



University of Dundee

Dynamic response of multi-unit floating offshore wind turbines to wave, current and wind loads

Lamei, Azin; Hayatdavoodi, Masoud; Riggs, H. Ronald; Ertekin, R. Cengiz

DOI:
[10.1063/5.0172543](https://doi.org/10.1063/5.0172543)

Publication date:
2024

Document Version
Peer reviewed version

[Link to publication in Discovery Research Portal](#)

Citation for published version (APA):
Lamei, A., Hayatdavoodi, M., Riggs, H. R., & Ertekin, R. C. (2024). Dynamic response of multi-unit floating offshore wind turbines to wave, current and wind loads. *Journal of Renewable and Sustainable Energy*, 16(2), Article 023307. <https://doi.org/10.1063/5.0172543>

General rights

Copyright and moral rights for the publications made accessible in Discovery Research Portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

1 **Dynamic response of multi-unit floating offshore wind turbines to wave, current**
2 **and wind loads**

3 A. Lamei,¹ M. Hayatdavoodi,^{1,2} H.R. Riggs,³ and R. C. Ertekin^{2, 4}

4 ¹*School of Science and Engineering, University of Dundee, Dundee DD1 4HN,*
5 *UK*

6 ²*College of Shipbuilding Engineering, Harbin Engineering University, Harbin,*
7 *China*

8 ³*Civil and Environmental Engineering Department, University of Hawaii, Honolulu,*
9 *HI 96822, USA*

10 ⁴*Ocean and Resources Engineering Dep., University of Hawaii, Honolulu,*
11 *Hawaii 96822, USA*

12 (*Electronic mail: m.hayatdavoodi@dundee.ac.uk)

13 (Dated: 14 February 2024)

14 Motion of a multi-unit wind-tracing floating offshore wind turbine (FOWT) to combined
15 wave-current and wind are obtained in the frequency-domain. The linear diffraction wave
16 theory with a Green function for small current speeds and the blade-element momentum
17 method are used for the hydrodynamic and aerodynamic analysis, respectively. Finite-
18 element method is coupled with the hydrodynamic and aerodynamic equations to obtain
19 the elastic responses of the FOWT to the environmental loads. The wind-tracing FOWT
20 consists of three 5 MW wind turbines installed at the corners of an equilateral triangular
21 platform. The platform is connected to the seabed through a turret-bearing mooring system,
22 allowing the structure to rotate and face the dominant wind direction, hence the multi-unit
23 FOWT is called the wind-tracing FOWT. In this study, rigid-body responses of the wind-
24 tracing FOWT to waves and wind are compared with those to combined wave, current
25 and wind loads for several current speeds and various wave heading angles. For a chosen
26 current speed and wave heading angle, hydro- and aeroelastic responses of the wind-tracing
27 FOWT to combined waves, current and wind are obtained and compared with those of the
28 rigid structure. Discussion is provided on the effect of wave-current interaction on the
29 motion and elastic responses of the wind-tracing FOWT. The numerical results show that
30 under the rated wind speed, the motion of the wind-tracing FOWT is mainly governed by
31 the wave-induced hydrodynamic forces and moments and the presence of current results
32 in larger elastic motion of the FOWT to the environmental loads.

33 I. INTRODUCTION

34 Offshore wind turbines with floating substructures experience complex dynamics compared
35 with those on bottom-fixed foundations. The motion of the floating platform is due to the simulta-
36 neous effect of waves and current, hydrostatic and mooring restoring forces, and loads due to the
37 aerodynamic load on the rotor and the tower of the wind turbine. As a result of the platform mo-
38 tion of a FOWT, the direction of the incoming flow to the rotor and consequently the aerodynamic
39 load on the wind turbine changes. In addition to the environmental loads, the elasticity of the wind
40 turbine, *i.e.* its blades and tower, and the floating substructure is of great importance for the power
41 output of a FOWT and its motion analysis.

42 To study the motion of a FOWT due to the environmental loads and its power output, an ac-
43 curate understanding of the dynamics of the floating substructure and its coupling effect on the
44 wind turbine is essential. High-fidelity methods such as computational fluid dynamics (CFD) pro-
45 vide detailed description of the fluid interaction (water and air) with the FOWTs, see Tran and
46 Kim (2016), Cheng *et al.* (2019) and Zhou *et al.* (2022), among others. However, in practice,
47 CFD calculations, due to the heavy computational demand, can only be applied for a simplified
48 presentation of the wave and wind-interaction with a FOWT such that either the substructure or
49 the entire FOWT is rigid and its motion is constrained to limited degrees of freedom. Hence,
50 high-fidelity methods are not the best option to gain an understanding of the rigid-body and elastic
51 motions of a new concept of FOWTs exposed to the environmental loads. For instance, to the au-
52 thors' knowledge, high-fidelity methods are not applied to investigate elastic motion of multi-unit
53 FOWTs, where multiple towers are installed on a single platform, subject to simultaneous effect
54 of aerodynamic and hydrodynamic loads.

55 On the other hand, in design and concept stages of FOWTs, medium- and low-fidelity meth-

66 ods are more desirable as they provide an efficient and fast solution of the dynamic motion of
67 FOWTs due to the environmental loads. Although these methods assume that the motion of the
68 substructure is small, they can include the flexibility of the entire structure of the FOWTs in their
69 motion analysis, see Lamei *et al.* (2023a) and Lamei *et al.* (2023b). In multi-unit FOWTs, due
60 to the large size of the substructure, and the coupling effect between the wind turbines and the
61 substructure, hydro- and aeroelasticity analyses of the entire structure are equally important and
62 should not be neglected. A review of various approaches for the analysis of responses of FOWTs
63 to hydrodynamic and aerodynamic loads can be found in Lamei and Hayatdavoodi (2020).

64 To date, in high-, medium- and low-fidelity methods, little attention is given to wave-current
65 interaction with single- or multi-unit FOWTs. In addition to the direct effect of current on struc-
66 tures, depending on the current speed, its direction and velocity profile, current interaction with
67 incoming waves can change the wave properties. Hence, the wave-induced hydrodynamic forces
68 on a floating structure might be different when current is present. See Kumar and Hayatdavoodi
69 (2023a,b) for more details on the effect of current on periodic waves in deep and shallow waters,
70 respectively.

71 The wave-current interaction with a FOWT can be significant considering its mooring system
72 and the geometrical characteristics of the floating platform. For instance, wave-current interaction
73 with a SPAR FOWT results in vortex-induced vibration due to their long cylindrical substructure.
74 Chen *et al.* (2018), Qu *et al.* (2020) and Silva *et al.* (2021) studied the wave-current interaction
75 with a SPAR FOWT using low-fidelity methods and showed that depending on the layout of the
76 mooring system, current effects, in the absence of waves, are confined to static displacement of
77 the mooring lines. However, the wave-current interaction can influence the dynamic motion of the
78 SPAR FOWT significantly.

79 Several concepts of multi-unit FOWTs with two and three wind turbines have been proposed

80 and their motion to combined wave and wind loads are investigated; see for instance Bae and Kim
81 (2014, 2015), Bashedy and Ozcelik (2020) and Lamei *et al.* (2023a), among others. Multi-unit
82 FOWTs undergo larger hydrodynamic loads due to their large substructures in comparison with
83 single-unit FOWTs. Accordingly, depending on the characteristics of the floating platform and the
84 layout of their mooring system, wave-current interaction may significantly change the hydrody-
85 namic forces and moments on the structure. In a study by Kang *et al.* (2017), the elastic motion
86 of a quadrilateral platform, with four wind turbines, exposed to combined wave, current and wind
87 loads was investigated, and a strong coupling between the elastic motion of the substructure to non-
88 linear waves and the mooring lines was observed. Hence, it is essential to include the wave-current
89 interaction in the hydrodynamic analysis of multi-unit FOWTs. To the authors' knowledge, other
90 than Kang *et al.* (2017), no other study has considered the effect of wave-current-wind interaction
91 with multi-unit FOWTs.

92 The present study is concerned with comparing rigid-body and elastic motion of the wind-
93 tracing FOWT to combined wave, current and wind loads. The wind-tracing FOWT, introduced
94 by Wong (2015), consists of three 5 MW NREL wind turbines that are supported by a triangular
95 floating platform. The wind-tracing platform is moored with a turret-bearing mooring system that
96 allows the structure to rotate with respect to the turret such that the wind turbines are aligned
97 with the dominant aerodynamic loads on the wind turbines. To identify the preferred location of
98 the turret, Lamei *et al.* (2023b) conducted a parametric study involving the turret location and
99 compared the rigid-body responses of the wind-tracing FOWT to waves aligned and misaligned
100 with wind.

101 Recently, Lamei *et al.* (2023b,c) developed and introduced a numerical coupling approach for
102 FOWTs in which wave-current-wind interaction with the structure is considered. The numerical
103 approach allows the inclusion of the elasticity of the entire structure, *i.e.* blades, tower and the

104 floating substructure of the FOWTs, and can be applied to both single- and multi-unit FOWTs, see
105 Lamei *et al.* (2023a). The hydro- and aerodynamic analyses are based on low-fidelity methods,
106 the linear diffraction theory for small current speeds and the blade-element momentum method.
107 Furthermore, to include the flexibility of the entire structure, dynamic analysis of the FOWT is
108 coupled with the finite-element method. Using this approach, Lamei *et al.* (2023b) conducted a
109 comparative study on the rigid body responses and elastic motions of single-unit FOWTs, namely
110 a SPAR, a barge and a semisubmersible FOWT to combined wave, current and wind loads. It
111 was shown that motion of the SPAR FOWT undergoes largest changes when current is present in
112 comparison with its motion exposed to waves and wind.

113 Studying the effect of wave-current-wind interaction on the motion of multi-unit FOWTs is of
114 interest. The wind-tracing FOWT involves unique characteristics in the design of its substructure
115 and mooring layout that are missing in single-unit FOWTs. Therefore, in the present study, the
116 approach by Lamei *et al.* (2023c) is applied to study the dynamic motion of the wind-tracing
117 FOWT to wave, current and wind loads. Due to the presence of multiple wind turbines on the
118 wind-tracing substructure and the large floating platform, hydro- and aeroelastic responses of the
119 multi-unit FOWT might be significant. Thus, it is essential to investigate the elastic deformation
120 of the entire structure of the wind-tracing FOWT and its effect on dynamic motion of the FOWT to
121 various environmental loads. Furthermore, by considering both rigid-body and elastic responses,
122 one can evaluate the importance of elasticity analysis of the FOWT and identify the environmental
123 conditions where its elastic responses are significant.

124 The theory and the developed numerical approach on hydro- and aeroelastic analysis of FOWTs
125 to the environmental loads are discussed in Section II. Section III presents the rigid-body responses
126 of the wind-tracing FOWT to the environmental loads for various current speeds and misalign-
127 ments of incoming waves with wind and current. Under the same environmental loading, elastic

128 motion of the wind-tracing FOWT and the structural responses along the towers and the pontoons
129 are presented and discussed in Section V. Finally, concluding remarks on the effect of the wave-
130 current-wind interaction on the elastic motion of the wind-tracing FOWT are provided in Section
131 VI.

