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# Softening-based interface model and nonlinear load-settlement response analysis of piles in saturated and unsaturated multi-layered soils

### Citation for published version:

Pham, TA, Nadimi, S & Sutman, M 2024, 'Softening-based interface model and nonlinear load-settlement response analysis of piles in saturated and unsaturated multi-layered soils', *Computers and Geotechnics*, vol. 171, 106331. <https://doi.org/10.1016/j.compgeo.2024.106331>

### Digital Object Identifier (DOI):

[10.1016/j.compgeo.2024.106331](https://doi.org/10.1016/j.compgeo.2024.106331)

### Link:

[Link to publication record in Edinburgh Research Explorer](#)

### Document Version:

Peer reviewed version

### Published In:

Computers and Geotechnics

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1                   **Softening-based interface model and nonlinear load-settlement**  
2 **response analysis of piles in saturated and unsaturated multi-layered soils**  
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29 **Abstract**

30 This work presents a simplified method for the nonlinear analysis of the load-displacement response of piles  
31 in multi-layered soils. A new interface model based on the disturbed state concept (DSC) is put forth to simulate the  
32 interface shear stress-displacement relationship by considering the nonlinear hardening-softening behaviour. In the  
33 new model, input parameters can be conveniently calibrated using conventional interface shear tests or on-site tests.  
34 The good agreement between predictions and experimental data from interface direct shear tests validated the  
35 performance of the proposed DSC model. The DSC model performed better in terms of predictions when compared  
36 to the hyperbolic one. Next, the soil-structure interface model and bearing capacity theory are coupled to provide a  
37 theoretical framework for the analysis of pile load-transfer in saturated and unsaturated multi-layered soils, where the  
38 DSC model is employed to represent base resistance as well as skin friction. This work also discusses the profile of  
39 steady-state in-situ matric suction, soil-water characteristic curve, and pore-water pressure of unsaturated soils. The  
40 proposed method has the advantage of being used in practice as it is simple to obtain input parameters from laboratory  
41 tests, as well as Standard Penetration or Cone Penetration Tests. The proposed framework is finally applied to the  
42 analysis of four well-documented case studies. The proposed approach and the static load test results from the field  
43 measurements are found to be in satisfactory agreement, indicating that the proposed method performs well. The  
44 proposed method is suggested to be utilised for preliminary analysis, planning a suitable programme of loading tests,  
45 as well as optimizing the pile design by back analysis of the load test results.

46  
47 **Keywords:** Piles; settlement; soil-structure interaction; bearing capacity; softening model; analytical method;  
48 disturbed state.

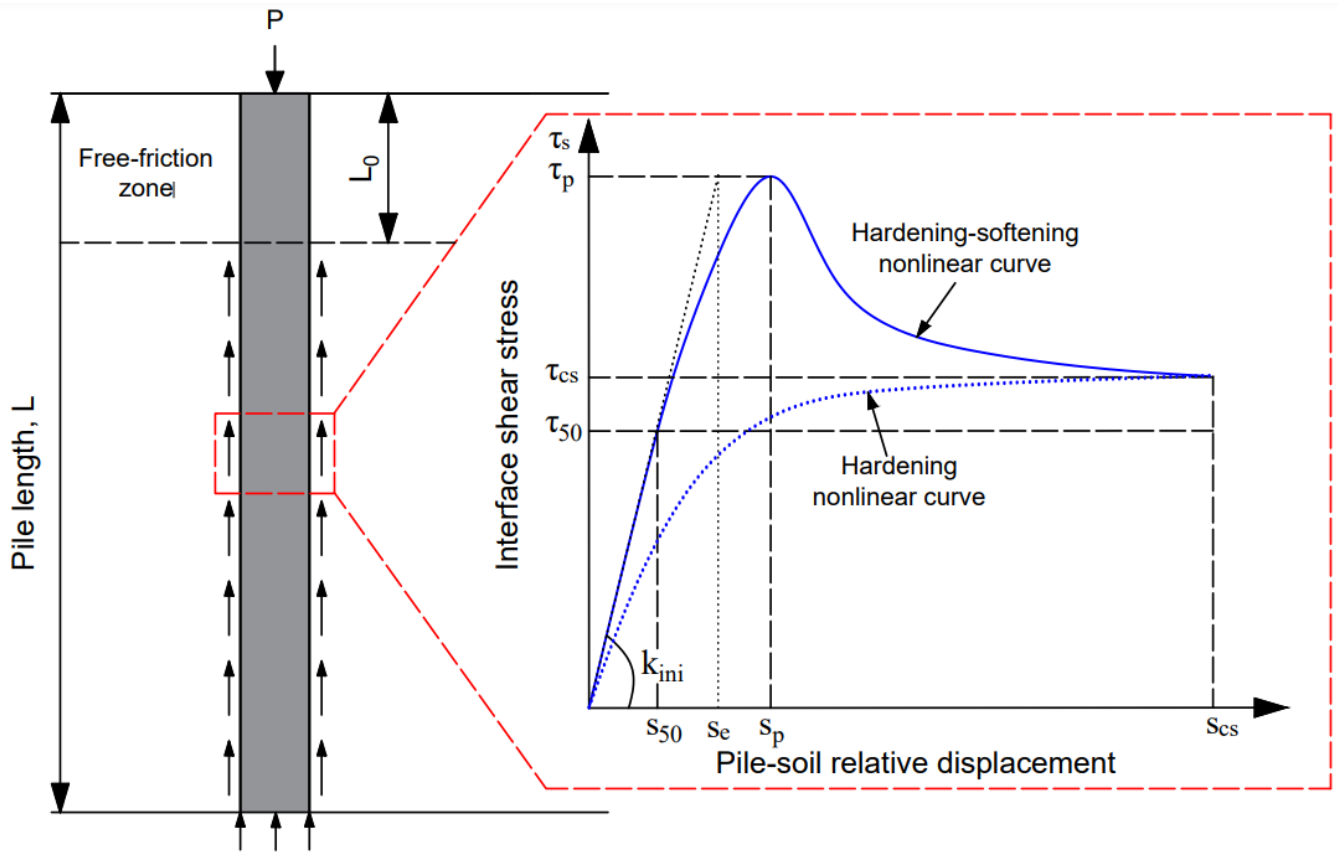
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## 64 **1. Introduction**

