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Customer-to-customer returns logistics: Can it mitigate the negative impact of online returns?☆

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ABSTRACT

Customer returns are a major problem for online retailers due to their economic and environmental impact. This paper investigates a new concept for handling online returns: customer-to-customer (C2C) returns logistics. The idea behind the C2C concept is to deliver returned items directly to the next customer, bypassing the retailer's warehouse. To incentivize customers to purchase C2C return items, retailers can promote return items on their webshop with a discount. We build the mathematical models behind the C2C concept to determine how much discount to offer to ensure enough customers are induced to purchase C2C return items and to maximize the retailer's expected total profit. Our first model, the base model (BM), is a customer-based formulation of the problem and provides an easy-to-implement constant-discount-level policy. Our second model formulates the real-world problem as a Markov decision process (MDP). Since our MDP suffers from the curse of dimensionality, we resort to simulation optimization (SO) and reinforcement learning (RL) methods to obtain reasonably good solutions. We apply our methods to data collected from a Dutch fashion retailer. We also provide extensive numerical experiments to claim generality. Our results indicate that the constant-discount-level policy obtained with the BM performs well in terms of expected profit compared to SO and RL. With the C2C concept, significant benefits can be achieved in terms of both expected profit and return rate. Even in cases where the cost-effectiveness of the C2C returns program is not pronounced, the proportion of customer-to-warehouse returns to total demand becomes lower. Therefore, the system can be defined as more environmentally friendly. The C2C concept can help retailers financially address the problem of online returns and meet the growing need for reducing their environmental impact.

1. Introduction

Free product returns are an essential part of customer service in retail. Under European law, consumers who buy a product online are entitled to a full refund if they cancel their purchase within 14 days of receipt. As a result, many purchases are returned to online retailers' warehouses every day, with return rates ranging from 5% to 40% [1, Dutch Broadcast Foundation]. Similar figures have been observed in the United States [data source: 2]. The total monetary value of returns for U.S. online retailers amounted to $212 billion in 2022 [3].

Processing a returned product is a costly and time-consuming activity. It typically consists of collection, screening, and sometimes repair [4]. It is estimated that one returned item costs an online retailer between €10 and €15 [1], taking into account warehousing, labor, packaging and transportation costs (both to and from the warehouse). In addition to requiring additional warehouse logistics capabilities, returns also result in reduced product availability and sub-optimal reordering policies. Moreover, retailers must process large volumes of returns during peak return periods, such as right after Christmas. This strains warehouse performance and incurs additional costs to manage the processing capacity. It is clear that the increasing number of daily returns over the past decade is suppressing the profitability of the online retail sector. Moreover, high return volumes have a negative environmental impact due to the additional transportation required. Returns cause an increase in express parcel volumes and therefore CO_2

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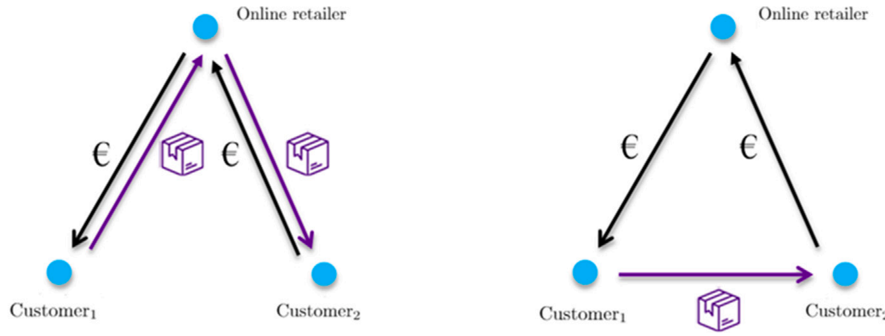

Fig. 1. Flow of goods and money under the conventional (left) and the C2C (right) returns programs.

emissions. In addition, each return requires new packaging materials when items are sold again.

Return rates are particularly high in fashion e-commerce. Diggins et al. [5] mention that return rates between 20% and 40% are common, with values as high as 74% [6]. Since customers typically return fashion products because they are the wrong size or color, "a significant number of the returned products are in perfect condition for resale" [4]. This means there is no need for an immediate return to the warehouse.

The volume of returns and the associated costs show that there is still no proper solution to the returns problem that online retailers face on a daily basis. With this research, we investigate a new concept as a possible solution to the customer returns problem: the Customer-to-Customer (C2C) Returns Logistics concept. Unlike existing approaches that focus on preventing returns, we take a different approach and aim to minimize the impact once a return occurs. The main idea behind the concept is to send returned items directly to other customers requesting the same product, rather than first sending them back to the retailer's warehouse. A customer is offered the option of receiving the product directly from a customer who wishes to return it, hereafter called the 'returnee', or the conventional option of receiving the product directly from the retailer's warehouse. To compensate for the reduced unboxing experience and potentially longer and variable delivery time, the option to purchase a returned item is offered in exchange for a discount on the corresponding product. The online retailer, who may also be an external party, has a management and operational role as the product flows between customers and when offering discounts. All transportation costs and cash flows are the retailer's responsibility. This means that customers pay and receive money from the retailer, even though products are shipped between customers. A visualization of the C2C concept in its basic form is shown in Fig. 1.

The returnee could be incentivized to participate in the C2C returns program in an economic way, e.g. by offering loyalty points, or in a moral way, by making it clear that this is a 'greener' option compared to a direct return. The environmental benefits come from (i) eliminating non-value-adding transportation by eliminating shipments from customer-to-warehouse, (ii) reducing packaging waste as C2C shipments do not require new packaging, and (iii) preventing unused returned items from ending up in landfills [see 7,8].

The concept is similar to e-commerce sites that sell used clothing, such as Vinted (vinted.com). It is also similar to retailers who sell open-box items at a discount next to their new items. Unlike Vinted, in our case (i) the retailer organizes the process and coordinates it with their direct sales, and (ii) the clothes are not used and can still be considered as good as new, thus serving a different market.

The contribution of our research is both practical and theoretical. From a practical point of view, this research analyzes a new concept, the C2C concept, which has the potential to reduce the physical and financial burden for retailers in handling returns. In our research, we contacted several retailers who showed interest in implementing the concept. One of the co-authors of this article is following these real-world implementations and conducting promising experiments. Their progress can be followed on https://itgoesforward.com/. From a theoretical point of view, we provide the mathematical models that underpin the C2C concept. First, we propose a stylized base model and provide some theoretical results on the structure of the profit function, the optimal discount level, and the profitability of the C2C returns program. We propose a constant-discount-level policy for the problem using our base model. Then, we formulate the real-world problem as a Markov decision process (MDP). Since our MDP suffers from the curse of dimensionality, we resort to simulation optimization and reinforcement learning algorithms to find reasonably good solutions. We evaluate the financial prospects of the C2C concept for different levels of product demand volumes, return rates, customer participation scenarios, and other model variables. Our methods are applied to data collected from a Dutch fashion retailer. The data consist of online sales and return data from May 2017 to May 2019 and include 2.6 million data points. We find that the C2C concept could lead to higher profits compared to a conventional returns program. In our case study, we evaluate best-case and worst-case scenarios. In the best-case scenario, the C2C returns program generates additional demand and we observe a profit increase of 34%. We show that the value of the C2C returns program is non-significant in the worst-case scenario where C2C demand fully substitutes regular demand. Interestingly, in the cases of minor savings, the share of customer-to-warehouse returns in total demand decreases by 6%–11%, which suggests that the C2C concept is still beneficial from an environmental perspective. In a more extensive set of experiments, where we consider a full factorial combination of possible parameter values, this behavior is confirmed.

The remainder of this paper is organized as follows: Section 2 reviews the research most relevant to our paper. Section 3 elaborates on the C2C returns program. Section 4 presents mathematical models and theoretical results. Section 5 presents solution methods to solve our MDP. Section 6 reports our comparative results for the performance of the algorithms, the value of the C2C program, and the impact of problematic returns in our case study. Section 7 contains extensive numerical experiments to establish generality. Finally, Section 8 concludes the paper with critical insights from the study and future extensions of our work.

2. Literature review

Our research contributes to the literature on consumer returns management and C2C sharing economy. Abdulla et al. [9] provide a recent overview on managerial decision-making related to return policies and consumer behavior. In the literature, the main research questions include: under what conditions should companies allow returns, how much restocking fee to charge, how to collect returns, and what to do with them. Below, we summarize the most relevant papers related to our research.

An early seminal work by Shulman et al. [10] proposes an analytical model to examine how consumers' purchase and return decisions are affected by a retailer's pricing and restocking fee decisions. The analysis of Nageswaran et al. [11] helps explain the reasons behind omnichannel retailers' decisions to offer full refunds or charge fees for online returns. Wagner and Martínez-de Albéniz [12] demonstrate that lenient return policies can boost sales but also increase retailers' cost. Yang and Ji [13] and Yang et al. [14] examine the financial impact of buy-online-return-to-store mechanisms. We contribute to this stream by proposing a separate logistics channel to sell returned products and investigate how much discount we should offer to make this system profitable.

Online returns are associated with increased opportunities for fraudulent returns [15]. Fraudulent returns are actions carried out with the purpose of defrauding a retailer by exploiting its return policy. Some examples of fraudulent returns are empty box fraud (e.g., claiming that an empty box was received), wardrobing (also known as free renting or opportunistic return which occur when customers intentionally rent a product for short-term use), damage fraud (e.g., claiming that the product is defective but it was damaged by the customer), shipping-related fraud (e.g., claiming that the parcel was never received). Altug et al. [16] propose tactics that can help retailers address the negative financial implications of opportunistic returns. In our paper, we initially consider all customers as trustworthy. However, we also include a break-even analysis of the rate of problematic returns in our case study.

Among e-commerce returns, fashion products have remarkably high return rates. Factors affecting the product returns in the fashion and apparel industry have been studied by several authors using either different data sets [17–20] or different focus groups [21,22]. Fashion products are often differentiated in terms of size and fit, and a single customer may order multiple sizes. As a result, many returns are of excellent quality, similar to garments hanging in a shop and being tried on by different customers. In many countries, returned garments can even be sold as new. A lot of fashion is seasonal, which means that the selling season is short. Therefore, returns need to be processed quickly. de Leeuw et al. [4] provide a detailed analysis of the returns process for fashion retailers. Difrancesco et al. [23] develop a queueing model and investigate how to set the return duration, whether to refurbish returned products or sell them on the secondary market in order to maximize profit. They cross-check their analytical results with a data set coming from one of Europe's largest online fashion retailers, Zalando.

Our paper relates to C2C sharing economy research. Mont et al. [24] provide an analysis of different concepts, theories, and understandings of the term sharing economy. Hawlitschek et al. [25] investigate key factors contributing to the success of the sharing economy. Factors that make customers engage in C2C sharing include cost-saving, trust in other users, environmental sustainability, and a modern lifestyle. These factors would also pave the way for the success of the C2C returns program. Some papers provide conditions under which the supply chain partners are better or worse off under the presence of a C2C marketplace. Jiang et al. [26] incorporate a C2C marketplace as a decision maker, setting its own profit maximizing transaction fee, and while accounting for consumers' valuation uncertainty about goods. Li et al. [27] examine retailer's optimal return policy under the presence of a C2C (second-hand goods) marketplace. None of these papers consider selling returns at a discount and shipping them directly from C2C.

Several studies have explored the intersection of the sharing economy and sustainability. Xue et al. [28] investigate the environmental impact of encouraging the reuse and reselling of secondhand goods through C2C platforms, finding that such platforms can be beneficial for both the environment and profitability when the environmental impact of a product primarily occurs during production or disposal. Similarly, Vedantam et al. [29] compare the financial and environmental impacts of two business models for an apparel company: (i) trade-in with resale and (ii) C2C resale marketplace. They conclude that the C2C resale marketplace model could be more environmentally friendly if the environmental impact from production outweighs that from usage activities like laundry and ironing. We assess environmental implications of the C2C returns program by examining changes in the ratio of customer-to-warehouse returns relative to total demand, which we consider indicative of reductions in non-value-adding return shipments and packaging waste. Pang and Li [30] show the presence of a C2C resale marketplace might increase the demand for new items and such a demand expansion effect can aggravate environmental impact. Even though the C2C returns program might also generate more shipments due to demand expansion for returned items, it might prevent returned items from ending up in landfills [7], similarly to secondhand markets such as Craiglist [8].

3. Customer-to-customer returns program

In this section, we describe the C2C returns program, elaborate on the relation between program design and customer participation, and discuss potential implementation issues. Section 3.1 offers an in-depth explanation of how the C2C returns work and presents previous implementations of similar concepts. Section 3.2 discusses incentives for customer participation, elaborates on the potential environmental benefits, and discusses how to model customer demand. Finally, Section 3.3 describes potential implementation issues and suggests how to resolve them.

3.1. How the C2C returns work

The C2C returns program consists of several distinct steps. First, the returnee is asked to evaluate the condition of the item (e.g., like new, defective, stained, etc.). If the item is deemed good as new and the customer expresses interest in participating in the C2C returns program, they are instructed to repackage the item and hold it for a designated period of time, known as the *time window for matching* (typically a few days). Therefore, inspection, handling, and repackaging operations are outsourced to the returnee. During the time window for matching, the item is listed on the retailer's webshop as a discounted C2C item. Simultaneously, potential customers visiting the webshop are presented with two options: (i) purchasing an item that will be shipped from the warehouse (W2C), or (ii) purchasing a discounted item that will be shipped from a customer (C2C). In the case of a C2C sale, the QR code on the item's packaging is linked to the purchaser's address and the returnee is asked to hand in the item within a designated period of time, known as the *time window for handing-in* (also a few days). From the return drop-off location, the item is shipped along with other parcels, potentially by third-party logistics providers. A software enables the matching and tracking of items.

It is important to note that returnees are only given the option to return items using the C2C returns program if they indicate that the item is like new. Damaged items will always be returned using the current conventional returns program, i.e. customer-to-warehouse (C2W). Indeed, the item must be easily inspectable by the returnee. Apparel, small appliances, and products aimed at environmentally-conscious customers are suitable as C2C returns.

If a customer who has purchased a C2C item is dissatisfied with their purchase, they can return it to the warehouse. However, we propose not selling the same item a second time as C2C in order to facilitate quality control and packaging refurbishment by the retailer. In this paper, we present the mathematical models that underpin the C2C returns program, focusing on decision models that determine the optimal discount level to maximize the expected profit for the retailer. We also assess the program's overall profitability and environmental impact.

Implementations of concepts similar to the C2C returns program have been observed at a few companies including Veepee, a French retailer specializing in flash sales, and Prada Vida, a sustainability-focused e-commerce brand based in Winnipeg, Manitoba, Canada. Prada Vida's implementation was facilitated by a Canadian startup called Frate.co, which enables *peer-to-peer return services* for retailers. In these implementations, returns are shipped and sold between customers without them ever going back to the warehouse. The idea is the same as *return article forwarding*, which is the term used by It Goes Forward (see https://itgoesforward.com) to define the C2C returns program. While Veepee has discontinued its so-called *Re-turn system*, Frate has reported significant benefits from its implementation. Veepee's Re-turn system experienced several problems that could have been mitigated with the C2C returns program introduced in this paper. Both returnees and purchasers experienced problems such as difficulties with refunds, receiving incorrect or damaged items, and inadequate customer service responses [see customer reviews section in 31]. Our C2C returns program addresses these issues by offering free returns and free shipping, allowing returns within the 14-day period even if the item is not resold, and considering dispute resolution services. Frate's success story with Prada Vida [32] includes approximately 57% of returns being successfully shipped and sold to another customer, with C2C returns being repurchased within an average of 2 days. In addition, 0% of purchased C2C returns are re-returned. Frate also claims an average 32% increase in profit, 40% decrease in returns, and 23% decrease in emissions. We note that the findings in our paper are in line with those reported by Frate.

