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APPROXIMATE SOLUTION TO FRACTIONAL ORDER SOIL TRANSMITTED HELMINTH INFECTION MODEL USING LAPLACE ADOMIAN DECOMPOSITION METHOD

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Abstract:

In this study, we proposed a fractional order compartmental model based on the Caputo derivative to describe the dynamics of soil-transmitted helminth infection. We employed the Laplace Adomian Decomposition Method (LADM) to derive series solutions for each equation within the system of non-linear differential equations that comprise the epidemiological model. Our findings indicate that the infinite series generated by LADM converges to the exact solution of the system. We performed numerical simulations of the fractional order compartmental deterministic model using MATLAB to validate the approximate results. Additionally, comparing the solutions of the fractional order model with the classical model reveals that the fractional order model offers greater flexibility, allowing the system to be adjusted to achieve various desired outcomes in different compartments by varying the fractional order to values such as 0.75, 0.8, 0.85, 0.9, 0.95, and 1.

Keywords:

Fractional order, Caputo derivative, Laplace Adomian decomposition method.

1. Introduction

Soil-transmitted helminths (STHs) are a group of parasitic nematodes that predominantly thrive in tropical and subtropical climates, particularly in low- and middle-income countries (LMICs) [1]. The most common STHs are intestinal parasites including *Ascarislumbricoides* (roundworm), *Trichuristrichiura* (whipworm), and hookworm species such as *Necatoramericanus and Ancylostoma spp.* (including A. duodenale and A. ceylanicum) [2]. Nearly one-quarter of the global population is estimated to be infected with STHs [3]. Transmission primarily occurs through eggs or larvae present in the feces of infected individuals or those that hatch in the soil after defecation. Adult worms residing in the gut of an infected person produce thousands of eggs daily, which can contaminate environments or food in areas lacking adequate sanitation [4].Consequently, STH infections can lead to complications such as gut blood loss, nutrient malabsorption, loss of appetite, and anemia due to the depletion of iron and other essential proteins [5]. In children, these infections can result in serious issues including anemia, growth retardation, impaired cognitive development, school absenteeism, and a significant number of disability-adjusted life years (DALYs) lost [6].Although STH infections are usually chronic and debilitating, they are not regarded as significant contributors to global mortality.

Treatment for STH infections with anthelmintic drugs such as mebendazole or albendazole is inexpensive and generally well tolerated. The efficacy of these drugs varies across different STH species: both drugs are highly effective against *Ascarislumbricoides* (up to 100%), but they are less effective against hookworm and *Trichuristrichiura*[7].Over the past twenty years, mathematical modeling has played a crucial role in understanding the dynamics of infectious diseases. Several models using integer-order nonlinear differential equations have been developed to explore the transmission dynamics of soil-transmitted helminth infections. For instance, Lambura et al. [8] developed and evaluated a deterministic compartmental model to study the dynamics of soil-transmitted helminth infections. Their approach incorporated optimal control strategies to raise awareness among the susceptible population and included interventions such as mass drug administration and improved sanitation practices. Oguntolu et al. [9] introduced an innovative deterministic mathematical model to thoroughly examine the dynamics of helminth infection transmission via soil. They incorporated two time-dependent control strategies: the rate of hygiene awareness in the infectious class and the rate of hygiene awareness in the susceptible class. The simulation results suggest that if both control strategies are implemented effectively, the burden of helminth infections can be significantly reduced. Truscott et al. [10] attempted to model the population-level helminth transmission cycle and analyzed the impact of prophylactic chemotherapy on its management and eradication.

In recent years, fractional calculus has garnered significant attention from researchers because the fractional derivative is a crucial tool for explaining the dynamic behavior of various physical systems [11]. Unlike integer-order models, which only describe local properties, fractional-order models provide a comprehensive description of the entire system and offer a better representation of real systems with memory effects [11].Fractional-order models are more realistic and practical than classical integer-order models. In the context of biological problems, the Caputo and Riemann-Liouville derivatives are considered singular kernel fractional derivatives. Additionally, there are non-singular options, such as the Mittag-Leffler and Atangana-Baleanu operators, which also play a significant role [11].The Laplace Adomian Decomposition Method (LADM) combines the Adomian Decomposition Method (ADM) with Laplace transforms. ADM is an effective technique for solving systems of ordinary differential equations, while the Laplace transform method has been proposed for modeling nonlinear biological responses. It is claimed that this approach yields accurate results and can solve any linear or nonlinear differential equation [12].

The Laplace-Adomian Decomposition Method has been applied to numerous mathematical models (see, for instance, [13, 14, 15, 16, 17]).Fazal et al. [18] determined the numerical solution of a fractional-order epidemic model of a childhood disease using the Laplace-Adomian Decomposition Method. Atokolo et al. [11] presented a fractional-order sterile insect technology (SIT) model to mitigate the spread of Zika virus disease in a population. They used the Laplace-Adomian Decomposition Method (LADM) to determine an analytical (approximate) solution to the model. It was revealed that LADM produced a solution in the form of an infinite series, which converges to the exact value. Yunus et al. [19] investigated the spread and control of COVID-19 in Nigeria using the Caputo fractional-order derivative via the Laplace-Adomian Decomposition Method. Their study revealed that the effect of the Caputo fractional derivative on the transmission dynamics of the disease indicated a greater individual recovery rate at an integer order. This finding reflects the full implementation of factors such as treatment, vaccination, and efforts to reduce disease transmission.

Consequently, in this study, we extended the work of Oguntolu et al. [9] by considering a fractional-order model using the Caputo derivative to understand the dynamics of soil-transmitted helminth infections. We utilized the Laplace-Adomian Decomposition Method to determine an approximate solution to the model.

2. Preliminaries and Formulation of Fractional Order Model

In this section, we look at the preliminary information about what Caputo fractional order derivative is, giving necessary definition associated with the concepts.

Definition 1. The Caputo fractional order derivative of a function y on the interval $[0,T]$ is defined by:

$$ {}^{c}D_{0+}^{\beta} y(t) = \frac{1}{\Gamma(n-\beta)} \int_0^t (t-s)^{n-\beta-1} y^{(n)}(s)\, ds, \tag{1} $$

Where $n=[\beta]+1$ and $[\beta]$ represents the integer part of β . In particular for $0<\beta<1$ the fractional order derivative becomes:

$$ {}^{c}D^{\beta} q(t) = \frac{1}{\Gamma(n-\beta)} \int_0^t \frac{y(s)}{(t-s)^{\beta}} y^{(n)}(s) ds $$

Definition 2*.The Laplace transform of Caputo derivatives can be defined as:*

$$L\left[{}^{c}D^{\beta}y(t)\right]=s^{\beta}y(s)-\sum_{k=0}^{n-1}s^{\beta-i-1}y^{k}(0),\ n-1<\beta<n,n\in N. \tag{2}$$

For arbitrary $c_i\in\Re,\ i=0,1,2,...,n-1,$ and $n=[\beta]+1$ where $[\beta]$ represents the non-integer parts of β.

