of competition on welfare is ambiguous. Decreasing competition is welfare increasing for both borrower type as long as $\sigma \in [\bar{\sigma}, \infty)$ and the participation constraint of G is not binding:*

$$G\ borrowers: \quad [\beta_G - \frac{m_c}{\theta_G}]\overbrace{[\beta^B - \frac{mc}{\theta^G}]^{-1}\sigma(AI_B - AI_G)}^{Lower\ L\ distortion} > \overbrace{1}^{Increase\ mark-up} - \overbrace{\sigma AI_G}^{Cross-Subsidy} \tag{2.27}$$

$$B\ borrowers: \quad \overbrace{\sigma AI_B}^{Cross-Subsidy} > \overbrace{1}^{Increase\ mark-up} \tag{2.28}$$

The less high default borrowers there are (n^B) relative to high default borrower (n^G) or the lower is their default probability relative to low default borrower ($\frac{\theta^B}{\theta^G}$ high), the better decreasing competition is. The reason being that pooling is not costly for safer borrowers in that case. The results also depends on the difference in willingness to pay. The mode similar they are, the more effective is the cross-subsidy to lower the credit constraint since lenders can increase interest rate more without the treat of high WTP to copy the low WTP contract. Notice that those parameter, if they affect positively the effectiveness of decreasing competition, they also lower the threshold at which decreasing competition is not welfare increasing anymore. The results are summarized in the following graphs.

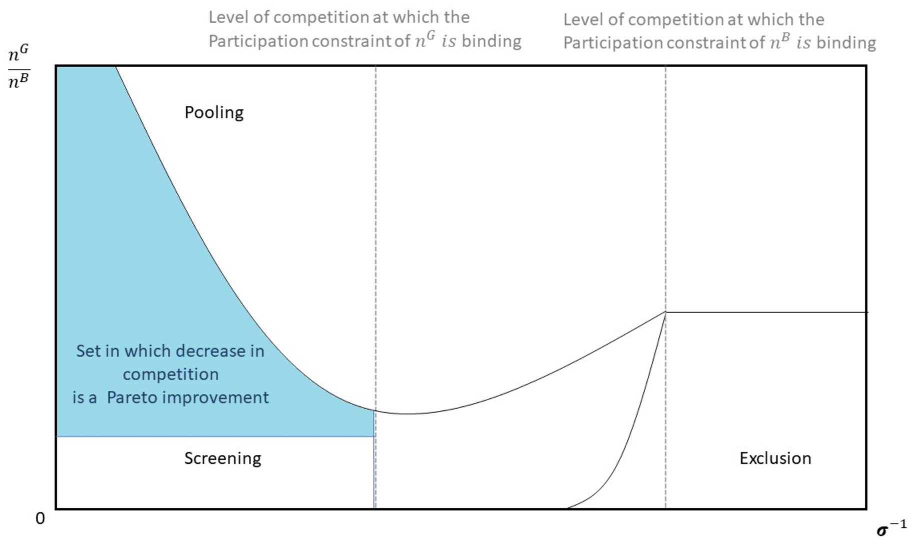


FIGURE 2.3: Pareto set

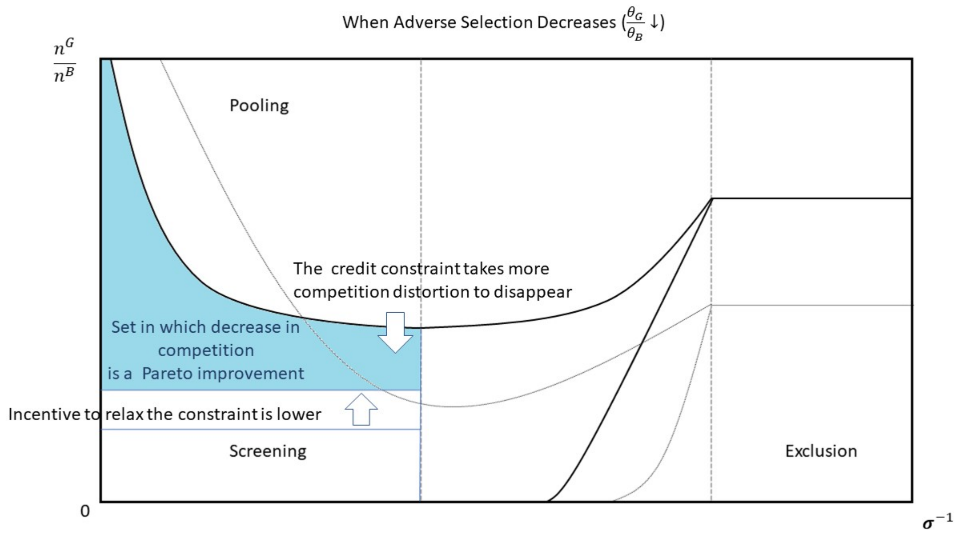


FIGURE 2.4: Pareto set when adverse selection decreases

2.6.2 Policy Interventions

In this section, I analyze the positive and normative impact of marginal cost policies (i.e. capital requirements), preferences and marginal costs policies (i.e. government guarantee schemes).

For simplicity, as they do not bring anything to the policy analysis, I assume that the price elasticity of both borrowers are the same ($\sigma := \sigma_G(1-\frac{n_G}{B}) = \sigma_B(1-\frac{n_B}{B})$).

The key point of those policies is that they are designed differently for each specific product. For tractability, it will be useful to model those policies as a piecewise linear function over some product characteristics. As a result, it is convenient to also introduce discontinuities in the marginal costs at the same thresholds so that the first best products are different for each borrower. This assumption makes sure that the demand is continuous. Formally, We consider a situation in which the marginal cost of lending at high X is higher above a certain threshold $\bar{T}$:

$$mc(X) = mc_1 + 1_{X>\bar{T}}(mc_2 - mc_1) \tag{2.29}$$

Borrowers have preference such that:

$$\beta_G \geqslant \frac{mc_1}{\theta_G},\ \beta_G < \frac{mc_1}{\theta_G} \tag{2.30}$$

$$\beta_B > \frac{mc_2}{\theta_B},\ \beta_B \geqslant \frac{mc_2}{\theta_B} \tag{2.31}$$

To enlighten the impact of the poly on the distortions, I will consider that they are not large enough to change the above ordering.

Policy experiment 1: Effect of changes in capital requirements

Capital requirements are often based on Loan-to-Values ratios (for instance in Basel III). For this reason, I model capital requirements base on LTV. I consider that the marginal cost have now the form:

$$mc_c = mc - \omega^l \frac{1}{LTV} 1_{LTV<L\bar{T}V} - (\omega^h - \omega^l)\frac{1}{LTV} 1_{LTV \geqslant L\bar{T}V} \tag{2.32}$$

$\omega\frac{1}{LTV}$ being the capital requirements and ω capturing how the capital requirements vary with the loan leverage. A positive ω^l (or ω^h) implies that capital requirements are increasing in LTV when LTV is below (above) a threshold. Using the fact that $LTV := \frac{L}{H}$, this specific functional form is equivalent to redefining the γ parameter as $(1-\theta_i)(\gamma - \frac{\omega}{(1-\theta_i)})H$ in our our previous model.

Assumptions 5: Let us assume that the increase in capital requirements is not high enough to make the NPV of lending negative.

For simplicity of the exposition, Let us assume that $\frac{1}{\epsilon} < \frac{mc}{\theta}$ so that the optimal contract features screening on loan size and a maximum amount of deposit D. This last assumption do not impact the results.

Using Proposition 2 and Proposition 2, the equilibrium distortions can be written:

$$L_G = \underbrace{\frac{\tilde{\beta}_B^L - \frac{\tilde{mc}^L+\omega^h}{\theta_B}}{\tilde{\beta}_B^L - \frac{\tilde{mc}^L+\omega^l}{\theta_G}}(\bar{H}-\bar{D})}_{Fair\ price\ effect} + \underbrace{\frac{AI_B - AI_G}{\tilde{\beta}_B^L - \frac{\tilde{mc}^L+\omega^l}{\theta_G}}}_{Asymmetric\ information\ discount\ effect} \leqslant \bar{H} - D_G^{PI}\ when\ \sigma\ high\ enough \tag{2.33}$$

$$Asymmetric\ information\ discount\ given\ to\ type\ B:$$

$$IR_B := \frac{1}{\sigma}\frac{\tilde{\beta}_G - \frac{mc^L+\omega^l}{\theta_G}}{(\tilde{\beta}_B - \tilde{\beta}_G)}\frac{\theta_G}{\theta_B}\frac{n_G}{n_B} > 0 \tag{2.34}$$

$$Asymmetric\ information\ premium\ paid\ by\ type\ G:$$

$$IR_G := \frac{1}{\sigma}\frac{\tilde{\beta}_G - \frac{\tilde{mc}^L+\omega^l}{\tilde{\theta}_G}}{(\tilde{\beta}_B - \tilde{\beta}_G)} > 0 \tag{2.35}$$

Proposition 5: *(i) And increase in low LTV capital requirements (ω^l) have an ambiguous effect on contract characteristics distortion. Under high competition the distortion tend to increases. Under low competition, it decreases.*

(ii) The welfare of the low LTV market segment changes in the same direction as the low LTV. The welfare in the other market segment is ambiguous; welfare tends to decrease following the policy intervention when competition is low; it increases otherwise.

PROOF: Appendix D.7

(i) The ambiguous result is due to two opposing effects. I call the first effect the "Fair price effect". Providing a government guarantee to high X loan will lower the cost of high X lending. Under a high level of competition, the decrease in the cost of lending must be passed through to the high X interest rate. This, in turn, relaxes the distortion in the other market segments as borrowers shopping in high X markets then have fewer incentives to choose low X contracts.

The second effect goes into an opposite direction. I call it the "Asymmetric information discount effect". Under a low level of competition, the decrease in the cost of lending to the high X market segment increases profits in this market. As shown in Proposition 2, this creates incentives to distort the other market segments to enable extracting more surplus from high X loans.

(ii) The effect on the high X is ambiguous because, under low level of competition, the increase in profits makes banks less willing to provide an information rent as they want to extract more surplus from this market segment.

The same two channels do not appear when changing ω^h) in my model. This is because of the kink in the housing utility function. Without this, both channels would be present as well.

Policy experiment 2: Effect of the UK mortgage guarantee scheme (Guarantee of high LTV loans)

As shown in Figure (**??**) the COVID-19 pandemic has led to a reduction in the availability of high loan-to-value (X) mortgage products. This is particularly true for mortgage buyer willing to put 5% of deposits. In order to help those borrowers climb the property ladder, the government has introduced a government guarantee scheme. This scheme provides a guarantee to lender that compensates them for a portion of their losses in the event of foreclosure. This scheme was available to any first time buyers as long as their property value was less that £600,000 and their loan had an X above 91%.[20]

20. Details on the government guarantee scheme is provided at: "https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment$_{d}ata/file/$965665/210301$_{Budg}$

Effect on demand: Using the micro-foundation in section **??**, we have that the willingness to pay for loan size is driven by:

$$\tilde{\beta}_i = 1 + \frac{1}{\theta_i + (1-\theta_i)es} \tag{2.36}$$

es is a positive parameter driving how costly it is to default. As defaulting is more likely to happen for low survival probability borrowers (i.e. low θ_i), the government guarantee is likely to be more beneficial to those borrowers and thus increase the spread between borrowers' willingness to pay. As the preferences are now a function of the policy and thus of the product chosen, we use the notation β_i^j for the preference of borrower j when choosing contract i. We consider that choosing a non-guaranteed contract lowers the willingness to pay by γ. We allow for the effect to differ based on the contract chosen and denote it $\beta_i^j := \gamma_i^j \beta_j$.

Effect on supply: Let us consider the following scenario: the government guarantee scheme is beneficial to banks I model this as an increase g in the survival probability for loans:

$$\theta R + \underbrace{g^l R}_{extra\ profits\ from\ government\ guarantee} 1_{LTV<L\bar{T}V} - (g^h - g^l)\frac{1}{LTV} 1_{LTV \geqslant L\bar{T}V}$$

With those assumptions, the equilibrium distortion can be written:

$$L_G = \underbrace{\frac{\gamma_B^B \beta_B - \frac{\tilde{m}c^L}{\theta_B + g^h}}{\gamma_G^B \beta_B - \frac{\tilde{m}c^L}{\theta_G + g^l}} \bar{L}}_{Fair\ price\ effect} + \underbrace{\frac{AI_B - AI_G}{\gamma_G^B \beta_B - \frac{\tilde{m}c^L}{\theta_G + g^l}}}_{Asymmetric\ information\ discount\ effect} \leqslant \bar{H} - D_G^{PI}\ when\ \sigma\ high\ enough \tag{2.37}$$

$$Asymmetric\ information\ discount\ given\ to\ type\ B:$$
$$IR_B := \frac{1}{\sigma} \frac{\gamma_G^G \beta_G - \frac{\tilde{m}c^L}{\theta_B + g^l}}{(\gamma_G^B \beta_B - \gamma_G^G \beta_G)} \frac{\theta_G + g_G}{\theta_B + g_B} \frac{n_G}{n_B} > 0 \tag{2.38}$$

$$Asymmetric\ information\ premium\ paid\ by\ type\ G:$$
$$IR_G := \frac{1}{\sigma} \frac{\beta_G - \frac{\tilde{m}c^L}{\theta_G}}{(\gamma_G^B \beta_B - \gamma_G^G \beta_G)} > 0 \tag{2.39}$$

Proposition 6: *(i) An increase in the government guarantee has an ambiguous effect on contract characteristics distortion. Under high competition, X increases. Under low competition, X decreases. The bigger is the effect on the relative preferences, the more likely X increases.*
(ii) The welfare of the low X market segment changes in the same direction as the low X. The welfare in the other market segment is ambiguous; welfare tends to decrease following the policy intervention when competition is low; it increases otherwise.

PROOF: Appendix D.7

2.7 Conclusion

The main contribution of the paper is to provide a the first analysis of the screening externality and of policy intervention in a market where principals use menu to screen agents. The main technical contribution of the paper is to build a well behaved screening models in which the equilibrium exist without the use of equilibrium concepts refinements.

Chapter 3

Regulation complexity in the banking market

3.1 Introduction

Prudential regulations in the banking sector have grown increasingly complex. For example, since 2008, European rules have become more interconnected, longer and more numerous. As a result, the average number of words a reader has to process starting from a single rule increased from about 600 words to over 25,000 (Amadxarif et al. (2019)).

On the one hand, designing comprehensive rules can mitigate market inefficiencies such as moral hazard by changing market participants' incentives. On the other hand, policymakers are growing concerned that fine-tuning regulations also increase the amount of resources banks need to spend to understand, interpret and implement the rules. Hence, regulations may increase banks' operating costs and create a barrier to entry or growth. This concern is supported by recent surveys[1] documenting that the average bank spends the equivalent of a three percent yearly interest rate on their total asset size to comply with regulations. Consequently, central banks are currently considering simplifying the prudential regulation regime, especially for small banks (see for instance Sam Wood (2019) speech).

In this paper, I provide the first formal analysis of the above-mentioned trade-off. I apply my analysis to the case of heterogeneous capital requirements regimes. To that end, I build a novel model of banking in which setting individualized capital requirements allows to better deal with each lender's excessive lending behaviour. However, creating different capital requirements also increases lenders' fixed cost of understanding and interpreting the regulation. Changes in the fixed cost endogenously affect the market structure and bank interest rate markups. Those changes feed back into lenders' incentives to over-lend. Due to this general equilibrium effect, I show that increasing capital requirement heterogeneity can have a negative impact on the friction it was designed to reduce. Motivated by this theoretical result, I develop a sufficient statistic approach to empirically assess the impact of regulation complexity on welfare.

In my set-up, lenders engage in excessive lending because they do not bear the full cost of defaulting due to deposit insurance (as in, for instance, Bahaj and Malherbe (2020)). In that context, capital requirements can reduce the deposit insurance friction by changing banks' marginal lending cost.

The main novelty of the analysis is to consider the complexity of prudential regulations, defined here as the number of different capital requirements set by the central bank. I capture the benefit of lender-specific capital by allowing lenders to have heterogeneous productivity and offer differentiated loan products. As a result, the level of capital requirement that undo the deposit insurance friction is lender-specific.

1. See the CRD or the BSA surveys on reporting and staff costs from the Bank of England.

However, individualizing capital requirements increase lenders' fixed cost of operating a bank. This cost can be interpreted as a cost of understanding (Becker (1965) on time allocation), interpreting (Basak and Buffa (2019) on complexity and operational risk) or implementing the rules. This cost has been generally abstracted from policy discussions and modelling but has been shown to be substantial (see Sam Woods 2019 speech).

To analyze the general equilibrium effect of capital requirement heterogeneity on competition through banks' entry and exit decisions, I adapt the approach developed in the international trade literature (see Melitz (2003) and Melitz and Ottaviano (2008)) to the banking sector. More specifically, I use Ottaviano, Tabuchi, and Thisse (2002) demand system. This demand system captures two dimensions of competition: (i) how substitutable products from different banks are and (ii) how efficient are competitors in producing their products. As a result, the interest rate mark-ups are a function of both the number of operating banks and their productivity. A key assumption is that each lender offers differentiated loan products so that the number of banks captures how likely a competitor is to offer a similar or more adapted product. Overall, a bank operating in a market with numerous and/or very productive competitors has incentives to lower its interest rates to stay competitive.

In that setup, regulation complexity creates two opposing forces on competition and banks' profits through lenders' entry and exit decisions. The higher cost of understanding the law creates a barrier to entry. So, all else equal, fewer banks enter the market. The smaller number of firms means fewer product types are offered, pushing lenders' incentives to increase markups. However, the higher complexity cost tends to also push the less productive banks out of the market. This channel increases competition as the average competitor becomes more productive. The net effect on lenders' markups and profits is thus ambiguous and depends on lenders' productivity distribution and the degree of product differentiation.

This market structure effect feeds back into the deposit insurance friction. Stronger competition lowers the interest rate markup, increasing banks' default probability and/or the loss given default. Consequently, the value of deposit insurance is higher and so is the amount of excessive lending generated by it. Overall, if designing lender-specific capital requirements increase lenders' cost of operating due to its complexity, this can impact positively or negatively mark-ups, changing the value of deposit insurance and thus the amount of moral hazard it generates.

Given the ambiguous impact of increasing capital requirement complexity on welfare, I derive a sufficient statistic approach and discuss how it could be implemented in practice using available data.

This paper is related to the literature on capital requirements, the literature on complexity and the international trade literature.

The traditional approach in the literature considers that the role of capital requirements is to mitigate the moral hazard problem generated by limited liability.[2] The vast majority of the literature abstract away from capital requirement heterogeneity by using a representative bank approach. We contribute to this literature by developing a framework which captures the impact of capital requirement heterogeneity on competition.

Our modelling of the cost and benefits of prudential regulation is related to the literature on complexity. Most of the existing literature in finance and economics focused on how firms can use complexity as a means to influence a consumer's purchasing decisions so that sellers can extract more surplus from their customers.[3] In finance, obfuscation through complexity has been studied mainly in the context of financial securities.[4] In contrast, my model applies to financial regulation. My approach to complexity is similar to Oehmke and Zawadowski (2019) as I consider that complexity can be value-enhancing.

Finally, this paper is related to the international trade literature on firms' decisions to engage in international trade (see Melitz (2003) seminal paper). The literature applies this approach to non-financial firms. I contribute to the literature by adapting the approach to the specificities of the banking sector to study the interaction between capital requirements regulations and the market structure.

3.2 Baseline Model

There are three periods, indexed by $t \in \{0, 1, 2\}$. The economy is populated with two types of agents: Borrowers and Investors. There is a continuum of identical borrowers of mass 1 and a continuum of investors. Both agent types are risk neutral and maximize their expected profits in period 2. They have access, in all periods, to a risk-free outside investment opportunity yielding a net return of 0.

Borrowers are penniless, while Investors start with an endowment in period 0. Each Investor can become an entrepreneur in period 0 by creating a bank to lend to borrowers in period 1. Banks have heterogeneous productivity and offer differentiated loan products. Investors observe their productivity draw after deciding to create a bank and can choose to exit the banking business after observing their productivity draw at the end of period 0.

Borrowers' loans are contracted in period 1 and repaid in period 2.

2. See for instance, Keeley (1990), Hellmann, Murdock, and Stiglitz (2000), or Malherbe (2020) and Bahaj and Malherbe (2020) for recent examples

3. SeeCarlin (2009), Carlin and Manso (2011), Ellison and Wolitzky (2012), Piccione and Spiegler (2012), Hefti (2018).

4. See for example Carlin, Kogan, and Lowery (2013), Ghent, Torous, and Valkanov (2019), Ganglmair and Wardlaw (2017).

In section 3.5 I introduce a Social Planner that sets capital requirements and provides deposit insurance.

3.2.1 Borrowers

For illustration, I interpret borrowers as firms, but the functional forms are also adapted to household borrowers (see section 3.3 for a household borrower micro-foundation). Firm i borrows an amount x from bank j in period 1 and produce $Z \cdot F(x)$ in period 2. Z is a random variable whose realization occurs in period 2. The productivity shock is Z = 1 with probability θ_i and 0 with probability $1 - \theta_i$. When Z=1, the firm repays the face value of the debt $R(x)$. The borrowers maximizes $E_Z[Z \cdot (F(x) - R(x))]$. The aggregate loan demand for a particular bank j is denoted $q^j(\cdot)$. I drop the j for simplicity of the notation in the next section.

3.2.2 Investors-Banks

There is a continuum of banks of mass N. The mass of banks will be pinned down by investors' entry and exit decisions into the banking business in period 0.

Period 1 and 2 — In the first period, a bank set its interest rate knowing its demand for loan $q(\cdot)$. The total amount lent q is funded with capital (κ) and deposits (d) so that:

$$q = d + \kappa \tag{3.1}$$

Banks face capital regulation. Capital (κ) must be bigger than a risk-weighted (λ) share of the amount lent (q):

$$\kappa \geqslant \lambda q \tag{3.2}$$

In the second period, the bank gets repaid $R \cdot X$ with X being a random variable in [0,1] with a CDF $M(\cdot)$. X represents the share of the face value R that is repaid. The bank then uses $R \cdot X$ to repay its debt (d), capital (C(X)) and cost of operating the bank $q\xi + \mu$. ξ represents the inverse productivity of the bank. It can be interpreted as the amount of labour and capital the bank need to manage q loans.

The bank cannot set C(X) smaller than 0 and thus default if $d + q\xi + \mu > RX$, so the threshold ($\bar{x}$) for X under which the bank default is:

$$\bar{x} := \frac{d + \mu}{R} + \xi \tag{3.3}$$

Deposits and capital are raised from risk-neutral Investors. They can be interpreted as Entrepreneurs that did not create a bank in period 0. They have access to an investment opportunity with a net return equal to zero. There is deposit insurance. Capital and debt market are thus priced according to:

$$\kappa := \frac{\int_{\bar{x}}^{1} C(X)dM(X)}{\delta} \tag{3.4}$$

$$d := \frac{\int_{0}^{1} d \cdot \tilde{r}\ dM(X)}{\delta} \tag{3.5}$$

with $C(X) \geqslant 0$ with an outside net rate of 0 in period 1 (so the discount rate δ is equal to 1) we get that the interest rate paid on debt is $\tilde{r} = 1$.

Banks are risk neutral, in period 1, they maximize their expected profits in the second period, knowing the demand and the pricing:[5]

$$max_{\{q,R,c(X)\geqslant 0,d\}} \int_{\bar{x}}^{1} R \cdot X - d - C(X) - \mu - q\xi\ dM(X) \tag{3.6}$$

$$\begin{aligned} s.t.\ q &= d + \kappa \\ \kappa &\geqslant \lambda q \\ \kappa &= \int_{\bar{x}}^{1} C(X)dM(X) \\ \underbrace{q}_{credit\ supply} &= \underbrace{q(R)}_{credit\ demand} \end{aligned}$$

Denoting the pricing schedule $R = q \cdot r$, the survival probability $p := \int_{\bar{x}}^{1} dM(X)$, using the budget constraint, the collateral requirement and the pricing equation, the problem in period 1 simplifies to:

$$max_{\{r\}} \overbrace{q(r)}^{Amount\ lent} \overbrace{[r \cdot E[X] - (1+\xi)] - \mu}^{Economic\ Surplus} \tag{3.7}$$

$$+ \overbrace{(1-p)\underbrace{q(r)[(1-\lambda+\xi) - r \cdot \underline{X}] + \mu}_{Loss\ given\ Default}}^{Deposit\ Insurance\ Value}]$$

The first term $(q(r)[r \cdot E[X] - (1+\xi)] - \mu)$ represents the bank's profits from the deposit insurance friction. On average, a share $E[X]$ of the clients do not default and repay the face value of their debt $q \cdot r$. The costs of the bank consist of the marginal

5. due to the deposit insurance, deposits are cheaper than collateral so the capital requirement will be binding

cost of funding plus the marginal cost of managing q loans $(1+\xi)$ multiplied by the amount lent q, and the fixed cost of banking μ.

The second term $((1-p)q[(1-\lambda+\xi)-r\cdot\underline{X}]+\mu)$ is the distortion coming from the deposit insurance. As in Merton (1974), this can be interpreted as the value of a put option. It is the value of the deposit insurance. The value is the loss given default $(q[(1-\lambda+\xi)-r\cdot\underline{X}]+\mu)$ multiplied by the probability of default $(1-p)$. The higher is the capital requirements (λ), the more the losses are absorbed by equity holders and thus the lower is the deposit insurance distortion. For a given amount lent and rate, low-productive banks lose more upon default as they face a higher cost of lending. Similarly, a low-interest rate, everything else equal, prevents the non-performing loans to be paid by gains from borrowers that do not default $(r\underline{X})$.

The value of insurance can be negative if capital requirements are too high. To minimize the interest rate distortion coming from the deposit insurance, one should set the capital requirements to $((1+\xi)-r\cdot\underline{X})$. This is different than minimizing the value of the insurance — which would yield $((1+\xi)-r\cdot\underline{X})+\frac{\mu}{q})$ — as the fixed cost has no impact on the interest rate distortion. This is not a general statement, as this would not be the case if the default rate were a function of the interest rate. The fixed cost parameter entering the deposit insurance will, however, impact entry and exit decisions. The central property needed for this paper's results is that the capital requirement minimizing the deposit insurance distortion is a function of the interest rate or any other variables impacted by the market structure.

Period 0 — Entry decision: Each investor starts with an endowment f. They maximize their expected profit in period 2 and have a discount rate denoted δ and is equal to 1. They have access to an investment opportunity in periods 0 and 1 with a net return equal to 0. At the beginning of period 0, before knowing his productivity, an entrepreneur chose to start a bank and pay the cost f if the expected gross return of running a bank is higher than his opportunity cost (i.e. f since the net rate is 0). Denoting $\pi(\xi)$ the profits when $\mu=0$ for a bank with productivity ξ^{-1}, the expected profit for an investor investing f in period 0 that does not know the productivity draw is thus:

$$E_{\xi}[\pi(\xi)-\mu+(1-p)\mu-f]\geqslant 0 \tag{3.8}$$

Entrepreneurs are homogeneous when they make their entry choice. With the demand specified in the next section, profits will be a function of the number of banks. At equilibrium, this condition is binding and entrepreneurs play a mixed strategy.

Exit decision: At the end of period 0, after knowing his bank's productivity realization and after having paid the fixed cost f, an entrepreneur chooses to exit the market a bank if it yields negative expected profits. Formally, an entrepreneur stays in the market if:

$$\pi(\xi) - \mu + (1-p)\mu \geqslant 0 \tag{3.9}$$

3.2.3 Introducing Capital Requirement Complexity

Following the Natural Language Processing literature, I link the complexity of prudential regulations to the number of different capital requirements (denoted $(\lambda_i)_i$) used by the social planner and how different they are (captured by the variance of λ). This modelling captures the idea that using different regulation tools makes the rule book lengthier and harder to process (Amadxarif et al. (2019)) as it increases its lengths, the number of exceptions or cross-references. Overall, it makes it more likely for a bank to make a mistake in reporting (Basak and Buffa (2019) for sophistication and operational risk) and increase the time one has to spend to understand the law or increase the need to hire a specialist to comply to the rules.

Formally, the complexity costs are captured by making the fixed cost f and the marginal cost of lending μ introduced in the previous section increasing functions of the complexity of capital requirements.

The benefit of having heterogeneous (i.e. complex) capital requirements come from deposit insurance and banks' heterogeneity in their lending cost (ξ). As shown in equation (3.7), this modelling implies that the value of the deposit insurance differs across banks. Therefore, the capital regulation (λ) that minimizes the deposit insurance distortion needs to be tailored to each bank's productivity.

In practice, the need for heterogeneous capital requirements could also result from banks' different portfolio riskiness (see Paravisini and Rappoport (2020) for evidence on banks' specialization), banks' different systemic importance, or changes over time in banks' moral hazard behaviour. As the paper's focus is not quantitative, modelling on one channel through which heterogeneous capital requirement is potentially welfare-improving is enough to illustrate the theoretical trade-off.

In the appendix E.1, I provide a micro-foundation of complexity based on a limited attention framework as Gabaix (2019). It leads to a fixed cost function that is increasing in the number of capital requirements and their variance.

All in all, the model captures the following complexity trade-off. On the one hand, setting very detailed prudential regulations allows mitigating the deposit insurance friction. On the other hand, implementing a rule book addressing all those specificities can be counterproductive. Indeed, this creates a cost as banks have to take time to understand, implement and comply with those rules.

3.3 Functional form assumptions

To solve the model tractable, I make the following functional form assumptions.

Individual and aggregate default— Let us consider that there are two states of the world (good and bad) that occur with probability p and $(1-p)$. In each state of the world, borrowers' default probability is iid so that a fraction $\bar{X} > 1/2$ of borrowers do not default in the first state (good state), and $\underline{X} < \bar{X}$ default in the other state (bad state). Before the aggregate state is known, the individual default probability is $\theta := p(1-\bar{X}) + (1-p)(1-\underline{X})$.

Thus, from the point of view of the banks, the share of borrowers defaulting is:

$$X := \begin{cases} \bar{X} & \text{with probability p} \\ \underline{X} & \text{with probability 1-p} \end{cases} \tag{3.10}$$

The deposit insurance subsidy is thus: $(1-p)[(1-\lambda)q - \underline{X}R]$.

Loan Contracts — The banks offer a linear pricing schedule $(x, r \cdot x)$, x being the principal and $r \cdot x$ the face value. Borrowers choose x. Banks choose the pricing schedule r.

Loan demand — As in Melitz and Ottaviano (2008), we consider a representative borrower that has a quadratic utility for loan quantity (q) coming from various banks. Each bank (ω) produce one variety of loan that cost ($r(\omega)q(\omega)$).

Melitz and Ottaviano (2008) micro-found their demand functional form using the following consumer maximization problem

$$\begin{aligned} max_{q(\omega)} \; & q_0 + \alpha \int q(\omega)d\omega - \gamma\frac{1}{2}\int q(\omega)^2 d\omega - \nu\frac{1}{2}(\int q(\omega)d\omega)^2 \\ s.t. \; & \int r(\omega)q(\omega)d\omega + q_0 = E \end{aligned} \tag{3.11}$$

q_0 is the numeraire.

Alternatively, I propose the following micro foundation in order to apply the demand for firms. The production function of a firms is defined as $F(q) := \alpha \int q(\omega)d\omega - \gamma\frac{1}{2}\int q(\omega)^2 d\omega - \nu\frac{1}{2}(\int q(\omega)d\omega)^2$. The firm thus maximizes:

$$max_{q(\omega)} \; \underbrace{\alpha \int q(\omega)d\omega - \gamma\frac{1}{2}\int q(\omega)^2 d\omega - \nu\frac{1}{2}(\int q(\omega)d\omega)^2}_{Production} - \underbrace{\int r(\omega)q(\omega)d\omega}_{Cost} \tag{3.12}$$

γ (> 0) indexes the degree of product differentiation between loan varieties. In the limit of $\gamma = 0$, goods become perfectly homogenous, so that consumers only care about the total amount of differentiated goods they consume, not which specific variety. As γ increases, consumers care more and more about the distribution of consumption over all varieties, so that goods become more and more differentiated.