132 II. THEORY AND NUMERICAL SOLUTION

133 The theory and numerical solution of wave-current-wind interaction with FOWTs is presented
134 and discussed in detail by Lamei *et al.* (2023a,b,c). In this section, the coupling approach is
135 discussed focusing on multi-unit FOWTs.

136 Shown in Fig. 1, a moving Cartesian coordinate system is defined with its origin on the still-
137 water-level (SWL) and the z -axis positive pointing upwards. The coordinate system is in steady
138 translation with the current speed, U_c along the x -axis. In this study, it is assumed that the incoming
139 wind is orthogonal to the rotor-plane area and codirectional with the x -axis. Furthermore, the
140 incoming current can be in the positive or negative x -directions. Therefore, to model the wave-
141 current misalignment, the wave heading angle, β , changes.

142 A. Structural analysis

143 A three-dimensional finite-element model of the blades, the towers and the floating platform
144 is generated with shell elements. Given the material properties of the entire structure, the mass
145 distribution and the structural stiffness matrix are calculated by use of the finite-element method.
146 Furthermore, the mooring lines are modelled with spring elements with a constant stiffness matrix.

147 The structural deformation of a FOWT is determined by use of a reduced-basis approach, with
148 a subset of m dry modes from the total possible modes of the flexible structure, N . In this approach,

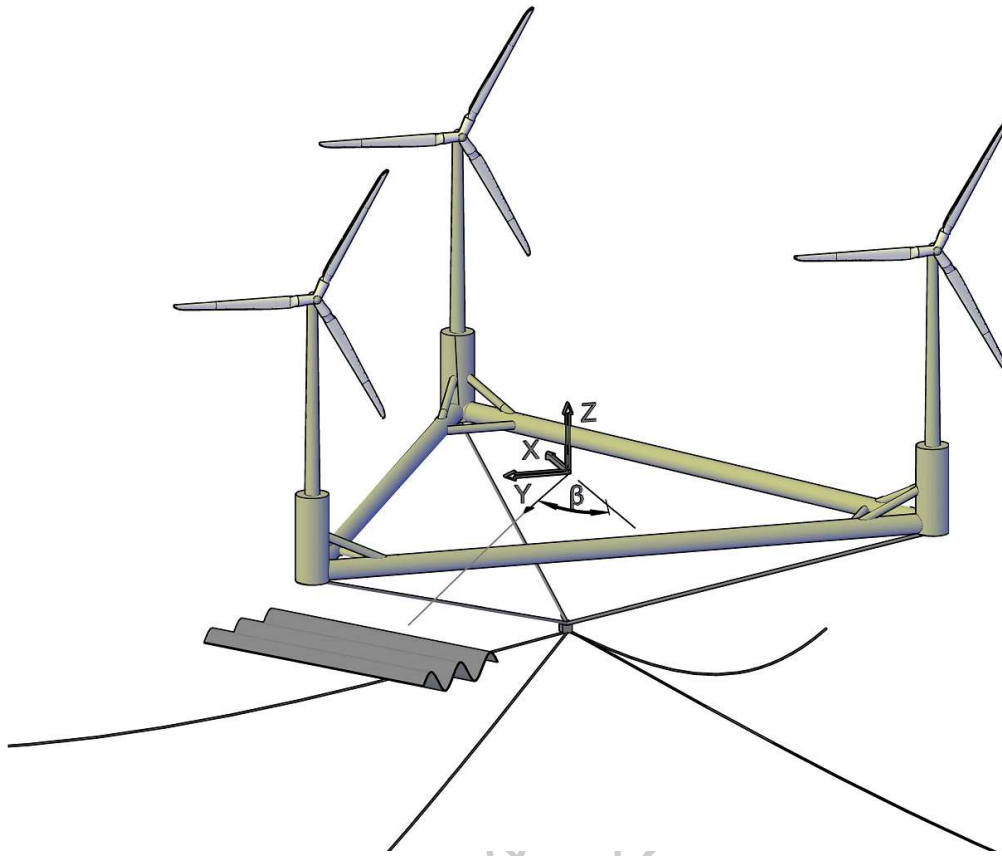


FIG. 1. Schematic of the wind-tracing FOWT, β is the wave heading angle, given with respect to the x -axis.

149 m modes are chosen such that they are sufficient to represent the flexibility of the blades, the tower
150 and the floating platform. Hence, the flexibility of the FOWTs is defined by a linear superposition
151 of m modes and are included in the total degrees of freedom of the FOWTs in their equation of
152 motion.

153 The hydrostatic restoring coefficients of flexible FOWTs are computed by an explicit formula-
154 tion that accounts for the change in hydrostatic pressure and the internal stresses of the structure,
155 see Huang and Riggs (2000) for more details.

156 B. Wave-current interaction with a floating structure

157 Wave-current interaction with a FOWT is studied within the context of the linear diffraction

158 theory for small current speeds, $\tau = \frac{U_c \omega_e}{g} \leq 0.25$, where U_c is the current speed, ω_e is the en-
 159 counter wave frequency, and g is the gravitational acceleration. In this approach, the fluid is
 160 inviscid and incompressible and the flow is irrotational. Waves are assumed of small-amplitude
 161 and the motions and rotations of the structure due to waves and current are linearly proportional to
 162 the wave amplitude. In this method, the current direction is parallel to the x -axis, and waves can
 163 propagate with an arbitrary angle, β , with respect to the x -axis, see Fig. 1. ω is the incoming wave
 164 frequency and the encounter frequency in the moving coordinate system is given as

$$\omega_e = \omega - |U_c| k \cos(\beta), \quad (1)$$

165 where k is the incoming wave number.

166 Assuming that the current speed is small, the total velocity potential can be given as the sum
 167 of the steady velocity potential $\bar{\phi}_s$ due the local steady flow by current, and a harmonic velocity
 168 potential:

$$\Phi(x, y, z, t) = |U_c| (\bar{\phi}_s - x) + \Re\{\phi e^{i\omega_e t}\}. \quad (2)$$

169 in which ϕ is the sum of incident wave velocity potential, ϕ^I , the linear velocity potential due to
 170 the wave diffraction-radiation, ϕ^L , and the linear term representing the interaction of the steady
 171 flow with the wave diffraction-radiation, ϕ^N , *i.e.*

$$\phi = \phi^I + \phi^L + \phi^N. \quad (3)$$

172 The incoming waves velocity potential is known analytically and ϕ^L and ϕ^N are obtained given
 173 the boundary conditions on the wetted surface of the body and the free surface at the vicinity of the
 174 floating structure. ϕ^L and ϕ^N satisfy homogeneous and non-homogeneous boundary conditions on

175 the free surface, respectively:

$$-\frac{\omega_e^2}{g}\phi^L + 2i\tau\frac{\partial\phi^L}{\partial x} + \frac{\partial\phi^L}{\partial z} = 0, \quad \text{on } S_F, \quad (4)$$

$$-\frac{\omega_e^2}{g}\phi^N + 2i\tau\frac{\partial\phi^N}{\partial x} + \frac{\partial\phi^N}{\partial z} = Q, \quad \text{on } S_F, \quad (5)$$

176 where Q is defined by

$$Q = 2i\nabla\bar{\phi}_s\nabla(\phi^I + \phi^L) - i(\phi^I + \phi^L)\frac{\partial^2\bar{\phi}_s}{\partial z^2}, \quad \text{on } S_F. \quad (6)$$

177 Furthermore, the boundary conditions of ϕ^L and ϕ^N on the body surface are given in diffraction
178 and radiation problems. The boundary conditions in diffraction problems are:

$$\frac{\partial\phi^L}{\partial n} = -\frac{\partial\phi^I}{\partial n}, \quad \frac{\partial\phi^N}{\partial n} = 0, \quad \text{on } S_B. \quad (7)$$

179 In the radiation problem, the boundary conditions are satisfied in translational and rotational de-
180 grees of freedom of the floating structure, and, if the hydroelastic motion of the body to waves and
181 current is of interest, the computed generalised modes of the structure. The boundary conditions
182 of ϕ^L and ϕ^N on the body surface for the radiation problem are:

$$\frac{\partial\phi^L}{\partial n} = n_j, \quad \frac{\partial\phi^N}{\partial n} = \frac{im_j}{k}, \quad j = 1, 2, \dots, m, \text{ on } S_B, \quad (8)$$

183 where m_j terms represent the change in the current-induced local steady flow due to the motion
184 and deformation of the body. Formulations of the m_j terms for rigid-body and the generalised
185 modes are presented in Wu (1984), Wu and Taylor (1990), Wu (1991) and Chen and Malenica
186 (1998). Finally, the steady velocity potential, $\bar{\phi}_s$ is obtained by its boundary conditions on the

187 body and free surface:

$$\frac{\partial \bar{\phi}_s}{\partial z} = 0, \quad \text{on } S_F, \quad (9)$$

$$\frac{\partial \bar{\phi}_s}{\partial n} = n_1, \quad \text{on } S_B, \quad (10)$$

188 where n is the normal vector on the body surface, $n = (n_1, n_2, n_3)$, pointing out of the fluid. Given
 189 the boundary conditions on the wetted surface of the body and the free surface, the steady velocity
 190 potential, $\bar{\phi}_s$ and harmonic velocity potentials, ϕ^L and ϕ^N , are determined by boundary-integral
 191 method using the Green function for small current speeds. The Green function for wave-current
 192 interaction with a floating structure with small current speeds have been developed generally in
 193 two approaches, (i) perturbation method, *i.e.* expansion of the Green function with respect to
 194 τ (see *e.g.* Nossen *et al.* (1991), Ertekin *et al.* (1994), Padmanabhan and Ertekin (2003) and
 195 Padmanabhan and Ertekin (2011)) or (ii) a Green function given as a sum of a term due to the
 196 wave diffraction-radiation problem and terms due to the presence of the local steady flow by the
 197 current, (see Noblesse *et al.* (1995), Chen and Malenica (1998) and Monroy *et al.* (2012)). In this
 198 study, the latter approach for the Green function is implemented, see Noblesse *et al.* (1995) and
 199 Chen and Malenica (1998) for the formulation of the Green function and the boundary-integral
 200 equations.

