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Control of Flexible Structures Using Model Predictive Control and Gaussian Processes

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Abstract. There is a recognised need to address issues of vibration control by making use of recent developments in data-driven modelling. The present study considers the difficulties imposed by the limitations of the actuator in the range of active vibration control. The paper proposes and examines a data-based Gaussian process (GP) model of a proof mass actuator in a flexible structural framework, aiming to improve control performance. This requires incorporating an inverse GP of static nonlinearity within the Wiener-Hammerstein model. The model starts with designing model predictive control (MPC) for a cantilever beam, in which the aim is to identify the optimal control force. Utilising the GP is the second step towards quantifying the uncertainty and limitation of the proof mass actuator by designing an inverse GP for the static nonlinearity. This quantification forwards to an MPC controller using a steady-state target optimisation tracking approach, in which this controller provides the optimal voltage required to eliminate vibration within the controller's limitations. The numerical outcome shows that the proposed scheme was capable of supplying the necessary voltage, which eliminated the structure's vibration within an actuator's limits. The results of this work encourage additional research into the developed strategy, particularly in the context of experimental real-time implementation.

1. INTRODUCTION

In recent years, there has been an increasing demand for data-driven modelling techniques to handle uncertainty and nonlinearity in practical engineering problems [1]. Particularly, one of the challenges in this area is the control of flexible structures in fields such as aerospace engineering and civil infrastructure. Even though active vibration control plays a critical role in structural systems [2], these flexible structural systems are easily subjected to oscillations and deformations that can affect their performance and stability. While several studies have tackled the control performance issues using data-driven control techniques [3], the key questions have not been fully answered in an active vibration setting. Therefore, there is an urgent need to investigate whether the control performance of an active vibration system can be improved using data-driven control, particularly with model predictive control (MPC) and Gaussian process (GP) techniques.

One of the most effective advanced control algorithms in industry is MPC. The foundation of the MPC concept is the use of the system model to predict the system output for a number of steps ahead at the system's current values. The system's prediction with regard to a predefined cost function, while fulfilling the constraints of the state and input systems, is then used to determine the optimal values of the future control input. The advantages of MPC stem primarily from its ability to handle input and state constraints [4], as well as multivariable processes, and it operates in discrete time, allowing it to be implemented directly in real time [5].

The general idea of designing an active control by using GP-MPC for vibration purposes is depicted in Figure 1. The efficacy of this approach is that within the MPC, the model of the system is represented by the Wiener-Hammerstein model. This approach is constructed from three distinct blocks: two MPC models of linear time invariant (LTI) dynamic blocks, and one nonlinear block. They are all coupled in series with the nonlinear block being placed between the linear blocks as shown in Figure 1. The nonlinear block in the model is considered, in this paper, as static nonlinearity and it has been represented by a GP. The benefits of this proposed model are as follows. First, the Wiener-Hammerstein model helps to accurately represent various complex dynamic systems, including nonlinear systems [6], which can be useful for control systems. Second, the GP can incorporate uncertainty which can be insightful for active vibration control. Finally, these previous aspects could be helpful in terms of the limitations of the actuator when implementing active vibration control, which is the primary focus of this paper.

The remainder of the manuscript is structured as follows: Section 2 describes the problem formulation and specifies the part of the problem addressed in this paper. Section 3 elaborates on the control tactics, where the Wiener-Hammerstein model is utilised. Section 4 presents a case study of a cantilever beam and proof mass actuator. Finally, Section 5 provides the final observations and outlines future work.