When interpreting F as a production function, the γ parameter can be interpreted as a preference for dispersed ownership as in Zhong (2020) or a need for different type of loans. Indeed, this parameter, when high, the cost of having a concentrated creditor is high. The ν parameter can be understood as decreasing return to scale.

α and ν (> 0) index the substitution between the differentiated varieties and the homogenous good. Both parameters shift out the demand for differentiated varieties relative to the homogenous good.

Both micro-foundations (3.11) and (3.12) yield the following loan demand for (q) for a particular bank:

$$r = \alpha - \gamma q - \nu Q \tag{3.13}$$

with Q being the aggregate demand ($\int q(\omega) d\omega$). This gives:

$$q(r) = \frac{\alpha}{\nu N + \gamma} - \frac{1}{\gamma} r + \frac{\nu N}{\nu N + \gamma} \frac{1}{\gamma} \bar{r} := \frac{1}{\gamma}[c_d - r] \tag{3.14}$$

with $\bar{r}$ the average interest rate. As the average price $\bar{r}$ goes down, or as the number of competitors N increases, the environment gets more competitive, and the price elasticity of demand increases.

Complexity — To simplify the Social planner problem, I restrict its choice to a uni-dimensional variable $\mathcal{C}$ on the $[0, 1]$ real line instead of choosing the number of different capital requirements as well as their value. By restricting the planner choice, I take into account, in a reduced form way, contract incompleteness: the social planner may not have enough information on banks' productivity and is thus limited in their regulatory design.

To simplify the formulas, I set $\underline{X} = 0$ and define the capital requirements as:

$$\lambda(\mathcal{C}) := \mathcal{C} \cdot (1 + \xi) \text{ or } \lambda(\mathcal{C}) := (2 - \mathcal{C}) \cdot (1 + \xi), \text{with } \mathcal{C} \in [0, 1] \tag{3.15}$$

When $c = 1$, the deposit insurance friction is equal to zero, when $c = 0$ the deposit insurance friction is maximum. The variance of capital requirements is thus an increasing function of c and hence so is the complexity cost ($f(c)$). The first formulation implies that complexity increases the cost of lending by going from too low capital requirements. The second formulation implies the opposite: at low level of complexity the capital requirements are too high.

3.4 Model Analysis

3.4.1 Banks' FOC

The bank first order conditions imply that the interest rate is:

$$r^*(m) = \frac{1}{2}[c_d + m] \tag{3.16}$$

Banks' profits after the complexity cost has been paid are:

$$\pi^*(c) = \frac{1}{4\gamma}[max\{c_d - m, 0\}]^2 \tag{3.17}$$

c_d is a parameter that captures how competitive is the banking market ($c_d = \frac{\gamma\alpha+\nu N\bar{r}}{\nu N+\gamma}$). It depends on how differentiated the products are (γ) and the number of different products that are offered (N). m is a parameter that captures the potential surplus generated by lending to a given borrower ($m := \frac{C}{A}$). It is a function of the marginal surplus $A := EX - (1-p)\underline{X}$ generated by the firm and the marginal cost of lending $C := 1 + \xi - (1-p)(1 - \lambda + \xi)$.

3.4.2 Entry and exit conditions at equilibrium

To determine the general equilibrium of this economy, we need to solve for the total number of banks (N), and the cost threshold ($\bar{\xi}$). To do so, we use the free entry condition and the exit conditions.

Let us assume that the regulation cost is non monetary (so it has no subsidy effect). The entry and exit conditions are thus:

Entry condition: Before knowing its productivity draw, the bank choose to enter if it is profitable in expectation. Banks are homogeneous ex-ante and play a mixing strategy on their entering decision. The free entry condition gives us at equilibrium:

$$E_\xi[\pi(\xi, N, \lambda(\mathcal{C})) - f(\mathcal{C})|\xi \leqslant \bar{\xi}, N] = 0 \tag{3.18}$$

$\pi(\xi, N, \lambda(c))$ is the profit of a bank with productivity draw ξ when operating in a market of size N and with firms with productivity draws on $[0, \bar{\xi}]$ and capital requirement policy $\lambda(c)$. $\pi(\xi, N, \lambda(c))$ is a decreasing function of N and $\lambda(c)$. As a result, $E_\xi[\pi(\xi, N, \lambda(c))|\xi \leqslant \bar{\xi}]$ also decreases in N. When the cost t increases, all other things equal, more firms enter the market.

Proposition 1: Entry *All other things equal, an increase in complexity (c) creates a barrier to entry (increases in $f(c)$)). An increase in complexity also sets capital requirements closer to their optimal value. This may increase or decrease the cost of lending depending on whether capital requirements for a particular bank were too high or too low (decreases or increase in $\lambda(c)$)*[6]*. Overall, everything else equal, complexity lowers incentives to enter the market if both effects go in the same direction or if the increase in the fixed cost dominates the impact of the potential decrease in the marginal cost of lending. An increase in complexity (c) thus directly reduces or increases the market size (N).*

Exit condition: After learning its productivity and paying the complexity cost bank exit the market if its expected profit is negative when it learns its productivity draw. At equilibrium, banks exit the market if their productivity draw is below $\bar{\xi}^{-1}$. This threshold is given by:

$$\pi(\xi, \bar{\xi}, N, \lambda(\gamma)) := \frac{1}{4\gamma}[max\{c_d(N, \bar{r}(\bar{\xi})) - C(\xi, \lambda(\mathcal{C})), 0\}]^2 = 0 \tag{3.19}$$

with $c_d(N, \bar{r}) = \frac{\gamma\alpha + \nu N\bar{r}}{\nu N + \gamma}$, $C(\xi, \lambda(c)) := \frac{1+\xi-(1-p)(1-\lambda+\xi)}{EX-(1-p)\underline{X}}$

Proposition 2 : Exit *All other things equal, an increase in complexity may increase or decrease the cost of lending (decreases or increase in $\lambda(\mathcal{C})$). If it leads to an increase, the least productive banks exit the market. This, in turn, decreases the average price $\bar{r}$, which further amplifies the exit of less productive banks. An increase in complexity leading to an overall increase in the cost of lending, thus increasing the average productivity of operating banks for the bank .*

3.4.3 Equilibrium

Assuming that ξ follows a Pareto distribution: $G(c) = (\frac{c}{c_m})^k$ on $[0, c_m]$, we get:

$$\bar{\xi}^* = \frac{\gamma^{\frac{1}{2}}}{b}\{ \overbrace{D}^{Product\ differentiation\ effect} - \overbrace{[4(1-(1-p))\mu]^{\frac{1}{2}}}^{Productivity\ Threshold\ effect} \} \tag{3.20}$$

$$N^* = \frac{2\gamma(k+1)}{\nu} \frac{\alpha - a - D}{[(1-(1-p))\mu 4\gamma]^{\frac{1}{2}} k + D} \tag{3.21}$$

6. This second effect depends on whether complexity increases or decreases capital requirements for a given bank

D is the product differentiation effect: $D := \gamma^{\frac{1}{2}}[2(k+1)(k+2)(c_m \frac{\gamma^{\frac{1}{2}}}{b})^k [f(\mathcal{C}) + \mu(1-(1-p))]]^{\frac{1}{k+2}}$.

$a := \frac{1-(1-p)(1-\lambda(\mathcal{C}))}{EX-(1-p\underline{X})}$ is the marginal cost of lending common to all bank.

$b := \frac{1-(1-p)(1-\tilde{\lambda}(\mathcal{C}))}{EX-(1-p\underline{X})}$ is the bank specific marginal cost of lending (The optimal collateral requirement are a linear function of the productivity, we thus denote: $\lambda := \lambda(\mathcal{C}) + \tilde{\lambda}(\mathcal{C})\xi$).

Proposition 3 : Interaction *An increase in the cost of lending (μ, λ or f) decreases entry (N^*). However, it has an ambiguous effect on the productivity threshold ($\bar{\xi}^*$). The reason is that the channels described on propositions 1 and 2 have opposite effect on banks' profits. When the productivity shocks are Pareto distributed, the sign of the derivative captures the overall effect on competition with respect to complexity of the average productivity in the market ($\partial_c\{\frac{b}{2}E[\xi|\xi \leqslant \bar{\xi}^*]\}$).*

The ambiguous effect of an increase in the cost of lending comes from two opposite effects on entry and exit mentioned in propositions 1 and 2. First, increasing the cost of banking lowers the number of entrants. Second, it also forces less productive banks to leave the market. However, those two effects interact: a smaller number of banks makes competition less tough, which allows less productive banks to stay in the market. The overall effect of complexity on the number of banks and average productivity - and hence on competition - is ambiguous.

General equilibrium effect on productivity and entry— Lower product differentiation or higher marginal cost of lending ($\frac{\gamma^{\frac{1}{2}}}{b}$) makes less productive firm leave the market. Indeed, in that case, having a high productivity draw is very important to make positive profits as the cost of lending is the main determinant of banks' market share and profits.

The term $[f + \mu(1-(1-p))]]^{\frac{1}{k+2}}$ represents the impact of the number of entrants on competition and thus profits. A lower entry cost (f) or fixed cost of banking (μ) creates more incentives for banks to enter, which make the competition tougher so that less productive firms have to exit the market.

The term $\frac{[4(1-(1-p))\mu\gamma]^{\frac{1}{2}}}{b}$ goes into an opposite direction. Indeed, a higher fixed cost of banking μ also makes, keeping the number of bank constant, banking costlier, which forces bad banks out of the market.

The distribution of productivity as well as the marginal cost of lending b change the relative force of those two effects $[2(k+1)(k+2)(c_m \frac{\gamma^{\frac{1}{2}}}{b})^k]^{\frac{1}{k+2}}$. When k or c_m is high[7], the productivity is more skewed toward bad productivity realizations. That

7. when k is equal to zero, that is, the productivity is uniformly distributed, the two effects cancel each other

is, potential entrants are likely to be less productive. The chance of getting a bad productivity shock and not being profitable enough is high (the fixed cost paid in period 2 is also more important as bad productivity banks are smaller). The impact of the fixed cost is thus very high, and less banks enter the market, making it more profitable for banks to stay. If products are not very differentiated (i.e. low γ), the impact of the number of varieties on the market is less important for profits than banks' productivity. Similarly, if the marginal cost of banking is high (high b) banks have to draw higher productivity to stay in the market, making entry less likely. The impact of entry on the productivity threshold - captured by $[f + \mu(1-(1-p))]]^{\frac{1}{k+2}}$ - have a lower impact on the productivity threshold when the distribution is skewed toward bad productivity, when the cost of banking is high and when products are close substitutes. [8]

Equilibrium interest rate, quantity and subsidy are thus:

$$r^*(\xi) = a + b\frac{1}{2}[\bar{\xi}^* + \xi] \tag{3.22}$$

$$q^*(\xi) = b\frac{\bar{\xi}^* - \xi}{2\gamma} \tag{3.23}$$

$$s^*(\xi) := q^*(\xi)(1-p)[1 - \lambda + \xi - r\underline{X}] \tag{3.24}$$

In my model, an increase in complexity (c) leads to higher entry cost (f), fixed cost of lending (μ) and marginal cost of lending (λ).

For banks that stay in the market, complexity has a direct effect through to the marginal cost of lending (a and b), which in turn affects the interest rate ($r^*(\xi) = a + b\frac{1}{2}[\bar{\xi} + \xi]$). It also has an indirect effect as it changes other entry and exit decision and cost of lending ($\bar{\xi}$).

The indirect effect is ambiguous. An increase in the cost of banking decreases the number of banks entering the market (N), which in turn increases the profits of the banks that do enter. The profit increases allow less productive banks to stay in the market (higher cut-off $\bar{\xi}^*$). The banking market is then less competitive, which creates upward pressure on the interest rate r. However, the increase in complexity (c) also increases the cost of banking (λ) and a decrease in profits. In turn, least productive banks leave the market (i.e. lower cut-off $\bar{\xi}$). As a result, this creates downward pressure on the interest rate as banks face more productive competitors. Overall, the effect of complexity on interest rates depends on the relative force of these effects. In my model, this is captured by the sign of the derivative concerning the productivity threshold ($\partial_c\{\frac{b}{2}\bar{\xi}^*\}$).

8. Notice that the marginal cost of lending that is independent of the productivity (a) does not impact the productivity threshold as the effects cancel each other due to the linear demand

The overall impact of complexity on a bank interest rate is $\partial_c(a + \frac{b}{2}[\bar{\xi} + \xi])$.

Effect of increasing complexity — Increasing complexity has a positive direct impact on the friction coming from deposit insurance: However, this policy also increases the fixed cost of entering into banking: it decreases banks' profits through two channels. First, banking becomes costlier as the banks have to pay a bigger fixed cost. Second, this entry cost impacts the market structure. Indeed, with a higher entry cost, there are fewer banks (good for profits), but they are more productive (bad for profits). This effect can increase the deposit insurance friction by increasing the default probability and/ or the loss given default.

3.5 Welfare

Looking at how complexity impacts the rate offered by banks is not sufficient as one has to also take into account the diversity of products offered as well as the cost of deposit insurance. This is captured by the aggregate welfare function. It is composed of the Total Production F minus the cost of lending. The cost of lending include the cost of the deposit insurance that has to be paid though taxes creating a dead-weight loss. This cost is modeled by the variable τ.

$$max_{\mathcal{C}} W(\mathcal{C}) := max_{\mathcal{C}} \underbrace{F(q(\mathcal{C})) - N(\mathcal{C}) \int_0^{\bar{\xi}(\mathcal{C})} q(\mathcal{C}, \xi) dG(\xi)}_{NPV} - \underbrace{\tau N(\mathcal{C}) \cdot NIC}_{Net\ Insurance\ Cost} \tag{3.25}$$

$NIC := (1-p)\int_0^{\bar{\xi}(\mathcal{C})} q(\mathcal{C}, \xi)[(1 - \lambda(\mathcal{C}) + \xi - r(\mathcal{C})\underline{X})]dG(\xi)$ is the average expected cost of providing deposit insurance.

Using the firm maximization problem, we get the following equation for the NPV:

$$NPV(\mathcal{C}) := \frac{1}{2}(\nu + \frac{\gamma}{N(\mathcal{C})})^{-1}(\alpha - \bar{r}(\mathcal{C}))^2 + \frac{1}{2}\frac{N(\mathcal{C})}{\gamma}\bar{\sigma}^2(\mathcal{C}) \tag{3.26}$$

With $\bar{\sigma}^2(\mathcal{C}) := \int_0^{\bar{\xi}} (\bar{r} - r^*(\xi))^2 dG(\xi)$, $\bar{r}(\mathcal{C}) = a + b\frac{1}{2}[\bar{\xi} + \bar{\xi}(\frac{k}{k+1})]$.

The NPV has the following properties. First, it rises with a decrease in the average interest rate $\bar{r}$. Second, it also rises with an increase in the price variance, σ^2, as consumers reoptimize their consumption across varieties. Finally, as in the CES case, firms exhibit a "love for variety", as welfare increase with the number of varieties available, N (holding the distribution of prices, $\bar{p}$ and $\bar{\sigma}^2$, constant).

Using the fact that $\underline{X} = 0$ and $\lambda(\mathcal{C}) := \mathcal{C} \cdot (1 + \xi)$, with $\mathcal{C} \in [0, 1]$ the insurance cost can be written:

$$\tau N(\mathcal{C}) \cdot NIC(\mathcal{C}) := (1-p)[\overbrace{\tau}^{Marginal\ Insurance\ Cost} \underbrace{(1-p)}_{prob\ bank\ default} \overbrace{(1-\mathcal{C})}^{regulationbenefits} \underbrace{N(\mathcal{C})\frac{b}{2\gamma}(1 + \bar{\xi} - \bar{\sigma}_{\xi}^{2})]}_{Loss\ given\ default} \tag{3.27}$$

with $\bar{\sigma}_{\xi}^{2} := \int_{0}^{\bar{\xi}} (\bar{\xi} - \xi)^{2} dG(\xi)$

The NIC has the following properties. First, the more variety (N high), the bigger is the aggregate demand as borrowers face a love for variety. Second, the bigger is the variance of productivity, the lower is competition so the interest rate goes up and the loss given default goes down. Finally, the higher the cut-off $\bar{\xi}$, the bigger the distortion is as less productive banks lose more upon default as they face a higher lending cost.

Proposition 4 : Welfare *As shown in proposition 3, an increase in complexity decreases or increases entry. A decrease is detrimental to welfare as the number of differentiated loans decreases. However, this also positively affects the deposit insurance as the aggregate loan amount decreases. Similarly, depending on how the marginal and fixed cost varies with complexity, the average productivity of banks may decrease or increase. An increase in average productivity is good for borrowers as they face lower interest rates. However, it also increases the net insurance cost as the loss given default increases (banks earn less (σ decreases)). This increase in loss given default is mitigated by the decrease in the average cost of banking (increase in productivity).*

3.6 Sufficient Statistic

Given the ambiguous effect of complexity on welfare, I propose a sufficient statistic approach to estimate the impact of regulation complexity on welfare in practice. The sufficient statistic approach consists of writing the derivative of the welfare function as a function of measurable elasticities. For instance, using the theoretical model presented above and an increase in complexity, is welfare increasing if the derivative of welfare with respect to complexity $\frac{dW}{d\mathcal{C}}$ is positive.

One of the benefits of this analysis is that we do not need to calculate all the primitives of the model or to solve it. In addition, the formula is valid for small complexity variations, which is likely to hold in the data.

In appendix E.5, I show how to derive a sufficient statistic using a model with households, firms and banks. As in Chetty (2006), we assume that consumers have quasi-linear utilities. This modelling abstracts from changes in the stochastic discount factor (or assumes that they are negligible). The sufficient statistic formula is:

$$\frac{dW}{d\mathcal{C}} = m^B \int_{\bar{\xi}}^{\infty} \overbrace{Pr(X \leqslant \bar{x}_{\xi})}^{Prob\ that\ banks\ \xi\ default} [q(\xi)\frac{d(1-\lambda_{\xi})}{d\mathcal{C}}]dN(\xi) \tag{3.28}$$

$$+ m^B \int_{\bar{\xi}}^{\infty} \overbrace{E[X|X < \bar{x}_{\xi}]\bar{Z}F'(q(\xi))}^{Marginal\ productivity\ of\ firms\ when\ bank\ default} \frac{dq(\xi)}{d\mathcal{C}}dN(\xi) \tag{3.29}$$

$$+ \frac{dN}{d\mathcal{C}}\{ \overbrace{E_t[\mathbf{1}_{X \geqslant \bar{x}_{\xi}} XR(\bar{\xi})]}^{expected\ return\ on\ bank\ \bar{\xi} assets} - q(\bar{\xi})\lambda(\bar{\xi})\} + N\frac{df}{d\mathcal{C}} \tag{3.30}$$

$$- m^B \int_{\bar{\xi}}^{\infty} \underbrace{\frac{d(Pr(X \leqslant \bar{x}_{\xi})q(\xi)(1-\lambda_{\xi}))}{d\mathcal{C}}dN(\xi)}_{changes\ in\ deposit\ insurance\ \ cost} \tag{3.31}$$

The sufficient statistic requires estimating five elasticities. The first one is the impact of the polity on capital requirements ($\frac{d(1-\lambda_{\xi})}{d\mathcal{C}}$). Capital requirements are observable by central banks. The second elasticity is the elasticity of loan size to complexity $\frac{dq(\xi)}{d\mathcal{C}}$. Again, data on loan size are typically available in a central bank credit register. The third and fourth ones are the effect of complexity on the number of banks in the market ($\frac{dN}{d\mathcal{C}}$) and the fixed cost ($\frac{df}{d\mathcal{C}}$). The number of banks is directly observable, and the latter can be recovered using reporting cost surveys. The last elasticity is the impact on banks' default probabilities ($\frac{d(Pr(X \leqslant \bar{x}_{\xi})}{d\mathcal{C}}$). Default probabilities can be measured using, for instance, Credit Default Swaps as in Giglio (2016).

The first and second lines capture the direct benefits of deposit insurance: it provides a cheap funding source. The third line is the sufficient statistic for the effect of complexity on banks' entry. The last line is the direct cost of deposit insurance. Its associated sufficient statistic in the marginal effect of complexity on the deposit insurance cost.

3.7 Conclusion

This paper shows that increasing the complexity of capital requirements to reduce banks' moral hazard behaviour can have a positive impact on competition. However, a more competitive market tends to exacerbate moral hazard distortions by increasing the implicit subsidy the government has to provide upon banks' default. To estimate the ambiguous impact of regulation complexity on welfare, I propose a novel sufficient statistic approach based on my theoretical model. This approach could be used to analyse the impact of changes in regulation complexity but can be applied to any regulation that may impact the marginal cost and the fixed cost of lending.

Bibliography

Agarwal, Sumit, Itzhak Ben-David, and Vincent Yao. 2017. "Systematic mistakes in the mortgage market and lack of financial sophistication." *Journal of Financial Economics* 123 (1): 42–58.

Agarwal, Sumit, John Grigsby, Ali Hortaçsu, Gregor Matvos, Amit Seru, and Vincent Yao. 2020. *Searching for approval.* Technical report. National Bureau of Economic Research.

Aiyar, Shekhar, Charles W Calomiris, John Hooley, Yevgeniya Korniyenko, and Tomasz Wieladek. 2014. "The international transmission of bank capital requirements: Evidence from the UK." *Journal of Financial Economics* 113 (3): 368–382.

Akerlof, George A. 1978. "The market for "lemons": Quality uncertainty and the market mechanism." In *Uncertainty in economics,* 235–251. Elsevier.

Amadxarif, Zahid, James Brookes, Nicola Garbarino, Rajan Patel, and Eryk Walczak. 2019. "The language of rules: textual complexity in banking reforms."

Andersen, Steffen, Cristian Badarinza, Lu Liu, Julie Marx, and Tarun Ramadorai. 2021. "Reference dependence in the housing market." *Available at SSRN 3396506.*

Andersen, Steffen, John Y Campbell, Kasper Meisner Nielsen, and Tarun Ramadorai. 2020. "Sources of inaction in household finance: Evidence from the Danish mortgage market." *American Economic Review* 110 (10): 3184–3230.

Arcidiacono, Peter, and Robert A Miller. 2011. "Conditional choice probability estimation of dynamic discrete choice models with unobserved heterogeneity." *Econometrica* 79 (6): 1823–1867.

Bahaj, Saleem, and Frederic Malherbe. 2020. "The forced safety effect: How higher capital requirements can increase bank lending." *Journal of Finance, Forthcoming.*

Basak, Suleyman, and Andrea M Buffa. 2019. "A theory of model sophistication and operational risk." *Available at SSRN 2737178.*

Becker, Gary S. 1965. "A Theory of the Allocation of Time." *The economic journal,* 493–517.

Benetton, Matteo. 2018. *Leverage regulation and market structure: A structural model of the uk mortgage market.* Technical report. Working Paper.

Benetton, Matteo, Alessandro Gavazza, and Paolo Surico. 2021. "Mortgage pricing and monetary policy."

Berry, Steven, James Levinsohn, and Ariel Pakes. 1995. "Automobile prices in market equilibrium." *Econometrica: Journal of the Econometric Society,* 841–890.

Bisin, Alberto, and Piero Gottardi. 2006. "Efficient competitive equilibria with adverse selection." *Journal of political Economy* 114 (3): 485–516.

Bridges, Jonathan, David Gregory, Mette Nielsen, Silvia Pezzini, Amar Radia, and Marco Spaltro. 2014. "The impact of capital requirements on bank lending."

Brunnermeier, Markus, and Martin Oehmke. 2009. "Complexity in financial markets." *Princeton University* 8.

Buchak, Greg, Gregor Matvos, Tomasz Piskorski, and Amit Seru. 2018. "Beyond the balance sheet model of banking: Implications for bank regulation and monetary policy." *Columbia Business School Research Paper,* nos. 18-75.

Burdett, Kenneth, and Kenneth L Judd. 1983. "Equilibrium price dispersion." *Econometrica: Journal of the Econometric Society,* 955–969.

Busse, Meghan, and Marc Rysman. 2005. "Competition and price discrimination in yellow pages advertising." *RAND Journal of Economics,* 378–390.

Campbell, John Y. 2013. "Mortgage market design." *Review of finance* 17 (1): 1–33.

Carlin, Bruce I. 2009. "Strategic price complexity in retail financial markets." *Journal of financial Economics* 91 (3): 278–287.

Carlin, Bruce Ian, Shimon Kogan, and Richard Lowery. 2013. "Trading complex assets." *The Journal of finance* 68 (5): 1937–1960.

Carlin, Bruce Ian, and Gustavo Manso. 2011. "Obfuscation, learning, and the evolution of investor sophistication." *The Review of Financial Studies* 24 (3): 754–785.

Chetty, Raj. 2006. "A general formula for the optimal level of social insurance." *Journal of Public Economics* 90 (10-11): 1879–1901.

Chetty, Raj, and Amy Finkelstein. 2013. "Social insurance: Connecting theory to data." In *handbook of public economics,* 5:111–193. Elsevier.

Chiappori, Pierre-André, and Bernard Salanie. 2000. "Testing for asymmetric information in insurance markets." *Journal of political Economy* 108 (1): 56–78.

Cloyne, James, Kilian Huber, Ethan Ilzetzki, and Henrik Kleven. 2019. "The effect of house prices on household borrowing: a new approach." *American Economic Review* 109 (6): 2104–36.

Coen, Jamie, Anil K Kashyap, and May Rostom. 2021. "Price discrimination and mortgage choice."

Crawford, Gregory S, Rachel Griffith, Alessandro Iaria, et al. 2016. *Demand estimation with unobserved choice set heterogeneity.* Technical report. CEPR Discussion Papers.

———. 2021. "Demand estimation with unobserved choice set heterogeneity."

Crawford, Gregory S, Nicola Pavanini, and Fabiano Schivardi. 2018. "Asymmetric information and imperfect competition in lending markets." *American Economic Review* 108 (7): 1659–1701.

Crawford, Gregory S, Oleksandr Shcherbakov, and Matthew Shum. 2019. "Quality overprovision in cable television markets." *American Economic Review* 109 (3): 956–95.

Dasgupta, Partha, and Eric Maskin. 1986. "The existence of equilibrium in discontinuous economic games, I: Theory." *The Review of economic studies* 53 (1): 1–26.

DeFusco, Anthony A, Huan Tang, and Constantine Yannelis. 2022. "Measuring the welfare cost of asymmetric information in consumer credit markets." *Journal of Financial Economics* 146 (3): 821–840.

Dixit, Avinash K, and Joseph E Stiglitz. 1977. "Monopolistic competition and optimum product diversity." *The American economic review* 67 (3): 297–308.

Einav, Liran, Amy Finkelstein, and Mark R Cullen. 2010. "Estimating welfare in insurance markets using variation in prices." *The quarterly journal of economics* 125 (3): 877–921.

Einav, Liran, Amy Finkelstein, and Neale Mahoney. 2021. "The IO of selection markets." In *Handbook of Industrial Organization,* 5:389–426. 1. Elsevier.

Einav, Liran, Amy Finkelstein, and Pietro Tebaldi. 2019. "Market design in regulated health insurance markets: Risk adjustment vs. subsidies." *Unpublished mimeo, Stanford University, MIT, and University of Chicago* 7:32.

Ellison, Glenn, and Alexander Wolitzky. 2012. "A search cost model of obfuscation." *The RAND Journal of Economics* 43 (3): 417–441.

Farinha Luz, Vitor. 2017. "Characterization and uniqueness of equilibrium in competitive insurance." *Theoretical Economics* 12 (3): 1349–1391.

Fox, Jeremy T, Kyoo il Kim, Stephen P Ryan, and Patrick Bajari. 2012. "The random coefficients logit model is identified." *Journal of Econometrics* 166 (2): 204–212.

Gabaix, Xavier. 2019. "Behavioral inattention." In *Handbook of Behavioral Economics: Applications and Foundations 1,* 2:261–343. Elsevier.

Ganglmair, Bernhard, and Malcolm Wardlaw. 2017. "Complexity, standardization, and the design of loan agreements." *Available at SSRN 2952567.*

Gaynor, Martin, Carol Propper, and Stephan Seiler. 2016. "Free to choose? Reform, choice, and consideration sets in the English National Health Service." *American Economic Review* 106 (11): 3521–57.

Ghent, Andra C, Walter N Torous, and Rossen I Valkanov. 2019. "Complexity in structured finance." *The Review of Economic Studies* 86 (2): 694–722.

Giglio, Stefano. 2016. *Credit default swap spreads and systemic financial risk.* Technical report. ESRB Working Paper Series.

Guerrieri, Veronica, Robert Shimer, and Randall Wright. 2010. "Adverse selection in competitive search equilibrium." *Econometrica* 78 (6): 1823–1862.

Handel, Ben, Igal Hendel, and Michael D Whinston. 2015. "Equilibria in health exchanges: Adverse selection versus reclassification risk." *Econometrica* 83 (4): 1261–1313.

Handel, Benjamin R, Jonathan T Kolstad, and Johannes Spinnewijn. 2019. "Information frictions and adverse selection: Policy interventions in health insurance markets." *Review of Economics and Statistics* 101 (2): 326–340.

Hanemann, W Michael. 1984. "Discrete/continuous models of consumer demand." *Econometrica: Journal of the Econometric Society,* 541–561.

Hefti, Andreas. 2018. "Limited attention, competition and welfare." *Journal of Economic Theory* 178:318–359.

Hellmann, Thomas F, Kevin C Murdock, and Joseph E Stiglitz. 2000. "Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough?" *American economic review* 91 (1): 147–165.

Hertzberg, Andrew, Andres Liberman, and Daniel Paravisini. 2018. "Screening on loan terms: evidence from maturity choice in consumer credit." *The Review of Financial Studies* 31 (9): 3532–3567.

Hoteling, H. 1929. *Stability in Competition Economic Journal, 39.*

Karlan, Dean, and Jonathan Zinman. 2009. "Observing unobservables: Identifying information asymmetries with a consumer credit field experiment." *Econometrica* 77 (6): 1993–2008.

Keeley, Michael C. 1990. "Deposit insurance, risk, and market power in banking." *The American economic review,* 1183–1200.

Lacker, Jeffrey M. 2001. "Collateralized debt as the optimal contract." *Review of Economic Dynamics* 4 (4): 842–859.

Lancaster, Kelvin J. 1966. "A new approach to consumer theory." *Journal of political economy* 74 (2): 132–157.

Landais, Camille, Arash Nekoei, Peter Nilsson, David Seim, and Johannes Spinnewijn. 2020. "Risk-based selection in unemployment insurance: Evidence and implications."

———. 2021. "Risk-based selection in unemployment insurance: Evidence and implications." *American Economic Review* 111 (4): 1315–55.

Lee, Robin S, and Ariel Pakes. 2009. "Multiple equilibria and selection by learning in an applied setting." *Economics Letters* 104 (1): 13–16.

Lester, Benjamin, Ali Shourideh, Venky Venkateswaran, and Ariel Zetlin-Jones. 2019. "Screening and adverse selection in frictional markets." *Journal of Political Economy* 127 (1): 338–377.

Malherbe, Frederic. 2020. "Optimal capital requirements over the business and financial cycles." *American Economic Journal: Macroeconomics* 12 (3): 139–174.

McFadden, Daniel. 1981. "Econometric models of probabilistic choice." *Structural analysis of discrete data with econometric applications* 198272.

Melitz, Marc J. 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity." *econometrica* 71 (6): 1695–1725.

Melitz, Marc J, and Gianmarco IP Ottaviano. 2008. "Market size, trade, and productivity." *The review of economic studies* 75 (1): 295–316.

Merton, Robert C. 1974. "On the pricing of corporate debt: The risk structure of interest rates." *The Journal of finance* 29 (2): 449–470.

Morrow, W Ross, and Steven J Skerlos. 2011. "Fixed-point approaches to computing Bertrand-Nash equilibrium prices under mixed-logit demand." *Operations research* 59 (2): 328–345.

Mussa, Michael, and Sherwin Rosen. 1978. "Monopoly and product quality." *Journal of Economic theory* 18 (2): 301–317.

Nelson, Scott. 2020. "Private information and price regulation in the us credit card market." *Unpublished Working Paper.*

Nevo, Aviv. 2001. "Measuring market power in the ready-to-eat cereal industry." *Econometrica* 69 (2): 307–342.

