# Nonparametric Statistical Methods for the CostEffectiveness Analysis 

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## 1. Introduction

Cost-effectiveness analyses are commonly used techniques in health services research. In a cost-effectiveness analysis, two groups of subjects are often compared; e.g., new intervention against existing intervention or treatment A versus treatment $B$. As its name suggests, a cost-effectiveness analysis considers both cost and effectiveness of the interventions. The two measures commonly used in a cost-effectiveness analysis are the incremental costeffectiveness ratio (ICER) and the net health benefit (NHB). Each measure has its own advantages and disadvantages as we summarize in the list below. In this paper, we do not recommend one approach over the other. For this reason, we discuss statistical methods for both measures.

Formally, let the cost and effectiveness for subject $j$ who receives treatment $i$ be $C_{i j}$ and $E_{i j}$, respectively. Let us further assume that $\left(C_{i j}, E_{i j}\right)$ comes from a bivariate distribution with mean $\left(\mu_{C_{i}}, \mu_{E_{i}}\right)$, variance $\left(\sigma_{C_{i}}^{2}, \sigma_{E_{i}}^{2}\right)$ and covariance $\sigma_{E_{i} C_{i}}$. That is, for $\mathrm{i}=1,2$,

$$
\binom{C_{i 1}}{E_{i 1}},\binom{C_{i 2}}{E_{i 2}}, \ldots,\binom{C_{i n_{i}}}{E_{i n_{i}}} \sim_{i . i . d}\left(\binom{\mu_{C_{i}}}{\mu_{E_{i}}},\left(\begin{array}{cc}
\sigma_{C_{i}}^{2} & \sigma_{C_{i} E_{i}}  \tag{1}\\
\sigma_{C_{i} E_{i}} & \sigma_{E_{i}}^{2}
\end{array}\right)\right)
$$

If $\lambda$ is the willingness-to-pay per unit of the effectiveness (Stinnett and Mullahy, 1998), the ICER and the NHB are defined by

$$
I C E R=\frac{\mu_{C_{1}}-\mu_{C_{2}}}{\mu_{E_{1}}-\mu_{E_{2}}}, \quad N H B=\left(\mu_{E_{1}}-\mu_{E_{2}}\right)-\frac{1}{\lambda}\left(\mu_{C_{1}}-\mu_{C_{2}}\right)
$$

respectively
Using sample data, these two quantities can be estimated as:

$$
I \widehat{C E} R=\frac{\bar{C}_{1}-\bar{C}_{2}}{\bar{E}_{1}-\bar{E}_{2}}, \quad \widehat{N H B}=\left(\bar{E}_{1}-\bar{E}_{2}\right)-\frac{1}{\lambda}\left(\bar{C}_{1}-\bar{C}_{2}\right),
$$

respectively, where $\bar{C}_{i}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}} C_{i j}$ and $\bar{E}_{i}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}} E_{i j}(\mathrm{i}=1,2)$.

## - Arguments for the use of the ICER include:

- The ICER has a natural interpretation as a price-per-unit of product (e.g., $\$ 15,000$ per quality-adjusted life year saved).
- Analysis on the ICER is independent of the cut-off (willingness-to-pay) $\lambda$ value. This cut-off value will vary from context to context depending on preferences and the available budgets (Briggs and Fenn, 1998).
- The ICER has an economic foundation based on utility theory. Maximizing expected utility leads to finding optimal incremental cost-effectiveness ratio (Garber and Phelps, 1997).


## - Arguments against the use of the ICER include:

- Interpretation of a negative ICER is problematic. First, the magnitude of negative ICERs is meaningless. An intervention that is preferred may have smaller (negative) ICER (Briggs and Fenn, 1998). Second, both quadrants II and IV on the cost-effectiveness plane (Figure 1) generate negative ICERs; however, decision-makings are exactly opposite (Stinnett and Mullahy, 1998).
- Confidence intervals for the ICER can include an undefined value. Figure 1 shows a cost-effectiveness plane with the confidence interval represented by the two broken lines. This interval includes the vertical axis where $\mu_{E_{1}}-\mu_{E_{2}}=0$ and an undefined value for the ICER (Willan, 2003).
[Figure 1 here]
- Statistically, $\widehat{I C E} R$ is not a sufficient statistic and $\widehat{I C E} R$ is biased for ICER (Zethraeus et al., 2003).
- Arguments for the use of the NHB include:
- The NHB is properly ordered. Treatment with the largest NHB is the most cost-effective (Willan, 2003).
- Interpretation for the NHB is not ambiguous like the negative ICER.
- Since the NHB is linear in both costs and effects, $\widehat{N H B}$ is unbiased for NHB. Also, asymptotic results may apply at smaller sample sizes.
- The NHB can be extended easily to more than two treatments.
- Arguments against the use of the NHB include:
- The NHB incorporates $\lambda$ into the analysis. This $\lambda$ may vary from setting to setting and the analyst's $\lambda$ may be different from the policy maker's $\lambda$.
- The NHB does not have a natural interpretation as price per unit of product like the ICER.

Nonparametric statistical methods have been developed for inferences on the two measures. However, due to the skew nature of cost data, existing methods have not adequately addressed the problems. In this paper, we are interested in studying the theoretical performance of normal-based intervals and constructing new confidence intervals for the ICER and the NHB.

### 1.1 Existing Methods

Confidence intervals for the ICER and the NHB have been constructed using several existing techniques. For the ICER, intervals based on the asymptotic theory, the Fieller's (Fieller, 1954), and several versions of the bootstrap have been adopted for inferences on the ICER. For reviews and evaluations of these techniques, see Briggs et al. (1999) and Fan and Zhou (2005).

For the NHB, similar techniques have been proposed. They include the normal interval based on large sample theory and the bootstrap intervals (Stinnett and Mullahy, 1998; Willan, 2001).


### 1.2 Limitations of Existing Methods

For the ICER, normal theory intervals and bootstrap intervals which assume normal sampling distribution have poor coverages (Briggs et al., 1999; Fan and Zhou, 2005). In term of coverage accuracy, Briggs et al. (1999) found the Fieller's interval was best while Fan and Zhou (2005) recommended the bootstrap-t interval. However, Fieller's interval requires finding roots of a quadratic equation and these can be unreal. In addition, if this quadratic equation has one root, the confidence interval will be half-open. In our simulation study presented later, we also found the Fieller's interval may not give best coverage. The bootstrap-t interval gives good coverage but is often too wide. When having such wide intervals, one has to consider the trade off between the coverage accuracy and the precision of the estimate.

For the NHB, inferences based on large sample theory may not be appropriate for highly skewed data. Our simulation study indicates that coverage of such normal theory intervals can differ significantly from the nominal value.

### 1.3 Proposed Approach

Beside the above methods, another approach is to modify the test statistics to reduce the effect of skewness. The method is based on the Edgeworth expansion (Hall, 1992a). We will follow this approach in this paper.

In Sections 2 and 3, we will develop the Edgeworth expansions for the studentized t-statistics for the ICER and the NHB, respectively. We will use the expansions to study the theoretical performance of existing normal theory intervals and to derive new transformational intervals to improve coverage accuracy. In Section 4, we will demonstrate the method via a simulation study. In Section 5, we will apply our method to a real cost data set. In

Section 6, we will summarize the methods and provide our recommendation.

## 2. Edgeworth expansion for the incremental cost-effectiveness ratio (ICER)

Cost-effectiveness analyses are often hindered by skewed cost data. In this section, we derive the Edgeworth expansion for the studentized t -statistic for the ICER. The expansion will then be used to guide inferences. In addition, we use the expansion to derive new transformational intervals for the ICER.

Assume our data is given as in equation (1). The ICER is defined by $\theta_{1} \equiv I C E R=\frac{\mu_{C_{1}}-\mu_{C_{2}}}{\mu_{E_{1}}-\mu_{E_{2}}}$ and is estimated by $\hat{\theta}_{1} \equiv I \widehat{C E} R=\frac{\bar{C}_{1}-\bar{C}_{2}}{E_{1}-E_{2}}$. The asymptotic variance of the $I \widehat{C E} R$ can be estimated by
$\hat{\sigma}_{1}^{2}=\frac{\left[\frac{S_{C_{1}}^{2}}{n_{1}}+\frac{S_{C_{2}}^{2}}{n_{2}}\right]}{\left(\bar{E}_{1}-\bar{E}_{2}\right)^{2}}+\frac{\left(\bar{C}_{1}-\bar{C}_{2}\right)^{2}\left[\frac{S_{E_{1}}^{2}}{n_{1}}+\frac{S_{E_{2}}^{2}}{n_{2}}\right]}{\left(\bar{E}_{1}-\bar{E}_{2}\right)^{4}}-\frac{2\left(\bar{C}_{1}-\bar{C}_{2}\right)\left[\frac{S_{E_{1} C_{1}}}{n_{1}}+\frac{S_{E_{2} C_{2}}}{n_{2}}\right]}{\left(\bar{E}_{1}-\bar{E}_{2}\right)^{3}}$
where, for $\mathrm{i}=1,2, \bar{C}_{i}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}} C_{i j}, \bar{E}_{i}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}} E_{i j}, S_{C_{i}}^{2}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}}\left(C_{i j}-\right.$ $\left.\bar{C}_{i}\right)^{2}, S_{E_{i}}^{2}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}}\left(E_{i j}-\bar{E}_{i}\right)^{2}$, and $S_{C_{i} E_{i}}=\frac{1}{n_{i}} \sum_{j=1}^{n_{i}}\left(C_{i j}-\bar{C}_{i}\right)\left(E_{i j}-\bar{E}_{i}\right)$.

Theorem 1 Let $\nu_{N}=n_{1} /\left(n_{1}+n_{2}\right)=n_{1} / N$. Assume $\nu_{N}=\nu+O\left(N^{-r}\right)$ for some $r \geq 0$. Let $T_{1}=\frac{\widehat{C E R-I C E R}}{\hat{\sigma}_{1}}$. Under regularity conditions (Hall, 1992a), the distribution of $T_{1}$ has the following expansion,

$$
\begin{equation*}
P\left(T_{1} \leq x\right)=\Phi(x)+N^{-1 / 2} q_{1}(x) \phi(x)+O\left(N^{-\min (1, r+1 / 2)}\right) \tag{3}
\end{equation*}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and distribution functions, respectively, of the standard normal variable, and, with $E(\cdot)$ is the expectation function,

$$
\begin{aligned}
& q_{1}(x)=\frac{A_{1}}{3}+\frac{2 A_{1}+3 A_{2}}{3} x^{2}, \\
& A_{1}=\frac{1}{2 \Delta_{1}^{6} Q_{1}^{3 / 2}}\left[\frac{P_{1}}{\nu^{2}}-\frac{P_{2}}{(1-\nu)^{2}}\right], \quad A_{2}=\frac{\left(\Delta_{2} \Gamma_{1}-\Delta_{1} \Gamma_{12}\right)}{\Delta_{1}^{3} Q_{1}^{1 / 2}}, \\
& P_{1}=E\left[\Delta_{1}\left(C_{1 j}-\mu_{C_{1}}\right)-\Delta_{2}\left(E_{1 j}-\mu_{E_{1}}\right)\right]^{3}, \\
& P_{2}=E\left[\Delta_{1}\left(C_{2 j}-\mu_{C_{2}}\right)-\Delta_{2}\left(E_{2 j}-\mu_{E_{2}}\right)\right]^{3},
\end{aligned}
$$

$$
\begin{aligned}
& Q_{1}=\frac{\Gamma_{2}}{\Delta_{1}^{2}}+\frac{\Gamma_{1} \Delta_{2}^{2}}{\Delta_{1}^{4}}-2 \frac{\Delta_{2} \Gamma_{12}}{\Delta_{1}^{3}}, \quad \Delta_{1}=\mu_{E_{1}}-\mu_{E_{2}}, \quad \Delta_{2}=\mu_{C_{1}}-\mu_{C_{2}}, \\
& \Gamma_{1}=\frac{\sigma_{E_{1}}^{2}}{\nu}+\frac{\sigma_{E_{2}}^{2}}{1-\nu}, \quad \Gamma_{2}=\frac{\sigma_{C_{1}}^{2}}{\nu}+\frac{\sigma_{C_{2}}^{2}}{1-\nu}, \quad \Gamma_{12}=\frac{\sigma_{E_{1} C_{1}}}{\nu}+\frac{\sigma_{E_{2} C_{2}}}{1-\nu} .
\end{aligned}
$$