201 Finally, first-order hydrodynamic excitation forces and moments are:

$$F_j^{exc} = -i \rho_w \omega_e \iint_{S_B} (\phi_j^I + \phi_j^L + \tau \phi_j^N) n_j dS + \frac{\rho_w g}{\omega_e} \iint_{S_B} \nabla (\bar{\phi}_s - x) \nabla (\phi_j^I + \phi_j^L) n_j dS. \quad (11)$$

202 The added-mass, a_{jk} , and hydrodynamic damping, b_{jk} , coefficients are obtained from R_{jk}

$$R_{jk} = -i\rho_w\omega_e^2 \iint_{S_B} (\phi_{jk}^L + \tau\phi_{jk}^N) n_k dS - i\rho_w g \iint_{S_B} \nabla(\bar{\phi}_s - x) \nabla(\phi_{jk}^L) n_k dS, \quad (12)$$

203 where

$$a_{jk} = \Re\left\{\frac{1}{\omega_e^2}R_{jk}\right\}, \quad b_{jk} = \Im\left\{\frac{1}{\omega_e}R_{jk}\right\}. \quad (13)$$

204 Given the excitation forces and moments on the floating structure (sum of the Froude-Krylov
205 and diffraction forces and moments) and those due to the added-mass and hydrodynamic damping
206 coefficients, the equations of motion of the floating structure to combined waves and current are

$$\xi_j[-\omega_e^2(M_{ij} + a_{ij}) + i\omega_e(b_{ij}) + (c_{ij,moor} + c_{ij})] = AF_i^{exc}, \quad i, j = 1, 2, \dots, m, \quad (14)$$

207 where c_{ij} and $c_{ij,moor}$ are the hydrostatic restoring and the mooring line stiffness coefficients,
208 respectively. The response amplitude operators (RAOs), $\frac{\xi_j}{A}$ of the floating structure to waves and
209 current are computed for a range of encounter frequencies, ω_e .

210 C. Aerodynamic loads on wind turbines

211 Assuming that the wind speed is constant and the incoming wind flow is orthogonal to the
212 rotor-plane area, the aerodynamic loads on the rotors of a FOWT are computed by use of the
213 steady blade-element momentum method (BEM). The theory is described in detail by Hansen
214 *et al.* (2006), among others, and its application in our numerical approach is discussed for single
215 and multi-unit FOWTs in Lamei *et al.* (2023a,b,c). In this approach, to include the wind-wave
216 misalignment, the wave heading angle, β , changes. Furthermore, the wake-interaction between

217 the rotors supported by the same structure is not considered.

218 To obtain the motion of a FOWT to the environmental loads in the frequency-domain, the aero-
 219 dynamic loads on the rotor and the tower are linearised with a harmonic function with encounter
 220 wave frequency, ω_e . Assuming that the thrust force on the rotors is an excitation force that is
 221 transferred to the tower tops and given that the towers commonly have circular cross-sections, the
 222 phase angle of the aerodynamic excitation force, $F_{j,W}$ is computed by the same formulation given
 223 by MacCamy and Fuchs (1954) for harmonic excitation forces on circular cylinders. Thus, the
 224 complex wind excitation force on the rotor is given as

$$F_{j,W} = |F_{j,W}| \cos(\omega_e t - \delta_{\text{aero}}), \quad j = 1, 2, \dots, m, \quad (15)$$

225 where δ_{aero} is the phase angle of the excitation force and it is given by

$$\delta_{\text{aero}}(k_e r_0) = -\tan^{-1} \left[\frac{Y_1'(k_e r_0)}{J_1'(k_e r_0)} \right], \quad (16)$$

226 where k_e is the encounter wave number, $J_p(k_e r)$ and $Y_p(k_e r)$ are the Bessel functions of the first
 227 and the second kind of order p , respectively, and r_0 is the top diameter of the tower.

228 Due to the motion of the substructures of FOWTs, rotors experience a relative motion with
 229 respect to the incoming wind flow. Therefore, in steady BEM, the thrust force on a rotor of a
 230 FOWT is given as a function of the relative incoming wind speed on the rotor, $V_{\text{rel}} = V_0 - V_h$,
 231 where V_0 is the incoming wind speed and V_h is the horizontal speed of the rotor hub, along the
 232 incoming wind direction. Therefore, the thrust on the rotor of a FOWT is:

$$T(V_{\text{rel}}) = \frac{1}{2} \rho_a A_r C_T(V_{\text{rel}}^2), \quad (17)$$

233 where ρ_a is the air density, A_r is the rotor-plane area, and C_T is the thrust coefficient. In the
 234 frequency-domain, the hub velocity in the x -direction can be given as $i\omega_e(\xi_1 + \xi_5(z_h - z_{cg}))$ for
 235 the rigid structure, where z_h and z_{cg} are the vertical coordinates at the hub and the centre of gravity,
 236 respectively. Assuming that V_h is small and the thrust force due to $O(V_{rel}^2)$ and higher terms are
 237 negligible, Eq. (17) is simplified as

$$T(V_{rel}) = \frac{1}{2}\rho_a A_r C_T (V_0^2) - \rho_a A_r C_T (V_0 i\omega_e(\xi_1 + \xi_5(z_h - z_{cg}))). \quad (18)$$

238 in which the first term in Eq. (18) represents the excitation aerodynamic force in surge, $F_{1,W}$, on
 239 a fixed rotor, *i.e.* $V_h = 0$. Furthermore, the second term on the right-hand side of Eq. (18) is due
 240 to the relative motion of the hub along the incoming wind direction, where $\rho_a C_T V_0$ represents the
 241 aerodynamic damping coefficient, $B_{aero,11}$, on a FOWT with a single rotor. For an arbitrary FOWT
 242 with n wind turbines, the aerodynamic damping matrix in the translational and rotational modes is

$$B_{aero} = \begin{bmatrix} n \times (\rho_a C_T A_r V_0) & 0 & 0 & 0 & -\sum_{j=1}^n \rho_a C_T A_r V_0 (z_h^j - z_{cg}) & -\sum_{j=1}^n \rho_a C_T A_r V_0 (y_h^j - y_{cg}) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\sum_{j=1}^n \rho_a C_T A_r V_0 (z_h^j - z_{cg}) & 0 & 0 & 0 & \sum_{j=1}^n \rho_a C_T A_r V_0 (z_h^j - z_{cg})^2 & 0 \\ -\sum_{j=1}^n \rho_a C_T A_r V_0 (y_h^j - y_{cg}) & 0 & 0 & 0 & 0 & \sum_{j=1}^n \rho_a C_T A_r V_0 (y_h^j - y_{cg})^2 \end{bmatrix}, \quad (19)$$

243 and the aerodynamic load vector is

$$F_W = \begin{bmatrix} \sum_{j=1}^n |F_{1,W}^j| \cos(\omega_e t - \delta_{aero}^j) \\ 0 \\ 0 \\ 0 \\ - \sum_{j=1}^n |F_{1,W}^j| \times (z_h^j - z_{cg}) \cos(\omega_e t - \delta_{aero}^j) \\ - \sum_{j=1}^n |F_{1,W}^j| \times (y_h^j - y_{cg}) \cos(\omega_e t - \delta_{aero}^j) \end{bmatrix}, \quad (20)$$

244 Furthermore, the drag force by the incoming wind on the towers are computed with an empirical
 245 relation

$$F_D = \frac{1}{2} C_d A_t V_0^2 \quad (21)$$

246 where $C_d = C_d(Re)$ is the drag coefficient with respect to the incoming wind Reynolds number,
 247 $Re = \frac{V_0 D}{\nu}$, with D the diameter of the tower, ν is the air kinematic viscosity at 20°, and A_t is
 248 the cross-sectional area of the tower. Similar to the wind excitation forces on the rotors, the phase
 249 angle of the aerodynamic drag force on the towers is computed by Eq. (16).

250 If the elasticity of the FOWT is of interest, the aerodynamic forces and damping coefficients
 251 in the generalised modes are obtained by use of the finite-element method. Finally, given the total
 252 aerodynamic damping matrix and the excitation load vector at the centre of gravity of the FOWT,
 253 Eqs. (19) and (20), respectively, the motion of the structure to combined wave, current and wind
 254 is determined in the frequency domain. Therefore, F_W and B_{aero} are added to the right-hand and
 255 the left-hand sides of the equations of motion of a floating structure, Eq. (14), respectively:

$$\xi_j [-\omega_e^2 (M_{ij} + a_{ij}) + i\omega_e (b_{ij} + B_{aero,mat}) + (c_{ij,moor} + c_{ij})] = A F_i^{exc} + F_{i,W} \quad (22)$$

$$i, j = 1, 2, \dots, m,$$

256 where the rigid-body responses, for $m = 6$, and the elastic motion, for $m > 6$, of the FOWTs to
257 waves, current and wind are obtained by solving Eq. (22) for the encounter wave frequency, ω_e .

258 **D. Numerical solution**

259 The numerical solution of wave-, current- and wind-interaction with a FOWT is implemented
260 in HYDRAN-XR, see NumSoft Technologies (2023). HYDRAN-XR is a potential-flow solver for
261 wave-induced hydrodynamic analysis that is integrated with the finite-element method to include
262 the elastic motion of the floating body. Lamei *et al.* (2023a) and Lamei *et al.* (2023c) further
263 enhanced HYDRAN-XR to include a Green function for combined waves and small current speeds
264 to account for wave-current-structure interaction, and BEM to determine aerodynamic loads on
265 FOWTs.

266 Given the finite-element model of the FOWTs, the mass and stiffness matrices of the structure
267 are calculated. Furthermore, the aerodynamic thrust force and aerodynamic damping effect on
268 the rotor are applied as nodal forces and dampers to the nodes on the blades, facing the incoming
269 wind. The pressure difference upstream and downstream the towers results in an aerodynamic drag
270 force. In the developed numerical model, the drag forces on the towers are modelled as distributed
271 nodal forces on their front faces, *i.e.* those areas of the towers that face the incoming wind. The
272 equivalent aerodynamic excitation load vector and damping matrix at the centre of gravity of the
273 structure are determined by use of the finite-element method. Next, the finite-element model of
274 the FOWT is conformed to a panel mesh with a one-to-one mapping over the wet surface of the
275 platform. Furthermore, panel mesh on the free surface at the vicinity of the floating structure is
276 generated. Next, wave-current-structure interaction is solved by use of a three-dimensional source
277 distribution, the Green function method. Finally, the equation of motion of the FOWT to combined
278 waves, current and wind, Eq. (22), is solved to determine the rigid-body responses and, if elasticity

279 is considered, the elastic motion of the structure, at a given encounter wave frequency, ω_e .

280 III. THE WIND-TRACING FOWT

281 The wind-tracing FOWT is a multi-unit FOWT that consists of three 5 MW NREL wind tur-
282 bines supported by the columns of the floating platform, see Lamei *et al.* (2023b). The columns
283 are connected to three pontoons with a length of $2.2 D_r$, where D_r is the rotor diameter. The
284 length of the pontoons are specified such that the aerodynamic wakes of the front rotors poten-
285 tially have minimal interference with the performance of the rear rotor. The unique characteristic
286 of the wind-tracing FOWT is in its mooring mechanism, the turret-bearing mooring system.

