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ACCUMULATED DAMAGE IN NONLINEAR CYCLIC STATIC AND DYNAMIC ANALYSIS OF REINFORCED CONCRETE STRUCTURES THROUGH 3D DETAILED MODELING.

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Abstract. Accurate nonlinear cyclic static and dynamic analysis of reinforced concrete structures is necessary when trying to capture the behavior of concrete structures during earthquake excitations. The development of an objective and robust 3D constitutive modeling approach that will be able to account for the accumulated material damage during the cyclic loading of concrete structures is of great importance in order to realistically describe the physical failure mechanisms [1]. The proposed method is based on the experimental results and the concrete modelling of Kotsovos and Pavlovic [2] as modified by Markou and Papadrakakis [3]. The objective of this research work is to propose a computationally efficient modeling method that accounts for the accumulated damage developed in both concrete and steel materials during cyclic static and dynamic excitations.

Two new damage factors are proposed herein that take into account the number of openings and closures of cracks during the nonlinear cyclic analysis, thus provide with the ability to account for the accumulated damage in both steel and concrete materials. Furthermore, a solution strategy that describes the behavior of concrete during the cyclic static and dynamic analysis is also presented.

The proposed numerical method is validated by comparing its numerical response with the corresponding experimental data of a beam-column frame joint and a two-storey reinforced concrete frame, which were tested under cyclic static and dynamic loading conditions, respectively. Based on the numerical findings, the proposed algorithm manages to accurately capture the experimental results, while the simulation of the understudy models was performed with computational robustness and efficiency. This numerical outcome demonstrates the potential of the proposed 3D detailed modeling approach to be implemented for the seismic assessment of full-scale reinforced concrete structures through nonlinear cyclic static and dynamic analysis.

1 INTRODUCTION

During the last decades, many numerical simulations of reinforced concrete (RC) structures under cyclic loading conditions have been proposed. Most models tend to introduce many material parameters that are associated with the nonlinear behavior of concrete structures. They place emphasis on post-peak material characteristics in order to describe phenomena such as ductility, confinement, concrete cracking and crushing of reinforced concrete structures. These models can describe only certain aspects of concrete behavior and their implementation is limited to examples of small practical interest. It is important to formulate a constitutive model which represents accurately the actual mechanical behavior of concrete structures. An efficient and robust algorithm is developed in order to satisfy this cause.

Most models use elastoplastic uniaxial constitutive laws in order to describe the mechanical behavior of concrete. Others use the "equivalent uniaxial strain" concept to combine the uniaxial laws with biaxial or triaxial behavior of concrete. Other approaches propose constitutive models based on biaxial or triaxial failure surfaces. In addition to that, many researchers use the compression field theory to treat the behavior of cracked RC elements subjected to shear. Furthermore, many models combine the plasticity formulations for the behavior of concrete under compression with the fracture energy based smeared crack approaches for the behavior of concrete behavior under monotonic and cyclic loading conditions.

A detailed literature review in regards to the modeling of RC structures under cyclic loading conditions can be found in [3]. Few of these models have been used successfully for 3D dynamic analysis. Spiliopoulos and Lykidis

[4] and Cotsovos [5] used the constitutive model of Kotsovos and Pavlovic [3] integrated in 27-noded hexahedral elements for cyclic and dynamic analysis of RC structures. The latter introduces some restrictions in regards to the number of cracks that are allowed to open in each iteration, in an attempt to obtain convergence during the analysis. Moreover, none of these researchers investigated the computational efficiency of their proposed models, which is deemed crucial when dealing with dynamic nonlinear analyses of RC structures. Recently, Moharrami et al. [6] proposed a 3D constitutive model, which combines the elastoplastic and smeared crack approaches in order to describe the cyclic and dynamic behavior of concrete structures.