Lemma 1. The following results hold for the fractional order differential equations;

$$I^{\beta}\left[{}^{c}D^{\beta}q\right](t)=h(t)+\sum_{0}^{n-1}\frac{h^{i}(0)}{i!}t^{i}. \tag{3}$$

For arbitrary $\beta>0,\ i=0,1,2,...,n-1,$ and $n=[\beta]+1$ where $[\beta]$ represents the non-integer parts of β.

3. Model Formulation

The total human population at time t, denoted by $N(t)$, is subdivided into five mutually exclusive compartments of susceptible individuals $(S(t))$, exposed individual $(E(t))$, infected individuals $(I(t))$, hygiene consciousness individuals $(H(t))$, and Recovered individuals $(R(t))$. So that

$$N(t)=S(t)+E(t)+I(t)+H(t)+R(t).$$

The population of the parasite in the environment is denoted by $M(t)$. The population of the susceptible individuals is generated by the recruitment of individuals into the population either by birth or migration at the constant rate Λ. Susceptible individual's acquires helminth infections following effective constant with contaminated food or environment at the rate λ given by

$$\lambda=\frac{\beta_C M}{K+M}$$

Where β_c is the rate at which contaminated food or skin penetrations are consumed and K is the carrying capacity of the parasites? Susceptible individuals that come in contact with the contaminated food or environment progressed to be exposed, and then progressed to being infected at the rate ρ. The infected individuals recovers from the infection class due to treatment at the rate τ, and recovers from the infection due to strong body immune system at the rate q, δ is the disease- induced death rate of infected individuals. The rate α_1 and α_2 are the hygiene conscious rate of susceptible and infected individuals respectively. The rate ε is the recovery rate of hygiene conscious individuals. The human natural death rate is the same in all human epidemiological compartments. Recovered individuals become susceptible again at the rate γ. Natural death occurs to all human compartments at the rate μ_h. The rate σ is the shedding rate of the parasite in the environment by infected individuals. The clearance rate of the parasite from the environment is given at the rate μ_m.

Based on the above assumptions and formulation, the soil-transmitted helminth infections model is governed by the following system of nonlinear differential equations (The flow diagram is shown in Figure 1, the associated state variables and parameters are well described in Table 1):

$$\frac{dS}{dt} = \Lambda - \frac{\beta_c MS}{K+M} - (\alpha_1 + \mu_h)S + \gamma R,$$

$$\frac{dE}{dt} = \frac{\beta_c MS}{K+M} - (\rho + \mu_h)E,$$

$$\frac{dI}{dt} = \rho E - (\alpha_2 + q + \tau + \delta + \mu_h)I, \quad (4)$$

$$\frac{dH}{dt} = \alpha_1 S + \alpha_2 I - (\varepsilon + \mu_h)H,$$

$$\frac{dR}{dt} = (q + \tau)I + \varepsilon H - (\gamma + \mu_h)R,$$

$$\frac{dM}{dt} = \sigma I - \mu_m M.$$

With initial conditions $S(0) > 0, E(0) \geq 0, I(0) \geq 0, H(0) \geq 0, R(0) \geq 0, \textit{ and } M(0) \geq 0$.

Table 1: Model variables and parameters

Variables	**Description**
$S(t)$	Susceptible Individuals
$E(t)$	Exposed Individuals
$I(t)$	Infected Individuals
$H(t)$	Hygiene Conscious Individuals
$R(t)$	Recovered Individuals
$M(t)$	Parasite population
Parameters	**Description**
Λ	Recruitment rate
α_1	Hygiene conscious rate of susceptible individuals
α_2	Hygiene conscious rate of infected individuals
β	Intake rate of egg in contaminated food or larvae that have penetrated the skin
ρ	Progression rate from Exposed to being infected
q	Recovered rate of infected individuals perhaps due strong body immune system
τ	Recovery rate due to treatment
γ	Rate at which recovered individuals becomes susceptible again
σ	Shedding rate of the parasite in the environment by contaminated individuals
ε	Recovery rate of Hygiene conscious infected individuals
μ_h	Natural death rate for humans
μ_m	Clearance rate of parasites
K	Density of the parasites (carrying capacity)
δ	Disease induce death rate.

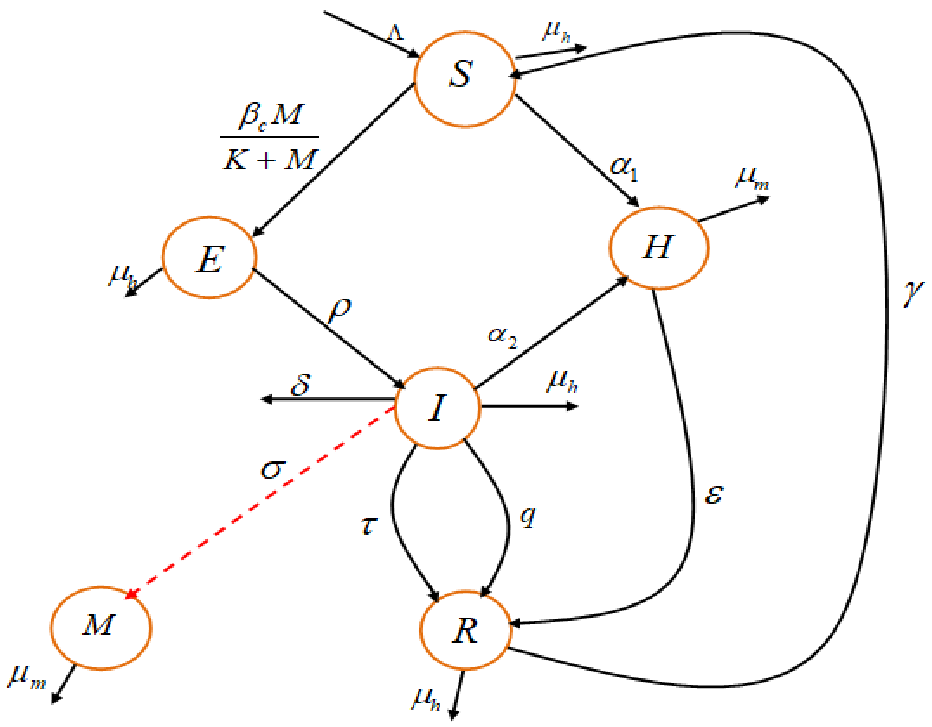

Figure 1: Schematic Diagram of the Soil-transmitted Helminth Infection.