287 Lamei *et al.* (2023b) conducted a parametric study on the layout of the turret-bearing mooring
288 system of the wind-tracing FOWT and identified the preferred location of the turret. The turret
289 is submerged $4 d$ under the platform, where $d = 16$ m is the draft of the wind-tracing FOWT, see
290 Fig. 2(b). Furthermore, in the xy -plane, the turret is located $1/6 L$ away from Column 1, where L
291 is the horizontal distance between Column 1 and Pontoon 3, see Fig. 2(a). The turret is connected
292 to the bottom of the columns and the seabed with three taut cables and four catenary mooring
293 lines, respectively. Here, the turret is modelled as $2 \times 2 \times 2$ m rigid box, and the taut cables are
294 connected to a universal joint at the top of the turret. Furthermore, the catenary mooring lines are
295 attached to the four corners at the bottom of the turret. This layout of the mooring lines allows the
296 platform to rotate about the z -axis freely with respect to the turret, and it is constrained in its roll
297 and pitch modes as it is connected to the catenary mooring lines at its four bottom corners.

298 The geometry and the material properties of the wind-tracing platform, the mass distribution
299 and the hydrostatic properties of the FOWT considered here, are presented in detail by Lamei *et al.*
300 (2023b), who also performed a mooring analysis of the wind-tracing FOWT. These properties are
301 summarised in Table I. Furthermore, Table II presents the computed wet natural periods of the

Response of multi-unit floating offshore wind turbines

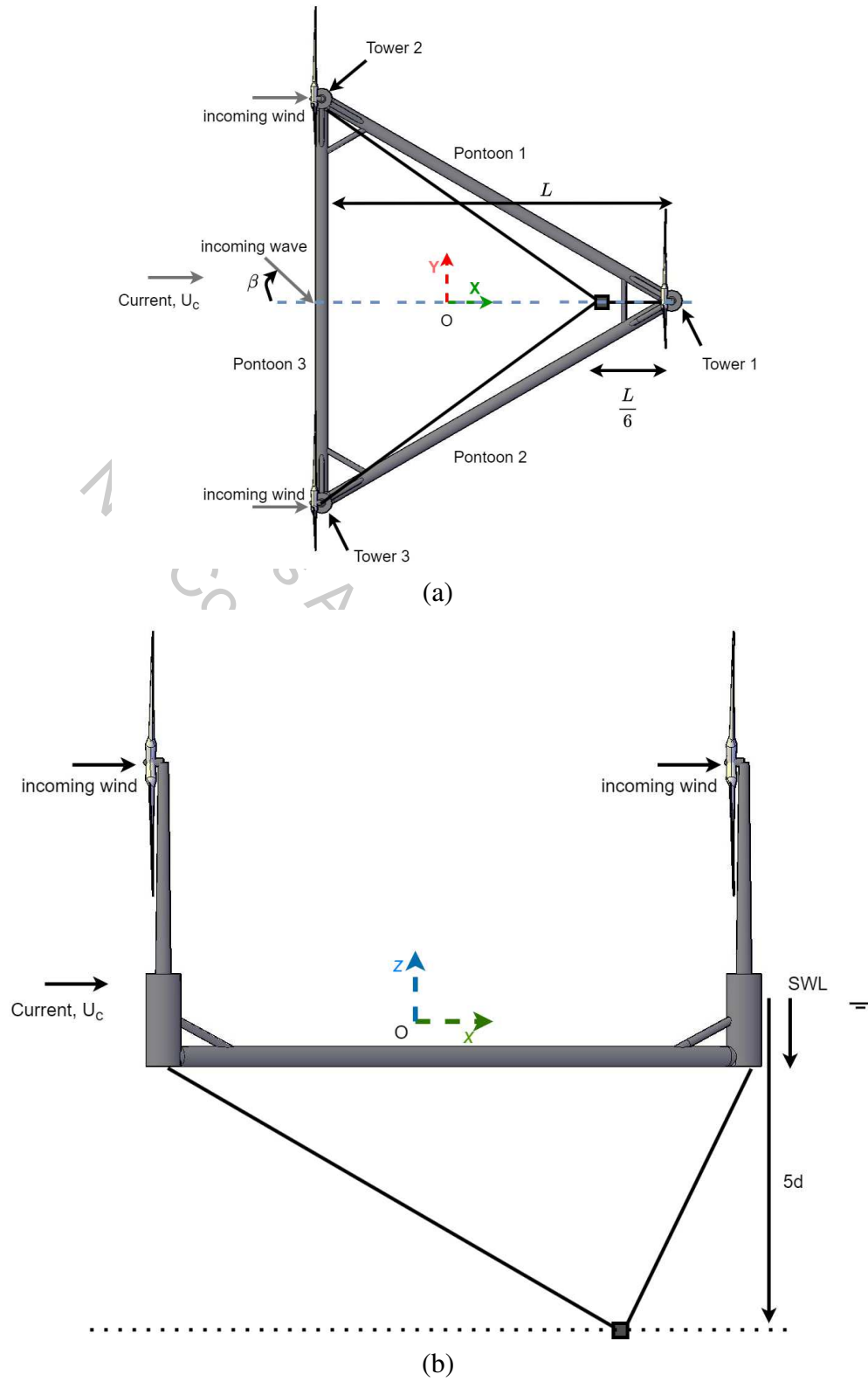


FIG. 2. (a) Top- and (b) side-view of the wind-tracing FOWT and its turret-bearing mooring system. The solid lines and the box represent the taut cables and the turret, respectively.

302 rigid and flexible FOWT in heave, roll and pitch modes.

TABLE I. The mass distribution of the wind-tracing FOWT and the properties of its mooring layout.

Mass distribution and hydrostatic properties	
Mass of the structure (without ballast)	23099 tonnes
Total mass of the turret	64.3 tonnes
Ballast mass	23718 tonnes
Displaced volume	$4.7001 \times 10^4 \text{ m}^3$
Centre of gravity (CG)	7.60 m below SWL
Centre of buoyancy (CB)	11.2 m below SWL
Roll inertia about CG	$3.55 \times 10^{11} \text{ kg-m}^2$
Pitch inertia about CG	$3.58 \times 10^{11} \text{ kg-m}^2$
Yaw inertia about CG	$6.85 \times 10^{11} \text{ kg-m}^2$
Mooring lines properties	
Centre of geometry of the turret	(20.03 m, 0 m, -80 m)
Taut diameter	153 mm
Taut axial stiffness	1481 MN/m
Catenary diameter	95 mm
Catenary wet weight	1942.4 N/m

303 Here, the wave-current-wind interaction with the wind-tracing FOWT under various environ-
 304 mental loadings are investigated. Firstly, assuming that the entire structure of the wind-tracing
 305 FOWT is rigid, its motion due to the wave-current-wind combination is obtained and discussed.
 306 Next, the elastic responses of the structure are obtained and compared with its rigid-body counter-

TABLE II. Wet natural periods of the rigid and flexible wind-tracing FOWT in heave, roll and pitch modes.

Mode	Rigid structure	Flexible structure
Heave	27.78 s	26.8 s
Roll	21.42 s	21.1 s
Pitch	20.44 s	21.1 s

307 parts. Shown in Fig. 2(a) and (b), the origin of the body-fixed coordinate system is at the centre of
308 gravity along a vertical line passing through the centre of geometry of the triangular platform and
309 7.60 m under the SWL. In the following sections, the incoming waves have unit amplitude and
310 the wind speed is fixed at $V_0 = 11.4$ m/s, the rated wind speed of the 5 MW NREL wind turbines.
311 Following the linear wave theory assumptions for wave-current interaction, the current speeds are
312 chosen such that $\tau < \frac{1}{4}$, see Section II. In this study, unless otherwise stated, the current speed
313 is constant at $U_c = 0.8$ m/s and always parallel to the x -axis. The current is assumed uniform
314 across the water depth, and may be codirectional or in opposite direction of the x -axis. Finally, in
315 the following sections, the results are presented as a function of the encounter wave period. The
316 simulations are carried out on a desktop machine with Intel Core *i5* 6500U, 3.20 GHz CPU and
317 32 GB memory and took approximately 6 days for 35 wave periods.

318 IV. RIGID-BODY RESPONSES

319 Prior to obtaining the responses of the wind-tracing FOWT, the wave-induced excitation forces
320 and moments on the structure are determined and compared with those when current is also
321 present. In this section, the current is always perpendicular to the structure, *i.e.* parallel to the x -
322 axis with $U_c = 0.8$ m/s and the waves are either in the following ($\beta = 0^\circ$) or opposing ($\beta = 180^\circ$)
323 directions. The horizontal and vertical excitation forces, F_x^{exc} and F_y^{exc} , and the excitation moment

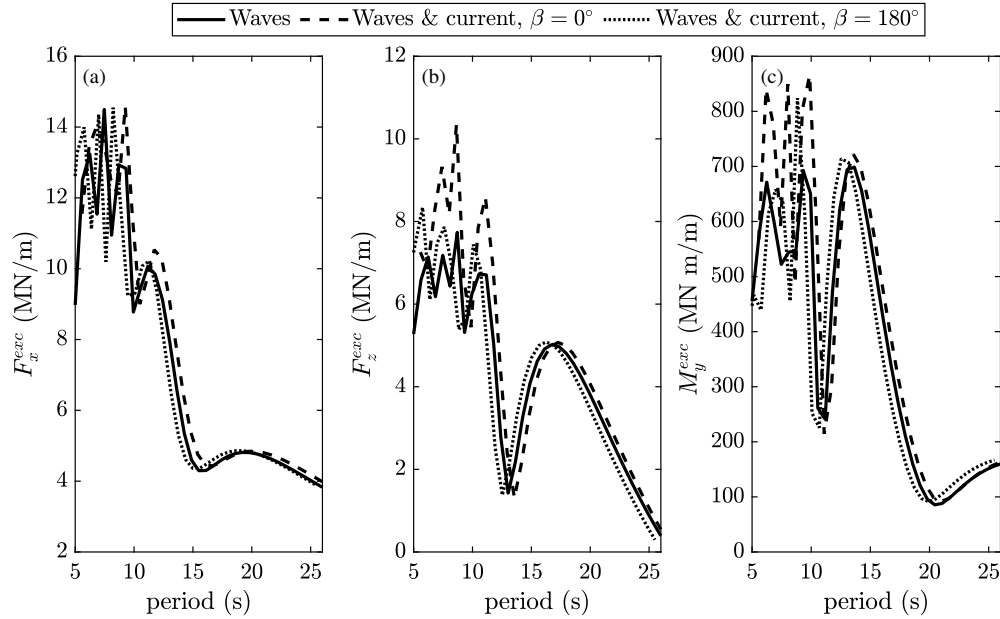


FIG. 3. Comparison of excitation forces in (a) surge F_x^{exc} , (b) heave F_z^{exc} , and (c) excitation moment in pitch, M_y^{exc} , on the wind-tracing FOWT due to waves with those due to combined waves and current, with $\beta = 0^\circ$ and 180° and $U_c = 0.8$ m/s.

