# Is the Number of Sick Persons in a Cohort Constant Over Time? 

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### 1.0 Introduction

In the United States, the population is aging and life expectancy is increasing. This trend has been expected to produce an increase in the number of persons at risk for costly ageassociated chronic diseases, injuries, and disability. However, the patterns of health and death and their associated costs in the aging time are unclear. It is important to understand the natural trajectory of health and morbidity over time. Ideally, we would follow a particular cohort from some point in time until death, and tabulate at each time how many persons are healthy (by some definition), how many are sick, and how many have died. In one longitudinal dataset, the number of sick persons in the cohort ("sick" was defined as being in fair or poor health, or alternatively as having at least one difficulty with activities of daily living) was approximately constant for nine years, based both on observed data and in synthetic cohorts created from transition probabilities using multi-state life table methods. 1234 This stability was unexpected, because there was substantial change over time in the number who were healthy or dead. We had expected that the number of sick persons would increase over time, although the number would eventually have to go to zero when everyone had died. Here, we attempt to describe and understand this stability, and to consider its implications for life expectancy, years of healthy life, and medical expenditures.

### 2.0 Methods

### 2.1 Study Design: The Cardiovascular Health Study.

The Cardiovascular Health Study (CHS) is a population-based longitudinal study of 5,888 adults aged 65 and older at baseline, designed to identify factors related to the occurrence of
coronary heart disease and stroke. ${ }^{5}$ Subjects were recruited from a random sample of the Medicare eligibility lists in four U.S. counties. Persons not expected to be able to participate for the next three years were ineligible, and about 59\% of those eligible agreed to enroll. ${ }^{6}$ Two cohorts were followed, one with 9 years of follow-up ( $\mathrm{n}=5201$ ) and the second (all African American, $\mathrm{n}=687$ ) with 6 years of follow-up. At baseline the mean age was 73 (range 65 to 105), $58 \%$ were women, and $84 \%$ were white. Data collection began in about 1990, and follow-up is virtually complete for all surviving subjects in the year 1999 (and for a few measures through 2004).

### 2.2 Health-Related Variables:

We selected 8 health-related variables that were measured every year and have been used elsewhere, ${ }^{7}$ and defined "sick" for each one. These definitions included having a Modified Mini Mental State Examination score below 80 (MMSE); ${ }^{8}$ having 1 or more difficulties with activities or instrumental activities of daily living (ADL and IADL); a score above 10 on the Center for Epidemiologic Studies Depression score (CESD); ${ }^{9}$ any days spent in bed in the previous two weeks (Bed Days); requiring more than 10 seconds to walk 15 feet (Timed Walk); and prevalent heart disease (CVD), defined as having angina, coronary heart disease, congestive heart failure, claudication, myocardial infarction, stroke, transient ischemic attack, angioplasty, or coronary artery bypass surgery at the survey time or earlier. A person who is sick by this definition (has CVD) cannot become healthy (no CVD) in the future. These measures include both self-report and clinical definitions, as well as one definitiion in which recovery is not possible. Findings that are similar for all of these disparate definitions of "sick" may be considered to hold in general.

Most of this paper examines self-rating of health (EVGGFP) as either excellent, very good or good (E/VG/G=healthy), or fair or poor (F/P=sick). EVGGFP was singled out because it was collected semi-annually, and continued to be collected by telephone even after the study clinic visits ended, providing up to 29 measures per person which could be used to calculate transition probabilities. The other variables in the CHS dataset, listed above, were collected only annually and were not collected after the end of the study. Data that were missing were imputed using simple methods have been shown to perform well in the CHS dataset. ${ }^{10}$ We define a transition pair as two EVGGFP values for the same person measured 1 year apart. The 5,888 CHS subjects contributed about 150,000 transition pairs, which were used to estimate the probability of moving from one state to another, at different ages.

### 2.3 Additional Data.

To increase the quantity and the age range of the transition data, we also used information from two large national surveys, the Medicare Current Beneficiary Study (MCBS, 1998-2002) ${ }^{11}$ and the Medical Expenditures Panel Survey (MEPS, 1996-2001). ${ }^{12}$ In MCBS, persons were followed 2 to 6 years, and about 41,000 persons contributed about 98,000 transition pairs. Unlike CHS, the MCBS sample had slightly more sicker persons than the general population. ${ }^{13}$ In MEPS, approximately 93,000 persons contributed about 224,000 transition pairs for ages 0 to 64 , and 29,000 transition pairs for age $\geq 65$. MEPS did not survey institutionalized persons. The small number of persons not surveyed for that reason were given "poor" health and we imputed the data missing for other reasons. MEPS ages above 90 (and later 85) were set to 90 (85). One person answered for everyone in the family, meaning that MEPS information was not usually "self" reported. We ignored the survey weights, to make the data consistent with the CHS data.