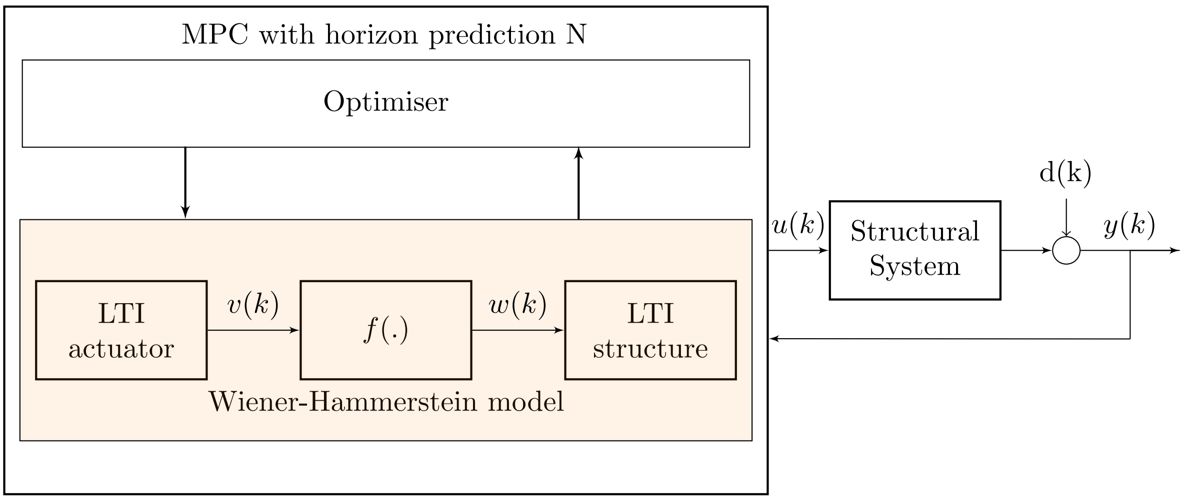

Figure 1: Overview of the proposed control system.

2. PROBLEM STATEMENT

Consider a single-input, single-output discrete-time non-linear structural system described by the Wiener-Hammerstein model in Figure 1. Mathematically, the model can be represented as a series of time-discretised systems as follows:

$$\begin{cases} x_p(k+1) = A_p x_p(k) + B_p u_p(k) \\ v(k) = C_p x_p(k) + D_p u_p(k) \end{cases} \quad \text{First linear system (actuator)} \tag{1}$$

$$\begin{cases} w(k) = f(v(k)) \end{cases} \quad \text{Nonlinear function} \tag{2}$$

$$\begin{cases} x_s(k+1) = A_s x_s(k) + B_s w(k) \\ y_s(k) = C_s x_s(k) \end{cases} \quad \text{Second linear system (structure)} \tag{3}$$

The first linear state space system (with subscript p) represents a proof mass actuator with state vector x_p, input vector u_p , and output vectors v. The second linear system is a model of the structure being controlled, in this case a cantilever beam, that can be represented by subscript s. More importantly, the nonlinear function f represents the static nonlinearity of the saturation of the proof mass actuator in an active vibration setting. The static nonlinearity $w(k)$ is modelled using input-output data and a GP as shown in Equation 4. The GP model is defined by a mean function, $m(x)$, and a covariance function, $k(x, x')$.

$$w(k) \sim \mathcal{GP}(m(x), k(x, x')) \tag{4}$$

In order to quantify the uncertainty or end stroke of the proof mass actuator, the proposed model has to go through three design stages:

(i) To design a constrained Linear Quadratic Model Predictive Control (LQ-MPC) for the regulator case. This step helps the model to identify the minimum required force $w(k)$.

(ii) To design the inverse GP model, which maps the minimum required force $w(k)$ to the actual force $v(k)$ through the static nonlinearity GP model.

(iii) The final stage is to design a constrained LQ-MPC using a steady-state target optimisation tracking approach. This step highlights the ability of the proof mass actuator to follow the required force input taking into account the static nonlinearity of the saturation of the actuator.

3. MODEL PREDICTIVE CONTROL

For the purpose of understanding the proposed model, the brief description of constrained LQ-MPC is presented by summarising the existing literature [7, 8].

3.1. Basic Constraint LQ-MPC Algorithm Components

MPC is widely recognised as a control strategy rather than a control algorithm. However, there are some basic ingredients that must be used to create the MPC algorithm. This section describes the fundamental elements of developing a regulator and tracking constrained LQ-MPC.

