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On modelling non-performing loans in bank efficiency analysis

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Abstract

This paper introduces a methodological framework for the examination of non-performing loans (NPLs) as reverse outputs under the extended strong disposability assumption, which does not require NPLs to be jointly produced with net loans, as it is implied when they are modelled as undesirable outputs. A directional distance function model with reverse outputs is used and is compared with the models that treat NPLs as an undesirable output under the weak disposability and the constrained weak disposability assumptions with uniform and non-uniform abatement factors. The model is applied at the case of European banks and for the sample to be representative the banks are chosen based on the European Banking Authority (EBA) stress test of 2021. The results indicate that the reverse output model have greater discriminatory power relative to all other models.

KEYWORDS

Bank efficiency, data envelopment analysis, extended strong disposability, non-performing loans, reverse outputs

1 | INTRODUCTION

Improving asset quality has been deemed as one of the key priorities for the European Central Bank, in response to high levels of non-performing loans (NPLs) for a number of banks in the Eurozone and their negative effect on lending the real economy (ECB, 2017). NPLs are bank loans that are either unlikely to be repaid or subject to late repayment, the latter defined according to ECB[1] as 'when more than 90 days have passed without the borrower paying the agreed instalments.'[2] In 2023, the combined stock of NPLs among banks deemed significant and those under direct ECB supervision amounted to €339 billion (ECB[3]). Furthermore, NPLs raise a number of issues for banks including problems with capital adequacy, reduced profitability, immobilised bank capital, increased funding costs, and limited ability to issue new credit and increased default and financial stability risks (Fredriksson & Frykström, 2019). Recently empirical studies have also demonstrated the negative impact of NPLs on bank efficiency (Barros et al., 2012; Phung et al., 2022). Despite the evident drawbacks associated with high NPL levels and the availability of various resolution methods, a significant portion of countries facing high NPLs remains unable to resolve this issue primarily due to a range of legal and administrative obstacles, as well as substantial information disparities between NPL buyers and sellers which hinder the effectiveness of the non-performing asset market (ECB, 2022). Nevertheless, a substantial portion of prior literature has overlooked the impact of NPLs, potentially resulting in biassed results where banks with high NPLs may erroneously appear as efficient, as a significant negative factor is omitted (Assaf et al., 2013).

One of the most prominent and widely employed methods for assessing bank efficiency is data envelopment analysis (DEA). DEA, developed by Charnes et al. (1978) for constant returns to scale (CCR model), is a mathematical programming tool designed for evaluating the efficiency of decision-making units (DMUs) with multiple inputs and outputs. Banker et al. (1984) further extended the original model to accommodate variable returns to scale (BCC model). In fact, banking stands as one of the first (Sherman & Gold, 1985) and afterwards the second most preferred field for DEA applications (see Emrouznejad & Yang, 2018). Perhaps the first attempt to include NPLs into bank efficiency analysis was Park and Weber (2006) who assumed strong disposability of desirable outputs and inputs, weak disposability of undesirable outputs and null-jointness. Strong disposability of desirable outputs and inputs means that banks can use more inputs to produce the same or even less quantities of NPLs and their desirable outputs, for example, net loans and investments. On the other hand, if banks want to reduce NPLs, keeping their inputs constant, they must also reduce net loans and investments proportionally. Null-jointness means that the only way of having a zero amount of NPLs is to produce a zero amount of desirable outputs or if some positive amount of desirable outputs is to be produced, then there will be some positive amount of NPLs. From a modelling standpoint, Park and Weber (2006) employed a directional distance function approach (DDF), initially introduced by Chambers et al. (1996). Following Park and Weber (2006), all previous studies, that is, Fukuyama and Weber (2008, 2010), Barros et al. (2012), Fujii et al. (2014), Lozano (2016), and Fukuyama and Matousek (2017), which take NPLs into account also used a DDF approach and assumed weak disposability of NPLs and null-jointness.

Besides banking, the notions of weak disposability and null-jointness have extensively been used in other industries where the by-product nature of undesirable outputs is considered as a physical law. This is particularly appealing for the case of environmental pollutants, such as CO_2, but there are several cases where undesirable outputs cannot be considered as by-products of desirable outputs and in this case, null-jointness no longer holds. In these cases, instead of considering them as undesirable, we can treat them as reverse outputs (Lewis & Sexton, 2004), that is, outputs for which higher numerical values represent lower achievements. Reverse outputs are compatible with the notion of extended strong disposability (Karagiannis & Kourtzidis, 2023), which implies that with more inputs is always possible to produce less of desirable outputs and more of undesirable outputs (Liu et al., 2010). This is distinct from both strong disposability, that is, with more inputs is always possible to produce no more of desirable and undesirable outputs, and weak disposability, that is., for given inputs it is possible to reduce undesirable outputs only if one also reduce desirable output proportionally. There are several examples where the notion of reverse outputs makes more sense than that of undesirable outputs; in healthcare, where successful operations is a good (forward) output and complications is the reverse output (Dyckhoff & Allen, 2001), in tax payments, where sales is the good (forward) output and tax payments is the reverse output (Dyckhoff & Allen, 2001), in football, where goals scored is a good (forward) output and goals conceded is a reverse output (Bouzidis & Karagiannis, 2012), and in the service sector (at least in the case where the market is not perfectly competitive), where the number of served customers is a good (forward) output and received complaints is a reverse output (Liu et al., 2010). In all of the aforementioned cases, it is possible to reduce the reverse outputs without reducing the good outputs.

In this paper, we argue that banks can also be considered as one of the cases where the notion of reverse outputs and extended strong disposability can be used. Specifically, net loans (and other earning assets) are considered as desirable (forward) outputs and NPLs as reverse outputs. In the context of the extended strong disposability assumption this implies that an increased amount of employment, fixed assets and deposits may result in more NPLs and less net loans and other earning assets, revealing the possibility of inefficiency. It also implies that an increase in NPLs is not necessarily associated with an increase in net loans and other earning assets. These are distinct from the assumptions made with weak disposability, where it is assumed that we can only reduce NPLs by decreasing proportionally net loans and other earning assets, and null-jointness, where it is assumed that there cannot be NPLs without net loans and other earning assets. As far as we are concerned, the extended strong disposability assumption is more reasonable for the banking industry. We operationalise the proposed modelling choices by means of a DDF, which to the best of our knowledge, is the first time that the notion of reverse output is used along with a DDF approach to evaluate banks' efficiency. In the empirical part of the paper, the proposed formulation is compared with the input and output-oriented models assuming weak disposability with uniform and non-uniform abatement factors.

The rest of the paper is organized as follows. Section 2 discussed the modelling of bank efficiency with a focus on the input–output framework, the orientation of the model and the disposability assumptions. Section 3 presents ten alternative models, eight with weak disposability of NPLs and different orientations for measuring in efficiency (some of which are new in the literature) and two with extended strong disposability that represent our prefer modelling choices. Section 4 presents the data and empirical analysis and Section 5 concludes the paper.