Data on life expectancy came from the Statistical Abstracts of the United States. ${ }^{14}$ Estimates of the health of Americans by age came from the National Health Interview Survey. ${ }^{15}$ Data on medical expenditures by age and health were estimated from MEPS 2002 data using the MEPSNET program. ${ }^{16}$

### 2.4 Transition probability calculations

We combined the transition pairs from all three datasets, calculated the transition probabilities for each age, and smoothed the curves over age using a loess smoother. (This smoother is esssentially a moving average in equally spaced data like ours with one observation per year). Estimates for ages below 65 were based on about 3400 transition pairs per year of age, and those over 65 were based on about 8000 transition pairs per year.

### 2.5 Projection of Future Health

We used the transition probabilities for EVGGFP to project the number of healthy, sick, and dead persons over time for a synthetic birth cohort that 98,000 healthy persons and 2000 sick persons at baseline, chosen to agree with national statistics. This was done using standard multistate life table methods,,$^{17}$ implemented in a spreadsheet and partly in a Stata program. ${ }^{18}$ We also created a synthetic cohort of 5201 persons whose initial distribution of age and health matched the CHS baseline population. An example of the calculations is given in section 3.2.2.

### 2.6 Analysis

We first examined whether the number of sick persons was constant over time in the actual CHS data, for all 8 definitions of being sick. We next used the estimated the transition probabilities to project the number of healthy, sick, and dead persons over time in the synthetic birth and CHS, using standard multi-state lifetable methods. We multiplied the expected
expenditures by the number of persons projected to be in each age and health state to estimate lifetime medical expenditures.

### 3.0 Findings

### 3.1 Number of healthy, sick, and dead persons in the CHS cohort

Figure 1 describes the first CHS cohort. It shows the number who were healthy (E/VG/G), sick (F/P), or dead in the 14 years after baseline. The number healthy decreased over time, the number dead increased, and the number who were sick was approximately constant. This was the graph that suggested the current investigation. The pattern was similar for men and women separately (not shown).
[Figure 1 about here]
Figure 2 shows the number healthy, sick, or dead using different definitions of "sick", as explained above, starting at baseline. (For convenience in plotting, the Y axis is the percent of 5,201 persons in each health state rather than the count). The number in parentheses is the mean number of persons per year based on this definition. The number sick was always approximately constant over time, even though "sick" is defined differently in each graph. The graph labeled EVGGFP-1 is equivalent to the first 9 years in Figure 1, and that labeled EVGGFP-2 is the same variable but for the second cohort, $\mathrm{n}=687$ and all African American. The number sick in EVGGFP-2 was higher than that in EVGGFP-1, but was also reasonably stable over time.
[Figure 2 about here]

### 3.2 Number healthy, sick and dead in a synthetic cohort

We used the estimated transition probabilities among the three health states (healthy, sick, and dead) to project the number of healthy, sick, and dead persons over time in the synthetic cohorts.

### 3.2.1 Health States and Transition Probabilities

Here, we define healthy as "E/VG/G" health, and sick as "F/P" health. The probabilities of transition among the three states were estimated from the 500,000 transition pairs, and are listed in the appendix table). Persons who are healthy at the first observation have the probability $\mathrm{P}(\mathrm{H} \mid \mathrm{H})$ of being healthy 1 year later; the probability $\mathrm{P}(\mathrm{S} \mid \mathrm{H})$ of being sick 1 year later; and $\mathrm{P}(\mathrm{D} \mid \mathrm{H})$ of being dead 1 year later. Similarly, persons who are initially sick have the associated probabilities $\mathrm{P}(\mathrm{H} \mid \mathrm{S}), \mathrm{P}(\mathrm{S} \mid \mathrm{S})$, and $\mathrm{P}(\mathrm{D} \mid \mathrm{S})$. The estimated transition probabilities change with age, as shown in Figure 3. The probabilities below age 65 are based only on MEPS data, while those for 65 and above are calculated from all three datasets. Despite the smoothing there is a discontinuity near age 65 and above age 95 for some of the probability estimates.
[Figure 3 about here]
The finding that $\mathrm{P}(\mathrm{H} \mid \mathrm{H})$ and $\mathrm{P}(\mathrm{H} \mid \mathrm{S})$ decrease with age reflects clinical experience that older adults become less likely to remain healthy or to recover from illness. $\mathrm{P}(\mathrm{D} \mid \mathrm{H}), \mathrm{P}(\mathrm{D} \mid \mathrm{S})$, and $\mathrm{P}(\mathrm{S} \mid \mathrm{H})$ increase monotonically with age, which also is not surprising. But we did not know in advance how the probability of remaining sick would change with age. For example, $\mathrm{P}(\mathrm{S} \mid \mathrm{S})=1$ $P(H \mid S)-P(D \mid S)$, but since $P(H \mid S)$ decreases with age while $P(D \mid S)$ increases, the effect of aging on $\mathrm{P}(\mathrm{S} \mid \mathrm{S})$ was not obvious. Here, $\mathrm{P}(\mathrm{S} \mid \mathrm{S})$ increases until about age 50 , and is fairly flat until about age 80, after which it declines, presumably because sick persons become more likely to die than
to remain sick. The trends over age 65 were reasonably similar in the three datasets with the most differences in the probability of remaining sick, $\mathrm{P}(\mathrm{S} \mid \mathrm{S})$ (not shown).