Oehmke, Martin, and Adam Zawadowski. 2019. *The tragedy of complexity.* Technical report. LSE and CEU Working Paper.

Ottaviano, Gianmarco, Takatoshi Tabuchi, and Jacques-François Thisse. 2002. "Agglomeration and trade revisited." *International Economic Review,* 409–435.

Paravisini, D, and V Rappoport. 2020. "P. Schnabl (2017) Specialization in bank lending: evidence from exporting firms."

Piccione, Michele, and Ran Spiegler. 2012. "Price competition under limited comparability." *The quarterly journal of economics* 127 (1): 97–135.

Polo, Taburet, and Vo. 2022. "Screening Using a Menu of Contracts: A Structural Model for Lending Markets." *Working Paper.*

Riley, John G. 1979. "Informational equilibrium." *Econometrica: Journal of the Econometric Society,* 331–359.

Robles-Garcia, Claudia. 2019. *Competition and incentives in mortgage markets: The role of brokers.* Technical report. Working paper.

Rothschild, Michael, and Joseph Stiglitz. 1976. "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information." *The Quarterly Journal of Economics.*

Salanié, Bernard, and Frank A Wolak. 2019. *Fast," robust", and approximately correct: estimating mixed demand systems.* Technical report. National Bureau of Economic Research.

Stiglitz, Joseph E. 1977. "Monopoly, non-linear pricing and imperfect information: the insurance market." *The Review of Economic Studies* 44 (3): 407–430.

Taburet, Arthur. 2022. "Screening using a menu of contracts: theory and policy implications for lending markets." *Working Paper.*

Tirole, Jean. 1988. *The theory of industrial organization.* MIT press.

Townsend, Robert M. 1979. "Optimal contracts and competitive markets with costly state verification." *Journal of Economic theory* 21 (2): 265–293.

Train, Kenneth. 1986. *Qualitative choice analysis: Theory, econometrics, and an application to automobile demand.* Vol. 10. MIT press.

Train, Kenneth E. 2009. *Discrete choice methods with simulation.* Cambridge university press.

Varian, Hal R. 1980. "A model of sales." *The American Economic Review* 70 (4): 651–659.

Wilson, Charles. 1980. "The nature of equilibrium in markets with adverse selection." *The Bell Journal of Economics,* 108–130.

Wollmann, Thomas G. 2018. "Trucks without bailouts: Equilibrium product characteristics for commercial vehicles." *American Economic Review* 108 (6): 1364–1406.

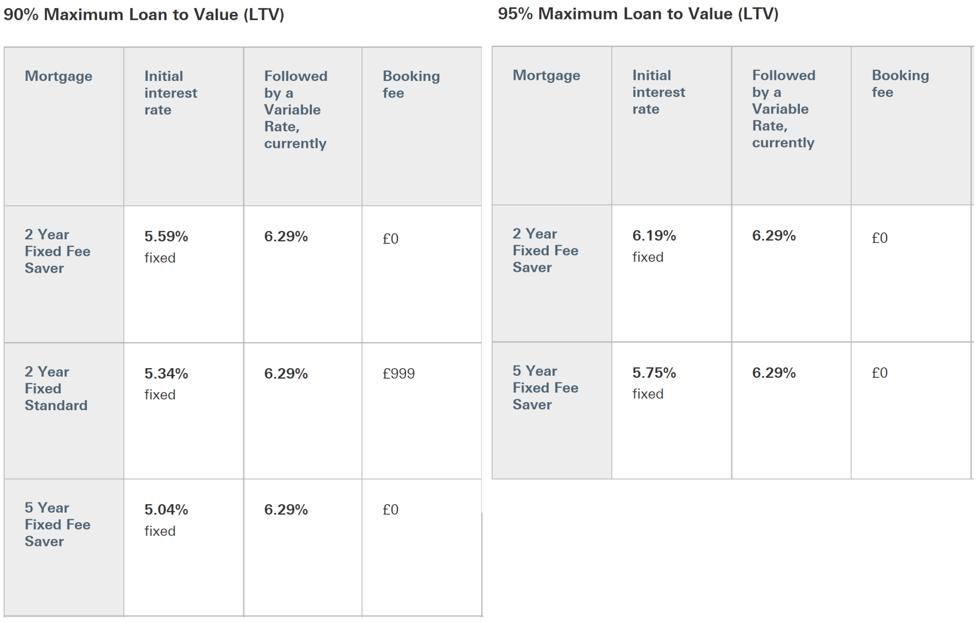
Yannelis, Constantine, and Anthony Lee Zhang. 2021. "Competition and selection in credit markets." *Available at SSRN 3882275.*

Zhong, Hongda. 2020. "A dynamic model of optimal creditor dispersion." *Journal of Finance.*

Appendix A

Chapter 1 Figures

A.1 Figures



90% Maximum Loan to Value (LTV)

Mortgage	Initial interest rate	Followed by a Variable Rate, currently	Booking fee
2 Year Fixed Fee Saver	5.59% fixed	6.29%	£0
2 Year Fixed Standard	5.34% fixed	6.29%	£999
5 Year Fixed Fee Saver	5.04% fixed	6.29%	£0

95% Maximum Loan to Value (LTV)

Mortgage	Initial interest rate	Followed by a Variable Rate, currently	Booking fee
2 Year Fixed Fee Saver	6.19% fixed	6.29%	£0
5 Year Fixed Fee Saver	5.75% fixed	6.29%	£0

FIGURE A.1.1: Extract of the Menu of contracts offered by HSBC in January 2023
Source: HSBC's website

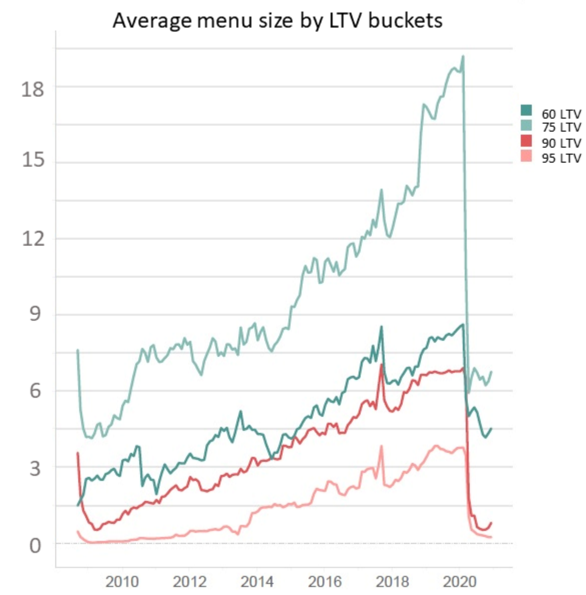


FIGURE A.1.2: Average number of advertised mortgage products for BtL, FtB and Remortgage
Source: Moneyfacts and Bank of England's calculations

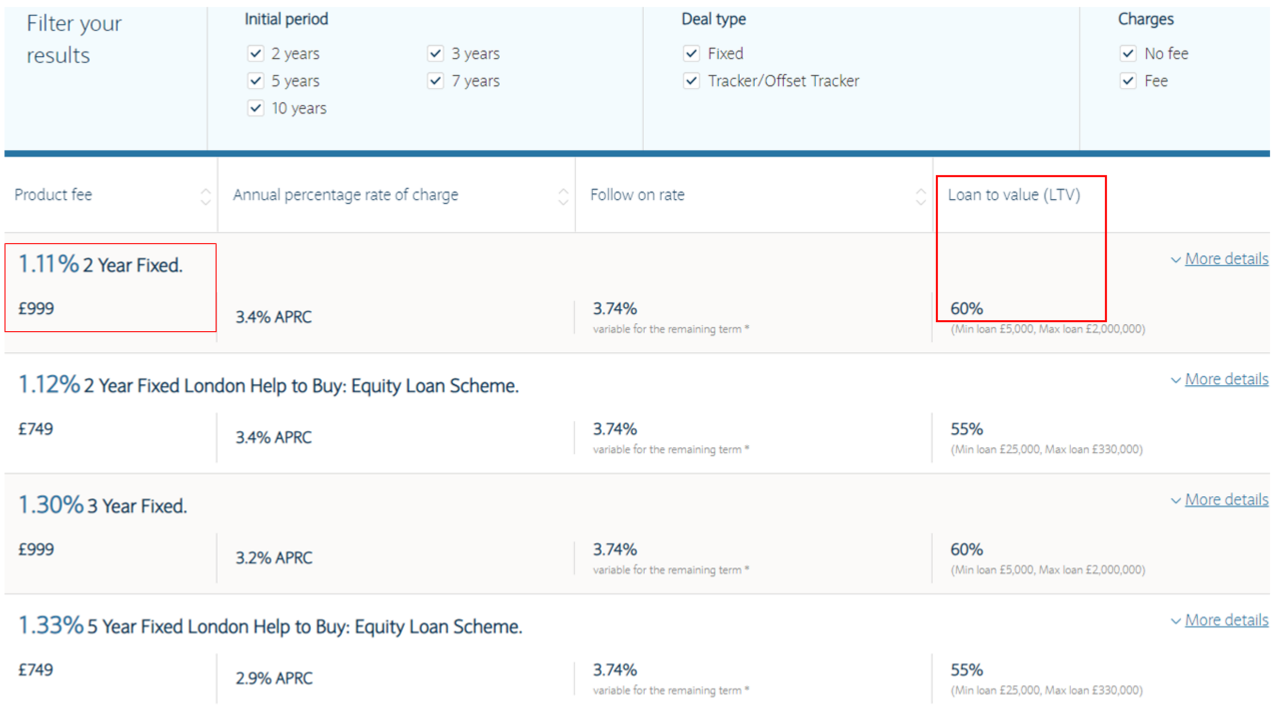


FIGURE A.1.3: Extract of the Menu of contracts offered by Barclays 17/01/2022
Source: Barclays' website

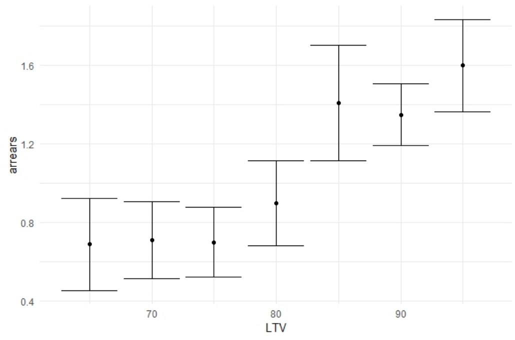


FIGURE A.1.4: Percentage of mortgages in arrears by LTV at origination
Source: PSD001-PSD007

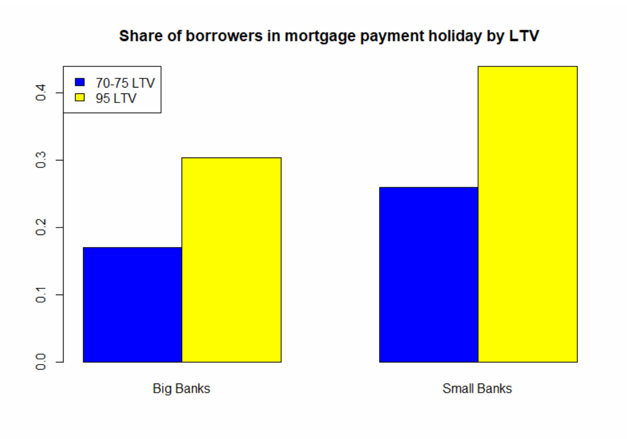


FIGURE A.1.5: Share of mortgages that asked for a payment deferral in 2020
Source: BoE survey, authors' own calculations

Appendix B

Chapter 1 Table

B.1 Tables

B.1.1 Descriptive statistics

TABLE B.1: Summary Statistics for 2018

Variable	Mean	SD	Min	Max
Loan Characteristics:				
Max LTV (percent)	82.5	10.8	50	95
Teaser rate period (years)	3.3	1.6	0	7
Maturity (years)	29.7	5.7	8	40
Fees (£)	503	631	0	2610
Rate (percent)	2.5	0.8	1.1	8
Loan amount (£ 1000)	164	129	35	864
Borrower Characteristics:				
Household income (£ 1000)	36	16	25	944
Loan applicants	1.56	0.5	1	2
Age (years)	31	7	18	75
Loan to Income	4.6	1.2	1.1	6.1
N	279,379			

TABLE B.2: Regression LTV on borrowers' characteristics

Variable	**Age**	**Yearly net income**	**Number of borrowers**	**Self employed**
intercept	33***	39,855	1.35***	0.085***
60-70% LTV	-0.7***	-82	0.04***	0.01***
70-75% LTV	-1.5***	3675***	0.007***	-0.005***
75-80% LTV	-1.3***	1793*	0.11***	0.006***
80-85% LTV	-1.7***	1941**	0.16***	0.007***
85-90% LTV	-2.4***	-2716***	0.22***	-0.024***
95+ ltv	-2.7***	-3842***	0.28***	-0.06***
N	1,077,291	1,077,291	1,077,291	1,077,291

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

TABLE B.3: Mortgage Holiday take up and arrears. A mortgage holiday is a payment deferral (up to 6 month)

	Mortgage Holiday by 2021	Arrears by 2020 (Origination: 2018)
Interest (in percent)	$1.23 \cdot 10^{-1***}$	$5.8 \cdot 10^{-3***}$
LTV> 90	$-3.5 \cdot 10^{-2}$	$-1.4 \cdot 10^{-3}$ ***
Fixed rate period (years)		$-9.9 \cdot 10^{-4***}$
Lender fees		$3.7 \cdot 10^{-6***}$
Income		$-1.2 \cdot 10^{-7***}$
Nb applicants		$-3.9 \cdot 10^{-3***}$
Age		$6.7 \cdot 10^{-5*}$
LTI		$-1.4 \cdot 10^{-3***}$
Time fixed effect	No	Yes
Bank fixed effect	Yes	Yes
Region fixed effect	No	Yes
Mean	26%	1.2%
Observations	53	279,379

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

TABLE B.4: Most common product characteristics

Variable	**2019**	**2021**
	high LTV (95)	
Average number of products (rounded)	8	0-2
Fixed rate period (years)	(5,3,2,0)	5 year more likely
Average lender fees (rounded)	(0, 750)	high fees more likely
	medium LTV (75-85)	
Average number of products (rounded)	12	16
Fixed rate period (years)	(5,3,2,0)	(5,3,2,0) + longer fixed rates
Average lender fees (rounded)	(0, 750, 1450)	(0, 750, 1450)

Source: PSD001 + Moneyfact

B.1.2 Estimation Results

TABLE B.5: Mixed logit (Origination: 2018)

	85 + LTV loans	70-85% LTV loans
Interest rate (percent)	$-5.4 \cdot 10^{-1}$	$-7.1 \cdot 10^{-1}$
	$(5 \cdot 10^{-1})$	$(4.4 \cdot 10^{-1})$
LTV (percent)	$2.3 \cdot 10^{-1***}$	$2.1 \cdot 10^{-1***}$
	$(1.2 \cdot 10^{-2})$	$(5 \cdot 10^{-2})$
Fixed rate period (years)	$-7.8 \cdot 10^{-1*}$	$-1.8 \cdot 10^{-1}$
	$(4 \cdot 10^{-1})$	$(1.9 \cdot 10^{-1})$
Lender fees (pounds)	$-9 \cdot 10^{-4***}$	$-7 \cdot 10^{-4***}$
	(1.610^{-4})	$(5 \cdot 10^{-5})$
Interest rate × Yearly Net Income (pounds)	$-4.5 \cdot 10^{-5***}$	$-3.2 \cdot 10^{-5***}$
	$(1.1 \cdot 10^{-5})$	$(1.7 \cdot 10^{-5})$
Standard deviation random coefficient Fixed rate period	2.5^{***}	1^{***}
	$(4.8 \cdot 10^{-1})$	$(2.7 \cdot 10^{-1})$
Standard deviation random coefficient LTV	$2.4 \cdot 10^{-1***}$	$4.8 \cdot 10^{-2***}$
	$(2.7 \cdot 10^{-2})$	$(2 \cdot 10^{-3})$
Region-Age-Nb applicants interaction terms for all product characteristics	Yes	Yes
Interest rate- Fixed rate period-fees random coefficient	Yes	Yes
Observations	279,379	230,680

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

TABLE B.6: Coefficient heterogeneity

	Interest rate (per cent)	**LTV (per cent)**	**Teaser rate (year)**	**Fees (pounds)**
		85+ loans		
Observable heterogeneity only				
First quartile	-1.1	$2.3 \cdot 10^{-1}$	$-7.8 \cdot 10^{-1}$	$-8 \cdot 10^{-4}$
Second quartile	$-8.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-1}$	$-7.8 \cdot 10^{-1}$	$-8 \cdot 10^{-4}$
Third quartile	$-6.3 \cdot 10^{-1}$	$2.3 \cdot 10^{-1}$	$-7.8 \cdot 10^{-1}$	$-8 \cdot 10^{-4}$
Observable and unobservable heterogeneity				
First quartile	-1.1	$1.5 \cdot 10^{-1}$	-2.4	$-8 \cdot 10^{-4}$
Second quartile	$-8.6 \cdot 10^{-1}$	$2.3 \cdot 10^{-1}$	$-7.8 \cdot 10^{-1}$	$-8 \cdot 10^{-4}$
Third quartile	$-6.3 \cdot 10^{-1}$	$3 \cdot 10^{-1}$	$9.2 \cdot 10^{-1}$	$-8 \cdot 10^{-4}$
		70-85 loans		
Observable heterogeneity only				
First quartile	-2.3	$1.6 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	$-7 \cdot 10^{-4}$
Second quartile	-1.9	$1.7 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	$-7 \cdot 10^{-4}$
Third quartile	-1.5	$1.8 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	$-7 \cdot 10^{-4}$
Observable and unobservable heterogeneity				
First quartile	-2.3	$1.3 \cdot 10^{-1}$	$-4.3 \cdot 10^{-1}$	$-7 \cdot 10^{-4}$
Second quartile	-1.9	$1.7 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	$-7 \cdot 10^{-4}$
Third quartile	-1.5	$2.1 \cdot 10^{-1}$	$9.1 \cdot 10^{-1}$	$-7 \cdot 10^{-4}$

Table B.7: WTP and elasticity heterogeneity

	Price elasticity	WTP LTV (percent)	WTP teaser rate (year)	WTP fees (pounds)
		85+ loans		
Observable heterogeneity only				
First quartile	2.8	$1.1 \cdot 10^{-1}$	$-6.5 \cdot 10^{-1}$	$-6.6 \cdot 10^{-4}$
Second quartile	3.9	$1.4 \cdot 10^{-1}$	$-4.9 \cdot 10^{-1}$	$-5 \cdot 10^{-4}$
Third quartile	5.4	$1.9 \cdot 10^{-1}$	$-3.7 \cdot 10^{-1}$	$-3.8 \cdot 10^{-4}$
Observable and unobservable heterogeneity				
First quartile	2.8	$5 \cdot 10^{-2}$	-1.1	$-6.6 \cdot 10^{-4}$
Second quartile	3.9	$1.3 \cdot 10^{-1}$	$-4.4 \cdot 10^{-1}$	$-5 \cdot 10^{-4}$
Third quartile	5.4	$2.4 \cdot 10^{-1}$	$5.3 \cdot 10^{-1}$	$-3.8 \cdot 10^{-4}$
		70-85 loans		
Observable heterogeneity only				
First quartile	2.6	$8 \cdot 10^{-2}$	$7 \cdot 10^{-2}$	$-6 \cdot 10^{-4}$
Second quartile	3.6	$1 \cdot 10^{-1}$	$9 \cdot 10^{-2}$	$-4.3 \cdot 10^{-4}$
Third quartile	5.1	$1.4 \cdot 10^{-1}$	$1.2 \cdot 10^{-1}$	$-3.2 \cdot 10^{-4}$
Observable and unobservable heterogeneity				
First quartile	2.6	$7 \cdot 10^{-2}$	$-3 \cdot 10^{-1}$	$-6 \cdot 10^{-4}$
Second quartile	3.6	$1 \cdot 10^{-1}$	$8 \cdot 10^{-2}$	$-4.3 \cdot 10^{-4}$
Third quartile	5.1	$1.4 \cdot 10^{-1}$	$5 \cdot 10^{-1}$	$-3.2 \cdot 10^{-4}$

Table B.8: Loan Demand (Origination: 2018)

	log(Loan size)	log(Loan size)
Interest rate (percent)	$-5.2 \cdot 10^{-2***}$	$-5.2 \cdot 10^{-2***}$
	$(3.9 \cdot 10^{-3})$	$(3.9 \cdot 10^{-3})$
LTV (percent)	$8.9 \cdot 10^{-4***}$	$8.8 \cdot 10^{-4***}$
	$(1.9 \cdot 10^{-4})$	$(3.9 \cdot 10^{-4})$
LTV=95 (percent)	$7.6 \cdot 10^{-2***}$	$1.5 \cdot 10^{-1***}$
	$(7.3 \cdot 10^{-3})$	$(2.1 \cdot 10^{-2})$
Fixed rate period (years)	$-1.7 \cdot 10^{-3*}$	$-8.5 \cdot 10^{-3***}$
	$(9 \cdot 10^{-4})$	$(2.4 \cdot 10^{-3})$
Lender fees (pounds)	$6.5 \cdot 10^{-5***}$	$6.9 \cdot 10^{-5***}$
	$(1.6 \cdot 10^{-6})$	$(1.6 \cdot 10^{-6})$
log(Income) (pounds)	$8 \cdot 10^{-1***}$	$8 \cdot 10^{-1***}$
	$(4 \cdot 10^{-3})$	$(4 \cdot 10^{-3})$
unobserved WTP fixed rate: $\hat{e}_{TR}$ (mean 0 sd normalized to 1)		$2 \cdot 10^{-1***}$
		$(2.3 \cdot 10^{-2})$
unobserved WTP LTV: $\hat{e}_{LTV}$ (mean 0 sd normalized to 1)		$8 \cdot 10^{-2***}$
		$(1.3 \cdot 10^{-2})$
Lender, Region, time fixed effect	Yes	Yes
Borrowers' characteristics control	Yes	Yes
Borrowers' WTP interaction terms	Yes	Yes
R^2	0.76	0.77
Observations	279,379	279,379

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table B.9: Default regression (mortgage originated in 2018)

	Arrears by 2020	**Arrears by 2020**
Interest (in percent)	$5.8 \cdot 10^{-3***}$	$3.9 \cdot 10^{-3***}$
	$(4.2 \cdot 10^{-4})$	$(3.4 \cdot 10^{-4})$
LTV	$-1.8 \cdot 10^{-5***}$	$1.2 \cdot 10^{-4}$
	$(2.7 \cdot 10^{-6})$	$(2.1 \cdot 10^{-6})$
Fixed rate period (years)	$-9.9 \cdot 10^{-4***}$	$-5.7\ 10^{-4*}$
	$(2.7 \cdot 10^{-6})$	$(1.6 \cdot 10^{-4})$
Lender fees	$3.7 \cdot 10^{-6***}$	$4 \cdot 10^{-6***}$
	$(7.5 \cdot 10^{-7})$	$(6.5 \cdot 10^{-7})$
Income	$-1.2 \cdot 10^{-7***}$	$-2.4 \cdot 10^{-7***}$
	$(1.1 \cdot 10^{-8})$	$(1.6 \cdot 10^{-8})$
Nb applicants	$-3.9 \cdot 10^{-3***}$	$-3.1 \cdot 10^{-3***}$
	$(2.8 \cdot 10^{-4})$	$(2.3 \cdot 10^{-4})$
Age	$6.7 \cdot 10^{-5*}$	$7.1 \cdot 10^{-5*}$
	$(1.1 \cdot 10^{-5})$	$(1.9 \cdot 10^{-5})$
$\hat{e}_{LTV}$ (sd normalized to 1)		$-5.4 \cdot 10^{-3***}$
		$(9.4 \cdot 10^{-5})$
$\hat{e}_{TR}$ (sd normalized to 1)		$-2.2 \cdot 10^{-4***}$
		$(5.1 \cdot 10^{-5})$
Time fixed effect	Yes	Yes
Lender fixed effect	Yes	Yes
Region fixed effect	Yes	Yes
Macroeconomics controls (monthly GDP)	Yes	Yes
Control for loan size	Yes	Yes
Mean	1.2%	1.2%
Observations	279,379	279,379

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

TABLE B.10: Marginal costs regression (LTV > 70)

	Marginal costs	**Interest rates**
Intercept	$8.5 \cdot 10^{-1***}$	$1.2 \cdot 10^{-2***}$
	$(2 \cdot 10^{-1})$	$(8.69 \cdot 10^{-3})$
$1_{LTV<85}$×LTV (percent)	$1.2 \cdot 10^{-2***}$	$1.4 \cdot 10^{-2***}$
	$(2.8 \cdot 10^{-3})$	$(1 \cdot 10^{-4})$
$1_{LTV>85}$×LTV (percent)	$1.8 \cdot 10^{-2***}$	$2 \cdot 10^{-2***}$
	$(1.5 \cdot 10^{-3})$	$(9.7 \cdot 10^{-5})$
95% LTV (dummy)	$9.8 \cdot 10^{-1***}$	$1.2 \cdot 10^{-1***}$
	$(9.1 \cdot 10^{-2})$	$(2.1 \cdot 10^{-3})$
Fixed rate period (years)	$4 \cdot 10^{-2**}$	$4.4 \cdot 10^{-2**}$
	$(1 \cdot 10^{-2})$	$(5.3 \cdot 10^{-4})$
High Fixed rate period (⩾5)	$1.8 \cdot 10^{-1***}$	$2.3 \cdot 10^{-1***}$
	$(5 \cdot 10^{-2})$	$(1.6 \cdot 10^{-3})$
Lender fees (pounds)	$-2.2 \cdot 10^{-4***}$	$-3.8 \cdot 10^{-4***}$
	$(1.8 \cdot 10^{-5})$	$(5.7 \cdot 10^{-7})$
High fees (1000-1500)	$-1 \cdot 10^{-1*}$	$-1.3 \cdot 10^{-1***}$
	$(4 \cdot 10^{-2})$	$(2.7 \cdot 10^{-3})$
Bank fixed effect	Yes	Yes
Average	2.12	2.42
N	278	647,433
R^2	0.88	0.76

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

TABLE B.11: Fixed cost results

	$\tilde{X}_c$=1 (1)
Profits (β)	4.47^{***}
	$(6.37 \cdot 10^{-2})$
Nbr of Product included (θ)	$7.8 \cdot 10^{7***}$
	$(5.04 \cdot 10^{3})$
Nbr of Product excluded ($\theta \cdot \lambda$)	$-2.4 \cdot 10^{7***}$
	$(5.04 \cdot 10^{3})$
Bank fixed effect	No
Time fixed effect	No
Observations	61

Note:
$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

B.1.3 Counterfactual Results

Table B.12: Product distortion (80+ LTV loans)

	Ideal LTV (percent)	Ideal teaser rate (year)	Ideal Fees (pounds)
Observable heterogeneity only (perfect information+perfect competition)			
First quartile	95	0	0
Second quartile	95	0	0
Third quartile	95	0	500
Observable and unobservable heterogeneity (perfect information+perfect competition)			
First quartile	90	0	0
Second quartile	95	2	0
Third quartile	95	5-7	500
Product choice distribution (data)			
First quartile	85	2	0
Second quartile	90	2	500
Third quartile	95	5-7	1000

Table B.13: LTV distortion perfect competition perfect information benchmark (70+ LTV loans)

Decile	10%	20%	30%	40%	50%	60%	70%	80%	90%
Product choice distribution (data)	75	75	80	85	90	90	90	90	95
Benchmark implied distribution (observable heterogeneity)	90	90	95	95	95	95	95	95	95
Benchmark implied distribution (observable + unobservable heterogeneity)	85-90	90	90	90	95	95	95	95	95

Table B.14: Interest rate decomposition (70-80+ LTV loans)

	Fair price (bps)	Perfect information mark-up (bps)	Asymmetric Information discount/premium (bps)
LTV	2***	$1 \cdot 10^{-1}$	$2 \cdot 10^{-1}$*
fees (500)	-16***	-9 ***	6***
fees (1000)	-29***	-20 ***	13***
fees (1500)	-35***	-30 ***	17***
teaser rate period (2 years)	-40***	-8	0
teaser rate period (5 years)	-20 *	-4	-10**
teaser rate period (7 years)	7	10	-20**
Average	202	65	-30

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table B.15: Intest rate decomposition (80+ LTV loans)

	Fair Price (bps)	Perfect information mark-up (bps)	Asymmetric Information discount/premium (bps)
LTV	12***	$8 \cdot 10^{-2}$	2***
fees (500)	-12***	-19 ***	20***
fees (1000)	-35***	-46 ***	41***
fees (1500)	-46***	-55 ***	45***
teaser rate period (2 years)	3	15***	-16 ***
teaser rate period (5 years)	15 ***	35***	-31 ***
teaser rate period (7 years)	27 **	43***	-40***
Average	231	116	-68

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Appendix C

Chapter 1 Appendix

C.1 Model

C.1.1 Borrowers

In this section, we model borrowers' decision to participate in the mortgage market and their choice of loan contract (section C.1.1). We also discuss how to extend the model to allow for the entry decision (section C.1.1). Our modeling yields the system of demand equations presented in section 1.4.2. For each equation, we discuss the interpretation of the parameters and provide possible micro-foundations in the Appendix.

Information structure: All parameters defined in this section are part of borrower i's information set at the time she makes her choice of contract and bank. We denote borrower i's information set at the time she makes her choice of contract by $\mathcal{I}_i^B$.

Choice of contract and bank

This section models borrowers' choice of bank and contract and discusses the mechanisms driving the demand.

Guided by the micro-foundations in Appendix (**??**) and the considerations discussed in the previous paragraph, we parametrize the indirect utility derived at the optimal borrowing amount given the loan characteristics X and price r as

$$U_i(L_i(X,r);X,r) := A_i(X)\frac{L_i(X,r)}{LTV} + V_i(Y_i), \tag{C.1}$$

where Y_i is the income of borrower i, A_i is a function of the product characteristics X, V_i is a function of income, and L_i is the optimal loan size as a function of product characteristics X and rate r. LTV is the loan-to-value of the contract, so $\frac{L_i(X,r)}{LTV}$ is the house price.

This parametrization is a generalized version of K. Train (1986). The main departure from K. Train (1986) is that we allow A_i to be a general function that varies with products' and borrowers' characteristics instead of a constant.

Using Roys' identity, the optimal loan size should satisfy $L_i(X,r) = -\frac{\partial_r U(L,X,r)}{\partial_Y U(L,X,r)}$. In Appendix (C.5.2), we show a parametrization of the function (A) that leads to the following demand system. We index by c a product (X_{cb}, r_{cb}) offered by bank b and relabel $L_i(c,b) := L(X_{cb}, r_{cb})$:

$$V_i(c,b) = \overbrace{\beta_{icb}X_{cb} - \alpha_{icb}r_{cb} + \xi_{cb}}^{\tilde{u}_i(c,b)} + \sigma_{icb}^{-1}\varepsilon_{icb} \tag{C.2}$$

$$ln(L_i(c,b)) = \tilde{\beta}_{icb}X_{cb} - \tilde{\alpha}_{icb}r_{cb} + \nu D_i + e_{icb}^L \tag{C.3}$$

$$with\ (\beta_{icb}, \alpha_{icb}, \sigma_i^{-1}, \tilde{\beta}_{icb}, \tilde{\alpha}_{icb}, e_{cb}^L)\ correlated,$$

where u_i is a monotonic transformation of the indirect utility U_i defined in equation (C.1).[1]

$(\beta_{icb}, \alpha_{icb}, \sigma_{icb}^{-1})$ drive how borrower i values the contracts' characteristics. $(\tilde{\beta}_{icb}, \tilde{\alpha}_{icb})$ parameterize the loan size demand elasticity. Since borrowers choose the loan size and contract jointly, $(\beta_{icb}, \alpha_{icb}, \sigma_{icb}^{-1})$ and $(\tilde{\beta}_{icb}, \tilde{\alpha}_{icb})$ derive from the same maximization problem and are thus potentially correlated. As illustrated in Appendix **??**, parameters $(\beta_{icb}, \alpha_{icb}, \sigma_{icb}^{-1})$ and $(\tilde{\beta}_{icb}, \tilde{\alpha}_{icb})$ are potentially a function of borrowers' default probability, the cost of defaulting — which depends on the loan being recourse or not — or how much the borrower values housing relative to consumption and how much savings the borrower has. For instance, high default may be less sensitive to the face value of the debt if borrowers expect that they will not have to repay it fully upon default (for example, if the loan is non-recourse). We formally discuss our modeling of preference heterogeneity in the default section C.1.2.

