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COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK

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Yasutaka Ogawa† and Jiro Yoshida‡

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**JEL Classifications: E22, D15, J11, R21**
**Keywords:** aging, the natural rate of interest, overlapping generations model, bequest motives, intergenerational transfer of housing, Japan

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### Abstract

This study quantifies the macroeconomic impact of population aging with a focus on large houses owned by elderly households for bequest motives, although younger generations may leave the inherited houses vacant. A quantitative overlapping generations model incorporates age-specific mortality rates and bequest motives to generate a hump-shaped age profile for consumption and an upward-sloping age profile for housing and savings. When calibrated to the Japanese economy, the model suggests that bequest-driven housing demand raises the output level but reduces consumption, the natural rate of interest, capital allocation to the goods sector, and housing affordability. These effects are more pronounced when households intend to bequeath housing rather than financial assets and when more houses become vacant upon inheritance.



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# I. Introduction

Population aging affects a wide range of markets and macroeconomy (for example, see Wise, 1989, and subsequent publications from the NBER Program on the Economics of Aging). Our study focuses on the ownership of large housing capacities by elderly households, part of which is underutilized and may eventually become vacant after being transferred to the younger generation (Seko et al., 2023). Because housing consumption increases with age in many countries (e.g., Yang, 2009; Cocco and Lopes, 2020), a large stock of accumulated housing will become a common feature of an aging society.

Despite the accumulation of housing stock, the younger generation may still face housing affordability problems if the housing stock is not readily available to them due to excess housing capacity held by the elderly and intergenerational mismatches in preferences. As a consequence, an aging society will allocate more capital to the construction industry, which has low productivity after decades of negative productivity growth in many countries (e.g., Goolsbee and Syverson, 2023; Garcia and Molloy, 2023). One possible consequence is low marginal product of capital (interest rates) due to the slow growth of overall productivity due to the composition effect.

Insights into the macroeconomic impact of aging can be gained by studying Japan, which is at the forefront of aging societies. Its potential support ratio (the number of working-age people (15-64) per elderly person (65+)) is the lowest at 2.0 in 2021, compared with 3.9 for the US, 5.3 for China, and 9.9 for India. In addition, Japan, like other countries, shows increasing housing consumption with age. The homeownership rate increases monotonically with the age of the household head from 3% for households under 25, to 83% for those 85 and older (2018 Housing and Land Survey). A similar upward-sloping age profile is also observed for housing wealth in Japan's 2019 National Survey of Family Income, Consumption and Wealth. Moreover, total factor productivity (TFP) in the construction and housing sector also declined in the four decades leading up to the 2000s (figure 1), in stark contrast to the growing productivity of other sectors. The Japanese economy is thus an ideal case for our study of how the ownership of large housing capacities by elderly households affects the macroeconomy.

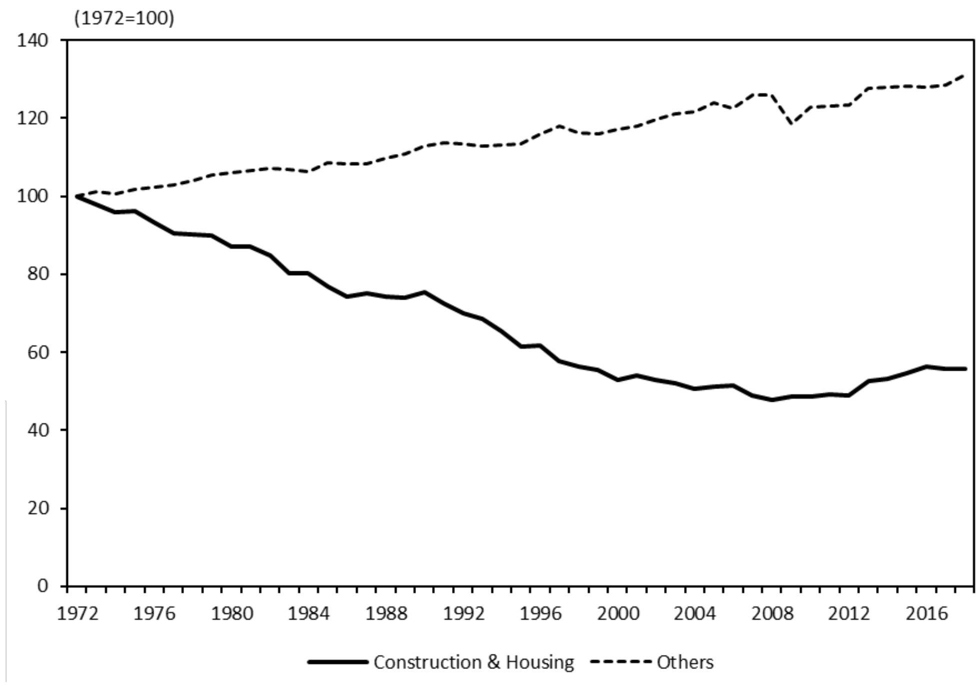



Figure 1: Total factor productivity level for the construction and housing sector in Japan. Source: Japan Industrial Productivity Database.

We quantify the effect of elderly households' accumulated housing by calibrating a quantitative, perfect-foresight, overlapping generations (OLG) model to the Japanese economy. The model that incorporates bequest motives and the housing production sector replicates well historical GDP growth rates, historical real interest rates, the current housing-to-consumption ratio, and the housing vacancy rate. It also replicates relatively well the age profile of consumption, housing assets, and savings. These replications and calibrations are based on the exogenously specified trajectory of fertility rates, mortality rates, sector-specific productivity, government spending, and government debt, in addition to constant parameters for age-specific labor productivity, the sectoral share of labor supply, and the pension replacement rate.

We analyze the natural rate of interest, gross domestic product, total factor productivity, capital allocation between two production sectors, and house prices between 1980 and 2050.

We first decompose the baseline prediction of the equilibrium outcome into the six exogenous factors: mortality, fertility, government debt, government spending, technology growth, and the technology gap between the two sectors. To identify the contribution of each factor, we "knock out" each factor at a time by keeping its 1980 value throughout the remaining years as Bielecki et al. (2020) do.[1] This decomposition analysis shows that the natural rate of interest will decline by 1.5-2.0 percentage points after 1980, mainly driven by stagnant technology growth and aging. The GDP growth rate will also decline by 4 percentage points for similar reasons. Housing value will increase from 2020, driven by the growing demand of the elderly.

The model also identifies the general-equilibrium effect of bequest-driven financial and housing asset holdings, the balance between financial and housing bequests, the efficiency of intergenerational housing transfers, and inheritance tax rates. First, bequest motives increase the level of GDP and the capital allocation to the housing sector but decrease consumption and the natural rate of interest. Houses are less affordable because prices are inflated by both the extra demand and the less efficient production of housing. Second, the above effects are exacerbated when housing bequests are preferred to financial bequests because these effects are primarily driven by the extra demand for housing driven by bequest motives. Third, the same effects will be further exacerbated if more inherited houses become vacant due to inefficient intergenerational transfers. Finally, under the premise that the government substitutes income taxes for inheritance taxes, changing the inheritance tax rate has only a small impact on the economic outcome. Overall, the efficiency of the Japanese economy can be improved by reducing the bequest-motivated housing demand of the elderly and improving the efficiency of house matching.

Our model has several special features. First, we introduce age-specific mortality rates. One challenge with a life-cycle model with a fixed life span is that the model predicts a flat age profile of consumption, whereas data show a hump-shaped age profile. By incorporating

[1] For the fertility rate, we use the average rate for 1980-1989 because the growth rate of the 20-year-old population is negative between 1970 and 1980 due to WWII.

actual mortality rates into our model, we generate a hump-shaped age profile of consumption. However, the actual mortality rates lead to a consumption peak much later (after age 80) than in the data (around age 50). Given that households take into account health-adjusted life expectancy (HALE) when making consumption decisions, we adjust the empirical mortality rates for healthy life expectancy. Our calibrated model shifts the consumption peak earlier, to around 70 years of age.

Second, we introduce motives for bequeathing housing. In the OLG model that includes housing consumption, households' first-order conditions require that the age profile of housing consumption be parallel to that of general consumption. However, the data show that housing consumption, unlike non-housing consumption, increases monotonically with age. Housing bequest motives in our model create an upward age profile of housing consumption.

Third, we also introduce motives for bequeathing financial assets. In an OLG model, savings usually peak just before retirement age and decline monotonically to zero as a household ages. However, the data show that households do not dissave after retirement age (French et al., 2023). Our model introduces intentional financial bequests, which help us explain the slow decumulation of financial assets after retirement.

However, our model does not include several factors. First, our perfect foresight model does not allow for precautionary saving, which could help explain the persistently high level of savings after retirement. Second, our rational model does not allow for behavioral biases, such as myopia and suboptimal housing choices. Behavioral biases could explain an early peak in consumption and large housing capacities, but we intend to study the rational benchmark. Third, our model does not take into account age-specific consumption goods. For example, middle-aged parents typically spend a significant amount on their children's education, resulting in a consumption peak well before the retirement age. Also, elderly households may prefer home care and other goods and services associated with their current home, which may explain the upward-sloping age profile of housing consumption. Fourth, our model omits financial constraints, such as required down payments and foreclosure costs, and frictions beyond house vacancies. The data suggest that households borrow and increase their housing wealth significantly when the head of the household is in their 30s and 40s, which is associated with first-time homeownership. Furthermore, older homeowners may not sell their homes and downsize if the outstanding balance of long-term recourse mortgages exceeds the market value of the house. One might think that the absence of downsizing is due to transaction costs, but excess housing capacity cannot be explained solely by transaction costs under perfect foresight because rational households will maintain undercapacity in their middle years to avoid costly downsizing in the future. Last, our model does not allow productivity to endogenously change by the resource allocated to each production sector. Relaxing this restriction may lead to a larger effect of housing choices of elderly households on growth.

Our study contributes to the literature in four ways. First, we show that homeownership is a new channel through which population aging affects the economy. This channel is based on the empirical regularity that the elderly are likely to own large housing capacities. Our model features the bequest motive of rational households to generate additional housing demand from elderly households, which leads to the allocation of capital to a less productive sector. Although other factors, such as a behavioral bias, could generate a similar result, we emphasize that downsizing constraints alone cannot generate an upward-sloping age profile of housing consumption if households are rational.

Second, our study provides quantitative insights into the impact of population aging on the Japanese economy, such as the real interest rate, GDP growth, tax rate, productivity growth, and house prices. By decomposing the changes in these variables between 1980 and 2050, we quantify the effect of the expected changes in mortality and fertility rates. These factors lead to lower interest rates and higher taxes, but they are almost neutral for GDP and TFP growth.

Third, our study shows the general equilibrium effect of age-specific housing demand on macroeconomic variables, whereas existing studies typically show partial equilibrium effects and age-independent housing choices. The increased housing capacity of the elderly due to bequest motives significantly affects consumption, output, productivity growth, and the natural rate of interest. Specifically, the additional demand for housing suppresses consumption and real interest rates, while it expands the level of output.

Fourth, our model demonstrates that an OLG model can replicate a hump-shaped age profile for consumption and an upward-sloping age profile for housing and savings by incorporating both age-specific mortality rates and bequest motives for financial and housing assets. It provides a benchmark modeling choice for future studies.

The remainder of this paper is organized as follows. Section II reviews the related literature, Section III describes our model, and Section IV explains the model calibration. Sections V and VI presents the results, followed by discussions in Section VII. Section VIII concludes the paper.

# II. Literature Review

## *A. Population aging*

Population aging can have a profound effect on the economy. A series of conference proceedings from the NBER Project on the Economics of Aging include a large number of contributions since 1989 (Wise, 1989, 1990, 1992, 1994, 1996, 1998b,a, 2001, 2004, 2005, 2009, 2011, 2012, 2014, 2017). These studies show that aging has not only an obvious impact on health care and the social security system but also a wide range of indirect effects on savings, consumption, and investment. Furthermore, Weil (1997) and Lee (2016) review the effect of aging on the macro economy. The literature generally predicts a tendency toward increased capital intensity, higher wages, and lower returns on capital. In an open economy setting, flows of capital and labor into the aging country are also expected to increase. The effect on output is generally expected to be negative (e.g., Sheiner et al., 2007; Sheiner, 2014; Börsch-Supan et al., 2014; Gagnon et al., 2021), but the literature does not completely agree on this prediction because capital deepening can overturn the effect (e.g., Cutler et al., 1990). In a recent study, Maestas et al. (2023) further point out that population aging slows economic growth by diminishing labor productivity growth in addition to employment growth. These results critically depend on conditions such as labor supply at older ages, per capita consumption of the elderly relative to younger ages, strength of public pension and health care systems, and health and vitality of the elderly.

### *B. Large-scale overlapping generations models*

Following the seminal work by Auerbach and Kotlikoff (1987), overlapping generations (OLG) models have been a leading tool to evaluate the macroeconomic effect of demographic change because they can incorporate empirical age distributions to replicate realistic life-cycle behavior. Recent OLG models predict a downward trend in the real interest rate due to population aging (e.g., Bielecki et al., 2020; Papetti, 2021). Longevity can result in more precautionary savings. In particular, Bielecki et al. (2020) is most closely related to the present study. They develop a quantitative OLG model calibrated to match the life-cycle profiles of the European economy. They particularly investigate the impact of demographics on the natural rate of interest (NRI) in the euro area, with a particular focus on the role played by economic openness, migrations and pension system design. They show that population aging explains approximately two thirds of the secular decline in the NRI between 1985 and 2030. They also find that an increase in the retirement age can reverse the downward trend in the interest rate, but a fall in the replacement rate exacerbates the trend. Papetti (2021) also confirms that demographics account for a decrease in the natural rate of interest in the euro area due to scarce labor input and more savings. However, the results depend on substitutability between labor and capital, intertemporal elasticity of substitution, higher productivity, and labor participation by older individuals. Gagnon et al. (2021) predict that GDP growth and the natural rate of interest in the U.S. will remain low. Carvalho et al. (2016) show that the natural rate of interest will be lower through potentially competing three channels: larger savings driven by longevity, larger capital per-worker, and a lower savings rate. Eggertsson et al. (2019) quantify the secular stagnation hypothesis and predict a natural rate in the U.S. ranging from $-1.5\%$ to $-2\%$. Sudo and Takizuka (2020) estimate that demographic changes account for approximately 270 basis points out of the 640 basis-point decline in the natural rate of interest during the last 50 years in Japan. Braun and Ikeda (2022) provide a quantitative theory of deflation and secular stagnation using Japanese data.

## *C. Macroeconomic models with bequests and housing*

A bequest is a type of intergenerational transfer in addition to public pensions and education, and plays a central role in investment decisions. Thus, it critically affects the consequences of population aging. Artle and Varaiya (1978) is an early study that incorporates homeownership and bequest motives in the life cycle consumption model. However, they generally predict a monotonic age profile of the optimal consumption, either sloping downward or upward, depending on financial constraints. Whether a bequest motive is a critical factor for the economy has been debated. Hurd (1989) estimates a life cycle model augmented by a bequest motive and finds that the bequest motive is statistically significant but economically trivial. Hurd (1987) argues that bequests are merely accidental resulting from precautionary responses to imperfect insurance markets. However, Bernheim (1991) finds that the typical household chooses to maintain a positive fraction of its resources in bequeathable forms even if insurance markets are perfect. Kopczuk and Lupton (2007) find that roughly three quarters of the elderly single population has a bequest motive and spends less on consumption expenditure, doubling the amount of bequeathed net wealth. Intentional bequests can emerge either from altruism (Tomes, 1981) or self-interested exchange with one's heirs (Bernheim et al., 1985). Bequests are also incorporated in quantitative macro models (e.g., Hurd, 1989; Hviding and Mérette, 1998; Fougère and Mérette, 1999; Kraft and Munk, 2011).

A model that features bequest motives in the utility function can also be seen as a version of models featuring wealth in the utility function (Michaillat and Saez, 2021; Nakajima, 2003; Barberis et al., 2001; Carroll, 1998; Dynan et al., 2002, 2004). Wealth in the utility function may resolve macroeconomic puzzles related to the savings rate and consumption around the zero lower bound. Wealth in the utility function is often incorporated in partial-equilibrium consumer models and dynamic stochastic general equilibrium (DSGE) models but not in OLG models.

Housing plays a critical role in consumption choices and investment decisions because of its dual role as a consumption good and an investment and collateral asset. The role of real estate as collateral is a key insight provided by Kiyotaki and Moore (1997), Bernanke et al. (1999), and subsequent studies on the effect of financial constraints on the financial crisis and monetary policy transmissions (e.g., Wong, 2021). Housing also affects household portfolio choices and equilibrium asset prices (e.g., Flavin and Yamashita, 2002; Piazzesi et al., 2007). Iacoviello (2005) and Iacoviello and Neri (2010) incorporate housing in DSGE models. They show that homeownership with mortgage financing significantly affects how demand and supply shocks propagate. Also, they find that slow technological progress in the housing sector is a major cause of house price appreciation. Furthermore, they show that housing is increasingly important in explaining consumption. More recent DSGE models also incorporate housing (Liu et al., 2013; Garriga et al., 2021; Adam and Woodford, 2021). Housing is also incorporated in multi-sector OLG models (Galor, 1992; Farmer and Wendner, 2003; Yang, 2009; Kraft and Munk, 2011; Waters, 2020; Nakajima, 2020). In particular, as in the present study, Nakajima (2020) focuses on the substitution between housing and non-housing capital and finds that preferential tax treatment for owner-occupied housing significantly decreases the optimal capital income tax rate. A low capital income tax rate improves welfare by narrowing a tax wedge between housing and non-housing capital. Nakajima and Telyukova (2020) study why retired homeowners dissave slowly. They show that the accumulated home equity during a housing boom cannot be easily cashed out. Cocco and Lopes (2020) study the role of housing wealth in retirement consumption and saving decisions by the elderly. They find that reverse mortgages combined with insurance against a forced home sale is Pareto improving.

# III. Model

## A. The Model Setup

Consider a model for a closed-economy populated by overlapping generations (OLG) of households that solve a standard life-cycle problem. There are four agents in this model economy: households, two-sector firms (general goods and housing sectors), and the fiscal authority. The model is mainly based on Bielecki et al. (2020) and Papetti (2021).

## B. Households

Each household consists of a single agent, who appears in our model at the age of 20, indexed as $j = 1$. Agents can live up to 104 years ($j = J = 85$) but may die at year $t$ according to an age- and time-dependent mortality rate $\omega_{j,t}$. Hence, at each point in time, the model economy is populated by 85 cohorts of overlapping generations. The size of the age-$j$ cohort is denoted by $N_{j,t}$.

The representative age-$j$ household at time $t$ maximizes its expected remaining lifetime utility until the maximum age $J$:

$$U_{j,t} = \sum_{i=0}^{J-j} \beta^i \Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right) \times$$
$$\left[ \left( \ln c_{j+i,t+i} + \chi \ln h_{j+i,t+i} \right) + \omega_{j+i,t+i} \psi \{ \nu \ln bi_{j+i,t+i} + (1-\nu) \ln h_{j+i,t+i} \} \right]. \quad (1)$$

where $\beta$ denotes the subjective discount factor, and $\omega_{j,t} \equiv 1 - N_{j+1,t+1}/N_{j,t}$ denotes the time-$t$ mortality rate between age $j$ and $j+1$. Thus, the term $\Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right)$ represents the probability of surviving for at least $i$ more years. The per-period utility function has two components. The first component represents felicity as a function of the consumption stream $c_{j,t}$ and housing services $h_{j,t}$ with the importance parameter $\chi$. The second component represents bequest motives conditional on death. Parameter $\psi$ determines the importance of bequest motives relative to felicity. Bequest motives are specified as the weighted average of the log utilities from the intentional financial bequest contribution $bi_{j,t}$ and the housing bequest $h_{j,t}$. Note that housing services in the felicity function are a linear transformation of housing assets in the bequest function. Parameter $\nu$ denotes the utility weight for the intentional financial bequest.