3.2. Benefits and incentives

To encourage greater participation in the C2C returns program, it is crucial to provide customers with appropriate incentives. Time windows for matching and handing-in require additional effort on the part of the returnee. Returnees should keep the item during the time window for matching. They should also be able to hand-in the item at short notice, either by going to a drop-off location or using a pick-up service. In the event of a long time window for matching and a short time window for handing-in, the hassle for the returnee increases, but so does the likelihood of a C2C sale. Previous research on customer returns developed endogenous models by introducing a hassle cost of return [see, 10,14,33]. In practice, it is difficult to quantify the hassle cost for returnees. In order to substantiate the C2C initiative, Wiersma [34] and Hsieh [35] conducted a rating-based conjoint analysis to collect and analyze customer preferences in the C2C returns program. According to their results, there are indications that the likelihood of returnees to participate in the C2C returns program decreases as the time window for matching increases and increases as the time window for handing-in increases. In our paper, we evaluate the impact of these relations on the retailer's expected profit through numerical experiments.

The C2C returns program is aligned with the United Nations Sustainable Development Goals (SDGs) [36]. More specifically, it seeks to achieve (i) Responsible consumption and production (SDG 12), through promoting the consumption of returned items to prevent them from being dumped or destroyed and reducing packaging waste, (ii) Climate action (SDG 13), by decreasing CO_2 emissions through eliminating one transport leg and avoiding that returned items end up in landfills. If returned items are thrown away, all the emissions generated during production and transport would be in vain. Moreover, the C2C returns program complies with various European Union (EU) directives and laws, such as the Corporate Sustainability Reporting Directive (CSRD), Packaging and Packaging Waste Directive (PPWD), and Green Claims Directive (GCD). Under CSRD, retailers are mandated to report their environmental impact and strategies for reduction. The C2C returns program enables retailers to demonstrate improvements in environmental impact related to returns. PPWD supports packaging reuse while the C2C returns program reduces the need for new packaging. GCD prohibits companies from outsourcing sustainability claims (for example, planting trees to offset CO_2 emissions), necessitating environmentally friendly operations. In our paper, the environmental impact of the C2C returns program is assessed by examining the change in the ratio of C2W returns to total demand. The proportional reduction in packaging waste per demand is consistent with our metric since C2C shipments do not require new packaging. Moreover, assuming average distance per shipment remains constant under the C2C returns program, our metric can represent the proportional change in CO_2 emissions generated by C2W return shipments.

Returnees can be encouraged to participate by promoting the environmental benefits of the C2C returns program. According to the recent questionnaire conducted by Wiersma [34] and Hsieh [35], a significant proportion of respondents are willing to participate in the C2C returns program for little or no monetary benefit. Self-esteem, altruism, and contributing to a better environment seem to be enough to motivate individuals to participate in the C2C returns program. In our paper, we ignore the effect of monetary benefits for returnees. We assume that returnees are motivated to engage in C2C returns solely by moral incentives.

For purchasers, the C2C service differs from the conventional service in terms of (i) delivery time, (ii) unboxing experience, (iii) condition of the product, (iv) additional discount offered. The C2C service offers the purchaser a potentially inferior experience for the first three elements. That is why the retailer offers the C2C returned items at a discounted price. The discount level should be high enough to induce enough customers to purchase C2C returns and as low as possible to maximize the expected profit. Wiersma [34] and Hsieh [35] show that the discount level (rather than delivery speed, product type, or product value) is the most important attribute for purchasers. In our paper, we express C2C demand as an non-decreasing function of the discount level.

We assume that a certain proportion of existing customers will be attracted by the discount on C2C returns. In other words, the C2C returns program will lead to demand substitution. However, there will also be new customers attracted only by the C2C returns. These customers would not otherwise purchase an item because they consider the conventional product price too high. In our paper, we model the two extremes by formulating best-case and worst-case scenarios. In the best-case scenario, all C2C purchasers are new customers and the C2C returns program increases total demand. In the worst-case scenario, C2C returns are considered substitutes and total demand remains the same. The best-case and worst-case scenarios represent pure demand expansion and pure cannibalization scenarios, respectively. We assess the profitability of the C2C returns program using these two scenarios. We note that the real situation is probably somewhere in between.

3.3. Potential issues and resolution

The operational issues that may arise in current conventional returns programs could also manifest in the C2C returns program. These include the delivery of incorrect items, as well as the loss or damage of items during transit. Another issue that can occur is return fraud. When an issue manifests, the current conventional returns program can result in disputes between the returnee, the third-party logistics provider, and the retailer. In the C2C returns program, the purchaser is an additional actor who is also a private individual. Currently, many goods or services are exchanged between private individuals, facilitated by a commercial party or platform. These companies make up the platform economy, of which Airbnb, DePop, eBay, Lyft, Uber, and Vinted are the most well-known. These platforms effectively resolve many disputes on a daily basis. There also exist specialized companies that can help in this respect (for example, *ReturnLogic*, *SEON* for conventional returns and *It Goes Forward* for C2C returns).

Some of the well-known tools that can be utilized to minimize, prevent, and resolve disputes between private individuals and/or a commercial party and a private individual are behavior monitoring (in order to identify and exclude service abusers), peer-to-peer reviews (in order to detect issues), microchip tags on items (in order to detect counterfeit items), monetary compensation, and return insurance [see e.g., 37–39]. In the C2C returns program, several actions can be taken to mitigate associated risks and efficiently resolve disputes. Firstly, returnees should be requested to submit a photo of the item prior to initiating the return process. Additionally, at the drop-off location, a detailed drop-off proof should be registered, including parcel characteristics like weight. Upon receipt of the item, purchasers should be encouraged to leave a review and rating. A dispute resolution process can be promptly initiated, if issues are reported through the review system within a few days of the purchaser receiving the C2C return item. Throughout this resolution process, various factors such as photos provided by the returnee/purchaser, drop-off proof, microchips on items, and the previous ratings/reviews/behavior of peers within the C2C returns program should be taken into consideration. These measures collectively aim to enhance the integrity and effectiveness of the C2C returns program.

According to a report by the National Retail Federation [3], for every \$100 in accepted returns, e-tailers in the United States lose \$10.70 due to return fraud. One could argue that the incidence of return fraud is potentially higher for items delivered through C2C transactions due to additional potential fraud types such as sending empty box or counterfeit items to C2C purchasers, or making premeditated C2C returns and purchases. In our mathematical models, our initial assumption is that all customers are trustworthy and that costly operational issues do not manifest. However, in our case study, we also report the break-even points for problematic return rates at which all the additional profit generated by the C2C returns program is offset due to operational issues or fraud.

4. Mathematical models and theoretical results

In this section, we define our problem, present our mathematical models, and provide our theoretical results. Our first model, the base model (BM), is a customer-based formulation of the problem. Our second model formulates the multi-customer problem as an MDP.

4.1. Base model

In this section, we present the base model under the conventional and the C2C returns program. The base model employs a customer-based approach by considering a single customer initially served from the warehouse. The purpose of the base model under the C2C returns program is to determine how much discount to offer to ensure that the retailer's expected total profit is maximized. The notations are summarized in the appendix (see Table A.1).

4.1.1. Conventional returns program

We consider an online retailer that sells an item in a webshop at price $P > 0$. Upon purchase, the item is shipped warehouse-to-customer (W2C), incurring shipping and handling cost S^{W2C}. Under the conventional returns program, the customer is allowed to return the item within T^{R} periods after delivery. A C2W return incurs shipping and handling cost S^{C2W}. We assume that the customer receives a full refund. We model the customer's return decisions as time-varying Bernoulli trials. Let u_i^{R} be the probability of return i periods after delivery with $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$. The probability of a C2W return is

$$p^{\mathrm{R}} = \sum_{i=1}^{T^{\mathrm{R}}} u_i^{\mathrm{R}} \prod_{j=1}^{i-1} \left(1 - u_j^{\mathrm{R}}\right) = 1 - \prod_{i=1}^{T^{\mathrm{R}}} \left(1 - u_i^{\mathrm{R}}\right),$$

where the equality follows from $\prod_{i=1}^{T^{\mathrm{R}}} \left(1 - u_i^{\mathrm{R}}\right)$ being the probability of no return. Under the natural assumption that $u_i^{\mathrm{R}} < 1$ for $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$, we have $p^{\mathrm{R}} < 1$. Let $R^{W2C} = P - S^{\mathrm{W2C}}$ be the revenue generated by a W2C delivery and $C^{C2W} = P + S^{\mathrm{C2W}}$ be the cost incurred due to a C2W return. Under the conventional returns program, the retailer's expected profit from a single customer is

$$\mathbb{E}[\Pi] = R^{W2C} - C^{C2W} p^{\mathrm{R}}. \tag{1}$$

This profit function excludes the costs associated with ordering, purchasing and inventory holding as they are not affected by the returns program.

4.1.2. C2C returns program

Under the C2C returns program, returnees can choose either a conventional C2W return or a C2C return within T^{R} periods after delivery. For a C2C return, we consider two cases: (i) the best-case scenario in which the C2C returns program potentially results in an additional customer, and (ii) the worst-case scenario in which a C2C return is viewed as a substitute by a webshop customer.

Return process

Consider a customer whose item was delivered W2C i periods ago with $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$. If the item has not been returned yet, the customer can (i) request a conventional C2W return with probability u_i^{C2W}, (ii) request a C2C return with probability u_i^{C2C}, (iii) keep the item one more period with probability $1 - u_i^{\mathrm{C2W}} - u_i^{\mathrm{C2C}}$. The probability of a C2C return is

$$p^{\mathrm{C2C}} = \sum_{i=1}^{T^{\mathrm{R}}} u_i^{C2C} \prod_{j=1}^{i-1} \left(1 - u_j^{C2C} - u_j^{C2W}\right).$$

Similarly, the probability of a conventional C2W return is

$$p^{\mathrm{C2W}} = \sum_{i=1}^{T^{\mathrm{R}}} u_i^{C2W} \prod_{j=1}^{i-1} \left(1 - u_j^{C2C} - u_j^{C2W}\right).$$

As in the conventional returns program, we make the natural assumption that $u_i^{\mathrm{C2W}} + u_i^{\mathrm{C2C}} < 1$ for $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$, implying that $p^{\mathrm{C2C}} + p^{\mathrm{C2W}} < 1$. Furthermore, a C2W return incurs cost C^{C2W}.

Matching process

After a C2C return request, the item is available for sale in the webshop. The retailer offers a discount $a \in [0, 1]$ on the selling price P for this item. Let the probability of selling a C2C return within a given period $q(a)$ be a function of the discount level a. The probability that a C2C return is sold within the time window for matching T^{M} is

$$p^{\mathrm{M}}(a) = 1 - (1 - q(a))^{T^{\mathrm{M}}}.$$

If the C2C return is sold at discount level a, the resulting revenue is $R^{\mathrm{C2C}}(a) = (1 - a)P$. The corresponding shipping cost is subtracted later during the hand-in process. If the return cannot be sold within T^{M} periods, a conventional C2W return is made at cost C^{C2W}.

Hand-in process

Following the matching of a C2C return to a C2C demand, the returnee must hand in the item within the time window for handing-in T^{H}. The C2C returnee hands in the item i periods after matching with probability u_i^{HI} for $i \in \{1, 2, \dots, T^{\mathrm{H}}\}$. Therefore, the probability of a hand-in within T^{H} periods after matching is

$$p^{\mathrm{HI}} = \sum_{i=1}^{T^{\mathrm{H}}} u_i^{\mathrm{HI}} \prod_{j=1}^{i-1} \left(1 - u_j^{\mathrm{HI}}\right).$$

If the C2C returnee hands in the item within T^{H} periods, then the item is shipped from C2C and a C2C shipping cost S^{C2C} is incurred, i.e., the hand-in cost is $C^{\mathrm{HI}} = S^{\mathrm{C2C}}$. If not, a new item is shipped from W2C to satisfy the C2C demand, resulting in shipping and handling cost S^{W2C}. Therefore, the cost associated with a late hand-in is $C^{\mathrm{LHI}} = S^{\mathrm{C2W}} + S^{\mathrm{W2C}}$. When the C2C returns program is launched, ensuring full refunds would encourage more customers to participate in the C2C returns program. In our model, we use full refunds for both late and on-time hand-ins.

Finalization process

The customer who purchased the C2C return is allowed to return it within T^{R} periods. However, a second C2C return is not allowed because the retailer must perform a quality check and renew the packaging. We model this final return process in the same way as a conventional returns program. Let u_i^{FR} be the probability of return i periods after delivery with $t \in \{1, 2, \dots, T^{\mathrm{R}}\}$. The probability the item is returned is

$$p^{\mathrm{FR}} = \sum_{i=1}^{T^{\mathrm{R}}} u_i^{\mathrm{FR}} \prod_{j=1}^{i-1} \left(1 - u_j^{\mathrm{FR}}\right),$$

where we make the assumption that $u_i^{\mathrm{FR}} < 1$ for $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$, implying that $p^{\mathrm{FR}} < 1$. The cost associated with a return made by a C2C customer who purchased an item at discount level a is $C^{\mathrm{C2C}}(a) = (1-a)P + S^{\mathrm{C2W}}$.

Expected profit

We express the expected profit for the best-case scenario (i.e., the number of customers potentially increases) and the worst-case scenario (i.e., the number of customers remains the same).

Given discount level a, the retailer's expected best-case profit $\mathbb{E}\left[\Pi_{\mathrm{B}}(a)\right]$ can be written as

$$\begin{aligned}\mathbb{E}\left[\Pi_{\mathrm{B}}(a)\right] =& R^{W2C} - C^{C2W}\left[p^{\mathrm{C2W}} + p^{\mathrm{C2C}}\left(1 - p^{\mathrm{M}}(a)\right)\right] + R^{\mathrm{C2C}}(a)p^{\mathrm{C2C}}p^{\mathrm{M}}(a) \\ &- \left[C^{\mathrm{HI}}p^{\mathrm{HI}} + C^{\mathrm{LHI}}\left(1 - p^{\mathrm{HI}}\right) + P\right]p^{\mathrm{C2C}}p^{\mathrm{M}}(a) - C^{\mathrm{C2C}}(a)p^{\mathrm{C2C}}p^{\mathrm{M}}(a)p^{\mathrm{FR}}.\end{aligned} \quad (2)$$

The first component of (2) is the revenue generated by a W2C delivery. The second component is the expected cost associated with a C2W return, which is incurred when the customer (i) requests a C2W return or (ii) announces a C2C return but the item cannot be sold within T^{M} periods. The third component represents the expected revenue from a C2C sale. The fourth component represents the expected costs associated with a late or on-time hand-in of a C2C return and the refund to the returnee. The fifth component is the expected cost associated with the return of a C2C demand. Fig. 2 summarizes the corresponding processes, costs, and revenues under the C2C returns program. The retailer's problem is to determine an optimal discount level a^* that maximizes the expected profit function given in (2).

Given discount level a, a webshop customer purchases a C2C return with probability $p^{\mathrm{C2C}}p^{\mathrm{M}}(a)$. In the best-case scenario, the total demand increases by the ratio $p^{\mathrm{C2C}}p^{\mathrm{M}}(a)$. In the worst-case scenario, the total demand remains the same. Hence, the retailer's expected worst-case profit $\mathbb{E}\left[\Pi_{\mathrm{W}}(a)\right]$ can be expressed by

$$\mathbb{E}\left[\Pi_{\mathrm{W}}(a)\right] = \frac{\mathbb{E}\left[\Pi_{\mathrm{B}}(a)\right]}{1 + p^{\mathrm{C2C}}p^{\mathrm{M}}(a)}. \quad (3)$$

Note that these profit functions are built based on the expected profit generated by a webshop customer, not based on an item in stock. By assuming ample stock in the warehouse, we make the connection to the multi-customer case in Section 4.2.

4.1.3. Theoretical results

In this section, we present our theoretical results on the relation between conventional and C2C returns programs and the optimal discount level under certain assumptions. Proofs of the theorems are provided in the appendix.