The ratio $\frac{\beta_{icb}}{\alpha_{icb}}$ represents borrower i's willingness to pay for a characteristic. Indeed, if a bank proposes a new high LTV contract, borrower i would be happy to take it (i.e., its utility would increase by taking the contract) as long as the price increase is below the borrower's willingness to pay. Formally, borrowers accept the new contracts if $U(L(LTV_2); r_2, LTV_2) \geqslant U(L(LTV_1); r_1, LTV_1) \iff \frac{\beta_{icb}}{\alpha_{icb}}(LTV_2 - LTV_1) > (r_2 - r_1)$.

ξ_{cb} captures the part of the average indirect utility that comes from unobserved (by the econometrician) contract characteristics. Borrowers' preference heterogeneity is captured by the variable $\sigma_i^{-1}\varepsilon_{icb}$.

1. $V_i(Y_i)$ is not present as $argmax_c U_i(L^*, X_c, r_c) = argmax_c U_i(L^*, X_c, r_c) - V_i(Y_i)$. For those that are skeptical about the discrete-continuous approach, one could end up with the same functional form by assuming that borrower i chooses product c and the optimal loan size $L_i(X_c, r_c)$:

$$max_{c \in M_{ib}} u_i(L_i(X_{cb}, r_{cb}), X_{cb}, r_{cb}) + \sigma_i^{-1}\varepsilon_{icb}$$

and make the assumption that $L_i(X_c, r_c)$ and $u_i(L_i^*(X_c, r_c), X_c, r_c)$ are linear in contract terms.

$\sigma_i^{-1}\varepsilon_{icb}$ captures demand shocks for a bank-product. σ_i^{-1} is a parameter driving the variance of the shock (ε_{icb}). The shock can be decomposed into deviations from the average borrower preferences for unobserved contract characteristics, plus an extra term (let us call it ε_{ib}), which can be interpreted as a search cost (see choice of bank section). Given the use of the variable ξ_{cb}, we consider, without loss of generality, that $E[\sigma_i^{-1}\varepsilon_{icb}] = 0$. We further assume that $E[\sigma_i^{-1}\varepsilon_{icb}|X_{cb}, r_{cb}, \beta_i, \alpha_i] = 0$ so that $\sigma_i^{-1}\varepsilon_{icb}$ represents the part of borrowers' demand that cannot be screened by banks when they use product characteristics (X_c, r_c) only (cf. proposition 1). The potential identification threat caused by this assumption not being valid will be discussed in the empirical section.

D_i are borrower i characteristics, such as income or age, that are observable by the bank and the econometrician. As shown in C.5.2, the Roy's identity micro-foundation of demand imposes a specific functional form for how the income element enters the loan demand. e^L_{icb} is a loan demand parameter that captures variables that are unobservable by the econometrician but can be partly observable by the lender.

Choice of contract and bank: A borrower chooses the bank b among the set of banks B that offer the best contract c within the available menu P_{ib}. Formally, given the specification of borrowers' preferences:

$$(c_i, b_i) = argmax_{\{b\in B,\ c\in P_{ib}\}}\ \tilde{u}_i(c, b) + \sigma_i^{-1}\varepsilon_{icb}. \tag{C.4}$$

The menu available to each borrower (P_{ib}) may be different as a result of rejections of borrowers' applications for a particular contract. The modeling of the choice of product in equation (C.4) is general enough to encompass the case in which borrowers have perfect knowledge of which applications would be successful and which would not. We favour the perfect information case interpretation as this case can be justified by the heavy use of brokers in this market. The imperfect information case is discussed in Appendix (C.4).

Equations (C.4) and (C.3) allow us to derive the product demand for each bank presented in section 1.4.2.

Distribution of the demand shocks assumption: We assume that individual i's preferences over banks and contracts ε_{icb} are drawn from an extreme value distribution and are independent across banks and products. The probability of individual i choosing contract c at bank j is thus[2]

$$Pr((c_i, b_i) = (c, b)|\alpha_{icb}, \beta_{icb}, \sigma_{icb}, X, \xi_{cb}) = \frac{exp(\sigma_i \tilde{u}_i(c, b))}{\sum_{x \in B, y \in \{P_{ib}\}} exp(\sigma_{icb} \tilde{u}_i(y, x))}, \quad (C.5)$$

where $\sigma \in \mathbb{R}^+$ captures the product elasticity. As σ_{icb} scales all the coefficients $(\beta_{icb}, \alpha_{icb})$ by the same amount, we will not be able to separate σ from $(\beta_{icb}, \alpha_{icb})$, so we will normalize σ_{icb} to 1 in the estimation section.

As discussed in Taburet (2022), when studying competition, it is useful to decompose the demand shock (ε_{icb}) into a bank shock $(\sigma_i \varepsilon_{ib})$ — representing search costs, for instance — and a contract shock (ε_{ic}) — representing inattention or unobserved preference heterogeneity, for instance. In that case, the demand shocks (ε_{icb}) are correlated across banks. σ_i can be interpreted as a competition parameter. Indeed, when σ tends to infinity, borrowers only care about the contract features offered by the banks (i.e., perfect competition). In that limited situation, banks have to price each loan at its fair price and thus make zero profits. When σ^{-1} tends to 0, each bank behaves as a monopolist with its borrowers.[3] Taburet (2022) analyzes a contract theory model with this demand form and shows that competition is an important driver of screening.

In this paper, as changing competition in the counterfactual exercise is not our focus, we will assume that the random shocks within a given bank are not correlated. Our approach limits the computational burden of estimating a nested logit with a random coefficient in a demand and supply setup. For the interested reader, we provide in Appendix (C.3) an extension of our demand model using a nested logit formulation that relaxes this assumption.

Choosing to enter the borrowing market

Borrower i chooses to participate in the market if the expected utility of entering the market and borrowing (V_i) is higher than the expected utility of not borrowing $(\bar{V}_i)$:

$$V_i \geqslant \bar{V}_i. \quad (C.6)$$

As shown in Andersen et al. (2021) and Benetton, Gavazza, and Surico (2021), borrowers' entry decision in the mortgage market is very inelastic to loan prices and characteristics.[4] Furthermore, Robles-Garcia (2019) and Benetton (2018) show that

2. As we show in Appendix (D.2), the model is also solved with a different functional form that yields a CES type of demand function instead of a logit one. This assumption is more common in theory but is not as empirically tractable.

3. This is the case when $(\varepsilon_{i,j})_j$ are not all equal.

4. They estimate the the entry decision in regular time, as opposed to a financial crisis. But it seems that even during the COVID-19 crisis, the number of borrowers did not drop on average.

the level of competition is high in the UK mortgage market, making it unlikely that banks will be able to extract the full surplus from borrowers. This motivates the assumption that the outside option $\bar{V}_i$ will be non-binding as well as the use of a static demand model.

In appendix C.7, we derive a nested logit version of the model in which borrowers actively choose to participate or not participate in the mortgage market. This extension yields a closed-form formula for the expected utility of participating in the market V_i, which can then be estimated. This modeling is convenient as it makes the logit coefficient independent of the assumptions on the set of potential mortgage buyers that did not enter the market.

C.1.2 Default

In this section, we model borrowers' repayment behaviour. We discuss the impact of borrowers' and lenders' information set on the expected default probabilities and define our screening test. Our modeling yields the default equations presented in section 1.4.2.

Default from borrowers' point of view

From the micro-foundations presented in Appendix (C.5), borrowers' demand elasticities parameters $\Gamma_{icb} := (\alpha_{icb}, \beta_{icb}, \tilde{\alpha}_{icb}, \tilde{\beta}_{icb}, \sigma_{icb})'$ might be a function of the default probabilities. Indeed, risky borrowers might be less sensitive to prices if they expect that they won't be forced to repay the full face value of the loan upon default. In that case, α_i would be a decreasing function of default. Alternatively, instead of being a default function directly, "price elasticity" α_i and default probability d_i might be influenced by the same fundamental parameter. For instance, a borrower with greater financial sophistication might find it less time-consuming to compare products and thus may end up with a cheaper product. The same borrowers might be more likely to make better financial decisions in general and thus may have a lower baseline default rate. For those reasons, we model default probabilities[5] (d_{icb}) and preferences Γ_{icb} the following way. Borrower i choosing contract c at bank b has a default probability d_{icb} (or equivalently a survival probability θ_{icb}) of

5. The logic behind our approach is as follows. The default probability is a function a of monthly repayment, the cost of defaulting and losing the house, the borrower's future income profile and the borrower's propensity to save. The loan size is an endogenous variable, so we replace it by its function defined in C.3. We linearize the expression around the contract and borrowers' characteristics. Then, we explicitly acknowledge that the choice of contract and loan size depends on default in equation (C.8).

$$d_{icb} = \beta^d(X_{cb}, r_{cb})' + \nu^d D_i + \rho \overbrace{PI_i^d + e_{ic}^d}^{borrower's\ private\ information} \tag{C.7}$$

$$\Gamma_{icb} = \mu + \nu^G D_i + \rho^G E[d_{icb}|\mathcal{I}_i^B] + e_i^G \tag{C.8}$$

where $E[d_{icb}|\mathcal{I}_i^B] := \beta^{di}(X_{cb}, r_{cb}) + \nu^{di} D_i + \underbrace{PI_i^d + e_i^{Ed}}_{borrower's\ private\ information}$.

and where $\beta^d(X_{cb}, r_{cb})'$ is the causal impact of contract characteristics on default due to moral hazard or burden of payment, D_i is a vector of borrowers' characteristics (such as income and age) that are observable by the bank, and (PI_i^d) is a vector of variables that influence borrower i's choice of bank-product and that are uncorrelated with contact characteristics. It captures adverse or advantageous selection. As mentioned in the previous paragraph, the impact of PI on default can be direct, as in the case of financial sophistication, or indirect through borrowers' private information about their default process. We assume that $cor(e_i^G, d_{icb}) = 0$. This is without loss of generality, as in the estimation, we will not be able to disentangle PI from e^G. The private information, for instance, contains information about the future income of the borrower or the borrower's level of risk aversion.

e_{ic}^d represents, for instance, the characteristics of borrower i (observable by the lender) that influence default but are not considered by borrowers when they make their loan decision (i.e., they do not enter Γ_i and cannot be recovered by banks).

As discussed in the previous section after equation C.5, without loss of generality, α_i has been normalized to one in equation C.8.

Borrowers' default from lenders' point of view

Now let us consider the lenders' point of view. Lenders would like to estimate the true model (equation C.7):

$$d_{icb} = \beta^d(X_{cb}, r_{cb}) + \nu^d D_i + \rho PI_i^d + e_{ic}^d. \tag{C.9}$$

However, by definition, banks (and the econometrician) cannot observe borrowers' private information (PI_i^d). Nonetheless, if borrowers with different private information (PI_i^d) consistently make different contract choices (i.e., $\rho^G \neq 0$), banks gain extra information about borrowers' default probabilities using a menu of products. Formally, using the menu of contracts, banks learn information about the borrowers' types ($\hat{\Gamma}_{icb} = E[\Gamma_i|i\ chose\ cb, \mathcal{I}_b^L]$) and use them as a proxy for (PI_i^d). $\mathcal{I}_b^L$ is the information set of bank b.[6] As using $\hat{\Gamma}_{icb}$ directly would bias the X estimates because Γ_{icb} is a function of X, we thus use $\hat{P}I_{icb} := E[PI^d + e_i^{Ed} + e_i^G | i \ chose \ cb, \mathcal{I}_b^L]$ instead. Banks' forecast of borrowers' default probability is thus

$$d_{icb} = \beta^d(X_{cb}, r_{cb}) + \nu^d D_i + \rho^d \hat{P}I_{icb}^P + \tilde{e}_{ic}^d, \tag{C.10}$$

where $\tilde{e}_{ic}^d$ is an error term uncorrelated with the characteristics of products. It can, for instance, be interpreted as the error term coming from the linearization of the true forecasting model used by banks.

Definition of a selection market: We denote $\rho_{x,y} := \rho_y^d \rho_{x,y}^d$ where ρ_y^d is the correlation between default and the y^{th} element of the private information component PI^d and $\rho_{x,y}^d$ is the correlation between the x^{th} preference parameter of Γ and the y^{th} element of PI^d. Given the assumption that the error term of the default regression is uncorrelated with observables and the private information component ($i.e., E[e_{icb}|X, r, PI] = 0$), we have $\rho_x := \sum_y \rho_{x,y} \neq 0$, which implies that the market is a selection market with respect to the contract characteristic associated with preference parameter x. That is, borrowers that prefer characteristic (x) tend to be more (less) likely to default if $\rho_x > 0$ ($\rho_x < 0$). We denote $\rho_{x/r}$ when, instead of the preference parameter for product x, we use the willingness to pay for product characteristic x.

Proposition 1: Test of screening and risk discrimination
It is possible to screen borrower a from borrower b with $\beta_{ax} \neq \beta_{bx}$ $\forall (a \neq b)$, *with* β_{ix} *he* x^{th} *element of vector* β_{ix}, *using contract characteristics* x *and rates if and only if* $\frac{\beta_{ac}}{\alpha_a} \neq \frac{\beta_{bc}}{\alpha_b} \forall (a \neq b)$.
We call this screening risk discrimination if $\rho_{x/r} \neq 0$, *with* $\rho_{x/r}$ *defined in the paragraph above.*

PROOF: Screening two borrowers (a, b) with different levels of willingness to pay $\beta_{ac} > \beta_{bc}$) works by offering a product (X, r) and $(X + \delta X_c, r + \tilde{r}\delta X_c)$ such that $\beta_{bc} < \tilde{r} \leqslant \beta_{ac}$. In that case, as shown in Appendix C.6, banks can almost perfectly screen the type of borrowers on their β_c characteristics as long as the variance of ϵ_{ic} tends to zero or there are no bounds on X_c and r_c. That is, it is possible to construct a menu of contracts such that $\forall i, \exists m \ / \ Pr(\beta_c = \beta_{ic} | choose \ contract \ m) \approx 1$.

6. The banks cannot use their menu to learn about demand shocks (ξ_i, e_i^L), search costs ϵ_{ib} and default shocks e_i^d as those characteristics are uncorrelated with how borrowers value the characteristics of contracts.

As a result, our empirical strategy aims to identify the distribution of preferences as well as its correlation with default ($\rho^d \neq 0$).

C.1.3 Lenders

This section models how lenders design their product menus and mortgage application rejection rules. Our modelling yields the supply equations presented in section 1.4.2. We start by presenting the lenders' maximization problem. We then informally and formally discuss lenders' product design incentives.

Information structure: Lenders know that borrowers behave according to equations (C.2, C.3, C.4, C.6, C.7). However, while the distribution of borrowers' parameters is known (Γ, e^L, $(\epsilon_{cb})_{cb}$, e^d), lenders cannot associate each draw of the distribution with a particular borrower. All other elements of the equations (C.2, C.3, C.4, C.6, C.7) such as the contract and banks' characteristics (X, ξ, r) associated with each loan amount L for all banks and the default parameters (β^d, ν^d, ρ), are known by lenders. We denote $\mathcal{I}_b^L$ as the information set of bank b.

Banks' behaviour

Bank b maximizes its expected profits by designing its menu of products $M_{bt} := (X_{cbt}, r_{cbt})_{c\ \in [\![1, C_{bt}]\!]}$ as well as its acceptance and rejection decision ($P_{ibt} \subset [\![1, C_{bt}]\!]$) for each borrower and contract. That is, banks choose the number of products C_{bt} they offer at time t, product characteristics X and prices r, and the subset of (indexes of) products from the menu available to borrower i (P_{ibt}). As discussed in section 1.4.1, we consider that banks use linear pricing for the loan size conditional on (X,r). This is optimal when, for instance, the only source of heterogeneity in loan demand comes from e^L or that screening is achieved with product choice rather than quantity. Banks face a fixed cost of changing their menu of products. They play a static game and take their competitors' contracts and pricing as given. In the empirical section, we also consider a two-stage game in which banks first choose their product and then compete on prices.

Formally, bank b maximizes:

$$max_{\{C_{bt},M_{bt}\in\mathcal{F}^{C_{bt}},P_{ibt}\}} \overbrace{E[\sum_{i=1}^{n}\sum_{c=1}^{C_{bt}} \underbrace{1_{\{(c_i,b_i)=(c,b)\}}1_{\{V_i\geqslant\bar{V}_i\}}}_{i\ chooses\ contract\ cb} \underbrace{NPV_{icb}}_{Net\ present\ value\ of\ lending} |\mathcal{I}_b^L]}^{Gross\ margin:\ \Pi_t^b} - \overbrace{F(M_{bt},M_{bt-1})}^{Fixed\ costs} \tag{C.11}$$

$$where\ (c_i,b_i) = \underbrace{argmax_{\{b\in B,\ c\in P_{ibt}\}}\{\tilde{u}_i(c,b)+\sigma_i^{-1}\varepsilon_{icb}\}}_{Borrower\ i\ optimal\ choice\ of\ bank-product}\ \forall i$$

where V_i is the expected utility of participating in the mortgage market and $\bar{V}_i$ is the outside option of not borrowing. The inequality ($V_i \geqslant \bar{V}_i$) conditions on borrower i's participation in the mortgage market. As discussed in section C.1.1, the effect of the contract term and prices on borrowers' entry and exit is negligible. We can thus ignore the inequality and take the number of borrowers as given in the estimation and counterfactual simulations.[7] Nonetheless, equations (C.17) and (C.18) in the appendix formally define the expected utility functional form that could be used to estimate the entry margin in an extension. This extension is important during an economic crisis when lending to some category of borrowers is a negative net present value project or when competition is low enough that banks may want to exclude the less profitable borrowers from the market to extract more surplus from the others.

NPV_{icb} is the net present value of lending to borrower i via contract c at bank b. The derivation of the formula is in Appendix (D.3). As in Benetton (2018) and Crawford, Pavanini, and Schivardi (2018), we consider that banks are risk neutral and that all borrowers refinance at the end of the teaser rate period, and we approximate the NPV by $NPV_{icb} := L_i(c,b)\cdot[(1-d_{icb})r_{cb} - mc_{cb}]F_{cb}$ where $L_i(c,b)$ is borrower i's loan demand conditional on choosing contract c at bank b (defined in equation C.3), d is the default probability (defined in equation C.7), r is the interest rate, F is the fixed rate period and mc_{cb} is the marginal cost of lending. The marginal costs are a function of product characteristics but will be estimated non-parametrically. $L_i(c,b)\cdot(1-d_{icb})r_{cb}F_{cb}$ is thus an approximation of the present value of lending to borrower i via contract c at bank b. An increase in the amount borrowed (L), the interest rate (r), the survival probability (1-d) or the fixed rate period (Fr) increases the present value of lending by increasing either the monthly payments or the periods during which borrower i provides monthly payments. $L_i(c,b)\cdot F_{cb}mc_{cb}$ is an approximation of the present value of the cost of lending via product c at bank b. An increase in the amount lent (L) increases, for instance, the amount of deposits required and thus increases the cost of lending by $F_{cb}mc_{cb}$. The marginal costs mc_{cb} are multiplied by

7. Taburet (2022) shows that this is the case when, for instance, all investments have a positive net present value, competition is high enough and $\bar{V}_i$ is constant across borrowers.

the fixed rate period F_{cb} to facilitate the comparison with the interest rate. That way, the marginal costs are expressed as if the bank funds its lending using debt with the same maturity as the fixed rate period of the loan. Not including F_{cb} would just lead to a renormalization of the fixed cost mc_{cb} in the estimation.

We introduce a fixed cost function $F(M_{bt}, M_{bt-1})$ in order to match the number of products offered by banks in the empirical analysis. Given our estimates on demand heterogeneity, the fixed cost function is required to prevent banks from offering a continuum of contracts. It is a function of a mathematical distance between bank b's current menu (M_{bt}) and its previous menu (M_{bt-1}). Its exact specification is provided in the overview of the setup section (section 1.4.2) and in the estimation section (section 1.5.4). The fixed cost prevents the model from being solved using the revelation principle (i.e., offering one contract per type of borrower).

In the next section (section C.9), we solve for the model without the fixed cost to provide intuition about the different mechanisms at play.

Feasible contracts: For the problem to be well defined even if the marginal costs turn out to be non-convex in product characteristics, we assume that contract characteristics are bounded: $\mathcal{F} := \{(r, X) : X \in [0, \bar{X}] \subset \mathbb{R}_+^{dim(X)}, r \geqslant 0\}$. This can reflect regulations — such as the maximum LTV of a contract — or the fact that the demand has some kinks. In the estimation and counterfactual exercise, we further assume that the set of possible combinations of product characteristics (X) is finite. We discuss in the identification and estimation section the conditions under which this assumption does not bias our results.

Discussion about the supply model assumptions

Any model simplifies the reality of focusing on a given economic phenomenon. In our structural model, we do not endogenize the house price upon default and do not model dynamic considerations in order to be able to model screening incentives in more detail. The counterfactual simulations thus consider that those elements — as well as unobserved product characteristics — remain constant. In this section, we discuss how those assumptions affect the interpretation of the supply parameters.

Collateral: In its current formulation, the model is set as if banks do not recover anything following borrowers' default. This assumption does not affect the demand estimation as we do not explicitly model the cost of default and instead rely on a revealed preference approach. However, it affects the interpretation of the marginal cost parameter that is recovered in the estimation section. To provide intuition for how to interpret the results given our assumption about collateral, let us introduce the following notation. Upon default, the mortgage originator can seize the lender's house and get $min\{\delta \cdot \frac{L}{LTV}, rL\}$. $\frac{L}{LTV}$ is the house value at the origination date, and δ is the ratio of the house price upon default over the one at origination. Default happens with probability d. The estimated marginal cost will capture the average loss given default conditional on LTV $E[mc - min\{\delta \cdot \frac{1}{LTV}, r\}d|LTV]$. Given our identification strategy, we cannot identify δ and mc separately. However, we discuss how one could do so using an integrating over approach in Appendix C.13.1.

Finally, although the use of collateral has been taken as given rather than derived from a first principle, conditions for collateralized debt to be the optimal contract is in Appendix *D*.2.1.

Static model of supply: The supply model used in this paper is static, as each period lenders maximize the expected profits generated by current lending activities only. This consideration is justified by the demand also being static. Static demand is heavily used in the literature and is a good approximation for mortgage markets as recent studies show that borrowers' entry and exit decisions — and thus their decisions on when to borrow — are almost never affected by mortgage prices and product offerings (Andersen et al. 2021 and Benetton, Gavazza, and Surico 2021). However, the use of the fixed cost function in the lenders' problem creates a dynamic relationship between current and past maximization problems and makes the use of a dynamic model natural.

The static supply approach can nonetheless be justified by the following considerations. First, our static modeling can be written as the hurdle rate approach, which is a good approximation of firms' product-offering decisions according to recent surveys (see Wollmann 2018). The hurdle rate approach assumes that firms choose to offer a set of products such that, for any other feasible set, the expected ratio of the added profits to added sunk costs does not exceed a set number (the hurdle rate).

Second, the only parameter affected by a dynamic modeling approach is the fixed cost function, which is not an object of interest of our analysis. Indeed, the marginal costs are not affected as they are identified from a model optimality condition that depends on the number of products being fixed. The counterfactual experiment is not affected by the use of the static model as long as the relationship between current and expected profits in the counterfactual experiment remains the same as in the data. The static estimation affects the economic interpretation of the size of the fixed cost. As a complementary approach, we show in Appendix C.13.3 how methods used in the dynamic demand estimation literature could be used in a dynamic version of our model to estimate the supply parameters. However, the dynamic estimation increases the computational burden of counterfactual experiments to the point where the counterfactual model would not be solvable with the current methods available.

C.2 Demand CES form

C.2.1 1 characteristic

$N := \frac{u^{\varepsilon}}{\sum u^{\varepsilon}}$ so $N'_a = \frac{(\varepsilon}{u} N_a[1-N_a]$, $\varepsilon \in [0,\infty)$

This can be microfounded by using the functional form:

$$choice : max\{ue^{\varepsilon^{-1}\epsilon}\} \iff max\{ln(u) + \varepsilon^{-1}\epsilon\}$$
$$Pr(choosing) : \frac{e^{ln(u^{\varepsilon})}}{\sum e^{ln(u^{\varepsilon})}} = \frac{u^{\varepsilon}}{\sum_b u_b^{\varepsilon}}$$

As in Dixit-Stiglitz, when the number of firm is large, we can abstract from $[1-N_a]$ in the derivative. In a symmetric equilibrium, $\frac{N_a}{N_b}$ equal to the share of type a versus type b borrowers. Using $\tilde{\varepsilon}_i := \varepsilon[1-\frac{N_i}{n}]$, n being the number of banks operating in the market:

$$u_a = S_a(X^*_{ac})\gamma_a$$
$$\gamma_a := \frac{\varepsilon_a}{\varepsilon_a + 1 - \frac{\theta_b N_b}{\theta_a N_a}\frac{[\alpha_{\tilde{c}b} - \frac{r_{\tilde{c}b}}{\theta_b}]}{(\alpha_{\tilde{c}a} - \alpha_{\tilde{c}b})}}$$

$$u_b = S_a(X^*_{ac})\gamma_b$$
$$\gamma_b := \frac{\varepsilon_b}{\varepsilon_b + 1 + \frac{[\alpha_{\tilde{c}b} - \frac{r_{\tilde{c}b}}{\theta_b}]}{(\alpha_{\tilde{c}a} - \alpha_{\tilde{c}b})}}$$

Pricing:

$$R_a = (1-\gamma_a)\alpha_a + \gamma_a \frac{r_c}{\theta_a}$$
$$R_b = (1-\gamma_b)\alpha_b + \gamma_b \frac{r_c}{\theta_b}$$

Distortion:

$$X^*_a := \bar{X}$$
$$X^*_b := \frac{\gamma_a[\alpha_a - \frac{r}{\theta_a}]}{\gamma_b[\alpha_b - \frac{r}{\theta_b}] + (\alpha_a - \alpha_b)}\bar{X} := \delta\bar{X}$$

Under perfect competition ($\varepsilon \to \infty$):

$$X_b := \frac{[\alpha_a - \frac{r}{\theta_a}]}{[\alpha_a - \frac{r}{\theta_b}]}\bar{X} \leqslant \bar{X},\ since\ \theta_a \geqslant \theta_b$$

As competition decreases ($\varepsilon \to 0$):

$$-\partial_\varepsilon \frac{X_b}{\bar{X}} := [\frac{-\partial_\varepsilon \gamma_a}{\gamma_a} + \frac{\partial_\varepsilon \gamma_b[\alpha_b - \frac{r}{\theta_b}]}{\gamma_b[\alpha_b - \frac{r}{\theta_b}] + (\alpha_a - \alpha_b)}]\delta \lesseqgtr 0$$
$$\partial_\varepsilon \frac{X_b}{\bar{X}} = [-\frac{\gamma_a}{\varepsilon}(1-\gamma_a) + \frac{\gamma_b}{\varepsilon}(1-\gamma_b)\frac{1}{\gamma_b + \frac{\alpha_b - \frac{r}{\theta_b}}{\alpha_a - \alpha_b}}]\delta$$

Want to extract more surplus from the high WTP borrower a but can deal better with the friction by using an information rent.

C.3 Nested logit

The product and bank choice can be written as a nested logit by making the assumption that the variable $\zeta_{ijc} := \xi_{icj} + \sigma_i^{-1}\varepsilon_{i,j}$ follows a generalized extreme value distribution. Indeed, assuming that borrower i draws $(\zeta_{ibc})_{bc}$ follows an extreme value distribution: $F((\zeta_{ibc})_{jc}) := exp(-\sum_{b\in J}(\sum_{c\in M_j} e^{-\zeta_{ibc}})^{\lambda_b}$.

Products within the same menu M_b have a correlation of approximately $1-\lambda_b$. In case in which all λ_b are equal to 1, the choice of bank-product has the logit form. In case all $\lambda_b = 0$, the random term within a nest are perfectly correlated. The choice of banks has a logit form, and within a nest, borrowers choose the product giving them the highest utility.

C.4 Imperfect Information about acceptance and rejections

When borrowers do not observe acceptance and rejection rules, denoting p_{icb} the probability of being accepted, the utility they derive from a contract $c \in C$ is:

$$p_{icb}u(cb) + (1-p_{icb})\beta[E_\varepsilon[V(c)] - cost] \tag{C.12}$$
$$V(c) = max_{\{x\in C\setminus c\}}[p_{ix}u(x) + (1-p_{ix})\beta E_\varepsilon[max_{\{x\in C\setminus c\}}V((c,x)) - cost] \tag{C.13}$$

$V(x)$ is the expected utility after being rejected from the contracts present in vector x. Since rejections are observed by other banks, the probability of being accepted in another contract may be lower upon rejections. Assuming that borrowers get a new extreme value draw after each rejection, once can calculate V in a closed form manner. To ease computational burden, one can assume that the probability of being accepted after the first rejection i 0 and replace $V(c)$ by an outside option that is borrower specific $\bar{u}_i$.

Assuming that the term $p_{icb}[\sigma_i^{-1}\varepsilon_{ibc}) - \bar{u}_i] + \bar{u}_i$ is extreme value distributed with a variance $\bar{\sigma}_i^{-1}$, the new model thus become equivalent to the perfect information case with all utility parameter scaled by p_{icb}:

$$p_{icb}u_i(cb) + (1 - p_{icb})\bar{u}_i \tag{C.14}$$

C.5 Micro-foundation borrowers ' utility mortgage market

In this section I micro-found borrowers' borrowers' indirect utility function used in the main section of the paper.

The assumptions about borrowers' utility function are made for tractability and do not impact the qualitative results.

C.5.1 Indirect utility functional form micro foundation

Toy Model consume in period 1, default and loose the house in period 2

$$\begin{aligned} u(C^*, H^*) :=& max_{\{C,L\}}\ \mu C_1 + \overbrace{(1 - \frac{\delta}{2}\frac{r}{Y_2}L)}^{survival\ probability}\ [\frac{\phi}{P_H}\frac{L}{ltv} + \mu C_2] \\ & pC_1 + (1 - ltv)\frac{L}{ltv} = Y_1 \\ & pC_2 = Y_2 - rL \end{aligned}$$

$\frac{\delta}{2}\frac{r}{Y_2}\frac{L}{ltv}$ represents the fact that you are more likely to default as you leverage This implies:

$$H^* = \frac{L^*}{ltv} = \frac{\overbrace{\frac{\phi}{P_H}}^{bigger\ house} - \overbrace{\frac{\mu}{p}(1 - ltv)}^{lower\ consumption\ period\ 1} - \overbrace{\mu r}^{lower\ consumption\ period\ 2}}{\underbrace{(\frac{\phi_c}{P_H} - \frac{\mu r}{p})\delta\frac{r}{Y_2}}_{Higher\ default}}$$

Thus:

$$V(Y_1, H^*) := u(C^*, H^*) = \frac{\mu}{p}[Y_1 + Y_2] + H^*\{(\frac{\phi_c}{P_H} - \mu r \cdot ltv)[\frac{\frac{\phi_c}{P_H} - \frac{\mu}{p}r + \frac{\mu}{p}(1 - ltv)}{2(\frac{\phi_c}{P_H} - \frac{\mu r}{p})}] - \frac{\mu}{p}\frac{\delta r}{2ltv}\}$$

Without consumption in period 2:

$$V(Y_1, H^*) := u(C^*, H^*) = \frac{\mu}{p}Y_1 + H^*[\frac{\frac{\phi_c}{P_H} + \frac{\mu}{p}(1 - ltv)}{2}]$$

C.5.2 Derivation of the Demand system

Borrowers maximize:

$$max_c u(L_{ci}, c) = max_c A_{ic}\frac{L_{ci}}{ltv} + V(Y)$$

A_c captures that default or consumption trade-off depends on contracts c features

$$max_c u(L_c, c) = max_c ln(A_{ic}) + ln(L_c) - ltv(ltv)$$
$$ln(A_{ic}) = \tilde{\beta}_i X_c + \sigma_i \varepsilon_{ic}$$

From Roy's Identity ($A_{ic}\ doesn't\ vary\ with\ Y, r$):

$$\frac{L}{ltv} = \gamma^{-1}\frac{[\partial_{D_i}\{\frac{L}{ltv}\}]A_{ic}}{V_Y(Y)}$$

Integrating with respect to DF_i (loan discount factor):

$$ln(L_c) = ln(ltv) + \gamma\frac{V_Y}{A_{ic}}DF_i + cst$$
$$with:\ cst := \beta_i X_c + \epsilon_i, with\ (cstr)$$

set $\frac{DF_i}{A_{ic}} = \nu D_i + (\beta_1 X_c + \gamma_c + \beta_2 X_i + \gamma_i)r_c$

In the regression, allow for some element of A_i to be proxied by income. That way the income element of β_i^b need not be equal to the one in β^L

$$Pr(i\ choose\ c) = \frac{exp(\beta_i^b X_c - \alpha_i^b r_c)}{\sum_j exp(\beta_i^b X_j - \alpha_i^b r_c)} \tag{C.15}$$

$$ln(L_{ci}) = \alpha_i^L r_c + \nu D_i + \beta_i^L X_c + \sigma_\epsilon \epsilon_i \tag{C.16}$$

with β_i^d, α_i^b correlated with α_i^L, β_i^L

C.6 Proof screening

Let us stat with considering that β_c can take only two values β_{ac} and β_{bc} with $\beta_{ac} < \beta_{bc}$, the preferences along the other dimensions are continuously distributed and independent of the contracts characteristics. We can then generalize the proof to any number of values.