### 3.2.2 Synthetic Cohorts

For those unfamiliar with life-table calculations, we present a simple example. Consider a cohort with a specified number of healthy and sick persons at baseline, and also specified transition probabilities among the three states: healthy, sick, and dead. For specificity, use the probabilities at age 65 (see Figure 3), which are as follows: $\mathrm{P}(\mathrm{H} \mid \mathrm{H})=.90 ; \mathrm{P}(\mathrm{S} \mid \mathrm{H})=.09$; $\mathrm{P}(\mathrm{D} \mid \mathrm{H})=.01 ; \mathrm{P}(\mathrm{H} \mid \mathrm{S})=.34 ; \mathrm{P}(\mathrm{S} \mid \mathrm{S})=.61$; and $\mathrm{P}(\mathrm{D} \mid \mathrm{S})=.05$. For an artificial cohort of 65-year-olds, of whom (say) 100 are healthy and 100 are sick at age 65, we can estimate the number who will be in each health state one year later, at age 66. Of the 100 healthy persons, the probabilities indicate $.09 * 100=9$ will be sick one year later, 1 will be dead, and 90 will still be healthy. Of those who start out sick, 61 will remain sick, 34 will become healthy, and 5 will die. Thus, at age 66 , there would be $90+34=124$ healthy persons, 70 sick persons, and 6 dead persons. These calculations can be repeated at age 66, using the transition probabilities specific to that age. The process can be continued until all subjects have died. At that point the total number of person-years spent in the healthy and sick states can be calculated, to yield estimates of the years of healthy life, years of sick life (morbidity), and years of life.

We estimated the number healthy, sick, and dead over time in a birth cohort of 100,000 persons, of whom $98 \%$ were healthy at age $0{ }^{15}$ Figure 4 shows the estimated number of healthy, sick, and dead persons over time. The number healthy declines and the number dead increases, as expected. The number sick increases slowly until about age 55, is fairly flat until about age 75 , and declines after that. The area under the Dead curve (divided by 100,000) is the average years
lost to death $=23.8$, and 100-23.8 is the average life expectancy, or 76.2 years. This is remarkably close to the life expectancy of the US population (77.3 years). The area below the Healthy curve is 66.9 years of healthy life from birth to age 100. The area below the Sick curve is 9.3 years of sick life or morbidity.
[Figure 4 about here]
The actual CHS cohort is a mix of birth cohorts and baseline health states. It is clear from Figure 4 that a cohort consisting primarily of persons aged 55 to 75 at baseline will have a fairly constant number of sick persons over time, while an excess of persons below 55 will result in an increase over time and an excess above 75 will result in a decrease. Using the starting age and health status distribution for the CHS cohort, the estimated number of sick persons in the synthetic CHS cohort was approximately constant for 14 years of follow-up, as was seen in Figure 1 for the real data. (not shown)

### 3.2.3 Medical Expenditures for the Birth Cohort

In Figure 4, the number of sick persons, reasonably expected to have the highest medical expenditures, stays constant from age 55-75, while the less expensive group (healthy) declines in size over time. If medical expenditures were a function only of health state, cost for the cohort would decrease after age 55, as the number of sick persons would be constant but the number healthy would decline. However, expenditures increase with age as well as with health status. ${ }^{19}$ We multiplied the estimated the mean cost by the projected number of persons in each health state and age. Total estimated annual costs for the birth cohort of 100,000 would increase monotonically with age after age 10, would be fairly constant at about $\$ 420$ to $\$ 430$ million per year from ages 61-73, and would decline after that.

### 4.0 Summary and discussion

### 4.1 Is the number of sick persons constant?

The number of sick persons was approximately constant in the real CHS data for every definition of "sick" that was considered. This finding is strengthened by the variety of variables that were considered. Timed walk was assessed at the clinic (not by self-report). Unlike the other variables, CVD (prevalent heart disease) was cumulative, in that $\mathrm{P}(\mathrm{H} \mid \mathrm{S})$ is zero. Persons in Cohort 2 (all African American) were substantially sicker than those in Cohort 1, but the number sick was still fairly constant over time.

We were able to reproduce Figure 1 from the transition probabilities and initial CHS age and health distribution. In addition, we projected that there would be a constant number sick in a birth cohort from about age 55 to 75 . The increase in the number dead and the decrease in the number healthy in the birth cohort, shown in Figure 4, were expected. It now seems obvious that the number sick must increase at earlier ages (since most persons are healthy at birth) and must decrease at the end (as most persons will be dead). The large range over which the estimated number of sick persons was constant was not expected.

This long period of stability may not have been noticed earlier, because there are few lengthy longitudinal series and most analyses have dealt with the \% of the living who were sick, rather than the \% of all the persons in the cohort. That is, many had noted that the \% of survivors who were sick increased over time, ${ }^{17}$ but our finding is equivalent to saying that the $\%$ of the initial cohort who were sick was constant for a long period.