2 | MODELLING OF BANK EFFICIENCY

Starting with Sherman and Gold (1985) there is an abundance of studies using DEA to evaluate bank efficiency. Researchers have examined various aspects of the banking industry including, among others: institutional performance at the bank level (e.g., Degl'Innocenti et al., 2017; Kourtzidis et al., 2021), performance of bank branches (e.g., Omrani et al., 2022; Portela & Thanassoulis, 2007), mergers and acquisition (e.g., Avrikan, 1999; Sherman & Rupert, 2006), financial stability (Ahmad et al., 2023; Degl'Innocenti et al., 2018), failure prediction (Li et al., 2022; Ravanos et al., 2023), ownership types (Fukuyama et al., 2023), interbank markets (Lartey et al., 2021), the impact of IFRS adoption (Mohsin et al., 2020), market power (Fukuyama & Tan, 2020) and bank size (Proaño-Rivera & Feria-Dominguez, 2023). However, the vast majority of the studies in the literature do not take into account NPLs, even though it is important to quantify the negative impact of NPLs on bank efficiency (Barros et al., 2012; Phung et al., 2022).

There are three issues that need to be addressed in order to properly formulate a model capable of taking into account NPLs: the input–output framework, the orientation of the model and the related disposability assumptions.

2.1 | Input–output framework

There is a consensus throughout the literature regarding the input and output variables used in a DEA model for bank efficiency (see the review paper of Fethi & Pasiouras, 2010). On the input side, most of the studies use fixed assets and either the number of employees or personnel expenses. On the output side, most of the studies use loans and other earning assets, both of which might be further disaggregated into more specific categories. There are however two issues regarding the modelling of inputs and outputs: the specification of deposits as inputs or outputs, and the inclusion of NPLs.

Regarding the former, there are three alternative approaches that can be summarized and explained by the deposits' dilemma (Berger & Humphrey, 1992). The Intermediation or Asset approach focuses on the core activity of banking institutions, which is the financial intermediation (they borrow funds from surplus units and lend them to deficit units). In this case, deposits are considered as inputs and loans as outputs. The Production or Value-added approach considers banks as traditional firms, which use labour and capital to produce output (deposits and loans). The User Cost approach where a financial product is considered as input or output based on its contribution to revenue. The majority of the previous literature adopts the Intermediation approach (Fethi & Pasiouras, 2010) and we also follow this approach.

Regarding the latter, a number of early studies included NPLs as a control variable in a second stage regression, which exogenously affects bank efficiency (e.g., Berger & DeYoung, 1997; Berger & Mester, 1997; Mester, 1996). The drawback of this approach is the implicit assumption that NPLs do not directly affect the production process (Assaf et al., 2013). Starting with the pioneer work of Park and Weber (2006), the alternative approach of incorporating NPLs into the analysis is to directly include them into the model as bad outputs.

2.2 | Orientation of the model

In the case of banks, managers have more control over inputs (e.g., personnel, expenses) rather than outputs (e.g., loans, income) (Hsiao et al., 2010). Therefore, a cost minimization framework would be more reasonable than a revenue maximisation one. Evidently, the vast majority of studies in bank efficiency use an input orientation. According to Fethi and Pasiouras (2010), 63.3% of the studies use an input-oriented model, 18.7% use an output-oriented model, 15.3% use both an input and an output-oriented model and 2.78% use a non-oriented model. On the contrary, the studies that incorporate NPLs into the analysis use a directional output distance function and thus, an output orientation (see Barros et al., 2012; Fujii et al., 2014; Fukuyama & Matousek, 2017, 2018; Fukuyama & Weber, 2008, 2010; Lozano, 2016; Park & Weber, 2006; Salim et al., 2017; Yang, 2014). Based on the literature and the reasoning provided above, we rely on an input-oriented model, but to be able to provide comparable results with previous studies accounting for NPLs we also apply an output-oriented model.

2.3 | Disposability assumptions

The choice of the underlying disposability assumption is closely related to the inclusion in the analysis of NPLs or not. Studies not accounting for NPLs reply on the strong disposability assumption, where it is assumed that with more employment, fixed assets and deposits a bank can always produce no more than given amount of net loans and other earning assets. On the other hand, starting with Park and Weber (2006), studies taking into account NPLS have so far relied on the weak disposability assumption and null-jointness, where it is assumed that for given inputs NPLs can only be reduced if net loans and other earning assets are reduced proportionally and that cannot be NPLs without some net loans and other earning assets. The DDF, introduced by Chambers et al. (1996), is one of only two functions, with the other being the hyperbolic distance function of Färe et al. (1985), which is able to simultaneously accommodate the expansion and contraction of both good and bad outputs, something which is not possible with the conventional CCR and BCC models. When comparing the two approaches, the DDF is much more widely used in the literature compared with the hyperbolic distance function.

A related issue with weak disposability assumption is whether a uniform (see Färe & Grosskopf, 2003) or a non-uniform (see Kuosmanen, 2005; Kuosmanen & Podinovski, 2009) abatement factor will be chosen. Notice that to the best of our knowledge this issue has not been examined in the banking efficiency literature. The former may result in some non-linearities that render the relevant linear programming models difficult to estimate.

In the context of bank efficiency, Fukuyama and Weber (2008) extended the approach of Park and Weber (2006) by estimating the shadow prices of NPLs and Fukuyama and Weber (2010) used it to construct a two-stage network DEA model, which takes NPLs into the account. Lozano (2016) and Fukuyama and Matousek (2017, 2018) have also developed network DEA models, which take into account NPLs. For the same purpose, Barros et al. (2012) used a weighted Russel DDF, which is able to determine each variable's contribution to inefficiency. Fujii et al. (2014) modified this model in order to calculate the TFP change of Indian banks. Yang (2014) proposed a modification of the DDF in order to decompose the technical efficiency of a bank into operating efficiency and risk management efficiency and included NPLs into the analysis as undesirable outputs. Salim et al. (2017) constructed a bias-corrected enhanced Russell-based DDF, which can be decomposed into desirable and undesirable output efficiency.

3 | MODELLING OF NPLs

In this section, we first present the three different disposability axioms on modelling bad outputs and then, we focus on modelling the NPLs. In the subsequent discussion, we assume a production process where m inputs $\left(x \in \mathbb{R}_+^m\right)$ are used to produce s outputs $\left(y \in \mathbb{R}_+^s\right)$ and p bad outputs $\left(b \in \mathbb{R}_+^p\right)$, which can be either undesirable or reverse outputs. This production process can be described by the technology set S, which includes all feasible combinations of inputs and outputs.

$$S = \left\{(x,y,b) \in \mathbb{R}_+^{m+s+p} : x \text{ can produce } (y,b)\right\}. \qquad (1)$$

Formally then the strong disposability axiom implies that if $(x,y,b) \in S$ and $x' \geq x$, $y' \leq y$ and $b' \leq b$ then $(x',y',b') \in S$ (Førsund, 2009). On the other hand, the weak disposability axiom implies that if $(x,y,b) \in S$ and $0 \leq \theta \leq 1$ then $(x,\theta y,\theta b) \in S$ and the null-jointness axiom that if $(x,y,b) \in S$ and $b = 0$ then $y = 0$ (Färe & Grosskopf, 2003, 2004a; Kuosmanen, 2005; Kuosmanen & Matin, 2011; Leleu, 2013; Podinovski & Kuosmanen, 2011). Lastly, the extended strong disposability axiom implies that if $(x,y,b) \in S$ and $x' \geq x$, $y' \leq y$ and $b' \geq b$ then $(x',y',b') \in S$ (Liu et al., 2010).