Using Bayes' rule:

$$Pr(\beta_c = \beta_{ic}|choose\ contract\ b) = \frac{Pr(choose\ contract\ b|\beta_c = \beta_{ic})Pr(\beta_c = \beta_{ic})}{\sum_{j\in\{a,b\}} Pr(choose\ contract\ b|\beta_c = \beta_{jc})Pr(\beta_c = \beta_{jc})}$$

We start by offering contract A with characteristics (X,r) that would be accepted by both borrowers and then offer another contract B with characteristic $X + \Delta X, r + \omega\Delta X_c$ with $\omega > max\{\frac{\beta_c}{\alpha}\}$. When $\Delta X = 0$ contract A have the following market share: $Pr(\sigma\xi_{jA} > \sigma\xi_{jB})$. Denoting f the pdf of $\xi_{jA} - \xi_{jB}$, By Increasing ΔX the probability of contract B being chosen by the high willingness to pay (WTP) borrower increases by: $\frac{\frac{\beta_{ac}}{\alpha} - \omega}{\sigma} f(0)$ while for the other it decreases by $\frac{\frac{\beta_c}{\alpha} - \omega}{\sigma} f(0)$.

We can generalize this proof by starting by separating the lowest WTP from all others, then the second lowest WTP from all other borrowers. When there are no bounds on r and on X, or when $\sigma \to 0$, borrowers can be almost perfectly screened $\forall\epsilon, \exists(\Delta X_m, \omega_m)_m : \forall i, \exists m : Pr(\beta_c = \beta_{ic}|choose\ contract\ m) > 1 - \epsilon$.

C.7 Nested logit extention

Following the nested logit approach, we use the following timing assumptions. Before knowing their individual i preferences over banks $(\varepsilon_{ib})_b$, borrowers choose to enter the borrowing market if their expected utility V_i is greater than the option of not borrowing. In the theoretical analysis section, this timing assumption is equivalent of assuming that borrower will not accept a contract if its utility $u_i(C^*_{ib})$ is lower than the option of not borrowing. This favours the interpretation that the value $\sigma_i\epsilon_{ib}$ models a search or sunk cost that has to be paid to lean about bank b menu rather than valuable characteristics of the bank. Formally, Borrowers enter the market if:

$$V_i := E_\varepsilon[max_{\{b\in B\}}\{\tilde{u}_i(C^*_{ib}, b) + \sigma_i\varepsilon_{ib}\}] - \sigma^{-1}ln(\#B) \geqslant \bar{V}_i \in \mathbb{R} \tag{C.17}$$

$\#B$ denotes the number of product available to borrower i. When all the banks offer the same contracts C^*_i, $E_\varepsilon[max_{\{b\in B\}}\{u_i(C^*_{ib})\}] = \sigma^{-1}ln(\#B) + E_\varepsilon[u_{\Gamma_i}(C^*_i)]$. The fist term captures the fact that if a lot of products are being offered, then it is more likely that borrowers find a product that have a high unobserved characteristic ε_{ib}. This is to get rid of this effect that I define V_i the entry condition this way.

This equation will not be binding except when competition is low enough.

Individual i preferences over banks $(\varepsilon_{i,j})_j$ is drawn from an extreme value distribution

$$E_\varepsilon[max_{\{b\in B\}}\{\tilde{u}_i(C^*_{ib}, b) + \sigma_i\varepsilon_{ib}\}] = \sigma^{-1}ln(\sum_{x\in B} exp(\sigma_i u_i(C^*_{ix}, x))) \quad (C.18)$$

C.8 Derivation Present Value of Lending

Given a loan size L, a maturity T and a per period compound interest rate r, the per period mortgage repayment C is given by the annuity formula:

$$C = \frac{Lr(1+r)^T}{(1+r)^T - 1} \quad (C.19)$$

Similarly, we can express the bank cost of lending an amount L as a constant rate (mc) and write it as an annuity to make it comparable to the interest rate (r):

$$D = \frac{Lmc(1+mc)^T}{(1+mc)^T - 1} \quad (C.20)$$

The marginal cost includes, among others, the interest rate banks need to pay on its deposits.

Using δ as the discount rate, the present value of lending the amount L, abstracting from default, can thus be written:

$$L\sum_{k=1}^{F}\delta^k[\frac{r(r+1)^T}{(r+1)^T-1} - \frac{mc(mc+1)^T}{(mc+1)^T-1}] + \gamma b\sum_{k=F+1}^{T}\delta^k[\frac{R(R+1)^{T-F}}{(R+1)^{T-F}-1} - \frac{mc(mc+1)^{T-F}}{(mc+1)^{T-F}-1}] \quad (C.21)$$

R is the reset rate and b is the remaining balance at the end of the teaser rate period. F is the fixed rate period, T is the maturity of the loan, γ is the share of people not refinancing and mc is the marginal cost of lending.

As in Crawford, Pavanini, and Schivardi (2018), assuming that banks consider the average default instead of the probability of defaulting in each period, for a constant discount rate ($\delta < 0$), denoting d a dummy equal to 1 if borrower default, the present value of lending up period F is:

$$C \cdot E[(1-d)] \cdot \sum_{k=1}^{F}\delta^k = Lr\frac{(1+r)^T}{(1+r)^T-1} \cdot E[(1-d)] \cdot \frac{1-\delta^F}{1-\delta}\delta \quad (C.22)$$

When T and F are large, $\frac{(1+r)^T}{(1+r)^T-1} \approx 1$ and $\delta^F \approx 0$, the net present value of lending is thus:

$$PV \approx L \cdot \{E[(1-d)]r\frac{\delta}{1-\delta} + \gamma E[(1-d)]R\frac{1-\delta^{T-F}}{1-\delta}\delta^F - [\frac{\delta}{1-\delta} + \gamma\frac{1-\delta^{T-F}}{1-\delta}\delta^F]mc\} \tag{C.23}$$

With ($\delta = 1$), the expression is instead:

$$PV \approx L \cdot [E[(1-d)]rF + \gamma RE[(1-d)](T-F) - [F + \gamma(T-F)]mc] \tag{C.24}$$

We further assume as in Benetton (2018) that $\partial_r \gamma = 0$ so that it does not enter inside the FoC of r_c and set γ_c to 0 (i.e., all borrower remortgage). We can thus also abstract from the discount rate if $\delta < 1$ as it is constant across mortgages, we thus get:

$$NPV_{icb} := L \cdot [E[(1-d)]r - mc] \text{ when } \delta < 1 \tag{C.25}$$

The above expression comes implies that banks do care about fixing the interest rate except from its impact on the cost of lending (mc), default (d) or on demand (L). This result comes from the assumption that $\delta^F \approx 0$. It may be problematic as for a given demand, interest rate, default and marginal cost, profits are likely to be increasing in F as the loan generates annuities for a longer period.

Relaxing the assumption $\delta^F \approx 0$ would however require an assumption about the discount rate used (for instance the bond of or deposit rates) or the use of non standard approaches like the integrating over one (see C.13.1). This last method is too computationally demanding for our set-up. We thus go with the first approach and assume that $\delta = 1$. We get:

$$NPV_{icb} := L \cdot [(1-d)r - mc]F \text{ when } \delta = 1 \tag{C.26}$$

Alternative approach:

Without using Crawford, Pavanini, and Schivardi (2018) assumption about default, the expression for the annuity would be would be, using d as the per period default probability:

$$C\sum_{k=1}^{t}((1-d)\delta)^t = Lr((1-d)\delta)\frac{(1+r)^T}{(1+r)^T-1}\frac{1-((1-d)\delta)^t}{1-((1-d)\delta)} \tag{C.27}$$

Using the same approximations as in Benetton (2018), $\frac{(1+r)^T}{(1+r)^T-1} \approx 1$ and $\partial_r \gamma = 0$, the expression for the NPV becomes:

$$NPV_{icb} := L \cdot [(1-d)\delta \frac{1-((1-d)\delta)^F}{1-\delta+d\delta} r - mc \frac{1-\delta^F}{1-\delta}] \text{ when } \delta < 1 \quad (C.28)$$

$$NPV_{icb} := L \cdot [(1-d)\frac{1-(1-d)^F}{d} r - mc \cdot F] \text{ when } \delta = 1 \quad (C.29)$$

Here again, as the discount rate is not observable, the NPV would require estimating both the discount rate δ and the marginal cost mc. In a low rate environment, the discount factor can be approximated by 1. Changing the definition of the NPV will impact the interpretation of the mc as discussed in C.1.3. Moreover, when d is small as in our empirical application and δ equal to 1, the expression becomes the same as in Crawford, Pavanini, and Schivardi (2018):

$$NPV_{icb} \underset{d \to 0}{\sim} L \cdot [(1-d)r - mc] \cdot F, \text{ when } \delta = 1 \quad (C.30)$$

C.9 Product introduction and exclusion incentives

This section provides an informal analysis of the different mechanisms driving the product offering and pricing in our setup. In particular, we discuss the impact of imperfect information on loan contracts. A formal analysis is provided in Appendixes C.10 and C.11. An in-depth analysis of the incentives to screen in this class of model is presented in Taburet (2022).

Under perfect information about borrowers' preferences and default probabilities (i.e., borrowers' type), the design of a different contract for each borrower type allows for catering to their heterogeneous needs (Tirole 1988). Absent a fixed cost of creating contracts, and as long as the same product can be sold at a different price to different borrower types, lenders should create as many loan contracts as borrower types. In that class of model, under classic assumptions, a high demand elasticity (competition) drives interest rates down and the loan size up.

With imperfect information, lenders may find it optimal to use contract menus to screen borrowers. As well established in the literature, screening may require distorting contracts away from their perfect information value to maintain borrowers' incentives to self-select. It is optimal for banks to distort — relative to the perfect information case — the contract features (i.e., product characteristics or pricing) that have the lowest impact on their profits.[8] In the monopoly case, for instance, banks have incentives to distort — relative to the perfect information case — the

8. The contract features also need to be heterogeneously valued by borrowers. If contract characteristics are valued similarly by borrowers, changing those terms for one or many contracts affects the incentives to choose a given contract similarly for all borrowers. Thus, it does not affect the distribution of borrower types choosing the contract.

contract designed to be selected by the less numerous borrower type or the one for which they make fewer profits on each loan. When some degree of competition is introduced, banks must also consider how contract terms affect the loan demand they face. When borrower demand elasticity is high and screening is feasible, this second consideration can force banks to price each borrower according to the borrower's own default probability (i.e., to screen) even when all borrower types would benefit from being pooled (Taburet 2022). Indeed, by failing to do so, a lender could take advantage of its competitors' offers to attract the most profitable borrowers only (i.e., screen). As a result, a high demand elasticity can drive the price of some contracts up — when it prevents high-default borrowers from being pooled with low-default borrowers — and the quantities down — when credit constraints are used to screen.

The above-mentioned considerations can lead to product introduction and exclusion relative to the perfect information case. To illustrate this point, let us consider a situation in which, under perfect information, banks' most profitable option is to offer one product only (e.g. a long-term loan) but price it differently depending on the customer. This heterogeneous pricing can be a result of, for instance, borrowers' price elasticity or default probability heterogeneity. However, under imperfect information about borrowers' heterogeneity, banks cannot sort borrowers with a menu composed of the same product priced differently, as each borrower will choose the cheapest one. Banks can thus choose to price the product using the average borrower characteristics or introduce one or many new products to make borrowers self-select. For instance, if high default probability borrowers find it relatively more costly to get a short-term contract, screening can be achieved by introducing short-term contracts. Those contracts will attract unobservably safer borrowers and can thus be offered at a lower price.

Under imperfect information, the impact of product introduction or exclusion on welfare is ambiguous. On the one hand, banks' product introduction provides more tools to screen borrowers. This can increase welfare as it lowers the asymmetric information level. For instance, under perfect competition, screening can maximize the sum of borrowers' utility. This happen when the utility loss caused by the net cost of screening (i.e., the contract terms' distortions, relative to the first best, that are required to sort borrowers) is lower than the utility losses coming from the net cost of being pooled (i.e., the spread between the fair price of lending to the average borrower and the perfect information pricing). On the other hand, as shown theoretically in Taburet (2022) and discussed in the following section, screening may be implemented even when the informationally constrained social planner would pool borrowers.

Inefficient product introduction or exclusion results from lenders not internalising that their screening behaviour affects the demand and thus the screening costs of other banks. This issue is analysed in Taburet (2022) and is called a contractual externality. A graphical illustration of this contractual externality is provided in the following section (C.11) in a simplified version of our setup. The contractual externality creates the following welfare trade-off between competition and adverse selection. A low competition level mitigates lenders' concerns over losing their market share; this can improve welfare by giving them more flexibility on how to use contract terms and prices to sort borrowers efficiently. However, a low competition level also incentivizes lenders to apply high markups, which can reduce welfare. Imperfect information and adverse selection thus interact: decreasing one imperfection may increase the other.

Overall, because of the contractual externality, the outcome of screening markets can be information constrained inefficient. Those markets can thus provide too many or too few products relative to the second best (i.e., what an imperfectly informed social planner could achieve). To measure this friction, we need to analyze banks' screening incentives. This analysis calls for an estimate of which contract feature distortions — relative to the perfect information case — have the lowest impact on banks' profits. The latter requires understanding how borrowers make their choice of banks and contracts, how borrowers' choices reveal information about default probabilities, measuring the present value of lending via a given contract and the cost of changing product characteristics and menu size.

C.10 Formal analysis of the model

Let us assume in a first step that preferences (Γ_i) are observable and solve for the optimal contracts before considering the case in which they are not. We consider throughout the exercise that the demand shocks $(\sigma^{-1}\varepsilon_{icb})_{icb}$ are extreme value distributed, independent and not observed by banks. This assumption is a tractable way of modelling competition among banks. It makes the demand function ϕ continuous, which allows to solve the model using the first order conditions and yields a closed form solution. As they are not at the center of our analysis, we also consider that there are no fixed cost of designing a contract.

Indexing contract by c $\in [\![1, C_{bt}]\!]$, the maximization problem (E.1) can be written:

$$max_{\{(X_c,r_c)_c \in \mathcal{F}^C, C\}} \sum_i n_i \sum_{c=1}^{C} \phi_{ibc}\pi_{ibc} \tag{C.31}$$

n_i is the number of type i borrowers.

ϕ_{ibc} the probability that borrower i chooses contract c. Our assumptions about $\sigma^{-1}\varepsilon_{icb}$ being independent implies that ϕ_{ibc} can be written $\frac{exp(\sigma(\beta_i X_{cb} - \alpha_i r_{cb}))}{\sum_{x \in B}\sum_{y \in P_{ix}} exp(\sigma(\beta_i X_{cb} - \alpha_i r_{cb}))}$. $\beta_i X_{cb} - \alpha_i r_{cb}$ is the average utility of borrower i when they get contract c at bank b. σ drives the product demand elasticity.

π_{ibc} is the expected profit on contract c when borrower i chooses contract c (i.e., $L_i(c,b) \cdot NPV_{icb}$ in the problem (E.1)).

Perfect information incentives: Under perfect information, banks offer one product per type. That is, P_{ix} is a singleton. We index the contracts by the borrower index i, drop the band index b in the notation and use the notation (X_{ix}) to denote the x element of vector (X_i). Using the first order conditions, the contract terms and prices given to borrower i (X_i, r_i), must satisfy:

$$Pricing: \quad \overbrace{n_i\phi_i\partial_{r_i}\pi_i}^{Intensive\ margin} \quad \overbrace{\underbrace{-\sigma\alpha_i n_i\phi_i(1-\phi_i)}_{number\ of\ lost\ customers} \quad \pi_i}^{Extensive\ margin} = 0 \tag{C.32}$$

$$Contract\ Characteristics: \quad \overbrace{n_i\phi_i\partial_{X_{ix}}\pi_i}^{Intensive\ margin} \quad + \overbrace{\sigma\beta_i n_i\phi_i(1-\phi_i)}^{Extensive\ margin}\pi_i = 0 \tag{C.33}$$

Changes in contract terms and price affects profits through an intensive and extensive margin channel. When increasing, for instance, interest rates, the bank increases profits on each loans (if $\partial_{r_i}\pi_i > 0$) but losses some customers $(-\sigma\alpha_i(1-\phi_i)\phi_i n_i\pi_i)$. $\sigma\alpha_i(1-\phi_i)\phi_i n_i$ is the number of customers lost. The extensive channel effect is stronger for highly price elastic borrowers (i.e., high α_i borrowers).

Neglecting the impact of contract terms (X) on default probabilities (d) to focus on adverse selection rather than moral hazard, considering a symmetric equilibrium, and relabeling $\sigma_i = \sigma(1 - \frac{1}{\mathbf{card}(B)})$ we get —rearranging the above equations — that the optimal equilibrium contract for borrower i is:

$$Optimal\ pricing: r_i = \frac{\overbrace{mc(X_i)}^{Fair\ price}}{1-d_i} + \overbrace{\frac{1}{\underbrace{\sigma_i\alpha_i}_{product\ demand\ elasticity} + \underbrace{\tilde{\alpha}_i}_{loan\ size\ demand\ elasticity}}}^{mark\ up} \tag{C.34}$$

$$Optimal\ characteristic: X_{ix} = \underbrace{\frac{\sigma_i\beta_{ix} + \tilde{\beta}_{ix}}{(\sigma_i\alpha_i + \tilde{\alpha}_i)}}_{\text{“}willingness\ to\ pay\text{”}} \quad \underbrace{\frac{1-d_i}{mc_x}}_{effective\ cost} \tag{C.35}$$

For intuition, we set the marginal cost of lending as the function $mc(X_i) := \sum_x mc_x \frac{X_{ix}^2}{2}$, with mc_x a known number that parameterize the cost of increasing the x^{th} characteristic of a product (i.e., its LTV, fixed rate duration...).[9]

9. Absent this functional form assumption, the formula would be: $\partial_{X_{ix}} mc_i = \frac{\sigma_i\beta_{ix}+\tilde{\beta}_{ix}}{(\sigma_i\alpha_i+\tilde{\alpha}_i)}(1-d_i)$

Equation (C.34) states that the contract price is the sum of a fair price ($\frac{mc(X)}{1-d}$) plus a "mark up" term[10] ($\frac{1}{\sigma_i \alpha_i + \tilde{\alpha}_i}$). The mark up is a function of both product and loan size demand elasticities. The interest rate elasticity of product demand depends on the variance of the demand shock parameters (σ). When competition is high ($\sigma \rightarrow \infty$), lenders price each contract at their fair price. The number of lenders ($\mathbf{card}(B)$) increases the product demand elasticity (σ_i) as it makes it more likely that the next bank is not too far away so that borrowers are more likely to change lenders (interpreting the demand shock as a distance from the closest bank branch like in Hoteling (1929)).

Labeling X_{ix} as the contract maximum loan-to-Value (LTV) for simplicity of the exposition, equation (C.35) states that banks provide high maximum LTV when: borrower i value positively this contract characteristic (i.e., $\sigma_i \beta_{ix} + \tilde{\beta}_{ix}$ high) and is not sensitive to a price increases ($\sigma_i \alpha_{ix} + \tilde{\alpha}_{ix}$ low), and when the cost of increasing the maximum LTV is low.[11] The cost is low when borrower i default probability (d) is low and when an increase in the maximum LTV of the contract is cheap to provide (low $\tilde{mc}_x$).

Banks thus have incentives to provide different products when borrowers' default probabilities and preferences are heterogeneous. Including fixed cost in the analysis would requires the heterogeneity to be larger (and the market size to be large enough) for new product to be offered.

Imperfect information incentives: Let us now consider the case where banks cannot observe borrowers' type. To focus on screening, we consider the situation in which all borrowers are observationally equivalent from banks' point of view. Using the first order condition of problem (C.31) and dropping the b index, we get:

$$Pricing: \overbrace{\sum_i n_i \phi_{ic} \partial_{r_c} \pi_{ic}}^{Intensive\ margin} - \overbrace{\sum_i n_i \alpha_i \phi_{ic} (\pi_{ic} - \sum_{j=1}^{C} \phi_{ij} \pi_{ij})}^{Extensive\ margin} = 0 \quad (C.36)$$

$$Contract\ characteristics: \overbrace{\sum_i n_i \phi_{ic} \partial_{X_{cx}} \pi_{ic}}^{Intensive\ margin} + \overbrace{\sum_i n_i \beta_{ix} \phi_{ic} (\pi_{ic} - \sum_{j=1}^{C} \phi_{ij} \pi_{ij})}^{Extensive\ margin} = 0 \quad (C.37)$$

10. The theoretical literature usually refers to the markup as the output price divided by the marginal cost. I instead define the mark up as the pricing above the marginal costs. The empirical IO literature sometimes uses the same terminology (Crawford, Pavanini, and Schivardi (2018)).

11. The ratio $\frac{\sigma_i \beta_{ix} + \tilde{\beta}_{ix}}{(\sigma_i \alpha_i + \tilde{\alpha}_i)}$ can be thought as a willingness to pay measure. Setting the loan demand parameters to 0, the fraction becomes borrowers' the willingness to pay for the x characteristic (i.e., $\frac{\beta_x}{\alpha}$). Setting the product choice parameters to zero we get ($\frac{\tilde{\beta}_X}{\tilde{\alpha}}$) which is the marginal loan demand increase following a change in X over minus the marginal increase in rates.

The intensive margin channel is the same as before, the extensive channel is different as banks must now consider the fact that they offer menus. The first order conditions with respect to contract c terms and pricing thus have an extra element capturing the probability that borrowers of type i choose another contract than contract c (ϕ_{ij}).

The extensive margin channel captures the effect of a marginal increase in, for instance, contract term X_{cx} on the number of borrowers choosing contract c. The increase primarily attracts borrowers that value this characteristics relatively more (i.e., those with a high β_{ix}). The increase in product c demand can come from borrowers that would have shopped in another bank absent the contract term increase ($\beta_{ix}\phi_{ic}(1-\phi_{ic})$), or borrowers that would have chosen another contracts at the same bank ($\beta_i\phi_{ic}(-\phi_{ix}) < 0$). The net effect on profits can be rewritten with the term $\pi_{ic} - \sum_x \phi_{ix}\pi_{ix}$, where $\sum_x \phi_{ix}\pi_{ix}$ is the expected profits on borrower i prior to the change in X_c. This enlightens the fact that attracting borrowers i into contract c is valuable when the bank makes more profit on contract c than its others contracts.

Rewriting the above system of equation as in the perfect information case and assuming that $cov((\sigma\alpha+\tilde{\alpha}), d) = 0$ for simplicity of the notation (the general formulas are in the appendix (**??**)), we get:

$$Optimal\ pricing: r_c = \overbrace{\frac{mc(X_c)}{E_{i|c}[1-d_i]}}^{Average\ fair\ price} + \overbrace{\frac{1}{E_{i|c}[(\sigma_i\alpha_i+\tilde{\alpha}_i)]}}^{Average\ markup} + \overbrace{\frac{E_{i|c}[\sigma_i\alpha_i E[\pi_i|ic]]}{E_{i|c}[(\sigma_i\alpha_i+\tilde{\alpha}_i)(1-d_i)]}}^{Asymmetric\ information\ discount/premiu} \tag{C.38}$$

$$Optimal\ characteristic: X_{cx} = \underbrace{\frac{E_{i|c}[(\sigma_i\beta_{ix}+\tilde{\beta}_{ix})(1-d_i)]}{E_{i|c}[\sigma_i\alpha_i+\tilde{\alpha}_i]mc_x}}_{optimal\ characteristics\ average\ type} + \underbrace{PD_{cb}}_{Product\ distortion} \tag{C.39}$$

$E_{i|c}[\beta_i] := \frac{\sum_i n_i\phi_{ic}\beta_i}{\sum_i n_i\phi_{ic}}$ is the average value of β of borrowers choosing contract c.

$E[\pi_i|ij] := \sum_{c=1,c\neq j}^{C}\phi_{ic}\cdot\sum_{c=1,c\neq j}^{C}\frac{\phi_{ic}}{\sum_{c=1,c\neq j}^{C}\phi_{ic}}\pi_{ic}$ is the probability of choosing another contract than contract j (i.e., $\sum_{c=1,c\neq j}^{C}\phi_{ic}$), multiplied by the expected profit on borrower i if they choose another contract than contract j (i.e., $\sum_{c=1,c\neq j}^{C}\frac{\phi_{ic}}{\sum_{c=1,c\neq j}^{C}\phi_{ic}}\pi_{ic}$).

PD are the product distortions, its formula is in the in the appendix (**??**).

The above first order conditions are useful for our empirical exercise as — in the spirit of the sufficient statistic literature — the right hand side variables can be replaced by their empirical estimates to decompose the equilibrium interest rates or product characteristics. This exercise is done in section 1.6 for the interest rate without neglecting the effect of product characteristics on default as we did here. However, as the right-hand side elements are equilibrium objects, those equations cannot be used to analyse the properties of the asymmetric information term at equilibrium and the screening externality. The complete theoretical analysis of the model is outside the scope of this paper; we thus refer to Taburet (2022) studies for a formal analysis of the incentives driving the number of contracts and the asymmetric information term. We discuss how to solve the model and interpret the results in the counterfactual section (section 1.7). In the next paragraph, we discuss how these formulas are related to the ones in classic screening models.

The pricing equation (C.38) has three terms. The first two terms are the equivalent of the perfect information pricing. The only difference is that banks consider the average default probability and the average price elasticity of borrowers choosing the contract. The last term of the pricing equation captures the pricing distortion that appears in classic screening models in order to sort borrowers. In the textbook monopoly model of screening this term only appears for the high willingness to pay borrower as other borrowers are at their participation constraints. In monopoly models, this term is called the information rent. The asymmetric information discount in our model plays the same role. Let us consider that contract c is designed for borrower of type i. Equation (C.38) states that, for a given menu, if i borrowers are likely to choose a contract that is not designed for them, and that this choice is costly for the bank (i.e., $E[\pi_{ic}|ix]$ negative), then banks provide a high discount compared to the perfect information situation in order to lower the probability of borrower i choosing the wrong contract. Alternatively, the bank can increase the rate of other contracts, in order to make the mistake less costly and less likely (increasing ($E[\pi_{ic}|ix]$).

Similarly, the optimal characteristic equation (C.39) contains a term that is the mirror of the perfect information case plus another one to maintains borrowers incentive to self-select. The last term of the pricing equation captures the product distortion that also appears in classic screening models.

As discussed in Proposition 1, screening two borrowers (a, b) with different willingness to pay for characteristic x $\beta_{ax} > \beta_{bx}$) works by offering a product (X, r) and $(X + \delta X_x, r + \tilde{r}\delta X_c)$ such that $\beta_{bc} < \tilde{r} \leqslant \beta_{ac}$ and $\delta > 0$. We formally test this in the empirical section. Theoretically, the price distortions of equations (C.38) has those characteristics for at least some special cases. For instance, when banks price close to marginal cost and preferences are positively correlated with default, the asymmetric information term is positive for contracts that attract high default borrowers[12] and negative for others. Similarly, we can show that contracts that attract high WTP

12. Indeed the expected profits if high default borrowers choose the low default contract is $E[\pi_{ic}|ix] < 0$

borrower and high default borrowers are more likely to feature high X characteristics in some special cases.[13]

As discussed in the informal discussion of the model, the asymmetric information distortions create new incentives for product introduction and exclusion. Indeed, even when the perfect information term is the same for all borrowers (i.e., $\frac{(\beta_{Xi}+\tilde{\beta}_{Xi})(1-d_i)}{\tilde{\alpha}_i+\tilde{\alpha}_i}$ constant across borrowers), banks have incentives to introduce a new product to sort borrowers on their price elasticity or on their default probability (i.e., PD_{cb} heterogeneous).

C.11 Screening Externality

In this section, we provide an analysis of the screening externality. In order to focus on a case in which the equilibrium properties (existence and uniqueness) of the model have been analyzed (Taburet (2022)), we make the following additional simplifying assumptions:

Simplifying assumptions with respect to the structural model:

A1: Two types of borrowers (i.e., $(\beta_i, \alpha_i, \tilde{\beta}_i, \tilde{\alpha}_i, \sigma_i)$ takes two values)

A2: Demand shocks ϵ_{icb} are perfectly correlated across product of the same bank

A3: Unitary demand for loan (i.e., $\tilde{\beta} = \tilde{\alpha} = 0$)

A4: No moral hazard (i.e., $\beta^d = 0$)

A5: Banks are identical (i.e., $mc_{cx} = mc_{cy}$ *and* $\xi_{cx} = \xi_{cy}, \forall x, y \in B$)

The above set of assumptions makes the screening model similar to a textbook one. The main departure from a classic model of screening is the introduction of the parameter σ that drives the competition level (i.e., the demand elasticities). It allows to get around technical issues related to characterizing the equilibrium of perfect competition screening models (see Taburet (2022)).

When demand shocks are perfectly correlated across product of the same bank, lenders can perfectly screen borrowers on their preferences. In this situation, the

13. When competition is high enough and under adverse selection, the product distortion term is positive for high WTP borrowers and negative for low WTP borrowers $\frac{E_{i|c}[\sigma\alpha_i E[\pi_{ic}|ic]]}{E_{i|c}[(\sigma\alpha_i+\tilde{\alpha}_i)(1-d_i)]}$ tend to $E_{i|c}[E[\pi_{ic}|ic]]/E_{i|c}[(1-d_i)]$ when normalizing the alpha parameters to 1 and $\frac{E_{i|c}[\sigma\beta_{Xi}E[\pi_{ic}|i]]}{E_{i|c}[(\sigma\beta_{Xi}+\tilde{\beta}_{Xi})}$ tends to $(1+\delta)E_{i|c}[E[\pi_{ic}|i]]/E_{i|c}[(1-d_i)]$ with $\delta > 1$ the overall effect is thus of the sign of $-E_{i|c}[E[\pi_{ic}|i]]$ and when banks price close to marginal cost $E[\pi_{ic}|i]]$ is positive for low default borrowers and negative for high default borrowers.

market share of product c at bank b of type i borrowers ϕ_{icb} can be written:

$$\frac{exp(\sigma\tilde{u}_i(c,b))1_{\{\tilde{u}_i(c,b)\geqslant\tilde{u}_i(x,b),\ \forall x\in P_{ib}\}}}{\sum_{x\in B}\sum_{y\in P_{ix}} exp(\sigma\tilde{u}_i(y,x))1_{\{\tilde{u}_i(y,x)\geqslant\tilde{u}_i(z,x),\ \forall z\in P_{ix}\}}} \tag{C.40}$$

$1_{\{\tilde{u}_i(c,b)\geqslant\tilde{u}_i(x,b),\ \forall x\in P_{ix}\}}$ is a dummy equal to one if the product c offered by bank b is the product in bank b menu that delivers the highest level of utility to borrowers i. This set of inequalities become the incentive compatibility constraints of a classic screening model when lenders use the revelation principle.