324 in pitch, M_y^{exc} are presented in Fig. 3. When current is present and codirectional with the incom-
 325 ing waves, the excitation forces in surge and heave and the excitation moment in pitch increase
 326 slightly. Furthermore, shown in Fig. 3, current interaction with opposing waves results in smaller
 327 excitation forces and moment on the floating structure compared with those due to waves and those
 328 due to current interaction with following waves. However, in general, the presence of current does
 329 not influence significantly the excitation forces and moments on the wind-tracing FOWT.

330 Next, the rigid-body responses of the wind-tracing FOWT in three environmental conditions,
 331 namely (i) waves only, (ii) combined waves and current (in the absence of wind) and (iii) combined
 332 waves and wind (in the absence of current) are presented in Fig. 4. It should be noted that the
 333 airfoil profile varies along the blade, resulting in an asymmetric blade. Hence, although the wind-
 334 tracing platform is symmetric, the complete structure, *i.e.* the platform and the wind turbines, is
 335 not symmetric and as a result its motion in roll is coupled with its motions in its heave and pitch
 336 modes. Commonly, surge, heave and pitch RAOs undergo a peak at approximately 22 s, which

337 is close to the roll and pitch natural frequencies of the rigid wind-tracing FOWT, see Table II.
338 Moreover, at $T_e = 27$ s, approximately the wet natural period of the rigid structure in heave (see
339 Table II), a peak is observed in waves- and wind-induced heave RAOs, and there is a trough in
340 heave RAOs to waves and combined waves and current. The peak observed in surge and heave
341 RAOs of the structure are due to their couplings with its pitch and roll motions. Furthermore,
342 it is observed that the addition of the aerodynamic loads results in slightly smaller surge RAOs
343 and larger pitch RAOs compared with the wave-induced and combined wave- and current-induced
344 surge and pitch motions for encounter wave periods $10 \text{ s} \leq T_e \leq 20 \text{ s}$. The total aerodynamic
345 thrust force and the moment in pitch mode on the wind-tracing FOWT are 2.45 MN and 284.9
346 MNm, respectively, which are significantly smaller than the wave-induced hydrodynamic forces
347 and moment, see Fig. 3. Hence, the small changes in surge and pitch RAOs of the structure when
348 aerodynamic loads are present can be explained by the effect of the aerodynamic damping due to
349 the operating rotors of the FOWT combined with the hydrodynamic damping coefficient. For the
350 same interval of encounter wave periods, wave-current interaction results in smaller heave motion
351 compared with those to waves only and combined waves and wind.

352 Shown in Figs. 3 and 4, the effect of current on excitation forces and moments, and the rigid-
353 body motion of the wind-tracing FOWT is relatively small. Similarly, considering the motion of
354 the wind-tracing FOWT to combined waves and wind, it is observed that wave-induced hydrody-
355 namic forces and moments are the dominating terms in dynamic motion of the structure.

357 **A. Effect of current speed**

358 As discussed earlier in this section, wave-current interaction with the wind-tracing FOWT and
359 current speed of 0.8 m/s, resulted in small changes in its dynamic motion compared with its wave-
360 induced rigid-body responses. In this section, we investigate the effect of current speed and its

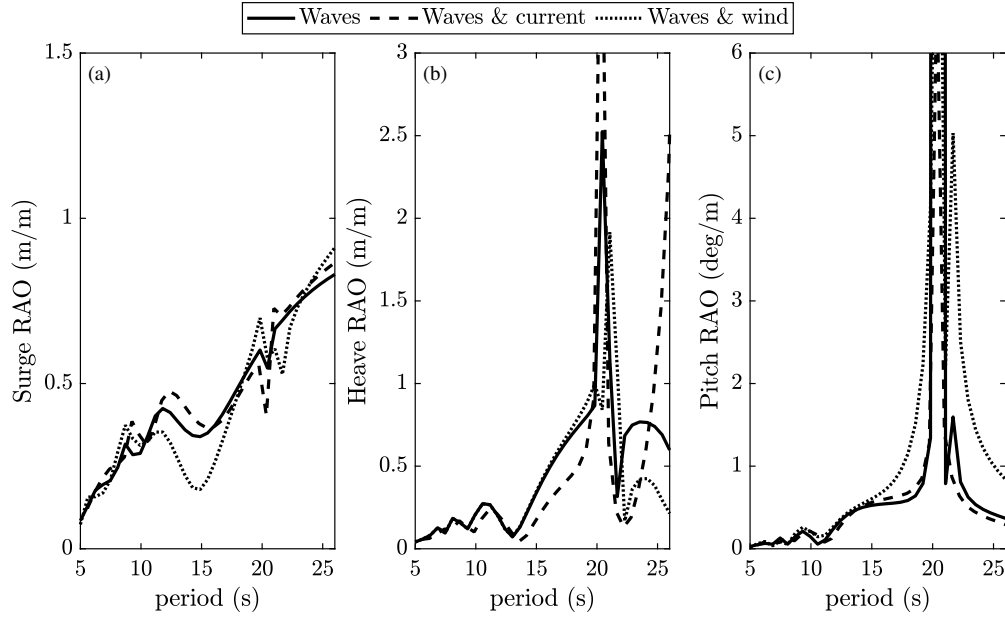


FIG. 4. Comparison of the rigid-body responses of the wind-tracing FOWT to (i) waves only, (ii) combined waves and current with current speed at $U_c = 0.8$ m/s and (iii) combined waves and wind, with wind speed of $V_0 = 11.4$ m/s.

361 direction with respect to the x -axis on the dynamic motion of the wind-tracing FOWT.

362 The wave-current-wind interaction with the wind-tracing FOWT is studied for various current
 363 speeds and discussed. For this purpose, six current speeds, ± 0.6 m/s, ± 0.8 m/s and ± 1.2 m/s
 364 are considered. The incoming waves are at zero wave heading angle, and positive and negative
 365 current speeds indicate that the current is in following or opposing direction of headsea waves,
 366 respectively. Shown in Fig. 5, surge, heave and pitch motions of the rigid wind-tracing FOWT to
 367 combined wave, current and wind are determined for the given current speeds and compared with
 368 those in the absence of current.

369 It is observed that for currents in the following direction of incoming waves, as the current
 370 speed becomes larger, the surge motion of the structure increases slightly for encounter wave
 371 periods smaller than approximately $T_e = 16$ s. Regarding the encounter wave periods smaller
 372 than $T_e \leq 20$ s, the heave motion of the wind-tracing FOWT is larger with current in opposing
 373 direction of incoming waves. Moreover, the largest effect of current speed is observed in heave

374 RAOs of the rigid wind-tracing FOWT, particularly at wave periods larger than $T_e = 22$ s. The
 375 significant increase in heave motion of the FOWT is observed in an interval of encounter wave
 376 periods between the pitch and heave natural periods of the structure, see Table II. Finally, for
 377 $T_e \leq 18$ s and $T_e \geq 22$ s, the effect of the current speed on the pitch motion of the wind-tracing
 378 FOWT is almost negligible, while it becomes more significant at approximately $18 \text{ s} \leq T_e \leq 22 \text{ s}$,
 379 around the wet natural period in pitch mode.

380 In general, by changing the current speed and its direction with respect to the incoming waves,
 381 the surge and heave motion of the wind-tracing FOWT undergo significant changes at encounter
 382 wave periods closer to the wet natural periods of the structure in pitch, roll and heave. However,
 383 the effect of current speed on the pitch motion of the FOWT is almost negligible.

385 B. Effect of wave direction

386 In this section, the effect of wave misalignment with current and wind loads on the motion of
 387 the wind-tracing FOWT is investigated. RAOs of the structure to combined waves, current and
 388 wind are presented in Fig. 6 for wave heading angles $\beta = 0^\circ, 30^\circ, 45^\circ, 90^\circ$ and 180° . The current
 389 and the wind speeds are constant at $U_c = 0.8$ m/s and $V_0 = 11.4$ m/s, respectively. In all cases, the
 390 current is codirectional with the x -axis and perpendicular to the structure.

391 Shown in Fig. 3, the motion of the structure is primarily governed by wave-induced forces and
 392 moments. In Fig. 6(c), relatively similar pitch RAOs of the FOWT are observed to codirectional
 393 and misaligned waves, current and wind for encounter wave periods up to approximately 20 s. At
 394 20 s, approximately the pitch wet natural period of the structure, the peak value of the pitch RAO
 395 to codirectional wave, current and wind is the largest compared with those for $\beta > 0^\circ$. At the same
 396 encounter wave period, shown in Fig. 3, the horizontal excitation force and moment in pitch by
 397 codirectional waves and current are larger than those to waves and current in opposite direction. It

Response of multi-unit floating offshore wind turbines

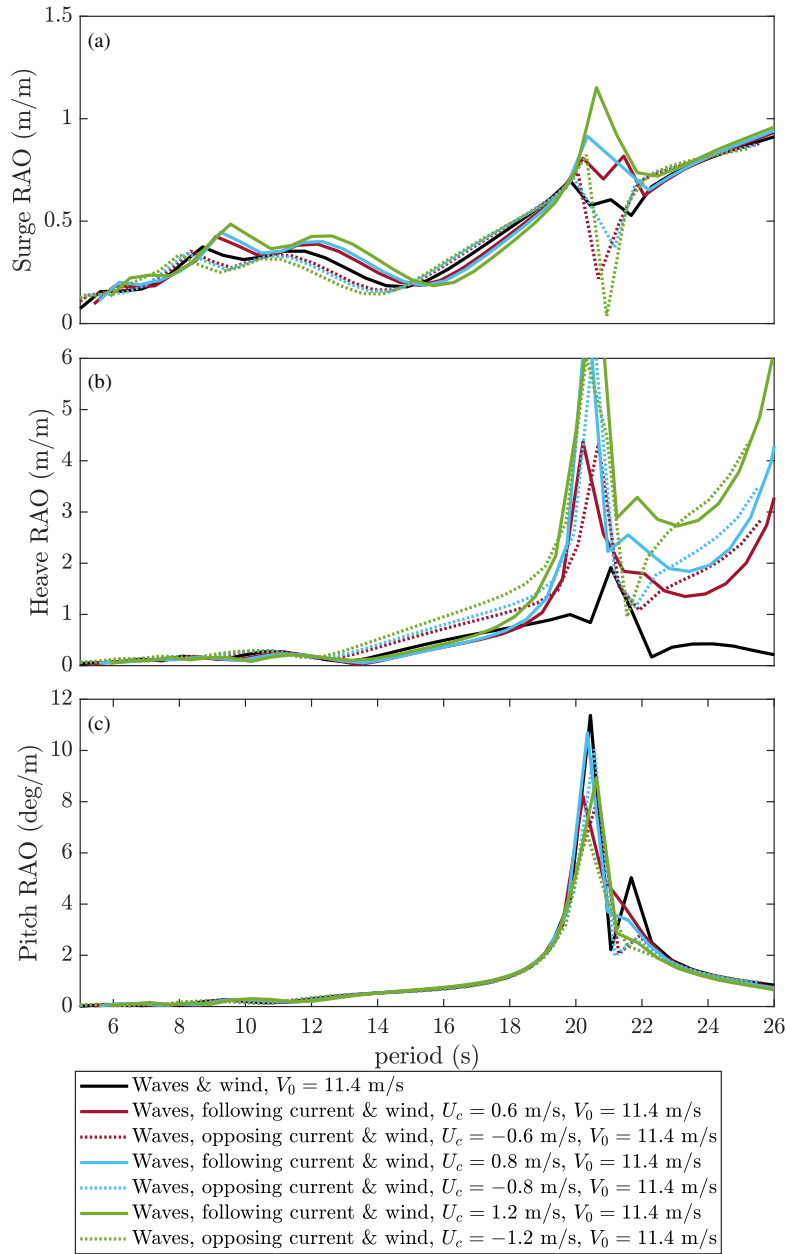


FIG. 5. Surge, heave and pitch motions of the rigid wind-tracing FOWT to waves and wind in the absence of current, and waves, current and wind with current speeds $U_c = \pm 0.6$ m/s, $U_c = \pm 0.8$ m/s and $U_c = \pm 1.2$ m/s and $V_0 = 11.4$ m/s.