In the rest of this section, we present ten alternative models for NPLs. Eight of them are with weak disposability of NPLs and different orientations for measuring efficiency, whereas two are non-linear and more challenging to estimate. Additionally, some of these models are novel in the literature, and these are the models with constrained weak disposability, and the input-oriented models with weak disposability. Two models assume extended strong disposability and represent our preferred modelling choices, and these are also new in the literature.

3.1 | Models with weak disposability

In this subsection we discuss the modelling issues related to weak disposability. First, under the assumption of constant or non-increasing returns to scale weak disposability imposes the minimal possible changes in the traditional DEA model, namely the inequality sign in the input constraints should turn into equality (Färe & Grosskopf, 2009). However, under the variable returns to scale (VRS), this is not enough and an abatement factor θ should be included. Färe and Grosskopf (2003) assumed the abatement factor θ to be uniform across all DMUs. In this case, a directional output distance function model with a uniform abatement factor and VRS is given as:

$$
\begin{aligned}
&\max \delta_k \\
&\text{s.t.}\,\theta \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, \quad r = 1,\dots,s \\
&\theta \sum_{j=1}^{n} \lambda_j b_{qj} = b_{qk} - \delta_k \times \vec{g}_b, \quad q = 1,\dots,p \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, \quad i = 1,\dots,m \\
&\sum_{j=1}^{n} \lambda_j = 1, \quad j = 1,\dots,n \\
&\lambda_j \geq 0, \quad 0 \leq \theta \leq 1
\end{aligned} \quad , \qquad (2)
$$

where δ refers to the efficiency score, g to the direction vector ($g \geq 0$) and λ to the intensity variables. This is a non-linear model that can be linearized by using $\sum_{j=1}^{n} z_j = \theta$(Sahoo et al., 2011). Then, (2) may be written as:

$$
\begin{aligned}
&\max \delta_k \\
&\text{s.t.} \sum_{j=1}^{n} z_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, \quad r = 1,\dots,s \\
&\sum_{j=1}^{n} z_j b_{qj} = b_{qk} - \delta_k \times \vec{g}_b, \quad q = 1,\dots,p \\
&\sum_{j=1}^{n} z_j x_{ij} \leq \theta x_{ik}, \quad i = 1,\dots,m \\
&\sum_{j=1}^{n} z_j = \theta, \quad j = 1,\dots,n \\
&z_j \geq 0, \quad 0 \leq \theta \leq 1
\end{aligned} \quad . \qquad (3)
$$

Similarly, the directional input distance function model with uniform abatement factor θ and VRS is given as:

$$
\begin{aligned}
&\max \varphi_k \\
&\text{s.t.} \sum_{j=1}^{n} z_j y_{rj} \geq y_{rk}, \quad r = 1,\dots,s \\
&\sum_{j=1}^{n} z_j b_{qj} = b_{qk}, \quad q = 1,\dots,p \\
&\sum_{j=1}^{n} z_j x_{ij} \leq \theta\left(x_{ik} - \varphi_k \times \vec{g}_x\right), \; i = 1,\dots,m \\
&\sum_{j=1}^{n} z_j = \theta, \quad j = 1,\dots,n \\
&z_j \geq 0, \; 0 \leq \theta \leq 1
\end{aligned} \quad , \qquad (4)
$$

where ϕ refers to the efficiency score. Model (4) is non-linear as well; however, it cannot be linearized using the Sahoo et al. (2011) transformation.

Kuosmanen (2005) and Kuosmanen and Podinovski (2009) criticized the use of a uniform abatement factor across all DMUs and proposed a model with a non-uniform abatement factor. The directional output distance function model with non-uniform abatement factors and VRS is given as:

$$
\begin{aligned}
&\max \delta_k \\
&\text{s.t.} \sum_{j=1}^{n} \theta_j \lambda_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, \; r = 1,\dots,s \\
&\sum_{j=1}^{n} \theta_j \lambda_j b_{qj} = b_{qk} - \delta_k \times \vec{g}_b, \; q = 1,\dots,p \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, \; i = 1,\dots,m \\
&\sum_{j=1}^{n} \lambda_j = 1, \; j = 1,\dots,n \\
&\lambda_j \geq 0, \quad 0 \leq \theta \leq 1
\end{aligned} \quad , \qquad (5)
$$

This is a non-linear model and to linearize it, Kuosmanen (2005) portioned the intensity variable λ_j as: $\lambda_j = \alpha_j + \mu_j$, where the former refers to the part of DMU's output which remains inactive and the latter to the part of DMU's output that is abated. The original abatement factor is given as: $\theta_j = \alpha_j / \left(\alpha_j + \mu_j\right)$. Then, Model (5) may be written as:

$$
\begin{aligned}
&\max \delta_k \\
&\text{s.t.} \sum_{j=1}^{n} \alpha_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, \; r = 1,\dots,s \\
&\sum_{j=1}^{n} \alpha_j b_{qj} = b_{qk} - \delta_k \times \vec{g}_b, q = 1,\dots,p \\
&\sum_{j=1}^{n} \left(\alpha_j + \mu_j\right) x_{ij} \leq x_{ik}, i = 1,\dots,m \\
&\sum_{j=1}^{n} \left(\alpha_j + \mu_j\right) = 1, j = 1,\dots,n \\
&\alpha_j, \mu_j \geq 0.
\end{aligned} \quad , \qquad (6)
$$

Similarly, the directional input distance function model with non-uniform abatement factors and VRS is given as:

$$\begin{aligned}&\max \varphi_k\\ &\text{s.t.} \sum_{j=1}^{n} a_j y_{rj} \geq y_{rk}, \quad r=1,\dots,s\\ &\sum_{j=1}^{n} a_j b_{qj} = b_{qk}, \quad q=1,\dots,p\\ &\sum_{j=1}^{n} \left(a_j+\mu_j\right) x_{ij} \leq x_{ik} - \varphi_k \times \vec{g}_x, \quad i=1,\dots,m\\ &\sum_{j=1}^{n} \left(a_j+\mu_j\right) = 1, \ j=1,\dots,n\\ &a_j, \mu_j \geq 0\end{aligned}, \tag{7}$$

Under the conventional weak disposability assumption, negative sloped segments of the efficient frontier are possible (Aparicio et al., 2013; Chen, 2014; Førsund, 2009; Kao & Hwang, 2019) and thus, we may have positive shadow prices for undesirable outputs. To overcome this problem, Leleu (2013) changed the equality constraint for the undesirable outputs in (3) to a 'less than or equal to' constraint. This formulation may be referred to as the constrained weak disposability model. Then, the directional output distance function model with a uniform abatement factor and VRS is given as:

$$\begin{aligned}&\max \delta_k\\ &\text{s.t.} \sum_{j=1}^{n} z_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, \quad r=1,\dots,s\\ &\sum_{j=1}^{n} z_j b_{qj} \leq b_{qk} - \delta_k \times \vec{g}_b, \quad q=1,\dots,p\\ &\sum_{j=1}^{n} z_j x_{ij} \leq \theta x_{ik}, \ i=1,\dots,m\\ &\sum_{j=1}^{n} z_j = \theta, \ j=1,\dots,n\\ &z_j \geq 0, \quad 0 \leq \theta \leq 1\end{aligned}, \tag{8}$$