The strong similarity of the patterns in Figure 2 suggests that we can estimate the stable level of the number sick for the variables other than EVGGFP. For example, in a figure similar to

Figure 4, but with sick defined as having ADL difficulties, we would expect the number sick in a birth cohort of 100,000 to increase over time, then to be stable for a long period, and finally to decrease, with about 756/5201*100,000 $=14,535$ persons with ADL difficulties each year during the stable period. This supposition needs to be verified, as does the age interval over which stability occurs.

The decrease in the medical expenditures of a birth cohort after about age 73 may provide an alternative way to think about future Medicare expenditures, although more detailed models are needed to provide exact expenditure estimates. The lifetime distribution of healthcare costs has been examined in a different way by Alemeyehy and Warner. ${ }^{20}$

### 4.2 Transition Probabilities

The transition probabilities were estimated from half a million transition pairs. Even so, there were relatively few sick and dying young persons and few healthy persons over age 90 . The data sets were drawn from different populations, in that positive selection bias at age 65 was a factor for CHS, the three study samples were drawn from somewhat different populations, and "healthy" was reported by the head of the household for the MEPS data. However, our only assumption was that the transition probabilities among the health states for each age were the same in all datasets, which could hold true even if the survey populations are different. Life expectancies calculated from the separate datasets (not shown) indicated that CHS and MEPS over-estimated survival, and MCBS underestimated survival, but the combined estimates were very close to the national figures. In the National Health Interview Survey, proxy respondents were found to give a more positive assessment of the health-related quality of life of others, which may explain why MEPS data produced more optimistic estimates. ${ }^{21}$ With these caveats, we next
discuss some features of the transition probabilities which have not, to our knowledge, been published in such detail and for such an age range.

Some will find it surprising that a simple measure like EVGGFP is such a strong predictor of mortality; that is, in Figure 3, $\mathrm{P}(\mathrm{D} \mid \mathrm{S})$ is considerably higher than $\mathrm{P}(\mathrm{D} \mid \mathrm{H})$. Considering only the age range where neither of these probabilities were based on small numbers, the estimated relative risk of death is above 7 from age 30 to 55, and then declines to about 2 at age 90 (data not shown).

It is not always recognized that even older adults have a meaningful probability of recovering from being sick. Figure 3 shows that for EVGGFP, $\mathrm{P}(\mathrm{H} \mid \mathrm{S})$ is about $70 \%$ under age 20, about $30 \%$ at age 65 , and then declines monotonically to about $10 \%$ at age 100 . This meaningful rate of recovery is of interest because some statistical procedures such as the quality-adjusted time without symptoms or toxicity (Q-Twist) need to assume that recovery is not possible, ${ }^{22}$ which is clearly not true in a general population. This recovery rate is consistent with rates reported by Gill and Hardy. ${ }^{23} 24$ Estimated recovery probabilities based on other definitions of being sick are available elsewhere. ${ }^{25}$

It is also interesting that so many healthy persons die. $\operatorname{Prob}(\mathrm{D} \mid \mathrm{H})$ is about $1 \%$ at age 65 but increases to about $20 \%$ at age 95 . As these are 1-year transitions, some healthy persons may have fallen sick and then died within the year. However, $18 \%$ of the CHS enrollees who died were healthy within 3 months of death, suggesting that a substantial number may die while assessing themselves as healthy. ${ }^{3}$

### 4.4 System Dynamics

This phenomenon of a constant number sick for an extended time may be understood better in terms of the behavior of a system which has three states (healthy, sick, and dead). A system evolves towards a distribution of health states in equilibrium with the transition probabilities. If the transition probabilities are constant, it is shown in Appendix 1 that the ratio of the number healthy to sick approaches:

$$
K=\frac{H}{S}=\frac{P(H \mid H)-P(S \mid S)}{2 P(S \mid H)}+\sqrt{\frac{[P(H \mid H)-P(S \mid S)]^{2}}{[2 P(S \mid H)]^{2}}+\frac{P(H \mid S)}{P(S \mid H)}}
$$

However, the transition probabilities do change with age, and inspection of Figure 3 shows that K must also decline with age (terms in the numerator decrease with age, and those in the denominator increase). A system that is in equilibrium at one age will be slightly out of equilibrium a year later, because K will have decreased. The system attempts to reach equilibrium by decreasing the number in the healthy state and/or increasing the number in the sick state. At younger ages, decreasing the number in the healthy state increases the number sick because deaths are negligible. After age 50, decreasing the number healthy does not increase the number sick because of the increasing death rate. At the oldest ages, the number becoming sick is lower than the number of sick dying (and recovering), and the number of sick persons declines.

### 4.5 Limitations

The longitudinal data from Figure 1 and Figure 2 came from a single study. Additional lengthy longitudinal datasets should be examined. There was not perfect agreement among the three datasets, particularly at the oldest ages. CHS and MEPS data over-estimated the published life expectancy, and MCBS data underestimated it.

