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Assessment of regression variabilities and biases: a demonstration in the context of structural health monitoring.

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Abstract. The challenge that is at the forefront of data-driven vibration-based structural health monitoring (VSHM) is the detrimental effect caused by environmental and operational variations (EOVs). Therefore, action must be taken in order to mitigate the effects of the EOVs without affecting the influence of damage. A number of regression-based approaches have been applied in VSHM, using measured environmental and operational parameters to model damage sensitive features (DSFs). In this work, a forward stepwise method is compared with Lasso regression for the purpose of regression model optimisation. The Akaike information criterion (AIC) and the variance of the covariance of the input variables are used to quantitatively assess the quality of the regression models. Additionally, the F-statistic is used to determine which DSFs should or should not be regressed. The results of the analysis showed that the reduced order forward stepwise regression had the lowest AIC and was the most appropriate model despite not having the most stable coefficient matrices. Ultimately, the choice of optimisation method has a significant impact on the quality of future predictions.

1. Introduction

Vibration-based structural health monitoring (VSHM) has been suggested as a potential alternative to costly visual inspections. However, the biggest downfall of VSHM is that the measurements are sensitive to changes in environmental and operational conditions. The changing conditions can effectively hide the presence of damage, making it more difficult to detect [1]. For this reason, a significant amount of work now focuses on mitigating the effects of the varying conditions without removing the influence of the damage [2, 3, 4, 5, 6, 7].

VSHM can be broken down into a number of sub-categories. Firstly, between model-based and data-driven. Model-based methods often require expensive computer models which are updated to match the behaviour of the structure as closely as possible. On the other hand, data-driven methods rely on models purely made of measured data. Data-driven can be split further into implicit and explicit models [8]. In implicit models, no information about the varying conditions are required. Alternatively, explicit models make use of information available on the varying conditions to create a cause-effect relationship with the measured damage sensitive features (DSFs) [9].

VSHM relies upon changes in the vibrational behaviour of the structure to detect damage. Observations are compared to a baseline model and the small changes can be detected through an outlier analysis. The VSHM implemented in this work follows a standard framework for explicit modelling. The layout of the framework is given in Fig. 1. The focus of this work lies in the design of the optimisation scheme, and how this affects the stability of the coefficient matrices.

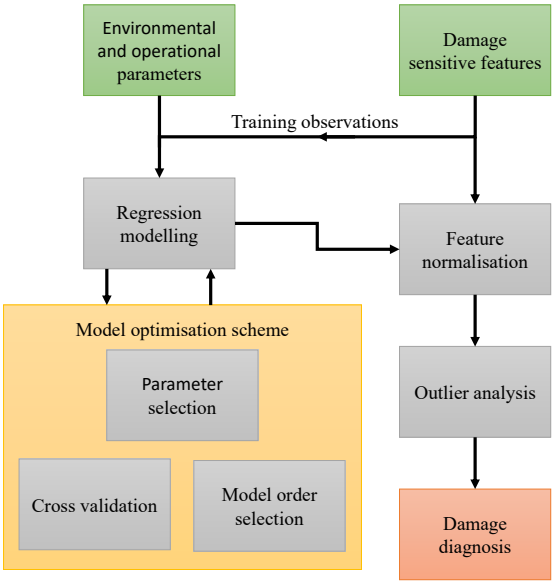

Figure 1. Schematic representation of an explicit VSHM framework.

Regression models have been a popular choice for explicit methods since they are flexible and can be implemented with varying degrees of complexity. This ranges from simple linear models [10] all the way to stochastic methods like Gaussian process regression [2]. The choice of model is dependent on the data that is being modelled. Overfitting the models to the data can lead to poor predictions of future observations [11]. The problem of overfitting is well established in the machine learning community. For example, when working with neural networks, the model will excessively weight around the training data [12].

Many methods have been implemented to avoid the problem of overfitting in regression-based approaches as best as possible. Complexity regularisation [13] introduces penalty functions to stop the neural networks growing too large, pruning [14] has been used to methodically remove sections of neural networks and subset selection [8] has been implemented to remove non-influential parameters. While these methods can help control overfitting, data sets with noisy predictors or responses can still cause problems. Additionally, machine learning approaches can also lack any interpretability.