Similarly, the corresponding directional input distance function is given as:

$$\begin{aligned}&\max \varphi_k\\ &\text{s.t.} \sum_{j=1}^{n} z_j y_{rj} \geq y_{rk}, \quad r=1,\dots,s,\\ &\sum_{j=1}^{n} z_j b_{qj} \leq b_{qk}, \quad q=1,\dots,p,\\ &\sum_{j=1}^{n} z_j x_{ij} \leq \theta\left(x_{ik} - \varphi_k \times \vec{g}_x\right), \quad i=1,\dots,m,\\ &\sum_{j=1}^{n} z_j = \theta, \quad j=1,\dots,n,\\ &z_j \geq 0, \quad 0 \leq \theta \leq 1.\end{aligned}, \tag{9}$$

Model (9) is non-linear and it cannot be transformed into a linear one.

If one wants to assume non-uniform abatement factors, then we can transform (6) by changing accordingly its input constraint assuming constrained weak disposability. This results in:

$$\begin{aligned}&\max \delta_k\\ &\text{s.t.} \sum_{j=1}^{n} a_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, \quad r=1,\dots,s\\ &\sum_{j=1}^{n} a_j b_{qj} \leq b_{qk} - \delta_k \times \vec{g}_b, \quad q=1,\dots,p\\ &\sum_{j=1}^{n} \left(a_j+\mu_j\right) x_{ij} \leq x_{ik}, \quad i=1,\dots,m\\ &\sum_{j=1}^{n} \left(a_j+\mu_j\right) = 1, \quad j=1,\dots,n\\ &a_j, \mu_j \geq 0\end{aligned}, \tag{10}$$

Similarly, the corresponding directional input distance function model is:

$$\begin{aligned}&\max \varphi_k\\ &\text{s.t.} \sum_{j=1}^{n} a_j y_{rj} \geq y_{rk}, \quad r=1,\dots,s\\ &\sum_{j=1}^{n} a_j b_{qj} \leq b_{qk}, \quad q=1,\dots,p\\ &\sum_{j=1}^{n} \left(a_j+\mu_j\right) x_{ij} \leq x_{ik} - \varphi_k \times \vec{g}_x, \quad i=1,\dots,m\\ &\sum_{j=1}^{n} \left(a_j+\mu_j\right) = 1, \quad j=1,\dots,n\\ &a_j, \mu_j \geq 0\end{aligned}, \tag{11}$$

3.2 | Models with extended strong disposability

The modelling of the reverse variables in DEA depends on both the orientation of the model and on whether the reverse variable is an input or an output. According to Lewis and Sexton (2004), to model an output as reverse one has to change the direction of the inequality sign in the relevant constraint. Then, the directional input distance function model with VRS is given as:

$$\begin{aligned} &\max \varphi_k \\ \text{s.t.} &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, \quad r=1,\dots,s \\ &\sum_{j=1}^{n} \lambda_j b_{qj} \leq b_{qk}, \quad q=1,\dots,p \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} - \varphi_k \times \vec{g}_x, \quad i=1,\dots,m \\ &\sum_{j=1}^{n} \lambda_j = 1, \quad j=1,\dots,n \\ &\lambda_j \geq 0 \end{aligned}, \qquad (12)$$

According to Färe and Grosskopf (2013), Aparicio et al. (2016) and Charles et al. (2016), a directional input distance function model under VRS is translation invariant as long as the direction vector is not DMU-specific. In this case, we can apply a positive affine transformation to the data without affecting the efficiency scores. This is equivalent to Seiford and Zhu (2002) transformation:

$$h(b_{qj}) = \sigma_j b_{qj} + c_j = -b_{qj} + c_j,$$

where $\sigma_j = -1$ and c_j is a sufficiently large scalar common for all DMUs which ensures that the transformed variable is positive. Consequently, Model (12) can be written as:

$$\begin{aligned} &\max \varphi_k \\ \text{s.t.} &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, r=1,\dots,s \\ &\sum_{j=1}^{n} \lambda_j (-b_{qj} + c_j) \geq -b_{qk} + c_k, q=1,\dots,p \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} - \varphi_k \times \vec{g}_x, i=1,\dots,m \\ &\sum_{j=1}^{n} \lambda_j = 1, j=1,\dots,n \\ &\lambda_j \geq 0. \end{aligned}, \qquad (13)$$

For comparability reasons, we also examine the output-oriented formulation of this model:

$$\begin{aligned} &\max \delta_k \\ \text{s.t.} &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk} + \delta_k \times \vec{g}_y, r=1,\dots,s \\ &\sum_{j=1}^{n} \lambda_j (-b_{qj} + c_j) \geq -b_{qk} + c_k + \delta_k \times \vec{g}_b, q=1,\dots,p \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik}, i=1,\dots,m \\ &\sum_{j=1}^{n} \lambda_j = 1, j=1,\dots,n \\ &\lambda_j \geq 0 \end{aligned}, \qquad (14)$$

Note that according to Färe and Grosskopf (2013), σ_j changes the unit of measurement for the b_{qk} and δ_k is independent of the unit of measurement, and hence $\vec{g}_b$ also needs to change into $\sigma_j \times \vec{g}_b$ in order to maintain the same units as $\sigma_j \times b_{qk}$. Consequently, the right hand side of the second constraint in (13) is $-b_{qk} + c_k - \delta_k \times \sigma_j \times \vec{g}_b = -b_{qk} + c_k - \delta_k \times (-1) \times \vec{g}_b = -b$ $+c_k + \delta_k \times \vec{g}_b$.

In summary, we introduce (13) and (14), representing the input and output orientations, respectively, for a model that assumes extended strong disposability and treat NPLs as a reverse rather than a bad output. Model (13) will be compared with Models (7) and (11), which are the input-oriented models under the weak and the constrained weak disposability, respectively, with a non-uniform abatement factor. It is worth noting that Models (4) and (9), equivalent to (7) and (11) but with a uniform abatement factor, cannot be linearised and they will not be utilized further in this paper. Thus, under VRS, the non-uniform abatement factor specification is linear for both the input and output-oriented models, whereas the uniform abatement factor specification is linear for only the output-oriented model. This holds for both radial and directional models (refer to Table 1). Furthermore, Model (14) will be compared with Models (3), (6), (8) and (10), which corresponds to models assuming weak disposability of undesirable outputs and a uniform abatement factor, weak disposability of undesirable outputs and non-uniform abatement factor, constrained weak disposability of undesirable outputs and a uniform abatement factor, and constrained weak disposability of undesirable outputs and non-uniform abatement factors, respectively.

TABLE 1 Linear and non-linear models under weak disposability assumption.