Because our goal was to understand what was happening, rather than to provide detailed probability or projection estimates, we made many simplifications. We restricted analysis to only 3 health states, and ignored gender, which is known to affect transition probabilities. ${ }^{4}$ We ignored other information which would have improved the probability estimates, such as the person's health one year before "now" or information about serious illness. Although such information would greatly improve estimates of transition for an individual, it was not needed for this paper, as we projected transitions for groups of persons and re-estimated the probability estimates for each age. The transition probabilities at each age should be appropriate for the group, since they were computed using data from general populations, which were likely close to the equilibrium distribution of health states. Some of the change over time in the probabilities is due to aging, and some due to the changing make-up of each health state. For example, if the healthy state at baseline comprised two equal subgroups, one with $90 \%$ of remaining healthy one year later and one with $70 \%$ remaining healthy, our estimated transition probability would be .8 , and $80 \%$ of the state would indeed remain healthy one year later, even though .8 was not the correct probability for any person in the group.

The cohort projections assume that the transition probabilities are stable over time (no birth cohort effect), which may not be the case. ${ }^{26} 27$ A cohort initiated in 2005 and followed to death might not have the same number in the health states if the probabilities change substantially in their lifetime. In spight of these concerns, the estimated probabilities were able to reproduce Figure 1, and the estimated life expectancy was close to published values. Validity of similar estimates has been demonstrated elsewhere. ${ }^{2,4}$

We calculated transition probabilities for only one measure of sickness, but Figure 2 suggests that similar results will be found when using other definitions of "sick". This conjecture needs verification. Although different populations may have different transition probabilities, we have in a sense considered some different transition probabilities by considering different definitions of being sick. Our understanding is limited in that the transition probabilities were calculated by brute force, without a theoretical model. Further research should include development of a theory-based model that also takes sex, prior health, and incident health events into account.

### 4.6 Conclusion

We found that in a birth cohort the number of persons in fair or poor health reaches a maximum and is approximately constant from ages 55-75, after which it declines. This results in a constant number of sick persons over time in some cohorts of older adults. This phenomenon is likely to hold for many other definitions of "sick" as well. This interesting finding may be useful in concepetualizing how the health and medical expenditures of a population or a cohort will change over time.

## Appendix 1

The following is a derivation of the equation in section 4.4.
Let $\mathrm{H}_{\mathrm{t}}, \mathrm{H}_{\mathrm{t}+1}=$ the number of healthy persons at times t and $\mathrm{t}+1$.
Let $\mathrm{S}_{\mathrm{t}}, \mathrm{S}_{\mathrm{t}+1}=$ the number of sick persons at times t and $\mathrm{t}+1$.
Assuming first that the transition probabilities do not change with age, we conjecture that the ratio of the number healthy to the number sick approaches a constant; that is, that there is a constant, K , such that eventually, for some time $t$,
[1] $\frac{H_{t}}{S_{t}}=\frac{H_{t+1}}{S_{t+1}}=K$
$\mathrm{H}_{\mathrm{t}+1}$ and $\mathrm{S}_{\mathrm{t}+1}$ can be calculated from $\mathrm{H}_{\mathrm{t}}$ and $\mathrm{S}_{\mathrm{t}}$ and the transition probabilities, as follows:
[2] $\mathrm{H}_{\mathrm{t}+1}=\mathrm{H}_{\mathrm{t}} * \mathrm{P}(\mathrm{H} \mid \mathrm{H})+\mathrm{S}_{\mathrm{t}} * \mathrm{P}(\mathrm{H} \mid \mathrm{S})$
[3] $\mathrm{S}_{\mathrm{t}+1}=\mathrm{H}_{\mathrm{t}} * \mathrm{P}(\mathrm{S} \mid \mathrm{H})+\mathrm{S}_{\mathrm{t}} * \mathrm{P}(\mathrm{S} \mid \mathrm{S})$
From equation 1, $\mathrm{H}_{\mathrm{t}}=\mathrm{K} * \mathrm{~S}_{\mathrm{t}}$. Substituting for $\mathrm{H}_{\mathrm{t}}$ in equations 2 and 3 yields
[4] $\mathrm{H}_{\mathrm{t}+1}=\mathrm{K}^{*} \mathrm{~S}_{\mathrm{t}} * \mathrm{P}(\mathrm{H} \mid \mathrm{H})+\mathrm{S}_{\mathrm{t}}{ }^{*} \mathrm{P}(\mathrm{H} \mid \mathrm{S})$
[5] $\mathrm{S}_{\mathrm{t}+1}=\mathrm{K} * \mathrm{~S}_{\mathrm{t}} * \mathrm{P}(\mathrm{S} \mid \mathrm{H})+\mathrm{S}_{\mathrm{t}} * \mathrm{P}(\mathrm{S} \mid \mathrm{S})$
From equation 1, the ratio of equation 4 to $5=\mathrm{K}, \mathrm{S}_{\mathrm{t}}$ cancels out, and
[6] $\frac{H_{t+1}}{S_{t+1}}=K=\frac{K * P(H \mid H)+P(H \mid S)}{K * P(S \mid H)+P(S \mid S)}$
This yields a quadratic equation
[7] $\mathrm{K}^{2^{*}} \mathrm{P}(\mathrm{S} \mid \mathrm{H})+\mathrm{K}^{*}(\mathrm{P}(\mathrm{S} \mid \mathrm{S})-\mathrm{P}(\mathrm{H} \mid \mathrm{H}))-\mathrm{P}(\mathrm{H} \mid \mathrm{S})=0$
Solving equation 7 for K yields
$[8] K=\frac{H}{S}=\frac{P(H \mid H)-P(S \mid S)}{2 P(S \mid H)}+\sqrt{\frac{[P(H \mid H)-P(S \mid S)]^{2}}{[2 P(S \mid H)]^{2}}+\frac{P(H \mid S)}{P(S \mid H)}}$
That is, once the ratio of healthy to sick reaches K , it will remain there.
The assumption that the probabilities do not change over time did not hold, as shown in Figure 3. Because the numerator terms in equation 8 decrease with age and the denominator terms increase, the value of $K$ at time $t$, $K_{t}$, will decrease over time. Even under extreme starting conditions, such as everyone being sick at birth, the ratio of the projected number healthy to sick quickly approaches the value of $K_{t}$ calculated from the transition probabilities for age $t$, and remains equal to the appropriate $\mathrm{K}_{\mathrm{t}}$ after that. This means that the expected ratio of the number healthy to the number sick depends on the initial conditions for a few years, but after that it can be estimated simply from equation 8, based on knowing the probabilities but without requiring lifetable calculations.