3.1.1. Prediction Making predictions is a key component of MPC, as it provides the state prediction associated with a sequence of future control inputs based on the starting state. Obtaining predictions begins with the model's form selection. The prediction in this study is based on the open loop state space model as shown in Equations 1 and 3. This prediction model is obtained by applying the model recursively a number of steps ahead, starting from the initial state, and then collecting the state prediction and input sequence in vector form as:

$$\mathbf{x(k)} := \begin{bmatrix} \mathbf{x}(k+1|k) \\ \mathbf{x}(k+2|k) \\ \vdots \\ \mathbf{x}(k+N|k) \end{bmatrix}, \mathbf{u(k)} := \begin{bmatrix} \mathbf{u}(k|k) \\ \mathbf{u}(k+1|k) \\ \vdots \\ \mathbf{u}(k+N-1|k) \end{bmatrix}, F := \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^N \end{bmatrix}, G = \begin{bmatrix} \mathbf{C} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CA} & \mathbf{C} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{N-1} & \mathbf{CA}^{N-2} & \cdots & \mathbf{C} \end{bmatrix} \tag{5}$$

where, $\mathbf{x}(k)$ can be represented in compact form according to:

$$\mathbf{x}(k) = Fx(k) + G\mathbf{u}(k) \tag{6}$$

where, $\mathbf{x}(k)$ is the state vector at time k, $\mathbf{u}(k)$ is the input vector at time k, and N is the prediction horizon.

3.1.2. Inequality constraints The ability of MPC to handle constraints is its most useful and important feature. The constraints can be either hard such as actuator saturation or soft such as performance specifications. Equation 7 represents the general linear inequality constraints on the states and inputs of the model.

$$\begin{aligned} \underbrace{\begin{bmatrix} C \\ -C \end{bmatrix}}_{P_x} x(k) &\leq \underbrace{\begin{bmatrix} x_{max} \\ -x_{min} \end{bmatrix}}_{q_x} \\ \underbrace{\begin{bmatrix} I \\ -I \end{bmatrix}}_{P_u} u(k) &\leq \underbrace{\begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix}}_{q_u} \end{aligned} \tag{7}$$

where P_x and P_u are, respectively, state constraints and input constraints, whereas q_x and q_u represents the limit of both state and input, respectively. After converting the cost function into a compact form with imposing the constrains, the constructed linear inequality equation becomes:

$$P_c \mathbf{u}(k) \leq q_x + S_c x(k) \tag{8}$$

Due to stability and convergence concerns, there is an additional linear inequality equation associated with a method known as dual mode. The control gains in this dual mode were constructed using the Linear Quadratic Regulator (LQR) approach to ensure a quick convergence to the origin [7, 8]. In a tracking problem, previous inequality equations and a dual mode method with a steady-state term were formulated.

$$\begin{aligned} \underbrace{\begin{bmatrix} C \\ -C \end{bmatrix}}_{P_x} z(k+j|k) &\leq \underbrace{\begin{bmatrix} x_{max} \\ -x_{min} \end{bmatrix}}_{q_x} - \underbrace{\begin{bmatrix} C \\ -C \end{bmatrix}}_{P_x} x_{ss} \\ \underbrace{\begin{bmatrix} I \\ -I \end{bmatrix}}_{P_u} v(k+j|k) &\leq \underbrace{\begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix}}_{q_u} - \underbrace{\begin{bmatrix} I \\ -I \end{bmatrix}}_{P_u} u_{ss} \end{aligned} \tag{9}$$

$$\underbrace{\mathbf{I_{nxn}} \otimes \begin{bmatrix} P_x \\ P_u K_\infty \end{bmatrix} \begin{bmatrix} (A+BK_\infty)^0 \\ \vdots \\ (A+BK_\infty)^{N-1} \end{bmatrix}}_{P_{x_N}} z(k+N|k) \leq \underbrace{\mathbf{I_n} \otimes \begin{bmatrix} q_x - P_x x_{ss} \\ q_u - P_u u_{ss} \end{bmatrix}}_{\widetilde{q}_{x_N}} \tag{10}$$

3.1.3. Performance index or cost function Obtaining an appropriate performance index is critical in developing a control law. Despite the existence of various performance indices in the literature [7], the linear quadratic cost function is appropriate in terms of penalising the state and input within the constraints imposed. The general LQ-MPC problem is as follows:

$$
\begin{aligned}
\mathbf{P_N}(x(k)) = \min_{u(k)} \sum_{i=0}^{N-1} [x(k+i|k)^T Q x(k+i|k) + u(k+i|k)^T R u(k+i|k)] + \underbrace{x(k+N|k)^T P x(k+N|k)}_{\text{terminal cost}} \\
\text{subject to} \quad & x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k) \\
& x(k|k) = x(k) \\
& P_x(x(k+i|k)) \leq q_x \\
& P_u(u(k+i|k)) \leq q_u \\
& P_{x_N}(x(k+N|k)) \leq \widetilde{q}_{x_N}
\end{aligned} \tag{11}
$$