398 is observed that the misalignment of incoming waves with current and wind has the largest effect
 399 on the surge motion of the floating structure. It can be seen that up to the encounter wave period
 400 of 16 s, the surge motion of the structure to codirectional waves, current and wind is the largest
 401 compared with other load cases. Furthermore, for wave periods approximately larger than 14 s,

402 the heave and pitch RAOs of the structure are the smallest to beam waves ($\beta = 90^\circ$) combined
 403 with the horizontal current and wind loads on the wind-tracing FOWT. The peak in surge RAOs
 404 is shifted to a larger wave period with wave heading angle $\beta = 180^\circ$. Moreover, with waves in
 405 opposing direction of current and wind, the large peaks observed in heave and pitch RAOs are
 406 shifted to larger encounter wave periods and are approximately at the wet natural period in the roll
 407 mode.

408 Shown in Fig. 6, the misalignment of incoming waves with the current and wind can signif-
 409 icantly change the surge RAOs of the wind-tracing FOWT. The effect of wave heading angle on
 410 the heave and pitch motion of the structure is significant in long wave periods larger than approx-
 412 imately $18 \text{ s} \leq T_e$.

413 V. FLEXIBLE-BODY RESPONSES

414 In this section, the hydro- and aeroelastic motion of the wind-tracing FOWT to the environ-
 415 mental loads are presented and discussed. Firstly, wave-current-wind interaction with the fully
 416 flexible wind-tracing FOWT is studied by presenting its motions in surge, heave and pitch modes.
 417 Next, nodal displacements along the towers and pontoons of the flexible FOWT are determined
 418 and compared with its rigid-body counterparts.

419 Shown in Fig. 7, rigid- and flexible-body responses of the wind-tracing FOWT to wave, current
 420 and wind loads are obtained and compared. Firstly, it can be seen that for $14 \text{ s} \leq T_e$, the rigid
 421 structure undergoes smaller surge motions than those of the flexible structure. Similarly, the heave
 422 motion of the rigid FOWT is smaller than the flexible wind-tracing FOWT for encounter wave
 423 periods $T_e \leq 20$, which is close to the roll and pitch natural periods of the flexible structure, see
 424 Table II. Finally, for the majority of the considered encounter wave periods, the rigid-body pitch
 425 RAOs to the combined waves, current and wind are larger than those of the flexible structure. The

Response of multi-unit floating offshore wind turbines

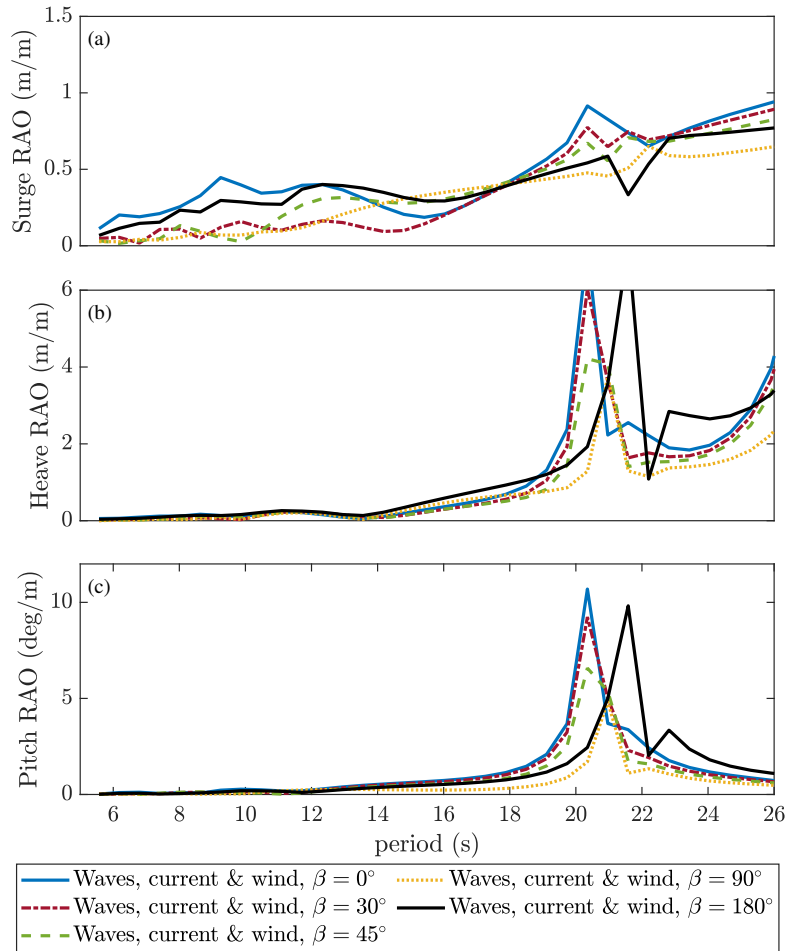


FIG. 6. (a) Surge, (b) heave and (c) pitch motions of the rigid wind-tracing FOWT to codirectional waves, current and wind, and waves misaligned with current and wind with $\beta = 30^\circ, 45^\circ, 90^\circ$ and 180° , $V_0 = 11.4$ m/s and $U_c = 0.8$ m/s.

426 peak of the pitch RAOs of the flexible structure is smaller than the one of the rigid wind-tracing
 427 FOWT, and as expected, occurs at approximately 21 s.

428 Next, the effect of flexibility of the wind-tracing FOWT on its structural responses to hydro-
 429 and aerodynamic loads is investigated. For this purpose, nodes at the leading edge of the towers,
 430 starting from their bases, $z = 22$ m, up to the tips of the towers, $z = 109$ m, facing the incoming
 431 wind, and nodes on the outer edge of the pontoons, facing outside the triangular platform at $z =$
 432 -12 m are considered. The horizontal nodal displacements along the towers are due to the motion
 433 of the structure in its surge and pitch modes, and if elasticity considered, its generalised modes.

Response of multi-unit floating offshore wind turbines

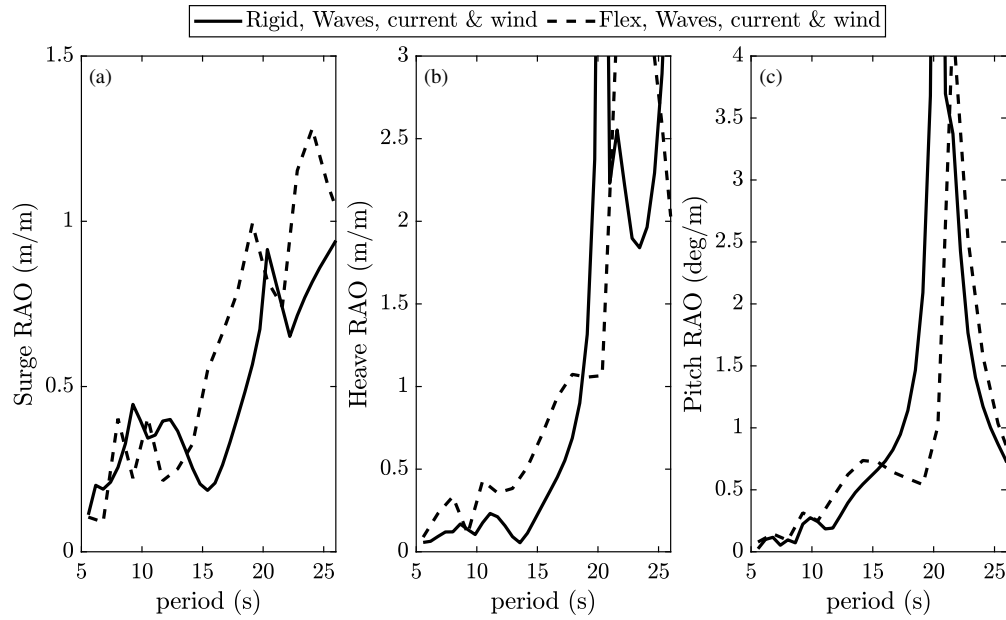


FIG. 7. (a) Surge, (b) heave and (c) pitch motions of the rigid and flexible wind-tracing FOWT to codirectional waves, current and wind, with $V_0 = 11.4$ m/s and $U_c = 0.8$ m/s.

434 Furthermore, the vertical nodal displacements along the pontoons are a result of the motion of
 435 the structure in its heave and roll and if elasticity considered, their coupling with the generalised
 436 modes of the structure. The vertical displacements along the pontoons are reported over their
 437 lengths. The length of Pontoons 1 and 2 are reported with respect to the column supporting Tower
 438 1, and the length of Pontoon 3 is given with respect to the column supporting Tower 2.

439 Shown in Figs. 8 and 9, the horizontal nodal displacements along the towers and the vertical
 440 nodal displacements along the pontoons of the rigid and flexible FOWTs are computed for codi-
 441 rectional waves and wind and compared with those due to combined waves, current and wind at
 442 three encounter wave periods, $T_e = 10$ s, 15 s and 24 s.

443 The structure of the wind-tracing FOWT, due to the asymmetric geometry of the blades and the
 444 rotors, is not symmetric with respect to the global x -axis. Therefore, the nodal displacements along
 445 Tower 2 are slightly smaller than those of Tower 3 in both rigid and flexible wind-tracing FOWT.
 446 The difference between the horizontal nodal displacements along these two towers is attributed

447 to the mode-shape of the complete structure. Commonly, the three flexible towers experience the
448 largest horizontal displacements at encounter wave period $T_e = 24$ s. This is expected, since at
449 approximately $T_e = 24$ s, the surge and pitch RAOs of the flexible wind-tracing FOWT are larger
450 than those of the rigid FOWT.