Appendix 2 shows the estimated probabilities. It also shows the equilibrium ratio (K) of healthy to sick, calculated from equation [8]. It also shows the \% of living persons who are healthy at that time, which is $\mathrm{K} /(\mathrm{K}+1) * 100$.

## Figure 1

Prevalence of 3 Health States by Years after Baseline CHS data, Cohort 1 only


Healthy=exc/very good/good, Sick=fair/poor

## Figure 2

## Actual Percent Healthy, Sick, and Dead

By Number of Years after Baseline Using Different Definitions of Sick

CHS data, all $\geq 65$
(The number in parentheses is the average number sick)


Figure 3
Estimated One-year Transition Probabilities by Age
Estimated Transition Probabilities



[^0]Figure 4
Estimated \# of healthy, sick and dead persons in birth cohort By Age (98\% were healthy at age 0)

## \# Healthy, Sick, Dead in Birth Cohort



## Appendix 2 <br> Estimated Transition Probabilities

| AGE | H\| ${ }^{\text {l }}$ | S\|S | H\|S | S\|H | D\|S | D \| H | H/S <br> EQUIL <br> RATIO | \% healthy <br> if alive |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | . 974 | . 260 | . 726 | . 025 | . 014 | . 000 | 29.06 | 96.67 |
| 1 | . 975 | . 274 | . 714 | . 024 | . 012 | . 000 | 29.82 | 96.76 |
| 2 | . 976 | . 284 | . 707 | . 023 | . 010 | . 000 | 30.55 | 96.83 |
| 3 | . 977 | . 289 | . 703 | . 023 | . 008 | . 000 | 31.24 | 96.90 |
| 4 | . 978 | . 292 | . 701 | . 022 | . 007 | . 000 | 31.85 | 96.96 |
| 5 | . 978 | . 294 | . 700 | . 022 | . 006 | . 000 | 32.29 | 97.00 |
| 6 | . 978 | . 296 | . 700 | . 022 | . 005 | . 000 | 32.48 | 97.01 |
| 7 | . 978 | . 297 | . 699 | . 022 | . 004 | . 000 | 32.36 | 97.00 |
| 8 | . 978 | . 299 | . 698 | . 022 | . 003 | . 000 | 31.91 | 96.96 |
| 9 | . 977 | . 301 | . 697 | . 022 | . 003 | . 000 | 31.17 | 96.89 |
| 10 | . 977 | . 303 | . 695 | . 023 | . 002 | . 000 | 30.21 | 96.80 |
| 11 | . 976 | . 305 | . 693 | . 024 | . 002 | . 000 | 29.09 | 96.68 |
| 12 | . 975 | . 308 | . 690 | . 025 | . 002 | . 000 | 27.90 | 96.54 |
| 13 | . 974 | . 310 | . 688 | . 026 | . 002 | . 000 | 26.68 | 96.39 |
| 14 | . 973 | . 313 | . 685 | . 027 | . 002 | . 000 | 25.46 | 96.22 |
| 15 | . 971 | . 315 | . 683 | . 028 | . 002 | . 000 | 24.28 | 96.04 |
| 16 | . 970 | . 318 | . 680 | . 029 | . 002 | . 000 | 23.15 | 95.86 |
| 17 | . 969 | . 321 | . 677 | . 031 | . 002 | . 000 | 22.06 | 95.66 |
| 18 | . 967 | . 326 | . 672 | . 032 | . 002 | . 000 | 21.00 | 95.45 |
| 19 | . 966 | . 331 | . 667 | . 033 | . 003 | . 001 | 19.99 | 95.24 |
| 20 | . 965 | . 337 | . 660 | . 035 | . 003 | . 001 | 19.03 | 95.01 |
| 21 | . 963 | . 344 | . 653 | . 036 | . 003 | . 001 | 18.12 | 94.77 |
| 22 | . 962 | . 352 | . 645 | . 037 | . 003 | . 001 | 17.25 | 94.52 |
| 23 | . 961 | . 361 | . 636 | . 039 | . 003 | . 001 | 16.45 | 94.27 |
| 24 | . 959 | . 370 | . 627 | . 040 | . 003 | . 001 | 15.72 | 94.02 |
| 25 | . 958 | . 379 | . 618 | . 041 | . 003 | . 001 | 15.06 | 93.78 |
| 26 | . 957 | . 387 | . 609 | . 042 | . 003 | . 001 | 14.48 | 93.54 |
| 27 | . 956 | . 396 | . 601 | . 043 | . 004 | . 001 | 13.95 | 93.31 |
| 28 | . 955 | . 404 | . 592 | . 044 | . 004 | . 001 | 13.47 | 93.09 |
| 29 | . 954 | . 413 | . 583 | . 045 | . 004 | . 001 | 13.02 | 92.86 |
| 30 | . 954 | . 423 | . 573 | . 046 | . 004 | . 001 | 12.58 | 92.64 |
| 31 | . 953 | . 432 | . 563 | . 047 | . 005 | . 001 | 12.16 | 92.40 |
| 32 | . 952 | . 441 | . 554 | . 048 | . 005 | . 001 | 11.73 | 92.15 |
| 33 | . 951 | . 451 | . 543 | . 048 | . 005 | . 001 | 11.30 | 91.87 |
| 34 | . 950 | . 462 | . 533 | . 049 | . 006 | . 001 | 10.86 | 91.57 |
| 35 | . 949 | . 472 | . 522 | . 051 | . 006 | . 001 | 10.42 | 91.25 |
| 36 | . 948 | . 483 | . 511 | . 052 | . 007 | . 001 | 9.99 | 90.90 |
| 37 | . 947 | . 494 | . 499 | . 053 | . 007 | . 001 | 9.57 | 90.54 |
| 38 | . 945 | . 504 | . 488 | . 054 | . 008 | . 001 | 9.16 | 90.16 |
| 39 | . 944 | . 514 | . 478 | . 055 | . 008 | . 001 | 8.78 | 89.77 |
| 40 | . 943 | . 524 | . 468 | . 056 | . 009 | . 001 | 8.42 | 89.38 |
| 41 | . 942 | . 533 | . 458 | . 058 | . 010 | . 001 | 8.08 | 88.98 |
| 42 | . 940 | . 542 | . 448 | . 059 | . 010 | . 001 | 7.75 | 88.57 |
| 43 | . 939 | . 551 | . 438 | . 060 | . 011 | . 001 | 7.43 | 88.14 |
| 44 | . 937 | . 560 | . 429 | . 062 | . 011 | . 001 | 7.11 | 87.68 |