For a proof, see Appendix A1.
From the expansion (3), we see that $q_{1}(x) / \sqrt{N}$, in absoluate value, plays an important role in the normal approximation of $T_{1}$. When $\left|q_{1}(x)\right| / \sqrt{N}$ is small, $T_{1}$ can be approximated by a normal distribution accurately. On the contrary, when $\left|q_{1}(x)\right| / \sqrt{N}$ is large, the second term in equation (3) is not ignorable, thus the normal approximation won't be as accurate. The term $q_{1}(x)$ can be large when either $A_{1}$ or $A_{2}$, or both are large. The quantity $A_{1}$ relates directly to the skewness of the cost data, thus can be large when the cost data is highly skewed. The term $A_{2}$ relates to the variance and covariance of the data, which can be large when the data is highly variable.

The expansion can be used to construct transformational confidence intervals for the ICER. The intervals will correct for the term $q_{1}(x)$ in the expansion (3). Following the similar ideas as in Hall (1992b) and Zhou and Dinh (2005), with

$$
\begin{aligned}
& g_{1}^{-1}(x)=(\hat{\gamma})^{-1}[1+3 \hat{\gamma}(x+\hat{\gamma} / N)]^{1 / 3}-(\hat{\gamma})^{-1}, \\
& g_{2}^{-1}(x)=\left(2 N^{-1 / 2} \hat{\gamma}\right)^{-1} \log \left[2 N^{-1 / 2} \hat{\gamma}\left(x+N^{-1} \hat{\gamma}\right)+1\right], \\
& g_{3}^{-1}(x)=[1+3(x+\hat{\gamma} / N)]^{1 / 3}-1,
\end{aligned}
$$

we proposed three transformational intervals,

$$
\begin{equation*}
\hat{\theta}_{1}-\hat{\sigma}_{1}\left[\frac{\hat{c}_{1}}{\sqrt{N}}+\sqrt{N} g_{i}^{-1}\left(\frac{z_{1-\alpha / 2}}{\sqrt{N}}\right)\right] \leq \theta_{1} \leq \hat{\theta}_{1}-\hat{\sigma}_{1}\left[\frac{\hat{c}_{1}}{\sqrt{N}}+\sqrt{N} g_{i}^{-1}\left(\frac{z_{\alpha / 2}}{\sqrt{N}}\right)\right] \tag{4}
\end{equation*}
$$

where, $\mathrm{i}=1,2,3, z_{\alpha}=\Phi(\alpha), \hat{\gamma}=\frac{2 \hat{A}_{1}+3 \hat{A}_{2}}{3}, \hat{c}_{1}=-\hat{A}_{1}-\hat{A}_{2}$,
$\hat{A}_{1}=\frac{1}{2 \hat{\Delta}_{1}^{\hat{Q}} \hat{Q}_{1}^{3 / 2}}\left[\frac{\hat{P}_{1}}{\nu_{N}^{2}}-\frac{\hat{P}_{2}}{\left(1-\nu_{N}\right)^{2}}\right], \quad \hat{A}_{2}=\frac{\left(\hat{\Delta}_{2} \hat{\Gamma}_{1}-\hat{\Delta}_{1} \hat{\Gamma}_{12}\right)}{\hat{\Delta}_{1}^{3} \hat{Q}_{1}^{1 / 2}}$,
$\hat{P}_{1}=\frac{1}{n_{1}} \sum_{j=1}^{n_{1}}\left[\hat{\Delta}_{1}\left(C_{1 j}-\bar{C}_{1}\right)-\hat{\Delta}_{2}\left(E_{1 j}-\bar{E}_{1}\right)\right]^{3}$,

$$
\begin{aligned}
& \hat{P}_{2}=\frac{1}{n_{2}} \sum_{j=1}^{n_{2}}\left[\hat{\Delta}_{1}\left(C_{2 j}-\bar{C}_{2}\right)-\hat{\Delta}_{2}\left(E_{2 j}-\bar{E}_{2}\right)\right]^{3}, \\
& \hat{Q}_{1}=\frac{\hat{\Gamma}_{2}}{\Delta_{2}^{2}}+\frac{\hat{\Gamma}_{1} \hat{\Delta}_{2}^{2}}{\Delta_{2}^{1}}-2 \frac{\hat{\Delta}_{2} \hat{\Gamma}_{12}}{\Delta_{1}^{3}}, \quad \hat{\Delta}_{1}=\bar{E}_{1}-\bar{E}_{2}, \quad \hat{\Delta}_{2}=\bar{C}_{1}-\bar{C}_{2}, \\
& \hat{\Gamma}_{1}=\frac{S_{E_{1}}^{2}}{\nu_{N}}+\frac{S_{E_{2}}^{2}}{1-\nu_{N}}, \quad \hat{\Gamma}_{2}=\frac{S_{C_{1}}^{2}}{\nu_{N}}+\frac{S_{C_{2}}^{2}}{1-\nu_{N}}, \quad \hat{\Gamma}_{12}=\frac{S_{E_{1} C_{1}}}{\nu_{N}}+\frac{S_{E_{2} C_{2}}}{1-\nu_{N}} .
\end{aligned}
$$

For the derivation, see Appendix A2.

## 3. Edgeworth expansion for the net health benefit (NHB)

In this section, we present the Edgeworth expansion for the studentized tstatistic for the NHB. The expansion provides a way to correct for the skewness in cost data and to derive new confidence intervals for the NHB.

Let the data be given as in equation (1), the NHB is defined by $\theta_{2} \equiv$ $N H B=\left(\mu_{E_{1}}-\mu_{E_{2}}\right)-\frac{1}{\lambda}\left(\mu_{C_{1}}-\mu_{C_{2}}\right)$ and can be estimated as $\hat{\theta}_{2} \equiv \widehat{N H B}=$ $\left(\bar{E}_{1}-\bar{E}_{2}\right)-\frac{1}{\lambda}\left(\bar{C}_{1}-\bar{C}_{2}\right)$. The asymptotic variance of $\hat{\theta}_{2}$ can be estimated by,

$$
\begin{equation*}
\hat{\sigma}_{2}^{2}=\frac{S_{E_{1}}^{2}}{n_{1}}+\frac{S_{E_{2}}^{2}}{n_{2}}+\frac{S_{C_{1}}^{2}}{\lambda^{2} n_{1}}+\frac{S_{C_{2}}^{2}}{\lambda^{2} n_{2}}-\frac{2 S_{C_{1} E_{1}}}{\lambda n_{1}}-\frac{2 S_{C_{2} E_{2}}}{\lambda n_{2}} \tag{5}
\end{equation*}
$$

Theorem 2 Let $\nu_{N}=n_{1} /\left(n_{1}+n_{2}\right)=n_{1} / N$. Assume $\nu_{N}=\nu+O\left(N^{-r}\right)$ for some $r \geq 0$. Let $T_{2}=\frac{\widehat{N H B}-N H B}{\hat{\sigma}_{2}}$. Under regularity conditions (Hall, 1992a), the distribution of $T_{2}$ has the following expansion,

$$
\begin{equation*}
P\left(T_{2} \leq x\right)=\Phi(x)+N^{-1 / 2} q_{2}(x) \phi(x)+O\left(N^{-\min (1, r+1 / 2)}\right) \tag{6}
\end{equation*}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and distribution functions, respectively, of the standard normal variable,

$$
\begin{aligned}
& q_{2}(x)=\frac{A}{6}\left(2 x^{2}+1\right), \\
& Q_{2}=\frac{\sigma_{E_{1}}^{2}}{\nu}+\frac{\sigma_{E_{2}}^{2}}{(1-\nu)}+\frac{\sigma_{C_{1}}^{2}}{\lambda^{2} \nu}+\frac{\sigma_{C_{2}}^{2}}{\lambda^{2}(1-\nu)}-\frac{2 \sigma_{C_{1} E_{1}}}{\lambda \nu}-\frac{2 \sigma_{C_{2} E_{2}}}{\lambda(1-\nu)} \\
& B_{1}=E\left(\frac{C_{1 j}-\mu_{C_{1}}}{\lambda}-\left(E_{1 j}-\mu_{E_{1}}\right)\right)^{3}, \quad B_{2}=E\left(\frac{C_{2 j}-\mu_{C_{2}}}{\lambda}-\left(E_{2 j}-\mu_{E_{2}}\right)\right)^{3},
\end{aligned}
$$

where $E(\cdot)$ is the expectation function, and

$$
\begin{equation*}
A=\frac{1}{Q_{2}^{3 / 2}}\left[\frac{B_{1}}{\nu^{2}}-\frac{B_{2}}{(1-\nu)^{2}}\right] \tag{7}
\end{equation*}
$$

For a proof, see appendix B.
Similar to the ICER, we see that the normal approximation to $T_{2}$ is accurate when $\left|q_{2}(x)\right| / \sqrt{N}$ is small. When $\left|q_{2}(x)\right| / \sqrt{N}$ is large, the second term in the expansion (6) can't be ignored and the normal approximation won't be as accurate. The quantity $q_{2}(x)$ relates directly to the skewness of the cost data, thus can be large when the cost data is highly skewed.