Modelling approach	Input-oriented	Output-oriented
Weak disposability of undesirable outputs and a uniform abatement factor	Non-linear	Linear
Weak disposability of undesirable outputs and a non-uniform abatement factors	Linear	Linear
Constrained weak disposability of undesirable outputs and a uniform abatement factor	Non-linear	Linear
Constrained weak disposability of undesirable outputs and non-uniform abatement factors	Linear	Linear
Extended strong disposability with reverse output	Linear	Linear

TABLE 2 Summary statistics.

	Employees	Fixed assets	Deposits	Net loans	Investments	NPLs
Mean	51056.00	5849351.67	332991785.39	385372047.18	213496475.76	10168055.67
St.dev.	51426.13	8900844.55	382054452.23	576469095.74	336996639.80	10066033.28
Median	28,051	2,392,442	160,586,690	187,314,600	62,555,460	5,278,000
Max	193,319	32,895,000	1,829,029,509	3,253,493,331	1,368,637,755	39,973,352
Min	3349	421,028	18,399,129	31,600,807	13,430,260	696,820
Skewness	1.53	2.27	2.24	4.09	2.29	1.33
Kurtosis	1.69	4.23	6.37	19.91	4.79	1.23

Abbreviation: NPLs, non-performing loans.

4 | EMPIRICAL ANALYSIS

4.1 | Data

In this section we study the NPLs of European banks. For our sample to be representative of the population of European banks, we choose the 50 banks which were included at the European Banking Authority (EBA) stress test of 2021, which cover 70% of the EU banking sector assets (EBA, 2021). This sample includes banks from 15 EU and European Economic Area countries and therefore covers the EU sufficiently, both in geographical and economic terms.

Regarding the input–output framework, this paper adopts the intermediation approach as it was discussed in Section 2.1 (Berger & Humphrey, 1992). In this case, banks treat deposits as inputs along with the number of employees and fixed assets. On the output side, banks produce net loans and investments, which are considered as forward (good) outputs and NPLs which are treated as reverse outputs. All data have been collected from Refinitiv Workspace and Datastream, and only banks with complete information were considered. As a result, the dataset is reduced, due to the missing information mainly on NPLs for a number of banks, to a total of 33 banks. Table 2 provides the summary statistics for all the variables used in this paper.

As discussed in the previous section, we use a DDF approach for the results to be comparable with the previous literature. The chosen directional vector is $g=\left(g_x,g_y,g_b\right)=(-1,0,0)$ which ensures that the (13) is translation invariant since the directional vector of (12) and the directional vector of the translated Model (13) are the same (Aparicio et al., 2016). According to Färe and Grosskopf (2004b) and Färe et al. (2005), the solution of the model with a directional vector of $(-1,0,0)$ can be interpreted as a slack, which in our case gives the net improvement in performance in terms of feasible decreases in inputs. Furthermore, an input orientation is adopted since the cost minimization seems the most reasonable strategy for the bank and it is also consistent with the majority of the previous literature. In addition, the dataset contains banks which are quite different in sizes, therefore a VRS assumption is applied.

4.2 | Results

The first part of this section presents the results of the reverse output model (13) and compares them with the results of the two alternative models in the literature, which are linear under the input orientation (see Table 1), namely the weak disposability with a non-uniform abatement factor, namely (7), and the constrained weak disposability with non-uniform abatement factors, namely (11).[4]

Figure 1 provides the visual representation of the results for the three models and Table 3 the summary

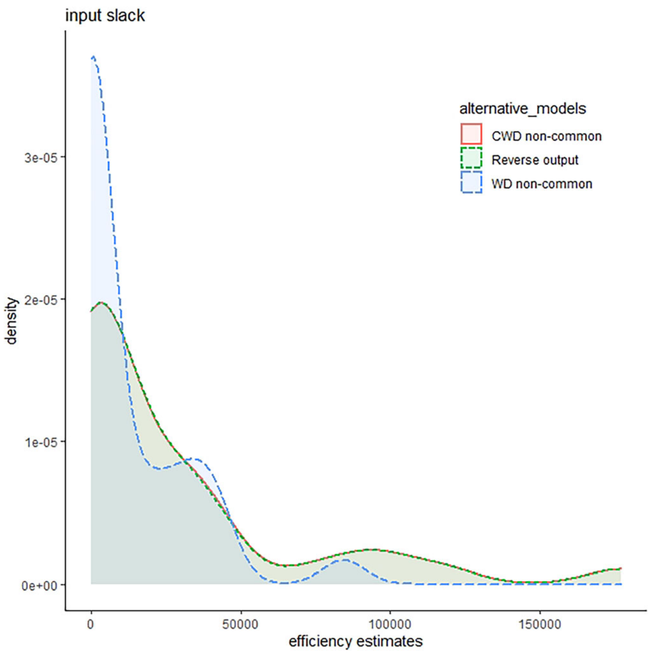

FIGURE 1 Kernel density plots for the input slacks. [Colour figure can be viewed at wileyonlinelibrary.com]

statistics. Both the kernels and the descriptive statistics reveal that (13) and (11) have very similar distributions of the efficiency scores with 14 and 13 fully efficient banks, respectively. Furthermore, (13) and (11) yield identical results for 31 banks and only 2 banks are found to have a different efficiency scores. This happens for this dataset because only a very small number of θs (abatement factors) are not equal to 1. On the other hand, (7) yields lower inefficiency scores with 18 banks deemed to be efficient. Moreover, with (7) the median bank is fully efficient. Thus, (11) and (13) have greater discriminatory power relative to (7).

Using the inefficiency scores, we can calculate the percentage targets for each input. This can be calculated at optimality as:

$$\text{Input percentage target} = \frac{\text{observed input} - \text{slack}}{\text{observed input}}. \quad (15)$$

Figure 2 displays the kernel density plots for the three inputs percentage targets used in this paper, and Table 4 presents their descriptive statistics. These plots generate a smooth curve that estimates the probability density function of a continuous variable, providing a useful representation of the distribution of such a variable. In Figure 2, we can observe a considerable room for employee reductions. Models (11) and (13) yield nearly identical results, whereas (7) produces higher percentage targets. The results are also confirmed by those reported in Table 4. Moving from left to right, we see that the three models suggest that banks should operate at 54%, 73% and 53% of the current input usage, respectively. Regarding fixed assets and deposits, the results are very similar for the three models, indicating limited room for improvement.

Another interesting aspect to examine is the average rank shift for each input percentage target, which is defined as the absolute value of the difference between the ranks of two models. For example, if a bank has been ranked first with (13) and twenty-first with (7), then the average rank difference would be 20. Table 5 verifies that ranking with (13) and (11) are very close to each other, while Model (7) is different as the average rank shift ranges from 4.03 to 6.12. Using a Mann–Whitney test, we verify that the differences between (13) and (11) are not statistically significant, whereas there are statistically significant differences between (7) and the other two models.[5]

As noted in Section 2.2, previous studies that incorporated the NPLs in the model and use a DDF approach also used an output orientation. In order to compare our results with theirs we now turn into output-oriented Model (14). The chosen directional vector is $g = \left(g_x, g_y, g_b\right) = (0, 1, -1)$ which ensures that (14) is translation invariant. According to Färe et al. (2005), the solution of the model can be interpreted as a slack, which in our case indicates by how much forward output(s) can be increased and reverse output(s) can be decreased, with given inputs and technology. We compare the results from the reverse output formulation (14) with four alternative models in the literature treating NPLs as undesirable outputs, namely the weak disposability with a uniform abatement factor Model (3), the weak disposability with non-uniform abatement factors Model (6), the constrained weak disposability with a uniform abatement factor Model (8), and the constrained weak disposability with non-uniform abatement factors Model (10).