|  |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  | H/S | EQUIL | \% healthy |
| AGE | H\|H | S\|S | H\|S | S\|H | D\|S | D\|H | RATIO | if alive |
| 45 | .936 | .569 | .419 | .063 | .012 | .001 | 6.80 | 87.18 |
| 46 | .934 | .578 | .409 | .065 | .013 | .001 | 6.49 | 86.66 |
| 47 | .933 | .587 | .399 | .066 | .014 | .001 | 6.20 | 86.10 |
| 48 | .931 | .595 | .389 | .068 | .015 | .001 | 5.91 | 85.53 |
| 49 | .929 | .603 | .380 | .070 | .017 | .002 | 5.65 | 84.95 |
| 50 | .927 | .610 | .372 | .071 | .018 | .002 | 5.40 | 84.37 |
| 51 | .925 | .617 | .364 | .073 | .019 | .002 | 5.18 | 83.81 |
| 52 | .923 | .622 | .358 | .075 | .020 | .002 | 4.98 | 83.27 |
| 53 | .921 | .627 | .352 | .077 | .021 | .003 | 4.80 | 82.75 |
| 54 | .919 | .630 | .347 | .078 | .022 | .003 | 4.64 | 82.26 |
| 55 | .917 | .633 | .343 | .080 | .024 | .003 | 4.50 | 81.81 |
| 56 | .914 | .634 | .340 | .082 | .026 | .004 | 4.38 | 81.41 |
| 57 | .912 | .635 | .338 | .083 | .028 | .004 | 4.28 | 81.06 |
| 58 | .910 | .634 | .336 | .085 | .030 | .005 | 4.20 | 80.76 |
| 59 | .908 | .634 | .334 | .086 | .032 | .005 | 4.13 | 80.50 |
| 60 | .906 | .632 | .332 | .087 | .035 | .006 | 4.07 | 80.27 |
| 61 | .904 | .631 | .331 | .089 | .038 | .007 | 4.01 | 80.06 |
| 62 | .902 | .629 | .330 | .090 | .041 | .007 | 3.96 | 79.85 |
| 63 | .900 | .626 | .330 | .092 | .044 | .008 | 3.91 | 79.65 |
| 64 | .898 | .624 | .329 | .093 | .047 | .009 | 3.86 | 79.43 |
| 65 | .895 | .621 | .329 | .095 | .050 | .010 | 3.80 | 79.18 |
| 66 | .892 | .618 | .328 | .097 | .054 | .011 | 3.73 | 78.88 |
| 67 | .889 | .615 | .327 | .099 | .057 | .012 | 3.65 | 78.52 |
| 68 | .885 | .613 | .326 | .102 | .061 | .013 | 3.56 | 78.08 |
| 69 | .881 | .611 | .324 | .105 | .066 | .014 | 3.46 | 77.56 |
| 70 | .876 | .609 | .321 | .109 | .070 | .015 | 3.34 | 76.94 |
| 71 | .871 | .608 | .317 | .113 | .075 | .017 | 3.21 | 76.23 |
| 72 | .864 | .607 | .313 | .117 | .081 | .018 | 3.07 | 75.42 |
| 73 | .858 | .606 | .308 | .122 | .087 | .020 | 2.92 | 74.52 |
| 74 | .850 | .605 | .302 | .127 | .093 | .022 | 2.78 | 73.54 |
| 75 | .842 | .604 | .297 | .133 | .099 | .025 | 2.63 | 72.47 |
| 76 | .833 | .603 | .291 | .139 | .106 | .028 | 2.49 | 71.35 |