Similar to the ICER, we can derive the three transformational confidence intervals for the NHB. The intervals will correct for the term $q_{2}(x)$ in the expansion (6) above. With,

$$
\begin{aligned}
& g_{1}^{-1}(x)=\left(\frac{1}{3} \hat{A}\right)^{-1}\left[1+3 \frac{1}{3} \hat{A}\left(x-N^{-1} \frac{1}{6} \hat{A}\right)\right]^{1 / 3}-\left(\frac{1}{3} \hat{A}\right)^{-1}, \\
& g_{2}^{-1}(x)=\left(2 \frac{1}{3} N^{-1 / 2} \hat{A}\right)^{-1} \log \left[2 \frac{1}{3} N^{-1 / 2} \hat{A}\left(x-N^{-1} \frac{1}{6} \hat{A}\right)+1\right], \\
& g_{3}^{-1}(x)=\left[1+3\left(x-N^{-1} \frac{1}{6} \hat{A}\right)\right]^{1 / 3}-1,
\end{aligned}
$$

the $(1-\alpha) 100 \%$ transformational confidence interval for the NHB is given by

$$
\begin{equation*}
\widehat{N H B}-\sqrt{N} g_{i}^{-1}\left(\frac{z_{1-\alpha / 2}}{\sqrt{N}}\right) \hat{\sigma}_{2} \leq N H B \leq \widehat{N H B}-\sqrt{N} g_{i}^{-1}\left(\frac{z_{\alpha / 2}}{\sqrt{N}}\right) \hat{\sigma}_{2} \tag{8}
\end{equation*}
$$

where, $\mathrm{i}=1,2,3, z_{\alpha}=\Phi(\alpha), \hat{A}=\frac{1}{\hat{Q}_{2}^{3 / 2}}\left[\frac{\hat{B}_{1}}{\nu_{N}^{2}}-\frac{\hat{B}_{2}}{\left(1-\nu_{N}\right)^{2}}\right]$, $\hat{Q}_{2}=\frac{S_{E_{1}}^{2}}{\nu_{N}}+\frac{S_{E_{2}}^{2}}{\left(1-\nu_{N}\right)}+\frac{S_{C_{1}}^{2}}{\lambda^{2} \nu_{N}}+\frac{S_{C_{2}}^{2}}{\lambda^{2}\left(1-\nu_{N}\right)}-\frac{2 S_{C_{1} E_{1}}}{\lambda \nu_{N}}-\frac{2 S_{C_{2} E_{2}}}{\lambda\left(1-\nu_{N}\right)}$,
$\hat{B}_{1}=\frac{1}{n_{1}} \sum_{j=1}^{n_{1}}\left(\frac{C_{1 j}-\bar{C}_{1}}{\lambda}-\left(E_{1 j}-\bar{E}_{1}\right)\right)^{3}, \quad \hat{B}_{2}=\frac{1}{n_{2}} \sum_{j=1}^{n_{2}}\left(\frac{C_{2 j}-\bar{C}_{2}}{\lambda}-\left(E_{2 j}-\bar{E}_{2}\right)\right)^{3}$.
4. Simulation study

### 4.1 ICER Simulation

In this section we report a simulation study to evaluate the method presented in Sections 2. In particular, we compared our new intervals given in equation (4) against the Taylor's interval (based on normal theory), the

Fieller's interval (recommended by Briggs et al. (1999)), the bootstrap percentile-t interval (recommended by Fan and Zhou (2005)), and the Hinkley's interval. Details of these methods are given in Appendix C.

We generated data from three bivariate distributions: bivariate normal, bivariate lognormal, and bivariate mixture (costs are lognormal and effectiveness data are normal). We chose different correlation structures for costs and effects and sample sizes as presented in Tables 1 and 2. The data were generated as following:

Bivariate normal:

$$
\begin{aligned}
& \binom{C_{1 j}}{E_{1 j}} \sim_{i . i . d} N_{2}\left(\binom{\mu_{1}=40,000}{\mu_{2}=60},\left(\begin{array}{cc}
\sigma_{1}^{2}=200,000 & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}=5
\end{array}\right)\right), \\
& \binom{C_{2 j}}{E_{2 j}} \sim_{i . . . d} N_{2}\left(\binom{\mu_{3}=30,000}{\mu_{4}=50},\left(\begin{array}{cc}
\sigma_{3}^{2}=100,000 & \sigma_{34} \\
\sigma_{34} & \sigma_{4}^{2}=10
\end{array}\right)\right),
\end{aligned}
$$

Bivariate mixture:

$$
\begin{aligned}
& \binom{C_{1 j}}{E_{1 j}}=\binom{e^{Y_{C 1 j}}}{Y_{E 1 j}},\binom{Y_{C 1 j}}{Y_{E 1 j}} \sim_{i . i . d} N_{2}\left(\binom{\mu_{1}=8}{\mu_{2}=4},\left(\begin{array}{cc}
\sigma_{1}^{2}=2 & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}=.5
\end{array}\right)\right), \\
& \binom{C_{2 j}}{E_{2 j}}=\binom{e^{Y_{C 2 j}}}{Y_{E 2 j}},\binom{Y_{C 2 j}}{Y_{E 2 j}} \sim_{i . . i . d} N_{2}\left(\binom{\mu_{3}=6}{\mu_{4}=3},\left(\begin{array}{cc}
\sigma_{3}^{2}=2 & \sigma_{34} \\
\sigma_{34} & \sigma_{4}^{2}=.5
\end{array}\right)\right),
\end{aligned}
$$

Bivariate lognormal:

$$
\begin{aligned}
& \binom{C_{1 j}}{E_{1 j}} \sim_{i . i . d} \\
& E X P\left[N_{2}\left(\binom{\mu_{1}=8}{\mu_{2}=4},\left(\begin{array}{cc}
\sigma_{1}^{2}=2 & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}=.5
\end{array}\right)\right)\right], \\
& \binom{C_{2 j}}{E_{2 j}} \sim_{i . i . d} \\
& E X P\left[N_{2}\left(\binom{\mu_{3}=6}{\mu_{4}=3},\left(\begin{array}{cc}
\sigma_{3}^{2}=2 & \sigma_{34} \\
\sigma_{34} & \sigma_{4}^{2}=.5
\end{array}\right)\right)\right] .
\end{aligned}
$$

[Tables 1 and 2 here]
The result of our simulation is presented in Tables 1 and 2. For data generated from normal distributions, all intervals give good coverages. Average interval lengths (presented in the parenthesis) are also comparable for
all methods. For data generated from bivariate mixture and bivariate lognormal distributions, however, confidence intervals based on normal theory (Taylor's) are obviously inadequate. Fieller's intervals are also insufficient in term of coverage accuracy in some cases. Hinkley's intervals are very similar to Fieller's intervals. Overall, the bootstrap-t intervals (boott) appear best in term of coverage accuracy. However, as mentioned before, bootstrap-t intervals are often too wide. When facing with such wide intervals, one has to balance between the coverage accuracy and the precision of the estimate. Our new intervals give better coverage than Taylor's interval. They are comparable and sometimes better than the Fieller's and Hinkley's intervals. $T_{3}$ intervals have comparable coverage and are narrower than bootstrap-t intervals. We also considered other correlation structures and sample sizes. The results were similar and are not reported here.

It is clear from equation (3), $\left|q_{1}(x)\right| / \sqrt{N}$ plays an important factor in the expansion. In our simulation study for the $95 \%$ confidence interval, we saw that when $\left|q_{1}(1.96)\right| / \sqrt{N}$ was large ( $\geq 0.3$ ), the coverages of normal based intervals were inadequate. Such intervals can be improved upon by the bootstrap-t or our new intervals $T_{3}$. On the other hand, when $\left|q_{1}(1.96)\right| / \sqrt{N}$ was small ( $<0.3$ ), normal based intervals gave reasonable coverage results.

In summary of our simulation in this section, we found the bootstrap- t intervals gave best coverage. Our intervals $T_{3}$ gave comparable coverage as bootstrap-t intervals, were about one-third narrower, could be computed easily (with a hand-held calculator), and also required less computing than the bootstrap-t intervals. $T_{3}$ intervals should be recommended for the ICER confidence intervals construction when data coming from skewed distributions
and sample sizes are small.

### 4.2 NHB Simulation

We evaluated our three new intervals given in equation (8) against the normal interval, the bootstrap bias-corrected interval, and the bootstrap percentile-t interval. Details of these intervals are given in Appendix D.

We also simulated data from three setups: bivariate normal, bivariate lognormal, and bivariate mixture (costs are lognormal and effectiveness data are normal). With $n_{1}, n_{2}, \sigma_{12}$, and $\sigma_{34}$ vary as presented in Tables 3 and 4, the data were generated as following:

Bivariate normal:

$$
\begin{aligned}
& \binom{C_{1 j}}{E_{1 j}} \sim_{i . i . d} N_{2}\left(\binom{\mu_{1}=40,000}{\mu_{2}=60},\left(\begin{array}{cc}
\sigma_{1}^{2}=200,000 & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}=5
\end{array}\right)\right) \\
& \binom{C_{2 j}}{E_{2 j}} \sim_{i . i . d} N_{2}\left(\binom{\mu_{3}=30,000}{\mu_{4}=50},\left(\begin{array}{cc}
\sigma_{3}^{2}=100,000 & \sigma_{34} \\
\sigma_{34} & \sigma_{4}^{2}=10
\end{array}\right)\right),
\end{aligned}
$$

Bivariate mixture:

$$
\begin{aligned}
& \binom{C_{1 j}}{E_{1 j}}=\binom{e^{Y_{C 1 j}}}{Y_{E 1 j}},\binom{Y_{C 1 j}}{Y_{E 1 j}} \sim_{i . i . d} \quad N_{2}\left(\binom{\mu_{1}=8}{\mu_{2}=4},\left(\begin{array}{cc}
\sigma_{1}^{2}=1 & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}=1
\end{array}\right)\right), \\
& \binom{C_{2 j}}{E_{2 j}}=\binom{e^{Y_{C 2 j}}}{Y_{E 2 j}},\binom{Y_{C 2 j}}{Y_{E 2 j}} \sim_{i . i . d} \quad N_{2}\left(\binom{\mu_{3}=6}{\mu_{4}=3},\left(\begin{array}{cc}
\sigma_{3}^{2}=2 & \sigma_{34} \\
\sigma_{34} & \sigma_{4}^{2}=.5
\end{array}\right)\right),
\end{aligned}
$$

Bivariate lognormal:

$$
\begin{aligned}
& \binom{C_{1 j}}{E_{1 j}} \sim_{i . i . d} \\
& E X P\left[N_{2}\left(\binom{\mu_{1}=8}{\mu_{2}=4},\left(\begin{array}{cc}
\sigma_{1}^{2}=1 & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}=1
\end{array}\right)\right)\right], \\
& \binom{C_{2 j}}{E_{2 j}} \sim_{i . i . d} \\
& E X P\left[N_{2}\left(\binom{\mu_{3}=6}{\mu_{4}=3},\left(\begin{array}{cc}
\sigma_{3}^{2}=2 & \sigma_{34} \\
\sigma_{34} & \sigma_{4}^{2}=.5
\end{array}\right)\right)\right] .
\end{aligned}
$$

[Tables 3 and 4 here]
For data generated from normal and mixture distributions, all intervals give good coverage. Average interval length (in parenthesis) are also comparable. For data generated from lognormal distribution, however, intervals
based on normal theory suffer deficiency in term of coverage accuracy. These coverages can be improved upon by the bootstrap-t and especially intervals based on $T_{3}$ transformation. $T_{3}$ intervals have several advantages over bootstrap methods including shorter interval lengths, easy to compute, and less computing time.

It is clear from equation (6) that coefficient $A=\frac{1}{Q^{3 / 2}}\left[\frac{B_{1}}{\nu^{2}}-\frac{B_{2}}{(1-\nu)^{2}}\right]$ in absolute value and sample sizes play an important role in the normal approximation. Our simulation suggests that when $|\hat{A}| / \sqrt{N}$ is small ( $\leq 0.3$ ), normal intervals give good coverages. On the contrary, when $|\hat{A}| / \sqrt{N}$ is large ( $>0.3$ ), normal intervals suffer deficiency and can be improved by the bootstrap-t or the $T_{3}$ intervals. Thus, when dealing with skewed data, intervals based on $T_{3}$ transformation should be recommended.