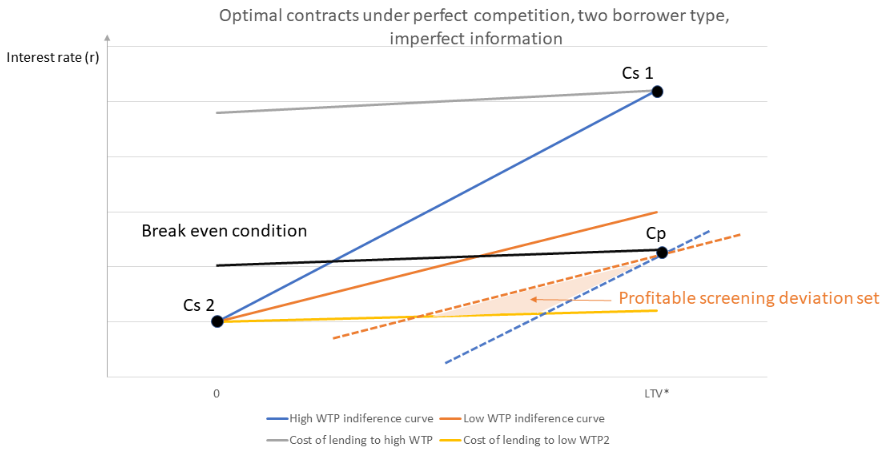


Figure C.11.1: Screening externality

Graphical analysis: For simplicity of the exposition, we present the results graphically for the extreme case in which the product demand elasticity parameter ($\sigma_i = \sigma_j\ \forall i,j$) tends to infinity (i.e., perfect competition). We use figure C.11.1 to illustrate that the contractual externality can lead to a sub-optimal equilibrium —in the second best sense — for all borrowers types. A formal analysis for any elasticity parameter value is provided in Taburet (2022).

In the limit case in which σ tend to infinity, the model tends to the seminal Rothschild and Stiglitz (1976) perfect competition screening model. This allows discussing the model behaviour using traditional tools developed by the literature. Yet, the fact that the demand function is continuous and that there is always a mass of borrowers with a low demand elasticity in our model as long as σ is finite (i.e., $\sigma^{-1} \neq 0$) implies that the model solution exist in pure strategy unlike in Rothschild and Stiglitz (1976). This feature allows to characterize the equilibrium and to show the existence of the screening externality.

In our set-up, it is optimal to screen using rates and only one other product characteristic (see Taburet (2022)).[14] We label this characteristic LTV and set the other ones to their first best value.

In Figure C.11.1, we plot on the (LTV, interest rate) plan the key elements of the model: borrowers' indifference curves (in orange and blue), the cost of lending to each borrower (in grey and yellow) and the set of pooling contracts such that banks would break even in a symmetric pooling equilibrium (the black line). Borrowers' indifference curves are upward slopping, meaning that borrowers like high LTV and dislike high rate (i.e., $\frac{\beta}{\alpha} > 0$). The closer to the bottom right corner of figure C.11.1 the indifference curve is located, the higher the borrower utility level is. The blue borrower indifference curve is steeper, meaning they have a higher WTP for LTV. This assumption implies that screening is possible: putting more down payment (i.e., decreasing the contract LTV) is relatively more costly to the high default borrower. The cost curves are upward sloping: providing high LTV loans is costlier for lenders.[15] The cost curve of the high default borrower type is above the one of the low default borrower type. There is thus adverse selection: lending to high WTP borrowers is more costly as they have a higher baseline default probability. The indifference curves slopes are steeper than the cost of lending slopes meaning that lending generates positive NPV.

Under perfect information, given that lending generates positive NPV, the optimal LTV level of each contract is equal to the maximum feasible LTV amount (denoted LTV^*).[16] LTV units are priced at the borrower's specific marginal cost (i.e., using the borrower's default probability) so that banks break even on each contract. Under imperfect information, offering a menu with the perfect information contracts would lead to negative profits for lenders as all borrowers would choose the cheapest contract. Lenders thus have to distort the first best contracts.

There is two potential (pure strategy) symmetric equilibriums in which banks make zero profits. In the first one, banks offer the pooling contract C^p. In the pooling equilibrium candidate, borrowers get the first best LTV level (denoted LTV^*), but LTV units are priced using the average borrower default probability. In the second potential equilibrium, banks offer the screening contracts (C_1^s, C_2^s). Screening restores perfect information pricing. However, maintaining borrowers' incentives to self-select requires the contract designed for the low default borrower to have a lower LTV level than in the first best. Screening is achieved as getting a low LTV is relatively more

14. This is because, in a linear set-up, the characteristic that allows sorting borrowers while having the lowest negative impact on profits is the same no matter the characteristic levels.

15. This holds in the data and can be due to the expected loss given default being higher for high LTV loans as the probability of not being able to repay the debt even after selling the house increases in the loan leverage.

16. In this model, each bank set the contract terms to maximizes the surplus generated by lending. Then it uses the interest rate to share the surplus between itself and the borrower. Product demand elasticities drive how much of the surplus borrowers can keep.

costly to high default borrowers. How low the LTV must be relative to the perfect information case depends on the spread between unobserved default probability (grey and yellow lines). When borrowers' default probability spread is large, the spread between interest rate must be is large as well, the high default borrower thus benefits more from pretending to be the other type. Thus, the LTV distortion must be high as well. The distortion is also decreasing in the WTP of the high WTP borrower (i.e., the slope of the blue indifference curve). Indeed, when the WTP for LTV is low, getting a lower LTV provides less disutility. The amount of LTV distortion required to prevent the blue borrower from pretending to be the other type is thus higher. As a result, even low spread between defaults can lead to a high product distortion and thus high welfare losses. This insight is important in our empirical application as default probabilities are low in normal times.

The pooling equilibrium candidate can Pareto dominate the screening one. High default borrowers are better off under pooling as they get a lower interest rate. Low default borrowers are better off as well when the cost of being pooled (i.e., getting higher rate) is lower than the costs of being screened (i.e., getting a lower LTV). In the figure, this is the case as the screening contracts (CS_1, CS_2) are above the indifference curve passing by the pooling contract (C^p).

However, under perfect competition, the pooling equilibrium candidate (C^p) cannot be an equilibrium even when it Pareto dominates the screening one (CS_1, CS_2). The reason is the following. Under the pooling contract, banks make more profits on the low default borrowers. This creates incentives for a competitor bank to offer a low rate, low LTV loan that will only attract highly profitable borrowers (those are called cream-skimming deviations in the literature). The deviation is profitable because, since high default borrowers are better off under pooling contracts, a lender can benefit from its competitors' pooling menus to screen the low default borrowers at a low cost. The orange triangular area in Figure C.11.1 represents this set of profitable deviations. While the screening deviation benefits both low default borrowers and the deviating screening bank if everything else stays equal, it also affects the demand and, thus the cost of screening of other banks (contractual externality). The reaction from pooling banks losing their low default customers is to increase the interest rate on their pooling contract or to start screening as well. High default borrowers utility decreases, screening them form low default borrowers thus becomes costlier. All in all, the threat of screening deviations prevents borrowers from being pooled when the competition level is high.

Under perfect competition and when the number of high default borrowers is low, the screening equilibrium candidate cannot be an equilibrium in models used in the literature — such as Rothschild and Stiglitz (1976). The reason is the following. When lenders screen but pooling Pareto dominates screening, a lender can make profits by offering a single contract that is preferred by both borrower types (i.e., to pool) and attract the full market size. This is illustrated in figure C.11.2 in the appendix. This pooling deviation is profitable if the number of high default borrowers in the market is low enough and that screening is costly (for instance, when pooling Pareto dominates screening).

Overall, the existence of both pooling and screening deviations prevents from making an inference about whether the equilibrium of screening market is optimal in the second best sense in Rothschild and Stiglitz (1976).

In our model, when the demand elasticity is finite, there exists a unique pure strategy equilibrium as pooling deviation attracts too many high default borrowers to be profitable (see Taburet (2022) for the proof).[17] The intuition is the following. With logit demands, deviations do not attract the full market size. Indeed, even with a very low variance of the random utility shock, there is always a mass of borrowers with low demand elasticity. As a result, in our model, the relative number of high versus low default borrowers attracted by pooling deviations not only depends on the relative market sizes but also on how attractive the pooling deviation is to those borrowers. Given that pooling deviations requires increasing a characteristic (the LTV in our example) that is relatively more valuable to the high default borrower, the proportion of those borrowers it attracts is higher. This makes pooling deviations not profitable.

Formally, the above intuition can be formulated via two equations. The first one compares how much more utility high WTP borrowers (indexed by B) derive from a given contract compared to low WTP borrowers (indexed by G):[18]

$$\tilde{u}_B = \tilde{u}_G + (\beta_B - \beta_G)LTV \tag{C.41}$$

The second equation is the ratio of good versus bad borrowers a pooling deviation attracts. It can be written:

$$Relative\ number\ of\ borrowers\ attracted\ by\ any\ pooling\ deviation:$$

$$\frac{n_B + \sigma d\tilde{u}_B n_B}{n_G + \sigma d\tilde{u}_G n_G} = \overbrace{\frac{n^B}{n^G}}^{Rotchild\ Stiglitz\ Demand} + \overbrace{\frac{\sigma(\beta_B - \beta_G)dLTV)}{1 + \sigma d\tilde{u}_G}\frac{n^B}{n^G}}^{Logit\ Demand\ extra\ term} > \frac{n^B}{n^G} \tag{C.42}$$

Pooling deviations imply that the contract LTV and the utility of G borrowers increase so: $dLTV > 0$ and $d\tilde{u}_G > 0$. Thus, $\frac{\sigma(\beta_B - \beta_G)dLTV}{1+\sigma d\tilde{u}_G}\frac{n^B}{n^G} > 0$. For banks to gain from this deviation, $\frac{dLTV}{d\tilde{u}_G}$ needs to be high enough to increase profits on the G

17. More generally, the existence and uniqueness of a pure strategy Nash equilibrium solution holds for any finite demand elasticity level.

18. For simplicity of the notation, we set other characteristics than LTV to zero. This is without loss of generality as pooling deviations implies increasing only the LTV, so the other characteristics disappear once we use the infinitesimal version of the equation (C.41) in equation (C.42).

market segment. However, this condition also implies that too many B borrowers are attracted for it to be profitable.[19]

Overall, when competition is high enough (σ high enough),[20] banks are forced to screen even when pooling Pareto dominates screening. In the situation depicted by figure C.11.1, borrowers would get the screening contract (CS_1, CS_2) and not the pooling one (C^p) even when they benefit from being pooled. Graphically (see figure C.11.3), the demand function we use is such that the break-even condition (i.e., the black line) when lenders deviate is above the indifference curve of the low WTP borrower passing by the CS_2 contract. That is, the pooling deviation is not profitable as it does not attract enough low default borrowers. This property carries through in our numerical simulation.

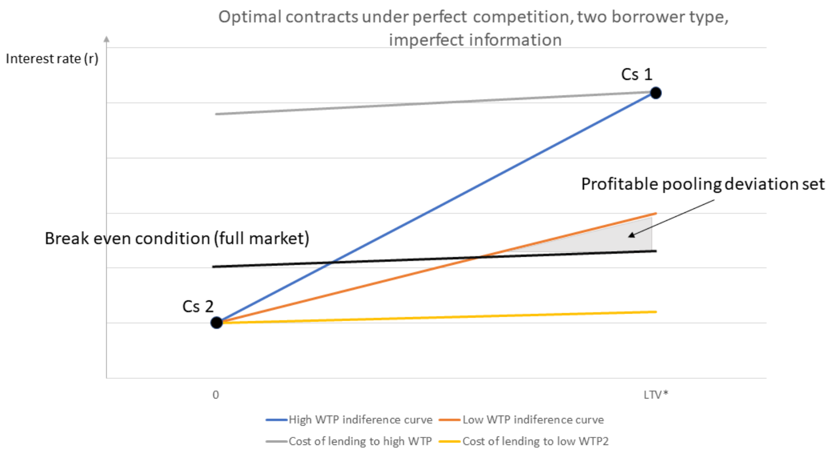


FIGURE C.11.2: Pooling deviation: perfect competition ($\sigma^{-1} = 0$)

C.11.1 Logit Identification Intuition

In this section, I look at a linearized version of the logit model (similar to Salanié and Wolak (2019)) and study the identification of the parameters. Contrarily to the mixed logit model, the estimation is not computationally demanding and can thus support

19. The sketch of the proof is the following. Let us consider that lenders offer the break even pooling rate assuming they will attract $\frac{n^B}{n^G}$. The break even condition implies that $\frac{dLTV}{d\tilde{u}_G} > 0$, this makes the pooling deviation not profitable as lenders B borrower ration is higher than $\frac{n^B}{n^G}$. Similarly, using the shares $\frac{n^B+\epsilon}{n^G}$ implies that the pooling deviation attracts $\epsilon \cdot A$ with $A(0) > 1$ and $A(\epsilon)$ convex. This also makes the pooling deviation not profitable.

20. Formally, there is a threshold value above which the pure strategy equilibrium does not exists (see Taburet (2022)) for a formal analysis. It that case, the model has to be solved using mixed strategies (see Lester et al. (2019) for a mixed strategy characterisation). The intuition that pooling is not feasible carries through.

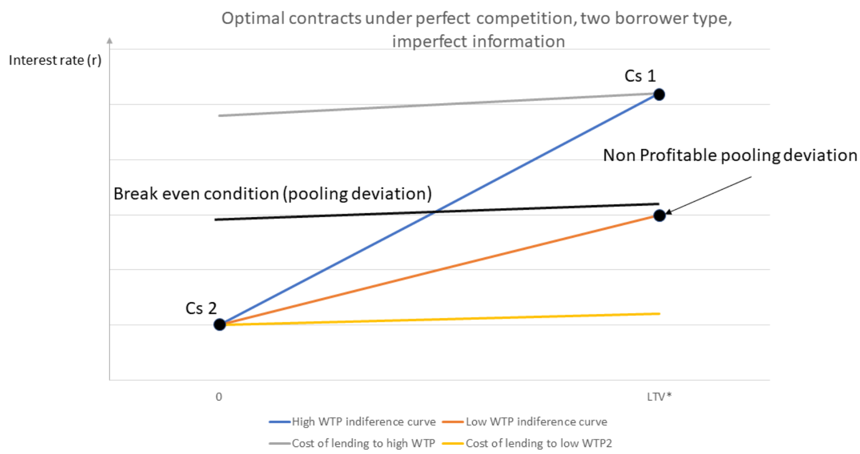


FIGURE C.11.3: Pooling deviation: close to perfect competition ($\sigma \to \infty$)

a lot of unobservable heterogeneity. A proof of identification of the parameters in the non linearized model is provided in Fox et al. (2012).

Linear approximation of the model

Now let us generalize the utility by including a random preference shock $\epsilon_i + \epsilon_{ict}$ as well as unobserved (to the econometrician) product characteristics ξ_{ct}.

$$u_{ict} + \sigma\epsilon_{ict} = \beta X_{ct} + \tilde{\beta}\xi_{ct} + \alpha_t + \beta_i X_{ct} + \tilde{\beta}_i \xi_{ct} + \epsilon_i + \sigma\epsilon_{ict}, \ \epsilon_{ict} \ iid, \ EV$$

$$\implies Pr(i \ choose \ c|\beta_i) = s_{ic}(X_{ct}, \xi_{ct}) \approx \frac{exp(\sigma^{-1}[\beta X_{ct} + \tilde{\beta}\xi_{ct}])}{\sum_{j\in J} exp(\sigma^{-1}[\beta X_{jt} + \beta\xi_{jt}])}(1 + \sigma^{-1}\beta_i X_{ct}\bar{s}_{ct} + \sigma^{-1}\tilde{\beta}_i \xi_{ct}\bar{s}_{ct}) + \nu$$

with $\bar{s}_{ct} := (1 - \frac{\sum_j X_{jt} exp(\sigma^{-1}[\beta X_{jt} + \beta\xi_{jt}])}{X_c \sum_j exp(\sigma^{-1}[\beta X_{jt} + \beta\xi_{jt}])})$, and ν_{ict} being an approximation error that is assumed to be negligible (higher order approximation terms) and with mean 0.

Identification of the model:

By construction, we have $E[\beta_i|X] = E[\tilde{\beta}_i|X] = 0$, and without loss of generality, σ can be normalized to 1 and δ_0 can be normalized to 0, so:

$$E_i[s_{ic}(X_{ct}, \xi_{ct})|X, Y] = a := \frac{exp(\delta_{ct})}{\sum_j exp(\delta_{jt})} + E_i[\nu_{ict}|X]$$

$$E_c[\delta_{ct}|X, Y] = \beta(X_{ct} - X_{0t}) + E_c[\tilde{\beta}(\xi_{ct} - \xi_{0t})|X, Y]$$

$$V_i[s_{ic}(X_{ct}, \xi_{ct})|X, Y, \xi] = a(\sigma^2_{\beta_i|X,Y,\xi} X_{ct}\bar{s}_{ct} + \sigma^2_{\tilde{\beta}_i|X,Y,\xi}\xi_{ct}\bar{s}_{ct} + \rho_{\beta_i,\tilde{\beta}_i|X,Y,\xi}\sigma_{\beta_i|X,Y,\xi}\sigma_{\tilde{\beta}_i|X,Y,\xi} X_{ct}\xi_{ct}\bar{s}^2_{ct}) + V_i[\nu$$

δ_{ct} identified from market share of product c at time t

α_t and β identified from a linear regression of δ_{ct} on $(1, X)$

$\sigma_{\beta_i}, \sigma_{\tilde{\beta}_i}, \rho_{\beta_i,\tilde{\beta}_i}$ identified from the variance of product decisions for individuals having the same observable Y and that got accepted to the same menu J.

Including Loan demand

Loan demand and the choice of banks are joint decisions as they come from the same maximization problem. They thus share some common parameters. K. Train (1986) provides a way to formally link the two decisions by specifying the indirect utility function.

Assuming that the indirect utility function of taking a loan using contract c is:

$$V_{ict} = \mu_i \frac{Y_i^{1-\phi}}{1-\phi} + exp(u_{ict})exp(\sigma\epsilon_{ict})$$

Y_i being the income of borrower i. $\sigma\epsilon_{ict}$ is a perturbation parameter that is equal to zero once the borrower chose the bank. It s a scaling parameter that allows for the product elasticity to be different than the loan elasticity. It can be interpreted as search costs.

Using Roys'identity ($L_{ict} = -\frac{\partial_r V_{ict}}{\partial_Y V_{ict}}$), we get:[21]

$$log(L_{ict}) = log(\frac{\mu_i}{\beta_i}) + \phi log(Y_i) + u_{ict}$$

This leads to the moment condition:

$$\begin{aligned} E[log(L_{ict})|choose\ c] &= E[log(\frac{\mu_i}{\beta_i}) + \phi log(Y_i) + u_{ict}|choose\ c] \\ &= \beta X_{ct} + \tilde{\beta}\xi_{ct} + \alpha_t + E[\beta_i X_{ct} + \tilde{\beta}_i\xi_{ct}|choose\ c] + E[log(\frac{\mu_i}{\beta_i}) + \epsilon_i|choose\ c] \end{aligned}$$

$E[\beta_i X_{ct} + \tilde{\beta}_i\xi_{ct}|choose\ c]$ can be calculated using the expression from the previous section. It allows to control for the selection bias by capturing the fact that, for instance, borrowers with a high propensity to borrow may be more price elastic and end up choosing a cheaper bank.

$E[log(\frac{\mu_i}{\beta_i})|choose\ c] = \alpha_c$

$\epsilon_i := \nu D_i + e_i$, with $E[e_i] = E[e_i|choose\ c] = 0$ as the choice of contract does not reveal information about e_i (or it isd captured by the product c fixed effect α_c).

21. ideally Y_i should be the present value of income coming from the intertemporal budget constraint while r should be the present value of the loan cost

This leads to:

$$\begin{aligned} E[log(L_{ict})|choose\ c] &= \beta X_{ct} + \tilde{\beta}\xi_{ct} + \alpha_t + \underbrace{E[\beta_i X_{ct} + \tilde{\beta}_i \xi_{ct}|choose\ c]}_{\neq 0} + \alpha_c + \nu D_i + \underbrace{E[e_i|choose\ c]}_{=0} \\ &= \delta_{ct} + E[\beta_i X_{ct} + \tilde{\beta}_i \xi_{ct}|choose\ c] + \nu D_i \end{aligned}$$

Identification of the model:

$$\begin{aligned} &E_i[s_{ic}(X_{ct}, \xi_{ct})|X, Y] = a := \frac{exp(\sigma^{-1}\delta_{ct})}{\sum_j exp(\sigma^{-1}\delta_{jt})} + E_i[\nu_{ict}|X] \\ &V_i[s_{ic}(X_{ct}, \xi_{ct})|X, Y, \xi] = a(\sigma^2_{\beta_i|X,Y,\xi} X_{ct}\bar{s}_{ct} + \sigma^2_{\tilde{\beta}_i|X,Y,\xi} \xi_{ct}\bar{s}_{ct} + \rho_{\beta_i,\tilde{\beta}_i|X,Y,\xi}\sigma_{\beta_i|X,Y,\xi}\sigma_{\tilde{\beta}_i|X,Y,\xi} X_{ct}\xi_{ct}\bar{s}^2_{ct}) + V_i[\nu \\ &E[log(L_{ict})|choose\ c] = \delta_{ct} + E[\beta_i X_{ct} + \tilde{\beta}_i \xi_{ct}|choose\ c] + \nu D_i + E[e_i|choose\ c] \\ &E_c[\delta_{ct}|X, Y] = \alpha_t + \alpha_c + \beta X_{ct} + E_c[\tilde{\beta}\xi_{ct}|X, Y] \end{aligned}$$

Thanks to the additional loan demand equation,[22] we are able to identify the scaling parameter σ^{-1} as well as the time and contract fixed effect of the utility function.

C.12 Unobserved Choice set

When the consideration set of the borrower is unobserved (due to acceptance and rejections for instance), this can bias the results. Indeed, wrongly including a desirable product can bias the results as the model parameter will have to be twisted to rationalize the fact that the product is never chosen.

In the case of unobserved choice set heterogeneity, the model has to be estimated using either the integrating over of sufficient set method (see Crawford, Griffith, Iaria, et al. (2021)).

22. The assumption that ε is equal to 0 during the loan demand equation is not necessary for identification as they will be absorbed in the variance of the ϵ_i parameter otherwise

I will use the sufficient set approach. This relies on restricting the menu offered (M) to a subset (S) of the real one. This lead to consistent estimates:

$$
\begin{aligned}
&Pr(i\ choose\ j|M = M^*, S = S^*, (1-d_i)) \\
&= \frac{Pr(i\ choose\ j|M = M^*, (1-d_i))}{\sum_{m\in S^*\cap M^*} pr(i\ choose\ m|M = M^*, (1-d_i)) + \sum_{m\in S^*\setminus M^*} Pr(i\ choose\ m|M = M^*, (1-d_i))} \\
&= \frac{Pr(i\ choose\ j|M = M^*, (1-d_i))}{\sum_{i\in S^*\cap M^*} pr(i\ choose\ j|M = M^*, (1-d_i))} \\
&= Pr(i\ choose\ j|M = M^*, S = S^*\cap M^*, (1-d_i)) \\
&= \frac{exp((1-d_i)X_j)}{\sum_{m\in S^*} exp((1-d_i)X_m)} \frac{\sum_{m\in S^*} exp((1-d_i)X_m)}{\sum_{m\in S^*\cap M^*} exp((1-d_i)X_m)} \\
&= \frac{exp((1-d_i)X_j - ln(\pi))}{\sum_{m\in S^*} exp((1-d_i)X_m)} \\
&= Pr(i\ choose\ j|M = S^*, S = S^*, (1-d_i))\ iif\ S^*\subset M^*
\end{aligned}
$$

with $ln(\pi) := ln(\frac{\sum_{m\in S^*\cap M^*} exp((1-d_i)X_m)}{\sum_{m\in S^*} exp((1-d_i)X_m)})$ equal to 0 when the subset $S^*\subset M^*$. That is, the preferences of the mispecified model ($Pr(i\ choose\ j|M = S^*)$) coincide to the one of someone that had access to M but the econometrician restrics it to $S^*\subset M^*$ ($Pr(i\ choose\ j|M = M^*, S = S^*)$).

Random Coefficient The above proof is the same for random coefficient models. However, the subset S need to be included in $\cap_{\theta\in\Omega} M(\theta)$, with Ω being the set of values that can take the random coefficient θ. The set $M(\theta)$ can depend on the random draw if for instance willingness to pay (θ) is a function of default probability and bank accept and reject some application based on default probabilities.

The distribution of random coefficient identified is he one conditional of being accepted to the a menu S^* that included the sub-menu S.

C.13 Estimation Procedures

C.13.1 Estimation of the marginal costs (Integrating Over approach)

The first approach consist of writing the probability of seeing a given choice of contract contract as the probability of choosing it given the choice set, times the probability of being offered the choice set, and estimate the parameters of interests maximizing the likelihood. The functional form for the probability of being offered the choice set can comes from a bank maximization problem or not (reduced form approach).

Formally, the likelihood of the data is:

$$
\begin{aligned}
& Pr(i\ choose\ c_j|X,\theta,\gamma) && \text{(C.43)} \\
= & \Sigma_{CS_i \in \mathcal{C}} Pr(i\ choose\ c_j|X,\gamma,\theta,CS_i) \cdot Pr(Choice\ set\ offered\ to\ i\ is\ CS_i|X,\theta,\gamma)
\end{aligned}
$$

with observables X, borrowers preference θ and banks preference γ. The main technical difficulty of this approach is the curse of dimensionality coming from the set $\mathcal{C}$ being large. One can reduce the dimensionality of $\mathcal{C}$ by excluding some combination of contracts that are considered to have a zero probability of being offered.

Possible parameterization for: $Pr(Choice\ set\ offered\ to\ i\ is\ CS_i|X,\theta,\gamma)$

As is Gaynor, Propper, and Seiler (2016), one can parametrize the second part of equation ($C.43$) in a reduced form way. For instance, Gaynor, Propper, and Seiler (2016) assumes that the choice k at bank j to borrower i is included in the choice set if:

$$\Pi_{i,k} \geqslant max_j\{\Pi_{i,j}(\gamma)\} - \lambda_j$$

$\Pi_{i,j}$ being banks preferences for offering a particular contract. While this approach helps reducing the dimensionality of the problem, it assumes banks preference for a particular product depends on the the menu that is offers. In the screening context, this is however the element we are interested about.

Alternatively, the second part of equation ($C.43$) comes from the following maximization problem: the choice set offered by bank j is the one that maximizes its expected profits given its expectation about how other banks act:

$$argmax_{\{CS_{i,j}\}\in\mathcal{C}_j}\{E_{Ci,-j}[\Pi(CS_{i,j} \times CS_{i,-j};\theta,\gamma)] + \epsilon_{CS_{i,j}}\}$$

$\epsilon_{CS_{i,j}}$ is a menu cost shock.

Functional form assumptions: If $CS_{i,j}$ is a discrete set, and $\epsilon_{CS_{i,j}}$ iid gumbel distributed, the probability of seeing the menu c by bank j is:

$$\frac{E_{Ci,-j}[\Pi(c \times CS_{i,-j};\theta,\gamma)]}{\Sigma_{x\in\mathcal{C}_j} E_{Ci,-j}[\Pi(x \times CS_{i,-j};\theta,\gamma)]}$$

The probability of seeing the menu $M \in \mathcal{C}$ assuming rational expectations (or that bank offer the offer of other banks) is:

$$\prod_j \{ \frac{\Pi_j(M;\theta,\gamma)]}{\Sigma_{x\in\mathcal{C}_j}\Pi_j(x \times M_{-j};\theta,\gamma)]} \}$$

The maximum likelihood of the data of borrower i is thus:

$$max_\gamma \ \Sigma_{CS_i \in \mathcal{C}} [\frac{exp(V_{c_j}(\hat{\theta}))}{\Sigma_{c_x \in CS_i} exp(V_{c_x}(\hat{\theta}))} \cdot \prod_j \{ \frac{\Pi_j(CS_i;\hat{\theta},\gamma)]}{\Sigma_{x\in\mathcal{C}_j(CS_{i,-j})}\Pi_j(x \times CS_{i,-j};\hat{\theta},\gamma)]} \}] \quad \text{(C.44)}$$

Since the supply side has a logit form, the identification of the parameter γ is subject to the same restrictions as the one from demand. This formulation is convenient as we observe the price schedule for each bank so the only unobservables are the bank preferences γ.

Curse of dimensionality: Since computing this choice set can be computationally challenging, once can assume that the banks offers all LTV product between a minimum and maximum threshold: the choice set CS thus consist of setting the maximum and minimum (denoted for bank j $LTVmax_j$ and $LTVmin_j$). The market share for a particular LTV market segment and unobserved heterogeneity v is thus:

$$s_j(LTV, v, CS_i) = \begin{cases} \frac{exp(V(LTV,v,j))}{\sum_b \sum_{LTVmin_b}^{LTVmax_b} exp(V(LTV,v,b))dLTV} & \text{if } LTV \in CS_{i,j} \\ 0 & \text{Otherwise} \end{cases}$$

with v being the unobserved willingness to pay. Let us denote $p(CS_{i,-j})$ banks probability of seeing this equilibrium. I denote $\mathcal{C}_{-j}(CS_{i,j})$ the set of potential menu offered by other banks when banks j offer the menu $CS_{i,j}$.

Profit on borrower of unobserved type v is:

$$\pi_j(v) := [\beta R_{LTV}(v) - MC_{LTV}] \cdot L(v)$$

Normalizing by β, the parameter γ to estimate is thus $\frac{MC_{LTV}}{\beta}$

C.13.2 Solving for counter rates for the fixed cost estimation

I need to solve the counterfactual equilibrium condition in which the bank stay with the old contract: The FoC for the interest rate of the $\underline{LTV}$ contract ($\underline{R}$) can be written (assuming that banks can see other banks offers):

$$E_v\{s(v,\underline{R},c)\{\pi_{\underline{R}}(v,\underline{R}) + \sigma\pi(v,\underline{R})\frac{s_{\underline{R}}(v,\underline{R},c)}{s(v,\underline{R},c)} \tag{C.45}$$

$$-\frac{P_{\underline{R}}(v,c)}{P(v,c)}\sum_{LTV_j}\sigma\pi(v,R_{LTV})\frac{s(v,R_{LTV},c)}{s(v,\underline{R},c)}\} = 0 \tag{C.46}$$

The equation ($C.46$) can be solved by discretizing the integral and using a fixed point approach on with ($X := \pi(R,\gamma)$ and $T(X) := [\mathbf{\Lambda}(R)^{-1}\mathbf{\Gamma}(R)]X - \mathbf{\Lambda}(R)^{-1}\mathbf{\Phi}(R)$).

$$\boldsymbol{\pi}(R,\gamma) = [\mathbf{\Lambda}(R)^{-1}\mathbf{\Gamma}(R)]\boldsymbol{\pi}(R,\gamma) - \mathbf{\Lambda}(R)^{-1}\mathbf{\Phi}(R)$$

$\boldsymbol{\pi}(R,\gamma)$ is a row vector of size $(J_f \cdot v)$. It is composed of $\begin{pmatrix}\pi^1(R,\gamma)\\ ..\\ \pi^v(R,\gamma)\end{pmatrix}$, with $\pi^v(R,\gamma)$ being a row vector (of size J_f) composed of the profit of bank f on each market segment j (with $j \leqslant J_f$) individuals with unobserved type v.

$\mathbf{\Lambda}(R)$ is a diagonal squared matrix of size $(J_f \cdot v)$ with $w^v \cdot \Lambda^v(R)$ in its diagonal. w^v is a scalar representing the share of people of type v (discretised), $\Lambda^v(R)$ is a squared diagonal matrix of size J_f composed of the market share of the J_f markets $(DiagS^v)$. This diagonal matrix is multiplied by the derivative of utility matrix. It is a diagonal matrix with elements $V^v(j) := \partial_{R(j)}V(R(j),v)$ in its diagonal. We thus have $\Lambda^v(R) = V^v DiagS^v$.