Assumption 1. The probability of a return under the conventional returns program is equal to the probability of a return under the C2C returns program, i.e., $p^{\mathrm{R}} = p^{\mathrm{C2C}} + p^{\mathrm{C2W}}$.

This assumption seems reasonable since it is likely that a customer first decides to return an item and then, given this decision, they decide which returns program (conventional or C2C) to use. The following parameters play an important role in our theorems and proofs:

$$\begin{aligned}\phi_B &= C^{\mathrm{HI}}p^{\mathrm{HI}} + C^{\mathrm{LHI}}\left(1 - p^{\mathrm{HI}}\right) - (1 - p^{\mathrm{FR}})S^{C2W}, \\ \phi_W &= \phi_B + \mathbb{E}(\Pi).\end{aligned}$$

The term ϕ_B consists of the hand-in cost ($C^{\mathrm{HI}}p^{\mathrm{HI}} + C^{\mathrm{LHI}}\left(1 - p^{\mathrm{HI}}\right)$) and the benefit of eliminating the shipment from C2W (which is $(1 - p^{\mathrm{FR}})S^{\mathrm{C2W}}$ because it only happens if the C2C purchase is not returned). Thus, ϕ_B and ϕ_W can be interpreted as the difference in operational cost when using the C2C returns program rather than the conventional returns program for the best-case and worst-case scenarios, respectively. If the difference is positive, there are additional costs. If not, there are savings from using the C2C returns program instead of the conventional returns program.

Theorem 1. *Under Assumption 1 and discount level $a \in [0,1]$, the C2C returns program is more profitable than the conventional program for the best-case scenario if and only if*

$$P(1-a)\left(1 - p^{FR}\right) \geq \phi_B, \quad (4)$$

and for the worst-case scenario if and only if

$$P(1-a)\left(1 - p^{FR}\right) \geq \phi_W. \quad (5)$$

It is interesting to note that the profitability of the C2C returns program in comparison to the conventional program is independent of the probability $p^{\mathrm{M}}(a)$. In other words, it is independent of the functional relation between the C2C demand and the discount level. In the best-case scenario, one needs to check whether discount level a is such that the revenue from selling the item to the C2C customer (i.e., $P(1-a)\left(1 - p^{\mathrm{FR}}\right)$) exceeds the difference in operational cost. Consequently, in case $\phi_B < 0$, the C2C returns program is always (i.e., regardless which discount level a is chosen) more profitable, while the opposite is true in case $\phi_B > P\left(1 - p^{\mathrm{FR}}\right)$ (i.e., even the highest revenue corresponding to discount level $a = 0$ cannot compensate for the cost). A similar reasoning holds for the worst-case scenario, but the revenue must also compensate for $\mathbb{E}(\Pi)$, since in this scenario a conventional customer is substituted with a C2C customer. To summarize the discussion, there exists a discount level $a \in [0,1]$ for which the C2C returns program is more profitable than the conventional returns program for the best-case (or worst-case) scenario if and only if $P\left(1 - p^{\mathrm{FR}}\right) \geq \phi_B$ (resp. $P\left(1 - p^{\mathrm{FR}}\right) \geq \phi_W$).

Finally, conditions (4) and (5) provide an upper bound on the optimal discount level a, assuming that a firm only prefers to offer the C2C returns program if it is more profitable than the conventional program. To obtain tractable expressions for the optimal discount level, we need the following assumption.

Assumption 2. The probability of selling a C2C return $q(a)$ is a linear function of discount level a, i.e., $q(a) = q_0 + a\left(1 - q_0\right)$ where q_0 is the probability of selling the C2C return at its original price P and $0 \leq q_0 < 1$.

While it is possible to specify any functional form for $q(a)$, we consider a simple linear function to simplify analysis of our base model [see, e.g., 40,41, for other studies that make a similar assumption].

Theorem 2. *The best-case profit function $E[\Pi_B(a)]$ is unimodal on the interval $[0,1]$.*

Theorem 2 immediately provides a method for numerically finding the optimal best-case discount level a_B^*. It is well known that golden-section search is a method that can efficiently find the optimum of a unimodal function at any desired precision. Although we conjecture that $E[\Pi_{\mathrm{W}}(a)]$ is also unimodal, we are unable to prove this formally. However, the next property turns out to be useful in the sense that it reduces the search space for finding the optimal discount level in the worst-case model, in addition to being of interest on its own.

Theorem 3. *The optimal worst-case discount level a_W^* is lower than or equal to the best-case discount level a_B^*, i.e., $a_W^* \leq a_B^*$.*

For certain parameter ranges (see Appendix, Theorem A.1), we have an analytical expression for the optimal discount levels, although in

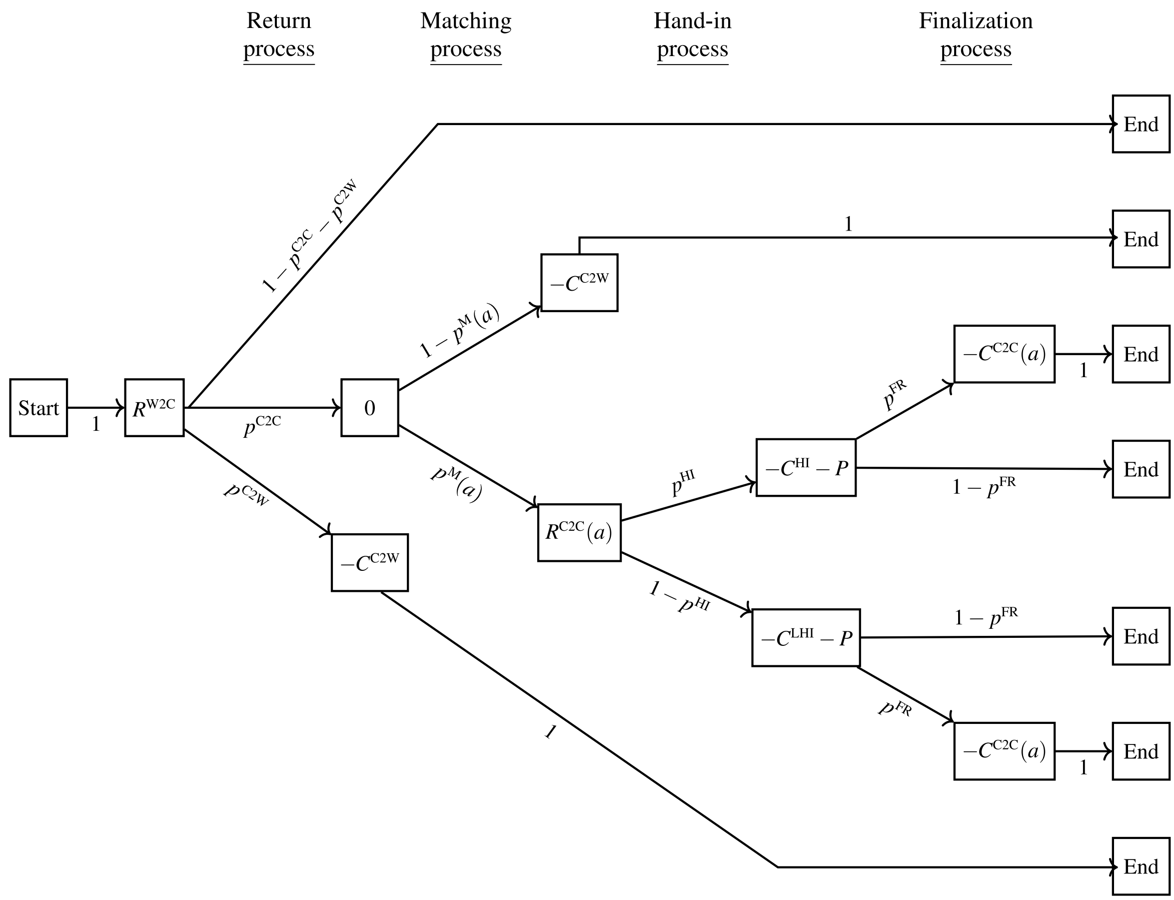

Fig. 2. Processes, costs, and revenues under the C2C returns program.

many cases one must solve a high-degree polynomial equation for which there is no closed-form solution. The ways to find a close-to-optimal discount level are to (i) use a brute force method, for example, by simply enumerating over a finite set of discount levels, for instance, $a = 0\%, 1\%, \dots, 100\%$, or (ii) use a golden-section search, which leads to a locally but not necessarily globally optimal solution (since $E[\Pi_{\mathrm{W}}(a)]$ is conjectured to be unimodal).

4.2. Multi-customer model

In this section, we present the multi-customer model under both conventional and C2C returns programs. The multi-customer model extends the base model to multiple arriving and returning customers during a selling season of T periods. Under the C2C returns program, the main difference from the base model is the explicit modeling of the C2C returns-demand matching process and the influence of discount levels on both C2C and W2C demand.

4.2.1. Conventional returns program

We assume that customers arrive according to a Poisson process with rate λ_t during $[t, t+1)$ for $t \in \{0, 1, \dots, T-1\}$. The demand size of each customer is one unit. We assume that there is sufficient stock in the warehouse. Demand occurring during $[t, t+1)$ is satisfied at the end of period t, generating revenue R^{W2C} per customer. Let $\boldsymbol{x}_t = \left(x_{t,i} \mid i \in \{1, \dots, T^{\mathrm{R}}\}\right)$ be a T^{R} - dimensional vector of integers, where $x_{t,i}$ represents the number of customers satisfied i periods ago and who have not returned by period t. For items purchased in period $t-i$, the number of C2W returns during $[t, t+1)$ follows a binomial distribution with parameters $x_{t,i}$ and u_i^{R} with $i \in \{1, \dots, T^{\mathrm{R}}\}$. Each C2W return incurs cost C^{C2W}. Due to the relation between Bernoulli trials and the binomial distribution, this model scales up the base model given in Section 4.1.1 in a stationary demand, infinite horizon setting (see Theorem 4).

4.2.2. C2C returns program

We formulate the multi-customer problem under the C2C returns program as a finite-horizon discrete-time Markov Decision Process (MDP). We present our MDP using "The Five Elements of a Sequential Decision Problem" defined by [42], namely state information, actions, exogenous information, transition function, and objective function.

The sequence of events is as follows. At the beginning of period t, state s_t is observed and decision a_t is made. During $[t, t+1)$, exogenous information $\boldsymbol{w}_t$ is faced. At the end of period t, costs and revenues are evaluated.

State information.

At period t, state information $s_t = \left(\boldsymbol{x}_t, \boldsymbol{y}_t\right)$ is composed of

(i) $\boldsymbol{x}_t = \left(x_{t,i} \mid i \in \{1, \dots, T^{\mathrm{R}}\}\right)$ where $x_{t,i}$ represents the number of W2C demands that are satisfied i periods ago and not returned by period t,
(ii) $\boldsymbol{y}_t = \left(y_{t,i} \mid i \in \{1, \dots, T^{\mathrm{M}}\}\right)$ where $y_{t,i}$ represents the number of C2C returns that are announced i periods ago and not matched to a C2C demand by period t.

Actions.

Action $a_t \in [0, 1]$ is the discount level applied to C2C demands during $[t, t+1)$.

Exogenous information.

Exogenous information $\boldsymbol{w}_t = \left(d_t^{\mathrm{W2C}}, d_t^{\mathrm{C2C}}, \boldsymbol{r}_t^{\mathrm{C2W}}, \boldsymbol{r}_t^{\mathrm{C2C}}\right)$ known at the end of period t is composed of

(i) d_t^{W2C}, the number of newly arrived W2C demands,
(ii) d_t^{C2C}, the number of newly arrived C2C demands,
(iii) $\boldsymbol{r}_t^{\mathrm{C2W}} = \left(r_{t,i}^{\mathrm{C2W}} \mid i \in \{1,\dots,T^{\mathrm{R}}\}\right)$ where $r_{t,i}^{\mathrm{C2W}}$ is the number of newly announced C2W returns of items purchased at period $t-i$,
(iv) $\boldsymbol{r}_t^{\mathrm{C2C}} = \left(r_{t,i}^{\mathrm{C2C}} \mid i \in \{1,\dots,T^{\mathrm{R}}\}\right)$ where $r_{t,i}^{\mathrm{C2C}}$ is the number of newly announced C2C returns of items purchased at period $t-i$.

Note that d_t^{W2C} and d_t^{C2C} would depend on discount level a_t and current period t. Typically, d_t^{W2C} would decrease and d_t^{C2C} would increase with discount level a_t in period t. We model the two extremes using the following best-case and worst-case demand scenarios.

In the best-case demand scenario, W2C demand remains the same and total customer demand increases by C2C demand. We model this scenario as follows. We assume that W2C demand is Poisson with rate λ_t during $[t, t+1)$ where λ_t. C2C demand d_t^{C2C} follows a binomial distribution with parameters $\sum_{i=1}^{T^{\mathrm{M}}} y_{t,i}$ and $q(a_t)$ during $[t, t+1)$.

In the worst-case demand scenario, total customer demand remains the same, i.e., W2C demand decreases by C2C demand. Both W2C and C2C demand depend on the current discount level. We model this scenario as follows. Customers arrive according to a Poisson process with rate λ_t during $[t, t+1)$ where λ_t. A webshop customer is willing to purchase a C2C return with probability $q(a_t)$. The number of customers who are willing to purchase a C2C return $\tilde{d}_t^{\mathrm{C2C}}$ follows a binomial distribution with parameters d_t and $q(a_t)$ where d_t is the number of newly arrived customers during $[t, t+1)$. If a new customer arrives and there are no C2C returns available for purchase, then purchasing a C2C return is not an option for them. Therefore, we have $d_t^{\mathrm{C2C}} = \min\{\tilde{d}_t^{\mathrm{C2C}}, \sum_{i=1}^{T^{\mathrm{M}}} y_{t,i}\}$ and $d_t^{\mathrm{W2C}} = d_t - d_t^{\mathrm{C2C}}$.

The probability of a customer return depends on the time elapsed since delivery. We assume that newly announced C2W and C2C returns $(r_{t,i}^{\mathrm{C2W}}, r_{t,i}^{\mathrm{C2C}})$ follow a multinomial distribution with parameters $x_{t,i}$ and $(u_i^{\mathrm{C2W}}, u_i^{\mathrm{C2C}})$ during $[t, t+1)$.

Transition function.

The transition function $S(.)$ defines the transition from state $\boldsymbol{s}_t$ to $\boldsymbol{s}_{t+1} = S(\boldsymbol{s}_t, a_t, \boldsymbol{w}_t)$ after taking action a_t and facing exogenous information $\boldsymbol{w}_t$. We split the transition function into $S^1(.)$ and $S^2(.)$ where $\boldsymbol{x}_{t+1} = S^1(\boldsymbol{x}_t, a_t, d_t^{\mathrm{W2C}}, \boldsymbol{r}_t^{\mathrm{C2W}}, \boldsymbol{r}_t^{\mathrm{C2C}})$ and $\boldsymbol{y}_{t+1} = S^2(\boldsymbol{y}_t, a_t, d_t^{\mathrm{C2C}}, \boldsymbol{r}_t^{\mathrm{C2C}})$. Functions $S^1(.)$ and $S^2(.)$ generate T^{R} and T^{M} - dimensional vectors, respectively.

At the transition from period t to $t+1$, state information is advanced by 1 period. During $[t, t+1)$, the number of W2C demands is d_t^{W2C} and the number of returns is $\boldsymbol{r}_t^{\mathrm{C2W}} + \boldsymbol{r}_t^{\mathrm{C2C}}$. Transition function $S^1(.)$ can be defined as

$$\left(S^1(\boldsymbol{x}_t, a_t, d_t, d_t^{\mathrm{C2C}}, \boldsymbol{r}_t)\right)_i = \begin{cases} d_t^{\mathrm{W2C}} & \text{for } i = 1, \\ x_{t,i-1} - r_{t,i-1}^{\mathrm{C2W}} - r_{t,i-1}^{\mathrm{C2C}} & \text{for } i = 2, 3, \dots, T^{\mathrm{R}}, \end{cases}$$

where $(.)_i$ is the i^{th} element of the vector inside the brackets.