where P is the terminal weight matrix, and Q and R are the state and control weight matrices respectively. Also, P_{x_N} and $\widetilde{q}_{x_N}$ are stacked inequality constraints and its limit for the terminal state of the system. Once this optimisation problem is solved, the control law can be formed based on the receding horizon principle. This states that the system is only excited by the first control value in the optimised sequence, before the optimisation problem is re-solved. This repeated action of applying the first optimised control in the sequence defines a feedback control law:

$$u(k) = u^*(k|k) = K_N(x(k)) \tag{12}$$

3.1.4. Feedforward steady state target optimization The last three sections are the main MPC components for regulator tasks. MPC tracking, however, requires some extra tools in order to ensure feasibility and stability. The steady-state model is the first adjustment to the open loop state space form of Equation 1. The idea here is that steady-state model helps to identify how far the current state is from the steady-state. Hence, Equation 13 represents the steady-state model, and this expected steady-state model [x_{ss},u_{ss}] can be solved by Equation 14. The aim of LQ-MPC tracking is to regulate x to x_{ss} and u to u_{ss} which will accomplished by identifying and computing the deviation variable (z) and target optimiser. In addition, the inequality equation of constraints has to be considered in these changes as stated above.

$$
\begin{aligned}
x_{ss} &= Ax_{ss} + Bu_{ss} \\
y_{ss} &= Cx_{ss} + Du_{ss} + d_k
\end{aligned} \tag{13}
$$

$$\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix} = \begin{bmatrix} C & D \\ A-I & B \end{bmatrix}^{-1} \begin{bmatrix} y_{ss} - d \\ 0 \end{bmatrix} \tag{14}$$

3.2. Control law of the constrained LQ-MPC tracking problem
The control law of LQ-MPC in Equation 11 must be reformulated to consider the tracking problem:

$$
\begin{aligned}
\mathbf{P_N}(x(k) = \min_{u(k)} \sum_{i=0}^{N-1} [z(k+i|k)^T Q z(k+i|k) + v(k+i|k)^T R v(k+i|k)] + z(k+N|k)^T P z(k+N|k) \\
\text{subject to} \quad & z(k+i+1|k) = Az(k+i|k) + Bv(k+i|k) \\
& z(k|k) = z(k) \\
& P_x(z(k+i|k) + x_{ss}) \leq q_x \\
& P_u(v(k+i|k) + u_{ss}) \leq q_u \\
& P_{x_N}(z(k+N|k)) \leq \widetilde{q}_{x_N}
\end{aligned} \tag{15}
$$

The solution of this optimisation problem provides the control sequences which can be formed as the feedback control law as demonstrated above. However, the control law of tracking is coupled with the input in the steady-state as shown here:

$$v(k) = u_{ss} + v^*(k|k) \tag{16}$$

Two points need to be considered. First, designing MPC with free offset tracking under changing target is itself a research problem since the majority of research in literature assumed the target is fixed [9]. Second, $y_{ss} = r$ is a critical condition to ensure offset free tracking with constraint satisfaction.

4. CASE STUDY

This work considers the dynamics of a cantilever beam, and the author in [10] described those dynamics in great detail. In order to form a collocated system, this system typically consists of a uniform beam with a point force actuator and a displacement sensor at the tip. For convenience of use, Table 1 shows all of the beam's essential parameters. Furthermore, the proof mass actuator that was taken into consideration in this work is based on the ADD-45 inertia actuator from Micromega Dynamics [11], whose dynamic is described in terms of a transfer function:

$$\frac{f_{act}(s)}{v_{in}(s)} = \frac{5s^2}{s^2 + 15.843s + 2785.6} \tag{17}$$

The proposed control model operates in a discrete domain, so these dynamics were discretised using the zero hold method at a sampling rate of 0.001 seconds.

Table 1: Essential Parameters of the actuator dynamics and control system.