3.1. The Laplace-Adomian Decomposition Method

Using the Caputo fractional derivative on the Helminth model (4), we obtained

$$
\begin{aligned}
{}^cD^\beta S(t) &= \Lambda - \frac{\beta_c MS}{K+M} - (\alpha_1 + \mu_h)S + \gamma R,\\
{}^cD^\beta E(t) &= \frac{\beta_c MS}{K+M} - (\rho + \mu_h)E,\\
{}^cD^\beta I(t) &= \rho E - (\alpha_2 + q + \tau + \delta + \mu_h)I, \quad (5)\\
{}^cD^\beta H(t) &= \alpha_1 S + \alpha_2 I - (\varepsilon + \mu_h)H,\\
{}^cD^\beta R(t) &= (q+\tau)I + \varepsilon H - (\gamma + \mu_h)R,\\
{}^cD^\beta M(t) &= \sigma I - \mu_m M.
\end{aligned}
$$

Where $\beta = (0,1]$, with initial conditions, $S(0) = n_1$, $E(0) = n_2$, $I(0) = n_3$, $H(0) = n_4$, $R(0) = n_5$, and $M(0) = n_6$.

Applying Laplace transform on both sides of system (5), we obtained the following system;

$$\mathcal{L}\left\{{}^{c}D^{\beta}S(t)\right\}=\mathcal{L}\left\{\Lambda-\frac{\beta_{c}MS}{K+M}-(\alpha_{1}+\mu_{h})S+\gamma R\right\},$$

$$\mathcal{L}\left\{{}^{c}D^{\beta}E(t)\right\}=\mathcal{L}\left\{\frac{\beta_{c}MS}{K+M}-(\rho+\mu_{h})E\right\},$$

$$\mathcal{L}\left\{{}^{c}D^{\beta}I(t)\right\}=\mathcal{L}\left\{\rho E-(\alpha_{2}+q+\tau+\delta+\mu_{h})I\right\}, (6)$$

$$\mathcal{L}\left\{{}^{c}D^{\beta}H(t)\right\}=\mathcal{L}\left\{\alpha_{1}S+\alpha_{2}I-(\varepsilon+\mu_{h})H\right\},$$

$$\mathcal{L}\left\{{}^{c}D^{\beta}R(t)\right\}=\mathcal{L}\left\{(q+\tau)I+\varepsilon H-(\gamma+\mu_{h})R\right\},$$

$$\mathcal{L}\left\{{}^{c}D^{\beta}M(t)\right\}=\mathcal{L}\left\{\sigma I-\mu_{m}M\right\}.$$

This implies that,

$$s^{\beta}\mathcal{L}\left\{S(t)\right\}-s^{\beta-1}S(0)=\mathcal{L}\left\{\Lambda-\frac{\beta_{c}MS}{K+M}-(\alpha_{1}+\mu_{h})S+\gamma R\right\},$$

$$s^{\beta}\mathcal{L}\left\{E(t)\right\}-s^{\beta-1}E(0)=\mathcal{L}\left\{\frac{\beta_{c}MS}{K+M}-(\rho+\mu_{h})E\right\},$$

$$s^{\beta}\mathcal{L}\left\{I(t)\right\}-s^{\beta-1}I(0)=\mathcal{L}\left\{\rho E-(\alpha_{2}+q+\tau+\delta+\mu_{h})I\right\}, (7)$$

$$s^{\beta}\mathcal{L}\left\{H(t)\right\}-s^{\beta-1}H(0)=\mathcal{L}\left\{\alpha_{1}S+\alpha_{2}I-(\varepsilon+\mu_{h})H\right\},$$

$$s^{\beta}\mathcal{L}\left\{R(t)\right\}-s^{\beta-1}R(0)=\mathcal{L}\left\{(q+\tau)I+\varepsilon H-(\gamma+\mu_{h})R\right\},$$

$$s^{\beta}\mathcal{L}\left\{M(t)\right\}-s^{\beta-1}M(0)=\mathcal{L}\left\{\sigma I-\mu_{m}M\right\}.$$

Using the initial conditions, equation (7) becomes

$$\mathcal{L}\left\{S(t)\right\}=\frac{n_{1}}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\Lambda-\frac{\beta_{c}MS}{K+M}-(\alpha_{1}+\mu_{h})S+\gamma R\right\}\right],$$

$$\mathcal{L}\left\{E(t)\right\}=\frac{n_{2}}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\frac{\beta_{c}MS}{K+M}-(\rho+\mu_{h})E\right\}\right],$$

$$\mathcal{L}\left\{I(t)\right\}=\frac{n_{3}}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\rho E-(\alpha_{2}+q+\tau+\delta+\mu_{h})I\right\}\right], (8)$$

$$\mathcal{L}\left\{H(t)\right\}=\frac{n_{4}}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\alpha_{1}S+\alpha_{2}I-(\varepsilon+\mu_{h})H\right\}\right],$$

$$\mathcal{L}\{R(t)\} = \frac{n_5}{s} + \left[\frac{1}{s^\beta}\mathcal{L}\{(q+\tau)I + \varepsilon H - (\gamma + \mu_h)R\}\right],$$

$$\mathcal{L}\{M(t)\} = \frac{n_6}{s} + \left[\frac{1}{s^\beta}\mathcal{L}\{\sigma I - \mu_m M\}\right].$$

Assuming that the solution $S(t), E(t), I(t), H(t), R(t)$, and $M(t)$ are in the form of infinite series given by

$$S(t) = \sum_{n=0}^{\infty} S_n, E(t) = \sum_{n=0}^{\infty} E_n, \; I(t) = \sum_{n=0}^{\infty} I_n, \; H(t) = \sum_{n=0}^{\infty} H_n, \; R(t) = \sum_{n=0}^{\infty} R_n, \text{ and}$$

$$M(t) = \sum_{n=0}^{\infty} M_n. \tag{9}$$

The non-linear term involved in the Helminth model (1) is $M(t)S(t)$, and it is decomposed by the Adomian polynomial as follows;

$$M(t)S(t) = \sum_{n=0}^{\infty} X_n. (10)$$

Where X_n is an Adomian polynomial given by

$$X_n = \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\left[\sum_{k=0}^{n}\lambda^k M_k(t)\sum_{k=0}^{n}\lambda^k S_k(t)\right]\Bigg|_{\lambda=0}. \tag{11}$$