## 5. Application

In this section, we applied the methods evaluated above to a real data set. In 2002, Katon et al. (2002) conducted a randomized study to assess whether a collaborative care intervention would increase the number of anxiety-free days for patients with panic disorder, compared to the usual primary care setting. The collaborative care intervention included a systematic patient education and approximately 2 visits with an on-site consulting psychiatrist. To demonstrate our methods, we consider total outpatient non-mental health costs and for measure of effectiveness, we consider the number of days a patient experiences panic disorder during the one-year study period.

The summary statistics are presented in Table 5. Distributions of costs and effectiveness are presented in Figure 2. Costs in both groups are highly skewed with coefficient of skewness 4.93 for the control group and 3.46 for
the intervention group. On average, the control group incurred $\$ 1181.95$ and 77.1 days of panic attack more than the intervention group. The estimated ICER is 15.33 indicating that the intervention arm is dominant.
[Tables 5, 6, and Figure 2 here]

Confidence intervals for the ICER and the NHB are presented in Table 6. Both bootstrap-t and $T_{3}$ intervals are positive showing that the intervention is significantly dominant (the control group incurred more cost and more days of panic disorder). As anticipated, both Fieller and Hinkley intervals are similar and $T_{3}$ interval has shorter length than bootstrap-t. The estimated $\hat{q}(1.96) / \sqrt{N}=.9$, indicating the normal based interval is inadequate. Based on our simulation results, we would recommend using the $T_{3}$ interval as the confidence interval for the ICER.

A common willingness-to-pay is $\lambda=\$ 50,000$, using this value, the estimated NHB is 77.08 . Confidence intervals for the NHB are presented in Table 6. All intervals are relatively similar, especially $T_{1}, T_{2}$, and normal intervals. The coefficient $\hat{A} / \sqrt{N}$ is 0.05 in this setting suggesting that the normal theory interval is adequate. All of these confidence intervals are strictly positive indicating, again, that intervention arm is cost-effective.

## 6. Discussion

In this paper, we derived the Edgeworth expansions for the studentized statistics for the ICER and the NHB and proposed new confidence intervals based on these Edgeworth expansion. We demonstrated via simulation studies that our methods have comparable coverage accuracy and are narrower compared with the current recommendation. In particular, for the ICER, when data
were generated a from skewed distribution, our new intervals gave better coverages than the Taylor's interval. They are comparable and sometimes better than the Fieller's, and Hinkley's intervals. We found that Hinkley's method, that has not been adopted for the ICER, was similar to the Fieller's method in term of coverage accuracy and interval length. Intervals based on $T_{3}$ transformation were comparable to the bootstrap-t intervals in term of coverage but were about one-third narrower.

For the NHB, we saw that intervals based on $T_{3}$ transformation gave good coverages in all cases considered and they were comparable to the bootstrap-t intervals. However, our intervals were narrower than the bootstrap-t intervals and required less computing in term of bootstrap resampling.

When dealing with highly skewed data, our intervals based on the transformation $T_{3}$ should be recommended.

The remaining question is what one should choose between the ICER and the NHB. As it has been pointed out by other authors, and is summarized in our introduction section, each measure has its own advantages as well as disadvantages. The purpose of this paper is not to recommend one measure over the other, but to present new statistical methodology for both. We leave it to the readers to decide which measure is more appropriate for their works.

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authors and do not necessarily represent the authors' institutions.

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## Appendix A

Derivation of the ICER Edgeworth expansion

## A1. Proof of theorem 1

Let $X=\left(X_{1}, X_{2}, X_{3}, X_{4}, X_{5}, X_{6}, X_{7}, X_{8}, X_{9}, X_{10}\right)$ and

$$
Y_{i}=\frac{\left(C_{1 i}-\mu_{C_{1}}\right)}{\sigma_{C_{1}}}, \quad U_{i}=\frac{\left(E_{1 i}-\mu_{E_{1}}\right)}{\sigma_{E_{1}}}, \quad Z_{i}=\frac{\left(C_{2 i}-\mu_{C_{2}}\right)}{\sigma_{C_{2}}}, \quad W_{i}=\frac{\left(E_{2 i}-\mu_{E_{2}}\right)}{\sigma_{E_{2}}}
$$

where

$$
\begin{aligned}
& X_{1}=\bar{Y}, \quad X_{2}=n_{1}^{-1} \sum_{i=1}^{n_{1}} Y_{i}^{2}, \quad X_{3}=\bar{U}, \quad X_{4}=n_{1}^{-1} \sum_{i=1}^{n_{1}} U_{i}^{2} \\
& X_{5}=\bar{Z}, \quad X_{6}=n_{2}^{-1} \sum_{i=1}^{n_{2}} Z_{i}^{2}, \quad X_{7}=\bar{W}, \quad X_{8}=n_{2}^{-1} \sum_{i=1}^{n_{2}} W_{i}^{2}, \\
& X_{9}=n_{1}^{-1} \sum_{i=1}^{n_{1}} Y_{i} U_{i}, \quad X_{10}=n_{2}^{-1} \sum_{i=1}^{n_{2}} Z_{i} W_{i} .
\end{aligned}
$$

Let $k(X)=\hat{\theta}_{1}-\theta_{1}, \quad h(X)=\hat{\sigma}_{1}^{2}, \quad f(X)=\frac{k(X)}{h(X)^{1 / 2}}$, and $\quad T_{1}=N^{1 / 2} f(X)$.
Let $V=E(X)$, then $f(V)=0$. By Taylor expansion, we obtain,

$$
f(X)=\frac{\partial f(V)}{\partial V}(X-V)+\frac{1}{2}(X-V)^{\prime} \frac{\partial^{2} f(V)}{\partial V^{2}}(X-V)+\ldots
$$

Let $J_{1}=N^{1 / 2}\left(\frac{\partial f(V)}{\partial V}(X-V)+\frac{1}{2}(X-V)^{\prime} \frac{\partial^{2} f(V)}{\partial V^{2}}(X-V)\right)$. Let $A_{1}$ and $A_{2}$ be given as in Theorem 1. The first three moments of $J_{1}$ are given by

$$
\begin{aligned}
& E J_{1}=-\left(A_{1}+A_{2}\right) N^{-1 / 2}, \quad E J_{1}^{2}=1+O\left(N^{-1}\right) \\
& E J_{1}^{3}=-\left(7 A_{1}+9 A_{2}\right) N^{-1 / 2}+O\left(N^{-\min (1, r+1 / 2)}\right) .
\end{aligned}
$$

Let $K_{1 N}, K_{2 N}, K_{3 N}$ be the first three cumulants of $J_{1}$. Then

$$
\begin{aligned}
& K_{1 N}=-\left(A_{1}+A_{2}\right) N^{-1 / 2}, \quad K_{2 N}=1+O\left(N^{-1}\right) \\
& K_{3 N}=-\left(4 A_{1}+6 A_{2}\right) N^{-1 / 2}+O\left(N^{-\min (1, r+1 / 2)}\right)
\end{aligned}
$$

Let $\chi_{N}(t)$ be the characteristic function of $J_{1}$. Applying Taylor's expansion to function $f(x)=e^{x}$, we obtain
$\chi_{N}(t)=\exp \left(-\frac{t^{2}}{2}\right)\left[1+N^{-1 / 2}\left(-\left(A_{1}+A_{2}\right)(i t)-\frac{\left(4 A_{1}+6 A_{2}\right)}{6}(i t)^{3}\right)+O\left(N^{-\min (1, r+1 / 2)}\right]\right.$
Let $r_{1}(i t)=-\left(A_{1}+A_{2}\right)(i t)-\frac{\left(4 A_{1}+6 A_{2}\right)}{6}(i t)^{3}$, we can write

$$
\chi_{N}(t)=\exp \left(-\frac{t^{2}}{2}\right)\left[1+N^{-1 / 2} r_{1}(i t)+O\left(N^{-\min (1, r+1 / 2)}\right)\right]
$$

Since $\chi_{N}(t)=\int e^{i t x} d P\left(J_{1} \leq x\right)$ and $e^{-t^{2} / 2}=\int e^{i t x} d \Phi(x)$, this suggests that

$$
P\left(J_{1} \leq x\right)=\Phi(x)+N^{-1 / 2} R_{1}(x)+O\left(N^{-\min (1, r+1 / 2)}\right)
$$

where $R_{1}(x)$ is such a function that its Fourier-Stieltjes transform equals to $r_{1}(i t) e^{-t^{2} / 2}$. Applying integration by part, we obtain,

$$
R_{1}(x)=\left[\left(A_{1}+A_{2}\right)+\frac{4 A_{1}+6 A_{2}}{6}\left(x^{2}-1\right)\right] \phi(x) .
$$

Therefore,

$$
P\left(J_{1} \leq x\right)=\Phi(x)+N^{-1 / 2} q_{1}(x) \phi(x)+O\left(N^{-\min (1, r+1 / 2)}\right) .
$$

Since $T_{1}=J_{1}+O\left(N^{-1}\right)$, Theorem 1 follows.

## A2. Confidence intervals construction

Let $c_{1}=-\left(A_{1}+A_{2}\right)$ and $c_{2}=-\left(7 A_{1}+9 A_{2}\right)$. Following Hall (1992a), let $T_{1}^{\prime}=T_{1}-N^{-1 / 2} \hat{c}_{1}$, then

$$
\begin{aligned}
& P\left(T_{1}^{\prime} \leq x\right)=P\left(T_{1}-N^{-1 / 2} \hat{c}_{1} \leq x\right)=P\left(T_{1} \leq x+N^{-1 / 2} \hat{c}_{1}\right) \\
& \quad=\Phi\left(x+N^{-1 / 2} \hat{c}_{1}\right)+N^{-1 / 2} q_{1}\left(x+N^{-1 / 2} \hat{c}_{1}\right) \phi\left(x+N^{-1 / 2} \hat{c}_{1}\right)+O\left(N^{-\min (1, r+1 / 2)}\right)
\end{aligned}
$$

Applying Taylor expansion, with $\gamma=\frac{2 A_{1}+3 A_{2}}{3}$, and $\hat{\gamma}$ is its estimate, we get

$$
P\left(T_{1}^{\prime} \leq x\right)=\Phi(x)+N^{-1 / 2} \hat{\gamma}\left(x^{2}-1\right) \phi(x)+O\left(N^{-\min (1, r+1 / 2)}\right)
$$

Similar to Zhou and Dinh (2005), intervals (4) follow.