During $[t, t+1)$, the number of C2C returns is $\sum_{j=1}^{T^{\mathrm{R}}} r_{t,j}^{\mathrm{C2C}}$ and the number of C2C demands is d_t^{C2C}. At the end of period t, C2C returns are matched to C2C demands according to the first-in first-out (FIFO) rule. In other words, the matching starts from the oldest C2C returns. Returns announced T^{M} periods ago are matched first, those announced $T^{\mathrm{M}} - 1$ periods ago are matched second, etc., until the C2C demand is fully satisfied. Let $(x)^+ = \max(0, x)$. Define $d_{t,i}^{\mathrm{C2C}} = \left(d_t^{\mathrm{C2C}} - \sum_{j=i}^{T^{\mathrm{M}}} y_{t,j}\right)^+$ as the remaining number of C2C demands after matching customers who announced a C2C return i periods ago and onward. Transition function $S^2(.)$ can be defined as

$$\left(S^2(\boldsymbol{y}_t, a_t, d_t^{\mathrm{C2C}}, \boldsymbol{r}_t^{\mathrm{C2C}})\right)_i = \begin{cases} \sum_{j=1}^{T^{\mathrm{R}}} r_{t,j}^{\mathrm{C2C}} & \text{for } i = 1, \\ \left(y_{t,i-1} - d_{t,i}^{\mathrm{C2C}}\right)^+ & \text{for } i = 2, 3, \dots, T^{\mathrm{M}}. \end{cases}$$

Objective function.

As in the base model, we consider costs, revenues, hand-in and return probabilities. The expected immediate profit at the end of period t, given state $\boldsymbol{s}_t$, action a_t, and exogenous information $\boldsymbol{w}_t$ is

$$\begin{aligned} \mathbb{E}\left[R(\boldsymbol{s}_t, a_t, \boldsymbol{w}_t)\right] = {} & R^{\mathrm{W2C}} d_t^{\mathrm{W2C}} - C^{\mathrm{C2W}} \left[\sum_{i=1}^{T^{\mathrm{R}}} r_{t,i}^{\mathrm{C2W}} + \left(y_{t,T^{\mathrm{M}}} - d_t^{\mathrm{C2C}}\right)^+\right] + R^{\mathrm{C2C}}(a_t) d_t^{\mathrm{C2C}} \\ & - \left(C^{\mathrm{HI}} p^{\mathrm{HI}} + C^{\mathrm{LHI}}(1 - p^{\mathrm{HI}}) + P\right) d_t^{\mathrm{C2C}} - C^{\mathrm{C2C}}(a_t) p^{\mathrm{FR}} d_t^{\mathrm{C2C}}. \end{aligned} \tag{6}$$

Eq. (6) follows the same reasoning as (2).

Let $V_t(\boldsymbol{s}_t)$ be the maximum expected total profit in state $\boldsymbol{s}_t$ following the optimal policy from period t onward. The optimal discount levels at period $t = 0, 1, \dots, T-1$ can be obtained by

$$a_t^* = \underset{a_t \in [0,1]}{\operatorname{argmax}} \{\mathbb{E}_{\boldsymbol{w}_t}\left[\mathbb{E}\left[R(\boldsymbol{s}_t, a_t, \boldsymbol{w}_t)\right] + V_{t+1}(\boldsymbol{s}_{t+1})\right]\},$$

where $V_T(\boldsymbol{s}) = 0$ for all states $\boldsymbol{s}$.

4.2.3. Theoretical results

In this section, we provide our theoretical results on the relation between the base and multi-customer models under certain assumptions.

Assumption 3. W2C demand follows a Poisson distribution with constant rate λ.

Theorem 4. *Let $\mathbb{E}[\hat{\Pi}]$ be the long-run average profit of the conventional multi-customer system defined in Section 4.2.1. Under Assumption 3, we have*

$$\mathbb{E}[\hat{\Pi}] = \lambda \mathbb{E}[\Pi].$$

Theorem 4 implies that the base model and the multi-customer model are equivalent to each other in a stationary demand, infinite horizon setting. More specifically, when the expected profit of the base model is scaled up by demand rate λ, we obtain the long-run average profit of the multi-customer model.

Let a constant-discount-level policy with parameter $a \in [0, 1]$ be a policy with $a_t = a$ for all $t = \{1, 2, \dots, T\}$.

Theorem 5. *Let $\mathbb{E}[\hat{\Pi}(a)]$ be the long-run average profit of the C2C multi-customer system as defined in Section 4.2.2 under a constant-discount-level policy with parameter a. Assume $T^M = 1$. Under Assumption 3 and the best-case scenario, we have*

$$\mathbb{E}[\hat{\Pi}(a)] = \lambda \mathbb{E}[\Pi_B(a)].$$

Theorem 5 implies that if $T^{\mathrm{M}} = 1$, then the base model and the multi-customer model are equivalent to each other in an infinite horizon setting and the best-case scenario. We note that this result does not hold for $T^{\mathrm{M}} > 1$ due to the existence of the FIFO rule in the matching process. Furthermore, this result does not hold for the worst-case scenario, since W2C demand would not follow Poisson distribution.

5. Solution methods

The base model (BM) introduced in Section 4.1.2 simplifies the real-world problem under the C2C returns program. The multi-customer model presented in Section 4.2.2 is more comprehensive, being an MDP with unbounded state space and continuous action space. For the multi-customer model, finding an optimal policy using exact algorithms is computationally intractable. Therefore, we propose heuristic approaches to find reasonably good solutions.

First, we consider a state-independent and time-independent constant-discount-level policy with parameter a^{BM}, where a^{BM} is the optimal discount level for the base model in the best-case and worst-case demand scenarios, found using Theorems 2 and 3, respectively. Note that using Theorem 2, we can obtain the optimal constant-discount-level policy for $T^{\mathrm{M}} = 1$ in a stationary demand, infinite horizon,

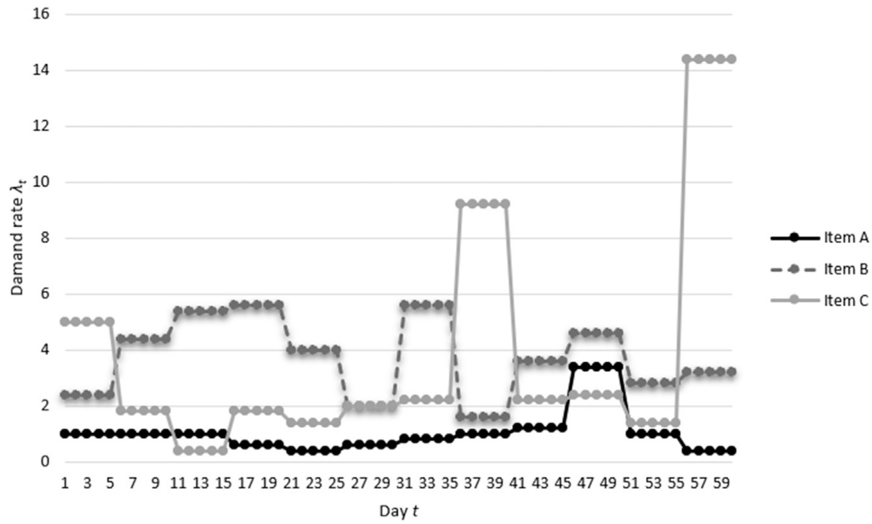

Fig. 3. Demand rate λ_t for $t = 0, 1, \ldots, T-1$.

best-case demand setting (Theorem 5). For all other cases, Theorems 2 and 3 would not lead to the optimal constant-discount-level.

Second, we determine a constant-discount-level policy using a simple simulation optimization (SO) procedure. Given resolution N, we evaluate the performance of a finite set of constant-discount-level policies with parameter $a \in \{0, 1/N, 2/N, \ldots, N-1/N\}$ by simulation, using common random variables. We determine the discount level a^{SO} that provides the greatest expected total profit, where we also make statistical comparisons.

Third, we implement the reinforcement learning (RL) method introduced by Kearns et al. [43]. The algorithm we implemented can be found in Figure 1, page 198 of [43]. This algorithm is designed for finding near-optimal solutions to MDPs with infinitely large state spaces. It is based on the idea of *sparse sampling*, leading to a non-stationary stochastic policy. Given any state s_t at period t, the algorithm uses a *simulator* of the MDP to draw samples for many state–action pairs, and uses these samples to compute a good action from s_t, which is then executed. More precisely, for state s_t at period t, a finite subset of actions A are considered and a randomly sampled look-ahead tree of depth H and sample size C is constructed. Using this look-ahead tree, we formulate a sub-MDP. The optimal action for this sub-MDP is obtained by *dynamic programming*. The complexity of the per-state computations (i.e., the number of simulated transitions for the development of the look-ahead tree) is $O\left((|A|C)^H\right)$. We note that for our problem, there is no guarantee that the sub-MDP contains enough information to compute a near-optimal action from state s_t. The number of calls to the simulator required to obtain a near-optimal solution is often extremely large [44]. In exchange for this limitation, the running time of the algorithm has no dependence on the number of states. We use the solution obtained by this algorithm as a benchmark solution to assess the performance of the constant-discount-level policies obtained by the base model and simulation optimization for real-world cases introduced in Section 6. We refer to Kearns et al. [43] for more details about this algorithm.

6. Case study

In this section, we assess the value of the C2C returns program at a fashion retailer in the Netherlands. For the C2C returns program, we illustrate the performance of the BM solution compared to those of SO and RL and we report the break-even points for the rate of problematic returns.

Table 1
Parameters of optimistic and pessimistic scenarios.

Inputs		Scenarios	
		Optimistic (O)	Pessimistic (P)
Demand		Best-case	Worst-case
Returnees' participation ratio	γ	0.75	0.25
Time window for handing-in (days)	T^{H}	7	14
Time window for matching (days)	T^{M}	3	5
Return rate for C2C purchases (€)	u_i^{FR}	μ_i^{R}	$1.5 \times \mu_i^{\mathrm{R}}$
C2C shipping cost (€)	S^{C2C}	4	6

6.1. Data

We analyze data from our partner retailer from May 2017 to May 2019, consisting of 2.6 million data points. We consider 3 items (items A, B and C) sold on the retailer's webshop during a selling season of $T = 60$ days. Customers are allowed to return items to the retailer within $T^{\mathrm{R}} = 30$ days after delivery. Historical data shows that demand is non-stationary and return probabilities depend on the time elapsed since delivery. Demand rate λ_t in day $t = 1, 2, \ldots, T$ and return probability u_i^{R} for an item purchased $i = 1, 2, \ldots, 18$ days ago are as reported in Figs. 3 and 4, respectively. Item A has low demand (expected demand $\lambda = 1.03$ units per day) and high returns ($p^{\mathrm{R}} = 0.42$), item B has high demand ($\lambda = 3.77$) and high returns ($p^{\mathrm{R}} = 0.44$), and item C has high demand ($\lambda = 3.68$) and low returns ($p^{\mathrm{R}} = 0.28$). For items A, B, and C, return probabilities u_i^{R} are negligible for $i = 18, \ldots, T^{\mathrm{R}}$. Based on commercial prices, we set shipping and handling costs as $S^{\mathrm{W2C}} = €6$ and $S^{\mathrm{C2W}} = €8$. Items A, B, and C are sold for €34.99, €29.99, and €19.99, respectively.

We consider C2C returns to be fully refunded to encourage customers to participate in the C2C returns program. Based on expert knowledge, we define optimistic and pessimistic scenarios for the C2C returns program as shown in Table 1. Solution methods SO and RL can incorporate any functional form for the relation between C2C demand and discount level a. For the sake of simplicity, we use Assumption 2 with $q_0 = 0.05$.

In an optimistic (O) scenario, W2C demand remains the same and total customer demand increases by C2C demand. This corresponds to the best-case demand described in Section 4.2.2 (exogenous information), where λ_t is as shown in Fig. 3 for $t = 0, 1, \ldots, T-1$. The time window for handing-in is set at a reasonable level, i.e., $T^{\mathrm{H}} = 7$ days. Consequently, the delivery lead time for C2C purchases is reasonable (7 days maximum), paving the way for best-case demand. In order

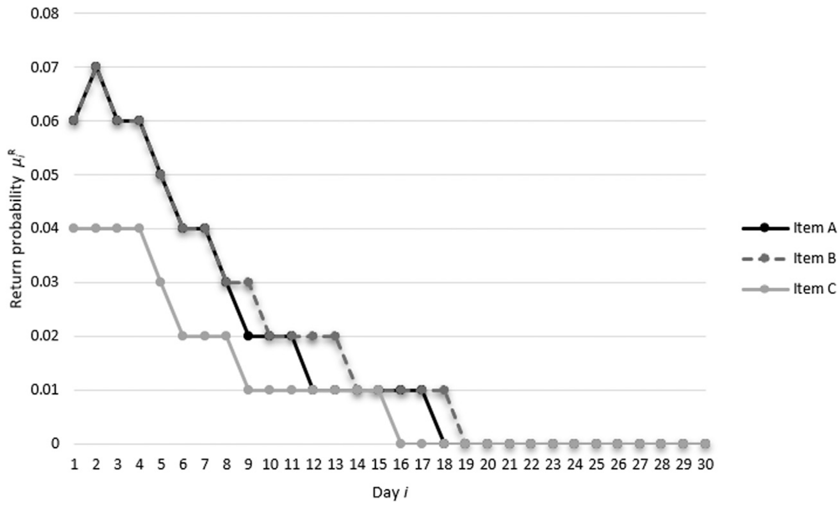

Fig. 4. Return probability u_i^{R} for $i = 1, 2, \ldots, T^{\mathrm{R}}$.

to encourage returnees to participate in the C2C returns program, we consider a short time window for matching by setting $T^{\mathrm{M}} = 3$ days. Under a short time window for matching and a reasonable time window for handing-in, the majority of returnees (75%) are assumed to make a C2C return. We set $u_i^{\mathrm{C2C}} = \gamma \times u_i^{\mathrm{R}}$ and $u_i^{\mathrm{C2W}} = (1-\gamma) \times u_i^{\mathrm{R}}$ with $\gamma = 0.75$ and u_i^{R} as shown in Fig. 4 for $i = 1, 2, \ldots, T^{\mathrm{R}}$. In an optimistic scenario, a delivery from C2C incurs $S^{\mathrm{C2C}} = €4$, which is lower than a standard W2C shipping cost. Return probabilities for W2C and C2C demands are considered the same, i.e., $u_i^{\mathrm{R}} = u_i^{\mathrm{FR}}$ for $i = 1, 2, \ldots, T^{\mathrm{R}}$.

In a pessimistic (P) scenario, total customer demand remains the same, i.e., W2C demand decreases by C2C demand. We model this situation like the worst-case demand described in Section 4.2.2 (exogenous information). In this scenario, the time window for handing-in is set to $T^{\mathrm{H}} = 14$ days, which could cause a long delivery lead time for customers. We impose a long time window for matching by setting $T^{\mathrm{M}} = 5$ days. Due to the hassle a long matching period creates for returnees, we consider that the majority of returnees (75%) choose to make C2W returns, i.e. we set $\gamma = 0.25$. In a pessimistic scenario, a delivery from C2C incurs $S^{\mathrm{C2C}} = €6$. In addition, return probabilities for C2C demand are set to be much higher than those of W2C demand, where $u_i^{\mathrm{FR}} = 1.5 \times u_i^{\mathrm{R}}$ for $i = 1, 2, \ldots, T^{\mathrm{R}}$.