The methodology implemented in this study uses multivariate nonlinear regression since the results are more interpretable. Two different optimisation methods are used for performing subset selection and to prevent overfitting of model orders. The first is a nonlinear forward stepwise regression [9] and the second is Lasso regression [15]. Since the Lasso is a backward stepwise type of regression, it makes a good comparison for the forward stepwise method. The novelty of this work comes from the comparison of the two methods, as well as the introduction of new metrics for assessing the quality of the coefficient matrices. The work in this paper complements forthcoming work on the topic of forward stepwise regression [16].

2. Methodology

2.1. Multivariate nonlinear regression

The DSFs extracted from the vibration responses from the structure can be expressed as a function relating to the environmental and operational parameters (EOPs). DSFs are modelled as a function of a coefficient matrix (W) and a multivariate representation of the EOPs ($\mathbf{f}(\boldsymbol{\xi})$). $\mathbf{f}(\boldsymbol{\xi})$ is expressed as the Kronecker product of the input variables. The coefficient matrix can then be estimated using the least squares estimate along with the DSFs and the multivariate representations. Once the coefficient matrix is established, the DSFs for each observation can be estimated. EOP-corrected DSFs are calculated as the difference between the original DSF and the estimated DSF, as shown in Eq. 1. A full explanation of this process can be found in previous work [9].

$$\tilde{\boldsymbol{\alpha}}_n = \boldsymbol{\alpha}_n - \hat{\boldsymbol{\alpha}}_n \tag{1}$$

where $\tilde{\boldsymbol{\alpha}}_n$ are the corrected DSFs, $\boldsymbol{\alpha}_n$ are the measured DSFs and $\hat{\boldsymbol{\alpha}}_n$ are the estimated DSFs.

When working with a large number of DSFs, it is impossible to individually design each regression model. Combining this fact with the necessity of avoiding overfitting, a well designed optimisation scheme is required. In this work, two different optimisation methods are tested, a forward stepwise method and Lasso regression. In their essence, the forward stepwise method starts with one EOP and builds up the model while the Lasso starts with all EOPs and works backwards. To evaluate which method is superior, two crucial metrics are introduced.

2.2. Nonlinear forward stepwise regression

In any forward stepwise regression, the starting point is always modelling the training data using a single variable. In this form of nonlinear forward stepwise regression, each input variable is tested up to a given order. The functional representation of the first step is given by Eq. 2.

$$\mathbf{f}(\boldsymbol{\xi}_n) = \mathbf{f}_i(\xi_{n,i}) \tag{2}$$

where the dimension of $\mathbf{f}_i(\xi_{n,i})$ is increased from 1 to the maximum order being tested.

For each EOP and order tested, the leave-one-out cross validation error (LOOCV) is calculated to give a measure of the accuracy of the model. The LOOCV (MSE_{LOO}) is calculated as shown in Eq. 3. At the end of of the first step, the EOP and corresponding order with the lowest LOOCV is progressed to the following stage.

$$MSE_{LOO} = \frac{1}{N_T} \cdot \sum_{n=1}^{N_T} (\tilde{\boldsymbol{\alpha}}_n)^2 \cdot \left(\sum_{n=1}^{N_T} (\boldsymbol{\alpha}_n)^2 \right)^{-1} \tag{3}$$

In the second step, the remaining EOPs are tested across the given range of orders along with the selected EOP and order from the first step. The functional representation is now written in the form in Eq. 4.

$$\mathbf{f}(\boldsymbol{\xi}_n) = \mathbf{f}_a(\xi_{n,a}) \otimes \mathbf{f}_i(\xi_{n,i_{-a}}) \tag{4}$$

where $\mathbf{f}_a(\xi_{n,a})$ is the functional representation from the first step and $\mathbf{f}_i(\xi_{n,i_{-a}})$ is the functional representation of the introduced EOP, excluding ξ_a which was selected in the first step.

As before, the combination with the lowest LOOCV is selected and progressed to the next step. The functional representation of the third step is given in Eq. 5.

$$\mathbf{f}(\boldsymbol{\xi}_n) = \mathbf{f}_a(\xi_{n,a}) \otimes \mathbf{f}_b(\xi_{n,b}) \otimes \mathbf{f}_i(\xi_{n,i_{-a,b}}) \tag{5}$$

where $\mathbf{f}_a(\xi_{n,b})$ is the functional representation from the second step and $\mathbf{f}_i(\xi_{n,i_{-a,b}})$ is the functional representation of the introduced EOP, excluding ξ_a and ξ_b which was selected in the first two steps.