Element (Symbols)	Value (Units)
Height (h)	25 mm
Width (b)	1 mm
Length (L)	350 mm
Modulus of Elasticity	210 GPa
Density of steel (ρ)	7850 (kg/m^3)
Prediction Horizon (N)	7
Weighted matrix (Q)	100I(n)
Weighted matrix (R)	10
Damping ratio (ζ)	1%

4.1. LQ-MPC for cantilever beam

As shown in Figure 2, the control strategy in this first section of the Wiener-Hammerstein model was established in such a way as to direct the controller to reach the limit of the control input, which is 45 N. To achieve this, the MPC controller, as shown in Table. 1, was penalising the state more heavily than the control input. Normally, the best weighted matrix for state in LQ-MPC is the transpose of the output matrix times the output matrix ($C^T C$). The response of the structural system is depicted in Figure 3, where it takes the highest force needed to eliminate the vibration.

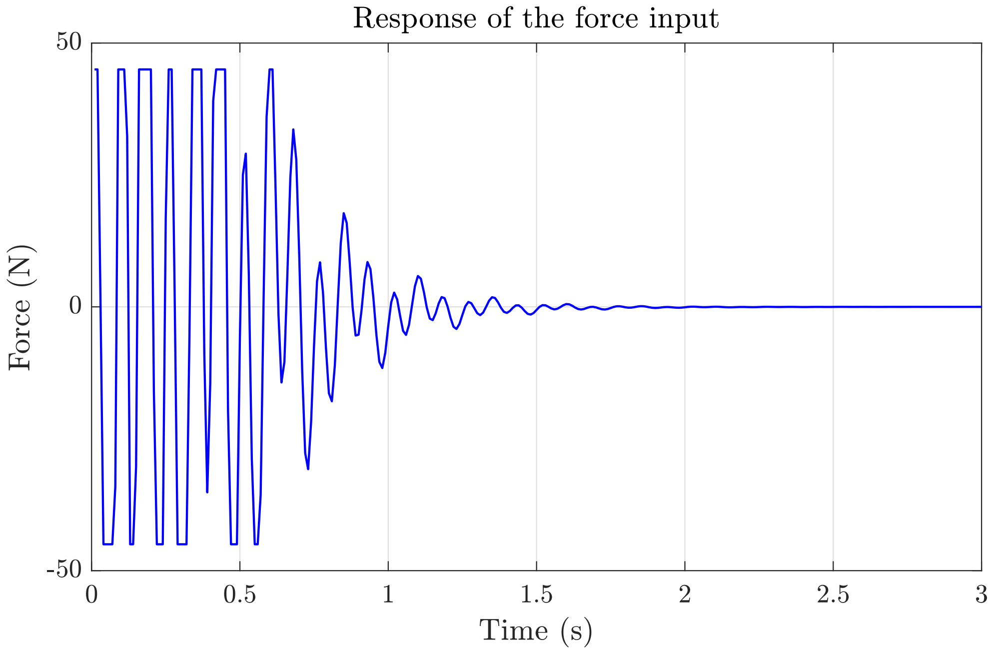

Figure 2: Control input of the cantilever beam.