6.2. Algorithm settings

Solution methods BM, SO, and RL presented in Section 5 were coded in C++ and experiments were carried out on the Lisa cluster computer, which is installed and maintained by SURFsara, the Netherlands [45]. First, we determine discount level a^{BM} using the BM for the best-case and worst-case scenarios (i.e. Theorems 2 and 3) and evaluate the performance of the corresponding constant-discount-level policy by simulation. Second, we obtain discount level a^{SO} using SO with $N = 20$. Third, we evaluate non-stationary stochastic policies with RL. For RL, we define the set of possible discount levels as $A = \{0.0, 0.15, 0.25, 0.50\}$. We restrict the per-state computational complexity $(|A|C)^H$ between 160,000 and 250,000 by taking $H = 2$, $C = 120$; $H = 3$, $C = 15$; and $H = 4$, $C = 5$. We name the corresponding algorithms H2C120, H3C15, and H4C5, respectively. We note that algorithms H2C120, H3C15, and H4C5 are in increasing order of depth and decreasing order of per-state computational complexity. In addition, we assess the performance of the system in a conventional returns program by simulation. Each evaluation uses 1000 replications and common random variables. We introduce a cool-down period of $T^{\mathrm{R}} = 30$ days, during which customer returns are allowed, but W2C and C2C demands do not occur.

6.3. Results

6.3.1. Performance of the base model

We define the relative difference in the expected total profit of different solutions compared to the BM as

$$\Delta\Pi^{\mathrm{SM}} = \frac{\Pi^{\mathrm{SM}} - \Pi^{\mathrm{BM}}}{\Pi^{\mathrm{BM}}}, \tag{7}$$

where Π^{SM} and Π^{BM} are the expected total profits obtained by solution methods SM $\in$ {SO, H2C120, H3C15, H4C5} and BM, respectively. For our numerical experiments, $\Pi^{\mathrm{BM}} > 0$ and $\Pi^{\mathrm{SM}} > 0$ for all SM $\in$ {SO, H2C120, H3C15, H4C5}. With one-tailed paired t-tests, we check whether we can reject the null hypothesis $\Pi^{\mathrm{BM}} = \Pi^{\mathrm{SM}}$ in favor of $\Pi^{\mathrm{BM}} < \Pi^{\mathrm{SM}}$ (resp. $\Pi^{\mathrm{BM}} > \Pi^{\mathrm{SM}}$) at the significance level of 5% for cases where $\Delta\Pi^{\mathrm{SM}} > 0$ (resp. $\Delta\Pi^{\mathrm{SM}} < 0$).

Table 2 shows that BM and SO provide very similar solutions. The relative difference in expected total profit $\Delta\Pi^{\mathrm{SO}}$ is negligible (0.16% at most) even when there is a reasonable difference between constant-discount-levels a^{BM} and a^{SO} ($a^{\mathrm{BM}} - a^{\mathrm{SO}} = 3.3\%$ at most for Item A - pessimistic scenario). We observe that the expected total profit is not very sensitive to the discount level.

We observe that SO outperforms the RL algorithms. The trade-off between depth H and sample size C is case-specific. The RL algorithms lead to non-stationary stochastic policies, which are more general than the constant-discount-level policies considered in BM and SO. However, the proposed approach is computationally expensive and does not guarantee near-optimality. The computational time required to evaluate the RL algorithms averages 34 h per instance. SO evaluates the finite set of discount levels within 40 s for each instance. SO works offline and provides a policy that is easy to implement and understand. The RL method is an online approach, and needs to be computed at the beginning of each day. The resulting policy could result in a different discount level for each day, which may be perceived negatively by customers.

6.3.2. Value of the C2C returns program

We measure the value of the C2C returns program in terms of expected total profit by

$$\Delta\Pi^{\mathrm{C2C}} = \frac{\Pi^{\mathrm{C2C}} - \Pi^{\mathrm{CON}}}{\Pi^{\mathrm{CON}}}, \tag{8}$$

where Π^{CON} is the expected total profit under the conventional returns program and $\Pi^{\mathrm{C2C}} = \max\{\Pi^{\mathrm{BM}}, \Pi^{\mathrm{SO}}, \Pi^{\mathrm{H2C120}}, \Pi^{\mathrm{H3C15}}, \Pi^{\mathrm{H4C5}}\}$. In our numerical experiments, Π^{CON}, we always have $\Pi^{\mathrm{C2C}} > 0$. We define the return rate as the ratio of the number of C2W returns to the total

Table 2
Performance of solution methods.

Item	Setting	a^{BM} (%)	a^{SO} (%)	$\Delta\Pi^{SO}$ (%)	$\Delta\Pi^{H2C120}$ (%)	$\Delta\Pi^{H3C15}$ (%)	$\Delta\Pi^{H4C5}$ (%)
A	O	31.9	30	0.03	−2.55	−1.23	−2.13
	P	1.7	5	0.16	0.06[a]	0.01[a]	−0.05[a]
B	O	31.2	30	0.01	−2.27	−1.18	−1.91
	P	4.6	5	−0.08	−0.26	−0.26	−0.37
C	O	33.1	35	−0.04	−1.64	−1.03	−1.51
	P	11.6	10	0.07	−0.41	−0.42	−0.40

[a] H_0 is not rejected.

Table 3
Value of the C2C returns program.

Item	Setting	Π^{CON} (€/day)	Π^{C2C} (€/day)	$\Delta\Pi^{C2C}$ (%)	ρ^{CON} (%)	ρ^{C2C} (%)	$\Delta\rho^{C2C}$ (%)
A	O	7.59	9.48	24.97	41.81	24.13	42.30
	P	7.59	7.61	0.35[a]	41.81	39.48	5.59
B	O	17.94	23.97	33.61	44.29	25.00	43.56
	P	17.94	18.13	1.08	44.29	39.54	10.72
C	O	15.23	17.89	17.43	27.77	17.12	38.34
	P	15.23	15.55	2.06	27.77	24.74	10.91

[a] H_0 is not rejected.

demand. The total demand consists of W2C demand (conventional purchase) and C2C demand (C2C purchase). The number of C2W returns consists of items delivered from W2C or from C2C. The value of the C2C returns program in terms of the expected return rate $\Delta\rho^{C2C}$ is measured by

$$\Delta\rho^{C2C} = \frac{\rho^{CON} - \rho^{C2C}}{\rho^{CON}}, \tag{9}$$

where ρ^{CON} and ρ^{C2C} are the expected return rates under the conventional and the C2C returns programs, respectively. For the C2C returns program, we consider the solution with the greatest expected total profit.

As shown in Table 3, in optimistic scenarios, the value of the C2C returns program is significant in terms of expected profit and expected return rate. We observe an increase in expected profit of up to 34% (from a daily profit of €18 to €24) and a reduction in the expected return rate of up to 44% (the ratio of C2W deliveries to total demand drops from 44% to 25%). We note that the expected profits reported in Table 3 do not include the costs of ordering, purchasing, and holding inventory since we assume they are the same for both returns programs.

In optimistic scenarios, the total demand increases by 12%–24% because C2C always generates additional demand. However, the system can be interpreted as more environmentally friendly considering the ratio of C2W deliveries to total demand. In pessimistic scenarios, the total demand remains the same. In the most pessimistic scenario (Item A), expected total profits Π^{C2C} and Π^{CON} are not significantly different. (One-tailed paired t-tests show that we cannot reject the null hypothesis $\Pi^{C2C} = \Pi^{CON}$ in favor of $\Pi^{CON} < \Pi^{C2C}$ at the significance level of 5%.) However, the relative reduction in the expected return rate is 6%. This shows that the C2C returns program can help to reduce return rates and thus provide a more environmentally friendly system, even if the increase in profit is not statistically significant.

6.3.3. *Impact of problematic returns*

One could argue that the incidence of return fraud or the likelihood of operational issues are potentially higher for items delivered through C2C transactions. In this section, we present the break-even point for problematic returns. To ensure a fair comparison between conventional and C2C returns programs, we assume that the costs associated with problematic returns for conventional C2W returns remains constant across both programs. Therefore, our analysis only focuses on issues associated with items sold through C2C.

The daily average monetary value of returns from C2C purchasers MVR can be calculated by

$$MVR = \frac{P p^{FR} \sum_{t=1}^{T} d_t^{C2C}}{T}.$$

Table 4
Break-even points of problematic returns rate.

Item	Setting	$\Pi^{C2C} - \Pi^{CON}$(€/day)	MVR (€/day)	BEP (%)
A	O	1.89	1.71	110.6
	P	0.03	0.35	7.6
B	O	6.03	6.28	96.1
	P	0.19	2.36	8.2
C	O	2.65	1.28	207.5
	P	0.31	0.61	51.9

The break-even point, denoted as BEP, indicates the threshold at which the profit generated by the C2C returns program equals the financial loss caused by problematic C2C returns. That is,

$$BEP = \frac{\Pi^{C2C} - \Pi^{CON}}{MVR}.$$

Table 4 shows the break-even points for various scenarios examined in our case study. We note that in pessimistic scenarios, characterized by factors like demand cannibalization and higher return rates from C2C purchasers, the additional profit generated by the C2C returns program can be easily nullified by the loss incurred due to problematic returns for items A and B. In these instances, the break-even points fall below the benchmark return fraud rate of 10.7% [3]. It is important to highlight that items A and B are relatively high-priced products, priced at €34.99 and €29.99, respectively.

However, in optimistic scenarios such as those concerning items A and C, the C2C returns program remains profitable even if all returns from C2C purchasers are problematic. A break-even point exceeding 100% can be interpreted as not only losing all the value in returned items due to fraud and costs associated with operational issues, but also suffering reputational damage. For item C, priced at €19.99, even in the most pessimistic scenario, the C2C returns program remains profitable as long as the rate of problematic returns is less than 51.9% in value.

Based on our findings, it can be inferred that the C2C returns program is likely to maintain profitability for lower-priced items, even in pessimistic scenarios.

7. Numerical experiments

In this section, we extend our numerical experiments on the performance of the BM and the value of the C2C returns program to a wide range of instances to claim generality. We focus on the performance of SO compared to BM due to its good performance, as reported in Section 6.

The setup of the experiments corresponds to the optimistic and pessimistic scenarios presented in Section 6. The algorithm settings are as shown in Section 6.2. We consider the same assumptions and

Table 5
Performance of SO and the value of the C2C returns program.

Inputs		a^{BM} (avr. %)	a^{SO} (avr. %)	$\Delta\Pi^{SO}$ (avr. %)[a]	H_0 not rejected /total instances	$\Delta\Pi^{C2C}$ (avr. %)	$\Delta\rho^{C2C}$ (avr. %)
Demand assumption	Best-case	23.61	23.46	0.07	106/192	58.93	24.92
	Worst-case	18.26	18.15	0.66	187/192	36.65	21.51
γ	0.75	20.60	21.69	0.37	140/192	70.16	33.43
	0.25	21.27	19.92	0.52	153/192	25.42	13.00
T	60	20.94	21.12	0.53	153/192	43.61	21.23
	180	20.94	20.49	0.35	140/192	51.97	25.20
T^{H}	14	23.11	22.89	0.54	141/192	55.42	24.28
	7	18.77	18.72	0.36	152/192	40.16	22.15
T^{M}	5	18.78	18.83	0.53	154/192	52.88	24.53
	3	23.09	22.79	0.35	139/192	42.70	21.90
u_i^{FR}	u_i^{R}	23.98	23.39	0.51	164/192	65.38	26.52
	$1.5 \times u_i^{R}$	17.90	18.23	0.36	129/192	30.20	19.91
S^{C2C}	4	23.48	23.18	0.52	152/192	56.72	24.41
	6	18.40	18.44	0.37	141/192	38.86	22.02
λ	5	20.94	19.10	0.65	99/128	51.37	25.13
	3	20.94	20.23	0.28	97/128	48.89	23.92
	1	20.94	23.09	0.41	97/128	43.11	20.60

[a] For the instances where H_0 is not rejected.

performance measures (see (7), (8), (9)). We set the selling price $P =$ €20, consider period-independent return probabilities $u_i^{R} = 0.02$ for $i = 1, 2, \ldots, T^{R}$ with $T^{R} = 30$ days ($p^{R} = 0.45$) and constant demand rates λ. Input parameters that are varied are as shown in Table 5. Performing a full factorial analysis with these input parameters, we obtain 384 instances.

Table 5 reports average (avr.) values across all instances setting input parameters given in the corresponding rows. Using one-tailed paired t-tests, we check whether the null hypothesis H_0: $\Pi^{BM} = \Pi^{SO}$ can be rejected in favor of $\Pi^{BM} > \Pi^{SO}$ at the significance level of 5%. The 5th column in Table 5 reports $\Delta\Pi^{SO}$ for the instances where H_0 cannot be rejected. The 6th column in Table 5 reports the number of instances where the null hypothesis cannot be rejected. We note that for the instances where H_0 can be rejected, the average $\Delta\Pi^{SO}$ is −0.12%.

In our numerical experiments, SO and BM provide similar discount levels and expected total profits. For the instances where SO outperforms BM, the relative difference in expected total profit is on average 0.45% with a maximum of 3.11%. Discount level a^{SO} is very similar to a^{BM} on average (20.81% vs. 20.94%) but the difference between the minimum and the maximum values of a^{SO} is higher than that of a^{BM} (5.00%–40.00% vs. 7.92%–33.51%). Both solutions often behave similarly. The proposed discount level increases with the hand-in time window T^{H} and decreases with the C2C shipping cost S^{C2C} and final return probabilities u_i^{FR}. If operational costs for the C2C returns program become lower (higher), higher (lower) discounts can be offered. The proposed discount levels decrease with the time window for matching T^{M}. This is because the likelihood of observing C2C sales becomes higher for the longer time window for matching. We note that a^{BM} is not affected by the changes in horizon length T or demand rate λ, by definition.

We observe that expected total profits Π^{C2C} and Π^{CON} are significantly different for all instances. As shown in columns 7–8 of Table 5, the C2C returns program can be highly valuable both in terms of expected profit and expected return rate. The C2C returns program is more valuable when the selling season is long, C2C customers can tolerate long waiting times, the final return probability is not higher than the initial return probability, or the demand rates are high. In the most pessimistic scenario, where the increase in profit is 3.81%, the decrease in the expected return rate is 6.1%.

8. Conclusion

Online returns are a major problem for retailers around the world. Handling these returns is costly, puts profit under pressure, and contributes to CO_2 emissions. In this paper, we investigate the C2C returns program, where returns bypass the retailer's warehouse and are delivered straight to the next customer. In the C2C returns program, when customers return an item, they are asked to keep it for a few days. During those days, the item is promoted on the retailer's website at a discount and the CO_2 emissions saved are highlighted. When the item is sold, the returnee receives a notification to ship the package. Payments and refunds are handled by the online retailer or an external operator. A quick response (QR) label links the returnee to the new customer. The new customer inspects the item upon receipt, scans the product's QR label on the package, and writes a review of the item. The review is added to the returnee's profile, where it contributes to their reputation. One of the co-authors of this article is following real-world implementations of this concept. For further information, see https://itgoesforward.com/.

Our paper presents the mathematical models behind the C2C concept. The goal is to determine optimal discount levels to offer so that the retailer's expected profit is maximized. First, we propose a customer-based model and show how to determine a constant-discount-level policy. Second, we formulate the real-world problem as an MDP. Due to the curse of dimensionality, determining the optimal policy is computationally intractable. We use simulation optimization and reinforcement learning algorithms to find reasonably good solutions. We analyze historical real-world demand and returns data from a fashion retailer and assess the performance of different solution methods and the value of the C2C returns program in different scenarios.

Our numerical experiments show that the base model performs well compared to simulation optimization and reinforcement learning algorithms. In general, the base model outperforms reinforcement learning. The base model and simulation optimization provide similar solutions. We observe that the retailer's expected profit is not very sensitive to the discount level. Our extensive numerical experiments show that the relative difference in expected profit between the base model and simulation optimization is on average 0.45%, with a maximum of 3.11%.