An age-$j$ household has the following budget constraint:

$$c_{j,t} + a_{j,t} + bi_{j,t} + P_t^h hi_{j,t} = (1 + r_t)\, a_{j-1,t-1} + \mathbf{1}_{j \geq JR} pen_t \\ + (1 - \mathbf{1}_{j \geq JR}) \left[ (1 - T_t) \left\{ \sigma_j^g w_t^g \xi_j^g + \left(1 - \sigma_j^g\right) w_t^h \xi_j^h \right\} + (1 - T^a)\, i_t - T^h P_t^h i_t^h \right]. \quad (2)$$

Retirees, as indicated by $\mathbf{1}_{j \geq JR}$, receive pension benefits $pen_t$ that are independent of their age. Participation in the pension system is mandatory, so that all workers have to make a social security contribution that is levied as part of personal income tax at the rate of $T_t$. Working-age households $(1 - \mathbf{1}_{j \geq JR})$ inelastically supply labor until reaching the age of 64 ($j = JR = 45$), either in the general goods production sector $g$ or housing production sector $h$. Labor income in sector $s = \{g, h\}$ depends on the sector-specific gross wage rate $w_t^s$ and the exogenously determined age-specific labor productivity $\xi_j^s$. Working-age households also inherit financial assets $i_t$ and housing assets $i_t^h$ evaluated at the market price of housing $P_t^h$ after paying inheritance taxes at rates $T^a$ and $T^h$, respectively.

All households can smooth their consumption by adjusting liquid assets $a_{j,t}$, which are claims on productive capital and government debt. We allow households to borrow across different cohorts, so that asset holdings $a_{j,t}$ can be negative at any age $j$. Households invest $bi_{j,t}$ in the intentional financial bequest account $b_{j,t}$. At death, agents leave the accumulated intentional financial bequest, the outstanding liquid asset, and the outstanding housing asset as bequests. Intentional bequest assets ($b_{j,t}$) are invested in productive capital and earn the real interest rate $r_t$, which is equivalent to the natural rate of interest in this real model without short-term business cycles:

$$b_{j,t} = (1 + r_t)\, b_{j-1,t-1} + bi_{j,t}, \tag{3}$$

$$b_{0,t} = a_{0,t} = a_{J,t} = 0. \tag{4}$$

Households can adjust housing with housing investment $hi_{j,t}$ at the market price of housing $P_t^h$. The sign of $hi_{j,t}$ can be negative if a household reduces the size of its dwelling. This smooth adjustment of housing by the representative household reflects the aggregate individual adjustments made by renters and homeowners. Although the direct interpretation of the budget constraint is that a household is a homeowner who spends to purchase housing and pays no periodic user costs, we do not assume that all households are homeowners. The present specification is equivalent to an alternative one that includes periodic user (rental) costs but not purchase costs because there are no market imperfections such as asymmetric information and incomplete contracts. The user cost rate equals the difference between the total gross return to housing assets $1+r_t+\delta_h$ and the house price appreciation rate $P_{t+1}^h \big/ P_t^h$ (See Appendix I). An advantage of the present specification is that house prices are directly specified.

Occupied and vacant houses ($h_{j,t}$ and $v_{j,t}$) are accumulated according to the following equations, respectively.

$$h_{j,t} = (1 - \delta_h)\, h_{j-1,t-1} + hi_{j,t} + \phi\, (1 - \mathbf{1}_{j \geq JR})\, i_t^h, \tag{5}$$

$$v_{j,t} = (1 - \delta_h)\, v_{j-1,t-1} + (1 - \phi)\, (1 - \mathbf{1}_{j \geq JR})\, i_t^h, \tag{6}$$

$$h_{0,t} = v_{0,t} = 0. \tag{7}$$

Housing assets depreciate at the rate of $\delta_h$. Parameter $\phi$ denotes the intergenerational house matching efficiency, as defined by the proportion of inherited housing $i_t^h$ used by younger generations.

Inherited financial assets and housing are defined by:

$$i_t = (1 + r_t) \frac{\sum_{j=1}^{J} \left(N_{j,t-1} - N_{j+1,t}\right) \left(a_{j,t-1} + b_{j,t-1}\right)}{\sum_{j=1}^{JR} N_{j,t}}, \tag{8}$$

$$i_t^h = (1 - \delta_h) \frac{\sum_{j=1}^{J} \left(N_{j,t-1} - N_{j+1,t}\right) h_{j,t-1}}{\sum_{j=1}^{JR} N_{j,t}}, \tag{9}$$

where $i_t$ and $i_t^h$ denote inherited financial assets and housing, respectively. Bequeathed financial assets and housing are equally allocated to all working-age households.

In our model, demographic processes are governed by exogenous changes in the size of the youngest cohort $n_{1,t} = N_{1,t}/N_{1,t-1} - 1$ and the mortality rate $\omega_{j,t}$. Then, the total number of living agents $N_t$ and the population growth rate $n_t$ are given by:

$$N_t = \sum_{j=1}^{J} N_{j,t}, \tag{10}$$

$$n_t = \frac{N_t}{N_{t-1}} - 1. \tag{11}$$

The number of agents in each cohort is given by:

$$N_{j,t} = \left(1 - \omega_{j-1,t-1}\right) N_{j-1,t-1}. \tag{12}$$

Table I summarizes the timing convention for each variable. The household's first-order conditions are derived in Appendix A.

## *C. Firms*

Firms consist of general goods and housing sectors: $s = \{g, h\}$. A representative firm in sector $s$ hires capital and labor and produces non-differentiated output $Y_t^s$ according to the

| Period | $t$ | |
|---|---|---|
| Date | $t-1$ (Beginning of period $t$) | $t$ (End of period $t$) |
| Financial assets | $A_{t-1}$, $a_{j-1,t-1}$ | $A_t$, $a_{j,t}$ |
| Housing | $H_{t-1}$, $h_{j-1,t-1}$ | $H_t$, $h_{j,t}$ |
| Intentional bequests | $B_{t-1}$, $b_{j-1,t-1}$ | $B_t$, $b_{j,t}$ |
| Capital | $K_{t-1}$ | $K_t$ |
| Government debt | $D_{t-1}$ | $D_t$ |
| Population and cohort size | $N_t$, $N_{j,t}$ | |
| Consumption | $C_t$, $c_{j,t}$ | |
| Bequest investment | $bi_{j,t}$ | |
| Housing investment | $hi_{j,t}$ | |
| Mortality rate | $\omega_{j,t}$ | |
| Goods production | $Y_t^g$ | |
| Housing production | $Y_t^h$ | |
| Capital for goods sector | $K_t^g$ | |
| Capital for housing sector | $K_t^h$ | |
| Government spending | $G_t$ | |
| Inherited financial assets | $i_t$ | |
| Inherited housing | $i_t^h$ | |
| Interest rate | $r_t$ | |

Table I: Timing Convention for Each Variable

following Cobb-Douglas production function:[2]

$$Y_t^s = \left(K_t^s\right)^{\alpha_s} \left(\zeta_t^s L_t^s\right)^{1-\alpha_s}, \tag{13}$$

where $\zeta_t^s$ denotes labor-augmenting technology growing at the rate of $g_t^s$:

$$\zeta_t^s = \left(1 + g_t^s\right) \zeta_{t-1}^s. \tag{14}$$

Let $\theta_t$ denote the technology gap between the housing and general goods sectors:[3]

$$\theta_t \equiv 1 - \frac{\zeta_t^h}{\zeta_t^g}. \tag{15}$$

[2]Inelastically-supplied land could be separately included in the housing production function, in which case decreasing returns to scale will contribute to lower productivity for larger production scale.

[3]Labor-augmenting technologies can be directly compared across these sectors because they use the same kind of labor input.

Then, the housing-technology growth rate equals the general-technology growth rate adjusted for the change in the technology gap:

$$1 + g_t^h = (1 + g_t^g)\,\frac{1 - \theta_t}{1 - \theta_{t-1}}. \tag{16}$$

The periodic profit flow is:

$$\pi_t^s = P_t^s Y_t^s - w_t^s L_t^s - (r_t + \delta_k)\, K_t^s, \tag{17}$$

where $P_t^g$ equals unity.

## *D. Government*

The government purchases general goods $G_t$ in the market and spends on pensions $pen_t$. It rolls over public debt by paying off the outstanding debt $(1 + r_t)\, D_{t-1}$ and issuing a new debt $D_t$. It also levies taxes from ordinary income at a rate $T_t$, inherited housing at a rate $T^h$, and inherited financial assets at a rate $T^a$. The government's budget constraint at time $t$ is:

$$\sum_{j=1}^{JR-1} T_t N_{j,t} \left[\sigma_j^g w_t^g \xi_j^g + \left(1 - \sigma_j^g\right) w_t^h \xi_j^h\right] + \left(T^h P_t^h i_t^h + T^a i_t\right) \sum_{j=1}^{JR-1} N_{j,t} + D_t = G_t + pen_t \sum_{j=JR}^{J} N_{j,t} + (1 + r_t)\, D_{t-1} \tag{18}$$

Pensions per retired household are determined as the product of the replacement rate $\rho_t$ and the economy-wide average wage:

$$pen_t = \rho_t \frac{w_t^g \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g + w_t^h \sum_{j=1}^{JR-1} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h}{\sum_{j=1}^{JR-1} N_{j,t}}, \tag{19}$$

where $\sigma_j^g$ is the proportion of $j$-aged household choosing to work for the general goods sector.

The paths of public debt, government purchases, the replacement rate, and inheritance tax rates are exogenously determined, whereas the income tax rate $T_t$ adjusts so that the government budget constraint (18) is satisfied. Thus, for the government, three types of taxes are perfect substitutes.

## *E. Market clearing and other aggregate conditions*

The financial market clears with

$$D_t + K_t = A_t + B_t \tag{20}$$

$$K_{t-1} = K_t^g + K_t^h \tag{21}$$

Labor market clears with:

$$L_t^g = \sum_{j=1}^{J} \sigma_j^g N_{j,t} \xi_j^g, \tag{22}$$

$$L_t^h = \sum_{j=1}^{J} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h. \tag{23}$$

New housing market clears with:

$$Y_t^h = \sum_{j=1}^{J} N_{j,t} h i_{j,t}. \tag{24}$$

The good market clears as:

$$Y_t^g = \sum_{j=1}^{J} N_{j,t} c_{j,t} + K_t - (1 - \delta_k) K_{t-1} + G_t. \tag{25}$$

The aggregate consumption, assets and housing are:

$$C_t = \sum_{j=1}^{J} N_{j,t} c_{j,t}, \tag{26}$$

$$A_t = \sum_{j=1}^{J} N_{j,t} a_{j,t}, \tag{27}$$

$$B_t = \sum_{j=1}^{J} N_{j,t} B_{j,t}, \tag{28}$$

$$H_t = \sum_{j=1}^{J} N_{j,t} h_{j,t}, \tag{29}$$

$$V_t = \sum_{j=1}^{J} N_{j,t} v_{j,t}. \tag{30}$$

## *F. Exogenous process*

The model economy is driven by several exogenous forces. Demographic processes are characterized by the growth rate of the 20-year-old cohort $n_{1,t}$ and the age-specific mortality risk $\omega_{j,t}(j = 1, ..., J)$. The fiscal authority exogenously determines its purchases of general goods $G_t$, the replacement rate in the pensions system $\rho_t$, and the path of public debt $D_t$. TFP is also exogenous for the general goods and housing sectors. The age-specific mortality rate $\omega_{j,t}(j = 1, ..., J)$ is adjusted for healthy life expectancy.

Table II shows the list of the exogenous variables.

## *G. Equilibrium*

For all periods $t = 0, 1, ..., \infty$, given exogenously given demographics $\{N_{j,t}\}_{j=1}^{J}$, $N_t$, $n_t$ and variables $n_{1,t}$, $\rho_t$, $\zeta_t^g$, $\zeta_t^h$, $G_t$, $\{\omega_{j,t}\}_{j=1}^{J}$, $\left\{\xi_j^g\right\}_{j=1}^{J}$, $\left\{\xi_j^h\right\}_{j=1}^{J}$, and $\left\{\sigma_j^g\right\}_{j=1}^{JR-1}$, the equilibrium of this economy is a sequence of aggregates $C_t$, $A_t$, $B_t$, $H_t$, $V_t$, prices $w_t^g$, $w_t^h$, $P_t^h$, $r_t$, transfers $T_t$, $pen_t$, and quantities $\{c_{j,t}, a_{j,t}, b_{j,t}, bi_{j,t}, h_{j,t}, hi_{j,t}, v_{j,t}\}_{j=1}^{J}$, $i_t$, $i_t^h$, $K_t$, $K_t^g$, $K_t^h$, $L_t^g$, $L_t^h$, $Y_t^g$, $Y_t^h$, such that:[4]

[4]The full set of equations that characterize the equilibrium is listed in Appendix B.

| Variable | Description | Type |
|---|---|---|
| $n_{1,t}$ | Fertility rate (the growth rate of 20-year-old cohort) | periods x 1 |
| $\omega_{j,t}$ | Age-specific modified mortality rate | periods x cohorts |
| $\zeta_t^g$ | Labor-augumenting technology in general production sector | periods x 1 |
| $\theta_t$ | Technology gap between the housing and general production sectors | periods x 1 |
| $\mathbf{1}_{j\geq JR}$ | Retirement indicator | periods x cohorts |
| $\rho_t$ | Pension replacement rate | periods x 1 |
| $\sigma_j^g$ | Age-specific proportion for working in general production sector | cohorts x 1 |
| $\xi_j^g$ | Age-specific effectiveness of labor in general production sector | cohorts x 1 |
| $\xi_j^h$ | Age-specific effectiveness of labor in housing sector | cohorts x 1 |
| $\overline{G/Y}$ | Government spending ratio | periods x 1 |
| $\overline{D/Y}$ | Government debt ratio | periods x 1 |

Table II: List of Exogenous Variables

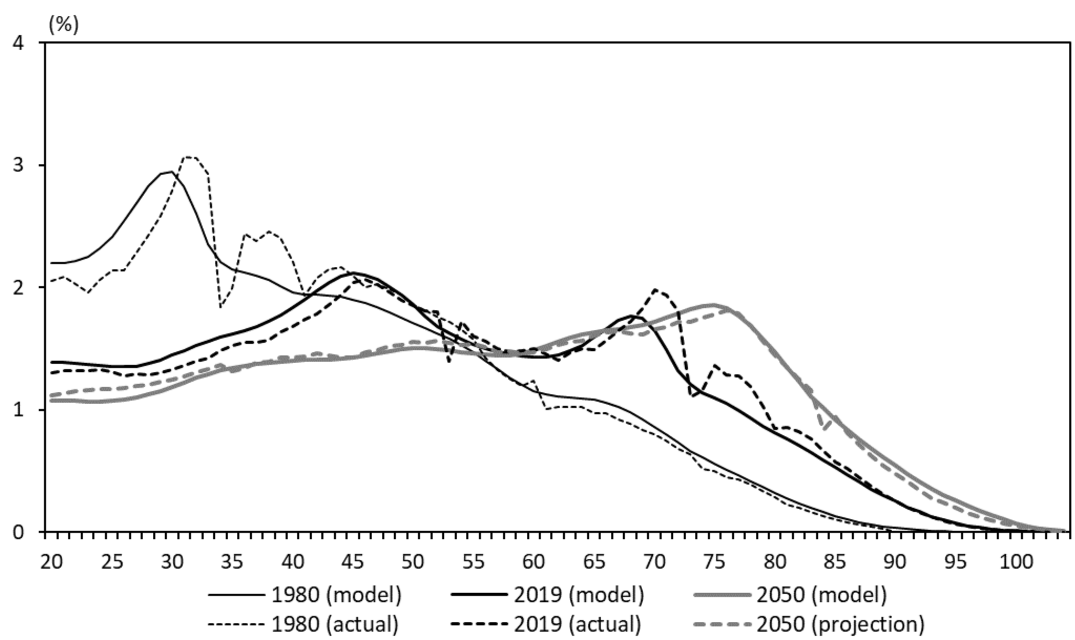
1. Demographics develops as (10)-(12) and the aggregation satisfies (26)-(30);
2. Households maximize the life-time utility (1) subject to constraints (2)-(9);
3. Firms maximize profits (17) given the production technology (13)-(16);
4. The government balances budget (18) and (19) by setting the income tax rate;
5. Factor markets (20)-(23), the housing market (24) and the general goods market (25) clear.

# IV. Baseline Calibration

## A. *Exogenous variables and structural parameters*

We calibrate the model to the Japanese economy using the data from 1925 to 2095. We exogenously specify a demographic scenario, technological progress, and the fiscal conditions of the government. The data source for calibration is summarized in Appendix C, the time series graph of each exogenous variable is shown in Appendix D , and the detailed solution method is outlined in Appendix E.

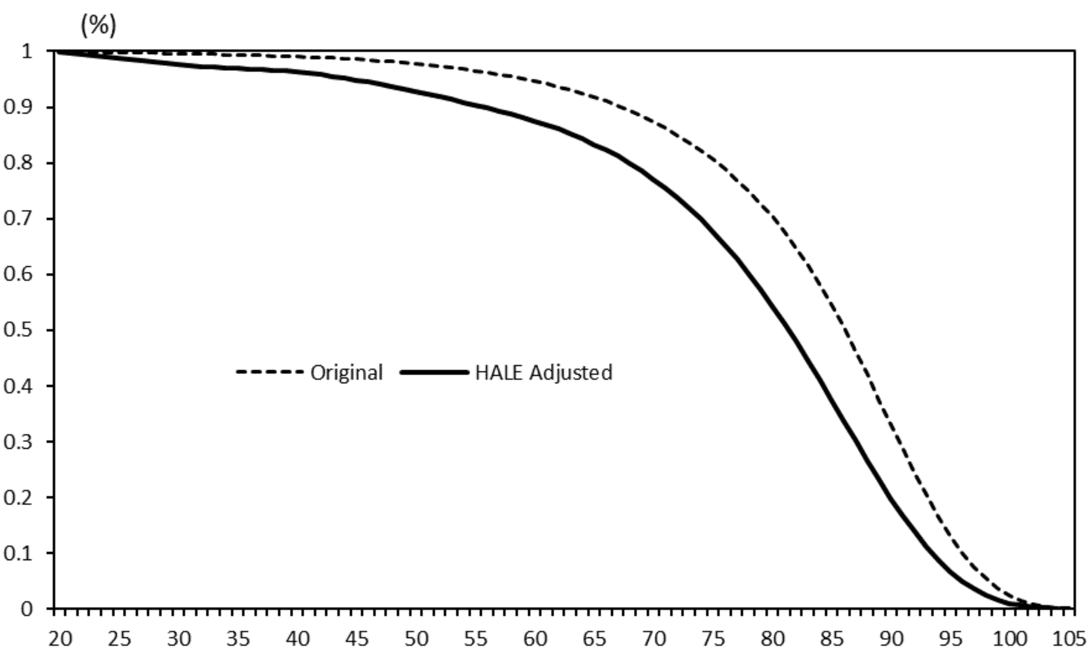
For our demographic scenario, we use past data and projections on mortality rates and the size of the 20-year-old cohort from the Population Projection for 2020 through 2050 from the National Institute of Population and Social Security Research. We smooth the mortality rate and the cohort size by using the stochastic trend component from the Hodrick-Prescott (HP) filter. We further adjust the mortality rate $\omega_{j,t}$ for the health-adjusted life expectancy (HALE) by using the Life Tables for 2019 and the Comprehensive Survey of Living Conditions from the Ministry of Health, Labour and Welfare. In Japan, HALE is shorter than the actual life expectancy by 8 to 12 years (figure 3). We assume that mortality rates are constant for 1925-1970 at the rate in 1970. Regarding the growth rate of the 20-year-old cohort $n_{1,t}$, we use the Population Estimates between 1925 and 2014 and the Population Projection between 2015 and 2050 from the Statistics Bureau. After 2050, we assume that both the mortality rate and the fertility rate stabilize at the 2050 level. Figure 2 contrasts the actual age distribution of the population with the model-generated age profile for 1980, 2019, and 2050. The model replicates the actual demographics quite well.