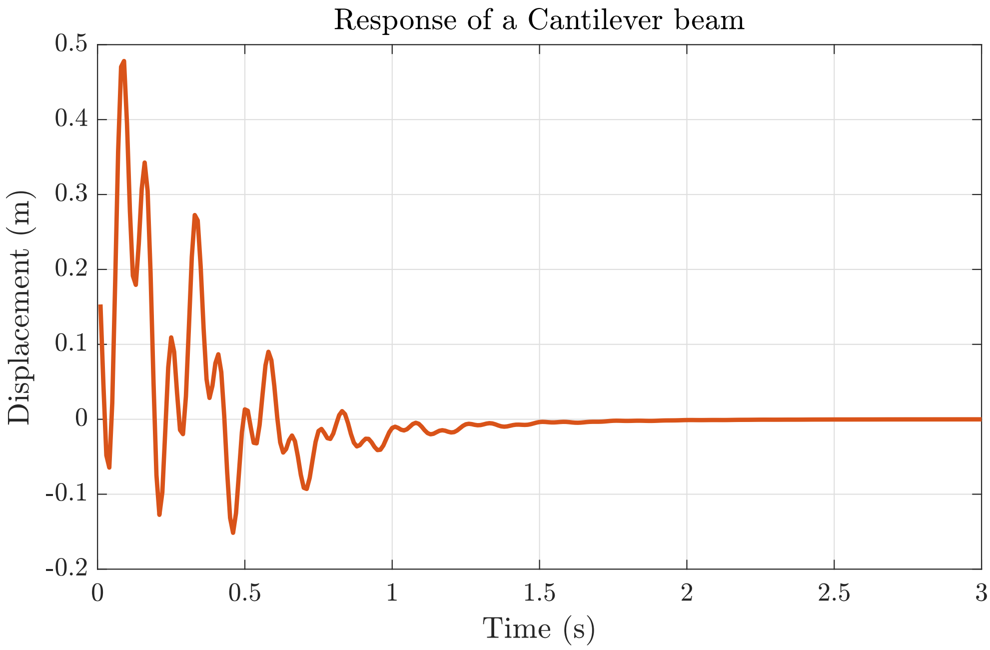

Figure 3: Response of the cantilever beam.

4.2. GP for nonlinear function

The structural control input was acquired and provided to the GP. Based on the actuator's saturation, the GP was trained. As shown in Figure 4, the GP model was trained on the inverse of the static nonlinearity in order to identify the force space and region of uncertainty. The GP

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prediction of the minimum force is shown in Figure 5 with the region of uncertainty highlighted. It is obvious that due to a lack of information, the GP predicts a higher force than the saturation limits. When data changes rapidly, this GP model is also unable to capture the relationship between the data.

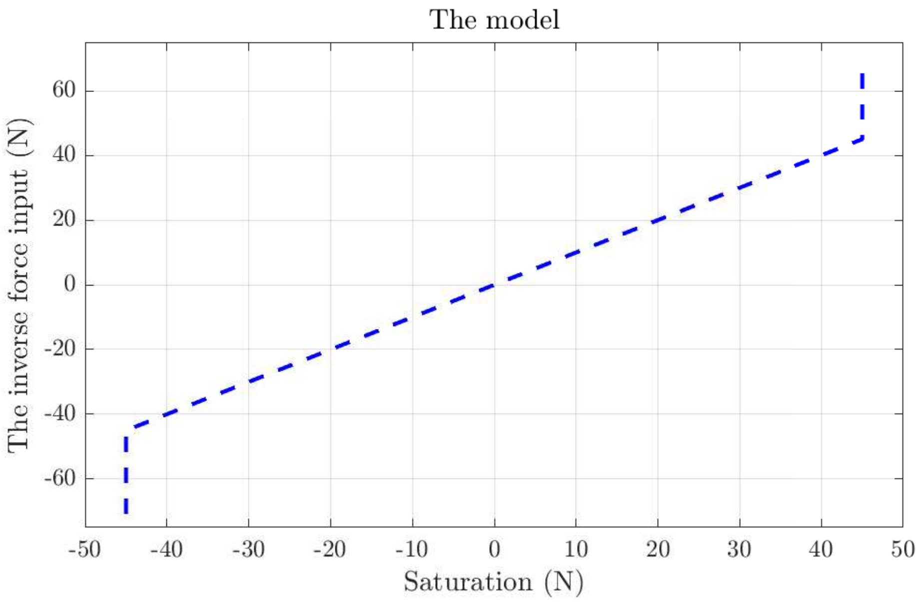

Figure 4: The model of GP considered the static nonlinearity of the system.