The first three polynomials are given by

$$X_0 = M_0(t)S_0(t),$$

$$X_1 = M_0(t)S_1(t) + M_1(t)S_0(t), (12)$$

$$X_2 = M_0(t)S_2(t) + M_1(t)S_1(t) + M_2(t)S_0(t).$$

Substituting equation (9) and (10) into equation (8), we obtained

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\} = \frac{n_1}{s} + \left[\frac{1}{s^\beta}\mathcal{L}\left\{\Lambda - \frac{\beta_c \sum_{n=0}^{\infty} X_n}{K + \sum_{n=0}^{\infty} M_n} - (\alpha_1 + \mu_h)\sum_{n=0}^{\infty} S_n + \gamma \sum_{n=0}^{\infty} R_n\right\}\right],$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty}E_n\right\}=\frac{n_2}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\frac{\beta_c\sum_{n=0}^{\infty}X_n}{K+\sum_{n=0}^{\infty}M_n}-(\rho+\mu_h)\sum_{n=0}^{\infty}E_n\right\}\right],$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty}I_n\right\}=\frac{n_3}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\rho\sum_{n=0}^{\infty}E_n-(\alpha_2+q+\tau+\delta+\mu_h)\sum_{n=0}^{\infty}I_n\right\}\right], \quad (13)$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty}H_n\right\}=\frac{n_4}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\alpha_1\sum_{n=0}^{\infty}S_n+\alpha_2\sum_{n=0}^{\infty}I_n-(\varepsilon+\mu_h)\sum_{n=0}^{\infty}H_n\right\}\right],$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty}R_n\right\}=\frac{n_5}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{(q+\tau)\sum_{n=0}^{\infty}I_n+\varepsilon\sum_{n=0}^{\infty}H_n-(\gamma+\mu_h)\sum_{n=0}^{\infty}R_n\right\}\right],$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty}M_n\right\}=\frac{n_6}{s}+\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{\sigma\sum_{n=0}^{\infty}I_n-\mu_m\sum_{n=0}^{\infty}M_n\right\}\right].$$

Solving the Laplace transform of the right hand side of equation (13) and equating the two sides of equation (13) yields the following iterative algorithm;

$$\mathcal{L}(S_0)=\frac{n_1}{s},$$

$$\mathcal{L}(E_0)=\frac{n_2}{s},$$

$$\mathcal{L}(I_0)=\frac{n_3}{s},$$

$$\mathcal{L}(H_0)=\frac{n_4}{s},$$

$$\mathcal{L}(R_0)=\frac{n_5}{s},$$

$$\mathcal{L}(M_0)=\frac{n_6}{s},$$

$$\mathcal{L}(S_1)=\left(\Lambda-\frac{\beta_c X_0}{K+M_0}-(\alpha_1+\mu_h)S_0+\gamma R_0\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(E_1)=\left(\frac{\beta_c X_0}{K+M_0}-(\rho+\mu_h)E_0\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(I_1)=\left(\rho E_0-(\alpha_2+q+\tau+\delta+\mu_h)I_0\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(R_1)=\left((q+\tau)I_0+\varepsilon H_0-(\gamma+\mu_h)R_0\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(M_1)=\left(\sigma I_0-\mu_m M_0\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(S_2)=\left(\Lambda-\frac{\beta_c X_1}{K+M_1}-(\alpha_1+\mu_h)S_1+\gamma R_1\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(E_2)=\left(\frac{\beta_c X_1}{K+M_1}-(\rho+\mu_h)E_1\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(I_2)=\left(\rho E_1-(\alpha_2+q+\tau+\delta+\mu_h)I_1\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(H_2)=\left(\alpha_1 S_1+\alpha_2 I_1-(\varepsilon+\mu_h)H_1\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(R_2)=\left((q+\tau)I_1+\varepsilon H_1-(\gamma+\mu_h)R_1\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(M_2)=\left(\sigma I_1-\mu_m M_1\right)\frac{1}{s^{\beta+1}},$$

$$\vdots \qquad \qquad \vdots$$

$$\mathcal{L}(S_{n+1})=\left(\Lambda-\frac{\beta_c X_n}{K+M_n}-(\alpha_1+\mu_h)S_n+\gamma R_n\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(E_{n+1})=\left(\frac{\beta_c X_n}{K+M_n}-(\rho+\mu_h)E_n\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(I_{n+1})=\left(\rho E_n-(\alpha_2+q+\tau+\delta+\mu_h)I_n\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(H_{n+1})=\left(\alpha_1 S_n+\alpha_2 I_n-(\varepsilon+\mu_h)H_n\right)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(R_{n+1}) = ((q+\tau)I_n + \varepsilon H_n - (\gamma+\mu_h)R_n)\frac{1}{s^{\beta+1}},$$

$$\mathcal{L}(M_{n+1}) = (\sigma I_n - \mu_m M_n)\frac{1}{s^{\beta+1}},\ n \geq 1 \qquad (14)$$

Taking the inverse Laplace transform of (14) and considering the first three terms, we obtained:

$$S_0 = n_1, E_0 = n_2, I_0 = n_3, H_0 = n_4, R_0 = n_5, M_0 = n_6,$$

$$S_1 = \left(\Lambda - \frac{\beta_c n_6 n_1}{K+n_6} - (\alpha_1+\mu_h)n_1 + \gamma n_5\right)\frac{t^\beta}{\Gamma(\beta+1)},$$

$$E_1 = \left(\frac{\beta_c n_6 n_1}{K+n_6} - (\rho+\mu_h)n_2\right)\frac{t^\beta}{\Gamma(\beta+1)},$$

$$I_1 = (\rho n_2 - (\alpha_2+q+\tau+\delta+\mu_h)n_3)\frac{t^\beta}{\Gamma(\beta+1)},$$

$$H_1 = (\alpha_1 n_1 + \alpha_2 n_3 - (\varepsilon+\mu_h)n_4)\frac{t^\beta}{\Gamma(\beta+1)},$$

$$R_1 = ((q+\tau)n_3 + \varepsilon n_4 - (\gamma+\mu_h)n_5)\frac{t^\beta}{\Gamma(\beta+1)},$$