451 At $T_e = 10$ s, the towers of the flexible wind-tracing FOWT experience larger nodal displace-
452 ments compared with the rigid towers in both load conditions. When current is present, shown
453 in Fig. 7, the pitch RAOs of the flexible structure is slightly larger than the rigid structure. As a
454 result, we observe that flexible Tower 2 and 3 undergo larger nodal displacements compared with
455 their rigid-body counterpart. Moreover, the horizontal nodal displacement along Tower 1 is the
456 largest in the absence of current. The different nodal displacements along the towers at a wave
457 period can be attributed to the asymmetry of the wind-tracing FOWT and its mode-shapes.

458 At $T_e = 15$ s, the presence of current results in small horizontal nodal displacements along
459 flexible Tower 1 in comparison with its rigid counterparts to waves and wind with and without
460 current. Similarly, when current is present, nodal displacements along Tower 2 is the smallest,
461 when the FOWT is flexible. Shown in Fig. 5, it can be seen that when current is added, the surge
462 RAOs of the structure are slightly smaller than its combined waves- and wind-induced surge mo-
463 tion, hence smaller horizontal nodal displacements along the rigid and flexible towers are observed
464 when wave-current interaction is considered. However, at $T_e = 15$ s, it is observed that the current
465 interaction does not influence the nodal displacements along Tower 3 significantly. Furthermore,
466 at $T_e = 24$ s, flexible towers undergo the largest nodal displacements to waves and wind in the
467 absence of current. In conclusion, at the encounter wave periods considered in Fig. 8, the addition
468 of current mostly resulted in smaller horizontal nodal displacements along the three towers.

469 The vertical nodal displacements along pontoons 1, 2 and 3 to waves and wind, and combined
470 waves, current and wind are presented in Fig. 9. For the first two encounter wave periods, $T_e = 10$

471 s and 15 s, smaller differences between the nodal displacements along the rigid and flexible pon-
472 toons are observed compared with those at $T_e = 24$ s. Considering encounter wave period $T_e = 10$
473 s, the vertical nodal displacements along the three rigid pontoons are almost negligible compared
474 with their flexible counterparts. However, it can be seen that the addition of current loads results in
475 larger nodal displacements of flexible pontoons 1 and 2. As discussed earlier, the structure of the
476 wind-tracing FOWT is not symmetric, and therefore identical vertical displacements along Pon-
477 toons 1 and 2 are not observed. At $T_e = 15$ s, it is seen that the largest vertical nodal displacements
478 occur by the flexible pontoons to combined wave, current and wind loads. Similar behaviour is
479 observed at $T_e = 24$ s by flexible pontoons 2 and 3, which their vertical nodal displacements in-
480 duced by waves, current and wind are significantly larger than others. Furthermore, $T_e = 24$ s is
481 close to the wet natural period of both flexible and rigid wind-tracing FOWTs in their heave and
482 roll modes. Large vertical displacements along the three pontoons at this encounter wave period,
483 $T_e = 24$ s, could be explained by the resonance behaviour of the structure in its heave and roll
484 natural periods. Nevertheless, it can be seen that the wave-current interaction results in larger
485 hydroelastic motion of the three pontoons at the considered encounter wave periods in Fig. 9.

486 In this section, the effect of flexibility of the wind turbines and the floating platform of the
487 wind-tracing FOWT on its motion to the environmental loads is discussed. It is shown that surge
488 and pitch RAOs of the flexible FOWT are larger and smaller than their rigid-body counterparts,
489 respectively. Finally, shown in Figs. 8 and 9, the effect of wave-current interaction on the nodal
490 displacements along the towers and the pontoons of the wind-tracing FOWT is presented. It is
491 shown that the presence of current has more significant effect on elastic responses of the pontoons
492 compared with the towers.

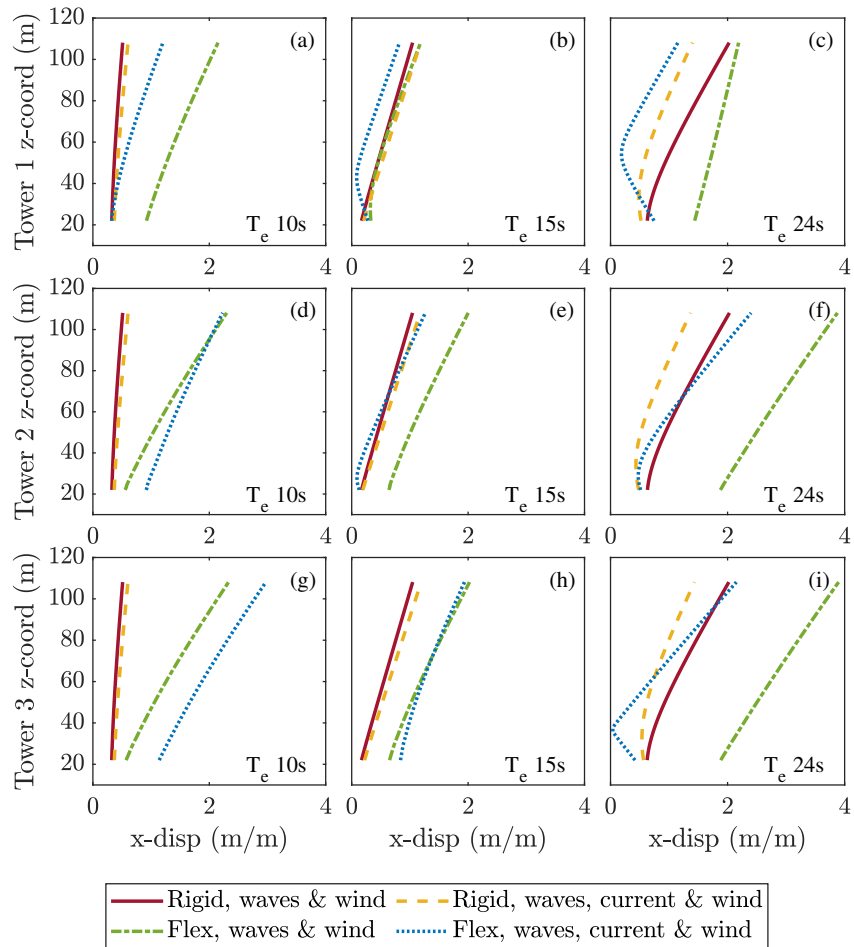


FIG. 8. Horizontal nodal displacements along tower 1 (a), (b) and (c), tower 2 (d), (e) and (f) and tower 3 (g), (h) and (i) at encounter wave periods 10 s, 15 s and 24 s with $V_0 = 11.4$ m/s and $U_c = 0.8$ m/s.

494 **VI. SUMMARY AND CONCLUDING REMARKS**

495 This study investigates the wave-current-wind interaction with the multi-unit wind-tracing
 496 FOWT. Dynamic motion of the wind-tracing FOWT to combined wave, current and wind loads is
 497 determined by a hydro-aero-elastic numerical coupling approach in the frequency-domain.

498 In this approach, the governing equations of motions of a floating structure to waves and current
 499 in the frequency domain is extended to include the aerodynamic loads on the wind turbines of the
 500 FOWT and to obtain elastic motion of the structure, if the elasticity is considered.

501 The rigid- and flexible-body responses of the wind-tracing FOWT to wave, current and wind are

Response of multi-unit floating offshore wind turbines

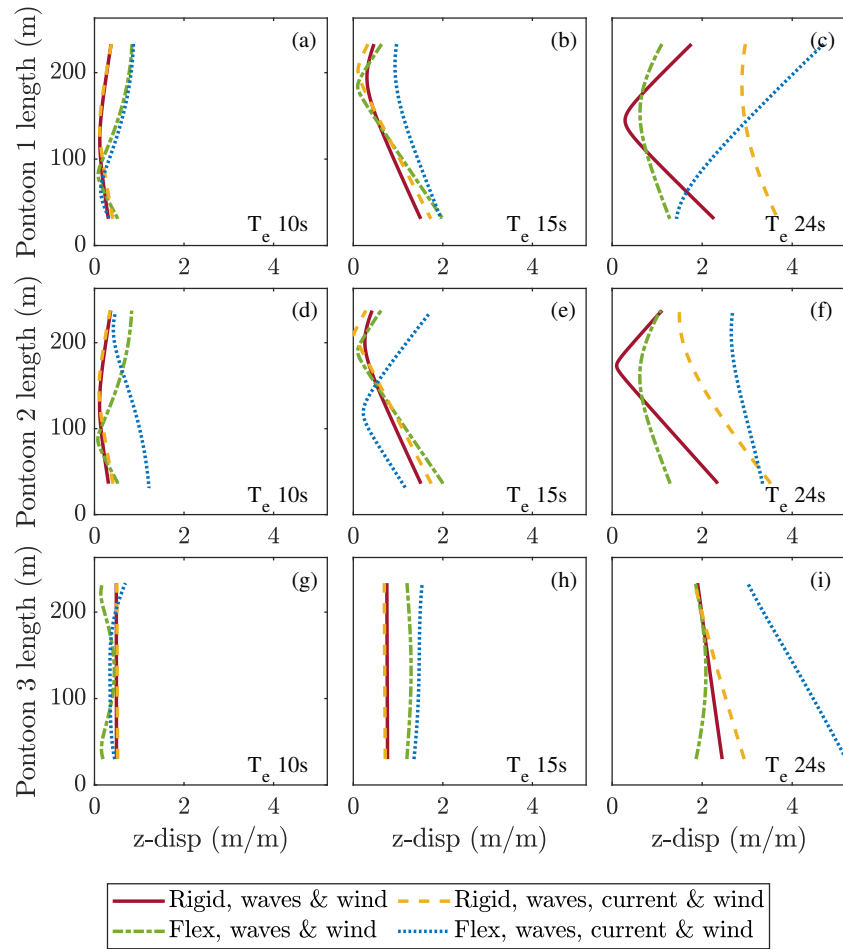


FIG. 9. Vertical nodal displacements along pontoon 1 (a), (b) and (c), pontoon 2 (d), (e) and (f) and pontoon 3 (g), (h) and (i) at encounter wave periods 10 s, 15 s and 24 s with $V_0 = 11.4$ m/s and $U_c = 0.8$ m/s.

502 calculated and discussed. Firstly, the motion of the rigid FOWT to waves only, combined waves
 503 and current and combined waves and wind are compared, and it is seen that its responses are mainly
 504 dominated by wave-induced hydrodynamic forces and moments. Next, the rigid-body responses
 505 of the FOWT to several current speeds and misalignments between waves with current and wind
 506 are presented. It is observed that when aerodynamic loads are present, increasing the current
 507 speed results in larger surge and heave motion of the FOWT, whereas the pitch RAOs undergo
 508 negligible changes. Furthermore, for encounter wave periods smaller than the pitch natural period
 509 of the structures, wave misalignment with current and wind has insignificant effect on the heave
 510 and pitch motion of the structure. However, the wind-tracing FOWT undergoes the largest pitch

511 motion when waves, current and wind are codirectional.