$\mathbf{\Gamma}(R)$ is a block diagonal squared matrix of size $(J_f \cdot v)$. Each block $w^v \cdot \mathbf{\Gamma}^v(R)$ is the product of the market share vectors $(\Phi^v(j) := s(R(j),v))$ multiplied by the derivative of utility matrix V^v defined above. So $\mathbf{\Gamma}^v(R) = V^v\Phi^v\Phi^{v\prime}$.

$\mathbf{\Phi}(R)$ is a row vector of size $(J_f \cdot v)$. It is composed of the Φ^v vectors defined above $\begin{pmatrix}w^1\Phi^1\\ ..\\ w^v\Phi^v\end{pmatrix}$.

$\mathbf{\Lambda}(R)^{-1}\mathbf{\Phi}(R)$ is thus a vector composed of $\frac{1}{V'(R,v)}$, it is constant if $V'(R,v)$ linear. Otherwise, we need:$d(\frac{1}{V'(X,v)}, \frac{1}{v \cdot V'(Y,v)}) \leqslant kd(X,Y)$. Using the taylor expension and using the fact that $1/V'(R,v)$ is decreasing and concave in R: $d(\frac{1}{V'(X,v)}, \frac{1}{V'(Y,v)}) \leqslant d(\frac{V''(Y)}{(V'(Y))^2}(X-Y))$.

In the case of $V := exp(a-\beta R)$ we need $|\frac{exp(vX)}{vexp(a)}| < 1$ in the region we are looking for.

$[\mathbf{\Lambda}(R)^{-1}\mathbf{\Gamma}(R)]$ is a block diagonal matrix with elements: $(DiagS^v)^{-1}(V^v)^{-1}V^v\Phi^v\Phi^{v\prime} = DiagS^{v-1}\Phi^v\Phi^{v\prime}$ in its diagonal. Since it is composed of the market shares or the product of market shares, it is strictly smaller than 1.

C.13.3 Menu Adjustment Costs: Dynamic approach

I want to estimate:

$$Pr(d_{jt}(M,\tilde{M})),\ with\ d_{jt}(M,\tilde{M}) := 1_{\{V_{jt}(M)-sc(M,\tilde{M}))\geqslant V_{jt}(\tilde{M})+e_{Mtj}-e_{\tilde{M}tj}\}}$$
$$and\ V(M_{t-1},(e_t)) = max_{M\in\mathcal{M}_j}\underbrace{\Pi_j(M) - sc_M + \beta E[V(M,(e_{t+1}))]}_{v(M,M_{t-1})} + e_{Mtj}$$

With $(e_{Mtj} - e_{\tilde{M}tj})$ are iid and EVD I get:

$$Pr(d_{jt}(M,\tilde{M})) = \frac{exp(u_{\tilde{M}}(M_{t-1},M_t))}{1+\sum_{m\neq\tilde{M}} exp(u_{\tilde{M}}(M_{t-1},m))} \tag{C.47}$$

with:

$$\begin{aligned} u_{\tilde{M}}(M,M_{t-1}) &:= \Pi(M) - \Pi(\tilde{M}) - [sc(M,M_{t-1}) - sc(\tilde{M},M_{t-1})] \\ &- \beta[\underbrace{log(Pr(\tilde{M}|M)) - log(Pr(\tilde{M}|\tilde{M}))}_{Observable\ in\ data}] \end{aligned} \tag{C.48}$$

The last term $(log(Pr(\tilde{M}|M)) - log(Pr(\tilde{M}|\tilde{M})))$ comes from using the EV assumption and rewriting the value function as in Arcidiacono and Miller (2011) (cf. **Proof**)

I parametrize:

$$sc(M',M) = \sum_c \theta'\tilde{X}_c[\underbrace{\mathbf{1}_{c\in M',c\notin M}}_{Inclusion} + \underbrace{\lambda\mathbf{1}_{c\in M,c\notin M'}}_{Exclusion}] \tag{C.49}$$

Proof:

$$\begin{aligned} v(M,M_{t-1}) =& \Pi(M) - sc(M,M_{t-1}) + \beta[log(\sum_m exp(v(m,M))) + cst] \\ =& \Pi(M) - sc(M,M_{t-1}) + \beta[log(v(\tilde{M},M)) - log(Pr(\tilde{M}|M)) + cst] \end{aligned} \tag{C.50}$$

and noting that, as in (Arcidiacono and Miller (2011)):

$$\begin{aligned} u_{\tilde{M}}(M, M_{t-1}) &:= v(M, M_{t-1}) - v(\tilde{M}, M_{t-1}) \\ &= \Pi(M) - \Pi(\tilde{M}) - [sc(M, M_{t-1}) - sc(\tilde{M}, M_{t-1})] \\ &- \beta[\underbrace{log(Pr(\tilde{M}|M)) - log(Pr(\tilde{M}|\tilde{M}))}_{Observable\ in\ data}] \end{aligned} \tag{C.51}$$

C.14 Perfect information Benchmark

Replacing the interest rate offered to borrower i for contract X_i using the promised utility constraint $r_i = \frac{\beta_i X_i}{\alpha_i} - \bar{u}_i$, we get:

$$max_{\{(X_i)\}} \sum_i n_i \overbrace{\text{Ł}_i[(1-d_i)\cdot(\frac{\beta_i}{\alpha_i}X_i - \bar{u}_i) - mc_i)]}^{NPV_i} \tag{C.52}$$

with the loan demand L being expressed as a function of the promised utility $\text{Ł}_i = exp(-\tilde{\alpha}\bar{u}_i)exp((\tilde{\beta} - \tilde{\alpha}\frac{\beta_i}{\alpha_i})X_i)$. The marginal cost of lending, denoted mc_i, is a continuous function of contract terms.

To provide intuition about the model's behaviour, let us consider that X_i is a vector in $\mathbb{R}^c$ with c being the number of contract characteristics. Using the first order conditions with respect to characteristic $x \in X_i$, the perfect information contract feature should satisfy:

$$\text{Ł}_i[\underbrace{((\tilde{\beta}_{ix} - \tilde{\alpha}\frac{\beta_{ix}}{\alpha_i})[(1-d_i)\cdot(\frac{\beta_i}{\alpha_i}X_i - \bar{u}_i) - mc_i]}_{Increase\ lending} + \underbrace{(1-d_i)\frac{\beta_i}{\alpha_i}}_{rate\ increase}] = \text{Ł}_i[\underbrace{\frac{\partial d_i}{\partial x}\cdot(\frac{\beta_i}{\alpha_i}X_c - \bar{u}_i)}_{Increase\ default} + \underbrace{\frac{\partial mc_i}{\partial x}}_{increase\ marginal\ c} \tag{C.53}$$

Equation (C.53) illustrate that increasing product characteristic x (LTV, teaser rate...) affects profits though an intensive and extensive margin channel. Indeed by increasing a valuable characteristic x, the bank increases the amount borrowed by $L_i \cdot (\tilde{\beta}_{ix} - \alpha\frac{\beta_{ix}}{\alpha_i})$, making $(1-d_{ic})r - mc_c = [(1-d_{ic})\cdot(\frac{\beta_i}{\alpha_i}X_c - \bar{u}_i) - mc_c]$ additional profits on each extra unit sold. When the characteristic x is valued positively by borrowers $\beta_{ic} > 0$, increasing it also raises the surplus (and thus the rate) generated by each unit lent by $(1-d_{ic})\frac{\beta_i}{\alpha_i}$. Two right hand side terms model the cost of increasing product characteristic x. First, it increases default probability $\frac{\partial d_i}{\partial x}$, which decrease the profits by $\text{Ł}_{ic}[\frac{\partial d_i}{\partial x}r]$. Second, it increases the marginal cost of lending by $\frac{\partial mc_i}{\partial x}$.

The assumption about the promised utility level $\bar{u}$ affects the equilibrium contract characteristics through the intensive margin channel $((\tilde{\beta}_{ix} - \alpha\frac{\beta_{ix}}{\alpha_i})[(1-d_{ic})\cdot(\frac{\beta_i}{\alpha_i}X_c - \bar{u}_i) - mc_c])$ and through the default channel $(\frac{\partial d_i}{\partial x}(\frac{\beta_i}{\alpha_i}X_c - \bar{u}_i))$. Given our parameters estimates, the intensive margin channel so that decreasing the promised utility level $\bar{u}$ (i.e., decreasing competition) leads to an increase in the net benefits of increasing a valuable characteristic such as LTV. The LTV level is thus minimized under perfect competition. As a result, we report the perfect competition perfect information in Table B.13 to provide a lower bound on the amount of product distortion relative to the perfect information benchmark.

C.15 Empirical model

consume in period 1, default and loose the house in period 2

$$max_{\{C,L\}} \; \mu C + \frac{\phi}{P_H} \overbrace{(1 - \frac{\delta}{2}\frac{r}{Y_2}L)}^{survival\ probability} \frac{L}{ltv}$$
$$pC + (1 - ltv)\frac{L}{ltv} = Y_1$$

$\frac{\delta}{2}\frac{r}{Y_2}\frac{L}{ltv}$ represents the fact that you are more likely to default as you leverage This implies:

$$H^* = \frac{L^*}{ltv} = \frac{\frac{\phi}{P_H} - \frac{\mu}{p}(1 - ltv)}{\frac{\phi_c}{P_H}\delta\frac{r}{Y_2}}$$

Thus:

$$V(Y_1, H^*) := u(C^*, H^*) = \frac{\mu}{p}Y_1 + H^*[\frac{\frac{\phi_c}{P_H} - \frac{\mu}{p}(1 - ltv)}{2}]$$

The mortgage guarantee scheme, launched on 1 April 2021, involves the government ‘guaranteeing’ 95% mortgages for buyers with 5% deposits.

The scheme was announced in the March 2021 Budget and is designed to encourage banks to start offering 95% mortgages again, after nearly every single one was withdrawn during the pandemic.

Under the terms of the scheme, the government guarantees the portion of the mortgage over 80% (so, with a 95% mortgage, the remaining 15%). This might sound complicated, but in practice it just means the government will partially compensate the lender if a homeowner defaults on (fails to pay) their mortgage.

The scheme is quite similar to the Help to Buy mortgage guarantee scheme, which ran from 2013 to 2016 and was used by 105,000 buyers.

This scheme works for home with a value below 600,000 pounds.

Appendix D

Chapter 2 Appendix

D.1 Micro foundation demand

At time t=0, a borrower living T periods, with income Y_t and savings s_0 choose how much to consume (C), how much to borrow (L) to buy a house of size H at price pH, using a contract with a given X (X), interest rate rate (r) and maturity (M). Assuming that the borrower looses its house upon default, the problem can be written:

$$
\begin{aligned}
U = maL_{L,X} \sum_{0<t\leqslant M} \theta^t [C_t + H] + \theta^M \sum_{M<t\leqslant T} (H + E[Y_t | Y_M = \bar{Y}])& \\
s.t.\ C_0 + L(1-X) = \bar{Y} + s_0& \\
C_t + rL = Y_t,\ t \in [1, M], if\ Y_t \neq 0& \\
C_t = Y_t,\ t \in [M+1, T]& \\
pH = \frac{L}{L}& \\
C_0 \geqslant 0\ (positive\ consumption\ constraint)& \\
L \leqslant \gamma \bar{Y}\ (LTI\ constraint)&
\end{aligned}
$$

$$
Y_t = \begin{cases} \bar{Y} & \text{with probability } \ \theta < 1 \ \text{ if } Y_{t-1} = \bar{Y} \\ 0 & \text{Otherwise} \end{cases}
$$

$C_t = 0, H = 0,\ t \in [1, M], if\ Y_t = 0.$

Solution $(when\ U^*(X, r, M) > E[Y_t | Y_0 = \bar{Y}])$:

$$L^* = \frac{\bar{Y} + s_0}{1 - X} \text{ or } L^* = \gamma \bar{Y}$$

$$U^*(X, r, M) = cst + L^*[\underbrace{\frac{1}{X \cdot p}[\overbrace{(\frac{1-\theta^M}{1-\theta})}^{prob\ keep\ house\ during\ loan} + \overbrace{(1-\theta)^M (T-M)}^{prob\ keep\ house\ at\ maturity}]}_{house\ utility}$$

$$- \underbrace{(1-X)}_{down\ payment}] - \underbrace{(\frac{1-\theta^M}{1-\theta} - 1)r}_{lower\ future\ consumption}]$$

Using a first order approximation, we can get an indirect utility that is linear in X.

D.2 Demand CES form

$N := \frac{u^\varepsilon}{\sum u^\varepsilon}$ so $N'_a = \frac{\varepsilon}{u} N_a [1 - N_a]$, $\varepsilon \in [0, \infty)$

This can be microfounded by using the functional form:

$$choice : max\{ue^{\varepsilon^{-1}\epsilon}\} \iff max\{ln(u) + \varepsilon^{-1}\epsilon\}$$

$$Pr(choosing) : \frac{e^{ln(u^\varepsilon)}}{\sum e^{ln(u^\varepsilon)}} = \frac{u^\varepsilon}{\sum_b u_b^\varepsilon}$$

As in Dixit-Stiglitz, when the number of firm is large, we can abstract from $[1-N_a]$ in the derivative. In a symmetric equilibrium, $\frac{N_a}{N_b}$ equal to the share of type a versus type b borrowers. Using $\tilde{\varepsilon}_i := \varepsilon[1 - \frac{N_i}{n}]$, n being the number of banks operating in the market:

$$u_a = S_a(X^*_{ac})\gamma_a$$
$$\gamma_a := \frac{\varepsilon_a}{\varepsilon_a + 1 - \frac{\theta_b N_b}{\theta_a N_a} \frac{[\alpha_{\tilde{c}b} - \frac{r_{\tilde{c}b}}{\theta_b}]}{(\alpha_{\tilde{c}a} - \alpha_{\tilde{c}b})}}$$

$$u_b = S_a(X^*_{ac})\gamma_b$$
$$\gamma_b := \frac{\varepsilon_b}{\varepsilon_b + 1 + \frac{[\alpha_{\tilde{c}b} - \frac{r_{\tilde{c}b}}{\theta_b}]}{(\alpha_{\tilde{c}a} - \alpha_{\tilde{c}b})}}$$

with$S_i(X) := \beta'_i X - \frac{mcX}{\theta_i}$ is the surplus generated by lending

Pricing:

$$R_a = (1-\gamma_a)\alpha_a + \gamma_a \frac{r_c}{\theta_a}$$
$$R_b = (1-\gamma_b)\alpha_b + \gamma_b \frac{r_c}{\theta_b}$$

Distortion:

$$L_a^* := \bar{X}$$
$$L_b^* := \frac{\gamma_a[\alpha_a - \frac{r}{\theta_a}]}{\gamma_b[\alpha_b - \frac{r}{\theta_b}] + (\alpha_a - \alpha_b)} \bar{X} := \delta \bar{X}$$

Under perfect competition ($\varepsilon \to \infty$):

$$L_b := \frac{[\alpha_a - \frac{r}{\theta_a}]}{[\alpha_a - \frac{r}{\theta_b}]} \bar{X} \leqslant \bar{X}, \; since \; \theta_a \geqslant \theta_b$$

As competition decreases ($\varepsilon \to 0$):

$$-\partial_\varepsilon \frac{L_b}{\bar{X}} := [\frac{-\partial_\varepsilon \gamma_a}{\gamma_a} + \frac{\partial_\varepsilon \gamma_b[\alpha_b - \frac{r}{\theta_b}]}{\gamma_b[\alpha_b - \frac{r}{\theta_b}] + (\alpha_a - \alpha_b)}]\delta \lesseqgtr 0$$
$$\partial_\varepsilon \frac{L_b}{\bar{X}} = [-\frac{\gamma_a}{\varepsilon}(1-\gamma_a) + \frac{\gamma_b}{\varepsilon}(1-\gamma_b)\frac{1}{\gamma_b + \frac{\alpha_b - \frac{r}{\theta_b}}{\alpha_a - \alpha_b}}]\delta$$

Want to extract more surplus from the high WTP borrower a but can deal better with the friction by using an information rent.

D.2.1 Conditions for collateralized debt to be the optimal contract

Up to now, I assumed that the contract offered by the bank is a debt contract. This can be micro founded by assuming that the wage realization is not costlessly observable as in Townsend (1979). For the optimal contract to be collateralized, an additional assumption (assumption 2 bellow) is needed. This assumption is an adaption to the mortgage market of the one used in Lacker (2001). It states that the bank can use collateral and seize it upon default if this is more efficient than spending the verification cost. For the collateral to be seized upon default only, the bank needs to value it less than the borrower.

To summarize, the assumptions are:

ASSUMPTION 1: *Ex post Private information.* In this paper, I model this assuming that banks cannot observe the second period cash flow (W) of the borrower,[1] borrowers can thus lie about their income and hide it from the bank. The bank can spend some amount to verify it.

ASSUMPTION 2: *The house can be used as collateral (i.e. housing is observable). Using collateral to deal with ex post private information is less costly than verifying cash flows. The borrowers values the house more than the bank.* This assumption makes sure that in the optimal mortgage contract, the bank ask for cash instead of housing when possible. In this paper, the reason for the borrower to prefer the house more than the bank, is that borrowers value house more than its selling price and that banks have a utility over cash rather than house.

At t=2, borrowers observe privately their income realization ($\tilde{W}$) and choose whether to fill for default or not. Borrowers default when they cannot repay (i.e. $\delta H + W - R < 0$) or when its better for them to strategically default (i.e. $H + u(W - R) < u(K + W)$ or $u(\delta H - R + W) < u(K + W)$. K is the amount that the bank give back (or ask) after seizing the house and selling it. K has thus to be lower or equal to $\delta H - R$ so that all inequalities are satisfied. In our model, since borrower value more cash upon default than banks ($\alpha > 1$), the constraint is binding $K = \delta H - R$. Notice that, the bank wants to prevent strategic default when the borrowers is in negative equity ($\delta H < R$) it has to punish the the borrower by seizing more than the house (i.e. $K = \delta H - R$ can be negative).

D.3 Derivation Present Value of lending

Given a loan size L, a maturity T and a per period compound interest rate r, the per period mortgage repayment C is given by the annuity formula:

$$C = \frac{Lr(1+r)^T}{(1+r)^T - 1} \tag{D.1}$$

Similarly, we can express the bank cost of lending an amount L as a constant rate (mc) and write it as an annuity to make it comparable to the interest rate (r):

$$D = \frac{Lmc(1+mc)^T}{(1+mc)^T - 1} \tag{D.2}$$

1. or it is costly to do so

The marginal cost includes, among others, the interest rate banks need to pay on its deposits.

Using δ as the discount rate, the present value of lending the amount L, abstracting from default, can thus be written:

$$L\sum_{k=1}^{F}\delta^k[\frac{r(r+1)^T}{(r+1)^T-1}-\frac{mc(mc+1)^T}{(mc+1)^T-1}]+\gamma b\sum_{k=F+1}^{T}\delta^k[\frac{R(R+1)^{T-F}}{(R+1)^{T-F}-1}-\frac{mc(mc+1)^{T-F}}{(mc+1)^{T-F}-1}] \tag{D.3}$$

R is the reset rate and b is the remaining balance at the end of the teaser rate period. F is the fixed rate period, T is the maturity of the loan, γ is the share of people not refinancing and mc is the marginal cost of lending.

As in Crawford, Pavanini, and Schivardi (2018), assuming that banks consider the average default instead of the probability of defaulting in each period, for a constant discount rate ($\delta < 0$), denoting d a dummy equal to 1 if borrower default, the present value of lending up period F is:

$$C\cdot E[(1-d)]\cdot\sum_{k=1}^{F}\delta^k = Lr\frac{(1+r)^T}{(1+r)^T-1}\cdot E[(1-d)]\cdot\frac{1-\delta^F}{1-\delta}\delta \tag{D.4}$$

When T and F are large, $\frac{(1+r)^T}{(1+r)^T-1}\approx 1$ and $\delta^F\approx 0$, the net present value of lending is thus:

$$PV\approx L\cdot\{E[(1-d)]r\frac{\delta}{1-\delta}+\gamma E[(1-d)]R\frac{1-\delta^{T-F}}{1-\delta}\delta^F-[\frac{\delta}{1-\delta}+\gamma\frac{1-\delta^{T-F}}{1-\delta}\delta^F]mc\} \tag{D.5}$$

With ($\delta = 1$), the expression is instead:

$$PV\approx L\cdot[E[(1-d)]rF+\gamma RE[(1-d)](T-F)-[F+\gamma(T-F)]mc] \tag{D.6}$$

We further assume as in Benetton (2018) that $\partial_r\gamma = 0$ so that it does not enter inside the FoC of r_c and set γ_c to 0 (i.e. all borrower remortgage). We can thus also abstract from the discount rate if $\delta < 1$ as it is constant across mortgages, we thus get:

$$NPV_{icb} := L\cdot[E[(1-d)]r - mc] \text{ when } \delta < 1 \tag{D.7}$$

The above expression comes implies that banks do care about fixing the interest rate except from its impact on the cost of lending (mc), default (d) or on demand (L). This result comes from the assumption that $\delta^F\approx 0$. It may be problematic as for a given demand, interest rate, default and marginal cost, profits are likely to be increasing in F as the loan generates annuities for a longer period.

In empirical applications, relaxing the assumption $\delta^F \approx 0$ would however require an assumption about the discount rate used (for instance the bond of or deposit rates) or the use of non standard approaches like the integrating over one (see Polo, Taburet, and Vo 2022). We thus go with the first approach and assume that $\delta = 1$. We get:

$$NPV_{icb} := L \cdot [(1-d)r - mc]F \text{ when } \delta = 1 \tag{D.8}$$

Alternative approach:

Without using Crawford, Pavanini, and Schivardi (2018) assumption about default, the expression for the annuity would be would be, using d as the per period default probability:

$$C\sum_{k=1}^{t}((1-d)\delta)^t = Lr((1-d)\delta)\frac{(1+r)^T}{(1+r)^T-1}\frac{1-((1-d)\delta)^t}{1-((1-d)\delta)} \tag{D.9}$$

Using the same approximations as in Benetton (2018), $\frac{(1+r)^T}{(1+r)^T-1} \approx 1$ and $\partial_r\gamma = 0$, the expression for the NPV becomes:

$$NPV_{icb} := L \cdot [(1-d)\delta\frac{1-((1-d)\delta)^F}{1-\delta+d\delta}r - mc\frac{1-\delta^F}{1-\delta}] \text{ when } \delta < 1 \tag{D.10}$$

$$NPV_{icb} := L \cdot [(1-d)\frac{1-(1-d)^F}{d}r - mc \cdot F] \text{ when } \delta = 1 \tag{D.11}$$

When d is small and δ equal to 1, the expression becomes the same as in Crawford, Pavanini, and Schivardi (2018):

$$NPV_{icb} \underset{d\to 0}{\sim} L \cdot [(1-d)r - mc] \cdot F, \text{ when } \delta = 1 \tag{D.12}$$

D.4 Lemmas

Lemma H1: At least one IC constraint is binding. *Given the assumption that the sets* $\bar{N}PV_i := \{f : \beta_{if} > \frac{mc_f}{\theta_i}\}$, $\underline{N}PV_i = \{f : \beta_{if} < \frac{mc_f}{\theta_i}\}$ *and* $NPV_i = \{f : \beta_{if} = \frac{mc_f}{\theta_i}\} = \varnothing$ *are the same for all borrower i, the first best contracts are not incentive compatible except on the sets* $\{(\sigma, \theta, V) : r_j(X^*) = r_i(X^*), \forall(i,j)\}$, X^* *defined in equation (??). One incentive compatibility contraint at least is thus* *binding.*

Proof Lemma 1: The two ICC can be written $(\beta_a - \beta_b)' L_a \geqslant \bar{u}_a - \bar{u}_b \geqslant (\beta_a - \beta_b)' L_b$ when $L_a = L_b$ as in the FB, this implies that $\bar{u}_a - \bar{u}_b = (\beta_a - \beta_b)' X^*$ which is the case when $r_a(\bar{X}) = r_b(\bar{X})$

Lemma 1 bis: When both borrowers have a positive NPV, there is always at least one contract with $X = \bar{X}$.

Proof Lemma H1 bis: if it is not binding, then the other one is binding from Lemma 1. If that is the case the FoC on L_G of the promised utility problem gives that $L_B = L_G = \frac{\bar{u} - u}{\beta - \underline{\beta}}$. In that case both constraints are binding. However, if $L_G = \frac{\bar{u} - u}{\beta - \underline{\beta}} < \bar{X}$ the banks can do a Pareto improvement by setting $L_B = \bar{X}$.

Lemma 2: When both IC constraints are binding, bank pool borrowers.

Proof Lemma H2: for a given $\bar{u}_i$ the IC implies: $(\beta_{ac} - \beta_{bc}) L_b^b = (\beta_{ac} - \beta_{bc}) L_a^b$, this equation is satisfied for $L_a^b = L_b^b$. Since the surplus of the profit of the bank is $\sum_i S_i(L_i) - \bar{u}_i$ and the maximum surplus is generated by $L_i = \bar{X}$ then the profit is maximized when $L_a^b = L_b^b = \bar{X}$. This is the unique maximum under the conditions of Lemma 1.

Lemma 3: Which characteristics X are used to screen. *Bank use interest rate and the characteristics $(L_f)_f$ to screen. In order to screen, the bank will favor the non binding characteristics c that have the lowest $\tilde{\lambda}_c$ value. $\tilde{\lambda}_c$ is defined as:*

$$\tilde{\lambda}_c := \frac{\overbrace{\beta_{bc} - \frac{mc}{\theta_b}}^{\text{Surplus Increases}}}{\underbrace{(\beta_{ac} - \beta_{bc})}_{\text{IR Increases}}} \tag{D.13}$$

Proof Lemma H3: solve for the optimization problem using the lagrangian. As long as the lower bound $\underline{X}$ on banks preferred characteristics for screening is low enough, banks screen using only one product characteristics and rates. When this condition is not satisfied, Banks use their preferred screening device until it reach the bound, then it moves to the second preferred and so on.

Simplification: *That $\sigma_G \geqslant \sigma_B$, (IC_B) is always binding and (IC_B) can be written as a (non-binding) monotonicity constraint $L_B \geqslant L_G$. When borrowers have* *the same outside option of not borrowing (i.e.* $\bar{V}_G = \bar{V}_B$*), the participation constraint of borrower B (*$u_B \geqslant \tilde{V}_B$*) is redundant.*

The maximization problem defined in (2.10) can thus be written:

$$maL_{\{L_i \in [0,\bar{X}], \bar{u}_i \in \mathbb{R}\}} \sum_{i \in \{G,B\}} \overbrace{n_i \cdot N_i^b(\bar{u}_i) \cdot 1_{\bar{u}_i \geqslant \bar{V}_i}}^{Demand} \qquad \overbrace{\theta_i[\underbrace{\beta_i L_i - \frac{mcL_i}{\theta_i}}_{surplus:\ S_i(L_i)} - \bar{u}_i]}^{expected\ profit\ on\ each\ loan:\ \theta R - mc'X} \tag{D.14}$$

$$s.t.\ (IC_B) : \bar{u}_B = \bar{u}_G + \underbrace{(\beta_B - \beta_G)}_{>0} L_G$$

Proof: Lemmas appendix (D.4). Lemma H3 in the appendix (D.4) shows how to select f. The maximization problem () derives from the problem ($problemPU1$). I use the fact that ICB is binding and write the problem in terms of promised utility $\bar{u}$ to make it similar to the monopoly case as in *Stiglitz*(1977).

I postpone the discussion of how screening works to section (2.4.1) and (2.4.2). Here I discuss the role of the assumptions used in Lemma 1.

Assumption (ii) allows banks to screen perfectly borrower's type, assumptions (i)-(iii) makes the problem similar to the textbfook model of screening in which at least one participation constraint is binding. Assumptions (v)-(vi) and $\sigma_G \geqslant \sigma_B$ insure that the binding IC is the high default borrower. (v)-(vi) are made in order to simplify the exposition. It makes sure that the borrower type that benefits from pretending to be the other type is always the B type. Indeed, under high level of competition, borrower B benefits from the lower price due to low default. Under low level of competition, he benefits from the lower price due to the lower WTP and higher price elasticity of the other borrower. This is done to simplify the analysis, drivers behind the screening incentives do not depends on this assumption.

As shown in Lemma H3 in Appendix (D.4), banks screen using only one product characteristics ($L_c \in X$) and rate to screen. I index this characteristic by f. As explained in the section (2.4.2), this characteristic must satisfies $(\beta_B - \beta_G) > 0$. I provide in the appendix the conditions to determine which characteristic is used to screen. This conditions depends on how good is the screening device (i.e. borrowers' willingness to pay for the screening characteristics is very different) and costly it is to distort the product characteristic used to screen (i.e. how much surplus is lost by unit of characteristic distortion).

D.5 Existence and Uniqueness

The maximization problem of bank b is:

$$max_{\{u_B,u_G,L_G,L_G\}} N_G(u_G)[\theta_G(\beta_G F(L_G) - u_G) - mcL_G] + N_B(u_B)[\theta_B(\beta_B F(L_B) - u_B) - mcL_B] \tag{D.15}$$

$$s.t.\ u_B = u_G + (\beta_B - \beta_G)L_G \tag{D.16}$$

$$u_G \geqslant u_B - (\beta_B - \beta_G)L_B \tag{D.17}$$

where $N_i(u_i) := n_i \frac{exp(\sigma u_i)}{exp(\sigma u)+\sum_{j\neq b} exp(\sigma u_{ij})}$. n_i is the number of type i borrower in the market. $\frac{exp(\sigma u_i)}{exp(\sigma u)+\sum_{j\neq b} exp(\sigma u_{ij})}$ is the probability of attracting those borrowers given what the competitors offer $(u_{ij},\ jneb)$.

When assuming that F is piece wise linear and denoting $A_i = \beta_i - \frac{mc}{\theta_i}$ the surplus generated by the loan in that on $L \in [0, \bar{H}]$ and the loan size positive, we use the the additional constraint instead of (D.17)$0 \leqslant \frac{u_B - u_G}{\beta_B - \beta_G} \leqslant \bar{H} = L^B$.

The Hessian of the maximization problem is:

$$\begin{pmatrix} \sigma N_B(1-\frac{N_B}{n_B})\theta_B[(A-u)(1-2\frac{N}{n})\sigma - 2] & \sigma N_G(1-\frac{N_G}{n_G})A_G\frac{\theta_G}{\beta_G-\beta_B} \\ \sigma N_G(1-\frac{N_G}{n_G})A_G\frac{\theta_G}{\beta_G-\beta_B} & \sigma\theta_G N_G(1-\frac{N_B}{n_B})[(A_G L_G - u)(1-2\frac{N}{n})\sigma - \sigma A_G\frac{1}{\beta_G-\beta_B}(1-2\frac{N}{n}) \end{pmatrix} \tag{D.18}$$

Condition for concavity: The matrix is negative semi definite when sigma is low enough[2] (for instance $\sigma < max_i\{\beta_i - \frac{mc}{\theta_i}\}$), u positive and $\beta_G - \beta_B$ large enough (for instance, when $A_i < 1$ give the condition: $IR_B \cdot A_G \cdot \frac{N_G}{N_B}\frac{1-\frac{N_G}{n_G}}{1-\frac{N_G}{n_G}} < 4$, this holds for the number of bank being large enough to satisfy $1+(M-1)exp(\pm\sigma[u_G^o - u_G + (\beta_B - \beta_G)\bar{H}]) > \frac{4}{IR_B}/(\frac{4}{IR_B}-1)$ and $IR_B \cdot A_G < 4$. This bound is found by setting other bank doing a symmetric equilibrium and differentiating completely the loan from theirs by setting a maximum or a minimum information rent: $u_G^o - u_G + (\beta_B - \beta_G)\bar{H}]$,$u_G^o$ is the utility other banks offer to G. need to show that u_G is bounded). In that case, the solution exist and is unique.