Both our case study and numerical experiments report significant benefits from the C2C returns program. In the most optimistic scenario in our case study, we observe a 34% increase in expected profit and a 44% reduction in expected return rate. In pessimistic scenarios, where the increase in expected profit is not significant, the relative reduction in the expected return rate can be as high as 6%. Thus, the C2C returns program can make the system more environmentally friendly even when it is not highly cost-effective. Our analysis of the impact of problematic returns shows that the C2C returns program is likely to maintain profitability for lower-priced items even in pessimistic scenarios.

This research has shown promising initial results for the C2C concept. Future research can extend our work to consolidate the demand/return of multiple items, examine customer response to discount levels, and incorporate inventory control and item availability in stock. Another research opportunity could be to revisit the C2C concept. For example, C2C sales requests can be collected in advance and fulfilled when the corresponding item is available for a C2C delivery. This would eliminate the matching period and could prevent the inconvenience of returnees holding the item until it is sold to the next customer. Finally, future research can investigate the overall environmental impact of the C2C returns program under demand expansion. In this regard, a product life-cycle approach that considers the environmental impact in all phases including production, consumer use, collection, resale, and disposal would be suitable [see approaches proposed in, e.g., 46,47].

CRediT authorship contribution statement

Ayse Sena Eruguz: Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Methodology, Conceptualization. **Oktay Karabağ:** Visualization, Validation, Software, Methodology, Conceptualization. **Eline Tetteroo:** Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Carl van Heijst:** Visualization, Conceptualization. **Wilco van den Heuvel:** Supervision, Methodology, Conceptualization. **Rommert Dekker:** Supervision, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All required data is given in the paper.

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Appendix. Notations and proofs

A.1. Notations

See Table A.1 for notations.

A.2. Proofs

Proof of Theorem 1.

Under Assumption 1, substituting $p^{\mathrm{C2W}} = p^{\mathrm{R}} - p^{\mathrm{C2C}}$ into (1) and writing the difference between (1) and (2), we obtain:

$$E\left[\Pi_{\mathrm{B}}(a)\right] - E\left[\Pi\right] = p^{\mathrm{C2C}} p^{\mathrm{M}}(a)\left(P\,(1-a)\left(1-p^{\mathrm{FR}}\right) - \phi_B\right). \tag{A.1}$$

Note that $p^{\mathrm{C2C}} p^{\mathrm{M}}(a)$ is non-negative for all $a \in [0,1]$. Hence it follows that $E\left[\Pi_{\mathrm{B}}(a)\right] \geq E\left[\Pi\right]$ if and only if

$$P\,(1-a)\left(1-p^{\mathrm{FR}}\right) \geq \phi_B.$$

Similarly, by taking the difference of $E\left[\Pi_{\mathrm{W}}(a)\right]$ and $E\left[\Pi\right]$, we obtain:

$$\begin{aligned} E\left[\Pi_{\mathrm{W}}(a)\right] - E\left[\Pi\right] &= p^{\mathrm{C2C}} p^{\mathrm{M}}(a)\left(P\,(1-a)\left(1-p^{\mathrm{FR}}\right) - \phi_B - E\left[\Pi\right]\right) \\ &= p^{\mathrm{C2C}} p^{\mathrm{M}}(a)\left(P\,(1-a)\left(1-p^{\mathrm{FR}}\right) - \phi_W\right). \end{aligned} \tag{A.2}$$

Again, note that $p^{\mathrm{C2C}} p^{\mathrm{M}}(a)$ is non-negative for all $a \in [0,1]$, and hence we find that $E\left[\Pi_{\mathrm{W}}(a)\right] \geq E\left[\Pi\right]$ if and only if

$$P\,(1-a)\left(1-p^{\mathrm{FR}}\right) \geq \phi_W. \quad \square$$

The parameter

$$\beta = \frac{\left(T^{\mathrm{M}}-1\right)\phi_B}{\left(T^{\mathrm{M}}+1\right)\left(1-p^{\mathrm{FR}}\right)P}. \tag{A.3}$$

plays an important role in characterizing the shape (convex or concave) of the profit functions and hence in optimizing the discount level. The next lemma will be used in the proofs of Theorem A.1 and Theorem 2.

Lemma A.1. *Under Assumption 2 and the best-case scenario, the expected profit function of the C2C returns program $\mathbb{E}\left[\Pi_B(a)\right]$ can be characterized as follows:*

i. If $\beta \leq 0$, $\mathbb{E}\left[\Pi_B(a)\right]$ is concave on $a \in [0,1]$,
ii. If $0 < \beta < 1$, $\mathbb{E}\left[\Pi_B(a)\right]$ is concave on $a \in [0, 1-\beta]$ and convex on $a \in [1-\beta, 1]$,
iii. If $\beta \geq 1$, $\mathbb{E}\left[\Pi_B(a)\right]$ is convex on $a \in [0,1]$.

Let $T^M \in \mathbb{Z}^+ \setminus \{1\}$. Under Assumption 2 and the worst-case scenario, the expected profit function of the C2C returns program $\mathbb{E}\left[\Pi_W(a)\right]$ can be characterized as follows:

i. If $\phi_W \leq 0$, $\mathbb{E}\left[\Pi_W(a)\right]$ is concave on $a \in [0,1]$,
ii. if $\phi_W \geq 3P\left(1-p^{FR}\right)$, $\mathbb{E}\left[\Pi_W(a)\right]$ is convex on $a \in [0,1]$,
iii. if $0 < \phi_W < 3P\left(1-p^{FR}\right)$, the second-order condition is inconclusive to identify whether $\mathbb{E}\left[\Pi_W(a)\right]$ is convex or concave on $a \in [0,1]$.

Proof of Lemma A.1.

Under Assumption 2 the second order derivative of $E\left[\Pi_{\mathrm{B}}(a)\right]$ equals

$$\begin{aligned} \frac{d^2 E\left[\Pi_{\mathrm{B}}(a)\right]}{d a^2} &= p^{\mathrm{C2C}}\,(1-a)^{T^{\mathrm{M}}-2}\left(1-q_0\right)^{T^{\mathrm{M}}} T^{\mathrm{M}} \\ &\times \left(\left(T^{\mathrm{M}}-1\right)\phi_B - (1-a)\left(T^{\mathrm{M}}+1\right)\left(1-p^{\mathrm{FR}}\right)P\right), \end{aligned}$$

where $\phi_B = C^{\mathrm{HI}} p^{\mathrm{HI}} + C^{\mathrm{LHI}}\left(1-p^{\mathrm{HI}}\right) - (1-p^{\mathrm{FR}})S^{C2W}$.

Since $p^{\mathrm{C2C}}\left(1-q_0\right)^{T^{\mathrm{M}}} T^{\mathrm{M}}$ is a positive constant, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is concave (resp. convex) on an interval of $a \in [0,1]$ if and only if

$$(1-a)^{T^{\mathrm{M}}-2}\left(\left(T^{\mathrm{M}}-1\right)\phi_B - (1-a)\left(T^{\mathrm{M}}+1\right)\left(1-p^{\mathrm{FR}}\right)P\right) \tag{A.4}$$

is non-positive (resp. non-negative) on this interval. Since $0 \leq p^{\mathrm{FR}} < 1$, the sign of (A.4) is the same as the sign of

$$(1-a)^{T^{\mathrm{M}}-2}\left[\beta - (1-a)\right], \tag{A.5}$$

where

$$\beta = \frac{\left(T^{\mathrm{M}}-1\right)\phi_B}{\left(T^{\mathrm{M}}+1\right)\left(1-p^{\mathrm{FR}}\right)P}.$$

As $(1-a)^{T^{\mathrm{M}}-2} \geq 0$ for $a \in [0,1]$, the sign of (A.5) is defined by the sign of $[\beta - (1-a)]$. So, we consider the following cases to characterize the sign of the corresponding expression:

i. If $\beta \leq 0$, then (A.5) is non-positive. Thus, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is concave on $a \in [0,1]$,
ii. If $0 < \beta < 1$, then (A.5) non-positive for $a \in [0, 1-\beta]$ and (A.5) is non-negative for $a \in [1-\beta, 1]$. Thus, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is concave on $a \in [0, 1-\beta]$ and convex on $a \in [1-\beta, 1]$,

Table A.1
Notations.

Time-related parameters	
T	Length of selling season
T^{H}	Time window for handing-in
T^{M}	Time window for matching
T^{R}	Time window for returns
Indices	
i	Number of periods elapsed after delivery with $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$
t	Time period with $t \in \{0, 1, \dots, T-1\}$
Monetary parameters	
Π	Retailer profit from a single customer under conventional returns program
$\hat{\Pi}$	Retailer profit in a multi-customer system under conventional returns program
C^{HI}	Cost of on-time hand-in
C^{LHI}	Cost of late hand-in
C^{C2W}	Cost of a C2W return
P	Price of an item
R^{W2C}	Revenue generated by a W2C delivery
S^{C2C}	C2C shipping cost
S^{C2W}	C2W shipping and handling cost
S^{W2C}	W2C shipping and handling cost
Probability-related parameters	
p^{C2C}	Probability of a C2C return within time window for return T^{R}
p^{C2W}	Probability of a C2W return within time window for return T^{R}
p^{FR}	Probability of return of an item sold via C2C within time window for return T^{R}
p^{HI}	Probability of handing-in within T^{H} periods after matching
p^{R}	Probability of a return within time window for return T^{R}
q_0	Probability of selling the C2C return item at its original price P
u_i^{HI}	Probability of handing-in i periods after matching with $i \in \{1, 2, \dots, T^{\mathrm{H}}\}$
u_i^{R}	Probability of return i periods after delivery with $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$
u_i^{C2C}	Probability of C2C return i periods after delivery with $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$
u_i^{C2W}	Probability of C2W return i periods after delivery with $i \in \{1, 2, \dots, T^{\mathrm{R}}\}$
u_i^{FR}	Probability of return of an item sold via C2C i periods after delivery with $t \in \{1, 2, \dots, T^{\mathrm{R}}\}$
λ_t	Demand rate during $[t, t+1)$ for $t \in \{0, 1, \dots, T-1\}$
Decision variables	
a	Constant discount level on the selling price where $a \in [0, 1]$
a_t	Discount level applied to C2C demands during $[t, t+1)$
Dependent variables	
$\hat{\Pi}(a)$	Retailer's profit in a C2C multi-customer system at discount level a
$\Pi_{\mathrm{B}}(a)$	Retailer's best-case profit at discount level a
$\Pi_{\mathrm{W}}(a)$	Retailer's worst-case profit at discount level a
$C^{\mathrm{C2C}}(a)$	Cost of a C2C customer return purchased at discount level a
$p^{\mathrm{M}}(a)$	Probability that a C2C return item is sold within the time window for matching T^{M} at discount level a
$q(a)$	Probability of selling a C2C return item in a given period at discount level a
$R(\boldsymbol{s}_t, a_t, \boldsymbol{w}_t)$	Immediate profit at the end of time t, given state $\boldsymbol{s}_t$, action a_t, and exogenous information $\boldsymbol{w}_t$
$R^{\mathrm{C2C}}(a)$	Revenue generated when a C2C return item is sold at discount level a
Operators	
$\mathbb{E}[.]$	Expectation
$S(.), S^1(.), S^2(.)$	Transition functions
$V_t(.)$	Value function
State information	
$\boldsymbol{s}_t = (\boldsymbol{x}_t, \boldsymbol{y}_t)$	State information
$\boldsymbol{x}_t = (x_{t,i} \mid i \in \{1, \dots, T^{\mathrm{R}}\})$	T^{R} - dimensional vector of integers, where $x_{t,i}$ represents the number of customers satisfied i periods ago and not returned by time t
$\boldsymbol{y}_t = (y_{t,i} \mid i \in \{1, \dots, T^{\mathrm{M}}\})$	T^{M} - dimensional vector of integers, where $y_{t,i}$ represents the number of C2C returns that are announced i periods ago and not matched to a C2C demand by time t.
Exogenous information	
d_t^{C2C}	Number of newly arrived C2C demands
d_t^{W2C}	Number of newly arrived W2C demands
$\boldsymbol{r}_t^{\mathrm{C2C}} = \left(r_{t,i}^{\mathrm{C2C}} \mid i \in \{1, \dots, T^{\mathrm{R}}\}\right)$	T^{R} - dimensional vector of integers, where $r_{t,i}^{\mathrm{C2C}}$ is the number of newly announced C2C returns of items purchased at time $t-i$
$\boldsymbol{r}_t^{\mathrm{C2W}} = \left(r_{t,i}^{\mathrm{C2W}} \mid i \in \{1, \dots, T^{\mathrm{R}}\}\right)$	T^{R} - dimensional vector of integers, where $r_{t,i}^{\mathrm{C2W}}$ is the number of newly announced C2W returns of items purchased at time $t-i$
$\boldsymbol{w}_t = (d_t^{\mathrm{W2C}}, d_t^{\mathrm{C2C}}, \boldsymbol{r}_t^{\mathrm{C2W}}, \boldsymbol{r}_t^{\mathrm{C2C}})$	Exogenous information

iii. If $\beta \geq 1$, then (A.5) is non-negative. Thus, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is convex on $a \in [0,1]$.

Under Assumption 2, the second order derivative of $E\left[\Pi_{\mathrm{W}}(a)\right]$ is

$$\frac{d^2 E\left[\Pi_{\mathrm{W}}(a)\right]}{da^2} = \frac{p^{\mathrm{C2C}}(1-q_0)^{T^{\mathrm{M}}}(1-a)^{T^{\mathrm{M}}-2}}{\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)^3}$$
$$\Big[\left(\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)-\left(1+p^{\mathrm{C2C}}\left(2-p^{\mathrm{M}}(a)\right)\right)T^{\mathrm{M}}\right)\left(P(1-a)\left(1-p^{\mathrm{FR}}\right)-\phi_W\right)$$
$$-\ 2\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)P(1-a)\left(1-p^{\mathrm{FR}}\right)\Big].$$

The fraction given in the above expression is non-negative on $a \in [0,1]$. Accordingly, $E\left[\Pi_{\mathrm{W}}(a)\right]$ is concave (resp. convex) on $[0,1]$ if and only if

$$\left(\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)-\left(1+p^{\mathrm{C2C}}\left(2-p^{\mathrm{M}}(a)\right)\right)T^{\mathrm{M}}\right)$$
$$\times\ \left(P(1-a)\left(1-p^{\mathrm{FR}}\right)-\phi_W\right)-2\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)P(1-a)\left(1-p^{\mathrm{FR}}\right) \tag{A.6}$$

is non-positive (resp. non-negative) for any given $a \in [0,1]$. For any given a where $a \in [0,1)$, the sign of (A.6) is the same as the sign of

$$\left(1-\frac{\left(1+p^{\mathrm{C2C}}\left(2-p^{\mathrm{M}}(a)\right)\right)T^{\mathrm{M}}}{\left(1+p^{\mathrm{C2C}}\right)p^{\mathrm{M}}(a)}\right)\left(\frac{P(1-a)\left(1-p^{\mathrm{FR}}\right)-\phi_W}{2P(1-a)\left(1-p^{\mathrm{FR}}\right)}\right)-1. \tag{A.7}$$

Under the assumption that $T^{\mathrm{M}} \in \mathbb{Z}^+ \setminus \{1\}$, the first part of (A.7) is at most -1. Correspondingly, if

$$\left(\frac{P(1-a)\left(1-p^{\mathrm{FR}}\right)-\phi_W}{2P(1-a)\left(1-p^{\mathrm{FR}}\right)}\right) \tag{A.8}$$

is greater (smaller) than 0 (-1), the corresponding profit function is concave (convex).