Note: The health-adjusted life expectancy (HALE) is computed from the Life Tables for 2019 and the Comprehensive Survey of Living Conditions by the Ministry of Health, Labour and Welfare.

Figure 2: Distribution of Population by Age in 1980, 2019 & 2050

We also estimate the age-specific labor productivity for the general goods sector $\xi_j^g$ and housing sector $\xi_j^h$ and the age-specific proportion of workers in the general goods sector $\rho_j^g$,




Note: Data on mortality rates and the size of the 20-year cohort are provided by the National Institute of Population and Social Security Research. Mortality rates adjusted for health-adjusted life expectancy are held constant at the 1970 level for 1925-1970 and at the 2050 level for 2050 and later.

Figure 3: Original and HALE-Based Survival Rates

using the 2020 Basic Survey on Wage Structure by the Ministry of Health, Labour and Welfare. Our empirical measure of labor productivity is the estimated hourly labor income based on a regression on the constant, age, and age squared. The retirement age $JR$ is 65 for all periods.

For technological progress, we estimate the labor-augmenting technology for the general goods sector $\zeta_t^g$ and housing sector $\zeta_t^h$ using the JIP Database between 1972 and 2018 from the Research Institute of Economy. We use the stochastic trend component out of the Hodrick-Prescott filter. After 2019, we assume that the TFPs grow at the same rates as 2018.

For the fiscal condition of the government, we use the government spending ratio (of GDP) and government debt ratio (of GDP) from the Japanese System of National Accounts (JSNA) data for 1980-2021. We assume that those are constant for 1925-1980 at the rate in 1980 and for 2021-2095 at the rate in 2021. We also fix the replacement rate at 45%, in line with the literature.

Table III summarizes the parameters. The discount factor $\beta$, the capital depreciation rate $\delta_g$ and the housing depreciation rate $\delta_h$ are based on the literature. The inheritance tax rates for financial assets ($T^a$) and housing ($T^h$) capture the preferential tax treatment for housing. Inherited financial assets are heavily taxed up to the maximum tax rate of 55%, although there are some basic exemptions. In contrast, the assessed value of housing is reduced by 80% up to a 330 $m^2$ lot. Thus, many heirs who inherit only a house often pay no inheritance taxes at all. Instead of specifying the complex inheritance tax codes, we simply use a 40% effective tax rate for inherited financial assets and no effective tax for inherited housing. Other countries also have various forms of preferential treatment of housing in estate taxes, such as a step-up in basis. Thus, this difference in the effective tax rate is relevant for many countries.

We determined the importance of housing consumption $\chi$, the relative importance of bequest motives $\psi$ in the lifetime utility function, and the weight on financial bequests $\nu$

relative to housing bequests by matching the following statistics with the 2019 data: (1) the ratio of housing consumption to non-housing consumption ($p^h h/c$) for ages 20-64 and 65-104 and (2) the average saving-to-consumption ratio ($(a+b)/c$) for 65-104.[5]

The efficiency parameter for intergenerational housing transfer $\phi$ is set to 0.46 to match the inverse vacancy rate (20% of the inherited houses are used by the younger generation). The capital shares for general goods sector $\alpha_g$ and for the housing sector $\alpha_h$ are calibrated to the JSNA data for 1994-2018.

| Variable | | Value | How to decide |
|---|---|---|---|
| $\beta$ | Discount factor | 0.996 | Literature |
| $\chi$ | Importance of housing consumption | 0.385 | Match the average $P^h h/c$ among ages 20-64 and 65-104, and the average $(a+b)/c$ among ages 65-104 |
| $\psi$ | Importance of bequest motives | 22 | |
| $\nu$ | Importance of financial bequests | 0.6 | |
| $\delta_g$ | Depreciation rate for capital | 0.15 | Litereture |
| $\delta_h$ | Depreciation rate for housing | 0.03 | Litereture |
| $\phi$ | Proportion of reused housing | 0.46 | Match the average $h/v$ |
| $T^a$ | Inheritance tax rate for financial assets | 0.4 | Litereture |
| $T^h$ | Inheritance tax rate for housing | 0 | Litereture |
| $\alpha_g$ | Capital share in general production sector | 0.37 | Calibrate with JSNA |
| $\alpha_h$ | Capital share in housing sector | 0.31 | Calibrate with JSNA |

Table III: List of Structural Parameters

Table IV shows the parameter calibration results. The ratio of housing consumption to non-housing consumption in the model ($P^h h/c$) is identical to the data for both the 20-64 group (8.4) and the 65-104 group (13.3). However, these perfect matches and the realistic age profile of housing consumption come at the expense of a high saving-to-consumption ratio ($(a+b)/c = 15.2$) compared to the actual average ratio (6.4) for 65-104. A lower value of $\psi$ can decrease the saving-to-consumption ratio but worsens the housing-to-non-housing consumption ratio and the age profile of saving and housing consumption. Because the main focus of this study is on the effect of housing consumption, we accept a high saving-to-consumption ratio for the baseline. However, we perform a counterfactual analysis by changing the value of $\psi$ and $\nu$. The inverse vacancy rate from the model ($h/v = 5.0$)

[5] In the long run, these parameters may change as a result of population aging and low fertility rates. However, we assume constant parameters because reliable estimates are unavailable for potential future changes in bequest motives.

matches the data. In fact, if we remove vacant rental units from the data, the empirical inverse vacancy rate increases to 7.14.

| Ratios | Target Value | Model Value |
|---|---|---|
| Average $P^h h/c$ among age 20-64 | 8.4 | 8.4 |
| Average $P^h h/c$ among age 65-104 | 13.3 | 13.3 |
| Average $(a+b)/c$ among age 65-104 | 6.4 | 15.2 |
| Average $h/v$ | 5.0 | 5.0 |

Table IV: Calibration Results

## B. *Baseline Calibration*

Figure 4 illustrates the age profile of consumption, housing, and savings in 2019. In contrast to the basic model for the life-cycle and permanent income hypotheses (LCH/PIH), our model generates a hump-shaped age profile of per capita consumption due to variable mortality rates (panel 4a). The consumption growth for a household of age $j$ between $t$ and $t+1$ provides an insight into this cross-sectional age profile. Our model's Euler equation is as follows:

$$\frac{c_{j+1,t+1}}{c_{j,t}} = \beta\left(1 + r_{t+1}\right)\left(1 - \omega_{j,t}\right). \tag{31}$$

This equation shows that the consumption growth rate is determined not only by the standard factors (the subjective discount factor and the interest rate) but also the survival rate $(1-\omega_{j,t})$. Younger generations increase consumption each year based on the standard factors because changes in survival rates are negligible. However, the survival rate decreases rapidly towards the end of the lifespan. As a result, consumption growth becomes less than one when the survival rate is significantly low. Panel 4a shows a hump-shaped age profile of per capita consumption for both the model and data. One difference is that the peak occurs at around 50 years old in the data, whereas it occurs at around 70 years old in the model. One possible explanation for the discrepancy is the significant amount of educational expenses for children when the household head is around 50 years old. Our model does not account for age-specific consumption goods and services, which prevents it from replicating this early consumption peak. However, when consumption is weighted by the cohort size, the age profile of aggregate consumption is reasonably similar between the model and data (see panel 4b).

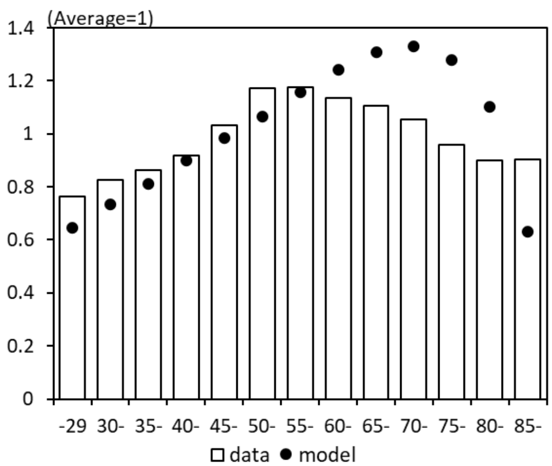
Housing consumption, unlike per capita consumption, increases with age, as shown in panel 4c. This upward-sloping age profile of housing consumption is also observed in many countries, including the U.S. (Yang, 2009). In the standard two-good model without bequest motives, the ratio of housing to non-housing consumption should be independent of age because both consumption and housing consumption have parallel age profiles (Yang, 2009). Our model replicates the realistic age profile of housing by incorporating bequest motives and age-dependent mortality rate.

Housing consumption is determined by the following first-order condition:

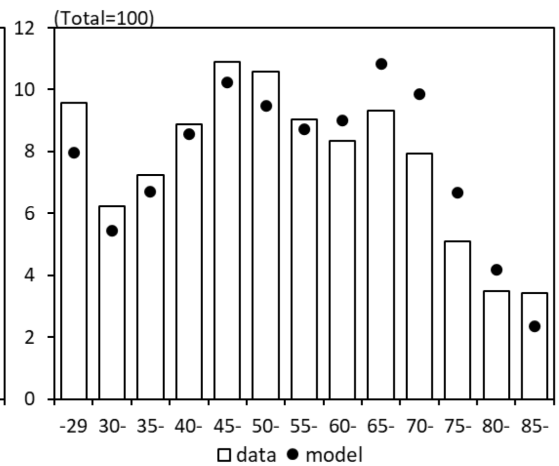
$$\frac{h_{j,t}}{c_{j,t}} = \left[\chi + \psi \left(1-\nu\right) \omega_{j,t}\right] \left[P_t^h - \frac{(1-\delta_h)}{(1+r_{t+1})} P_{t+1}^h\right]^{-1}. \tag{32}$$

As in the standard model, the ratio of housing to non-housing consumption increases with the importance of housing $\chi$ in the felicity function and decreases with housing costs. However, unlike the standard model, equation (32) includes the product of the mortality rate $\omega_{j,t}$, the bequest motive parameter $\psi$, and the housing bequest weight $(1-\nu)$ for retirees only $(\mathbf{1}_{j \geq JR})$. Elderly households increase their housing consumption relative to non-housing consumption as mortality increases due to their bequest motives. Note that housing consumption is a linear transformation of housing assets in the model. Thus, the age profile shown in panel 4d is based on housing assets due to data availability. The model agrees well with the data, except for a sharp increase for households in their 30s and 40s in the data. This sharp increase is likely due to first-time home purchases around child births, which the model does not account for. However, cohort-weighted aggregate age profile is quite similar between the model and the data (panel 4d).

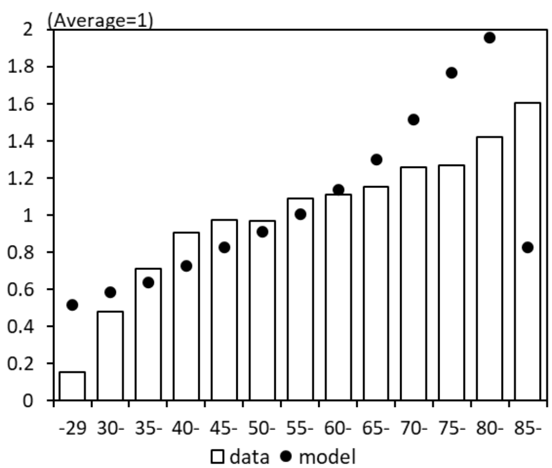
Savings exhibit a hump-shaped age profile in the standard life-cycle model because house-



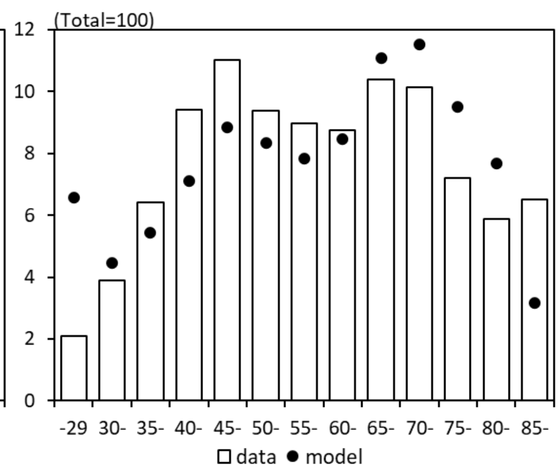

(a) Per-Capita Consumption



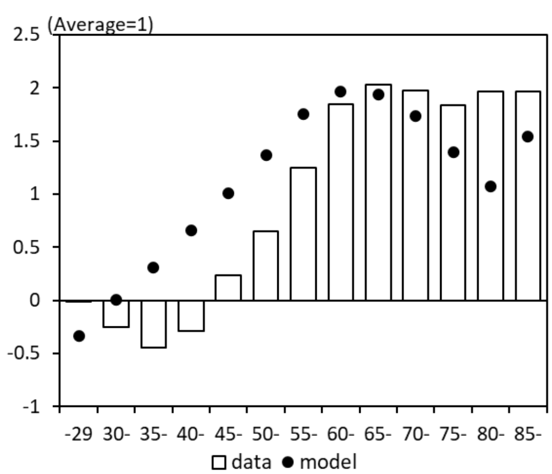

(b) Aggregate Consumption



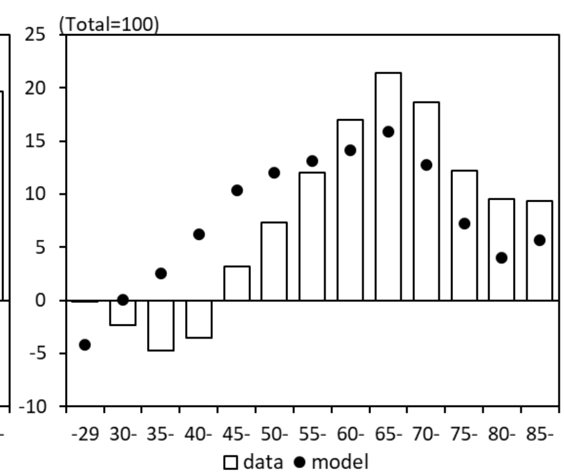

(c) Per-Capita Housing Assets




(d) Aggregate Housing Assets




(e) Per-Capita Financial Assets




(f) Aggregate Financial Assets

Figure 4: The Age Profile of Consumption, Housing and Liquid Assets: Data and Model

holds save during a high-income period and dissave during a low-income period to smooth consumption. In our model, liquid asset savings $a_{j,t}$ has that age profile. However, as French et al. (2023) discuss, retired households deccumulate their assets very slowly in the US and other countries. Our data confirm this tendency even for liquid assets (panel 4e); savings peak at the retirement age and remain at the same level throughout the remaining life. Our model addresses this issue by incorporating bequest motives. The second component of savings in the form of financial bequests $b_{j,t}$ increases sharply with mortality rates because of the following intra-temporal optimality condition:

$$\frac{bi_{j,t}}{c_{j,t}} = \psi \nu \omega_{j,t}. \tag{33}$$

Thus, the sum of liquid assets $a_{j,t}$ and financial bequests $b_{j,t}$ exhibits a high level of savings for the elderly. However, the data show negative savings for ages between 30 and 50. This borrowing corresponds to a sharp increase in housing assets. Thus, this dip in savings for middle-aged household heads likely corresponds to mortgages for first-time home purchases driven by the birth of children. Because our model does not account for such mortgage financing or a rapid change in the household size, it deviates from the data for these age groups.

Figure 5 contrasts the model results with the data for the dynamics of key aggregate variables. For annual real GDP growth rates (panel 5a), we use JSNA. We estimate the stochastic trend using the HP filter, setting parameter $\lambda$ equal 100. The model shows a decreasing trend in GDP growth as in the data, but the predicted change is smaller than the actual change. However, both the model and the data show no growth around 2010. Our model predicts negative real GDP growth rates after the 2000s.

For the real interest rate (panel 5b), we use the prime lending interest rate adjusted for the GDP deflator, provided by the World Bank. This rate corresponds to the average rate for safe government notes and risky borrowing by firms and households, because the model has a single interest rate for both lending and borrowing rates in the public and private sectors. This interest rate represents the natural rate of interest in our model, although it is higher than the rates estimated from the risk-free rates. The model predicts decreasing real interest rates until the 2010s, which is consistent with the data, except for the period of the Bank of Japan's quantitative and qualitative easing regime since 2013. From the 2010s, the natural rate of interest is stabilized around 2%.

The capital stock ratio (panel 5c) decreases between 1980 and 2020 in both the model and the data. The model predicts continued decline in the ratio until 2050. For household assets, we include land but exclude pension assets to make the data consistent with the model.

The real interest rate can be characterized by both household's Euler equation (equation (31)) and each production sector's first-order condition with respect to capital:

$$r_t + \delta_k = \alpha_g \left( \frac{K_t^g}{\zeta_t^g L_t^g} \right)^{\alpha_g - 1} = \alpha_h \left( \frac{K_t^h}{\zeta_t^h L_t^h} \right)^{\alpha_h - 1} P_t^h. \tag{34}$$

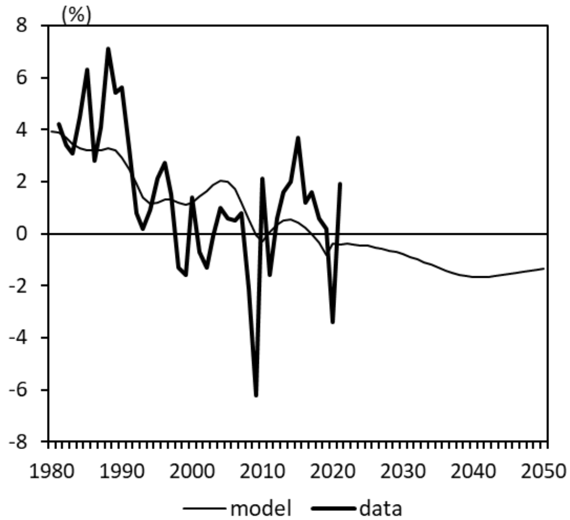
The secular decrease in the interest rate between the 1980s and 2010s inversely reflects the dynamics of capital $K_t^g$, which peaks in 2011. The interest rate also reflects the secular decrease in productivity $\zeta_t^g$.

Furthermore, the model demonstrates that the outcomes exhibit heterogeneous dynamics by age. The age-specific equilibrium dynamics in the baseline calibration are discussed in Appendix H.