$$M_1 = (\sigma n_3 - \mu_m n_6)\frac{t^\beta}{\Gamma(\beta+1)},$$

$$S_2 = \Lambda\frac{t^\beta}{\Gamma(\beta+1)} - \frac{\beta_c\left(n_6\left(\Lambda - \frac{\beta_c n_6 n_1}{K+n_6} - (\alpha_1+\mu_h)n_1 + \gamma n_5\right) + n_1(\sigma n_3 - \mu_m n_6)\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)}}{K + (\sigma n_3 - \mu_m n_6)\frac{t^\beta}{\Gamma(\beta+1)}}$$
$$-(\alpha_1+\mu_h)\left(\Lambda - \frac{\beta_c n_6 n_1}{K+n_6} - (\alpha_1+\mu_h)n_1 + \gamma n_5\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)} +$$
$$\gamma((q+\tau)n_3 + \varepsilon n_4 - (\gamma+\mu_h)n_5)\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$E_2 = \frac{\beta_c\left(n_6\left(\Lambda - \frac{\beta_c n_6 n_1}{K+n_6} - (\alpha_1+\mu_h)n_1 + \gamma n_5\right) + n_1(\sigma n_3 - \mu_m n_6)\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)}}{K + (\sigma n_3 - \mu_m n_6)\frac{t^{\beta}}{\Gamma(\beta+1)}} -$$

$$(\rho+\mu_h)\left(\frac{\beta_c n_6 n_1}{K+n_6} - (\rho+\mu_h)n_2\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$I_2 = \left(\rho\left(\frac{\beta_c n_6 n_1}{K+n_6} - (\rho+\mu_h)n_2\right) - (\alpha_2+q+\tau+\delta+\mu_h)\left(\rho n_2 - (\alpha_2+q+\tau+\delta+\mu_h)n_3\right)\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$H_2 = \left(\alpha_1\left(\Lambda - \frac{\beta_c n_6 n_1}{K+n_6} - (\alpha_1+\mu_h)n_1 + \gamma n_5\right) + \alpha_2\left(\rho n_2 - (\alpha_2+q+\tau+\delta+\mu_h)n_3\right)\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)} -$$

$$(\varepsilon+\mu_h)\left(\alpha_1 n_1 + \alpha_2 n_3 - (\varepsilon+\mu_h)n_4\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$R_2 = \left((q+\tau)\left(\rho n_2 - (\alpha_2+q+\tau+\delta+\mu_h)n_3\right) + \varepsilon\left(\alpha_1 n_1 + \alpha_2 n_3 - (\varepsilon+\mu_h)n_4\right)\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)} -$$

$$(\gamma+\mu_h)\left((q+\tau)n_3 + \varepsilon n_4 - (\gamma+\mu_h)n_5\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$M_2 = \left(\sigma\left(\rho n_2 - (\alpha_2+q+\tau+\delta+\mu_h)n_3\right) - \mu_m(\sigma n_3 - \mu_m n_6)\right)\frac{t^{2\beta}}{\Gamma(2\beta+1)}.$$

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. . .

$$S_{n+1} = \left(\Lambda - \frac{\beta_c X_n}{K+M_n} - (\alpha_1+\mu_h)S_n + \gamma R_n\right)\frac{t^{\beta}}{\Gamma(\beta+1)},$$

$$E_{n+1} = \left(\frac{\beta_c X_n}{K+M_n} - (\rho+\mu_h)E_n\right)\frac{t^{\beta}}{\Gamma(\beta+1)},$$

$$I_{n+1} = \left(\rho E_n - (\alpha_2+q+\tau+\delta+\mu_h)I_n\right)\frac{t^{\beta}}{\Gamma(\beta+1)},$$

$$H_{n+1} = \left(\alpha_1 S_n + \alpha_2 I_n - (\varepsilon+\mu_h)H_n\right)\frac{t^{\beta}}{\Gamma(\beta+1)},$$

$$R_{n+1} = \left(\left(q+\tau\right)I_n + \varepsilon H_n - \left(\gamma+\mu_h\right)R_n\right)\frac{t^{\beta}}{\Gamma\left(\beta+1\right)},$$

$$M_{n+1} = \left(\sigma I_n - \mu_m M_n\right)\frac{t^{\beta}}{\Gamma\left(\beta+1\right)}, \quad (15)$$

4. Numerical Schemes

In this section, we shall carry out the numerical solution of the model using the initial conditions

$n_1 = 20000, n_2 = 2000, n_3 = 300, n_4 = 1200, n_5 = 1, \ n_6 = 1000$ and parameters in Table 2.

Table 2: Parameter Values

Parameters	**Values**	**Source**
Λ	$\frac{1}{18250}$	[20]
α_1	0.4	[9]
α_2	0.6	[9]
β	0.4	[9]
p	$\frac{1}{10}$	[20]
q	$\frac{1}{28}$	[20]
τ	$\frac{16}{30}$	[9]
γ	$\frac{1}{7}$	[20]
σ	0.09	[20]
ε	0.03	[9]
μ_h	$\frac{1}{60\times365}$	[20]
μ_m	$\frac{13}{37500}$	[20]

K	10^5	[20]
δ	$\frac{35}{1000}$	[20]

The Laplace-Adomian decomposition method (LADM) gives us an approximate solution in terms of an infinite series presented as follows;

$$\begin{aligned} S(t) &= S_0 + S_1 + S_2 + ..., \\ E(t) &= E_0 + E_1 + E_2 + ..., \\ H(t) &= H_0 + H_1 + H_2 + ..., \quad (16) \\ I(t) &= I_0 + I_1 + I_2 + ..., \\ R(t) &= R_0 + R_1 + R_2 + ..., \\ M(t) &= M_0 + M_1 + M_2 + ..., \end{aligned}$$

So that we have

$$S(t) = 20000 - 8079.9845\frac{t^{\beta}}{\Gamma(\beta+1)} + \left(\frac{3018770\frac{t^{2\beta}}{\Gamma(2\beta+1)}}{10^5 + 26.653\frac{t^{\beta}}{\Gamma(\beta+1)}} + 3261.8759\frac{t^{2\beta}}{\Gamma(2\beta+1)} \right),$$

$$E(t) = 2000 - 120.8841\frac{t^{\beta}}{\Gamma(\beta+1)} - \left(\frac{3018770\frac{t^{2\beta}}{\Gamma(2\beta+1)}}{10^5 + 26.653\frac{t^{\beta}}{\Gamma(\beta+1)}} - 12.094\frac{t^{2\beta}}{\Gamma(2\beta+1)} \right),$$

$$I(t) = 300 - 161.2279\frac{t^{\beta}}{\Gamma(\beta+1)} + 182.045\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$H(t) = 1200 + 8143.9448\frac{t^{\beta}}{\Gamma(\beta+1)} - 3573.4237\frac{t^{2\beta}}{\Gamma(2\beta+1)}, \quad (17)$$

$$R(t) = 1 + 206.5714\frac{t^{\beta}}{\Gamma(\beta+1)} + 123.0523\frac{t^{2\beta}}{\Gamma(2\beta+1)},$$

$$M(t) = 1000 + 26.653\frac{t^{\beta}}{\Gamma(\beta+1)} - 14.5198\frac{t^{2\beta}}{\Gamma(2\beta+1)}.$$