## Appendix B

Proof of Theorem 2

Let $X$ be defined as in the proof of Theorem 1, $k(X)=\hat{\theta}_{2}-\theta_{2}, h(X)=$ $\hat{\sigma}_{2}^{2}, \quad f(X)=\frac{k(X)}{h(X)^{1 / 2}}$, and $T_{2}=N^{1 / 2} f(X)$. Let $V=E(X)$, then $f(V)=0$. By Taylor expansion, we obtain,

$$
f(X)=\frac{\partial f(V)}{\partial V}(X-V)+\frac{1}{2}(X-V)^{\prime} \frac{\partial^{2} f(V)}{\partial V^{2}}(X-V)+\ldots
$$

Let $J_{2}=N^{1 / 2}\left(\frac{\partial f(V)}{\partial V}(X-V)+\frac{1}{2}(X-V)^{\prime} \frac{\partial^{2} f(V)}{\partial V^{2}}(X-V)\right)$. With coefficient A given in Theorem 2, the first 3 moments of $J_{2}$ are:

$$
E J_{2}=-\frac{1}{2} A N^{-1 / 2}, E J_{2}^{2}=1+O\left(N^{-1}\right), E J_{2}^{3}=-\frac{7}{2} A N^{-1 / 2}+O\left(N^{-\min (1, r+1 / 2)}\right) .
$$

Let $K_{1 N}, K_{2 N}, K_{3 N}$ be the first three cumulants of $J_{2}$. Then,

$$
\begin{aligned}
& K_{1 N}=-\frac{1}{2} A N^{-1 / 2}+O\left(N^{-\min (1, r+1 / 2)}\right), \quad K_{2 N}=1+O\left(N^{-\min (1, r+1 / 2)}\right), \\
& K_{3 N}=-2 A N^{-1 / 2}+O\left(N^{-\min (1, r+1 / 2)}\right)
\end{aligned}
$$

Let $\chi_{N}(t)$ be the characteristic function of $J_{2}$. Applying Taylor expansion to an exponential function, we obtain

$$
\chi_{N}(t)=\exp \left(-\frac{t^{2}}{2}\right)\left\{1+N^{-1 / 2}\left(-\frac{1}{2} A(i t)-\frac{2 A}{6}(i t)^{3}\right)+O\left(N^{-\min (1, r+1 / 2)}\right)\right\}
$$

Letting $R_{2}(i t)=\left(-\frac{1}{2} A(i t)-\frac{2 A}{6}(i t)^{3}\right)$, we can write

$$
\chi_{N}(t)=\exp \left(-\frac{t^{2}}{2}\right)\left\{1+N^{-1 / 2} R_{2}(i t)+O\left(N^{-\min (1, r+1 / 2)}\right)\right\} .(*)
$$

Since $\chi_{N}(t)=\int_{-\infty}^{\infty} e^{i t x} d p\left(J_{2} \leq x\right)$ and $e^{-t^{2} / 2}=\int_{-\infty}^{\infty} e^{i t x} d \Phi(x)$, expression $\left(^{*}\right)$ suggests that

$$
P\left(J_{2} \leq x\right)=\Phi(x)+N^{-1 / 2} R_{2}(X)+O\left(N^{-\min (1, r+1 / 2)}\right)
$$

where $R_{2}(X)$ is such a function that its Fourier-Stieltjes transform equals to $r_{2}(i t) e^{-t^{2} / 2}$. Applying integration by part, we obtain

$$
R_{2}(x)=\frac{A}{2}+\frac{2 A}{6}\left(x^{2}-1\right) \phi(x)=\frac{A}{6}\left(2 x^{2}+1\right) \phi(x) .
$$

Therefore,

$$
P\left(J_{2} \leq x\right)=\Phi(x)+N^{-1 / 2} q_{2}(x) \phi(x)+O\left(N^{-\min (1, r+1 / 2)}\right)
$$

Since $T_{2}=J_{2}+O\left(N^{-1}\right)$, Theorem 2 follows.

## Appendix C

Details of Methods for the ICER Simulation

## C1. Taylor's interval

The variance of the $\widehat{I C E} R$ can be estimated as in equation (2). The asymptotic confidence interval (namely Taylor's interval) can be derived using the central limit theorem and is given by $\left(I \widehat{C E} R-z_{1-\alpha / 2} \hat{\sigma}_{1}, I \widehat{C E} R+z_{1-\alpha / 2} \hat{\sigma}_{1}\right)$.

## C2. Fieller's interval

The Fieller (Fieller, 1954) method assumes that cost and effectiveness follow a bivariate normal distribution. Thus, $\hat{\Delta}_{2}-R \hat{\Delta}_{1}$ is normally distributed with zero mean, with R denotes the ICER and $\hat{\Delta}_{1}=\bar{E}_{1}-\bar{E}_{2}$, and $\hat{\Delta}_{2}=\bar{C}_{1}-\bar{C}_{2}$. Consequently,

$$
\frac{\hat{\Delta}_{2}-R \hat{\Delta}_{1}}{\sqrt{\hat{\operatorname{Var}}\left(\hat{\Delta}_{2}-R \hat{\Delta}_{1}\right)}} \sim N(0,1) .
$$

The $(1-\alpha) 100 \%$ confidence interval for the ICER (denoted Fieller) can be obtained by equating the formula

$$
\frac{\hat{\Delta}_{2}-R \hat{\Delta}_{1}}{\sqrt{\hat{\operatorname{Var}}\left(\hat{\Delta}_{2}-R \hat{\Delta}_{1}\right)}}=z_{1-\alpha / 2}
$$

and solve for R . This is equivalent to solving the quadratic equation

$$
\begin{equation*}
X R^{2}-2 Y R+Z=0 \tag{C.1}
\end{equation*}
$$

where

$$
\begin{aligned}
& X=\left(\bar{E}_{1}-\bar{E}_{2}\right)^{2}-z_{1-\alpha / 2}^{2}\left[\frac{S_{E_{1}}^{2}}{n_{1}}+\frac{S_{E_{2}}^{2}}{n_{2}}\right], \\
& Y=\left(\bar{E}_{1}-\bar{E}_{2}\right)\left(\bar{C}_{1}-\bar{C}_{2}\right)-z_{1-\alpha / 2}^{2}\left[\frac{S_{E_{1} C_{1}}}{n_{1}}+\frac{S_{E_{2} C_{2}}}{n_{2}}\right], \\
& Z=\left(\bar{C}_{1}-\bar{C}_{2}\right)^{2}-z_{1-\alpha / 2}^{2}\left[\frac{S_{C_{1}}^{2}}{n_{1}}+\frac{S_{C_{2}}^{2}}{n_{2}}\right] .
\end{aligned}
$$

Confidence limits for the ICER are the solutions of the equation (C.1).

## C3. Bootstrap-t interval

The t-statistic $T=(\widehat{I C E} R-I C E R) / \hat{\sigma}_{1}$ is normally distributed based on the central limit theorem when the sample size is large. When this assumption is violated, an alternative is to approximate its distribution by a bootstrap method, namely the bootstrap-t. For each bootstrap sample b, the bootstrapt computes another t-statistic $T_{b}^{*}=\left(I \widehat{C E} R_{b}^{*}-I \widehat{C E} R\right) / \hat{\sigma}_{1_{b}}^{*}$. These t-statistics will then be sorted to find the $\alpha / 2$ and $(1-\alpha / 2)$ percentiles, denoted by $\hat{t}^{(\alpha / 2)}$ and $\hat{t}^{(1-\alpha / 2)}$. The resulting $(1-\alpha) 100 \%$ bootstrap-t interval (denoted boott) is $\left(I \widehat{C E} R-\hat{t}^{(1-\alpha / 2)} \hat{\sigma}_{1}, I \widehat{C E} R-\hat{t}^{(\alpha / 2)} \hat{\sigma}_{1}\right)$.

## C4. Hinkley's interval

The Hinkley (Hinkley, 1969) method also assumes the numerator and denominator of the ICER come from a bivariate normal distribution. The
distribution function $F(w)$ of the ICER is given by (Hinkley, 1969),

$$
\begin{aligned}
F(w)= & L\left(\frac{\Delta_{2}-\Delta_{1} w}{\sqrt{\Gamma_{1} \Gamma_{2}} a(w) / N},-\frac{\Delta_{1}}{\sqrt{\Gamma_{1} / N}} ; \frac{w \sqrt{\Gamma_{1} / N}-\rho \sqrt{\Gamma_{2} / N}}{\sqrt{\Gamma_{1} \Gamma_{2}} a(w) / N}\right) \\
& +L\left(\frac{\Delta_{1} w-\Delta_{2}}{\sqrt{\Gamma_{1} \Gamma_{2}} a(w) / N}, \frac{\Delta_{1}}{\sqrt{\Gamma_{1} / N}} ; \frac{w \sqrt{\Gamma_{1} / N}-\rho \sqrt{\Gamma_{2} / N}}{\sqrt{\Gamma_{1} \Gamma_{2}} a(w) / N}\right)
\end{aligned}
$$

where, $\Delta_{1}, \Delta_{2}, \Gamma_{1}, \Gamma_{2}, \Gamma_{12}$ are given in Theorem 1, and

$$
\begin{aligned}
& L(h, k ; \gamma)=\frac{1}{2 \pi \sqrt{1-\gamma^{2}}} \int_{h}^{\infty} \int_{k}^{\infty} \exp \left[-\frac{x^{2}-2 \gamma x y+y^{2}}{2\left(1-\gamma^{2}\right)}\right] d x d y \\
& a(w)=\left(\frac{w^{2}}{\Gamma_{2} / N}-\frac{2 \rho w}{\sqrt{\Gamma_{1} \Gamma_{2}} / N}+\frac{1}{\Gamma_{1} / N}\right)^{1 / 2}, \rho=\Gamma_{12} / \sqrt{\Gamma_{1} \Gamma_{2}}
\end{aligned}
$$

The $(1-\alpha) 100 \%$ confidence interval (denoted Hinkley) for the ICER is given by $\left(w_{1}, w_{2}\right)$ where $F\left(w_{1}\right)=\alpha / 2, \quad F\left(w_{2}\right)=1-\alpha / 2$.

## C5. Three new transformational intervals

The three new transformational intervals based on Edgeworth expansion are given in equation (4). They are denoted by $T_{1}, T_{2}$, and $T_{3}$ in our simulation.

## Appendix D

## Details of Methods for the NHB Simulation

## D1. Normal interval

The asymptotic variance of the NHB can be estimated as in equation (5). By the central limit theorem, the asymptotic normal confidence interval (denoted normal) for the NHB is given by $\left(\widehat{N H B}-z_{1-\alpha / 2} \hat{\sigma}_{2}, \quad \widehat{N H B}+z_{1-\alpha / 2} \hat{\sigma}_{2}\right)$.

## D2. BCa interval

For each bootstrap sample b, the estimate $\widehat{N H} B_{b}^{*}$ is computed. The bootstrap replicates are then ordered from smallest to largest. The bias-corrected
acceleration (BCa) confidence interval is given by $\left(\widehat{N H B}{ }^{*\left(\alpha_{1}\right)}, \widehat{\operatorname{NHB}}{ }^{*\left(\alpha_{2}\right)}\right.$ ) where

$$
\begin{aligned}
& \alpha_{1}=\Phi\left(\hat{z}_{0}+\frac{\hat{z}_{0}+z_{\alpha}}{1-\hat{\alpha}\left(\hat{z}_{0}+z_{\alpha}\right)}\right), \alpha_{2}=\Phi\left(\hat{z}_{0}+\frac{\hat{z}_{0}+z_{1-\alpha}}{1-\hat{a}\left(\hat{z}_{0}+z_{1-\alpha}\right)}\right), \\
& \hat{z}_{0}=\Phi^{-1}\left(\frac{\#\left\{\widehat{N H B}^{*}(b)<\widehat{N H B}\right\}}{B}\right) .
\end{aligned}
$$

The acceleration constant $\hat{a}=\frac{1}{6} \hat{\sigma}_{2}^{-3}\left(n_{1}^{-2} \hat{\gamma}_{1}-n_{2}^{-2} \hat{\gamma}_{2}\right)$ is given in Hall and Martin (1988) where
$\hat{\gamma}_{1}=\frac{1}{n_{1}} \sum_{j=1}^{n_{1}}\left[\left(E_{1 j}-\bar{E}_{1}\right)-\frac{\left(C_{1 j}-\bar{C}_{1}\right)}{\lambda}\right]^{3}, \hat{\gamma}_{2}=\frac{1}{n_{2}} \sum_{j=1}^{n_{2}}\left[\left(E_{2 j}-\bar{E}_{2}\right)-\frac{\left(C_{2 j}-\bar{C}_{2}\right)}{\lambda}\right]^{3}$.