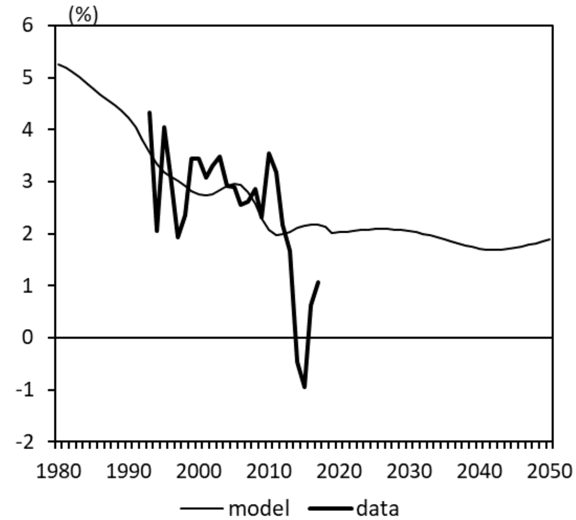
# V. Factor Decomposition

We specify our exogenous variables including demographic ratios between 1925 and 2100 and obtain the model outcome between 1980 and 2050.

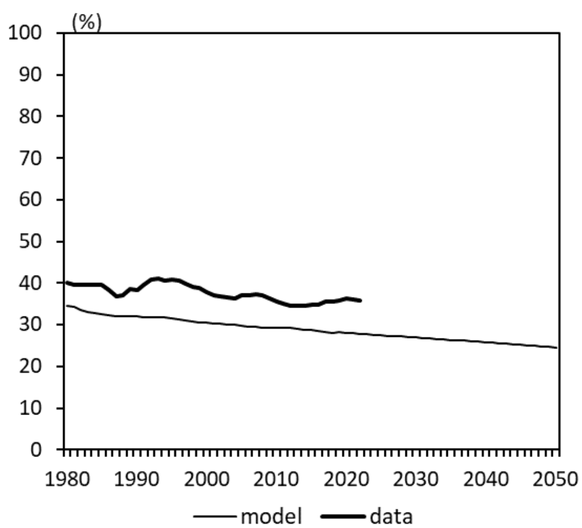
We obtain the model outcome for all equilibrium variables: aggregates $\{C_t, A_t, B_t, H_t, V_t\}_{t=0}^{\infty}$, prices $\left\{w_t^g, w_t^h, P_t^h, r_t\right\}_{t=0}^{\infty}$, transfers $\{T_t, pen_t\}_{t=0}^{\infty}$, and quantities $\Big\{\{c_{j,t}, a_{j,t}, b_{j,t}, bi_{j,t}, h_{j,t},$



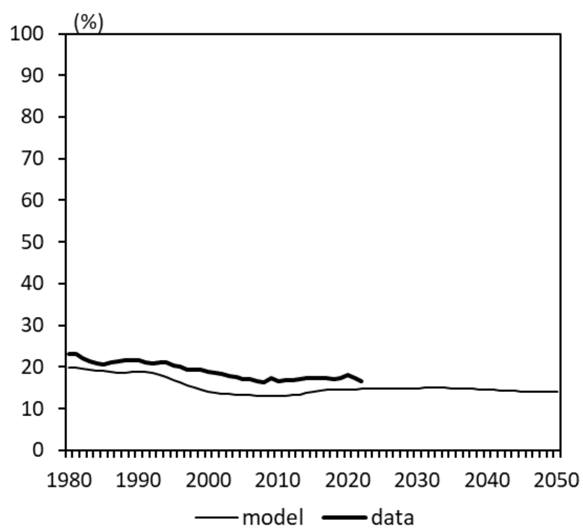

(a) Real GDP Growth




(b) Real Interest Rate




(c) Capital to Asset Ratio




(d) Housing Sector Share of GDP

Note: Panel (a): The annual real GDP growth rate from JSNA. Panel (b): The prime lending interest rate adjusted for the GDP deflator from the World Bank. Panel (c): Capital stock from JSNA. The ratio is calculated as $K_t/(A_t + B_t + (H_t + V_t)P_t^h)$. Panel (d): Sector share of housing sector in GDP from JSNA

Figure 5: Dynamics of Baseline Estimation: Data and Model

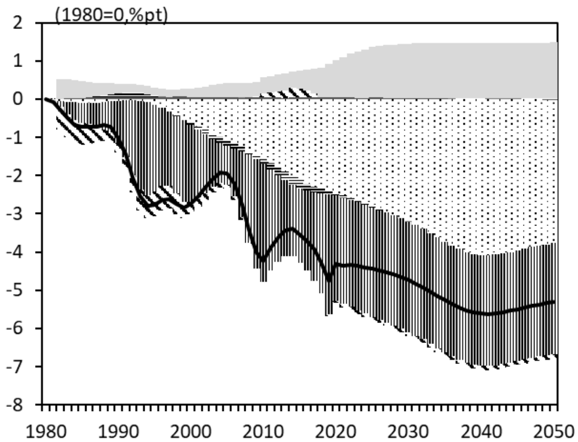
$hi_{j,t}, v_{j,t}\}_{j=1}^{J}, i_t, i_t^h, K_t^g, K_t^h, L_t^g, L_t^h, Y_t^g, Y_t^h\Big\}_{t=0}^{\infty}$. However, for brevity, we discuss GDP growth rate, the natural rate of interest, income tax rate, housing price, per capita housing value, and per capita financial assets.

To gain an insight into the dynamics of the equilibrium outcome, we decompose each variable into the effects of six exogenous factors: mortality rates, fertility rates, government debt, government spending, technology growth rates for the general goods sector, and technology gap between the housing and general goods sectors. We "knock out" each factor at a time by fixing the exogenous factor value at the 1980 level and measuring the difference between the baseline outcome and the outcome in the knock-out case. Residuals that cannot be attributed to any single factor are considered as interactions.

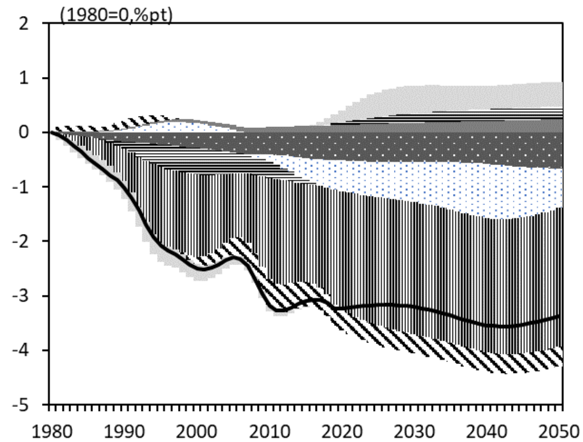
Figure 6 shows the results. Panel 6a shows that the model's GDP growth rate decreases by more than three percentage points between 1980 and 2050, mainly because low fertility rates reduce the working-age population. In addition, low technology growth rates in the general goods sector reduce GDP growth.

Panel 6b shows that after the 2000s, the natural rate of interest is approximately 1.7 percentage points below its 1980 level. The main factors driving the decline in interest rates are low technological growth and low mortality and fertility rates. These effects are consistent with those found in the literature (e.g., Kitao and Mikoshiba, 2020). After 2010, however, rising government spending financed by public debt will create demand for capital and raise interest rates.

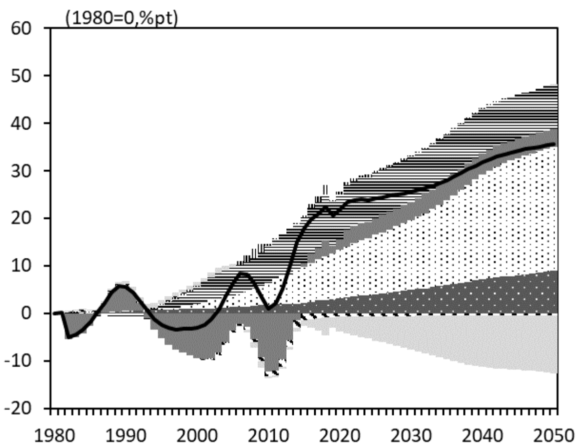
In panel 6c, income tax rates increase by approximately 37 percentage points by 2050, mainly because of declining fertility rates and increasing government spending. Note that we do not calibrate the model tax rate to the actual income tax rate because the model does not include other sources of government revenue. Thus, this large increase in tax rates should not be taken as a prediction, but as the necessary government revenue to finance the exogenously specified government spending, pension replacement rate, and government debt. If a 37 percentage point increase in the tax rate is unacceptable to voters, the Japanese



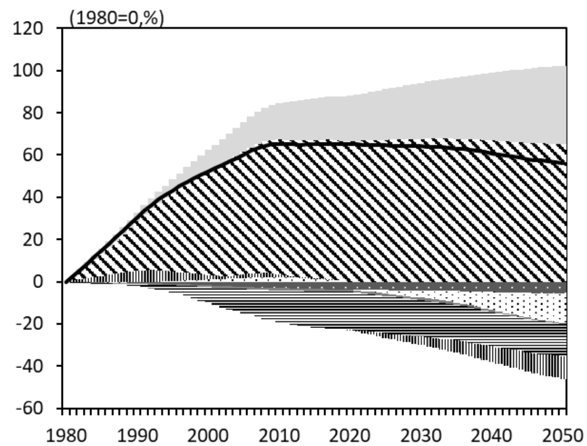

(a) GDP Growth Rate



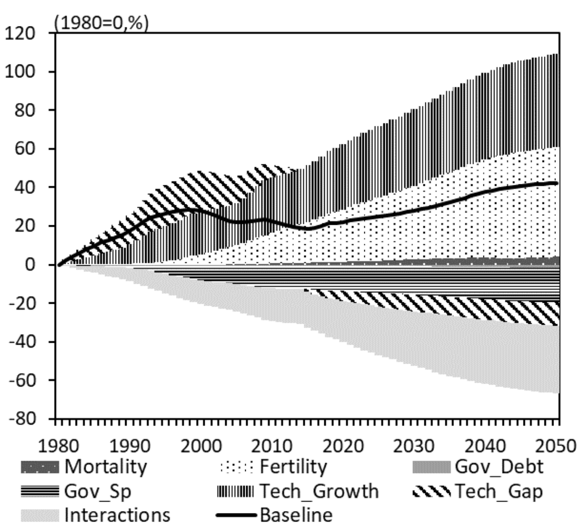

(b) Natural Rate of Interest



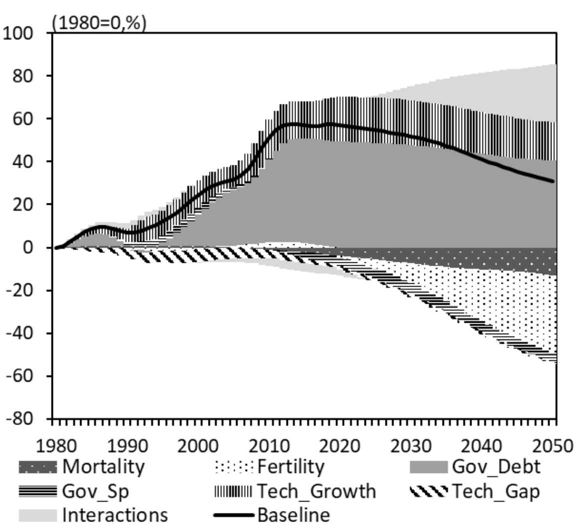

(c) Tax Rate




(d) Housing Price




(e) Per Capita Housing Value




(f) Per Capita Financial Assets

Figure 6: The Decomposition of Equilibrium Dynamics

government would naturally reduce spending, debt, and the pension replacement rate. To put this rate in perspective, the tax wedge for Japan in 2023 is 32.6%, which is significantly lower than Belgium's 53.0%.[6] Also, the national burden rate in 2020 is 47.9% for Japan, which is lower than the rates for the Scandinavian countries (65.9% for Denmark, 59.7% for Finland, 54.5% for Sweden, and 53.4% for Norway).

Panel 6d shows that the model housing price increases by 51% between 1980 and 2010, but then levels off. The main source of the increase in housing prices is the gap in TFP growth, a supply-side factor. As figure 1 shows, housing production became increasingly less efficient than other sectors until 2010, when TFP growth rates converged in the 2010s. As a result, the relative price of housing rose steadily for three decades. Another factor contributing to price increases is declining fertility rates, which create housing demand through population aging. However, our model does not replicate the rapid rise and fall of housing prices during the so-called "bubble economy" following the Plaza Accord in 1985. The actual national residential land price index showed a 266% premium over the 1980 value at its peak in 1991, and then declined steadily to a 33% premium in 2010. Thus, our model replicates the long-term real price changes relatively well, although it does not incorporate overly optimistic economic forecasts or monetary easing and tightening during this period.

Panel 6e shows that per capita housing value decreases by more than 10% between the 1990s and 2010s, but recovers by 2050.[7] There are competing effects on house values, which are the product of price and quantity. On the one hand, government spending has a negative effect on house values by suppressing the demand for housing. On the other hand, declining fertility rates, which cause population aging, have positive effects by increasing housing demand.

In panel 6f, the per capita value of financial assets (the sum of liquid assets $A_t$ and financial bequests $B_t$) increases by approximately 41.7% by 2020, mainly due to government

[6] See OECD data for tax wedge statistics: `https://www.oecd.org/tax/tax-policy/taxing-wages-japan.pdf`

[7] The per capita values of housing and financial assets in this figure do not include the effect of trend growth, but the trend adjustment does not change the dynamics significantly.

debt. In equilibrium, household savings are used to finance government debt. However, as the population declines mainly due to low fertility rates, total assets begin to decline after 2015.

# VI. Counterfactual Analysis

We conduct eight counterfactual simulations by changing the parameters as shown in Table V. We obtain the counterfactual values for all of the endogenous variables but discuss only the GDP level, the consumption level, the capital level, GDP growth rates, TFP growth rates, the natural rate of interest, house prices and quantities, and the capital share of the general goods sector.

| Parameter | | Baseline | CF1 | CF2 | CF3 | CF4 | CF5 | CF6 | CF7 | CF8 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\psi$ | Importance of bequest motives | 22 | 0 | 40 | - | - | - | - | - | - |
| $\nu$ | Importance of financial bequests | 0.6 | - | - | 0.3 | 0.9 | - | - | - | - |
| $\phi$ | Proportion of reused housing | 0.46 | - | - | - | - | 0 | 1 | - | - |
| $T^a$ | Inheritance tax rate for financial assets | 0.4 | - | - | - | - | - | - | 0 | 0.4 |
| $T^h$ | Inheritance tax rate for housing | 0 | - | - | - | - | - | - | 0 | 0.4 |
| $\beta$ | Discount factor | 0.996 | - | - | - | - | - | - | - | - |
| $\chi$ | Importance of housing consumption | 0.385 | - | - | - | - | - | - | - | - |
| $\delta_g$ | Depreciation rate for Capital | 0.15 | - | - | - | - | - | - | - | - |
| $\delta_h$ | Depreciation rate for housing | 0.03 | - | - | - | - | - | - | - | - |
| $\alpha_g$ | Capital share in general production sector | 0.37 | - | - | - | - | - | - | - | - |
| $\alpha_h$ | Capital share in housing sector | 0.31 | - | - | - | - | - | - | - | - |

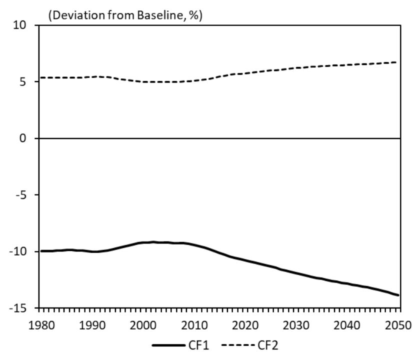
Table V: Counterfactual Parameter Values

First, we change the parameter $\psi$, which represents the importance of bequest motives. Starting with the baseline value of $\psi = 22$, we examine 0 (no motives) and 40 (stronger motives). Second, we change the parameter $\nu$, the weight on the financial bequest motive within the total bequest motives. We change the parameter from the baseline value of 0.6 to 0.3 and 0.9. Third, we change the parameter $\phi$, which represents the matching efficiency for intergenerational housing transfers, from the baseline value of 0.46 to 0 and 1. When $\phi = 0$, all bequeathed houses become vacant because the younger generation's housing demand does not match the inherited houses due to geographical and taste gaps. In contrast, when $\phi = 1$, all inherited houses are fully utilized by younger generations; there is no vacant housing in the economy. Matching efficiency can be improved by helping the elderly remodel their houses to match young households' tastes. Last, we change the inheritance tax rates. The baseline rates are 0 for housing and 0.4 for financial assets. We change them to $(0, 0)$ and $(0.4, 0.4)$ by removing preferential treatment of housing assets in the inheritance tax code.

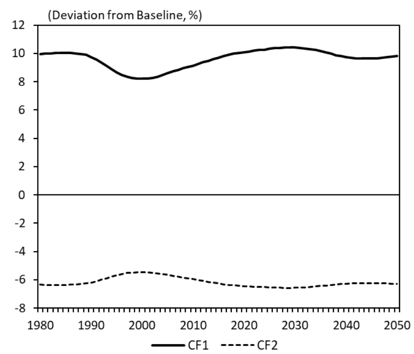
## *A. Bequest Motives*

Figure 7 shows the results when we change the importance of bequest motives. The bold line (CF1) corresponds to the case with no bequest motives, and the dotted line (CF2) corresponds to the case with greater bequest motives. The removal of bequest motives for both financial and housing assets reduces GDP by 9% to 14% (panel 7a) and the capital stock by 16% to 25% (panel 7c) because the demand for the less productive housing production is reduced. However, consumption increases by 9% to 13% (panel 7b) because a larger share of household income is spent on general consumption after each household optimizes housing consumption only for its own flow utility.