The proposed model in this research work, describes the triaxial behavior of concrete by using realistic assumptions without the need of introducing numerous concrete material parameters. The objective of the present paper is the formulation and numerical implementation of an accurate simulation of RC structures subjected to dynamic loading conditions. The uniaxial compressive and tensile strengths, the Young Modulus of elasticity and the Poisson's ratio are the only material parameters, which are needed to be defined for the analysis of concrete. The accuracy, numerical simplicity and the computational efficiency are the most important features in order to show the practical use of any model that can be easily implemented in predicting the nonlinear static and dynamic behavior of RC structures. Therefore, the proposed model adopts the numerical approach which was proposed in [3] (for cyclic loading conditions) thus it is further integrated herein for simulating the dynamic response of RC structures. The concrete model takes into account the effect of crack-closing through the use of a new damage factor. The numerical simulation is based on the proposed model by Markou and Papadrakakis [1], which was an extension of the model presented by Kotsovos and Pavlovic [2]. In order to account for the concrete's accumulated damage and its effect on the behavior of the steel rebars, a second damage factor is introduced for the steel material.

2 CONCRETE MATERIAL CONSTITUTIVE MODEL

The constitutive modelling of concrete has to describe a realistic behavior of concrete under generalized three dimensional states of stress. Therefore, it has to take into account the effect of out of plane small stresses that are usually ignored. The stress-strain relationships are expressed most conveniently by decomposing each state of strain and stress into hydrostatic and deviatoric components, where the proposed model uses two moduli of elasticity (bulk K and shear G) and an equivalent external stress (σ_{id}) in order to describe the constitutive relations as presented by the combined approach [2]. The bulk modulus K and the shear modulus G describe the non-linear σ_0 - $\varepsilon_{0(h)}$ and τ_0 - $\gamma_{0(d)}$ behavior combined with the use of σ_{id} in order to take into account the coupling effect of τ_0 - $\varepsilon_{0(d)}$ (h and d stand for hydrostatic and deviatoric components, respectively). The constitutive relations take the following form:

$$\varepsilon_0 = \varepsilon_{0(h)} + \varepsilon_{0(d)} = (\sigma_0 + \sigma_{id}) / (3K_s) \tag{1}$$

$$\gamma_0 = \gamma_{0(d)} = \tau_0 / (2G_s) \tag{2}$$

where K_s and G_s are the secant forms of bulk and shear moduli, respectively. The secant forms of bulk, shear modulus and σ_{id} are expressed as functions of the current state of stress which derived by regression analysis of the experimental data found in [2].

It is evident that when the deciatoric stress of an uncracked Gauss point of a concrete element, is less than the 50% of the corresponding ultimate strength, then the elastic constitutive matrix is used. Otherwise, the constitutive material matrix is updated using the tangent expressions of bulk and shear modulus [1]. The constitutive material matrix of the uncracked concrete is presented in eq. 3.

$$D = \begin{bmatrix} 2G_t + \mu & \mu & \mu & 0 & 0 & 0 \\ \mu & 2G_t + \mu & \mu & 0 & 0 & 0 \\ \mu & \mu & 2G_t + \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & G_t & 0 & 0 \\ 0 & 0 & 0 & 0 & G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & G_t \end{bmatrix}$$
(3)

The strength envelope of concrete is expressed by the value of the ultimate deviatoric stress by using the expressions of Willam and Warkne [7].

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$$\tau_{0u} = \frac{2\tau_{0c}(\tau_{0c}^2 - \tau_{0e}^2)\cos\theta + \tau_{0c}(2\tau_{0e} - \tau_{0c})\sqrt{4(\tau_{0c}^2 - \tau_{0e}^2)\cos^2\theta + 5\tau_{0e}^2 - 4\tau_{0c}^2\tau_{0e}^2}}{4(\tau_{0c}^2 - \tau_{0e}^2)\cos^2\theta + (2\tau_{0e} - \tau_{0c})^2}$$
(4)

Additionally, the new criterion of crack closing which was introduced in [5], uses the strains that caused the initial formation of the cracks so as to determine whether a crack that starts to close will be eventually closed in a numerical manner as well. The criterion of crack closure takes the following form:

$$\mathcal{E}_i \le a \cdot \mathcal{E}_{cr} \tag{5}$$

where ε_i is the current strain in the i-direction which is normal to the crack plane and ε_{cr} is the strain that caused the crack formation. Parameter *a* is a reduction factor, which takes the following form:

$$a = 1 - \frac{\mathcal{E}_{cr}}{\mathcal{E}_{\max}} = \frac{\mathcal{E}_{\max} - \mathcal{E}_{cr}}{\mathcal{E}_{\max}}$$
(6)