Let us abstract from $u_G \geqslant u_B - (\beta_B - \beta_G)L_B$, and replace L_G using $u_B = u_G + (\beta_B - \beta_G)L_G$. The objective function is strictly concave when $u_B > (\beta_B - \frac{mc}{\theta_B})L_B^* - 2$, L_B^* is the perfect information loan size, and $u_G > (\beta_G - \frac{mc}{\theta_G})\frac{1}{\beta_B - \beta_G + 1} - 2$. For a given competitor strategy, there is thus a unique solution on that satisfy the above utility

2. This assumption prevents lenders to end in a situation in which an equilibrium does not exist as in RS

constraints.

L_B^* is such that: $\beta_B F'(L_B^*) = mc$. Let us set F piece-wise linear or set a bound on L_B so that $L_B^* = \bar{H}$. Let us solve for the problem when $u_G \geqslant u_B - (\beta_B - \beta_G)L_B$ is not binding. The First order conditions yields:

$$\partial_{u_G} N_G(u_G)[\theta_G(\beta_G F(\frac{u_B - u_G}{\beta_B - \beta_G}) - u_G) - mc\frac{u_B - u_G}{\beta_B - \beta_G}] = N_G(u_G)[\theta_G + (\beta_G \theta_G F'(\frac{u_B - u_G}{\beta_B - \beta_G}) - mc)\frac{1}{\beta_B - \ldots} \tag{D.19}$$

$$\partial_{u_B} N_B(u_B)[\theta_B(\beta_B F(L_B^*) - u_B) - mcL_B^*] + N_G(u_G)[\theta_G(\beta_G F'(\frac{u_B - u_G}{\beta_B - \beta_G}) - mc)\frac{1}{\beta_B - \beta_G}] = N_B(u_B \ldots \tag{D.20}$$

Using dividing (D.19) by $\partial_{u_G} N_G(u_G)\theta_G$ and (D.20) by $\partial_{u_B} N_B(u_B)\theta_B$ which are always different than 0 when $\sigma \neq 0$, and taking their difference we get:

$$L_G = \delta\bar{H} + a[\frac{N_G}{\partial_{u_G} N_G}[1 + IR_G] - \frac{N_B}{\partial_{u_B} N_B} + \frac{N_G}{\partial_{u_B} N_B}[IR_B]] \tag{D.21}$$

Using the notation $a := [\beta_G - \frac{mc}{\theta_G} + \beta_B - \beta_G] = [\beta_B - \frac{mc}{\theta_G}]$, $\delta := [\beta_B - \frac{mc}{\theta_B}]/a$ $IR_G := (\beta_G F'(\frac{u_B - u_G}{\beta_B - \beta_G}) - \frac{mc}{\theta_G})\frac{1}{\beta_B - \beta_G}$ and $IR_B := \frac{\theta_G}{\theta_B} IR_G$.

Under the logit model, equation (D.21) simplifies to:

$$\frac{u_B - u_G}{\beta_B - \beta_G} = \delta\bar{H} + a[\frac{1}{\sigma_G(1 - \frac{N_G}{n_G})}(1 + IR_G) + \frac{N_G}{N_B \sigma_B (1 - \frac{N_B}{n_B})}(IR_B - \frac{N_B}{N_G})] \tag{D.22}$$

$$u_B = (1 + IR_G^{-1})[u_G + \frac{1}{\sigma_G(1 - N_G)}] \tag{D.23}$$

Existence: If all banks play the same strategy, using expression (D.24) we get the symmetric equilibrium candidate:

$$\frac{u_B - u_G}{\beta_B - \beta_G} = \delta\bar{H} + a[\frac{1}{\sigma_G(1 - \frac{1}{M_G})}(1 + IR_G)\frac{n_G M_B}{M_G n_B \sigma_B (1 - \frac{1}{M_B})}(IR_B - \frac{M_G n_B}{n_G M_B})] \tag{D.24}$$

M_i is the number of banks operating in the market.

We can then check that $\frac{u_B - u_G}{\beta_B - \beta_G} \leqslant \bar{H}$ (i.e. that $u_G \geqslant u_B - (\beta_B - \beta_G)L_B$ is satisfied). This is true when both σ parameters are large enough as ($\delta < 1$). In that case, we also have $u_B > (\beta_B - \frac{mc}{\theta_B})\bar{H} - 2$ and $u_G > (\beta_G - \frac{mc}{\theta_G})\frac{1}{\beta_G - \beta_G}$. For a given competitor strategy, there is thus a unique solution. The symmetric equilibrium candidate is thus indeed and equilibrium.

Uniqueness: There may exist other equilibrium that are not symmetric. We proof the uniqueness of the equilibrium comes from the following limit case (as in Dixit and Stiglitz (1977)):

Under the assumption that the number of banks is large $N_i \approx \frac{exp(\sigma u_{ic})}{MP_i}$, M being the number of banks, with $P_i = \sum_x exp(\sigma u_{ix})$ taken a given by the bank (as in Dixit and Stiglitz (1977)):

$$\frac{u_B - u_G}{\beta_B - \beta_G} = \delta \bar{H} + a[\frac{1}{\sigma} IR_G + \frac{P_B}{P_G}\frac{exp(-\sigma(u_B - u_G))}{\sigma} IR_B)]$$

This expression has a unique solution for any given $\frac{P_B}{P_G}$. Thus, the equilibrium must be symmetric in that case.

Condition for screening inefficient:

$$\frac{n_G}{n_B} IR_B := \frac{n_G \theta_G}{n_B \theta_B} \frac{\beta_G - \frac{mc}{\theta_G}}{\beta_B - \beta_G} > 1 \tag{D.25}$$

Equilibrium L_G amount:

$$L_G = \delta[\bar{H} + \frac{1}{\sigma}\tilde{\delta}\frac{1}{\beta_B - \beta_G}(1 + \frac{\theta_G n_G}{\theta_B n_B})] = \delta[\bar{H} + \frac{1}{\sigma}\frac{\tilde{\delta}}{\beta_B - \beta_G} + \frac{IR}{\sigma[\beta_B - \frac{mc}{\theta_B}]})] < \bar{H} \text{ for } \sigma \text{ high enough} \tag{D.26}$$

with: $\delta := \frac{\beta_B - \frac{mc}{\theta_B}}{\beta_B - \frac{mc}{\theta_G}}$ is the strength of the AS problem. $\tilde{\delta} := \frac{\beta_G - \frac{mc}{\theta_G}}{\beta_B - \frac{mc}{\theta_B}}$ is the relative profit measure.

When $\sigma = \frac{2}{A_B \bar{H}}$, $IR = 1^+$:

$$L_G = \delta[1 + \frac{\tilde{\delta}}{\beta_B - \beta_G}\frac{A_B}{2} + \frac{1^+}{2})] < \bar{H} \text{ For } \beta_B - \beta_G \text{ large enough or } A_B \text{ low enough (this imposes a conditic} \tag{D.27}$$

Given the condition for the function to be well behaved and screening to be inefficient, there exist a zone where banks screen. We can thus analyse the interior solution to understand the screening property trade-off. The level of inefficiency should be low enough so that the bound on the degree of competition is low enough so that banks to not have enough freedom to pool borrowers.

D.6 Participation constraint binding

When the low WTP participation constraint is binding[3]

3. it binds first

$\underline{\theta} \geqslant \bar{\theta}$ **and as long as** $0 \leqslant \underline{L}^*(\sigma) \leqslant L^*$ **(Screening)**

$$\bar{C}\begin{cases} \bar{L}^* = L^* \\ \bar{R} = \bar{\alpha}L^* - \bar{u} \end{cases}$$

$$\underline{C}\begin{cases} \underline{L}^*(\sigma) = \frac{\bar{u}-O^{NB}}{\bar{\alpha}-\underline{\alpha}} \\ \underline{R} = \underline{\alpha}\underline{L}^*(\sigma) - O^{NB} \end{cases}$$

With $\bar{u} = [\bar{\alpha} - \frac{r}{\bar{\theta}}]L^* - \sigma^{-1} + \sigma^{-1}\frac{\underline{n}}{\bar{n}}\frac{\bar{\theta}}{\underline{\theta}}[\underline{\alpha} - \frac{r}{\underline{\theta}}]\frac{1}{\bar{\alpha}-\underline{\alpha}}$

When $L^*(\sigma) \geqslant L^*$ (Pooling):

$$C\begin{cases} L = L^* \\ R = \underline{\alpha}\underline{L}^*(\sigma) - O^{NB} \end{cases}$$

When $L^*(\sigma) \leqslant 0$ (Exclusion):

$$C\begin{cases} L = L^* \\ R = \bar{\alpha}\bar{L}^* - \bar{u} \end{cases}$$

$$O \leqslant \bar{u} \leqslant (\bar{\alpha} - \underline{\alpha})L^* + O$$

$$\bar{u} = \bar{\alpha}L^* - \frac{r}{\bar{\theta}}L^* - \sigma^{-1}$$

D.6.1 When the low and high WTP participation constraint is binding: Monopoly case

Under monopoly, we have[4] :

$$\bar{L}^M = L^* \tag{D.28}$$

$$\underline{L}^M = \begin{cases} 0 & \text{When S satisfied with a strict inequality:} \\ (0, L^*) & \text{When S satisfied with an equality:} \\ L^* & \text{Otherwise} \end{cases} \tag{D.29}$$

4. The reason why screening does not arise in the monopoly case is that $L^* < \frac{W}{\underline{\alpha}}$, W being the maximum amount the borrower can pay in the next period

The rate are:

$$\underline{R}^M = \begin{cases} 0 & \text{When S satisfied with a strict inequality:} \\ \underline{\alpha} L - O & \text{Otherwise} \end{cases} \tag{D.30}$$

$$\bar{R}^M = \begin{cases} \bar{\alpha} L^* - O & \text{When S satisfied with a strict inequality:} \\ \bar{\alpha} L^* - O - \underbrace{(\bar{\alpha} - \underline{\alpha})\underline{L}}_{Information\ Rent} & \text{Otherwise} \end{cases} \tag{D.31}$$

The bank does exclude market participant, When[5]:

$S: \underbrace{\bar{N}(\bar{\alpha} - \underline{\alpha})\bar{\theta}}_{increase\ lending\ lowerICC} \geqslant \underbrace{\underline{N\theta}(\underline{\alpha} - r)}_{increase\ lending\ increases\ profits}$

By excluding market participants, the bank is able to charge the high willingness borrowers a higher price, but it losses the potential profits from lending to low WTP borrowers.

Under the case in which S is satisfied, we have Pooling until when competition is high enough so that $\underline{L}^*(\sigma) \leqslant L^*$ (Pooling)

D.7 Poof propositions

Proposition 4 $\partial_\delta L_G = \underbrace{\frac{\frac{mc_2}{(\theta_B+\delta_B)^2}}{\beta_B - \frac{mc_2}{\theta_G}} L_{90}}_{\text{“}perfect\ competition\text{” } effect} - \underbrace{IR_B \frac{1}{\theta_B + \delta_B}}_{\text{“}monopoly\text{” } competition\ effect}$

PROOF (ii): $u_i = S(L_i) - \frac{1}{\sigma} - IR_G 1_{i=G} + IR_B 1_{i=B}$. For borrowers G, $S(L_G)$ increases when L_G increases. In the other market segment: $\partial_\delta u_B = -\partial_\delta R_B = \frac{1}{\theta_B+\delta_B}[\underbrace{\frac{mcL_{95}}{\theta_B + \delta_B}}_{fair\ price} - \underbrace{IR_B}_{Information\ rent}]$

5. The general condition when the outside option is no 0 is: $\bar{N}[\bar{\theta}(\bar{\alpha} - \underline{\alpha})L^* + \bar{O}^{NB} - \underline{O}^{NB}] \geqslant \underline{N}[\underline{\theta}[\underline{\alpha} - \underline{O}^{NB})L^* - rL^*]]$

Appendix E

Chapter 3 Appendix

E.1 Microfoundation Complexity

Denoting $\xi(i)$ the function linking bank i to its inverse productivity realization ξ, $\lambda(\xi(i))$ the right capital requirements for bank i set by the regulator, Λ the menu of capital requirements and $\Lambda(i^-) := \{\lambda \in \Lambda : \lambda \neq \lambda(\xi(i))\}$ the subset of Λ representing wrong capital requirements for bank i, λ_j the elements of $\Lambda(i^-)$.

The cost of complexity for bank i ($f(\lambda(\xi(i)))$ can be micro-founded by the following problem in which the bank minimizes the expected cost of making a capital requirement mistake ($Pr(R \neq \lambda(\xi(i))|(t_j)_j)\iota$) by hiring some labour L to increase the attention $((t_j)_j)$ that can be allocated to reduce the probability of making a mistake $((Pr(R \neq \lambda(\xi(i))|(t_j)_j))$:

$$f(\lambda(\xi(i))) := min_{\{t_j,L\}} \overbrace{Pr(R \neq \lambda(\xi(i))|(t_j)_j)}^{probability\ of\ a\ mistake} \underbrace{\iota}_{cost\ of\ a\ mistake} + \underbrace{wL}_{labor\ cost} + \underbrace{f_e}_{fixed\ cost} \tag{E.1}$$

$$s.t. \sum_{j \in 1, \#\Lambda} t_j \leqslant T(L)$$

f_e is a fixed cost that is independent of the complexity of the law. R is the capital requirement reporting to the central bank. It is a random variable. The probability of a wrong reporting $Pr(R \neq \lambda(\xi(i)|(t_j)_j)$ is a decreasing function of time spent understanding each regulation $((t_j)_j)$. For instance, with $Pr(R \neq \lambda(\xi(i))) := \sum_{j \in 1, \#\Lambda(i^-)}[Pr(R = \lambda_j|t_j)] + Pr(R \notin \Lambda| \sum_{j \in 1, \#\Lambda} t_j)$, the probability of reporting the wrong capital requirement is a decreasing function of the time spend to understand the law. By assuming a limited budget for bankers as in Oehmke and Zawadowski (2019), the framework captures that complexity is inherently tied to bounded rationality (Brunnermeier and Oehmke (2009)) and inattention (Gabaix (2019)). The total time available T(L), is an increasing function of labour devoted to the task (L). This captures the idea that more complex rules require more time or more specialized and costly labour ι is the cost of an error in reporting, and w is the cost of labour. As the number of different capital requirements increases ($\#\Lambda$ increases), the more labour one need to hire in order to maintain a given level of attention (t_j) to each rule j.

The cardinality of Λ naturally proxies the concept of complexity defined above as there is a one-to-one mapping between the number of capital requirements regimes ($\#\Lambda$) and the time required to process the rule. Intuitively, this time understanding the rules is an increasing function of the number of sub-cases, but also an increasing function of how different the rules are. To capture this idea, I also consider the variance of capital requirements ($Var_\xi[\lambda(\xi(i))]$) to impact mistakes probability. Those two measures are in line with the Natural Language Processing that uses measures such as length and lexical diversity.

E.2 Model with period 1 complexity cost

he regulation is costly to implement for banks: it takes some time to understand, creates operational risks or it creates a need for new investments. We model it by a a cost ($t(\kappa)$ and $\tilde{t}(\kappa)$) that has to be incurred in period 0 1 or 2.

$$
\begin{aligned}
max_{\{x,R,c(X)\geqslant 0,d\}} q(x,R) \cdot \int_{a_0}^{1} R \cdot X - d - C(X) dM(X) \\
s.t.\ q(r)\cdot x + \underbrace{\tilde{t}(\kappa)}_{complexity\ cost} + \underbrace{q(r)\cdot x \cdot \xi}_{inverse\ productivity} = q(r)\cdot(d+c) \\
c \geqslant \lambda(\kappa) x \\
c = \int_{a_0}^{1} C(X) dM(X)
\end{aligned}
$$

The bank maximization problem in period 1 is:

$$
\begin{aligned}
max_{\{r\}} \quad & \underbrace{q(r)}_{Number\ of\ borrowers} \cdot [\underbrace{[r\cdot E[X] - 1(1+\xi)]}_{Economic\ Surplus} + \overbrace{(1-p)\underbrace{[(1-\lambda(c)+\xi) - r\cdot \underline{X}] - \tilde{t}(\kappa)}_{Loss\ given\ Default}}^{Deposit\ Insurance\ Value}] \\
& - \tilde{t}(\kappa) - \underbrace{t(\kappa)}_{fixed\ cost} \\
\Leftrightarrow max_{\{r\}} \quad & q(r)[Ar - C]
\end{aligned}
$$

with $A := EX - (1-p)\underline{X}$ is the benefits of lending, $C := 1 + \xi - (1-p)(1-\lambda+\xi)$ represent the cost of lending.

$$N(R) \cdot \int_a {}_0^1 R \cdot X - d - C(X)dF(X) \Leftrightarrow N(R) \cdot \int_a {}_0^1 R \cdot X - ddF(X) - c \Leftrightarrow N(R) \cdot \int_a {}_0^1 R \cdot X - (1-\lambda)xdF(X$$

E.3 Simple model sol

So $\bar{\xi} = \frac{c_d - a}{b}$, with $c_d = \frac{\gamma\alpha + \nu N \bar{r}}{\nu N + \gamma}$

The average price is:

$$\bar{r} := \frac{1}{2}[c_d + \bar{c}]$$
$$\bar{c} := \frac{1}{G(c_d)} \int_0^{c_d} (a + b\xi) dG(\xi)$$

Assuming that ξ follows a Pareto distribution: $G(c) = (\frac{c}{c_m})^k$ on $[0, c_m]$, we get the entry condition:

This gives:

$$N = \frac{2\gamma}{\nu} \frac{\alpha - c_d}{c_d - \bar{c}}$$

$\bar{c}$ is the average cost conditional on survival.

E.4 Solving the model

$$\pi = \frac{1}{4\gamma}(c_d - c)^2 - (1 - (1-p))\tilde{t} \tag{E.2}$$
$$r := \frac{1}{2}(c_d + c) \tag{E.3}$$
$$c_d = \frac{\gamma\alpha + \nu N \bar{r}}{\nu N + \gamma} \tag{E.4}$$
$$\bar{r} = \frac{c_d}{2} + \frac{1}{2}\int_0^{c_d} c dF(c) \tag{E.5}$$
$$c = a + b\xi \tag{E.6}$$

with $a := \frac{1-(1-p)(1-\lambda)}{EX-(1-p\underline{X})}$ $b := \frac{1-(1-p)(1-\tilde{\lambda})}{EX-(1-p\underline{X})}$

And the entry and exit decision:

$$E_\xi[\pi(\xi) - t(c)|\xi < \bar{\xi}] = 0 \tag{E.7}$$

$$\pi(\bar{\xi}) = 0 \tag{E.8}$$

Using E.2, E.8, E.6 *and* E.4, we get:

$$N = \frac{2\gamma}{\nu}\frac{\alpha - c_d}{c_d - \bar{c}}$$

with: $c_d = [(1-(1-p))\tilde{t}4\gamma]^{\frac{1}{2}} + a + b\bar{\xi}$ and $\bar{c} = (a + b \cdot E[\xi|\xi < \bar{\xi}])$

and:

$$\int_0^{\frac{[(1-(1-p))t4\gamma]^{\frac{1}{2}}}{b}+\bar{\xi}} (\frac{[(1-(1-p))t4\gamma]^{\frac{1}{2}}}{b} + \bar{\xi} - x)^2 dG_\xi(x) = 4\gamma\frac{t + \tilde{t}(1-(1-p))}{b^2} \tag{E.9}$$

with ξ Pareto distributed ($G(x) := (\frac{x}{c_m})^k$) on $[0, c_m]$, equation E.10 simplifies (using an IPP) to:

$$\bar{\xi}^* = [2\gamma(k+1)(k+2)c_m^k(\frac{t + \tilde{t}(1-(1-p))}{b^2})]^{\frac{1}{k+2}} - \frac{[4(1-(1-p))\tilde{t}\gamma]^{\frac{1}{2}}}{b}$$

$$N^* = \frac{2\gamma}{\nu}\frac{\alpha - [(1-p)\tilde{t}4\gamma]^2 - a - b\bar{\xi}}{[(1-p)\tilde{t}4\gamma]^2 + b(\bar{\xi} - E[\xi|\xi \leqslant \bar{\xi}])}$$

Generalization:

$$N = \frac{2\gamma}{\nu}\frac{\alpha - c_d}{c_d - \bar{c}}$$

with: $c_d = [(1-(1-p)t)4\gamma]^2 + a + b\bar{\xi}$ and $\bar{c} = (a + b \cdot E[\xi|\xi < \bar{\xi}])$ The exit equation give the productivity threshold as a function of the number of banks and the average cost $\bar{c}(\lambda)$, $c_d(N, \lambda)$. When N is high, the firm must more productive to operate, so $c_d(N, \lambda)$ is decreasing in N and increasing in λ. This formula a functional form comes from the linear demand assumption.

$$\frac{1}{4\gamma}\int_0^{c_d(N,\lambda)} (c_d(N,\lambda) - c)^2 dG_\xi(c) = t \tag{E.10}$$

Using an IPP (assuming that $G(0) = \int G(0) = 0$):

$$I(c_d) := [\int\int G]_0^{c_d} = t2\gamma \tag{E.11}$$

$I(c_d) := \int\int G$ is the primitive of the primitive of G, it is an increasing function of c_d.

E.5 Sufficient Statistic

E.5.1 Borrowers

Quasi-linear utilities.

$$V_t := max_{c_t,m_t,d_t,\kappa_t} u(c_t) + m_t + \beta E_t[V_{t+1}] \tag{E.12}$$

$$s.t.\ c_t + m_t + d_t + \kappa_t = W_t \tag{E.13}$$

d_t are deposits, κ_t are equity.

W_t is composed of firms profits Π_t^F banks' profits $\Pi_t^b(Z)$, returns from deposits $d_{t-1}r_t$ and capital $C_{t-1}(X_t)$

E.5.2 Firms

Borrowers own firms, I denote ψ_{t+1} borrowers SDF. Firms choose a loan contract (q, R). They use the amount q in the period t to produce $Z_{t+1}F(q)$ in period $t + 1$. They repay banks R_{t+1} in period t+1. Bank sets a contract so that the firm is indifferent between accepting or not. This way, the value of q is unaffected by the limited liability problem. Firms' profit is:

$$E_t[\Pi_{t+1}^F] = E_t[\psi_{t+1} Z_{t+1} F(q) - R_{t+1} | R_{t+1} < Z_{t+1}] \tag{E.14}$$

When Firms do not have enough money to repay R, they give their whole production $Z_{t+1}F(q)$. This implies that there exists a random variable X with a cdf M that models the share of the face value that is repaid.

To simplify, we assume the productivity level Z is either $\bar{Z}$ or 0 and $\bar{Z} > \tilde{r}_{t+1}q$. There is a continuum of firms of mass N with different Z_i that are iid and extreme value distributed. The probability that a share of firm X default is given by the cdf M.

The aggregate loan demand is thus:

$$q_t^d(R_{t+1}) := N q_{it+1}^d(R_{t+1}) \tag{E.15}$$

E.5.3 Banks

In period t, they finance loans with deposits and capital:

$$q_t = d_t + \kappa_t \tag{E.16}$$

They have capital requirements:

$$q_t \geqslant \lambda \kappa_t \tag{E.17}$$

In period t+1, a fraction X of their loan is repaid; lenders thus get XR, and R is the face value of their debt.

To get d and κ, they need to promise in period 2 the amounts $d_t r$ and $C(X)$. Denoting $\bar{x}$, the threshold at which lenders default ($\bar{x} := \frac{d_t r_{t+1}}{R}$ when the bank incurs no other irrepressible cost than deposits) and assuming that there is deposit insurance, we have:

$$d_t := \int_0^1 \psi_{t+1} d_t r_{t+1} dM(X) \tag{E.18}$$

$$\kappa_t := \int_{\bar{x}}^1 \psi_{t+1} C_{t+1}(X) dM(X) \tag{E.19}$$

Equation (E.18) implies $r = (\int_0^1 \psi_{t+1} d_t dM(X))^{-1}$ with, ψ_{t+1} being the SDF.

Deposit insurance implies that a deposit is cheaper than collateral. As a result, the inequality in E.17 is binding.

Denoting the demand for loan $q(R)$, lenders' problem is:

$$max_{q,R,c(X),d,\kappa,c(X)} \int_{\bar{x}}^1 \psi_{t+1}[XR - C(X) - d_t r] dM(X) \tag{E.20}$$

$$s.t.\ (E.18), (E.19), (E.17) \tag{E.21}$$

$$E_t[\Pi^F_{t+1}] = E_t[X][\bar{Z}F(q) - R] \geqslant O \tag{E.22}$$

This can be rewritten:

$$max_R \int_{\bar{x}}^1 \psi_{t+1} XR dM(X) - (\lambda + (1-\lambda)(1 - M(\bar{x})))q \tag{E.23}$$

$$E_t[\Pi^F] = E_t[\psi_{t+1} X][\bar{Z}F(q) - R] \geqslant O \tag{E.24}$$

The F.O.C yields[1]:

1. The marginal impact on $\bar{x}$ cancels out

$$E_t[\psi_{t+1}X|X > \bar{x}]\bar{Z}F'(q) = 1 - (1-\lambda)M(\bar{x}) \tag{E.25}$$

$$R = \bar{Z}F(q) + \frac{O}{E_t[\psi_{t+1}X]} \tag{E.26}$$

E.5.4 Equilibrium

The equilibrium values of (R^*, m^*, ϕ_{t+1}) are given by the following equations:

For R, given that $q(R)$ is defined in eq. (E.15):

$$\beta E[X|X > \bar{x}] = (\lambda - (1-\lambda)(1 - F(\bar{x})))q(R) \tag{E.27}$$

For m:

$$ZF(q) = u'^{-1}(1) + m + q(R) \tag{E.28}$$

for ψ_{t+1}

$$\psi_{t+1} = \beta u'(c_t) = \beta \tag{E.29}$$

Thus $r = \frac{1}{\beta}$

E.5.5 Welfare

This gives the Welfare function:

$$W := \beta\{E_t[u(u'^{-1}(1))] - u'^{-1}(1) + X\bar{Z}F(q) + \underbrace{\mathbf{1}_{X<\bar{x}}\frac{1}{\beta}q(1-\lambda)}_{deposit\ insurance\ friction} - T\} - q \tag{E.30}$$

The government sets $T(X) = \frac{1}{\beta}q(1-\lambda)$, the optimal loan demand q would be:

$$q = F'^{-1}(\frac{1}{\beta\bar{Z}E[X]}) \tag{E.31}$$

With the deposit insurance friction, the centralized demand would be:

$$q = F'^{-1}(\frac{1-(1-\lambda)M(\bar{x})}{\beta\bar{Z}E[X]}) \tag{E.32}$$

with $\bar{x} := \frac{dr}{R} = \frac{(1-\lambda)rq}{q\tilde{r}}$

The actual decentralized demand is:

$$q = F'^{-1}\left(\frac{1-(1-\lambda)M(\bar{x})}{\beta\bar{Z}E[X|X>\bar{x}](1-M(\bar{x}))}\right) \tag{E.33}$$

E.5.6 Sufficient Statistic

Welfare can thus be written:

$$W := max_q\{\beta\{E_t[u(u'^{-1}(1))] - u'^{-1}(1) + \mathbf{1}_{X\geqslant\bar{x}}X\bar{Z}F(q) + \underbrace{\mathbf{1}_{X\geqslant\bar{x}}\frac{1}{\beta}q(1-\lambda)}_{deposit\ insurance\ friction} - T - q\} \tag{E.34}$$

$$+ E_t[\mathbf{1}_{X<\bar{x}}X\bar{Z}F(q)] \tag{E.35}$$

The impact of a policy $\mathcal{C}$ affecting capital requirements is thus:

$$\frac{dW}{d\mathcal{C}} = \overbrace{Pr(X\leqslant\bar{x})}^{Prob\ that\ bank\ default}\left[q\frac{d(1-\lambda)}{d\mathcal{C}} + \overbrace{E[X|X<\bar{x}]\bar{Z}F'(q)}^{Marginal\ productivity\ when\ bank\ default}\overbrace{\frac{dq}{d\mathcal{C}}}^{Changes\ in\ lending}\right] \tag{E.36}$$

$$-\underbrace{\frac{d(Pr(X\leqslant\bar{x})q(1-\lambda))}{d\mathcal{C}}}_{changes\ in\ deposit\ insurance\ \ cost} \tag{E.37}$$

$q\frac{d(1-\lambda)}{d\mathcal{C}}$ captures the fact that an increase in capital requirements $q\frac{d(1-\lambda)}{d\mathcal{C}}$ increases the bank cost of lending.

The $E[X|X<\bar{x}]\bar{Z}F'(q)\frac{dq}{d\mathcal{C}}$ captures the fact that banks default a distortion in the optimal amount of lending.

$\frac{d(Pr(X<\bar{x})q(1-\lambda))}{d\mathcal{C}}$ captures the decrease in taxes to fund the deposit insurance.

E.5.7 Extension: add entry and exit of banks

A firm of type ξ entry condition is:

$$E_t[\psi_{t+1}\Pi^B_{t+1}(\xi)] \geqslant 0 \tag{E.38}$$

The marginal entrant is such that:

$$E_t[\psi_{t+1}\Pi^B_{t+1}(\bar{\xi})] = 0 \tag{E.39}$$

Denoting m^B the mass of banks, N^B the cdf and n^b the pdf, the number of banks in the market is:

$$n^B(1 - N^B(\bar{\xi})) \tag{E.40}$$

Assuming that the fixed cost f and the productivity cost $\xi^{-1}q$ are paid at time t, lenders offer differentiated products and compete for firms' loans.[2] The welfare function can be written:[3]

$$W := max_q\{\beta\{E_t[u(u'^{-1}(1))] - u'^{-1}(1) - m^B(1 - N^B(\bar{\xi}))f - m^B \int_{\bar{\xi}}^{\infty} \xi^{-1} q dN^B(\xi) \tag{E.41}$$

$$+ \mathbf{1}_{X \geqslant \bar{x}} X \bar{Z} F(q) + m^B \underbrace{\int_{\bar{\xi}}^{\infty} \mathbf{1}_{X \geqslant \bar{x}} \frac{1}{\beta} q(\xi)(1 - \lambda(\xi)) dN^B(\xi)}_{deposit\ insurance\ friction} - T - q\} \tag{E.42}$$

$$+ E_t[\mathbf{1}_{X < \bar{x}} X \bar{Z} F(q)] \tag{E.43}$$

The impact of a policy $\mathcal{C}$ affecting capital requirements now has the extra term:

$$\overbrace{\frac{dN}{d\mathcal{C}}}^{net\ impact\ of\ \mathcal{C}\ on\ bank\ entry} [-f - \bar{\xi}^{-1} q(\bar{\xi}) + E[\mathbf{1}_{X \geqslant \bar{x}} q(\bar{\xi})(1 - \lambda(\bar{\xi}))]] \tag{E.44}$$

$$= \frac{dN}{d\mathcal{C}}\{E_t[\mathbf{1}_{X \geqslant \bar{x}} X R(\bar{\xi})] - q(\bar{\xi})\lambda(\bar{\xi})\} \tag{E.45}$$

Using equation (E.39) to get the second line and denoting $\frac{dN}{d\mathcal{C}} := m^B n^B(\bar{\xi})$.

The sufficient statistic is thus:

2. Formally, the probability that a firm comes to the lender is a function $\Phi(E_t[\Pi^F_{t+1}])$ yields the same formula for q but a different one for R

3. The representative agent take N as given.

$$\frac{dW}{d\mathcal{C}} = m^B \int_{\bar{\xi}}^{\infty} \overbrace{Pr(X \leqslant \bar{x})}^{Prob\ that\ bank\ default} [q(\xi) \frac{d(1-\lambda(\xi))}{d\mathcal{C}}] dN(\xi) \tag{E.46}$$

$$+ \overbrace{E[X|X < \bar{x}]\bar{Z}F'(q)}^{Marginal\ productivity\ when\ bank\ default} \overbrace{\frac{dq}{d\mathcal{C}}}^{Changes\ in\ aggregate\ lending}] \tag{E.47}$$

$$+ \frac{dN}{d\mathcal{C}} \{E_t[\mathbf{1}_{X \geqslant \bar{x}} X R(\bar{\xi})] - q(\bar{\xi})\lambda(\bar{\xi})\} \tag{E.48}$$

$$- \underbrace{\frac{d(Pr(X \leqslant \bar{x})q(1-\lambda))}{dN\mathcal{C}}}_{changes\ in\ deposit\ insurance\ \ cost} \tag{E.49}$$