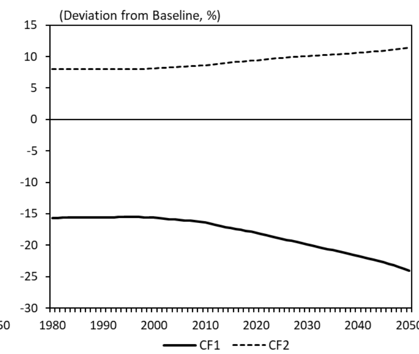
The growth rate of GDP (panel 7d) and average TFP (panel 7e) would have been $6 - 11$ percentage points higher until the 1990s if households did not have bequest motives. During this period, technological growth in the construction sector consistently lagged behind that of the general production sector. A lower demand for housing would have reduced the share of the housing production sector in the economy and thus increased the overall productivity of the economy. However, because we assume that technological progress in the two sectors will converge in the future, the effect of reduced housing production has a negligible impact



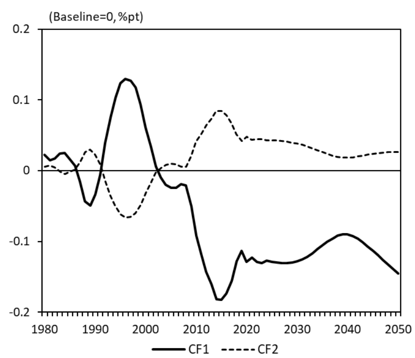

(a) GDP Level



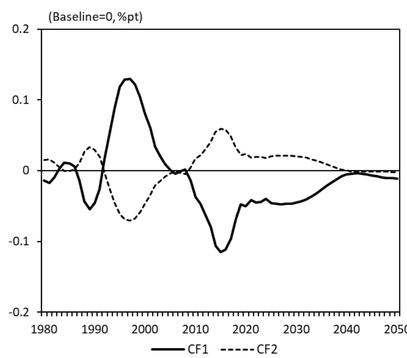

(b) Consumption Level



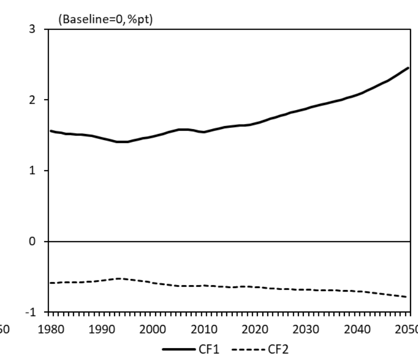

(c) Capital Level



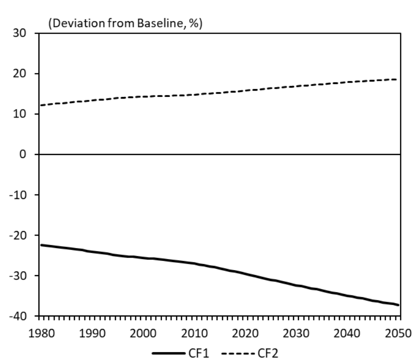

(d) GDP Growth



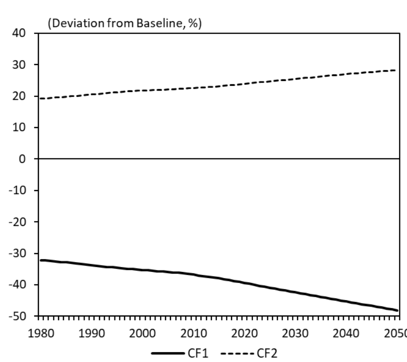

(e) TFP Growth



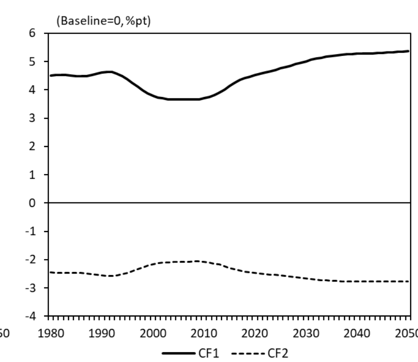

(f) Natural Rate of Interest




(g) House Prices




(h) Per Capita Housing Value




(i) Goods Sector Capital Share

Note: The figure shows deviations from the baseline dynamics for counterfactuals with $\psi = 0$ (no bequest motives, labeled CF1) and $\psi = 40$ (large bequest motives, labeled CF2). The baseline parameter for the importance of bequest motives is $\psi = 22$.

Figure 7: The Effect of Bequest Motives

on overall TFP growth after 2020. In contrast, GDP growth is depressed by the reduced demand for housing in the absence of bequest motives.

In the absence of bequest motives, the natural rate of interest is 2.6 percentage points higher than the benchmark rate in 2050 (panel 7f). Because the benchmark rate is projected to decrease from 3.6% to 1.9% over the same period, this upward adjustment will keep the natural rate of interest relatively constant. This upward shift corresponds to a higher marginal product of capital (MPK) due to a smaller scale of production and higher aggregate TFP.

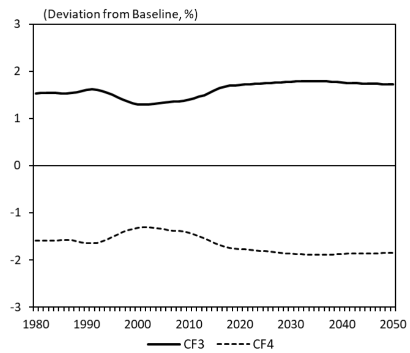
House prices are 16% to 26% lower without bequest motives (panel 7g); that is, housing becomes more affordable as bequest-driven housing demand is removed. In other words, the current equilibrium house prices are inflated by the additional demand of bequest motives. Similarly, the equilibrium level of the housing stock is 11% to 15% smaller without bequest motives (panel 7h). The reduction in both price and quantity also confirms that this change is caused by a negative shift in demand.

Furthermore, without bequest motives, 3% to 5% more capital is allocated to the general goods production sector (panel 7i). A reduction in the capital share of the housing sector makes the economy less exposed to the technological gap between the two sectors.

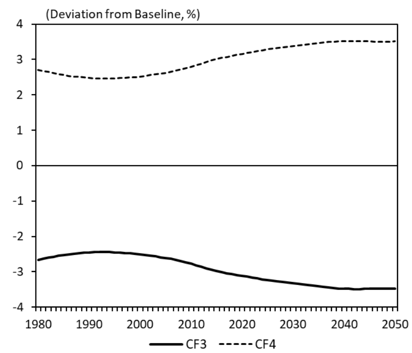
## B. *Utility Weight on Financial Bequests*

Figure 8 shows the results when we change the utility weight for financial bequests as opposed to housing bequests. The bold line (CF3) corresponds to a smaller weight on financial bequests, and the dotted line (CF4) corresponds to a larger weight.

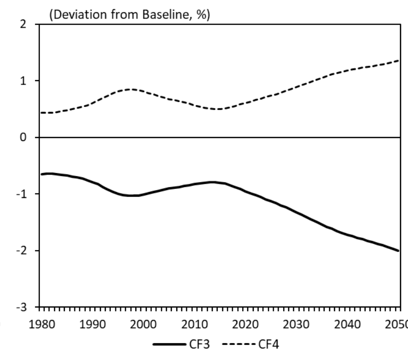
The results are qualitatively similar to those for a change in the importance of the total bequest motives. Thus, increasing the relative importance of financial bequests has a smaller but analogous effect to the case of decreasing the bequest motive itself. However, the effect on the natural rate of interest is opposite to that of changing bequest motives (panel 8f), although the magnitude is small because housing bequests and financial bequests are



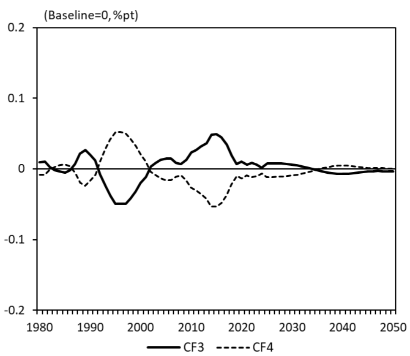

(a) GDP Level



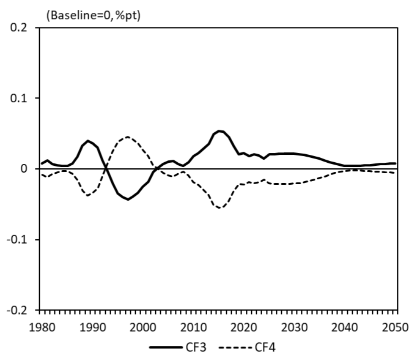

(b) Consumption Level



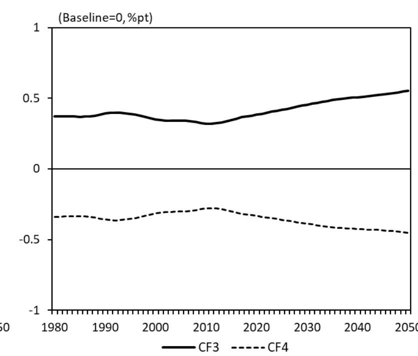

(c) Capital Level



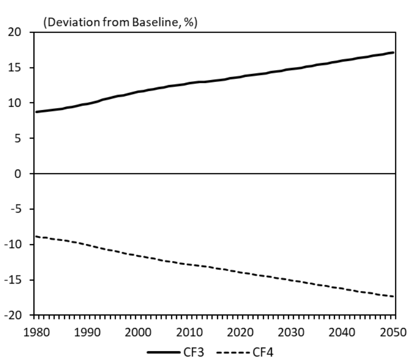

(d) GDP Growth



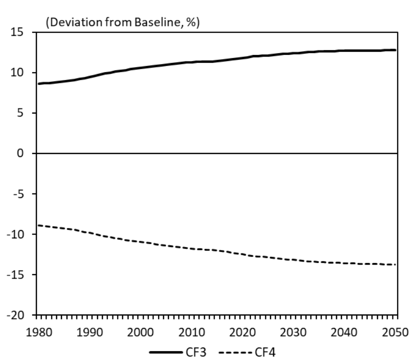

(e) TFP Growth



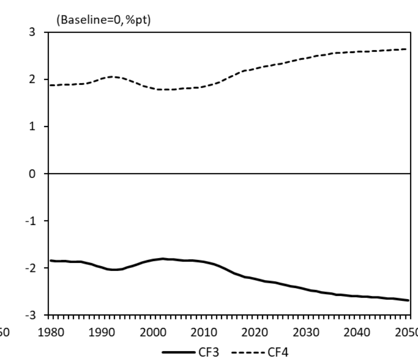

(f) Natural Rate of Interest




(g) House Prices




(h) Per Capita Housing Value




(i) Goods Sector Capital Share

Note: The figure shows deviations from the baseline dynamics for counterfactuals with $\nu = 0.3$ (a small weight on intentional financial bequests, labeled CF3) and $\nu = 0.9$ (a large weight on intentional financial bequests, labeled CF4). The baseline weight is $\nu = 0.6$.

Figure 8: The Effect of Relative Bequest Motives

substitutes in generating capital demand.

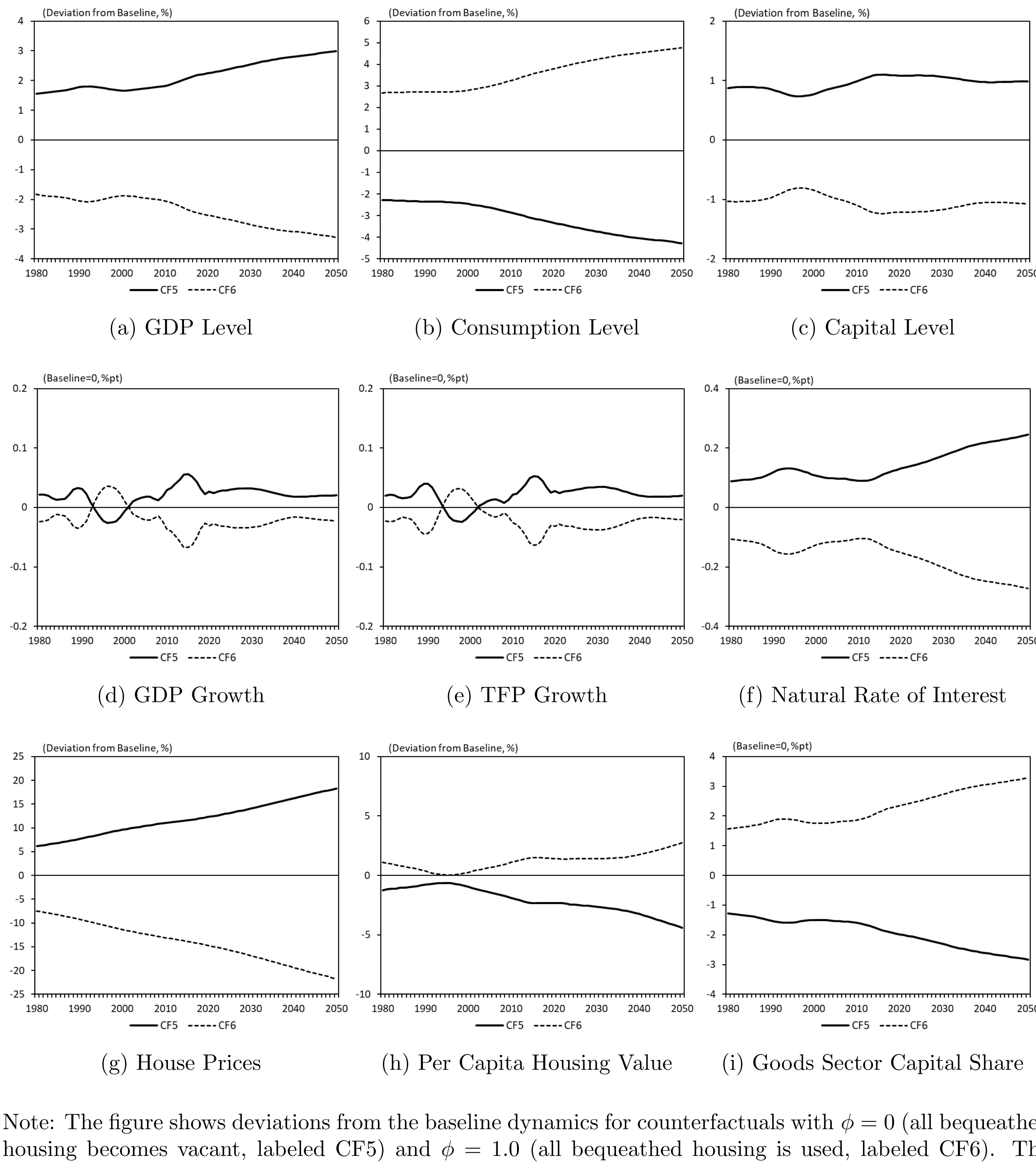
### C. *Matching Efficiency of Intergenerational Housing Transfers*

Figure 9 shows the effect of changing the house matching efficiency–the rate of successful transfers of bequeathed houses to younger generations. The bold line (CF5) corresponds to a completely inefficient transfer–all transferred houses become vacant, and the dotted line (CF6) corresponds to a completely efficient transfer–no vacancies. Efficient house matching can also be interpreted as indicating an environmentally sustainable society, in which all of the existing stock is reused. In such a sustainable economy, production activities are reduced.

Efficient house matching reduces GDP by $1.6-2.9\%$ (panel 9a) because less construction is needed. It also reduces the total capital stock employed in the production sectors (panel 9c). Thus, in a sustainable society, households can enjoy $2.4-4.7\%$ more consumption (panel 9b) without expanding the size of the economy or increasing capital intensity. Most importantly, efficient house matching makes housing affordable: house prices are $7-14\%$ lower, while more housing units are available (panels 9g and 9h). The opposite directions of price and quantity changes suggests that efficient house matching acts as a positive supply shift in the housing market; i.e., housing is supplied without using additional labor and capital. Also, a smaller presence of the less productive housing production sector would have increased GDP growth (panel 9d) and average TFP growth (panel 9e) during the period of stagnant construction productivity.

### D. *Inheritance Tax Rates*

Figure 10 shows the effect of changing inheritance tax rates. We explore two possibilities of removing the preferential treatment for housing. The bold line (CF7) corresponds to no effective inheritance taxes for both assets, and the dotted line (CF8) corresponds to the same 40% effective tax rate for both assets, possibly by removing the assessment reduction for housing.



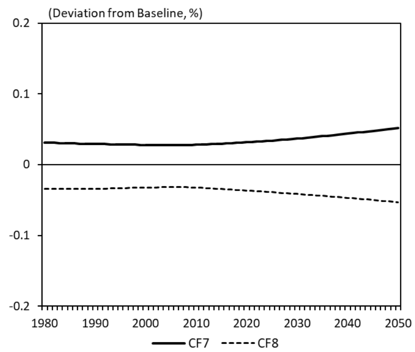

(a) GDP Level (b) Consumption Level (c) Capital Level

(d) GDP Growth (e) TFP Growth (f) Natural Rate of Interest

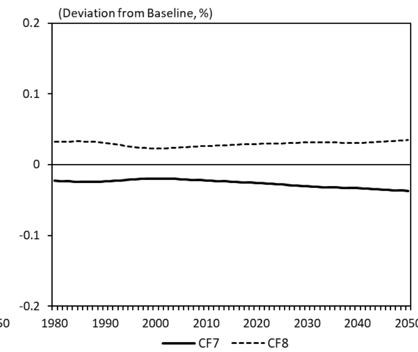
(g) House Prices (h) Per Capita Housing Value (i) Goods Sector Capital Share

Note: The figure shows deviations from the baseline dynamics for counterfactuals with $\phi = 0$ (all bequeathed housing becomes vacant, labeled CF5) and $\phi = 1.0$ (all bequeathed housing is used, labeled CF6). The baseline efficiency rate is $\phi = 0.46$.

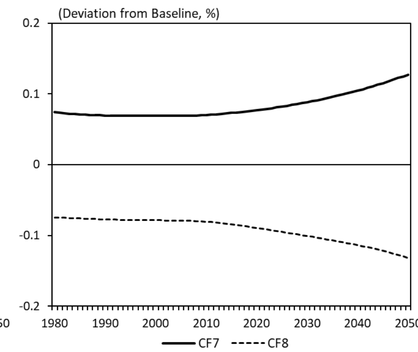
Figure 9: The Effect of Matching Efficiency



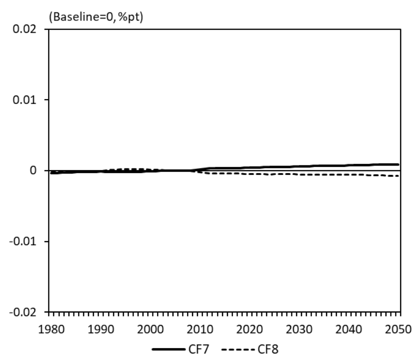

(a) GDP Level



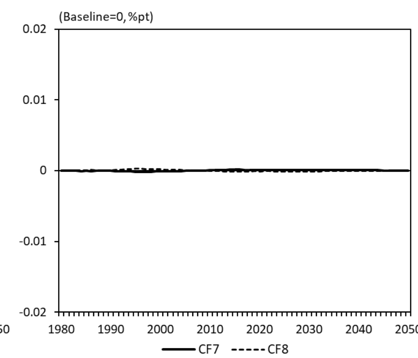

(b) Consumption Level



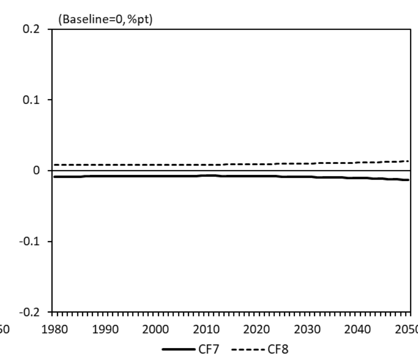

(c) Capital Level



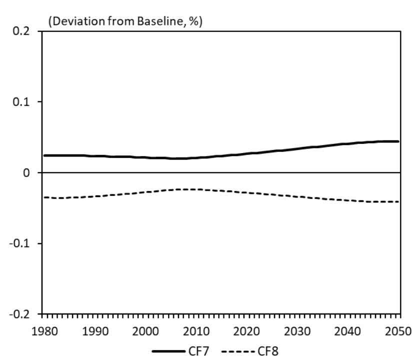

(d) GDP Growth



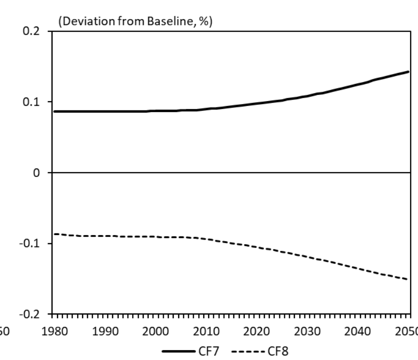

(e) TFP Growth



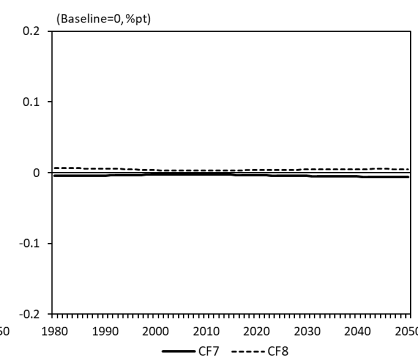

(f) Natural Rate of Interest




(g) House Prices




(h) Per Capita Housing Value




(i) Goods Sector Capital Share

Note: The figure shows deviations from the baseline dynamics for counterfactuals with $T^a = T^h = 0$ (no inheritance tax for both financial assets and housing, labeled CF7) and $T^a = T^h = 0.4$ (inheritance tax rates are 0.4 for both financial assets and housing, labeled CF8). The baseline tax rates are $T^a = 0.4, T^h = 0$.