This process is repeated until the addition of more EOPs no longer improves the prediction of the model, or only improves the model fractionally. The model with the lowest LOOCV from the previous stage is used as the final model for that DSF. The whole process is repeated for each DSF in the model.

2.3. Lasso regression

Lasso is a backward stepwise regression, and an appropriate comparison for the forward stepwise. The Lasso method is also based on the least squares estimate but introduces a penalty function in order to optimise the size of the regression coefficients. The form of the Lasso is given in Eq. 6a [17]. To understand better the Lasso method, it is easier to view it in its Lagrangian form, as in Eq. 6b

$$\hat{\mathbf{W}}_r = \underset{W}{\operatorname{argmin}} \sum_{n=1}^{N} \left(\alpha_{r,n} - W_0 - \sum_{l=1}^{L} \xi_{l,n} W_l \right)^2 \tag{6a}$$

$$\hat{\mathbf{W}}_r = \underset{W}{\operatorname{argmin}} \left\{ \frac{1}{2} \sum_{n=1}^{N} \left(\alpha_{r,n} - W_0 - \sum_{l=1}^{L} \xi_{l,n} W_l \right)^2 + \lambda \sum_{l=1}^{L} |W_l| \right\} \tag{6b}$$

where $\hat{\mathbf{W}}_r$ is the estimated coefficient matrix for the r-DSF, $\xi_{l,n}$ is the l-EOP of the n-observation, W_0 is a regression constant and W_l is the coefficient of the l-EOP. λ is the regularisation parameter which controls the rate at which the coefficient shrinks.

The regularisation parameter is used for the purpose of performing subset selection [15]. If λ is sufficiently small, the regression coefficients will become zero because of the geometry of the l_1 normalisation penalty [18]. If the coefficients reach zero, this implies that they have no influence on the DSF being modelled. Subset selection is performed through repetition of this idea.

2.4. System robustness metrics

2.4.1. Akaike information criterion. The Akaike Information Criterion (AIC) is a quantitative measure used to compare the predictive capability of different models [19]. In its essence, the AIC can be used to determine which model describes the most variation using the least number of input parameters. The AIC introduces a penalty function for the purpose of reducing overfitting by penalising higher order models. Eq. 7 gives the form used to calculate the AIC. The lower the AIC, the better the model.

$$AIC = -2 \ln(\Psi) + 2p_2 \tag{7}$$

where $\ln(\Psi)$ is the log-likelihood and p_2 the number of the input parameters of the given model.

There are other criteria that can be used to aid in the model selection process, such as the Bayesian Information Criterion (BIC) or the Deviance Information Criterion (DIC). The DIC is a generalisation of the AIC and typically suited to applications where the posterior is calculated through Markov chain Monte Carlo simulations, a reason why it is not applied here. The BIC is closely related to the AIC, but with a larger penalty function for the number of input parameters based on the sample size. However, the BIC aims to find the true model, a model that is unlikely to have been derived from the regression procedure. For the reasons above, the AIC is used as the most appropriate method given the data available in the case study.

2.4.2. Coefficient matrix stability. To understand the stability of the coefficient matrix, the variance of the covariance matrix is considered. The covariance matrix obtained from the least squares estimate. An estimate for the stability, variance-covariance, is given in Eq. 8. The expected outcome is that, the lower the variance-covariance, the more stable the coefficient matrix is. As such, the variability within the coefficient matrix is less.

$$\mathrm{Var}(\hat{\mathbf{W}})_r = (\mathbf{F}(\mathbf{X})^{\mathrm{T}}\mathbf{F}(\mathbf{X}))^{-1}\sigma_r^2 \tag{8}$$

where σ_r^2 is the variance, as calculated in Eq. 9, of the r-DSF.

$$\tilde{\sigma}_r^2 = \frac{1}{N - p_{2,r} - 1}\sum_{n=1}^{N}\tilde{\alpha}_{n,r}^2 \tag{9}$$

The $N - p_{2,r} - 1$, above, allows for an unbiased estimate of σ^2, such that $E(\hat{\sigma}^2) = \sigma^2$ [17].