The maximum strain ε_{max} is determined through the iterative Newton-Raphson procedure, whereas, in every internal Newton-Raphson iteration, the strains that are formed along the norm of the crack planes are calculated. Therefore, during an internal iteration when a crack is formed at a Gauss Point, it is assumed that $\varepsilon_{max} = \varepsilon_{cr}$. Then, in every i iteration (internal or external) the strains ε_i that are formed normal to the crack planes are checked if they are larger than the previously calculated ε_{max} . If this is the case, then ε_{max} is set equal to ε_i . For more details in regards to the closing crack criterion can be found in [5]. When the criterion of crack-closure is satisfied at a Gauss Point, which had prior to that only one crack formation, then a part of the stiffness is lost along the previously crack plane which was assumed to form in an orthogonal direction to the maximum principle tensile stress. Therefore the constitutive matrix takes the following form:

$$C' = \begin{bmatrix} a_n \cdot (1-D_c) \cdot (2G_t + \mu) & a_n \cdot (1-D_c) \cdot \mu & a_n \cdot (1-D_c) \cdot \mu & 0 & 0 & 0 \\ a_n \cdot (1-D_c) \cdot \mu & 2G_t + \mu & \mu & 0 & 0 & 0 \\ a_n \cdot (1-D_c) \cdot \mu & \mu & 2G_t + \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & a_s \cdot (1-D_c) \cdot \beta \cdot G_t & 0 & 0 \\ 0 & 0 & 0 & 0 & a_s \cdot (1-D_c) \cdot \beta \cdot G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & a_s \cdot (1-D_c) \cdot \beta \cdot G_t \end{bmatrix}$$
(7)

where β is a shear retention factor, a_n and a_s are constants with recommended values of 0.25 and 0.125, respectively. The factor, D_c is a proposed damage factor that describes the accumulated energy loss due to the number of times a crack has opened and closed. After a numerical investigation the proposed factor takes the following form:

$$D_{c} = e^{-(1-a)/f_{cc}} = e^{-\left(1-\left(1-\frac{\varepsilon_{cr}}{\varepsilon_{max}}\right)\right)/f_{cc}} = e^{-\left(\frac{\varepsilon_{cr}}{\varepsilon_{max}}\right)/f_{cc}}$$
(8)

where f_{cc} is the number a crack has closed which is updated in every iteration for every Gauss Point. A schematic representation of Eq. 8 can be seen in the Fig. 1.



Fig. 1. Schematic representation of the values of the damage factor D Eq. 8 as a function of the parameter a and f_{cc} .

Similarly, the constitutive matrix follows the above procedure in the cases when a crack is closed at a Gauss point, which had previously one, two or three cracks. After the crack closure, the stresses are corrected by using the following expression:

$$\sigma^{i} = \sigma^{i-1} + C' \cdot \Delta \varepsilon^{i} \tag{9}$$

Finally, when all the cracks have been closed (uncracked Gauss Point) and the reduction factor aof one of the previous cracks (any of the possibly three cracks) is larger than 0.5, then the constitutive matrix takes the following form:

$$C^{\prime\prime} = (1 - D_c) \cdot C \tag{10}$$

The proposed behavior of both cracked and uncracked Gauss points are described in the flow charts presented in Figs. 2 and 3.

Furthermore, a level of damage that is occurred due to the opening of cracks affects the contribution of steel reinforcement to surrounding concrete areas. In this way, a modification of the steel stress-strain relation of Menegotto-Pinto [8] is described. In this way, some pinching characteristics and the loss of bonding between steel reinforcement with the surrounding cracked concrete can indirectly be taken into account by reducing the stiffness contribution of steel reinforcement [9]. The average of all parameters a (Eq. 7) at the 8 Gauss Points within a single hexahedral element can determine the level of damage of the concrete hexahedron as shown in the following expression:

$$D_s = \left[1 - a_{Element}\right] \tag{11}$$

where,

$$a_{Element} = \frac{\sum_{i=1}^{m} a_i}{ncr}, \text{ ncr is the number of cracked Gauss Points}$$
(12)

In the case of unloading, when the structure reaches its initial deformation, a material deterioration of the steel reinforcement is computed based on the following proposed formulae:

$$E_s = (1 - D_s)E_s \tag{13}$$

The material deterioration is applied when $\sigma_s \cdot \epsilon_s < 0$, which describes the situations when crack closures and re-openings occur and the pinching phenomena are excessive. The modified material model is illustrated in Fig. 2.