Figure 10: The Effect of Inheritance Tax

The results suggest that the effects are negligible; for each variable, the effect is less than one percentage point. The reason is that these tax changes do not affect the household's decision. In our model, households cannot decide to forgo inheritance when tax rates are high. Thus, a change in inheritance tax rates could affect the macroeconomy only through its effect on households' after-tax wealth. For the government, however, inheritance taxes are a substitute for income taxes. As a result, the effect of a change in inheritance taxes is offset by the opposite effect of a change in income taxes on household wealth. Thus, if our model allows for households to choose whether to accept inheritances, the effect will be larger. Nevertheless, our analysis suggests that a reform of inheritance tax may have only a small effect compared with changes in household preferences.

# VII. Discussions

Bequest motives for housing create additional housing demand and increase GDP. Furthermore, capital demanded by the housing sector additionally increases general goods production. Thus, housing bequest motives increase aggregate demand. The effect of bequest motives continues to increase as society ages due to low fertility and mortality rates.

However, housing bequest motives have negative effects on the efficiency of capital usage. First, the growth rate of GDP and TFP is lower until the 2000s than in the counterfactual case of no bequest motives because the housing sector's productivity growth lagged the general goods sector's. The negative effect of bequest motives on TFP growth largely disappears after 2020 because we assume that technology growth rates converge between the two sectors. Thus, future aggregate TFP growth rates depend more on technological progress in the increasingly important housing sector. The effect of bequest motives on GDP growth is greater than the effect on TFP growth because housing demand increases due to the growing elderly population.

Second, with housing bequest motives, household assets are used less for productive capital than in the case of no-bequest motives, while housing services provide only small marginal utility. Without bequest motives, households hold more liquid assets, which are eventually invested in productive capital.

Third, greater housing production suggests the lower marginal product of capital or a lower natural rate of interest in two ways; the marginal product is diminishing with more capital employed, and the marginal product is lower in the housing sector because of lower productivity. Thus, a reduction in housing bequest motives will increase the marginal product of capital or interest rates.

Bequest motives for housing also suppress household consumption because households spend more on housing. Nevertheless, the elderly are satisfied with bequeathing housing to their heirs, despite suppressed consumption.

Bequest motives have an obvious direct effect on the housing market. They create additional demand and thus increase both price and quantity of housing. In other words, housing is less affordable for new buyers because older generations occupy their housing underutilized.

However, as long as bequeathed houses are efficiently transferred to the young generation, the crowding-out issue is minimal. One problem of housing bequest is that bequeathed houses do not perfectly match the young generation's housing demand. The young generation typically has different tastes in housing and lives in different cities from the old generation. In the calibrated baseline model, we incorporate such inefficiencies by assuming that 40% of bequeathed houses become vacant.

With such vacancies due to generational mismatch, usable housing stock does not accumulate as quickly as in the case of no wasted housing. Thus, the reduction of wasted housing works as an additional housing supply without the need for new housing production. Thus, a more efficient intergenerational transfer of housing will decrease house prices while increasing the aggregate housing quantity. As long as transfers are efficient, housing bequest motives have a small effect on aggregate housing affordability. In reality, there will be inequality between those who inherit housing and those who do not. We do not model household heterogeneity in housing assets.

Improving the matching efficiency can be achieved by improving house remodeling. Inefficiencies arise from taste and geographical gaps between generations. Thus, a better remodeling of housing at the time of intergenerational transfer will solve taste gaps. However, a geographical mismatch cannot be resolved by remodeling. To address geographical mismatch, it would be effective to facilitate multi-party exchanges beyond within-family inheritance. For example, if taxes are exempted for the transaction of an inherited house to a local buyer, housing assets will be better utilized.

The present study provides valuable insights, yet it omits several aspects of the economy. Future extensions will explore these additional aspects. First, the study only considers age heterogeneity, while omitting other forms of heterogeneity, such as income, wealth, and household size. As Mian et al. (2021) emphasize, the demand for saving by wealthy households has significant implications, particularly because age and wealth are positively correlated. Second, the study does not account for geographical heterogeneity. Even when intergenerational transfers appear to be efficient at the aggregate level, the geographical concentration of particular generations and housing assets may render transfers impracticable in reality. For instance, housing shortages for younger generations in Tokyo will not be resolved by houses in rural areas bequeathed by older generations. Third, the study does not consider the social cost of externalities due to vacant houses and the oversupply of housing that becomes vacant eventually. Fourth, the study specifies a log-linear utility function, which suggests a restrictive assumption on intra- and inter-temporal consumption substitutions. More realistic utility functions, such as the CES-CRRA utility and the Epstein-Zin recursive utility function, can suggest a more significant role for housing in the economy. Finally, it should be noted that the model is a closed economy, whereas in an open economy, flows of capital and labor are expected to increase and mitigate some of the effects.

# VIII. Conclusion

Japan is at the forefront of aging societies. Studying the macroeconomic effects of aging in Japan will provide valuable insights into the future of the global economy. This study focuses on the ownership of large houses by elderly households, some of which are wasted as vacant units after being transferred to the younger generation. Ownership of large houses by the elderly is a common phenomenon in many countries, and the housing sector tends to be less productive than other sectors. A natural prediction is that an aging society will allocate more capital to housing assets and a less productive housing sector. We quantify the impact of this prediction by calibrating a quantitative OLG model to the Japanese economy. The model with housing bequest motives explains well the age profile of housing assets.

The model shows that bequest driven demand for housing increases GDP, but decreases GDP and TFP growth when technological growth is slow in the housing sector. Households consume less and allocate less resources to productive capital. The natural rate of interest is lower because a larger scale of production reduces the marginal product of capital. Houses are less affordable because prices are inflated by the additional demand. These effects are exacerbated when inherited houses are more likely to become vacant and when the proportion of elderly people is higher. The efficiency of the Japanese economy can be increased by reducing bequest demand for housing and improving the efficiency of housing allocation.

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# Appendix A Household First-Order Conditions

A representative $j$-aged household at time $t$ maximizes its utility with respect to $c_{j,t}$, $a_{j,t}$, $bi_{j,t}$, $h_{j,t}$ and $hi_{j,t}$. The Lagrangian for $j = 1, ..., J$ is:

$$\begin{aligned}
\mathcal{L}_{j,t} = & \sum_{i=0}^{J-j} \beta^i \Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right) \times \\
& \{(\ln c_{j+i,t+i} + \chi \ln h_{j+i,t+i}) + \omega_{j+i,t+i} \psi \left[\nu \ln bi_{j+i,t+i} + (1-\nu) \ln h_{j+i,t+i}\right] \\
& + \lambda_{j+i,t+i} \left[c_{j+i,t+i} + a_{j+i,t+i} + bi_{j+i,t+i} + P^h_{t+i} \{hi_{j+i,t+i} + (1 - \mathbf{1}_{j+i \geq JR}) T^h_{t+i} i^h_{t+i}\}\right. \\
& - (1 + r_{t+i}) a_{j+i-1,t+i-1} - \mathbf{1}_{j+i \geq JR} pen_{t+i} \\
& \left. - (1 - \mathbf{1}_{j+i \geq JR}) \left[(1 - T_{t+i}) \{\sigma^g_{j+i} w^g_{t+i} z^g_{j+i} + \left(1 - \sigma^g_{j+i}\right) w^h_{t+i} z^h_{j+i}\} + \left(1 - T^a_{t+i}\right) i_{t+i}\right]\right] \\
& + \lambda^h_{j+i,t+i} \left[h_{j+i,t+i} - (1 - \delta_h) h_{j+i-1,t+i-1} - hi_{j+i,t+i} - \phi (1 - \mathbf{1}_{j+i \geq JR}) i^h_{t+i}\right] \} .
\end{aligned}$$

FOCs with respect to $c_{j+i,t+i}$, $a_{j+i,t+i}$, $bi_{j+i,t+i}$, $h_{j+i,t+i}$ and $hi_{j+i,t+i}$ are respectively

$$\beta^i \Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right) \left(\frac{1}{c_{j+i,t+i}} + \lambda_{j+i,t+i}\right) = 0$$

$$\iff \quad \lambda_{j+i,t+i} = -\frac{1}{c_{j+i,t+i}} \tag{A.1}$$

$$\beta^i \Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right) \lambda_{j+i,t+i} - \beta^{i+1} \Pi_{k=0}^{i} \left(1 - \omega_{j+k,t+k}\right) \lambda_{j+i+1,t+i+1} \left(1 + r_{t+i+1}\right) = 0$$

$$\iff \quad \lambda_{j+i,t+i} = \beta \left(1 - \omega_{j+i,t+i}\right) \lambda_{j+i+1,t+i+1} \left(1 + r_{t+i+1}\right) \tag{A.2}$$

$$\beta^i \Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right) \omega_{j+i,t+i} \frac{\psi \nu}{bi_{j+i,t+i}} + \beta^i \Pi_{k=0}^{i-1} \left(1 - \omega_{j+k,t+k}\right) \lambda_{j+i,t+i} = 0$$

$$\iff \quad \omega_{j+i,t+i} \frac{\psi \nu}{bi_{j+i,t+i}} = -\lambda_{j+i,t+i} \tag{A.3}$$

$$\beta^i \Pi_{k=0}^{i-1}\left(1-\omega_{j+k,t+k}\right)\omega_{j+i,t+i}\frac{\psi\left(1-\nu\right)}{h_{j+i,t+i}}+\beta^i \Pi_{k=0}^{i-1}\left(1-\omega_{j+k,t+k}\right)\left(\frac{\chi}{h_{j+i,t+i}}+\lambda^h_{j+i,t+i}\right)$$
$$-\beta^{i+1}\Pi_{k=0}^{i}\left(1-\omega_{j+k,t+k}\right)\lambda^h_{j+i+1,t+i+1}\left(1-\delta_h\right)=0$$

$$\iff \quad \left[\omega_{j+i,t+i}\psi\left(1-\nu\right)+\chi\right]\frac{1}{h_{j+i,t+i}}$$
$$+\lambda^h_{j+i,t+i}-\beta\left(1-\omega_{j+i,t+i}\right)\lambda^h_{j+i+1,t+i+1}\left(1-\delta_h\right)=0 \tag{A.4}$$

$$\beta^i \Pi_{k=0}^{i-1}\left(1-\omega_{j+k,t+k}\right)\left(\lambda_{j+i,t+i}P^h_{t+i}-\lambda^h_{j+i,t+i}\right)=0$$

$$\iff \quad \lambda_{j+i,t+i}P^h_{t+i}=\lambda^h_{j+i,t+i} \tag{A.5}$$

From A.1 and A.2, we have

$$c_{j+1,t+1}=\beta\left(1+r_{t+1}\right)\left(1-\omega_{j,t}\right)c_{j,t} \quad for\ j=1,...,J-1 \tag{A.6}$$

From A.1, A.2, A.4 and A.5, we have

$$\left[\omega_{j,t}\psi\left(1-\nu\right)+\chi\right]c_{j,t}=\left[P^h_t-\frac{\left(1-\delta_h\right)}{\left(1+r_{t+1}\right)}P^h_{t+1}\right]h_{j,t}$$
$$for\ j=1,...,J-1 \tag{A.7}$$

$$\left[\psi\left(1-\nu\right)+\chi\right]c_{J,t}=\left[P^h_t-\frac{\left(1-\delta_h\right)}{\left(1+r_{t+1}\right)}P^h_{t+1}\right]h_{J,t} \tag{A.8}$$

From A.1 and A.3, we have

$$bi_{j,t}=\omega_{j,t}\psi\nu c_{j,t} \quad for\ j=1,...,J-1 \tag{A.9}$$

$$bi_{J,t}=\psi\nu c_{J,t} \tag{A.10}$$

# Appendix B List of Model Equations

This section presents the full set of model equilibrium conditions that jointly determine the evolution of real allocations and prices. Endogenous variables are $\{N_{j,t},\ c_{j,t},\ a_{j,t},\ b_{j,t},\ bi_{j,t},\ h_{j,t},\ hi_{j,t},\ v_{j,t}\}_{j=1}^{J}$, $a_{0,t}$, $b_{0,t}$, $h_{0,t}$, $N_t$, $n_t$, $C_t$, $A_t$, $B_t$, $H_t$, $V_t$, $w_t^g$, $w_t^h$, $P_t^h$, $r_t$, $T_t$, $pen_t$, $i_t$, $i_t^h$, $K_t$, $K_t^g$, $K_t^h$, $L_t^g$, $L_t^h$, $Y_t^g$, $Y_t^h$, which are 705 variables. Note that $\omega_{J,t} = 1$.

## A Household

Budget Constraint:

$$\begin{aligned} c_{j,t} + a_{j,t} + bi_{j,t} + P_t^h hi_{j,t} &= (1 + r_t)\, a_{j-1,t-1} + \mathbf{1}_{j \geq JR} pen_t \\ &+ (1 - \mathbf{1}_{j \geq JR}) \left[ (1 - T_t) \left\{ \sigma_j^g w_t^g \xi_j^g + \left(1 - \sigma_j^g\right) w_t^h \xi_j^h \right\} + (1 - T^a)\, i_t - T^h P_t^h i_t^h \right] \\ &\qquad for\ j = 1, ..., J. \end{aligned} \tag{B.1}$$

Bequeathed Assets Accumulation:

$$b_{j,t} = (1 + r_t)\, b_{j-1,t-1} + bi_{j,t} \quad for\ j = 1, ..., J. \tag{B.2}$$

Housing Accumulation:

$$h_{j,t} = (1 - \delta_h)\, h_{j-1,t-1} + hi_{j,t} + \phi\, (1 - \mathbf{1}_{j \geq JR})\, i_t^h \quad for\ j = 1, ..., J. \tag{B.3}$$

Underutilized Housing Accumulation:

$$v_{j,t} = (1 - \delta_h)\, v_{j-1,t-1} + (1 - \phi)\, (1 - \mathbf{1}_{j \geq JR})\, i_t^h \quad for\ j = 1, ..., J. \tag{B.4}$$

Initial Holdings of Assets and Housing:

$$b_{0,t} = a_{0,t} = h_{0,t} = v_{0,t} = 0. \tag{B.5}$$

Terminal Asset Holdings:

$$a_{J,t} = 0. \tag{B.6}$$

FOC with respect to consumption:

$$c_{j+1,t+1} = \beta\left(1 + r_{t+1}\right)\left(1 - \omega_{j,t}\right)c_{j,t} \quad for\ j = 1, ..., J - 1. \tag{B.7}$$

FOC with respect to housing:

$$\left[\omega_{j,t}\psi\left(1 - \nu\right) + \chi\right]c_{j,t} = \left[P_t^h - \frac{\left(1 - \delta_h\right)}{\left(1 + r_{t+1}\right)}P_{t+1}^h\right]h_{j,t} \quad for\ j = 1, ..., J. \tag{B.8}$$

FOC with respect to intentional bequest:

$$bi_{j,t} = \omega_{j,t}\psi\nu c_{j,t} \quad for\ j = 1, ..., J. \tag{B.9}$$

## *B Demography*

Survival Rate:

$$\frac{N_{j,t}}{N_{j-1,t-1}} = \left(1 - \omega_{j-1,t-1}\right) \quad for\ j = 2, ..., J. \tag{B.10}$$

Growth Rate of Initial Young:

$$n_{1,t} = \frac{N_{1,t}}{N_{1,t-1}} - 1. \tag{B.11}$$

Total Population:

$$N_t = \sum_{j=1}^{J} N_{j,t}. \tag{B.12}$$

Population Growth:

$$n_t = \frac{N_t}{N_{t-1}} - 1. \tag{B.13}$$

## C Aggregation over households

Aggregate Consumption:

$$C_t = \sum_{j=1}^{J} N_{j,t} c_{j,t}. \tag{B.14}$$

Financial Assets:

$$A_t = \sum_{j=1}^{J} N_{j,t} a_{j,t}. \tag{B.15}$$

Bequest Assets:

$$B_t = \sum_{j=1}^{J} N_{j,t} b_{j,t}. \tag{B.16}$$

Housing Stock:

$$H_t = \sum_{j=}^{J} N_{j,t} h_{j,t}. \tag{B.17}$$

Underutilized Housing Stock:

$$V_t = \sum_{j=1}^{J} N_{j,t} v_{j,t}. \tag{B.18}$$

Financial Bequest:

$$i_t = (1 + r_t) \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t}) (a_{j,t-1} + b_{j,t-1})}{\sum_{j=1}^{JR} N_{j,t}}. \tag{B.19}$$

Housing Bequest:

$$i_t^h = (1 - \delta_h) \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t}) h_{j,t-1}}{\sum_{j=1}^{JR} N_{j,t}}. \tag{B.20}$$

## D Firms

Labor Demand by the Goods Sector:

$$w_t^g = (1 - \alpha_g) \zeta_t^g \left( \frac{K_t^g}{\zeta_t^g L_t^g} \right)^{\alpha_g}. \tag{B.21}$$

Labor Demand by the Housing Sector:

$$w_t^h = (1-\alpha_h)\,\zeta_t^h \left(\frac{K_t^h}{\zeta_t^h L_t^h}\right)^{\alpha_h} P_t^h. \tag{B.22}$$

Capital Demand:

$$r_t + \delta_k = \alpha_g \left(\frac{K_t^g}{\zeta_t^g L_t^g}\right)^{\alpha_g - 1} = \alpha_h \left(\frac{K_t^h}{\zeta_t^h L_t^h}\right)^{\alpha_h - 1} P_t^h. \tag{B.23}$$

General Goods Production:

$$Y_t^g = (K_t^g)^{\alpha_g} \left(\zeta_t^g L_t^g\right)^{1-\alpha_g}. \tag{B.24}$$

Housing Production:

$$Y_t^h = \left(K_t^h\right)^{\alpha_h} \left(\zeta_t^h L_t^h\right)^{1-\alpha_h}. \tag{B.25}$$

## *E Government*

Government Budget Constraint:

$$\begin{aligned} &\sum_{j=1}^{JR-1} T_t N_{j,t} \left[\sigma_j^g w_t^g \xi_j^g + \left(1-\sigma_j^g\right) w_t^h \xi_j^h\right] + \left(T^h P_t^h i_t^h + T^a i_t\right) \sum_{j=1}^{JR-1} N_{j,t} + D_t \\ &\qquad = G_t + pen_t \sum_{j=JR}^{J} N_{j,t} + (1+r_t)\, D_{t-1}. \end{aligned} \tag{B.26}$$

Pension Determination:

$$pen_t = \rho_t \frac{w_t^g \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g + w_t^h \sum_{j=1}^{JR-1} \left(1-\sigma_j^g\right) N_{j,t} \xi_j^h}{\sum_{j=1}^{JR-1} N_{j,t}}. \tag{B.27}$$

## *F Market Clearing*

Capital Market:

$$D_t + K_t = A_t + B_t, K_{t-1} = K_t^g + K_t^h. \tag{B.28}$$

Labor Market in the Goods Sector:

$$L_t^g = \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g. \tag{B.29}$$

Labor Market in the Housing Sector:

$$L_t^h = \sum_{j=1}^{JR-1} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h. \tag{B.30}$$

New Housing Market:

$$\sum_{j=1}^{J} N_{j,t} h i_{j,t} = Y_t^h. \tag{B.31}$$

General Goods Market:

$$Y_t^g = C_t + K_t - (1 - \delta_k) K_{t-1} + G_t. \tag{B.32}$$

# Appendix C Data Sources

This section summarizes the data source. Table A1 shows the exogenous variables and their data sources. The exogenous variables other than the fertility rate are filtered by the Hodric-Prscott filter with a smoothing parameter of 6.25.

| Description | Period | Sources |
|---|---|---|
| Fertility rate | 1925 to 2019 | Population Projections (Actual) |
| | 2020 to 2070 | Future Population Projections (2020 Projections) |
| | 2071 to 2100 | As of 2070 |
| Mortality rate | 1925 to 1970 | As of 1971 |
| | 1971 to 2019 | Life Table (Actual) |
| | 2020 to 2070 | Future Life Table (2020 Projections) |
| | 2071 to 2100 | As of 2070 |
| (HALE-adjusting) | 1925 to 2100 | Comprehensive Survey of Living Conditions (2019) |
| Labor-augumenting technology | 1925 to 1971 | As of 1972 |
| | 1971 to 2019 | JIP database (Actual) |
| | 2020 to 2100 | Assumed to be Zero |
| Technology gap | 1925 to 1969 | Assumed to be Zero |
| | 1970 to 2019 | JIP database (Actual) |
| | 2020 to 2100 | As of 2019 |
| Retirement indicator | 1925 to 2100 | 1 (Older than 65) |
| Replacement rate | 1925 to 2100 | Assumed to be 0.4 |
| Age-specific proportion for sectors | 1925 to 2100 | Basic Survey on Wage Structure (2019) |
| Age-specific labor productivity | 1925 to 2100 | Basic Survey on Wage Structure (2019) |
| Government spending ratio | 1925 to 1979 | As of 1980 |
| | 1980 to 2019 | JSNA (Actual) |
| | 2020 to 2100 | As of 2019 |
| Government debt ratio | 1925 to 1979 | As of 1980 |
| | 1980 to 2019 | JSNA (Actual) |
| | 2020 to 2100 | As of 2019 |

Table A1: List of Exogenous Variables

The HALE-Based mortality rate is calculated as follows:

$$HALE\,Based\,Motarity\,Rate_{j,t} = \frac{\kappa_{j+1}}{\kappa_j}\omega_{j,t}, \tag{C.1}$$

where $\kappa_j$ is the percentage of persons whose disability affects their daily life in the age-$j$ cohort.