## D3. Bootstrap-t interval

As described previously, For each bootstrap sample b, the bootstrap-t computes another t-statistic $T_{b}^{*}=\left(\widehat{N H} B_{b}^{*}-\widehat{N H B}\right) / \hat{\sigma}_{2 b}^{*}$. These t-statistics will then be sorted to find the $\alpha / 2$ and $(1-\alpha / 2)$ percentiles, denoted by $\hat{t}^{(\alpha / 2)}$ and $\hat{t}^{(1-\alpha / 2)}$. The resulting $(1-\alpha) 100 \%$ bootstrap-t interval (denoted boott) is $\left(\widehat{N H B}-\hat{t}^{(1-\alpha / 2)} \hat{\sigma}_{2}, \widehat{N H B}-\hat{t}^{(\alpha / 2)} \hat{\sigma}_{2}\right)$.

## D4. Three new transformational intervals

The three new transformational intervals based on Edgeworth expansion are given in equation (8). They are denoted by $T_{1}, T_{2}$, and $T_{3}$ in our simulation.


Figure 1. The $\Delta E-\Delta C$ Plane


Figure 2. Distributions of costs and effectiveness of two groups
$\square$
Table 1
 （ $n_{1}=n_{2}=50$ ）

| LEL＇0 | （ 6．60LII ）0モ06．0 | （ 977币9LI ）0¢ 280 | （ $7^{\prime}$ L88LI ）0906．0 |  | （ $\mathrm{C}^{\text {．} 987 Z I}$ ） 9668.0 | （ Ø．8\＆7ZI ） 9668.0 |  | $60^{-}$ | $7^{\circ} 0^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2980 |  | （ ¢． $7 \% 0 ¢ \tau$ ）8998．0 |  | （ 6．L986I ）L968．0 | （ L＇7¢8¢L ）¢¢68＊0 | （ 8．2888L ）9¢68＊0 | （ 6．9¢z¢I ）¢998．0 | $70^{-}$ | $6{ }^{-}$ |
| LTL：0 | （ \＆Lgcit ）Lg060 | （ \＆6\％git ）LZL8＊0 | （ 9676LI ）6906．0 | （ L＇L8L6I ）90L6．0 | （ I •8LIZI ） $9968{ }^{\circ}$ | （ 6.8 LIZI ） $9968^{\circ}$ | （ 8．889LI ）8898．0 | $70^{-}$ | $70^{-}$ |
| 098．0 | （ 8．Z币TEL ）\＆L06．0 | （ 6．78LEL ）¢998．0 | （ 9．6L6LZ ）98L60 | （［ $2 \mathrm{LZZ6L}$ ）6L68．0 | （ 9＊876¢I ）0868．0 |  | （ 067¢¢EI ）0898．0 | $6.0{ }^{-}$ | $6.0^{-}$ |
| \＆6900 | （ 066†て0I ）90680 | （ 8．L๖て0I ）LZ98．0 | （ 8\％7909I ）9068＊0 | （ 6．9EtLI ）0906＊0 | （ 6．8ILOI ）6028＊0 | （ 066ILOT ）8028＊0 | （ 9＊6L80I ）0øモ $8^{\circ} 0$ | 60 | $7 \cdot 0$ |
| 999.0 |  | （ LL．0¢L8 ）90币 ${ }^{\circ} 0$ | （ 8．97\％7I ）9098．0 | （ L L®799I ）9E8880 | （ ET00LI6）9678．0 | （ 78.0 LL 6 ）9678．0 | （ 96.9788 ）68¢880 | z\％ | 6.0 |
| $089{ }^{\circ}$ |  | （ 8＊0¢z0T ）$\ddagger 998.0$ | （ $\mathrm{F}^{\circ} 09$ LST ） $9888{ }^{\circ}$ |  | （ I 0¢LOI ） $7728 \cdot 0$ | （ $9 \cdot 0820 \mathrm{~L}$ ） 8 \％L8．0 | （ \＆89800 ）$\ddagger 8780$ | z＊ 0 | 7\％ |
| 889.0 | （ 88.9898 ）¢ $2980^{\circ} 0$ | （ 01＊8t98）9878．0 | （ 6．907ZI ）9898．0 | （ $8 \cdot 0989 \mathrm{~T}$ ）L2880 | （ LI＇\＆L06 ）L6z8＊0 | （ 86.8 L06 ） $9678^{\circ} 0$ | （ LL．8798）¢LZ8．0 | 6.0 | $\begin{aligned} & 6.0 \\ & \text { ә.nұx! } / 2 \end{aligned}$ |
| $607^{\circ} 0$ | （ 96．98\％）¢¢¢6．0 |  | （ ZI． $27 \%$ ）LLE6．0 | （ 20.877 ）9876．0 | （ L9．97\％）8Lも6．0 | （ $29.97 \%$ ）LLも6．0 | （ $7 \times 9.97 \%$ ） 2676.0 | 007－ | 00\％${ }^{-}$ |
| $607^{\circ} 0$ | （ ¢¢｀¢もて ）98も6．0 | （ LI•\＆¢\％） $28 \pm 6.0$ |  | （ LE．9¢Z ）LLも6．0 | （ LL．も¢Z ）\＆Lも6．0 | （ 9L．も¢Z ）\＆Lも6．0 | （ $\ddagger \in \subset \subset \subset \%$ ） 6876.0 | 008－ | 007－ |
| $607^{\circ} 0$ | （ 78. 帪 ）Lも¢ 6.0 | （ LE＇Z¢\％）8LШ6．0 | （ 99．も¢\％）98も6．0 | （ 89．9¢z ）96モ6．0 | （ $00 . \ddagger ¢ \%$ ）$\ddagger 8 \mp 6.0$ | （ $66 . \varepsilon ¢ \%$ ） $98 \pm 6 \% 0$ | （ 99．7¢\％）6Lも6．0 | 007－ | 008－ |
| $607^{\circ}$ |  | （ 2 「．0才て）69才6．0 | （ $\ddagger 97 \square \square$ ）ஏ ¢も6．0 | （ \＆9\％¢も ）L8も 6.0 | （ $66 . L \square 乙$ ）¢ ¢ 660 | （ 86．Lもち ）9Lも6．0 | （ $\downarrow$＊ $0 \searrow \square$ ）69も6．0 | 008－ | 008－ |
| $807^{\circ} 0$ |  | （ L0．8LZ ） $58 \pm 6.0$ | （ 61．9LZ ） $66 \pm 6{ }^{\circ} 0$ | （ ¢T．9LZ ）00¢6．0 | （ 79 もLて ）68モ6．0 | （ ヶ9．もちを ） 68 ¢ 6.0 | （ $9 \% \cdot \varepsilon$ LZ ） $08 \pm 6{ }^{\circ} 0$ | 007 | 007 |
| $807^{\circ} 0$ | （ ¢L｀giz ）¢tc6．0 |  | （ 8t．90を ）¢976．0 | （ \＆T＇LOz ）69ø6．0 | （ $92 . \mathrm{GOz}$ ） $8976 \cdot 0$ | （ LL｀g0Z ）\＆976．0 |  | 008 | $00 \%$ |
| $607^{\circ} 0$ | （ 07｀GIZ ）L6モ6．0 |  | （ $\ddagger 7.907$ ）$\ddagger$ ¢も6．0 | （ $67.20 Z$ ） $89 \pm 6.0$ | （ zL ¢0Z ）09も6．0 | （ $72 \cdot 907$ ） $7976 \cdot 0$ | （ 09＇も0 ）9Lも6．0 | 007 | 008 |
| 0IZ $\%$ | （ $\varepsilon z^{\circ} \mathrm{C} 07$ ）9z¢6＊0 | （ ¢9．モ6I ） 70960 | （ 62．96I ）66п6．0 |  |  | （ 97．96I ）¢0¢6．0 | （ 98． 66 L ）\＆L96．0 | 008 | $\begin{aligned} & 008 \\ & \text { [euı. } \end{aligned}$ |
| $\frac{N /}{\left\|(96 \cdot \mathrm{I})^{\mathrm{I} b \mid}\right\|}$ | $\varepsilon_{L}$ | ${ }^{Z} L$ | ${ }^{1} L$ | 77009 | Кәучи！Н | นәโІə！${ }^{\text {¢ }}$ |  | モ¢ | Z1． |

Table 2
Coverage of $95 \%$ 2-sided confidence intervals for the ICER for bivariate lognormal data