Figure 3 is a visual representation of the results for the five models and Table 6 presents the summary statistics. Both the kernels and the descriptive statistics reveal that (14) has a very different distribution relative to all others, whereas the other four models have a very similar distribution of efficiency scores to each other. Notice that (14) has again the lowest number of efficient DMUs (only 11) and therefore, a greater discriminatory power.

Using the inefficiency scores, we can also calculate the percentage targets for each output. This can be calculated at optimality as:

$$\text{Input percentage target} = \frac{\text{observed output}}{\text{observed output} + \text{slack}}. \quad (16)$$

Figure 4 illustrates the kernel density plots for the three output percentage targets, and Table 7 provides

TABLE 3 Descriptive statistics of slacks for input oriented models.

	Reverse output	WD non-uniform	CWD non-uniform
Mean	26587.41	11740.23	26717.26
St. dev	41520.03	19513.56	41502.10
Median	6066.71	0.0000	6066.71
Max	177296.82	84365.03	177296.82
Skewness	2.06	1.94	2.06
Kurtosis	6.97	6.89	6.96
# Of efficient DMUs	14	18	13

Note: WD non-uniform refers to weak disposability with non-uniform abatement factors and CWD non-uniform to constrained weak disposability with non-uniform abatement factors.

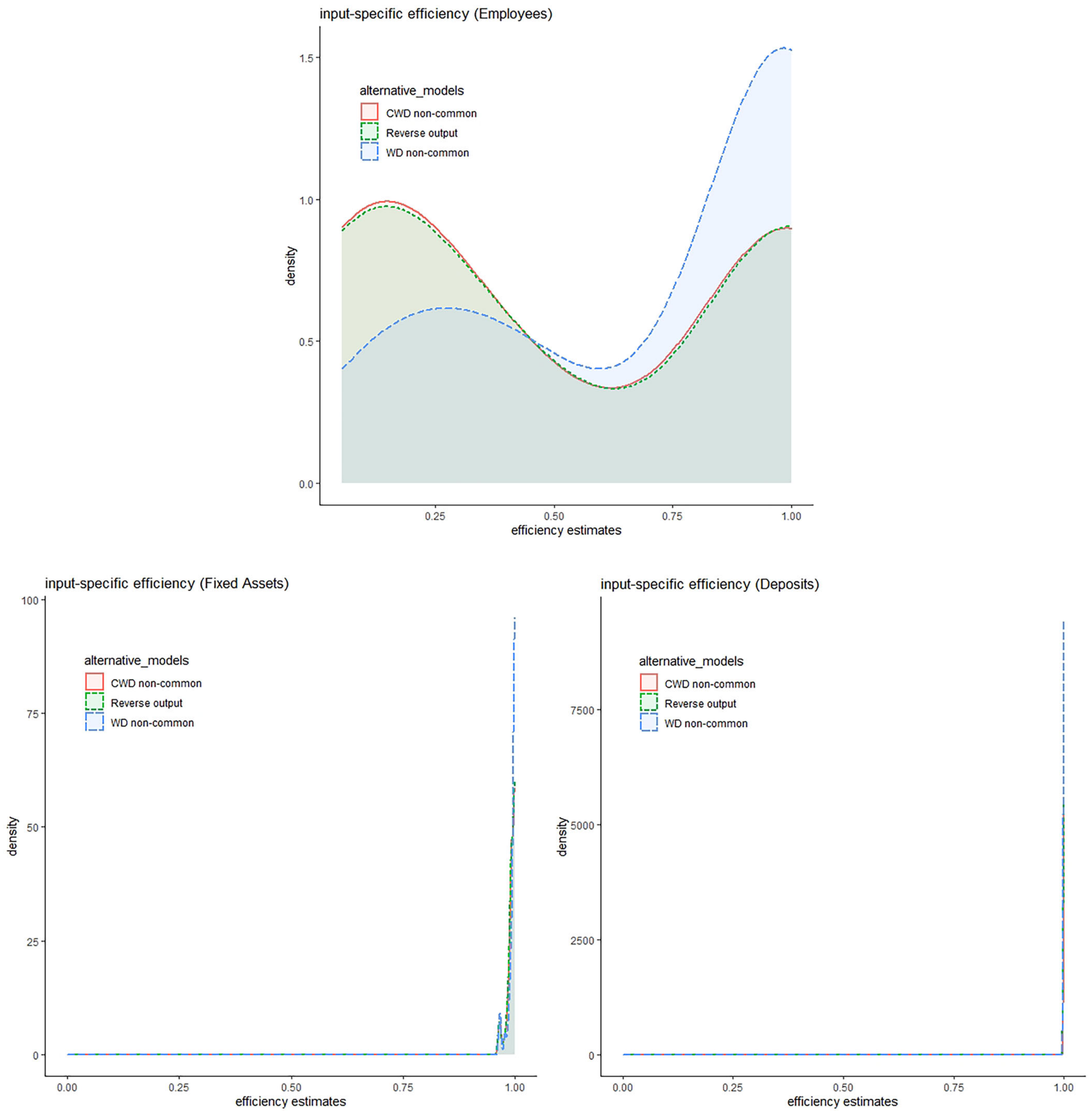

FIGURE 2 Distribution of input percentage targets. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 4 Descriptive statistics of the input percentage targets.

		#Eff	Mean	St. dev	Median	Min	Skew	Kurt
Reverse output	Employees	14	0.54	0.42	0.43	0.05	0.12	1.18
	Fix. Assets	14	0.99	0.01	0.99	0.96	−1.69	5.55
	Deposits	14	1.00	0.00	1.00	1.00	−1.86	6.89
WD non-uniform	Employees	18	0.73	0.35	1.00	0.10	−0.73	1.83
	Fix. Assets	18	0.99	0.01	1.00	0.97	−2.18	6.95
	Deposits	18	1.00	0.00	1.00	1.00	−2.55	9.99
CWD non-uniform	Employees	13	0.53	0.42	0.38	0.05	0.14	1.19
	Fix. Assets	13	0.99	0.01	0.99	0.96	−1.70	5.57
	Deposits	13	1.00	0.00	1.00	1.00	−1.86	6.86

Note: WD non-uniform refers to weak disposability with non-uniform abatement factors and CWD non-uniform to constrained weak disposability with non-uniform abatement factors.

TABLE 5 Average rank shift for the input percentage targets.