Considering the condition given in (A.8) for $a \in [0,1)$, we can characterize the corresponding profit function as follows:

i. If $\phi_W \leq 0$, then the expression given in (A.8) has a positive sign and so (A.7) is non-positive. Thus, we can conclude that $E\left[\Pi_{\mathrm{B}}(a)\right]$ is concave on $a \in [0,1]$.
ii. If $\phi_W \geq 3P(1-p^{FR})$, then the expression given in (A.8) is at most -1. Thus, we can conclude that (A.7) is non-negative and hence $E\left[\Pi_{\mathrm{B}}(a)\right]$ is convex on $a \in [0,1]$.
iii. If $0 < \phi_W < 3P(1-p^{FR})$, then the expression given in (A.8) is not sufficient to analytically determine the sign of the second derivative, and hence to identify whether the corresponding function is concave or convex. □

Theorem A.1. *Under Assumption 2 and the best-case scenario, the optimal discount level a^*_B that maximizes the expected profit function of the C2C returns program $\mathbb{E}\left[\Pi_B(a)\right]$ is obtained as follows:*

i. If $\beta < 1$ and there exists an $\hat{a} \in [0,1]$ satisfying

$$T^M\phi_B(1-\hat{a})^{T^M-1}-\left(T^M+1\right)\left(1-p^{FR}\right)P(1-\hat{a})^{T^M}+\frac{\left(1-p^{FR}\right)P}{\left(1-q_0\right)^{T^M}}=0, \tag{A.9}$$

then the optimal discount level is $a^*_B = \hat{a}$,

ii. Otherwise, the optimal discount level is $a^*_B = 0$.

Let $T^M \in \mathbb{Z}^+ \setminus \{1\}$. Under Assumption 2 and the worst-case scenario, the optimal discount level a^*_W that maximizes the expected profit function of the C2C returns program $\mathbb{E}\left[\Pi^W(a)\right]$ is obtained as follows:

i. If $\phi_W \leq 0$ and there exists an $\tilde{a} \in [0,1]$ satisfying

$$\frac{\left(1-p^M(\tilde{a})\right)T^M\left(\phi_W+P\left(1-\tilde{a}\right)\left(1-p^{FR}\right)\right)}{\left(1+p^{C2C}p^M(\tilde{a})\right)^2}-\frac{P\left(1-p^{FR}\right)p^M(\tilde{a})\left(1-\tilde{a}\right)}{1+p^{C2C}p^M(\tilde{a})}=0, \tag{A.10}$$

then the optimal discount level is $a^*_W = \tilde{a}$.

ii. If $\phi_W \leq 0$ and $\tilde{a} \in [0,1]$ satisfying (A.10) does not exist, or if $\phi_W \geq 3P\left(1-p^{FR}\right)$, then the optimal discount level is $a^*_W = 0$.

iii. If $0 < \phi_W < 3P\left(1-p^{FR}\right)$, then the first-order and second-order conditions are not sufficient for characterizing the optimal discount level a^*_W analytically.

Proof of Theorem A.1. Under Assumption 2, we can derive the optimal discount level a^*_{B} using the cases introduced for $E\left[\Pi_{\mathrm{B}}(a)\right]$ in Lemma A.1.

i. Let $\beta \leq 0$. In this case, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is concave on $a \in [0,1]$ from Lemma A.1. The first derivative of the expected profit function $E\left[\Pi_{\mathrm{B}}(a)\right]$ is:

$$\frac{dE\left[\Pi_{\mathrm{B}}(a)\right]}{da} = -p^{\mathrm{C2C}}\left((1-a)^{T^{\mathrm{M}}-1}\left(1-q_0\right)^{T^{\mathrm{M}}}\right.$$
$$\left.\times\ \left[T^{\mathrm{M}}\phi_B-\left(T^{\mathrm{M}}+1\right)\left(1-p^{\mathrm{FR}}\right)P(1-a)\right]+\left(1-p^{\mathrm{FR}}\right)P\right). \tag{A.11}$$

Note that p^{C2C} is a non-negative constant and $0 \leq q_0 < 1$. From the first-order condition, if there exists $\hat{a} \in [0,1]$ satisfying,

$$T^{\mathrm{M}}\phi_B(1-\hat{a})^{T^{\mathrm{M}}-1}-\left(T^{\mathrm{M}}+1\right)\left(1-p^{\mathrm{FR}}\right)P(1-\hat{a})^{T^{\mathrm{M}}}+\frac{\left(1-p^{\mathrm{FR}}\right)P}{\left(1-q_0\right)^{T^{\mathrm{M}}}}=0, \tag{A.12}$$

then $a^*_{\mathrm{B}} = \hat{a}$. Note that (A.11) is non-positive for $a = 1$. Therefore, if $\hat{a} \in [0,1]$ satisfying (A.12) does not exist, $E[\Pi(a)]$ is a decreasing concave function on $a \in [0,1]$. Thus, $a^*_{\mathrm{B}} = 0$.

ii. Let $0 < \beta < 1$. In this case, $\mathbb{E}\left[\Pi_{\mathrm{B}}(a)\right]$ is concave on $a \in [0, 1-\beta]$ and convex on $a \in [1-\beta, 1]$. To determine a^*_{B}, we should consider the first-order condition on the concave part $a \in [0,1-\beta]$ and the boundaries of $1-\beta$ and 1 on the convex part $a \in [1-\beta, 1]$. If there exist $\hat{a} \in [0, 1-\beta]$ satisfying (A.12), then

$$a^*_{\mathrm{B}} = \arg\max\left\{E\left[\Pi_{\mathrm{B}}(\hat{a})\right], E\left[\Pi_{\mathrm{B}}(1)\right]\right\}.$$

For $a \in [0,1]$, $\beta > 0$ implies $\phi_B > 0$. So, we obtain

$$E\left[\Pi_{\mathrm{B}}(a)\right]-E\left[\Pi(1)\right] = p^{\mathrm{M}}(a)\left(1-p^{\mathrm{FR}}\right)P(1-a)+\phi_B\left(1-p^{\mathrm{M}}(a)\right) \geq 0, \tag{A.13}$$

which implies that at $a = 1$ the minimal profit is obtained. Thus, we can conclude that $a^*_{\mathrm{B}} = \hat{a}$.

One can verify that (A.11) is non-positive for $a = 1-\beta$. Therefore, if $\hat{a} \in [0, 1-\beta]$ satisfying (A.12) does not exist, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is a decreasing concave function on $a \in [0, 1-\beta]$. From (A.13), $a = 1$ cannot be optimal. Thus, we can conclude that $a^*_{\mathrm{B}} = 0$.

iii. Let $\beta \geq 1$. In this case, $E\left[\Pi_{\mathrm{B}}(a)\right]$ is convex on $a \in [0,1]$ and we should consider the boundaries 0 and 1. From (A.13), $a^*_{\mathrm{B}} = 0$. Finally, by merging cases (i)–(iii) of this proof, Theorem A.1(i)–(ii) follows.

Similarly, using the cases introduced for $E\left[\Pi_{\mathrm{W}}(a)\right]$ in Lemma A.1, we can characterize the optimal discount level a^*_{W} as follows:

i. Let $\phi_W \leq 0$. In this case, $E\left[\Pi_{\mathrm{W}}(a)\right]$ is concave on $a \in [0,1]$ from Lemma A.1. The first derivative of the corresponding profit function is:

$$\frac{dE\left[\Pi_{\mathrm{W}}(a)\right]}{da}$$

$$= p^{\mathrm{C2C}}\left(\frac{\left(1-p^{\mathrm{M}}(a)\right)T^{\mathrm{M}}\left(P(1-a)(1-p^{\mathrm{FR}})-\phi_W\right)}{(1-a)\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)^2}-\frac{P\left(1-p^{\mathrm{FR}}\right)p^{\mathrm{M}}(a)}{\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)\right)}\right). \tag{A.14}$$

Note that p^{C2C} is a positive constant. From the first-order condition, if there exists $\tilde{a}\in[0,1]$ satisfying,

$$\frac{\left(1-p^{\mathrm{M}}(\tilde{a})\right)T^{\mathrm{M}}\left(P(1-\tilde{a})(1-p^{\mathrm{FR}})-\phi_W\right)}{(1-\tilde{a})\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(\tilde{a})\right)^2}=\frac{P\left(1-p^{\mathrm{FR}}\right)p^{\mathrm{M}}(\tilde{a})}{\left(1+p^{\mathrm{C2C}}p^{\mathrm{M}}(\tilde{a})\right)}. \tag{A.15}$$

then $a^*_{\mathrm{W}}=\tilde{a}$.

By using the fact that $p^{\mathrm{M}}(a)=1-(1-q(a))^{T^{\mathrm{M}}}$ and $q(a)=q_0+a\left(1-q_0\right)$ (according to Assumption 2), it can be verified that (A.14) is non-positive for $a=1$. Therefore, if $\tilde{a}\in[0,1]$ satisfying (A.15) does not exist, $E\left[\Pi_{\mathrm{W}}(a)\right]$ is a decreasing concave function on $a\in[0,1]$. Thus, $a^*_{\mathrm{W}}=0$.

ii. Let $\phi_W\geq 3P(1-p^{FR})$. In this case, $E\left[\Pi_{\mathrm{W}}(a)\right]$ is convex on $a\in[0,1]$ from Lemma A.1. We should consider the boundaries of 0 and 1 to find the optimal discount level. Correspondingly, we have:

$$a^*_{\mathrm{W}}=\arg\max\left\{E\left[\Pi_{\mathrm{W}}(0)\right],E\left[\Pi_{\mathrm{W}}(1)\right]\right\}. \tag{A.16}$$

By taking the difference between $E\left[\Pi_{\mathrm{W}}(0)\right]$ and $E\left[\Pi_{\mathrm{W}}(1)\right]$, it is possible to determine the conditions that identify which of these two discount levels would be the solution to the optimization problem given in (A.16). It follows that:

$$E\left[\Pi_{\mathrm{W}}(0)\right]-E\left[\Pi_{\mathrm{W}}(1)\right] = \frac{-p^{\mathrm{C2C}}P(1+p^{\mathrm{C2C}})(1-p^{\mathrm{FR}})+p^{\mathrm{C2C}}(1-q_0)^{T^{\mathrm{M}}}(P(1-p^{\mathrm{FR}})(1+p^{\mathrm{C2C}})-\phi_W)}{-(1+p^{\mathrm{C2C}})(1+p^{\mathrm{C2C}}(1-(1-q_0)^{T^{\mathrm{M}}}))}. \tag{A.17}$$

As $\phi_W\geq 3P(1-p^{FR})$, the expression given in (A.17) has a positive sign. This implies that $a=0$ is more profitable than $a=1$, and we can conclude that $a^*_{\mathrm{W}}=0$.

iii. Finally, let $0<\phi_W<3P(1-p^{FR})$. In this case, we are unable to determine the shape of the function analytically, and hence the first-order and second-order conditions are not sufficient for analytically characterizing the optimal discount level a^*_{W}. □

It follows that in the best-case scenario, one must solve the polynomial (A.9). If the degree of the polynomial is at most 4, i.e., if the matching time window $T^{\mathrm{M}}\leq 4$, a closed-form solution exists for finding $\hat{a}$ (see e.g. [48–50], and the references in there). For higher degrees, the optimal discount level can still be found numerically, since the function turns out to be unimodal (see Lemma A.1).

To interpret case ii. in the best-case scenario, note that $\beta>1$ implies $\phi_B>P(1-p^{\mathrm{FR}})$ (by using (A.3)), meaning that the C2C returns program is always worse then the conventional returns program (see Theorem 1). Hence, we want to have as few C2C customers as possible, which is achieved by setting $a=0$. Unfortunately, for the worst-case scenario, the interpretation of the ranges is less clear and there is a range of ϕ_W for which we cannot determine an analytical expression for the optimal discount level.

Proof of Theorem 2. As follows from the proof of Theorem A.1, the function $E\left[\Pi_{\mathrm{B}}(a)\right]$ is decreasing at $a=1$. Furthermore, we know from Lemma A.1 that (in the most general case) the first part of the function $E[\Pi_{\mathrm{B}}(a)]$ is concave and the second part is convex. Combining these observations gives the result. □

Conjecture A.1. *The function $E[\Pi_W(a)]$ is unimodal on the interval $[0,1]$.*

There are several reasons for this conjecture. First, note that $\phi_W\leq P(1-a)(1-p^{\mathrm{FR}})$ implies that the term in (A.8) is non-negative, which in turn implies that (A.7) is non-positive. Secondly, for $\phi_W\geq 3P(1-a)(1-p^{\mathrm{FR}})$ the term in (A.8) is at most -1, which in turn implies that (A.7) is non-negative. This implies that for any value of ϕ_W the function $E[\Pi_{\mathrm{W}}(a)]$ starts as concave or ends as convex (or both) on the interval $[0,1]$. Moreover, from the proof of Theorem A.1, the function $E\left[\Pi_{\mathrm{W}}(a)\right]$ is decreasing at $a=1$. Hence, if the function only switches once from concave to convex, then $E[\Pi_{\mathrm{W}}(a)]$ is unimodal. Finally, in all the parameter settings of our experiments, the best solution was always obtained by the golden section search, which also suggests the unimodality of the function.

Proof of Theorem 3. Recall that $\mathbb{E}\left[\Pi_{\mathrm{W}}(a)\right]=\frac{\mathbb{E}[\Pi_{\mathrm{B}}(a)]}{1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)}$, for which we know from the first part that $E[\Pi_{\mathrm{B}}(a)]$ is unimodal on $[0,1]$. That is, $E[\Pi_{\mathrm{B}}(a)]$ is increasing on $[0,a^*_B]$ and decreasing on $[a^*_B,1]$. Now let us focus on $p^{\mathrm{M}}(a)$ which can be written as

$$p^{\mathrm{M}}(a)=1-(1-q_0)^{T^M}(1-a)^{T^M}.$$

By analyzing the first derivative, it turns out that $p^{\mathrm{M}}(a)$ is a positive and increasing function on $[0,1]$, as well as $1+p^{\mathrm{C2C}}p^{\mathrm{M}}(a)$. Since we divide $\Pi_{\mathrm{B}}(a)$ by a positive and increasing function, $\Pi_{\mathrm{W}}(a)$ will be decreasing on $[a^*_B,1]$, which implies that the maximizer a^*_B of $\Pi_{\mathrm{W}}(a)$ should be found in the interval $[0,a^*_B]$, proving the result. □

Proof of Theorem 4. In this proof, we skip time index t due to stationarity under Assumption 3. The long-run average profit of the conventional multi-customer system can be written as

$$\mathbb{E}\left[\hat{\Pi}\right]=R^{W2C}\mathbb{E}\left[d^{\mathrm{W2C}}\right]-C^{C2W}\sum_{i=1}^{T^{\mathrm{R}}}\mathbb{E}\left[r_i^{\mathrm{C2W}}\right]=R^{W2C}\lambda-C^{C2W}\sum_{i=1}^{T^{\mathrm{R}}}\mathbb{E}\left[r_i^{\mathrm{C2W}}\right]. \tag{A.18}$$

By definition, x_1 is the number of customers who recently entered the system, i.e., d^{W2C}. This implies that x_1 follows the same distribution as d^{W2C}, i.e., $x_1\sim Poisson(\lambda)$.

The number of returns r_i^{C2W} follows a binomial distribution with parameters x_i and u_i^{R} for $i\in\left\{1,2,\ldots,T^{\mathrm{R}}\right\}$. This allows us to use the splitting property of the Poisson process to characterize the distribution of the returns stemming from x_1, i.e., $r_1^{\mathrm{C2W}}\sim Poisson\left(\lambda u_1^{\mathrm{R}}\right)$. Non-returned items will spend one more day in the system, thereby constituting x_2. Similarly, the number of non-returned items follows a Poisson distribution, i.e, $x_2\sim Poisson\left(\lambda\left(1-u_1^{\mathrm{R}}\right)\right)$. Advancing this argumentation for more periods, we can characterize the distribution and expectation of r_i^{C2W} as follows

$$r_i^{\mathrm{C2W}}\sim Poisson\left(\lambda u_i^{\mathrm{R}}\prod_{j=1}^{i-1}\left(1-u_j^{\mathrm{R}}\right)\right)\text{ and }\mathbb{E}\left[r_i^{\mathrm{C2W}}\right]=\lambda u_i^{\mathrm{R}}\prod_{j=1}^{i-1}\left(1-u_j^{\mathrm{R}}\right).$$

Incorporating this into (A.18), we have

$$\mathbb{E}\left[\hat{\Pi}\right]=R^{W2C}\lambda-C^{C2W}\sum_{i=1}^{T^{\mathrm{R}}}\mathbb{E}\left[r_i^{\mathrm{C2W}}\right]=R^{W2C}\lambda-C^{C2W}\lambda\sum_{i=1}^{T^{\mathrm{R}}}u_i^{\mathrm{R}}\prod_{j=1}^{i-1}\left(1-u_j^{\mathrm{R}}\right).$$

This implies

$$\mathbb{E}\left[\hat{\Pi}\right]=\lambda\left(R^{W2C}-C^{C2W}p^{\mathrm{R}}\right)=\lambda\mathbb{E}\left[\Pi\right]. \quad □$$

Proof of Theorem 5.