The solution of the model for $\beta = 1$, is given as follows;

$$S(t) = 20000 - 8079.9849t + \frac{1509385t^2}{10^5 + 26.653t} + 1630.9379t^2 + ...,$$

$$E(t) = 2000 - 120.8841t + \frac{1509385t^2}{10^5 + 26.653t} - 6.047t^2 + ...,$$

$$I(t) = 300 - 161.2279t + 91.0225t^2 + ...,$$

$$H(t) = 1200 + 8143.9448t + 1786.7119t^2 + ..., \quad (18)$$

$$R(t) = 1 + 206.5714t + 61.52615t^2 + ...,$$

$$M(t) = 1000 + 26.653t - 7.2599t^2 + ...$$

The solution of the model for $\beta = 0.95$, is given as follows;

$$S(t) = 20000 - 8245.6884t^{0.95} + \frac{1651948.123t^{1.9}}{10^5 + 27.1997t^{0.95}} + 1784.9819t^{1.9} + ...,$$

$$E(t) = 2000 - 123.3637t^{0.95} - \frac{1651948.123t^{1.9}}{10^5 + 27.1997t^{0.95}} - 6.6181t^{1.9} + ...,$$

$$I(t) = 300 - 164.5351t^{0.95} + 99.6197t^{1.9} + ...,$$

$$H(t) = 1200 + 8310.9958t^{0.95} - 1955.4684t^{1.9} + ..., \quad (19)$$

$$R(t) = 1 + 210.8087t^{0.95} + 67.3374t^{1.9} + ...,$$

$$M(t) = 1000 + 27.1997t^{0.95} - 7.9456t^{1.9} + ...$$

The solution of the model for $\beta = 0.9$,is given as follows;

$$S(t) = 20000 - 8400.8995t^{0.9} + \frac{1800638.2344t^{1.8}}{10^5 + 27.7116t^{0.9}} + 1945.6462t^{1.8} + ...,$$

$$E(t) = 2000 - 125.6852t^{0.9} - \frac{1800638.2344t^{1.8}}{10^5 + 27.7116t^{0.9}} - 7.2138t^{1.8} + ...,$$

$$I(t) = 300 - 167.6314t^{0.9} + 108.5863t^{1.8} + ...,$$

$$H(t) = 1200 + 8467.3995t^{0.9} - 2131.4785t^{1.8} + \ldots, \quad (20)$$

$$R(t) = 1 + 214.7758t^{0.9} + 73.3983t^{1.8} + \ldots,$$

$$M(t) = 1000 + 27.7116t^{0.9} - 8.6608t^{1.8} + \ldots$$

The solution of the model for $\beta = 0.85$, is given as follows;

$$S(t) = 20000 - 8544.8234t^{0.85} + \frac{1954275.9112t^{1.7}}{10^5 + 28.1863t^{0.85}} + 2111.6566t^{1.7} + \ldots,$$

$$E(t) = 2000 - 127.8385t^{0.85} - \frac{1954275.9112t^{1.7}}{10^5 + 28.1863t^{0.85}} - 7.8294t^{1.7} + \ldots,$$

$$I(t) = 300 - 170.5033t^{0.85} + 117.8514t^{1.7} + \ldots,$$

$$H(t) = 1200 + 8612.4628t^{0.85} - 2313.3448t^{1.7} + \ldots, \quad (21)$$

$$R(t) = 1 + 218.4554t^{0.85} + 79.661t^{1.7} + \ldots,$$

$$M(t) = 1000 + 28.1863t^{0.85} - 9.3998t^{1.7} + \ldots$$

The solution of the model for $\beta = 0.8$, is given as follows;

$$S(t) = 20000 - 8675.0967t^{0.8} + \frac{2111618.6346t^{1.6}}{10^5 + 28.6161t^{0.8}} + 2281.6703t^{1.6} + \ldots,$$

$$E(t) = 2000 - 129.7875t^{0.8} - \frac{2111618.6346t^{1.6}}{10^5 + 28.6161t^{0.8}} - 8.4597t^{1.6} + \ldots,$$

$$I(t) = 300 - 173.1027t^{0.8} + 127.3398t^{1.6} + \ldots,$$

$$H(t) = 1200 + 8743.7672t^{0.8} - 2499.5969t^{1.6} + \ldots, \quad (22)$$

$$R(t) = 1 + 221.7859t^{0.8} + 86.0746t^{1.6} + \ldots,$$

$$M(t) = 1000 + 28.6161t^{0.8} - 10.1565t^{1.6} + \ldots$$

The solution of the model for $\beta = 0.75$, is given as follows;

$$S(t) = 20000 - 8791.1926t^{0.75} + \frac{2270947.1150t^{1.5}}{10^5 + 28.999t^{0.75}} + 2453.8298t^{1.5} + \ldots,$$

$$E(t) = 2000 - 131.5244t^{0.75} - \frac{2270947.1150t^{1.5}}{10^5 + 28.999t^{0.75}} - 9.098t^{1.5} + \ldots,$$

$$I(t)=300-175.4193t^{0.75}+136.9480t^{1.5}+\ldots, \quad (23)$$

$$H(t)=1200+8860.7821t^{0.75}-2688.1996t^{1.5}+\ldots,$$

$$R(t)=1+224.754t^{0.75}+92.5692t^{1.5}+\ldots,$$

$$M(t)=1000+28.9991t^{0.75}-10.9229t^{1.5}+\ldots$$

4.1 Numerical Simulations

In this section, we carried out the numerical Simulations of the fractional order model (5) with a view to make comparison with classical integer order.