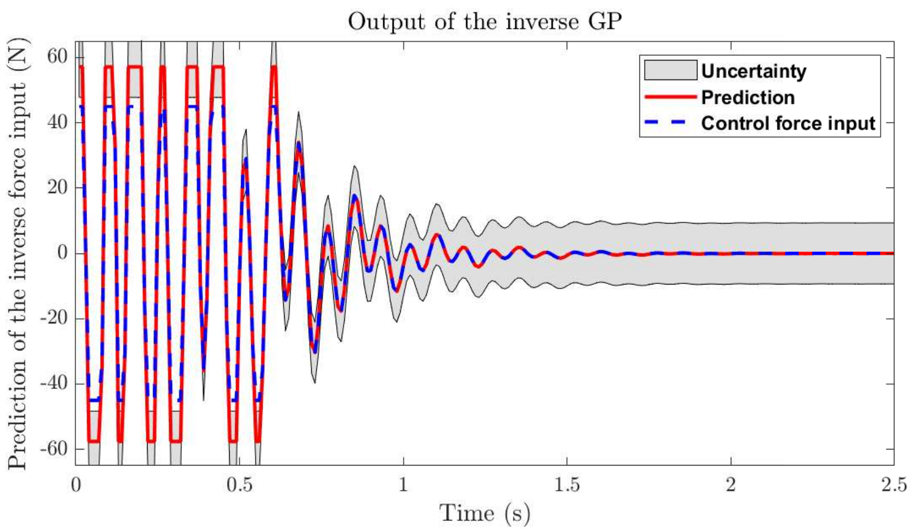

Figure 5: The prediction of the Gaussian process with the region of uncertainty.

4.3. MPC for proof mass actuator

After obtaining the inverse GP, LQ-MPC with steady-state optimiser tracking is established, with the mean of the GP at each sample serving as the reference. In the literature, this issue is

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known as MPC tracking with changing target. Figure 6 demonstrates how the MPC controller was able to provide the required voltage for the actuator to deliver the optimum force to the structure. Figure 7 compares the LQ-MPC regulator's control force with the actuator's output based on LQ-MPC tracking. It is obvious that an MPC controller for an actuator system faces feasibility problems due to the rapid change of the control signal. Based on the GP prediction of this problem, the controller was guided to either maximum or minimum control input. Both previous figures demonstrated that the proposed model was able to provide the control input into the structural system with static nonlinearity in the system taken into account.

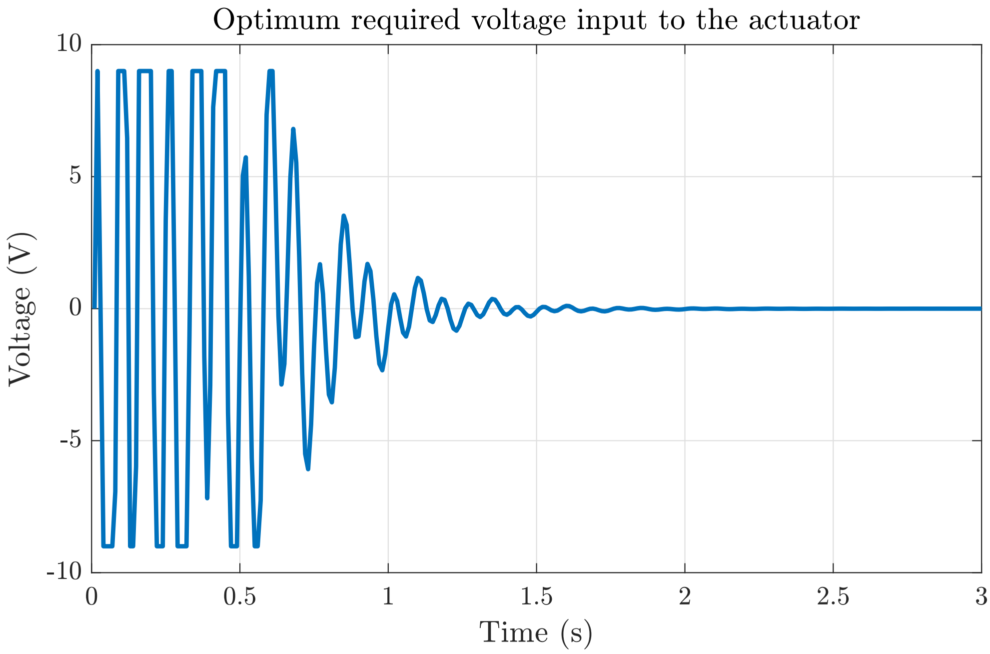

Figure 6: Optimum required voltage input to the actuator.