2.4.3. F-statistic. An F-test can be used to test a null hypothesis [20]. In this work, the null hypothesis is that the regressed DSFs provide no improvement in predictive quality over the mean of the original DSFs. The F-statistic is a way of determining which models provide statistically significant results. Since the mean of the DSFs is being used for comparison in each case, the order of the model will always equal one. The F-statistic is calculated using Eq. 11.

$$RSS = \sum_{n=1}^{N_T}(\alpha_n - \hat{\alpha}_n)^2 \tag{10}$$

$$F_{0,r} = \frac{RSS_1 - RSS_2}{RSS_2} \times \frac{N_T}{p_2 - 1} \tag{11}$$

where $F_{0,r}$ is the F-statistic of the r-DSF, RSS_1 is residual sum of squares of the mean of the DSF and RSS_2 is the residual sum of squares of the model under scrutiny. In both cases, the RSS is calculated using Eq. 10.

The critical F-statistic, γ_r, is used to determine whether a DSF should be regressed or not according to the hypotheses in Eq. 12 and Eq. 13. The F-distribution is used to calculate the critical F-statistic.

$$\mathrm{H}_0 : F_{0,r} \leq \gamma_r \rightarrow \text{Not regressed} \tag{12}$$

$$\mathrm{H}_f : F_{0,r} > \gamma_r \rightarrow \text{Regressed} \tag{13}$$

where H_0 is the null hypothesis and H_f is the alternate hypothesis.

2.5. Outlier analysis

The outlier analysis that was used for the purpose of identifying damaged observations was the Mahalanobis Squared Distance (MSD). The MSD has frequently been used in the context of VSHM [7, 8, 9]. Typically, the MSD is written in the form given in Eq. 14.

$$MSD(\tilde{\boldsymbol{\alpha}}_n, \boldsymbol{\Sigma}_{\mathbf{A}}) = \tilde{\boldsymbol{\alpha}}_n{}^{\mathrm{T}}\boldsymbol{\Sigma}_{\mathbf{A}}\tilde{\boldsymbol{\alpha}}_n \tag{14}$$

where $\boldsymbol{\Sigma}_{\mathbf{A}}$ is the covariance of the training data set.

However, the modified MSD proposed by Roberts et al. [16] was used instead. The modified MSD for the PCA-based DSFs are given in Eq. 15, the regressed DSFs in Eq. 16 and the combination of the two types of DSF in Eq. 17.

$$MSD(\tilde{\boldsymbol{\alpha}}_n, \boldsymbol{\Sigma}_{\boldsymbol{\beta}}) = \tilde{\boldsymbol{\alpha}}_n{}^{\mathrm{T}} \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1} \tilde{\boldsymbol{\alpha}}_n : \boldsymbol{\Sigma}_{\boldsymbol{\beta}} = \frac{1}{N_T - 1} diag\left(\mathbf{S}_j^2\right) \tag{15}$$

$$MSD(\tilde{\boldsymbol{\alpha}}_n, \boldsymbol{\Sigma}_{\epsilon}) = \tilde{\boldsymbol{\alpha}}_n{}^{\mathrm{T}} \boldsymbol{\Sigma}_{\epsilon}^{-1} \tilde{\boldsymbol{\alpha}}_n : \boldsymbol{\Sigma}_{\epsilon} = \frac{1}{N_T - 1} diag\left(\sum_{n=1}^{N_T} \tilde{\alpha}_{n,r}^2\right) \tag{16}$$

$$MSD(\tilde{\boldsymbol{\alpha}}_n, \boldsymbol{\Sigma}_{tot}) = \tilde{\boldsymbol{\alpha}}_n{}^{\mathrm{T}} \boldsymbol{\Sigma}_{tot}^{-1} \tilde{\boldsymbol{\alpha}}_n : \boldsymbol{\Sigma}_{tot} = \begin{bmatrix} \boldsymbol{\Sigma}_{\boldsymbol{\beta}} & 0 \\ 0 & \boldsymbol{\Sigma}_{\epsilon} \end{bmatrix} \tag{17}$$

where $\mathbf{S}_j$ are the eigenvalues of the PCA transformation of the j-accelerometer and $\tilde{\alpha}_{n,r}$ is the r-DSF of the n-observation.