In this proof, we skip time index t due to stationarity under Assumption 3. Considering $T^{\mathrm{M}}=1$, the long-run average profit of the C2C multi-customer system can be written as

$$\mathbb{E}\left[\hat{\Pi}(a)\right]=R^{\mathrm{W2C}}\mathbb{E}\left[d^{\mathrm{W2C}}\right]-C^{\mathrm{C2W}}\sum_{i=1}^{T^{\mathrm{R}}}\mathbb{E}\left[r_i^{\mathrm{C2W}}\right]-C^{\mathrm{C2W}}\mathbb{E}\left[\left(y_1-d^{\mathrm{C2C}}\right)^+\right] + \left(R^{\mathrm{C2C}}(a)-(C^{\mathrm{HI}}p^{\mathrm{HI}}+C^{\mathrm{LHI}}(1-p^{\mathrm{HI}}))-C^{\mathrm{C2C}}(a)p^{\mathrm{FR}}\right)\mathbb{E}\left[d^{\mathrm{C2C}}\right]. \tag{A.19}$$

Under Assumption 3, we have

$$\mathbb{E}\left[d^{\text{W2C}}\right] = \lambda. \tag{A.20}$$

Following the same reasoning as in the proof of Theorem 4, $x_1 \sim Poisson\ (\lambda)$. The number of returns $(r_i^{\text{C2W}}, r_i^{\text{C2C}})$ follows a multinomial distribution with parameters $(x_i, u_i^{\text{C2W}}, u_i^{\text{C2C}})$ for $i \in \{1, 2, \dots, T^{\text{R}}\}$. Thus, we can use the splitting property of the Poisson process to characterize the distributions of the number of C2W and C2C returns stemming from x_1, i.e., $r_1^{\text{C2W}} \sim Poisson\left(\lambda u_1^{\text{C2W}}\right)$ and $r_1^{\text{C2C}} \sim Poisson\left(\lambda u_1^{\text{C2C}}\right)$. The non-returned items will spend one more day in the system and constitute x_2. Similarly, the number of non-returned items follows a Poisson distribution, i.e, $x_2 \sim Poisson\left(\lambda\left(1 - u_1^{\text{C2C}} - u_1^{\text{C2W}}\right)\right)$. Advancing this argumentation for more periods, we can characterize the distributions and expectations of r_i^{C2W} and r_i^{C2C} as follows

$$r_i^{\text{C2W}} \sim Poisson\left(\lambda u_i^{C2W} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right)\right)$$
$$\text{and } \mathbb{E}\left[r_i^{\text{C2W}}\right] = \lambda u_i^{C2W} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right), \tag{A.21}$$

$$r_i^{\text{C2C}} \sim Poisson\left(\lambda u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right)\right)$$
$$\text{and } \mathbb{E}\left[r_i^{\text{C2C}}\right] = \lambda u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right). \tag{A.22}$$

Recall that the sum of r_i^{C2C} will constitute y_1. For each $i \in \{1, 2, \dots, T^{\text{R}}\}$, r_i^{C2C} follows a Poisson distribution. Moreover, the number of returns in different periods are independent from each other. Thus, we can use the merging property of the Poisson process to characterize the distribution and expectation of y_1. This leads to

$$y_1 \sim Poisson\left(\lambda \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right)\right)$$
$$\text{and } \mathbb{E}\left[y_1\right] = \lambda \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right). \tag{A.23}$$

According to (A.23), the number of C2C demands d^{C2C} is a Poisson distributed random variable. We can use the splitting property and derive the distribution and expectation of d^{C2C}. This leads to

$$d^{\text{C2C}} \sim Poisson\left(\lambda q(a) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right)\right), \tag{A.24}$$

$$\mathbb{E}\left[d^{\text{C2C}}\right] = \lambda q(a) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right). \tag{A.25}$$

Note that $\left(y_1 - d^{\text{C2C}}\right)^+$ denotes the number of items that are not sold within the matching period. By definition, d^{C2C} cannot exceed y_1. Thus, $\left(y_1 - d^{\text{C2C}}\right)^+ = y_1 - d^{\text{C2C}}$. Again, we can use the splitting property and establish the distribution and expectation of $\left(y_1 - d^{\text{C2C}}\right)^+$ as follows

$$\left(y_1 - d^{\text{C2C}}\right)^+ \sim Poisson\left(\lambda\left(1 - q(a)\right) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right)\right), \tag{A.26}$$

$$\mathbb{E}\left[\left(y_1 - d^{\text{C2C}}\right)^+\right] = \lambda\left(1 - q(a)\right) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right). \tag{A.27}$$

Using the results presented in (A.20)–(A.27), we can rewrite (A.19) as

$$\begin{aligned}
\mathbb{E}\left[\hat{\Pi}(a)\right] =& R^{\text{W2C}}\lambda - C^{\text{C2W}}\lambda \sum_{i=1}^{T^{\text{R}}} u_i^{C2W} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right) \\
&- C^{\text{C2W}}\lambda\left(1 - q(a)\right) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right) \\
&+ R^{\text{C2C}}(a)\lambda q(a) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right) \\
&- \left(C^{\text{HI}}p^{\text{HI}} + C^{\text{LHI}}(1 - p^{\text{HI}}) + C^{\text{C2C}}(a)p^{\text{FR}}\right) \\
&\times \lambda q(a) \sum_{i=1}^{T^{\text{R}}} u_i^{C2C} \prod_{j=1}^{i-1}\left(1 - u_j^{C2C} - u_j^{C2W}\right).
\end{aligned}$$

Recall that when T^{M}=1, the probability that a C2C return is sold within the assignment period is $p^{\text{M}}(a) = q(a)$, implying that

$$\begin{aligned}
\mathbb{E}\left[\hat{\Pi}(a)\right] =& \lambda\Big(R^{\text{W2C}} - C^{\text{C2W}}p^{\text{C2W}} - C^{\text{C2W}}p^{\text{C2C}}\left(1 - p^{\text{M}}(a)\right) + R^{\text{C2C}}(a)p^{\text{M}}(a)p^{\text{C2C}} \\
&- \left(\left(C^{\text{HI}}p^{\text{HI}} + C^{\text{LHI}}(1 - p^{\text{HI}})\right) + C^{\text{C2C}}(a)p^{\text{FR}}\right)p^{\text{M}}(a)p^{\text{C2C}}\Big) \\
=& \lambda\mathbb{E}\left[\Pi_{\text{B}}(a)\right].
\end{aligned}$$

□

References

[1] NOS. Retour roemoer. 2020, URL https://app.nos.nl/op3/pakjes/index.html. retrieved August 31, 2020.

[2] Statista. Return rate of online ordered clothing by U.S. consumers 2018. 2018, Statista. URL https://www.statista.com/. ID 962119.

[3] National Retail Federation. Consumer returns in the retail industry. 2022, URL https://cdn.nrf.com/sites/default/files/2022-12/AR3021-Customer%20Returns%20in%20the%20Retail%20Industry_2022_Final.pdf. retrieved July 21, 2023.

[4] de Leeuw S, Minguela-Rata B, Sabet E, Boter J, Sigurardottir R. Trade-offs in managing commercial consumer returns for online apparel retail. Int J Oper Prod Mgmt 2016;36(6):710–31.

[5] Diggins M, Chen C, Chen J. A review: Customer returns in fashion retailing. In: Choi TM, editor. Analytical modeling research in fashion business. Singapore: Springer; 2016, p. 31–48.

[6] Mostard J, Teunter R. The newsboy problem with resalable returns: A single period model and case study. European J Oper Res 2006;169(1):81–96.

[7] Abelvik-Lawson H. Why Amazon is throwing away millions of unused products – and how we can stop them? 2021, URL https://www.greenpeace.org.uk/news/stop-amazon-throwing-away-millions-unused-products/. retrieved February 13, 2024.

[8] Fremstad A. Does Craigslist reduce waste? Evidence from California and Florida. Ecol Econ 2017;132:135–43.

[9] Abdulla H, Ketzenberg M, Abbey J. Taking stock of consumer returns: A review and classification of the literature. J Oper Manag 2019;65(6):560–605.

[10] Shulman J, Coughlan A, Savaskan R. Optimal restocking fees and information provision in an integrated demand-supply model of product returns. Manuf Serv Oper Manag 2009;11(4):577–94.

[11] Nageswaran L, Cho S-H, Scheller-Wolf A. Consumer return policies in omnichannel operations. Manage Sci 2020;66(12):5558–75.

[12] Wagner L, Martínez-de Albéniz V. Pricing and assortment strategies with product exchanges. Oper Res 2020;68(2):453–66.

[13] Yang G, Ji G. The impact of cross-selling on managing consumer returns in omnichannel operations. Omega 2022;111:102665.

[14] Yang L, Li X, Xia Y, Aneja Y. Returns operations in omnichannel retailing with buy-online-and-return-to-store. Omega 2023;119:102874.

[15] Zhang D, Frei R, Senyo PK, Bayer S, Gerding E, Wills G, et al. Understanding fraudulent returns and mitigation strategies in multichannel retailing. J Retail Consum Serv 2023;70:103145.

[16] Altug M, Aydinliyim T, Jain A. Managing opportunistic consumer returns in retail operations. Manage Sci 2021;67(9):5660–78.

[17] Shang G, Pekgün P, Ferguson M, Galbreth M. How much do online consumers really value free product returns? Evidence from eBay. J Oper Manag 2017;53:45–62.

[18] Shang G, Ferguson M, Galbreth M. Where should I focus my return reduction efforts? Empirical guidance for retailers. Decis Sci 2019;50(4):877–909.

[19] Narang U, Shankar V. Mobile app introduction and online and offline purchases and product returns. Mark Sci 2019;38(5):756–72.

[20] Patel P, Baldauf C, Karlsson S, Oghazi P. The impact of free returns on online purchase behavior: Evidence from an intervention at an online retailer. J Oper Manag 2021;67(4):511–55.

[21] Lee D. An alternative explanation of consumer product returns from the postpurchase dissonance and ecological marketing perspectives. Psychol Mark 2015;32(1):49–64.

[22] Minnema A, Bijmolt T, Petersen J, Shulman J. Managing product returns within the customer value framework. In: Palmatier R, Kumar V, Harmeling C, editors. Customer engagement marketing. Cham: Springer International Publishing; 2018, p. 95–118.
[23] Difrancesco R, Huchzermeier A, Schröder D. Optimizing the return window for online fashion retailers with closed-loop refurbishment. Omega 2018;78:205–21.
[24] Mont O, Palgan Y, Bradley K, Zvolska L. A decade of the sharing economy: Concepts, users, business and governance perspectives. J Clean Prod 2020;269:122215.
[25] Hawlitschek F, Teubner T, Gimpel H. Consumer motives for peer-to-peer sharing. J Clean Prod 2018;204:144–57.
[26] Jiang L, Dimitrov S, Mantin B. P2P Marketplaces and retailing in the presence of consumers' valuation uncertainty. Prod Oper Manag 2017;(3):509–24.
[27] Li T, Xie J, Liu X. Consumer return policies in presence of a P2P market. Omega 2020;97:102092.
[28] Xue Y, Caliskan-Demirag O, Chen Y, Yu Y. Supporting customers to sell used goods: Profitability and environmental implications. Int J Prod Econ 2018;206:220–32.
[29] Vedantam A, Demirezen E, Kumar S. Trade-in or sell in my P2P marketplace: A game theoretic analysis of profit and environmental impact. Prod Oper Manag 2021;(11):3923–42.
[30] Pang C, Li L. Leveraging strategic consumer behavior in consumer-to-consumer resale: From economic and environmental perspectives. Omega 2023;120:102924.
[31] Veepee. Re-turn par Veepee : le retour de commande client à client. 2021, URL https://www.lesventesprivees.net/return-veepee.php. retrieved July 21, 2023.
[32] Frate. Fewer returns. Guaranteed. 2024, URL https://www.frate.co/. retrieved February 21, 2024.
[33] Xing W, Dan L, Jing C, Yongqin L. Managing customer returns strategy with the option of selling returned products. Int J Prod Econ 2020;230:107794.
[34] Wiersma S. Customer-to-customer return logistics: Exploring the consumer perspective to a new approach for managing returns. Vrije Universiteit Amsterdam, Faculty of Economics and Business Administration; 2021, URL https://www.ubvu.vu.nl/pub/fulltext/scripties/27_2693468_0.pdf.
[35] Hsieh M. Customer preferences for customer-to-customer return logistics and the differences in customer preferences between the Netherlands and Taiwan. Vrije Universiteit Amsterdam, Faculty of Economics and Business Administration; 2021, URL https://www.ubvu.vu.nl/pub/fulltext/scripties/27_2690479_0.pdf.
[36] United Nations. United Nations sustainable development: The 17 goals. 2024, URL https://sdgs.un.org/goals. retrieved February 13, 2024.
[37] Chen Z, Fan Z, Zhao X. Offering return-freight insurance or not: Strategic analysis of an e-seller's decisions. Omega 2021;103:102447.
[38] Räisänen J, Ojala A, Tuovinen T. Building trust in the sharing economy: Current approaches and future considerations. J Clean Prod 2021;279:123724.
[39] Zhang C, Yu M, Chen J. Signaling quality with return insurance: Theory and empirical evidence. Manage Sci 2022;68(8):5847–67.
[40] Adida E, Perakis G. Dynamic pricing and inventory control: Uncertainty and competition. Oper Res 2009;58(2):289–302.
[41] Cohen MC, Perakis G, Pindyck RS. A simple rule for pricing with limited knowledge of demand. Manage Sci 2021;67(3). 1329–1199.
[42] Powell W. Clearing the jungle of stochastic optimization. In: Bridging data and decisions. INFORMs tutorials in operations research, 2014, p. 109–37.
[43] Kearns M, Mansour Y, Andrew YNG. A sparse sampling algorithm for near-optimal planning in large Markov decision processes. Mach Learn 2002;49(2):193–208.
[44] Péret L, Garcia F. On-line search for solving Markov decision processes via heuristic sampling. In: Proceedings of the 16th European conference on artificial intelligence. Amsterdam, the Netherlands: IOS Press; 2004, p. 530–4.
[45] SURFsara. Lisa system. 2021, URL https://userinfo.surfsara.nl/systems/lisa/description. retrieved October 7, 2021.
[46] Agrawal V, Ferguson M, Toktay L, Thomas V. Is leasing greener than selling? Manage Sci 2012;58(3):523–33.
[47] Agrawal V, Bellos I. The potential of servicizing as a green business model. Manage Sci 2017;63(5):1545–62.
[48] Vetterling W, Press W. Numerical recipes in FORTRAN: The art of scientific computing. second ed.. New York, NY, USA: Cambridge University Press; 1992.
[49] Borwein P, Erdélyi T. Polynomials and polynomial inequalities. first ed.. NY, USA: Springer; 1995.
[50] Neumark S. Solution of cubic and quartic equations. first ed.. Oxford: Pergamon Press Ltd; 2014.