5. CONCLUSIONS

This paper is tackling the issue of designing an active vibration control based on the recent development of data-driven methods. The method utilised in this work is MPC and GP within the Wiener-Hammerstein model. The proposed model needs to go through three design phases in order to quantify the uncertainty or end stroke of the proof mass actuator. Designing a constrained Linear Quadratic Model Predictive Control (LQ-MPC) for the regulator case is the first step, which facilitates the model in determining the minimal force needed. The second step is to create the inverse GP model, which helps in mapping the minimal force necessary to force space using the GP model of static nonlinearity. Designing a constrained LQ-MPC utilising a steady-state target optimisation tracking approach is the last step. This step demonstrates the proof mass actuator's capacity to track the required force input while accounting for the saturation of the actuator's static nonlinearity. The numerical results suggested that the model was able to provide the required control input taking into account the limits of the actuator. The results of this work encourage additional research into the developed strategy, particularly in the context of experimental real-time implementation.

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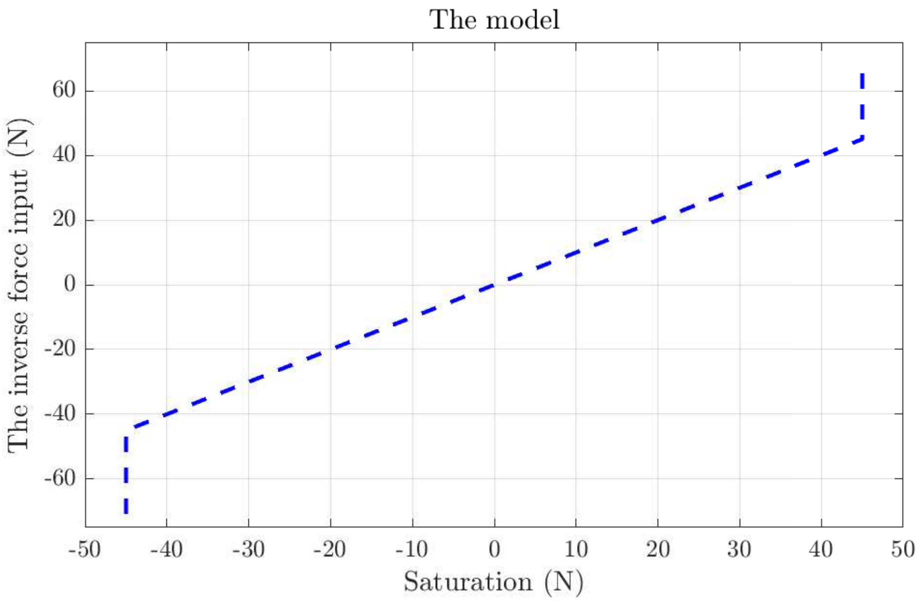

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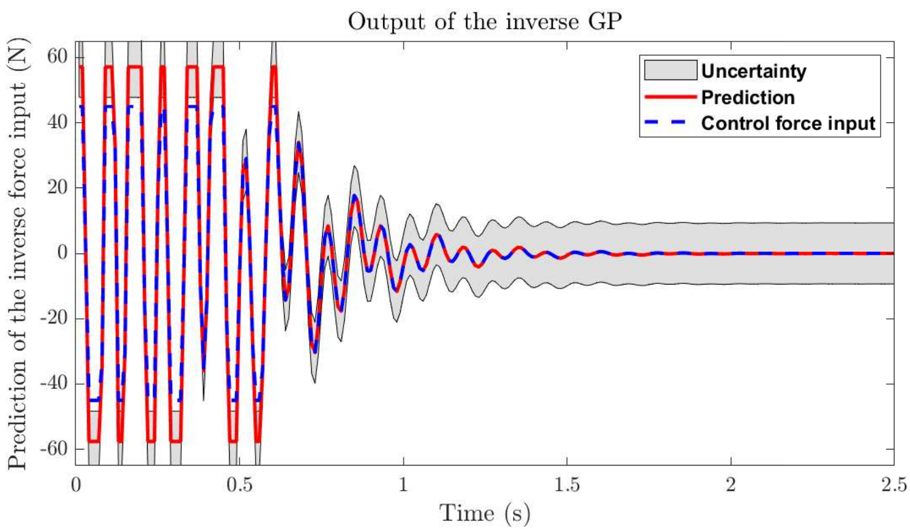

Figure 5: The prediction of the Gaussian process with the region of uncertainty.

4.3. MPC for proof mass actuator

After obtaining the inverse GP, LQ-MPC with steady-state optimiser tracking is established, with the mean of the GP at each sample serving as the reference. In the literature, this issue is