3. Case study

3.1. Experimental setup and monitoring campaign

The data that was used to demonstrate the proposed method was taken from an operational Vestas V27 wind turbine blade. Full details on the experiment can be found in [9], with a brief description given here for completeness. The blade was fitted with 12 accelerometers and 1 electro-mechanical actuator, as shown in Fig. 2. The blade was periodically struck by the actuator and the response measured using the accelerometers. Only the acceleromters on the leading and trailing edge are used, as well as the accelerometer adjacent to the actuator.

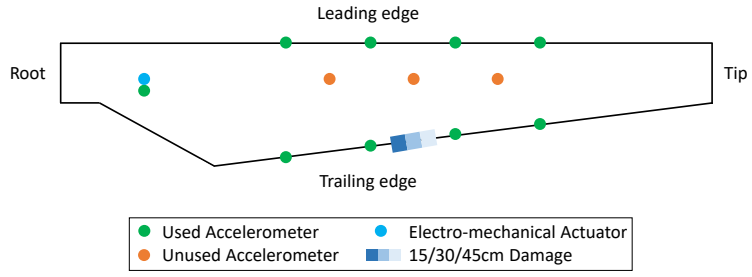

Figure 2. Locations of accelerometers and electro-mechanical actuator on Vestas V27 wind turbine blade (adapted from a previous work [9]).

Figure 3. Image of damage introduced to V27 wind turbine blade.

During the experimental campaign, different damage scenarios were tested. It started as undamaged, then a crack was introduced to the trailing edge of the blade, see Fig. 3, and finally repaired. The crack started at 15cm and was extended to 30cm and finally to 45cm. The total number of observations for each damage scenario is listed in Table. 1.

Table 1. Number of observations available for each damage scenario in the Vestas V27 wind turbine blade monitoring campaign.

Undamaged	15cm damage	30cm damage	45cm damage	Repaired
828	258	194	254	4320

In this work, the repaired observations were used for training and testing since there were significantly more of them. Additionally, the range of EOPs experienced within the repaired was greater than for the undamaged. 80% of the repaired observations (3456) were used for training and 20% for testing (864). The EOPs that were considered in this work were: temperature, wind speed, wind direction, pitch, yaw, azimuth angle, RPM, the maximum value of actuator hit and the standard deviation of actuator hit.

3.2. Damage sensitive feature extraction

The DSFs are based on a reduced order Fourier transformation of the acceleration responses. For each observation, the response is transformed into the frequency domain and the real component and real part of the imaginary component are concatenated. The vector is then reduced using principal component analysis. The information across all accelerometers are then concatenated to create a single DSF vector. A full explanation of the derivation is excluded for the sake of brevity, but can be found in previous works [9].

4. Results and discussions

In this section, reference is made to four different optimisation schemes for the design of regression models. The first (5 EOP) is a trial and error optimisation approach from previous work [9] where every combination of model order with 5 EOPs (temperature, wind speed, azimuth, actuator maximum and actuator standard deviation) is considered. The second (forward stepwise) is from the forward stepwise approach from Section 2.2. The third (reduced forward stepwise) is, again, from the forward stepwise approach but with the application of an order overfitting method. The fourth (Lasso) is from the Lasso regression method from Section 2.3. Depending on the optimisation scheme implemented, the model for each DSF will differ in which EOPs are included, and the order that they are included at.

4.1. Akaike information criterion

Firstly, the AIC is used to determine the models that explain the most variance using the fewest input parameters. The AIC for four different methods is shown in Fig. 4. In each case, the AIC has been sorted so that it is possible to determine which method has generally the lowest AIC.