| $n_{1}$ | $n_{2}$ | $\sigma_{12}$ | $\sigma_{34}$ | Taylor | Fieller | Hinkley | Boott | $T_{1}$ | $T_{2}$ | $T_{3}$ | $\frac{\left\|q_{1}(1.96)\right\|}{\sqrt{N}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 50 | 0.9 | 0.9 | 0.8086 ( 140.85) | 0.8129 ( 159.46) | 0.8131 ( 157.71) | 0.8678 ( 264.39 ) | 0.8353 ( 167.08) | 0.8142 ( 139.37) | 0.8628 ( 140.73) | 0.410 |
| 50 | 50 | 0.2 | 0.2 | 0.8556 ( 247.53 ) | 0.9037 ( 266.87 ) | 0.9037 ( 265.69) | 0.9030 ( 385.72) | 0.8961 ( 363.12) | 0.8609 ( 244.59) | 0.8944 ( 245.40) | 0.684 |
| 50 | 50 | 0.9 | 0.2 | 0.8361 ( 145.22) | 0.8482 ( 280.12 ) | 0.8486 ( 161.80) | 0.8740 ( 241.04 ) | 0.8478 ( 169.51 ) | 0.8389 ( 143.99) | 0.8855 ( 145.36 ) | 0.393 |
| 50 | 50 | 0.2 | 0.9 | 0.8620 ( 244.72) | 0.9065 ( 262.02 ) | 0.9065 ( 261.80 ) | 0.9095 ( 396.32) | 0.8974 ( 362.76) | 0.8683 ( 241.74) | 0.8991 ( 241.85 ) | 0.694 |
| 50 | 50 | -0.9 | -0.9 | 0.8698 ( 314.32) | 0.9242 ( 344.01) | 0.9242 ( 343.53) | 0.9120 ( 452.35) | 0.9337 ( 540.66) | 0.8785 ( 309.79) | 0.9111 ( 310.57) | 0.854 |
| 50 | 50 | -0.2 | -0.2 | 0.8685 ( 283.35 ) | 0.9179 ( 307.39) | 0.9182 ( 307.07) | 0.9148 ( 431.87) | 0.9169 ( 445.67) | 0.8760 ( 279.69) | 0.9076 ( 280.49) | 0.772 |
| 50 | 50 | -0.9 | -0.2 | 0.8690 ( 315.91) | 0.9191 ( 348.47) | 0.9190 ( 345.30) | 0.9063 ( 471.17) | 0.9287 ( 534.66) | 0.8770 ( 311.41 ) | 0.9068 ( 312.27) | 0.856 |
| 50 | 50 | -0.2 | -0.9 | 0.8649 ( 283.50 ) | 0.9142 ( 306.44) | 0.9142 ( 306.18) | 0.9138 ( 428.89) | 0.9180 ( 447.68) | 0.8714 ( 279.99) | 0.9028 ( 279.90) | 0.773 |
| 100 | 100 | 0.9 | 0.9 | 0.8427 ( 108.31) | 0.8416 ( 114.17) | 0.8416 ( 114.17) | 0.8944 ( 176.64) | 0.8643 ( 135.07) | 0.8482 ( 107.31 ) | 0.8776 ( 106.65 ) | 0.463 |
| 100 | 100 | 0.2 | 0.2 | 0.8829 ( 177.37 ) | 0.9140 ( 182.27 ) | 0.9139 ( 182.27) | 0.9210 ( 242.29 ) | 0.9067 ( 236.14 ) | 0.8861 ( 176.16) | 0.9088 ( 175.16 ) | 0.594 |
| 100 | 100 | 0.9 | 0.2 | 0.8583 ( 111.36) | 0.8636 ( 116.91 ) | 0.8636 ( 116.90 ) | 0.8956 ( 170.36) | 0.8731 ( 136.81 ) | 0.8637 ( 110.53) | 0.8937 ( 110.10) | 0.424 |
| 100 | 100 | 0.2 | 0.9 | 0.8823 ( 177.06) | 0.9098 ( 181.67 ) | 0.9097 ( 181.68) | 0.9186 ( 257.28 ) | 0.9030 ( 238.88 ) | 0.8858 ( 175.56) | 0.9046 ( 174.44 ) | 0.603 |
| 100 | 100 | -0.9 | -0.9 | 0.8899 ( 216.13 ) | 0.9254 ( 222.38 ) | 0.9255 ( 222.37 ) | 0.9268 ( 268.00 ) | 0.9264 ( 302.63) | 0.8946 ( 214.27) | 0.9144 ( 212.91 ) | 0.684 |
| 100 | 100 | -0.2 | -0.2 | 0.8912 ( 197.24 ) | 0.9231 ( 203.04 ) | 0.9232 ( 203.07) | 0.9268 ( 270.97) | 0.9198 ( 269.52 ) | 0.8959 ( 195.70) | 0.9146 ( 194.49 ) | 0.640 |
| 100 | 100 | -0.9 | -0.2 | 0.8977 ( 215.06 ) | 0.9268 ( 221.96 ) | 0.9269 ( 221.96) | 0.9258 ( 270.79 ) | 0.9272 ( 301.68) | 0.9017 ( 213.40 ) | 0.9186 ( 212.14 ) | 0.689 |
| 100 | 100 | -0.2 | -0.9 | 0.8975 ( 200.30) | 0.9250 ( 206.54 ) | 0.9250 ( 206.53) | 0.9269 ( 268.74 ) | 0.9186 ( 272.52) | 0.9017 ( 198.76 ) | 0.9183 ( 198.05 ) | 0.643 |
| 50 | 100 | 0.9 | 0.9 | 0.8021 ( 136.73) | 0.8024 ( 152.29 ) | 0.8023 ( 152.20) | 0.8757 ( 264.07 ) | 0.8325 ( 161.19) | 0.8081 ( 135.35 ) | 0.8452 ( 134.71 ) | 0.417 |
| 50 | 100 | 0.2 | 0.2 | 0.8520 ( 242.38 ) | 0.8980 ( 256.34 ) | 0.8979 ( 256.30) | 0.9059 ( 394.79) | 0.8957 ( 355.36) | 0.8557 ( 239.85 ) | 0.8829 ( 238.10 ) | 0.680 |
| 50 | 100 | 0.9 | 0.2 | 0.8166 ( 141.31 ) | 0.8216 ( 158.12 ) | 0.8216 ( 157.05 ) | 0.8747 ( 257.89 ) | 0.8411 ( 163.91 ) | 0.8229 ( 140.26) | 0.8558 ( 139.71 ) | 0.394 |
| 50 | 100 | 0.2 | 0.9 | 0.8565 ( 243.40 ) | 0.9013 ( 258.20) | 0.9013 ( 258.11) | 0.9130 ( 406.37) | 0.8967 ( 355.44) | 0.8614 ( 240.98) | 0.8846 ( 239.44) | 0.684 |
| 50 | 100 | -0.9 | 0.9 | 0.8672 ( 307.95) | 0.9190 ( 331.06 ) | 0.9190 ( 330.95) | 0.9165 ( 456.05) | 0.9296 ( 521.98) | 0.8746 ( 304.18) | 0.8984 ( 302.28) | 0.849 |
| 50 | 100 | -0.2 | 0.2 | 0.8628 ( 275.45 ) | 0.9105 ( 294.44 ) | 0.9105 ( 294.36 ) | 0.9174 ( 443.17) | 0.9176 ( 429.66 ) | 0.8691 ( 272.26) | 0.8907 ( 270.54) | 0.768 |
| 50 | 100 | -0.9 | 0.2 | 0.8638 ( 312.66) | 0.9148 ( 335.71) | 0.9146 ( 335.63) | 0.9146 ( 492.50) | 0.9282 ( 525.66) | 0.8711 ( 308.67) | 0.8954 ( 305.64) | 0.853 |
| 50 | 100 | -0.2 | 0.9 | 0.8649 ( 272.82) | 0.9144 ( 290.59 ) | 0.9144 ( 290.28) | 0.9211 ( 426.69) | 0.9168 ( 421.22) | 0.8717 ( 269.80) | 0.8942 ( 267.85 ) | 0.768 |
| 100 | 50 | 0.9 | 0.9 | 0.8520 ( 111.28) | 0.8524 ( 118.17) | 0.8525 ( 118.23) | 0.8929 ( 182.74) | 0.8664 ( 139.71 ) | 0.8578 ( 110.38) | 0.8883 ( 110.47) | 0.454 |
| 100 | 50 | 0.2 | 0.2 | 0.8875 ( 183.21 ) | 0.9171 ( 190.09 ) | 0.9172 ( 190.27) | 0.9171 ( 243.31 ) | 0.9084 ( 244.94 ) | 0.8924 ( 181.59) | 0.9175 ( 181.67) | 0.596 |
| 100 | 50 | 0.9 | 0.2 | 0.8753 ( 118.64 ) | 0.8813 ( 125.07) | 0.8812 ( 125.06) | 0.8961 ( 173.17 ) | 0.8786 ( 145.78 ) | 0.8772 ( 117.83 ) | 0.9071 ( 118.34 ) | 0.417 |
| 100 | 50 | 0.2 | 0.9 | 0.8838 ( 176.32) | 0.9118 ( 182.50 ) | 0.9119 ( 182.56 ) | 0.9158 ( 239.80) | 0.9059 ( 237.60) | 0.8887 ( 174.73) | 0.9099 ( 174.60 ) | 0.599 |
| 100 | 50 | -0.9 | -0.9 | 0.9000 ( 225.19 ) | 0.9296 ( 235.43 ) | 0.9296 ( 235.42) | 0.9175 ( 295.21 ) | 0.9219 ( 322.09) | 0.9045 ( 223.31 ) | 0.9244 ( 223.25 ) | 0.691 |
| 100 | 50 | -0.2 | -0.2 | 0.8918 ( 204.59 ) | 0.9271 ( 212.21 ) | 0.9271 ( 212.07) | 0.9182 ( 262.34 ) | 0.9167 ( 284.15 ) | 0.8962 ( 202.72) | 0.9220 ( 202.21) | 0.647 |
| 100 | 50 | -0.9 | -0.2 | 0.8968 ( 225.22 ) | 0.9295 ( 234.12 ) | 0.9295 ( 234.12 ) | 0.9196 ( 285.79) | 0.9232 ( 321.77) | 0.9020 ( 223.32 ) | 0.9234 ( 222.23) | 0.693 |
| 100 | 50 | -0.2 | -0.9 | 0.8988 ( 205.02) | 0.9286 ( 213.28 ) | 0.9285 ( 213.20 ) | 0.9201 ( 262.86 ) | 0.9158 ( 281.68) | 0.9027 ( 203.38) | 0.9239 ( 202.77) | 0.642 |

Coverage of 95\% 2-sided confidence intervals for the NHB for bivariate normal and mixture data ( $n_{1}=n_{2}=n$ )

| $n$ | $\sigma_{12}$ | $\sigma_{34}$ | Normal | Boott | BCa | $T_{1}$ | $T_{2}$ | $T_{3}$ | $\|\hat{A}\| / \sqrt{N}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Normal |  |  |  |  |  |  |  |  |  |
| 50 | 800 | 800 | $0.9468(2.14)$ | $0.9500(2.18)$ | $0.9426(2.11)$ | $0.9467(2.14)$ | $0.9469(2.14)$ | $0.9439(2.30)$ | 0.0001 |
| 50 | 800 | 200 | $0.9486(2.14)$ | $0.9512(2.18)$ | $0.9445(2.12)$ | $0.9479(2.14)$ | $0.9485(2.14)$ | $0.9455(2.30)$ | 0.0002 |
| 50 | 200 | 800 | $0.9465(2.14)$ | $0.9498(2.18)$ | $0.9428(2.12)$ | $0.9469(2.14)$ | $0.9464(2.14)$ | $0.9416(2.30)$ | 0.0002 |
| 50 | 200 | 200 | $0.9457(2.14)$ | $0.9493(2.18)$ | $0.9447(2.12)$ | $0.9459(2.14)$ | $0.9458(2.14)$ | $0.9440(2.30)$ | 0.0001 |
| 50 | -800 | -800 | $0.9495(2.15)$ | $0.9512(2.19)$ | $0.9463(2.12)$ | $0.9496(2.15)$ | $0.9496(2.15)$ | $0.9468(2.31)$ | 0.0003 |
| 50 | -800 | -200 | $0.9490(2.14)$ | $0.9530(2.18)$ | $0.9465(2.12)$ | $0.9490(2.15)$ | $0.9490(2.14)$ | $0.9453(2.31)$ | 0.0000 |
| 50 | -200 | -800 | $0.9480(2.14)$ | $0.9509(2.18)$ | $0.9436(2.12)$ | $0.9483(2.14)$ | $0.9474(2.14)$ | $0.9441(2.31)$ | 0.0004 |
| 50 | -200 | -200 | $0.9471(2.14)$ | $0.9496(2.18)$ | $0.9448(2.12)$ | $0.9475(2.14)$ | $0.9471(2.14)$ | $0.9447(2.31)$ | 0.0000 |
| Mixture |  |  |  |  |  |  |  |  |  |
| 50 | -0.8 | 0.8 | $0.9494(0.71)$ | $0.9514(0.72)$ | $0.9446(0.70)$ | $0.9507(0.71)$ | $0.9495(0.71)$ | $0.9445(0.76)$ | 0.014 |
| 50 | -0.2 | 0.2 | $0.9438(0.69)$ | $0.9480(0.70)$ | $0.9411(0.68)$ | $0.9441(0.69)$ | $0.9437(0.69)$ | $0.9452(0.74)$ | 0.002 |
| 50 | -0.8 | 0.2 | $0.9516(0.71)$ | $0.9539(0.73)$ | $0.9459(0.71)$ | $0.9520(0.72)$ | $0.9517(0.71)$ | $0.9457(0.77)$ | 0.015 |
| 50 | -0.2 | 0.8 | $0.9461(0.68)$ | $0.9503(0.70)$ | $0.9430(0.68)$ | $0.9460(0.68)$ | $0.9462(0.68)$ | $0.9435(0.74)$ | 0.001 |
| 50 | 0.8 | -0.8 | $0.9476(0.65)$ | $0.9508(0.67)$ | $0.9447(0.65)$ | $0.9483(0.65)$ | $0.9477(0.65)$ | $0.9414(0.70)$ | 0.005 |
| 50 | 0.2 | -0.2 | $0.9472(0.68)$ | $0.9506(0.69)$ | $0.9432(0.67)$ | $0.9471(0.68)$ | $0.9469(0.68)$ | $0.9402(0.73)$ | 0.001 |
| 50 | 0.8 | -0.2 | $0.9474(0.65)$ | $0.9515(0.66)$ | $0.9427(0.64)$ | $0.9468(0.65)$ | $0.9475(0.65)$ | $0.9444(0.70)$ | 0.009 |
| 50 | 0.2 | -0.8 | $0.9492(0.68)$ | $0.9518(0.69)$ | $0.9442(0.67)$ | $0.9499(0.68)$ | $0.9496(0.68)$ | $0.9451(0.73)$ | 0.004 |