The labor-augmenting technology in our Cobb-Douglas production function corresponds to productivity per worker, unlike productivity per quality-adjusted hour worked in the JIP database. Thus, we calculate the growth rate of labor-augmenting technology as follows:

$$\Delta Labor\, Augmenting\, Technology =$$
$$[1 + \Delta TFP + (1-\alpha)\,(\Delta Labor\, Quality + \Delta Man\, hour - \Delta Number\, of\, Worker)]^{1/(1-\alpha)} - 1, \tag{C.2}$$

where $\Delta$ indicates the annual rate of change for each period.

In our model calibration, we match the ratio of housing value to consumption. Housing wealth in our model conceptually includes both rental and owneroccupied housing (more precisely, there is no distinction between owners and renters). However, the measured gross housing wealth in the National Household Income and Expenditure Structure Survey is based only on owner-occupied housing. Thus, we adjust the per capita housing value in the data by multiplying a factor of 1.58, the inverse proportion of owner-occupied housing from the National Housing and Land Survey (2018).

The per capita value of each variable is calculated as follows to correct for the number of household members:

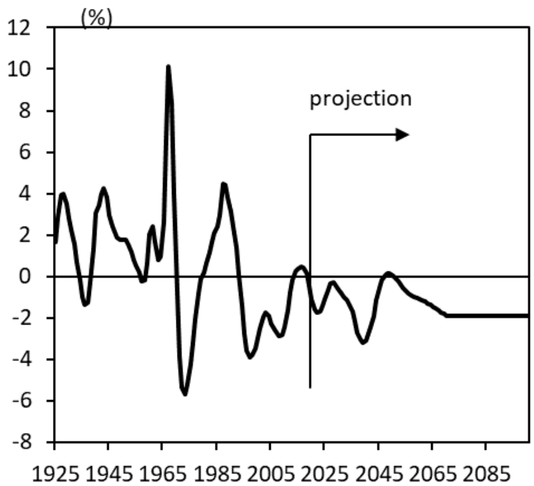
$$Value\ per\ capita = \frac{Value\ per\ household}{(Number\ of\ people\ per\ household)^{1/2}}. \tag{C.3}$$

| Valuable | Description | Source |
|---|---|---|
| $P^h h$ | Current residence/place of residence | National Household Income and Expenditure |
| $c$ | Consumer spending | Structure Survey (2019) |
| $(a+b)$ | Net financial assets (savings - debt) | |
| $v$ | Other than current residence/place of residence | |

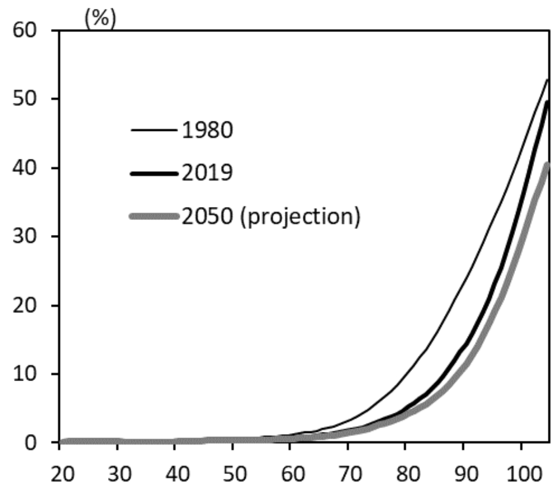
Table A2: List of Calibration Target

# Appendix D  Exogenous Variables

This section contains the graphs for each of the exogenous variables. The specification methodology is elaborated in IV.A.

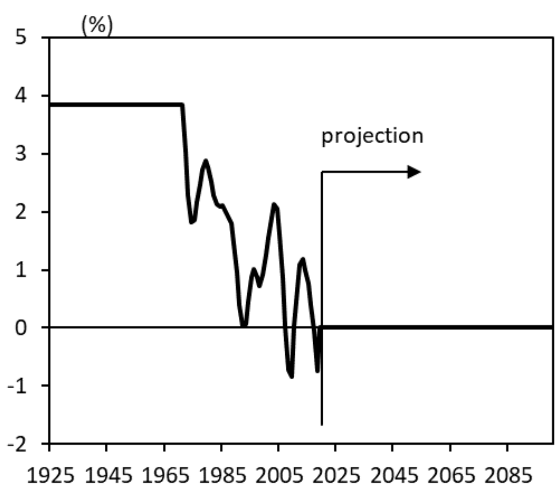



(a) Fertility Rate (Growth Rate of 20-year-olds)

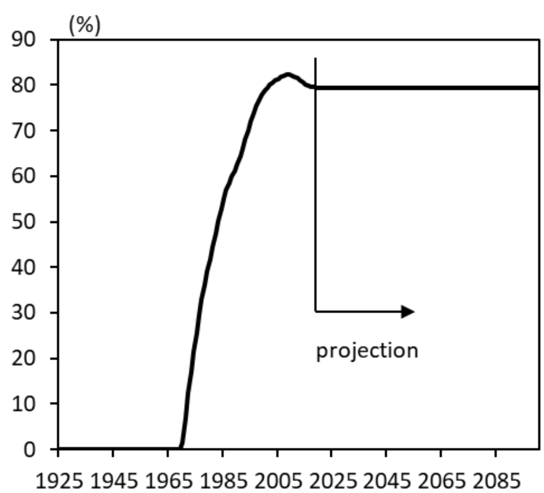



(b) Mortality Rate (over the Next 1-Year)

Figure A1: Demographics

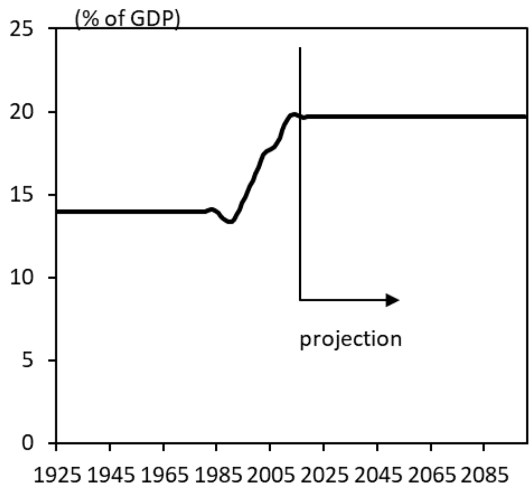



(a) Technology Growth in General Goods Sector

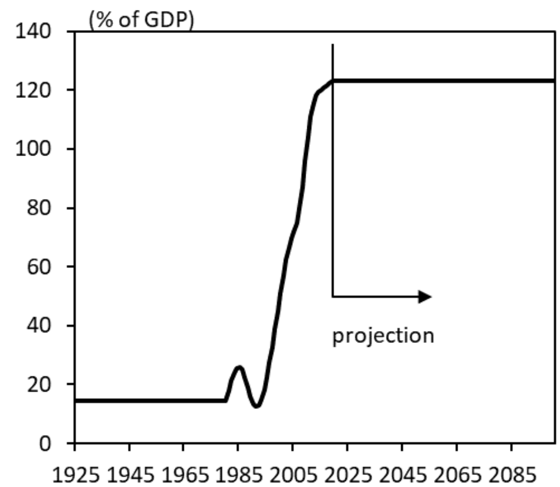



(b) Technology Gap Between the Housing and General Goods Sectors

Figure A2: Technology

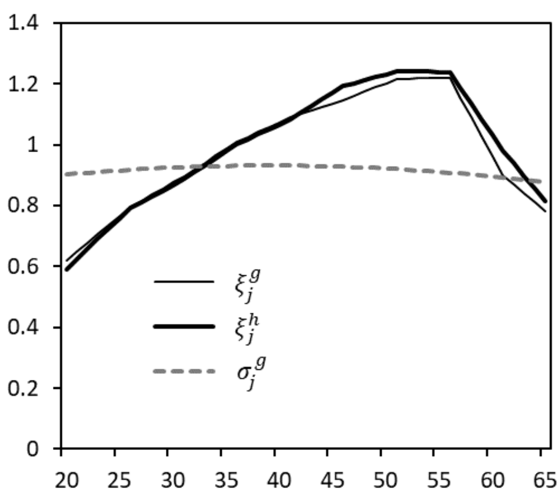



(a) Government Spending

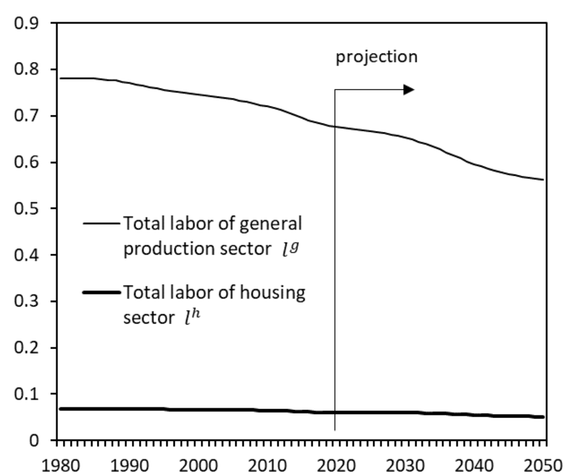



(b) Government Debt

Figure A3: Fiscal Condition




(a) Labor Properties




(b) Total Labor

Figure A4: Labor

# Appendix E Solution Method

We follow Bielecki et al. (2020) to solve our model. We first detrend the model (Appendix F) and estimate the steady state value for the initial (1925) and terminal (2095) states. Although we use the scenarios of our demographic and other exogenous forces from 1925 to 2095 (consisting of artificial data, past data and projections), our focus is on developments 1980 to 2050.

A steady state is defined as a stationary equilibrium where the exogenous demographic variables assume a fixed value forever. A steady state value is calculated by the following algorithm.

1. We set the initial value for the following aggregate variables.

| | $\frac{\hat{K}^g}{L^g}$ | $\frac{\hat{K}^h}{L^h}$ | $\hat{i}$ | $\hat{i}^h$ | $\hat{pen}$ |
|---|---|---|---|---|---|
| Initial values | 3 | 1.5 | 2.0 | 4.0 | 0.2 |

Table A3: Intial Value of the Algorithm

2. Calculate the factor prices $\hat{w}^g$, $\hat{w}^h$, $P^h$, $r$ and $T$.

3. Calculate the sum of lifetime discount factors and calculate the NPV of lifetime labor income and pension.

4. Calculate the initial consumption as in Appendix G and derive a sequence of other variables $\left\{N_j\,,\,\hat{c}_j,\,\hat{a}_j,\,\hat{bi}_j,\,\hat{h}_j,\,\hat{hi}_j\right\}_{j=1}^{J}$.

5. Evaluate the convergence of $\frac{\hat{K}^g}{L^g}$, $\frac{\hat{K}^h}{L^h}$, $\hat{i}$, $\hat{i}^h$ and $\hat{pen}$. If the convergence criteria are not met, guess the new values and repeat the process.

After deriving the initial and terminal states, we perform our deterministic simulation (with perfect foresight) for the entire period using MATLAB and Dynare.

# Appendix F  Detrending

We detrend the model by deflating all variables by the TFP $\zeta_t^g$. We define $\hat{x}_t \equiv x_t/\zeta_t^g$ for $x = \left\{c_j, a_j, bi_j, b_j, hi_j, h_j, v_j, C, A, B, H, V, w^g, w^h, pen, i, i^h, K, K^g, K^h, Y^g, Y^h\right\}$. The model equations listed in Appendix B are rewritten as follows.

$$\hat{c}_{j,t} + \left(1+g_{t+1}^g\right)\hat{a}_{j,t} + \hat{bi}_{j,t} + P_t^h\hat{hi}_{j,t} = \left(1+r_t\right)\hat{a}_{j-1,t-1} + \mathbf{1}_{j\geq JR}\hat{pen}_t$$
$$+\left(1-\mathbf{1}_{j\geq JR}\right)\left[\left(1-T_t\right)\left\{\sigma_j^g\hat{w}_t^g\xi_j^g + \left(1-\sigma_j^g\right)\hat{w}_t^h\xi_j^h\right\} + \left(1-T^a\right)\hat{i}_t - T^hP_t^h\hat{i}_t^h\right]$$
$$for\ j = 1, ..., J, \tag{F.1}$$

$$\left(1+g_{t+1}^g\right)\hat{b}_{j,t} = \left(1+r_t\right)\hat{b}_{j-1,t-1} + \hat{bi}_{j,t} \quad for\ j = 1, ..., J, \tag{F.2}$$

$$\left(1+g_{t+1}^g\right)\hat{h}_{j,t} = \left(1-\delta_h\right)\hat{h}_{j-1,t-1} + \hat{hi}_{j,t} + \phi\left(1-\mathbf{1}_{j\geq JR}\right)\hat{i}_t^h \quad for\ j = 1, ..., J, \tag{F.3}$$

$$\left(1+g_{t+1}^g\right)\hat{v}_{j,t} = \left(1-\delta_h\right)\hat{v}_{j-1,t-1} + \left(1-\phi\right)\left(1-\mathbf{1}_{j\geq JR}\right)\hat{i}_t^h \quad for\ j = 1, ..., J, \tag{F.4}$$

$$\hat{b}_{0,t} = \hat{a}_{0,t} = \hat{h}_{0,t} = \hat{v}_{0,t} = 0, \tag{F.5}$$

$$\hat{a}_{J,t} = 0, \tag{F.6}$$

$$\left(1+g_{t+1}^g\right)\hat{c}_{j+1,t+1} = \beta\left(1+r_{t+1}\right)\left(1-\omega_{j,t}\right)\hat{c}_{j,t} \quad for\ j = 1, ..., J-1, \tag{F.7}$$

$$\left[\omega_{j,t}\psi\left(1-\nu\right)+\chi\right]\hat{c}_{j,t} = \left[P_t^h - \frac{\left(1-\delta_h\right)}{\left(1+r_{t+1}\right)}P_{t+1}^h\right]\left(1+g_{t+1}^g\right)\hat{h}_{j,t} \quad for\ j = 1, ..., J-1, \tag{F.8}$$

$$\left[\psi\left(1-\nu\right)+\chi\right]\hat{c}_{J,t} = \left[P_t^h - \frac{\left(1-\delta_h\right)}{\left(1+r_{t+1}\right)}P_{t+1}^h\right]\left(1+g_{t+1}^g\right)\hat{h}_{J,t}, \tag{F.9}$$

$$\hat{bi}_{j,t} = \omega_{j,t}\psi\nu\hat{c}_{j,t} \quad for\ j = 1, ..., J-1, \tag{F.10}$$

$$\hat{bi}_{J,t} = \psi\nu\hat{c}_{J,t}, \tag{F.11}$$

$$\frac{N_{j,t}}{N_{j-1,t-1}} = \left(1-\omega_{j-1,t-1}\right) \quad for\ j = 2, ..., J, \tag{F.12}$$

$$n_{1,t} = \frac{N_{1,t}}{N_{1,t-1}} - 1, \tag{F.13}$$

$$N_t = \sum_{j=1}^{J} N_{j,t}, \tag{F.14}$$

$$n_t = \frac{N_t}{N_{t-1}} - 1, \tag{F.15}$$

$$\hat{C}_t = \sum_{j=1}^{J} N_{j,t}\hat{c}_{j,t}, \tag{F.16}$$

$$\hat{A}_t = \sum_{j=1}^{J} N_{j,t}\hat{a}_{j,t}, \tag{F.17}$$

$$\hat{B}_t = \sum_{j=1}^{J} N_{j,t}\hat{b}_{j,t}, \tag{F.18}$$

$$\hat{H}_t = \sum_{j=1}^{J} N_{j,t}\hat{h}_{j,t}, \tag{F.19}$$

$$\hat{V}_t = \sum_{j=1}^{J} N_{j,t}\hat{v}_{j,t}, \tag{F.20}$$

$$\hat{i}_t = (1 + r_t)\frac{\sum_{j=1}^{J}\left(N_{j,t-1} - N_{j+1,t}\right)\left(\hat{a}_{j,t-1} + \hat{b}_{j,t-1}\right)}{\sum_{j=1}^{JR} N_{j,t}}, \tag{F.21}$$

$$\hat{i}_t^h = (1 - \delta_h)\frac{\sum_{j=1}^{J}\left(N_{j,t-1} - N_{j+1,t}\right)\hat{h}_{j,t-1}}{\sum_{j=1}^{JR} N_{j,t}}, \tag{F.22}$$

$$\hat{w}_t^g = (1 - \alpha_g)\left(\frac{\hat{K}_t^g}{L_t^g}\right)^{\alpha_g}, \tag{F.23}$$

$$\hat{w}_t^h = (1 - \alpha_h)(1 - \theta_t)\left(\frac{\hat{K}_t^h}{(1 - \theta_t) L_t^h}\right)^{\alpha_h} P_t^h, \tag{F.24}$$

$$r_t + \delta_k = \alpha_g\left(\frac{\hat{K}_t^g}{L_t^g}\right)^{\alpha_g - 1} = \alpha_h\left(\frac{\hat{K}_t^h}{(1 - \theta_t) L_t^h}\right)^{\alpha_h - 1} P_t^h, \tag{F.25}$$

$$\hat{Y}_t^g = \left(\hat{K}_t^g\right)^{\alpha_g}\left(L_t^g\right)^{1-\alpha_g}, \tag{F.26}$$

$$\hat{Y}_t^h = \left(\hat{K}_t^h\right)^{\alpha_h}\left((1 - \theta_t) L_t^h\right)^{1-\alpha_h}, \tag{F.27}$$

$$\sum_{j=1}^{JR-1} T_t N_{j,t}\left[\sigma_j^g \hat{w}_t^g \xi_j^g + \left(1 - \sigma_j^g\right)\hat{w}_t^h \xi_j^h\right] + \left(T^h P_t^h \hat{i}_t^h + T^a \hat{i}_t\right)\sum_{j=1}^{JR-1} N_{j,t} + \left(1 + g_{t+1}^g\right) D_t$$

$$= G_t + p\hat{e}n_t \sum_{j=JR}^{J} N_{j,t} + (1 + r_t) D_{t-1}, \tag{F.28}$$

$$p\hat{e}n_t = \rho_t \frac{\sum_{j=1}^{JR-1} N_{j,t}\left[\sigma_j^g \hat{w}_t^g \xi_j^g + \left(1 - \sigma_j^g\right)\hat{w}_t^h \xi_j^h\right]}{\sum_{j=1}^{JR-1} N_{j,t}}, \tag{F.29}$$

$$D_t + \hat{K}_t = \hat{A}_t + \hat{B}_t, \hat{K}_{t-1} = \hat{K}_t^g + \hat{K}_t^h, \tag{F.30}$$

$$L_t^g = \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g, \tag{F.31}$$

$$L_t^h = \sum_{j=1}^{JR-1} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h, \tag{F.32}$$

$$\sum_{j=1}^{J} N_{j,t} \hat{h} i_{j,t} = \hat{Y}_t^h, \tag{F.33}$$

$$\hat{Y}_t^g = \hat{C}_t + \left(1 + g_{t+1}^g\right) \hat{K}_t - (1 - \delta_k) \hat{K}_{t-1} + G_t. \tag{F.34}$$