Fig. 2. Menegotto-Pinto steel model by taking into account the accumulated damage due to opening/closure of cracks.

3 NUMERICAL VALIDATION OF THE PROPOSED MODEL UNDER STATIC CYCLIC ANALYSIS.

The beam-column joint shown in Fig. 7, has been analyzed by Kusuhara and Shiohara [10] under static cyclic loading. The uniaxial compressive concrete strength was reported to be equal to $f_c=28.3$ MPa and the yielding stress of the steel reinforcement was 456 MPa for the 13 mm diameter bars in the beam section, while the yielding stress for the 13 mm diameter of the column section was 357 MPa. The Young modulus of elasticity for the longitudinal bar reinforcement was $E_S = 176$ GPa. For the stirrup reinforcement, 6 mm in diameter rebars were used with a yielding stress of 326 MPa and a Young modulus of elasticity equal to 151 GPa.



Fig. 3. (a) Geometry and reinforcement details of beam column joint [10] and (b) imposed displacement history of the interior frame joint.

The frame joint was subjected to different cyclic loading sets according to the experimental setup. The loading history that was modeled in this section, is presented in the form of imposed displacements in Fig. 3b, where 15 total displacement cycles can be seen. For the numerical model construction, the concrete domain was discretized with 8-noded hexahedral finite elements and the steel reinforcement was discretized with the beam finite element. A total number of 128 concrete (23cm x 15cm x 15cm) and 888 steel elements were used so as to discretize the entire frame joint, as illustrated in Fig. 4. The beam-column frame joint is supported according to the experimental configuration shown in Fig. 3, where the boundary conditions implemented within the developed model are shown in Fig. 4. As it can be seen in Fig. 4, the displacements were imposed at the top section of the column, while a 216 kN compressive force was also applied at the same section.



Fig. 4. RC beam-column frame joint. 3D views of the FE mesh of (a) concrete and (b) steel reinforcement elements.

The numerically curves are compared with the corresponding experimental curves in Fig 5. As it can be seen, the numerical results match very well with the experimental ones in terms of stiffness, strength and energy dissipation. Furthermore, Fig. 5 shows that the proposed model manages to capture efficiently the pinching effect in the case where both damage factors are implemented. It must be noted here that, during the analysis the steel rebars did not develop severe yielding but a bond degradation has occurred due to the opening of diagonal cracks

inside the joint according to the experimental results [32]. This observation indicates the importance of accounting the damage within the concrete domain and numerically transferring it to the steel rebar's material response through the proposed damage factor D_S .



Displacement(m)

Fig. 2. Beam-Column frame joint. Comparison between numerical and experimental results. Complete forcedisplacement history.

The required Newton-Raphson internal iterations per load increment are shown in Fig. 6. As it resulted, all the displacement increments required a reasonable number of internal iterations to reach convergence regardless the intense nonlinear behavior of the structure. 77% of the displacement increments require less than 5 internal iterations to converge, while 95% of the displacement increments require less than 10 internal iterations. A total of 173 seconds were required so as to solve 610 displacement increments. This illustrates the computational efficiency of the proposed algorithm and the overall stability of the nonlinear solution procedure.



Fig. 6. RC beam-column frame joint. Required Newton-Raphson iteration per displacement increment.

4 NUMERICAL IMPLEMENTATION FOR DYNAMIC ANALYSIS

A two-level RC frames denoted as H30 [11], which was subjected to dynamic loading conditions is investigated in this section. The iterative method uses an energy convergence tolerance criterion set to 10^{-4} , where the nonlinear Newmark integration method was used for the dynamic analysis. The geometric and reinforcement details are shown in Fig. 7a. The uniaxial cylinder compressive strength of concrete (f_c) was 50 MPa. The yielding stress (f_y) of steel reinforcement was 500 MPa and the masses of the frame were 2.87 and 2.62 tons at the lower and upper girders, respectively. The Frame H30 was designed by assuming a ductility of q = 5.