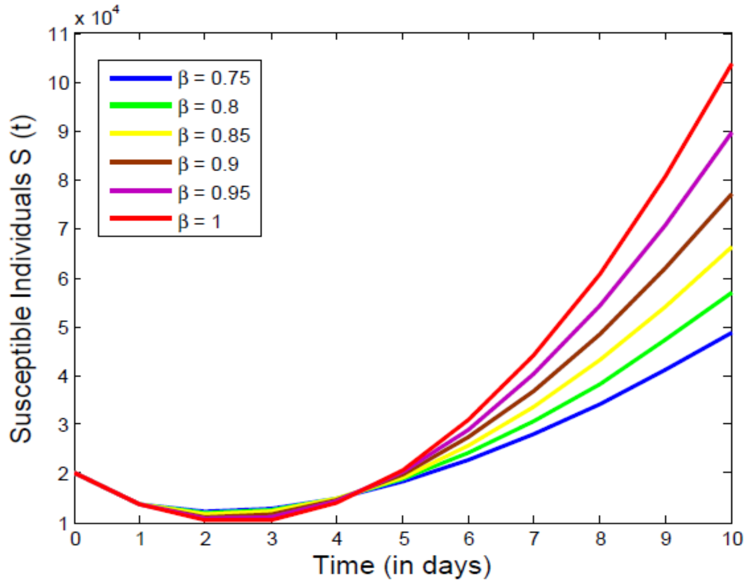

Fig 2. The simulation of the susceptible individuals with variation in β.

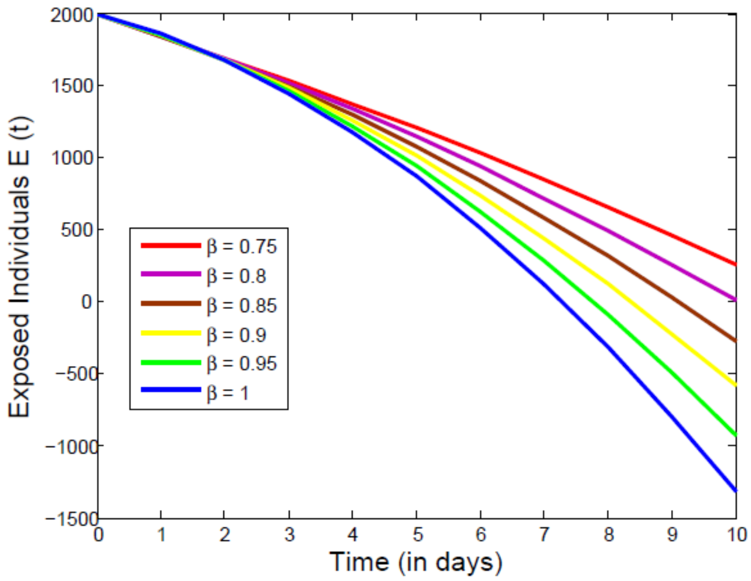

Fig 3. The simulation of the exposed individuals with variation in β.

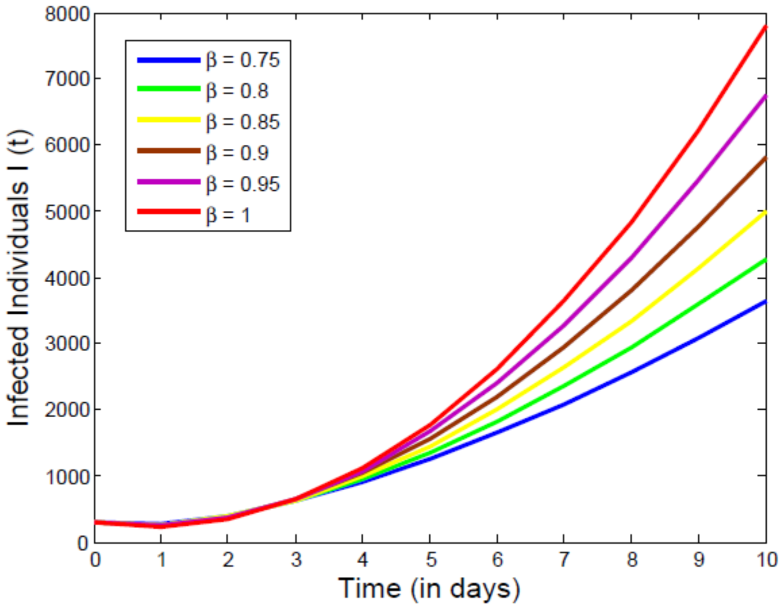

Fig 4. The simulation of the infected individuals with variation in β.

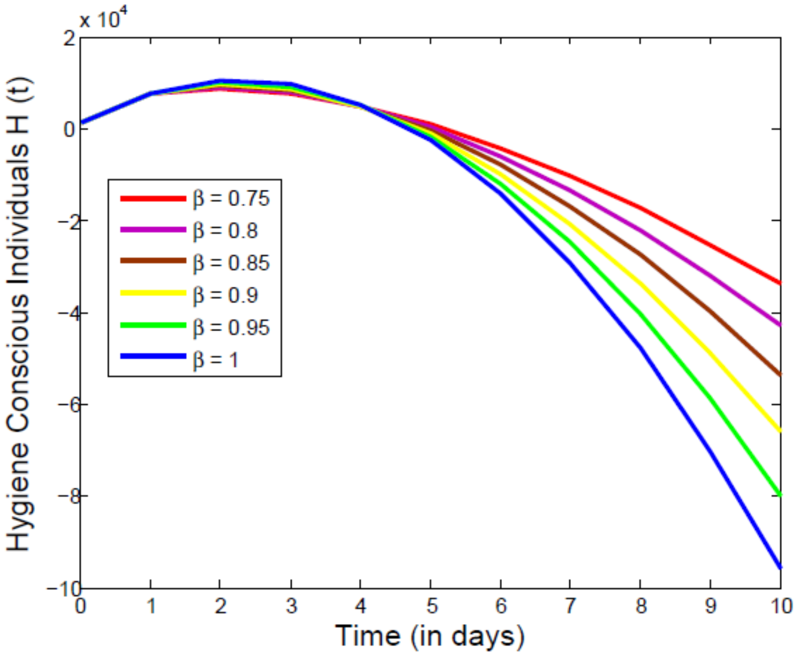

Fig 5. The simulation of the hygiene conscious individuals with variation in β.