| 77 | .823 | .602 | .285 | .146 | .114 | .031 | 2.35 | 70.18 |
| 78 | .813 | .600 | .279 | .152 | .122 | .035 | 2.22 | 68.99 |
| 79 | .802 | .597 | .272 | .159 | .131 | .040 | 2.11 | 67.81 |
| 80 | .790 | .593 | .266 | .165 | .140 | .045 | 2.00 | 66.68 |
| 81 | .778 | .589 | .260 | .171 | .151 | .050 | 1.91 | 65.59 |
| 82 | .766 | .583 | .254 | .177 | .162 | .057 | 1.82 | 64.55 |
| 83 | .753 | .577 | .248 | .182 | .175 | .065 | 1.74 | 63.56 |
| 84 | .739 | .570 | .242 | .188 | .188 | .073 | 1.67 | 62.61 |
| 85 | .725 | .562 | .236 | .192 | .203 | .082 | 1.61 | 61.69 |
| 86 | .711 | .553 | .229 | .197 | .218 | .092 | 1.55 | 60.81 |
| 87 | .697 | .544 | .222 | .201 | .234 | .103 | 1.50 | 59.99 |
| 88 | .683 | .534 | .214 | .203 | .252 | .114 | 1.45 | 59.25 |
| 89 | .669 | .524 | .205 | .206 | .270 | .125 | 1.41 | 58.56 |
| 90 | .656 | .514 | .196 | .207 | .290 | .137 | 1.38 | 57.91 |
|  |  |  |  |  |  |  |  |  |


| HGE | H\|H | S\|S | H\|S | S\|H | D\|S | D\|H | EQUIL <br> RATIO | \% healthy <br> if alive |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |  |  |
| 91 | .643 | .503 | .187 | .208 | .310 | .149 | 1.34 | 57.31 |
| 92 | .631 | .493 | .177 | .208 | .331 | .161 | 1.31 | 56.73 |
| 93 | .618 | .482 | .166 | .208 | .352 | .174 | 1.28 | 56.17 |
| 94 | .607 | .471 | .154 | .206 | .375 | .187 | 1.25 | 55.61 |
| 95 | .595 | .460 | .141 | .205 | .399 | .200 | 1.22 | 55.05 |
| 96 | .585 | .449 | .127 | .202 | .423 | .213 | 1.20 | 54.48 |
| 97 | .575 | .439 | .112 | .199 | .449 | .226 | 1.17 | 53.89 |
| 98 | .567 | .429 | .095 | .195 | .476 | .238 | 1.14 | 53.30 |
| 99 | .562 | .419 | .077 | .190 | .504 | .248 | 1.12 | 52.77 |
| 100 | .560 | .410 | .057 | .183 | .533 | .258 | 1.10 | 52.42 |

Age is age at time 0.
$\mathbf{H | H}$ is the estimated probability of being healthy next year for a person who is healthy this year.
H/S Equilibrium ratio is the ratio of $\#$ healthy to the $\#$ sick if the system is at equilibrium (See Appendix 1). \% healthy if alive is $\mathrm{H} /(\mathrm{H}+\mathrm{S}) * 100=$ ratio/(ratio+1)*100, if the system is at equilibrium.(See Appendix 1).

## Authors' Note

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[^0]:    Collection of Blostatistics