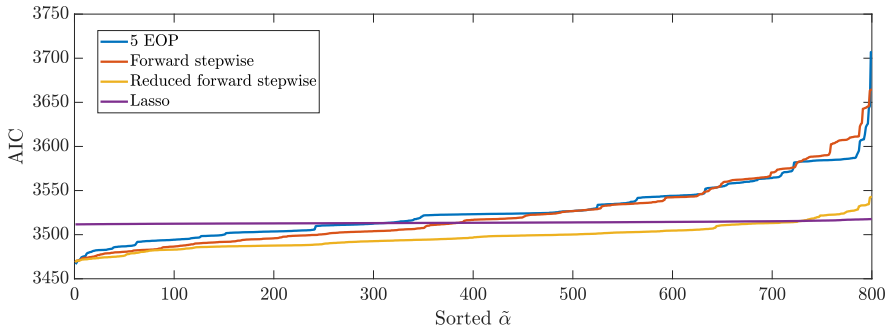

Figure 4. AIC values against assorted DSFs using the 5 EOP, forward stepwise, reduced forward stepwise and Lasso optimisation procedures. DSFs are sorted low to high according to their corresponding AIC value, order of DSFs are different for each optimisation method.

From Fig. 4, it can be seen that the reduced order forward stepwise method has the lowest AIC across the most DSFs. The regular forward stepwise method and the trial and error approach using 5 EOPs generally perform on a similar level. This demonstrates that the overfitting of model orders and fitting non-influential parameters has the same level of effect on the predictability of the models. In the case of the Lasso, the AIC is consistent across all DSFs. This is likely due to the coefficient matrices being more similar in size compared to the alternative methods, whilst also having similar predictive accuracy. While the Lasso has lower AIC for some DSFs, the reduced order forward stepwise method performs far better for a larger number of DSFs as it provides the lowest AIC across a large number of DSFs. Based on the results from Fig. 4, the method to use would be the reduced order forward stepwise.

4.2. Coefficient matrix stability

A secondary aspect of each model that was tested was the coefficient matrix stability. The values for the coefficient matrix stability from Eq. 8 are shown in Fig. 5.

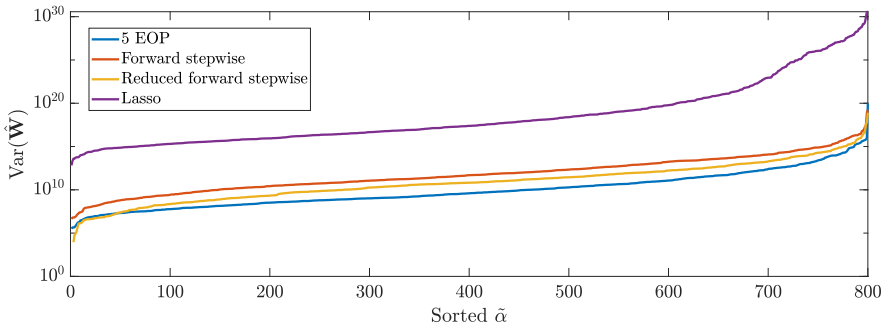

Figure 5. Coefficient matrix stability values against assorted DSFs using the 5 EOP, forward stepwise, reduced forward stepwise and Lasso optimisation procedures.

Fig. 5 shows that the trial and error (5EOP) approach has the most stable coefficient matrices. That is followed by the reduced order forward stepwise method, the forward stepwise method and, finally, the Lasso. While the trial and error approach has the highest stability, previous works [7, 9] have shown that this approach suffers in the prediction of future observations. On the other hand, the reduced order forward stepwise was able to predict future observations better through the methods within it that aimed to reduce overfitting. The result in Fig. 5 also shows that the reduction of model orders help with the matrix stability. Given previous results and those in Fig. 5, the best choice once again would be the reduced order forward stepwise method.

4.3. F-statistic

The F-statistic is used for determining whether the regression model predicts better the DSF than the mean of the DSF. Fig. 6 shows how the F-statistic for the reduced order forward stepwise method compares to its associated critical F-statistic. Highlighted in grey are the instances where the model satisfies the alternative hypothesis.