# Appendix G Initial Value Calculation

Set $\hat{w}_j = (1-\mathbf{1}_{j\geq JR})\left[(1-T)\left\{\sigma_j^g \hat{w}^g z_j^g + \left(1-\sigma_j^g\right)\hat{w}^h z_j^h\right\} + (1-T^a)\,\hat{i} - P^h T^h \hat{i}^h\right]$
$+\mathbf{1}_{j\geq JR} p\hat{e}n$ and $1+rg = \frac{1+r}{1+g^g}$. Then, the household budget constraint can be rewritten as:

$$\hat{c}_J + (1+g^g)\,\hat{a}_J + \hat{bi}_J + P^h \hat{hi}_J = \hat{w}_J + (1+r)\,\hat{a}_{J-1}$$

$$\begin{aligned}\iff\quad & \hat{c}_J + \hat{bi}_J + P^h\left\{(1+g^g)\,\hat{h}_J - (1-\delta_h)\,\hat{h}_{J-1} - \phi\,(1-\mathbf{1}_{J\geq JR})\,\hat{i}^h\right\}\\ =&\hat{w}_J + (1+rg)\left\{-\hat{c}_{J-1} - \hat{bi}_{J-1} - P^h\hat{hi}_{J-1} + \hat{w}_{J-1} + (1+r)\,\hat{a}_{J-2}\right\}\end{aligned}$$

$$\begin{aligned}\iff\quad & \hat{c}_J + (1+rg)\,\hat{c}_{J-1} + \hat{bi}_J + (1+rg)\,\hat{bi}_{J-1} + P^h\,(1+g^g)\,\hat{h}_J - P^h\,(1-\delta_h)\,\hat{h}_{J-1}\\ & + (1+rg)\,P^h\left\{(1+g^g)\,\hat{h}_{J-1} - (1-\delta_h)\,\hat{h}_{J-2} - \phi\,(1-\mathbf{1}_{J-1\geq JR})\,\hat{i}^h\right\}\\ =&\hat{w}_J + (1+rg)\,\hat{w}_{J-1} + P^h\phi\hat{i}^h\,(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1+r)\,a_{J-2}\end{aligned}$$

$$\begin{aligned}\iff\quad & \hat{c}_J + (1+rg)\,\hat{c}_{J-1} + (1+rg)^2\,\hat{c}_{J-2} + \hat{bi}_J + (1+rg)\,\hat{bi}_{J-1} + (1+rg)^2\,\hat{bi}_{J-2}\\ & + P^h\,(1+g^g)\left\{\hat{h}_J + (1+rg)\,\hat{h}_{J-1} + (1+rg)^2\,\hat{h}_{J-2}\right\}\\ & - P^h\,(1-\delta_h)\left\{\hat{h}_{J-1} + (1+rg)\,\hat{h}_{J-2} + (1+rg)^2\,\hat{h}_{J-3}\right\}\\ =&\hat{w}_J + (1+rg)\,\hat{w}_{J-1} + (1+rg)^2\,\hat{w}_{J-2}\\ & + P^h\phi\hat{i}^h\left\{(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1-\mathbf{1}_{J-1\geq JR}) + (1+rg)^2\,(1-\mathbf{1}_{J-2\geq JR})\right\}\\ & + (1+rg)^2\,(1+r)\,\hat{a}_{J-3}\end{aligned}$$

$$
\begin{aligned}
\iff\quad & \hat{c}_J + (1+rg)\,\hat{c}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{c}_1 \\
& + \hat{bi}_J + (1+rg)\,\hat{bi}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{bi}_1 \\
& + P^h\,(1+g^g)\left\{\hat{h}_J + (1+rg)\,\hat{h}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{h}_1\right\} \\
& - P^h\,(1-\delta_h)\left\{\hat{h}_{J-1} + (1+rg)\,\hat{h}_{J-2} + \cdots + (1+rg)^{J-1}\,\hat{h}_0\right\} \\
= & \hat{w}_J + (1+rg)\,\hat{w}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{w}_1 \\
& + P^h\phi\hat{i}^h\left\{(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1-\mathbf{1}_{J-1\geq JR}) + \cdots + (1+rg)^{J-1}\,(1-\mathbf{1}_{1\geq JR})\right\} \\
& + (1+rg)^{J-1}\,(1+r)\,\hat{a}_0.
\end{aligned}
$$

Recalling that $\hat{h}_j = \frac{(1+rg)\{\omega_j\psi(1-\nu)+\chi\}}{(r+\delta_h)P^h}\hat{c}_j$ and $\hat{bi}_j = \omega_j\psi\nu\hat{c}_j \quad for\ j = 1, ..., J-1$, we have

$$
\begin{aligned}
& \hat{c}_J + (1+rg)\,\hat{c}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{c}_1 \\
& + \psi\nu\left\{\hat{c}_J + (1+rg)\,\omega_{J-1}\hat{c}_{J-1} + \cdots + (1+rg)^{J-1}\,\omega_1\hat{c}_1\right\} \\
& + \frac{1+r}{r+\delta_h}\left[\{\psi\,(1-\nu)+\chi\}\,\hat{c}_J + (1+rg)\,\{\omega_{J-1}\psi\,(1-\nu)+\chi\}\,\hat{c}_{J-1}\right. \\
& \left. + \cdots + (1+rg)^{J-1}\,\{\omega_1\psi\,(1-\nu)+\chi\}\,\hat{c}_1\right] \\
& - \frac{1-\delta_h}{r+\delta_h}\left[(1+rg)\,\{\omega_{J-1}\psi\,(1-\nu)+\chi\}\,\hat{c}_{J-1} + \cdots + (1+rg)^{J-1}\,(\omega_1\psi\,(1-\nu)+\chi\}\,\hat{c}_1\right] \\
= & \hat{w}_J + (1+rg)\,\hat{w}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{w}_1 \\
& + P^h\phi\hat{i}^h\left\{(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1-\mathbf{1}_{J-1\geq JR}) + \cdots + (1+rg)^{J-1}\,(1-\mathbf{1}_{1\geq JR})\right\}
\end{aligned}
$$

$$
\begin{aligned}
\iff\quad & \left[1 + \left\{\nu + \frac{1+r}{r+\delta_h}\,(1-\nu)\right\}\psi + \frac{1+r}{r+\delta_h}\chi\right]\hat{c}_J \\
& + (1+rg)\,\{1+\omega_{J-1}\psi+\chi\}\,\hat{c}_{J-1} + \cdots + (1+rg)^{J-1}\,\{1+\omega_1\psi+\chi\}\,\hat{c}_1 \\
= & \hat{w}_J + (1+rg)\,\hat{w}_{J-1} + \cdots + (1+rg)^{J-1}\,\hat{w}_1 \\
& + P^h\phi\hat{i}^h\left\{(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1-\mathbf{1}_{J-1\geq JR}) + \cdots + (1+rg)^{J-1}\,(1-\mathbf{1}_{1\geq JR})\right\}.
\end{aligned}
$$

Further recalling that $\hat{c}_{j+1} = \beta(1+rg)(1-\omega_j)\hat{c}_j \quad for \ j = 1, ..., J-1$, we have

$$
\begin{aligned}
&(1+rg)^{J-1}\left[\beta^{J-1}(1-\omega_{J-1})(1-\omega_{J-2})\cdots(1-\omega_1)\left\{1+\psi\nu+\frac{1+r}{r+\delta_h}\psi(1-\nu)+\frac{1+r}{r+\delta_h}\chi\right\}\right.\\
&\left.+\beta^{J-2}(1-\omega_{J-2})\cdots(1-\omega_1)\{1+\omega_{J-1}\psi+\chi\}+\cdots+\beta(1-\omega_1)\{1+\omega_1\psi+\chi\}\right]\hat{c}_1\\
=&\hat{w}_J+(1+rg)\hat{w}_{J-1}+\cdots+(1+rg)^{J-1}\hat{w}_1\\
&+P^h\phi\hat{i}^h\left\{(1-\mathbf{1}_{J\geq JR})+(1+rg)(1-\mathbf{1}_{J-1\geq JR})+\cdots+(1+rg)^{J-1}(1-\mathbf{1}_{1\geq JR})\right\}
\end{aligned}
$$

$$
\begin{aligned}
\iff \quad &(1+rg)^{J-1}\left[\beta^{J-1}\left(\prod_{k=1}^{J-1}(1-\omega_k)\right)\left\{1+\psi\nu+\frac{1+r}{r+\delta_h}\psi(1-\nu)+\frac{1+r}{r+\delta_h}\chi\right\}\right.\\
&\left.+\sum_{j=1}^{J-1}\left\{\beta^{j-1}\left(\prod_{k=1}^{j-1}(1-\omega_k)\right)(1+\omega_j\psi+\chi)\right\}\right]\hat{c}_1\\
&=\sum_{j=1}^{J}(1+rg)^{J-j}\left\{\hat{w}_j+P^h\phi\hat{i}^h(1-\mathbf{1}_{j\geq JR})\right\}.
\end{aligned}
$$

The initial value of detrended consumption $\hat{c}_1$ is written as a function of the steady state prices.

# Appendix H Equilibrium Dynamics by Age

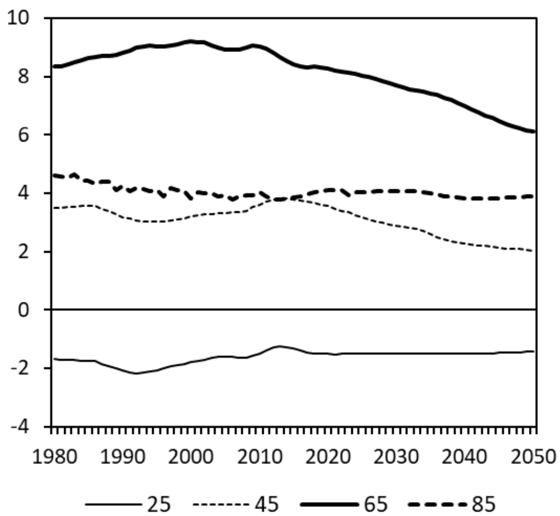
Figure A5 shows the age-specific dynamics of four detrended per-capita variables in the baseline case: financial assets (a), consumption (b), housing (c), and vacant housing (d). If aggregate TFP growth is added back, non-zero TFP growth rates between 1980 and 2019 will tilt the graphs. There is significant cross-sectional variation by age (ages 25, 45, 65, and 85). The 2019 values across ages correspond to figure 4, and the dynamics of population-weighted average correspond to figure 6.

In panel (a), per capita financial assets are negative for 25 years old throughout the simulation period, consistent with the standard life-cycle model. In contrast, an 85 years old person has the largest amount of financial assets, partly for bequest motives.

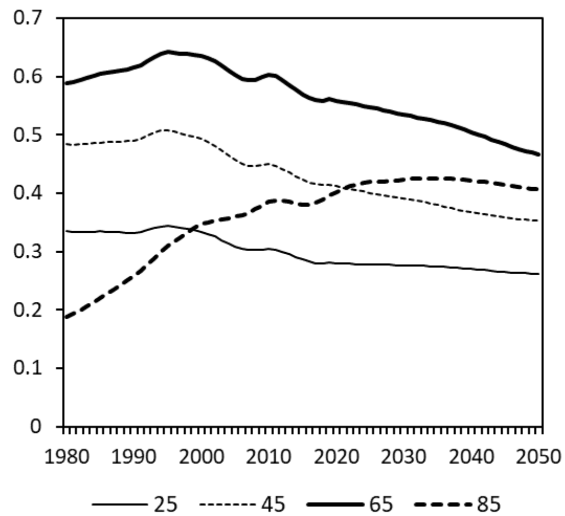
In panel (b), per capita consumption consistently decreases after 1990 for 25, 45, and 65 years old, whereas it increases for the 85 years old. In 1980, the peak of per capita consumption was much earlier in life than in 2050. Thus, the 85 years old already reduced consumption significantly because of increasing mortality rates. In 2050, however, the 85 years old will not be as old as in 1980 because of longevity. As the consumption peak is delayed, per capita consumption for 85 years old increases.

In panel (c), per capita housing consistently decreases after 1990 for 25, 45, and 65 years old because of lower real income. In contrast, housing for 85 years old is more stable over time because older households maintain large housing for bequest motives. Thus, the upward-sloping age profile of housing is exacerbated by population aging.

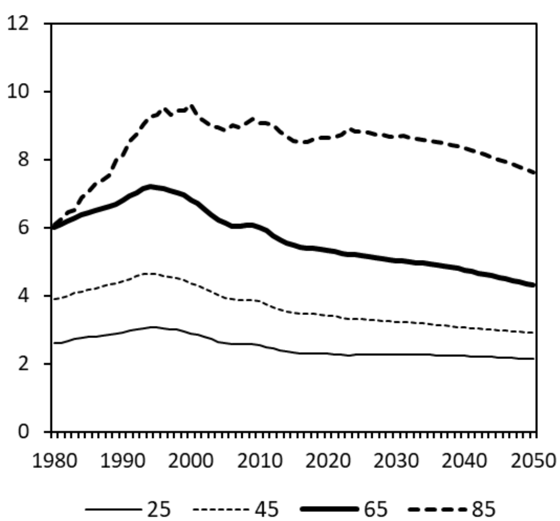
In panel (d), the largest amount of vacant housing is owned by 65 years old because a proportion $\phi$ of the inherited housing accumulates continuously between ages 20 and 64. Vacant housing increases rapidly from 2020 because inherited housing increases over time as population ages.



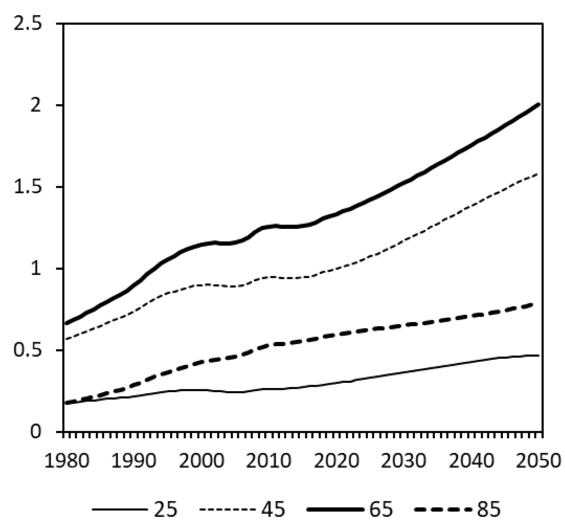

(a) Per Capita Financial Assets




(b) Per Capita Consumption




(c) Per Capita Housing




(d) Per Capita Vacant Housing

Figure A5: The Dynamics of Equilibrium Variables by Age

# Appendix I The User Cost of Housing

The present model specification is equivalent to an alternative one that includes periodic user (rental) costs but not purchase costs because there are no market imperfections such as asymmetric information and incomplete contracts. The user cost rate equals the total gross return to housing assets $1 + r_t + \delta_h$ less the house price appreciation rate $P^h_{t+1} / P^h_t$.

Figure A6 shows the user cost rate as the difference between the total gross interest rate $(r_t + \delta_h)$ and hosing price growth rate in the model. The implied user cost rate is approximately $8 - 10\%$, which is higher than the observed "cap rate" for residential properties.[8] This is because our model omits the land component of housing. Yoshida (2020) estimates that land accounts for $48 - 87\%$ of housing value in Tokyo and $28 - 70\%$ outside Tokyo, depending on the age of the structure. If, for example, non-depreciating land accounts for 60% of housing value, the implied depreciation rate of housing will be 4.5 percentage points lower. Thus, our model predicts that the observed cap rate would be approximately $3.5 - 5.5\%$, which is consistent with the data. The user cost was low in the late 1980s and early 1990s because of the large house price appreciation, but it increased by 2020 because of the lower house price appreciation rate.

[8] The cap rate for real estate investment corresponds to the user cost rate in our model, which does not include maintenance, insurance, and property taxes. The cap rate equals the ratio of net operating income to the property value.

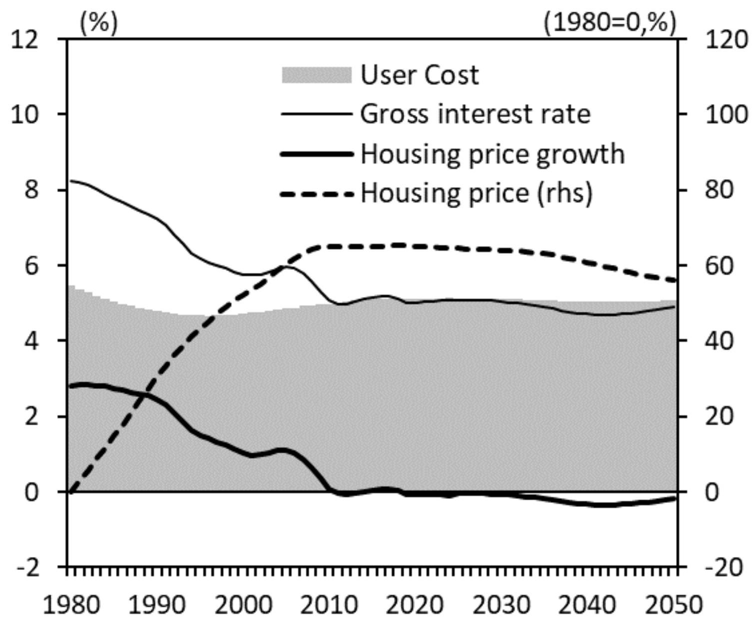



Figure A6: Gross Interest Rate and House Appreciation Returns