Table 4
Coverage of 95\% 2-sided confidence intervals for the NHB for bivariate lognormal data

| $n_{1}$ | $n_{2}$ | $\sigma_{12}$ | $\sigma_{34}$ | Normal | Boott | BCa | $T_{1}$ | $T_{2}$ | $T_{3}$ | $\|\hat{A}\| / \sqrt{N}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 50 | 0.8 | 0.8 | 0.9025 (60.88) | 0.9262 (76.77) | 0.9052 (64.40) | 0.8768 (85.87) | 0.8971 (61.06) | 0.9355 (64.61) | 0.332 |
| 50 | 50 | 0.2 | 0.2 | 0.8997 (60.95) | 0.9277 (77.09) | 0.9059 (64.41) | 0.8727 (85.80) | 0.8942 (61.13) | 0.9357 (64.69) | 0.331 |
| 50 | 50 | 0.8 | 0.2 | 0.8929 (60.88) | 0.9197 (76.94) | 0.8991 (64.39) | 0.8661 (86.31) | 0.8881 (61.07) | 0.9264 (64.61) | 0.333 |
| 50 | 50 | 0.2 | 0.8 | 0.8983 (60.81) | 0.9272 (77.22) | 0.9041 (64.35) | 0.8664 (85.99) | 0.8923 (60.99) | 0.9327 (64.53) | 0.332 |
| 50 | 50 | -0.8 | -0.8 | 0.9005 (61.05) | 0.9294 (77.31) | 0.9052 (64.62) | 0.8716 (86.03) | 0.8943 (61.23) | 0.9336 (64.78) | 0.333 |
| 50 | 50 | -0.2 | -0.2 | 0.9082 (61.25) | 0.9308 (77.52) | 0.9106 (64.79) | 0.8792 (86.34) | 0.9024 (61.43) | 0.9401 (65.00) | 0.332 |
| 50 | 50 | -0.8 | -0.2 | 0.9047 (61.07) | 0.9279 (77.31) | 0.9059 (64.67) | 0.8728 (86.41) | 0.8969 (61.25) | 0.9351 (64.80) | 0.331 |
| 50 | 50 | -0.2 | -0.8 | 0.8965 (61.05) | 0.9229 (77.24) | 0.9027 (64.57) | 0.8704 (86.23) | 0.8913 (61.23) | 0.9304 (64.79) | 0.331 |
| 100 | 100 | -0.8 | 0.8 | 0.9192 (44.42) | 0.9338 (51.20) | 0.9180 (46.43) | 0.8968 (56.67) | 0.9145 (44.48) | 0.9354 (45.59) | . 294 |
| 100 | 100 | -0.2 | 0.2 | 0.9207 (44.65) | 0.9352 (51.75) | 0.9186 (46.74) | 0.8981 (57.28) | 0.9178 (44.72) | 0.9367 (45.83) | 0.296 |
| 100 | 100 | -0.8 | 0.2 | 0.9230 (44.87) | 0.9330 (52.63) | 0.9200 (47.02) | 0.9011 (57.76) | 0.9196 (44.94) | 0.9374 (46.05) | . 295 |
| 100 | 100 | -0.2 | 0.8 | 0.9188 (44.66) | 0.9354 (51.90) | 0.9187 (46.72) | 0.8951 (57.22) | 0.9151 (44.73) | 0.9354 (45.83) | 0.297 |
| 100 | 00 | 0.8 | -0.8 | 0169 (44.37) | 0.9367 (51.53) | 0.9208 (46.41) | 0.8921 (56.85) | 0.9117 (44.44) | 0.9377 (45.54) | . 295 |
| 100 | 100 | 0.2 | -0.2 | 0.9185 (44.73) | 0.9344 (52.13) | 0.9183 (46.81) | 0.8971 (57.48) | 0.9150 (44.80) | 0.9378 (45.90) | 0.296 |
| 100 | 100 | 0.8 | -0.2 | 0.9185 (44.32) | 0.9363 (51.30) | 0.9215 (46.36) | 0.8955 (56.76) | 0.9145 (44.38) | 0.9378 (45.48) | 29 |
| 100 | 100 | 0.2 | -0.8 | 0.9191 (44.59) | 0.9319 (51.68) | 0.9167 (46.65) | 0.8939 (57.34) | 0.9153 (44.66) | 0.9377 (45.76) | 0.296 |
| 50 | 00 | 0.8 | 0.8 | 007 (60.80) | . 9292 (78.83) | 0.9066 (64.56) | 0.8671 (87.33) | 0.8945 (60.95) | 0.9234 (62.97) | 45 |
| 50 | 100 | 0.2 | 0.2 | 0.8947 (60.40) | 0.9290 (77.50) | 0.9033 (64.11) | 0.8625 (86.63) | 0.8887 (60.54) | 0.9241 (62.55) | . 344 |
| 50 | 100 | 0.8 | 0.2 | 0.8945 (60.35) | 0.9260 (77.26) | 0.8999 (64.02) | 0.8631 (86.71) | 0.8894 (60.49) | 0.9180 (62.50) | 0.346 |
| 50 | 100 | 0.2 | 0.8 | 0.8930 (60.21) | 0.9272 (77.03) | 0.9021 (63.85) | 0.8612 (86.55) | 0.8864 (60.35) | 0.9204 (62.35) | 0.347 |
| 50 | 10 | -0.8 | -0.8 | 0.8966 (60.87) | 0.9280 (78.80) | 0.9040 (64.72) | 0.8691 (87.30) | 0.8913 (61.01) | 0.9198 (63.03) | 46 |
| 50 | 100 | -0.2 | -0.2 | 0.9035 (61.33) | 0.9355 (79.30) | 0.9115 (65.20) | 0.8701 (88.56) | 0.8965 (61.47) | 0.9280 (63.50) | 0.349 |
| 50 | 100 | -0.8 | -0.2 | 0.9005 (60.36) | 0.9310 (77.31) | 0.9082 (64.17) | 0.8712 (86.40) | 0.8968 (60.50) | 0.9246 (62.51) | 0.346 |
| 50 | 100 | -0.2 | -0.8 | 0.8892 (60.77) | 0.9266 (79.15) | 0.9023 (64.59) | 0.8569 (88.00) | 0.8816 (60.92) | 0.9186 (62.93) | 0.347 |
| 100 | 50 | -0.8 | 0.8 | 0.9202 (45.19) | 0.9320 (51.53) | 0.9170 (46.98) | 0.9012 (56.46) | 0.9179 (45.26) | 0.9432 (46.92) | . 274 |
| 100 | 50 | -0.2 | 0.2 | 0.9212 (45.42) | 0.9322 (51.97) | 0.9177 (47.34) | 0.9006 (57.34) | 0.9181 (45.50) | 0.9436 (47.16) | 0.276 |
| 100 | 50 | -0.8 | 0.2 | 0.9192 (45.55) | 0.9269 (52.20) | 0.9142 (47.51) | 0.9008 (57.61) | 0.9159 (45.63) | 0.9388 (47.29) | 0.278 |
| 100 | 50 | -0.2 | 0.8 | 0.9233 (45.18) | 0.9289 (51.43) | 0.9162 (46.97) | 0.9084 (56.38) | 0.9204 (45.25) | 0.9408 (46.91) | 0.273 |
| 100 | 50 | 0.8 | -0.8 | 0.9221 (45.16) | 0.9304 (51.49) | 0.9177 (47.04) | 0.9019 (56.61) | 0.9198 (45.23) | 0.9432 (46.89) | 0.275 |
| 100 | 50 | 0.2 | -0.2 | 0.9239 (45.50) | 0.9323 (51.49) | 0.9186 (47.47) | 0.9053 (57.60) | 0.9213 (45.57) | 0.9442 (47.23) | 0.277 |
| 100 | 50 | 0.8 | -0.2 | 0.9230 (45.35) | 0.9349 (51.49) | 0.9192 (47.25) | 0.9046 (57.09) | 0.9198 (45.42) | 0.9455 (47.08) | 0.276 |
| 100 | 50 | 0.2 | -0.8 | 0.9286 (45.39) | 0.9351 (51.49) | 0.9226 (47.25) | 0.9102 (57.11) | 0.9271 (45.46) | 0.9457 (47.13) | 0.277 |

Table 5
Summary statistics for the two groups

|  | Estimated ICER $=15.33$, NHB $=77.08$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Control group $\left(n_{1}=54\right)$ |  |  |  |  |  |  |  | Intervention group $\left(n_{2}=53\right)$ |
|  | Mean | S.D. | Skewness | Mean | S.D. | Skewness |  |  |  |
| Cost (U.S.S) | 2507.42 | 4460.44 | 4.93 | 1325.48 | 1785.67 | 3.46 |  |  |  |
| Effectiveness (days | 211.52 | 139.68 | -.30 | 134.42 | 134.55 | .71 |  |  |  |
| with anxiety attack) |  |  |  |  |  |  |  |  |  |

Table 6
Summary of confidence intervals for the ICER and the NHB

|  |  | Methods | Confidence intervals |
| :--- | :--- | :--- | :---: |
| Interval length |  |  |  |
| ICER |  |  |  |
|  |  | Taylor | $(-3.44,34.10)$ |
|  | Fieller | $(-1.43,53.86)$ | 37.54 |
|  | Hinkley | $(-1.68,52.44)$ | 54.28 |
|  | Bootstrap-t | $(4.50,49.55)$ | 45.05 |
|  | $T_{1}$ | $(3.33,81.87)$ | 78.54 |
|  | $T_{2}$ | $(-1.99,35.22)$ | 37.21 |
|  | $T_{3}$ | $(0.35,38.38)$ | 38.03 |
| NHB |  |  |  |
|  | Normal | $(25.12,129.03)$ | 103.92 |
|  | Bootstrap-t | $(21.68,132.33)$ | 110.65 |
|  | BCa | $(20.21,125.52)$ | 105.31 |
|  | $T_{1}$ | $(26.78,130.87)$ | 104.10 |
|  | $T_{2}$ | $(25.46,129.39)$ | 103.92 |
|  | $T_{3}$ | $(32.84,144.42)$ | 111.58 |