		Reverse output	WD non-uniform	CWD non-uniform
Employees	Reverse output	-	6.12	0.52
	WD non-uniform	6.12	-	5.73
	CWD non-uniform	0.52	5.73	-
Fixed assets	Reverse output	-	4.55	0.58
	WD non-uniform	4.55	-	4.03
	CWD non-uniform	0.58	4.03	-
Deposits	Reverse output	-	4.97	0.45
	WD non-uniform	4.97	-	4.58
	CWD non-uniform	0.45	4.58	-

Note: WD non-uniform refers to weak disposability with non-uniform abatement factors and CWD non-uniform to constrained weak disposability with non-uniform abatement factors.

descriptive statistics for these targets. In Table 7, moving from left to right, we see that the results of (3) are identical to those of (8), and the results of (6) are identical to those of (10). This is because, in this dataset, we found no positive shadow prices for the NPLs. As a result, both formulations of the abatement factor (the weak and the constrained weak disposability models) give the same scores. Furthermore, the choice of a uniform or non-uniform abatement factor does not appear to significantly impact the resulting scores and rankings, as most θs are equal to 1. On the other hand, Figure 4 demonstrates that the kernel density of the distribution of the efficiency scores for (14) significantly deviates from the others. Using a Kruskal-Wallis test, it is confirmed that the differences between (3), (6), (8) and (10) are not statistically significant. However, the Mann–Whitney test confirms that there are statistically significant differences between (14) and the other four models.[6] Therefore, the treatment of NPLs as a reverse output has a significant impact on both on the efficiency scores and the rankings.

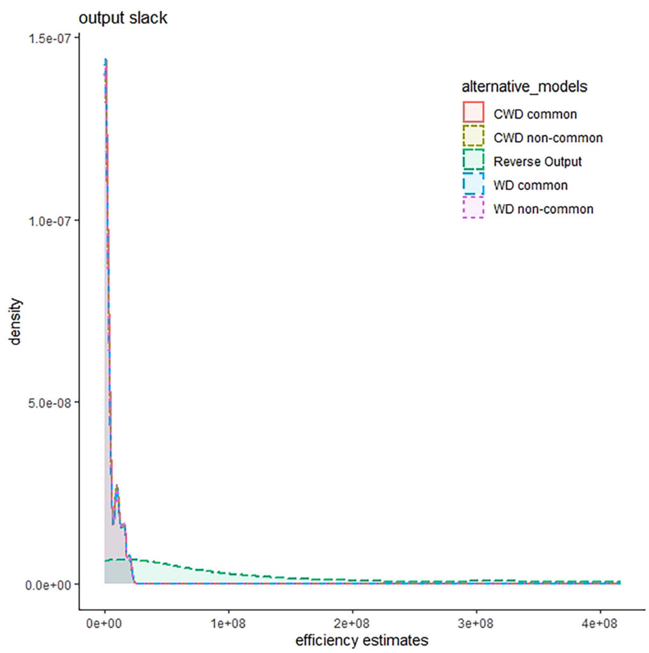

FIGURE 3 Kernel density plots for the output slacks. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 6 Descriptive statistics of slacks for output-oriented models.

	Reverse output	WD U	WD NU	CWD U	CWD NU
Mean	75,007,213.77	3,872,279.38	3,927,931.78	3,872,279.38	3,927,931.78
St. dev	105,145,329.33	5,566,763.35	5,554,980.03	5,566,763.35	5,554,980.03
Median	36,857,246.95	1,230,879.06	1,568,021.40	1,230,879.06	1,568,021.40
Max	415,629,735.02	20,365,140.94	20,365,140.94	20,365,140.94	20,365,140.94
Skewness	1.77	1.50	1.48	1.50	1.48
Kurtosis	5.49	4.20	4.18	4.20	4.18
# Eff DMUs	11	13	13	13	13

Note: WD U refers to weak disposability with a uniform abatement factor, WD NU to weak disposability with non-uniform abatement factors, CWD U to constrained weak disposability with a uniform abatement factor and CWD NU to constrained weak disposability with non-uniform abatement factors.

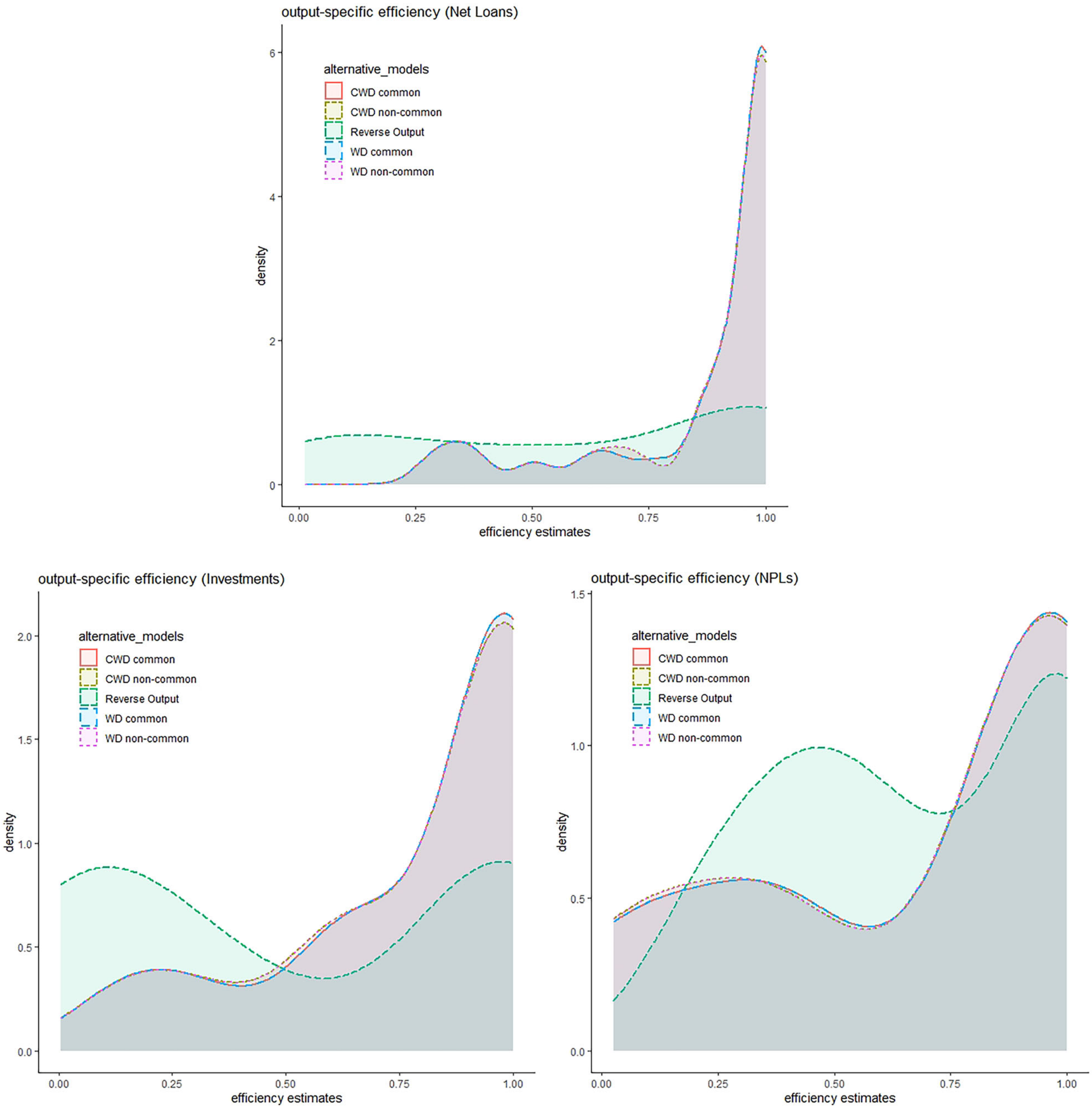

FIGURE 4 Distribution of output percentage targets. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 7 Descriptive statistics of the output percentage targets.