The frame was subjected to horizontal motions applied at its base as shown in Fig. 7b. The base motions foresaw of three sinusoidal events that were applied in sequence, as described in [11]. The three accelerograms exhibited a maximum magnitude of approximately one and two times the magnitude of the design ground acceleration of the frame, which was 0.30g.



Fig. 3. (a) Geometric and reinforcement details of RC specimen H30, (b) Hexahedral FE mesh and (c) embedded rebar elements.

For the numerical model (H30), a total of 2,384 embedded rebar elements were used, while for the RC slabs, 128 8-noded hexahedral elements were used (red elements). The mass density of the RC slab-elements has been set appropriately in order to take into account the mass of the structure based on the experimental setup [11]. The steel rebars were simulated as embedded beam elements within the hexahedral concrete elements as presented in [12].

The frame developed excessive cracking which occurred at the first dynamic cycles of the dynamic excitation (first 3 s of the experiment, see Fig. 9). This led to significant strength degradations mainly during the third round of dynamic loading (t > 17 s) due to the yielding and fracture of the longitudinal reinforcement at the base of the frame, which was also observed during the experiment [11]. The numerically computed displacements of the first and the second floor slabs of the RC frame are compared with the experimental data in Figs. 9 and 10.

The numerical results indicate that the proposed model managed to capture accurately the experimental data during the dynamic tests. During the last dynamic test, the model exhibited a relatively stiffer behavior compared to the experimental data. This can be attributed to severe damages developed at the base which led the specimen practically to fail due to excessive cracking and rebar failures. This explains the remaining deformation that can be noted during the last 5 seconds of the experiment (see Figs. 9 and 10). The numerical model managed to describe the overall behavior of the frame in a satisfactory manner, without any numerical instabilities.



Fig. 4. H30 frame. Comparison between the numerical and experimental results of the 1st level displacement response.

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Fig. 10. H30 frame. Comparison between the numerical and experimental results of the 2nd level displacement response.

The required internal iterations, for H30 numerical models, are shown in the Fig 11. As it resulted, the required internal iterations per dynamic step during the solution procedure were limited to an average of 2 to 3, underlining the numerical stability of the proposed method. Based on the numerical findings, a 78% of the dynamic steps required less than 5 internal iterations to achieve convergence. The total required time for solving the nonlinear dynamic problems was 565 s, which refers to the solution of 3,826 dynamic time increment steps.

Finally, Fig. 12 shows the crack patterns that were formed at the end of the dynamic event 3. The numerical crack patterns appear to be a denser than the experiment ones, which is attributed to the smeared crack method. However, there is a good agreement on the distribution, the location and the direction of the cracks predicted by the numerical model.



Fig. 11. Required Newton-Raphson internal iterations per dynamic step increment H30.



Fig. 12. H30 frame. Comparison of numerical and experimental crack patterns.

5 CONCLUSIONS

In this research work, a 3D detailed finite-element model was proposed for the nonlinear static cyclic and dynamic analysis of RC structures. The concrete material constitutive model describe a realistic behavior of

concrete under generalized three dimensional states of stress and treats cracking with the smeared crack approach. The numerical model has been integrated with a newly proposed concrete damage factor that was constructed by using the characteristics of cracking during the nonlinear static cyclic or dynamic analysis. Furthermore, a damage factor for the steel material that is also directly connected to the number of opening and closing of concrete cracks was introduced.

The proposed model was applied in a beam-column joint specimen, which was subjected to cyclic loading conditions and a two-level RC frame that was subjected to dynamic loading conditions. The numerical study revealed that the steel damage factor was crucial in the case of the static cyclic loading. Furthermore, the proposed concrete damage factor was found to play a controlling role during the dynamic analysis of the frame, while the ability of the proposed modeling approach in predicting the experimental data accurately and efficiently was illustrated. Finally, the numerical ability to capture the 3D cyclic behavior of reinforced concrete structures was presented herein, which is crucial when dealing with large-scale models [13].

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