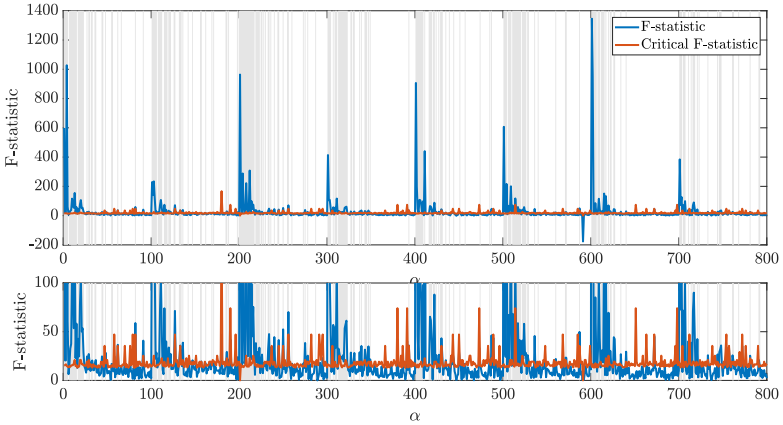

Figure 6. F-statistic and critical F-statistic across 800 DSFs (100 from each accelerometer) for the reduced forward stepwise optimisation method. The grey lines highlight when $F_{0,r} > \gamma_r$.

By studying Fig. 6, the most noticeable aspect across all accelerometers is that the DSFs that explain the most variance in the data are also the ones where the alternative hypothesis is satisfied. The implication of this is that they are the DSFs that are most influenced by the varying conditions, since the created models provide the most significant benefit. Similarly, the higher order DSFs most often satisfy the null hypothesis. This suggests that they may be a result of noise and, therefore, not affected by the varying conditions.

Given that a large number of DSFs satisfy the null hypothesis, it does not make sense to include all DSFs in the VSHM system. Instead, the regressed DSFs that satisfy the alternative hypothesis should be combined with the original DSFs that do not. This approach will reduce the level of variations in the system by choosing only the most influential regression models.

4.4. Outlier analysis

A control chart is a convenient way to present the outlier analysis across all the damage cases. With the use of a threshold, a binary damage diagnosis can be made. Fig. 7 shows the control chart for a DSF set made up of reduced order forward stepwise regression DSFs and original DSFs, based on the results from Fig. 6. The threshold between damage and undamaged, black dashed line, was chosen to be the value that explains 95% of the training data according to a Chi-squared distribution.

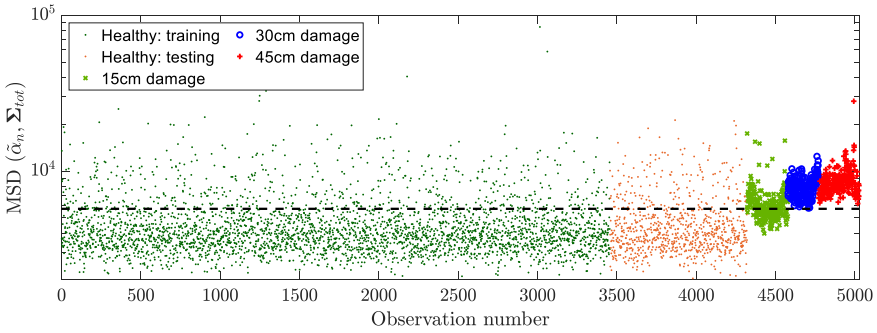

Figure 7. Control chart showing the result of combining reduced order forward stepwise regressed DSFs and original DSFs according to a critical F-statistic of 0.1%.

In the control chart in Fig. 7, the 30cm and 45cm damages are well detected. On the other hand, the smaller 15cm damage is not very well detected. However, this work aims to demonstrate the importance of carefully designed coefficient matrices and does not aim to optimise damage detection. The overlap between training and testing is a key factor in designing robust systems, and the control chart demonstrates this. The major negative aspect of Fig. 7 is that there are still a large number of outliers in the training and testing cases. Further work is required to quantify the effect of these observations [16].

5. Conclusions

A number of optimisation methods were applied to data taken from an operational wind turbine blade with artificially introduced damage. The aim of the work was to evaluate and compare different optimisation schemes using multivariate nonlinear regression for mitigating environmental and operational variations in the context of VSHM. The AIC was implemented first to quantify the predictive capability against the number of predictors. Following on, the variance-covariance was used to analyse the stability of the coefficient matrices, a concept previously unseen in VSHM. Finally, the F-statistic was used to determine which DSFs should or should not be regressed. The outcome was that the reduced order forward stepwise regression

method performed the best, owing to its low AIC values and stable coefficient matrices. A control chart was used to show the quality of the damage detection. Whilst the damage detection was not perfect, there was still separation between the undamaged observations and the 30cm and 45cm damaged observations.

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