		#Eff	Mean	St. dev	Median	Min	Skew	Kurt
Reverse output	Net loans	11	0.61	0.39	0.68	0.01	−0.35	−1.59
	Investments	11	0.53	0.43	0.42	0.00	0.02	−1.91
	NPLs	11	0.68	0.30	0.67	0.17	−0.20	−1.46
WD uniform	Net loans	13	0.87	0.21	0.99	0.28	−1.83	2.19
	Investments	13	0.80	0.29	0.97	0.11	−1.32	0.49
	NPLs	13	0.70	0.37	0.92	0.03	−0.78	−1.06
WD non-uniform	Net loans	13	0.87	0.21	0.99	0.28	−1.80	2.08
	Investments	13	0.80	0.29	0.97	0.11	−1.28	0.40
	NPLs	13	0.69	0.37	0.90	0.03	−0.76	−1.11
CWD uniform	Net loans	13	0.87	0.21	0.99	0.28	−1.83	2.19
	Investments	13	0.80	0.29	0.97	0.11	−1.32	0.49
	NPLs	13	0.70	0.37	0.92	0.03	−0.78	−1.06
CWD non-uniform	Net loans	13	0.87	0.21	0.99	0.28	−1.80	2.08
	Investments	13	0.80	0.29	0.97	0.11	−1.28	0.40
	NPLs	13	0.69	0.37	0.90	0.03	−0.76	−1.11

Note: WD U refers to weak disposability with a uniform abatement factor, WD NU to weak disposability with non-uniform abatement factors, CWD U to constrained weak disposability with a uniform abatement factor and CWD NU to constrained weak disposability with non-uniform abatement factors.

TABLE 8 Average rank shift for the output percentage targets.

		Reverse output	WD uniform	WD non-uniform	CWD uniform	CWD non-uniform
Net loans	Rev. output	-	1.73	1.73	1.73	1.73
	WD C	1.73	-	0.00	0.00	0.00
	WD NC	1.73	0.00	-	0.00	0.00
	CWD C	1.73	0.00	0.00	-	0.00
	CWD NC	1.73	0.00	0.00	0.00	-
Investments	Rev. output	-	1.79	1.67	1.79	1.67
	WD C	1.79	-	0.12	0.00	0.12
	WD NC	1.67	0.12	-	0.12	0.00
	CWD C	1.79	0.00	0.12	-	0.12
	CWD NC	1.67	0.12	0.00	0.12	-
NPLs	Rev. output	-	3.36	3.42	3.36	3.42
	WD C	3.36	-	0.18	0.00	0.18
	WD NC	3.42	0.18	-	0.18	0.00
	CWD C	3.36	0.00	0.18	-	0.18
	CWD NC	3.42	0.18	0.00	1.77	-

Note: WD U refers to weak disposability with a uniform abatement factor, WD NU to weak disposability with non-uniform abatement factors, CWD U to constrained weak disposability with a uniform abatement factor and CWD NU to constrained weak disposability with non-uniform abatement factors.

Table 8 reports the average rank shift for each output percentage target. The table verifies that the results of (3) are identical to those of (8), and the results of (6) are identical to those of (10), and all four of them are very close to each other. On the other hand, (14) is different from the other models, especially for the case of NPLs.

5 | CONCLUSIONS

This paper uses the notion of reverse outputs and extended strong disposability to consider bank NPLs. The appropriate modelling of NPLs is a key issue and of extreme importance to all the interested parties that would like to evaluate the efficiency of banks, such as shareholders, bank managers, competitors, regulators and credit rating companies. We argue that net loans and investments are forward (good) outputs while the NPLs are a reverse output. In this framework, forward and reverse outputs are not required to be produced jointly. Indeed, a bank could give a loan which will turn into NPL without the requirement to give a good loan on the same time, while keep its inputs constant. This is to be distinguished from the case of an environmental pollutant, where a unit of output cannot be produced without the production of the environmental pollutant and vice versa. In addition, even if the bank gives no new loans, existing loans could turn into NPLs, resulting in an increasing of NPLs without producing more net loans on the same time. This is also to be distinguished from the case of an environmental pollutant, where a good output cannot turn into an undesirable output.

We use a directional input distance function since the cost minimization strategy seems the most appealing strategy for the bank and is aligned with the majority of the literature. In addition, we examine the case of the output orientation in order our results to be comparable with those of previous studies accounting for NPLs. We compare and contrast our results with the alternative models, which treat the NPLs as an undesirable output.

The results for both input and output orientation models show that the model considering NPLS as a reverse output offers greater discriminatory power relative to all other models. Essentially, this allows the decision maker to better distinguish between banks which otherwise would be considered as fully efficient, thus making the comparison of rankings between banks more meaningful. Specifically, for the case of input orientation, the results of the reverse output model are almost identical to the constrained weak disposability model with non-uniform abatement factors and significantly different than the weak disposability model with non-uniform abatement factors. Thus, for the case of input orientation, the model with extended strong and constrained weak disposability has greater discriminatory power relative to that of the weak disposability model. On the contrary, for the case of output orientation, the reverse output model is significantly different than all the other models. Thus, for the case of output orientation, the model with extended strong disposability has greater discriminatory power relative to all the other models. These empirical findings support our choice of modelling NPLs as a reverse output.

This paper offers potential avenues for further research. Although this paper uses the unit directional vector, it is worth exploring alternative directional vectors, provided they are not DMU-specific. Otherwise, the model will no longer be translation invariant (Aparicio et al., 2016; Charles et al., 2016; Färe & Grosskopf, 2013), making it impossible to handle NPLs through variable transformation. Additionally, these models could be extended and compared on the basis of super-efficiency to address the issue of comparing efficient DMUs that are all situated on the boundary.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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ENDNOTES

[1] https://www.bankingsupervision.europa.eu/about/ssmexplained/html/npl.en.html.

[2] However, the specific time threshold for considering a loan as non-performing varies slightly across countries.

[3] https://www.bankingsupervision.europa.eu/press/pr/date/2023/html/ssm.pr23712~c5bce797c5.en.html.

[4] Recall that both the weak disposability and the constrained weak disposability models with a uniform abatement factor are not linear under input orientation (see [4] and [9]).

[5] Mann–Whitney test between (13) and (11) has a value of 535 and the p-value of the test is 0.900; between (7) and (11) has a value of 395 and the p-value of the test is 0.043; between (7) and (13) has a value of 405 and the p-value of the test is 0.057.

[6] Kruskal-Wallis test for (3), (6), (8), and (10) has a value of 0.007 and the p-value of the test is 1.000. Mann–Whitney test between (14) and (3) has a value of 385 and the p-value of the test is 0.035; between (14) and (6) has a value of 386 and the p-value of the test is 0.037; between (14) and (8) has a value of 385 and the p-value of the test is 0.035; between (14) and (10) has a value of 386 and the p-value of the test is 0.037.

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