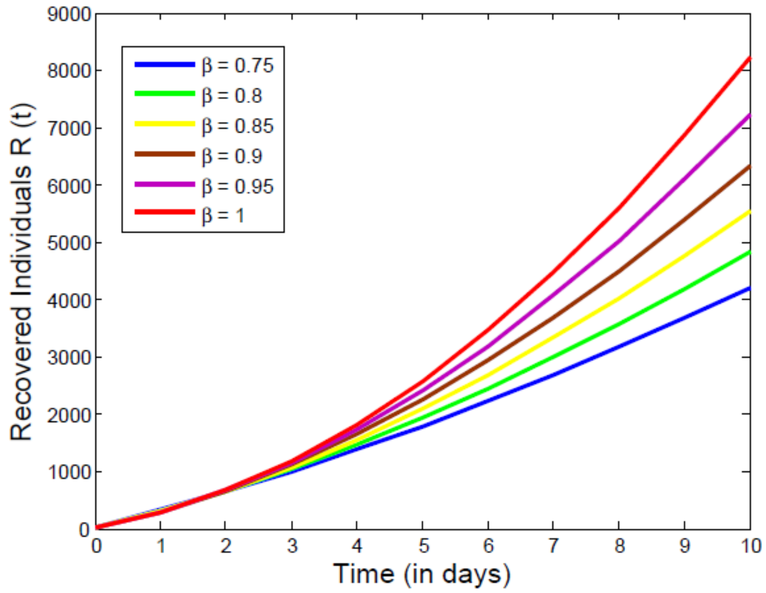

Fig 6. The simulation of the recovered individuals with variation in β .

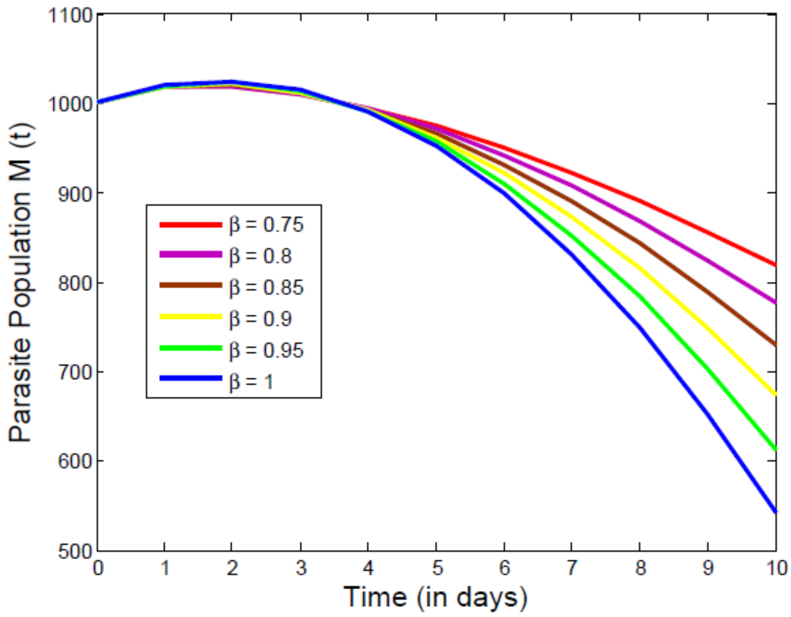

Fig 7. The simulation of the parasite population with variation in β .

Fig 2 depicts the simulation of the susceptible individuals while varying β, it is observed that as the order of the fractional derivative β is increasing, the number of the individuals susceptible to the soil-transmitted helminth infections increases. Fig 3 depicts the simulation of the exposed individuals while varying β , it is observed that as the order of the fractional derivative β is increasing, the number of the individuals exposed to the soil-transmitted helminth infections decreases. Fig 4 is the simulation of the infected individuals with variation in β , it is observed that as the order of the fractional derivative β is increasing, the number of the infected individuals decreases. A similarly result was observed for the hygiene conscious individuals in Fig 5.

Fig 6 is the simulation of the recovered individuals with variation in β, it is observed that as the fractional derivative β increases, the number of recovered individuals increases. Fig 7 which represents the simulation of the parasite population while varying β, it is observed in Fig 7 that as the fractional derivatives β is increasing, the parasite population reduces.

4.2 Convergence Analysis

The solution obtained in (17) is a series which is rapidly and uniformly convergent to the exact solution. To check convergence of series (17), we use classical techniques (see [10]), for sufficient conditions of convergence of this method, we give the following theorem by using idea in Shah et al [21].

Theorem 1. Let Y be a Banach space and $G:Y\to Y$ be a contractive non-linear operator, such that for all $y, y^1 \in Y, \left\|G(y)-G^1\left(y^1\right)\right\| \le k\left\|y-y^1\right\|, 0<k<1.$ Then, G has a unique point y such that $Gy = y$, where $y=\left(S,E,Q,A,I,M,I_H,R\right)$. The series given in (17) can be written by applying Adomian decomposition method as follows:

$$y_n = Gy_{n-1}, \quad y_{n-1}=\sum_{i=1}^{n-1} y_i, \quad n=1,2,3,...,$$

And assume that $y_0 \in B_r(y)$, where

$$B_r(y)=\left\{y^1 \in Y : \left\|y^1-y\right\|<r\right\},$$

Then, we have:

(i) $y_n \in B_r(y)$,

(ii) $\lim_{n\to\infty} y_n = y$.

Proof

(i) Using mathematical induction for $n=1$, we have:

$$\|y_0-y\|=\|G(y_0)-G(y)\|\le k\|y_0-y\|.$$

Let the result be true for $m-1$, then,

$$\|y_0-y\|\le k^{m-1}\|y_0-y\|.$$

We have:

$$\|y_m-y\|=\|G(y_{m-1})-G(y)\|\le k\|y_{m-1}-y\|\le k^m\|y_0-y\|.$$

i.e. $\|y_n - y\| \le k^n \|y_0 - y\| \le k^n r < r$.

$\Rightarrow \quad y_n \in B_r(y)$.

(ii) Since $\|y_n - y\| \le k^n \|y_0 - y\|$

and as $\lim_{n\to\infty} k^n = 0$,

therefore, we have:

$$\lim_{n\to\infty} \|y_n - y\| = 0$$

$$\Rightarrow \quad \lim_{n\to\infty} y_n = y$$

5. Conclusion

Inspired by the advantages of fractional order models over classical integer-order models, we extended the work of Oguntolu et al. [9] by reformulating their classical integer-order model into a fractional order model. We utilized the renowned Laplace Adomian Decomposition Method (LADM) to generate approximate solutions for the system of fractional order differential equations in series form. Our findings showed that these approximate solutions align qualitatively with other numerical solutions. As is required for numerical solutions to systems of non-linear differential equations, we demonstrated that our approximate solutions converge to the exact solutions. The numerical simulation of the fractional order model revealed that it provides greater flexibility, as the order of the fractional differential equations can be easily adjusted to reflect different responses in real time.

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