512 Finally, the elastic motion of the wind-tracing FOWT to wave, current and wind loads is ob-
513 tained and compared with its rigid-body counterpart. Furthermore, the importance of elasticity
514 analysis of the multi-unit wind-tracing FOWT is shown by comparing its rigid- and flexible-body
515 nodal displacements along its towers and pontoons to two environmental conditions, namely (i)
516 waves and wind and (ii) waves, current and wind. In general, it is observed that the presence of cur-
517 rent results in smaller nodal displacements along the towers. However, the wave-current-structure
518 interaction results in larger hydroelastic responses of the wind-tracing FOWT in comparison to
519 those when current is not present and hence larger vertical nodal displacements along the pon-
520 toons are obtained.

521 In conclusion, motion of the wind-tracing FOWT is largely governed by the wave-induced hy-
522 drodynamic forces and moments on the substructure and the effect of current with small speed
523 on the hydrodynamic excitation forces and moments on the structure is almost negligible. Conse-
524 quently, the wind-tracing platform would rotate with respect to the turret such that the moments
525 mainly by the wave-induced hydrodynamic loads on the turret are minimized. This indicates that
526 an appropriate understanding of rigid-body and elastic motion of the substructure plays an impor-
527 tant role in analysing the performance of the wind-tracing FOWT. Furthermore, it is also observed
528 that the aerodynamic damping effect by the three operating rotors on the platform influences sig-
529 nificantly the surge and pitch motion of the system. Finally, it is seen that when current is present,
530 the hydroelastic responses of the FOWT is affected considerably.

531 Dynamic analysis of multi-unit FOWTs, due to their unique design and geometrical charac-
532 teristics, mainly their platform and the mooring layout, are challenging to study. In this paper, it
533 is shown that the developed hydro-aero-elastic coupling approach implemented in HYDRAN-XR
534 can successfully model the environmental loads by waves, current and wind on multi-unit FOWTs

535 and determine their rigid-body and elastic motions. The developed numerical model provides in-
536 sight about motion of a FOWT concept with an arbitrary shape of the floating platform and number
537 of wind turbines. Therefore, the developed coupling approach can be used in design and concept
538 stages of FOWTs. Nevertheless, it should be noted that a thorough understanding of the effect
539 of nonlinear environmental loads on the performance of the structure, the effect of wind turbine
540 controllers on its power output, and the aerodynamic wake interaction between the wind turbines
541 on a single platform, among others are necessary for design purposes, and should be considered in
542 future studies.

543 **ACKNOWLEDGEMENTS**

544 The work of AL and MH is partially based on funding from the CBJ Ocean Engineering Corp.
545 of Hong Kong. This funding is gratefully acknowledged. Any findings and opinions contained
546 in this paper are those of the authors and do not necessarily reflect the opinions of the funding
547 company.

548 **Conflict of Interest:** The authors have no conflicts to disclose.

549 **Data Availability Statement:** The data that supports the findings of this study are available within
550 the article.

551 **REFERENCES**

552 Bae, Y. H. and Kim, M., “Coupled dynamic analysis of multiple wind turbines on a large single
553 floater,” *Ocean Engineering* **92**, 175–187 (2014).

554 Bae, Y. H. and Kim, M. H., “The dynamic coupling effects of a MUFOWT (multiple unit floating
555 offshore wind turbine) with partially broken blade,” *Journal of Ocean and Wind Energy* **2**, 89–97

- 556 (2015).
- 557 Bashetty, S. and Ozcelik, S., “Design and stability analysis of an offshore floating multi-turbine
558 platform,” in *2020 IEEE Green Technologies Conference (GreenTech)* (April 1-3, Oklahoma,
559 United States, 2020) pp. 1–6.
- 560 Chen, L., Basu, B., and Nielsen, S. R., “A coupled finite difference mooring dynam-
561 ics model for floating offshore wind turbine analysis,” *Ocean Engineering* **162**, 304–315,
562 doi:10.1016/j.oceaneng.2018.05.001 (2018).
- 563 Chen, X. B. and Malenica, Š., “Interaction Effects of Local Steady Flow on Wave Diffraction-
564 Radiation at Low Forward Speed,” *International Journal of Offshore and Polar Engineering* **8**,
565 101–109 (1998).
- 566 Cheng, P., Huang, Y., and Wan, D., “A numerical model for fully coupled aero-hydrodynamic
567 analysis of floating offshore wind turbine,” *Ocean Engineering* **173**, 183–196 (2019).
- 568 Ertekin, R. C., Liu, Y. Z., and Padmanabhan, B., “Interaction of incoming waves with a
569 steady intake-pipe flow,” *Journal of Offshore Mechanics and Arctic Engineering* **116**, 214–220,
570 doi:10.1115/1.2920154 (1994).
- 571 Hansen, M. O., Sørensen, J. N., Voutsinas, S., Sørensen, N., and Madsen, H. A., “State of the art
572 in wind turbine aerodynamics and aeroelasticity,” *Progress in Aerospace Sciences* **42**, 285–330
573 (2006).
- 574 Huang, L. L. and Riggs, H. R., “The hydrostatic stiffness of flexible floating structures for linear
575 hydroelasticity,” *Marine Structures* **13**, 91–106 (2000).
- 576 Kang, H. Y., Kim, M. H., Kim, K. H., and Hong, K. Y., “Hydroelastic analysis of multi-unit float-
577 ing offshore wind turbine platform (MUFOWT),” in *Proceedings of the International Offshore
578 and Polar Engineering Conference* (June 25-30, San Francisco, CA, 2017) pp. 554–560.

- 579 Kumar, A. and Hayatdavoodi, M., “Effect of currents on nonlinear waves in shallow water,”
580 Coastal Engineering **181**, 1–20, doi:10.1016/j.coastaleng.2023.104278 (2023a).
- 581 Kumar, A. and Hayatdavoodi, M., “On wave–current interaction in deep and finite water depths,”
582 Journal of Ocean Engineering and Marine Energy , 1–21, doi:10.1007/s40722–023–00278–x
583 (2023b).
- 584 Lamei, A. and Hayatdavoodi, M., “On motion analysis and elastic response of floating
585 offshore wind turbines,” Journal of Ocean Engineering and Marine Energy **6**, 71–90,
586 doi:10.1007/s40722–019–00159–2 (2020).
- 587 Lamei, A., Hayatdavoodi, M., and Riggs, H. R., “Hydro- and aero-elastic response of floating
588 offshore wind turbines to combined waves and wind in frequency domain,” Journal of Ocean
589 Engineering and Marine Energy , submitted and under review. (2023a).
- 590 Lamei, A., Hayatdavoodi, M., and Riggs, H. R., “Motion and elastic response of wind-tracing
591 floating offshore wind turbines,” Journal of Ocean Engineering and Marine Energy **9**, 43–67,
592 doi:10.1007/s40722–022–00250–1 (2023b).
- 593 Lamei, A., Hayatdavoodi, M., Riggs, H. R., and Ertekin, R. C., “Wave-current-wind interaction
594 with floating offshore wind turbines,” Renewable energy , submitted and under review. (2023c).
- 595 MacCamy, R. C. and Fuchs, R. A., *Wave forces on piles: A diffraction theory*, Tech. Memo. No.
596 69 (Beach Erosion Board. Army Corps of Engineers, 1-17, 1954).
- 597 Monroy, C., Giorgiutti, Y., and Chen, X.-B., “First and Second Order Wave-Current Interactions
598 for Floating Bodies,” in *Volume 1: Offshore Technology*, Vol. 1 (American Society of Mechanical
599 Engineers, 2012) pp. 373–382, doi:10.1115/OMAE2012–83409.
- 600 Noblesse, F., Taylor, D., and Chen, X. B., “Decomposition of free-surface effects into wave and
601 near-field components,” Ship Technology Research **42**, 167–185 (1995).

- 602 Nossen, J., Grue, J., and Palm, E., “Wave forces on three-dimensional floating bodies with small
603 forward speed,” *Journal of Fluid Mechanics* **227**, 135–160, doi:10.1017/S002211209100006X
604 (1991).
- 605 NumSoft Technologies,, “HYDRAN-XR, hydrodynamic response analysis with integrated struc-
606 tural finite element analysis, version 23.0,” Tech. Rep. (Numsoft Technologies, 2023).
- 607 Padmanabhan, B. and Ertekin, R. C., “On the interaction of waves with intake/discharge flows
608 originating from a freely-floating body,” *Journal of Offshore Mechanics and Arctic Engineering*
609 **125**, 41–47, doi:10.1115/1.1537724 (2003).
- 610 Padmanabhan, B. and Ertekin, R. C., “Interaction of waves with a steady intake/discharge flow
611 emanating from a 3D body,” *Journal of Offshore Mechanics and Arctic Engineering* **133**, 1–10,
612 doi:10.1115/1.4003057 (2011).
- 613 Qu, X., Li, Y., Tang, Y., Hu, Z., Zhang, P., and Yin, T., “Dynamic response of spar-type floating
614 offshore wind turbine in freak wave considering the wave-current interaction effect,” *Applied*
615 *Ocean Research* **100**, 1–19, doi:10.1016/j.apor.2020.102178 (2020).
- 616 Silva, L. S., Cazzolato, B., Sergiienko, N. Y., and Ding, B., “Nonlinear dynamics of a float-
617 ing offshore wind turbine platform via statistical quadratization — mooring, wave and current
618 interaction,” *Ocean Engineering* **236**, 1–13, doi:10.1016/j.oceaneng.2021.109471 (2021).
- 619 Tran, T. T. and Kim, D. H., “Fully coupled aero-hydrodynamic analysis of a semi-submersible
620 fowt using a dynamic fluid body interaction approach,” *Renewable Energy* **92**, 244–261 (2016).
- 621 Wong, C., “Wind tracing rotational semi-submerged raft for multi-turbine wind power generation,”
622 in *European Wind Energy Association Offshore 2015 Conference* (Copenhagen, Denmark, 2015)
623 pp. 1–10.
- 624 Wu, G., “A numerical scheme for calculating the mj terms in wave-current-body interaction prob-
625 lem,” *Applied Ocean Research* **13**, 317–319, doi:10.1016/S0141-1187(05)80055-0 (1991).

- 626 Wu, G. X. and Taylor, R. E., “The hydrodynamic force on an oscillating ship with low forward
627 speed,” *Journal of Fluid Mechanics* **211**, 333–353, doi:10.1017/S0022112090001598 (1990).
- 628 Wu, Y., *Hydroelasticity of floating bodies*, Ph.D. thesis (1984).
- 629 Zhou, Y., Xiao, Q., Liu, Y., Incecik, A., Peyrard, C., Wan, D., Pan, G., and Li, S., “Explor-
630 ing inflow wind condition on floating offshore wind turbine aerodynamic characterisation and
631 platform motion prediction using blade resolved cfd simulation,” *Renewable Energy* **182**, 1060–
632 1079, doi:10.1016/j.renene.2021.11.010 (2022).

Authors Accepted Manuscript;
Not Copy-edited by the journal.