65 Pile foundations provide high bearing capacity and have been widely employed to support a variety of  
66 applications such as building, deep excavations, slopes, tunnels, and embankments. The load-transfer mechanism of  
67 a single pile element includes skin friction resistance along the soil-pile interface and ground reaction at the pile base.  
68 There have been several attempts to develop techniques for determining the load-settlement behaviour of individual  
69 piles and pile groups, ranging from simplified analytical methods to sophisticated numerical modelling and full-scale  
70 in-situ experiments. As the behaviour of piles subjected to vertical force is a complex problem, the experimental  
71 approach is regarded as one of the most effective methods for analysing the behaviour of piles. The in-situ tests,  
72 however, are difficult, expensive, and time-consuming. In the majority of cases, design engineers prefer to use  
73 analytical approaches, particularly in the preliminary design phase.

74 The load transfer mechanism of piles frequently exhibits a highly nonlinear behaviour due to the intricate  
75 interactions between piles and the surrounding soils. The nonlinearity in the load-settlement response of the piles  
76 requires simultaneous consideration of both shaft and base resistance ([Castelli and Maugeri 2002](#); [Paik and Salgado 2003](#); [Park et al. 2012](#); [Seo et al. 2009](#); [Nadan and Patra 2014](#)). The existing methods have typically used the  
77 theoretical load-settlement relationship with the hyperbolic model to investigate the nonlinear behaviour of individual  
78 piles ([Hirayama 1990](#); [Flemming 1992](#); [Dithinde et al. 2011](#); [Dias and Bezuijen 2018](#)). The hyperbolic model,  
79 however, might only be appropriate in the unique scenario of strain hardening that is, the skin friction increases with  
80 the relative pile–soil displacement. In reality, skin resistance deterioration has been reported in full-scale field test  
81 results where skin friction softening was observed ([Reese and O'Neill 1988](#); [Caputo et al. 1991](#); [Briaud et al. 2000](#);  
82 [Zhu and Chang 2002](#); [Fioravante 2002](#); [Lee et al. 2003](#); [Fellenius et al. 2004](#); [Zhang et al. 2010](#); [Lehane et al. 2012](#);  
83 [Park and Lee 2015](#); [Bohn et al. 2017](#)). The typical characteristic of skin friction softening is that the skin friction  
84 increases with the pile-soil displacement before reaching its peak value and then decreases to a residual value ([Figure](#)  
85 [1](#)). As a result, the skin friction resistance in this situation cannot be reasonably predicted by the hyperbolic models  
86 ([Zhang and Zhang 2012](#)). Several models, including the dual exponential, the rational-function, and the three-phase  
87 piecewise-hyperbolic models, have been recently developed to characterise the softening behaviour of geomaterials  
88 and structures ([Seol et al. 2009](#); [Zhang and Zhang 2012](#); [Zhu et al. 2021](#); [Chen et al. 2022](#)). These models were,  
89 however, more frequently applied to the behaviour of the anchor-soil interface than to that of the pile. Moreover, they  
90 are less ideal for application in pile load-transfer analysis as they are defined piecewise and contain many assumed  
91 parameters. In the current theoretical load transfer modelling frameworks for piles, the interface stress-  
92 displacement model and its parameters were typically chosen based on the epistemic experience of specific  
93 researchers and/or back analysis of the in-situ test data, particularly of the pile load-displacement data ([Tang and](#)  
94 [Phoon 2018](#); [Guo et al. 2022](#); [Guo et al. 2023](#)). For this procedure, the tested structural boundary conditions at the  
95 pile head were substantially used. This parameter calibration method is indirect, semiempirical, and constrained by  
96 the outcomes of the in-situ tests. Therefore, further studies are necessary for modelling the characteristics of pile-soil  
97 interfaces, more accurately.  
98

99 In this study, a simplified method for the nonlinear analysis of the load-settlement response of piles in  
 100 saturated and unsaturated multi-layered soils which takes into account shaft resistance deterioration and base  
 101 resistance hardening is presented. Firstly, an adhesion-friction-based model is developed using the disturbed state  
 102 concept (DSC) to describe the behaviour of the soil-pile interface. The experimental outcomes of interface direct  
 103 shear tests are then used to verify the proposed DSC model. Next, based on the established integrated interface model,  
 104 a comprehensive load-transfer modelling framework of piles is developed for analysing the nonlinear load-settlement  
 105 behaviour. This work also discusses the profile of steady-state in-situ matric suction, soil-water characteristic curve,  
 106 and pore-water pressure of unsaturated soils that allows to apply the proposed method to piles in unsaturated soils.  
 107 The effectiveness and accuracy of the proposed approach are evaluated using well-documented field test results  
 108 gathered from the literature.



109  
 110 **Figure 1.** Mobilization of interface shear stress with pile-soil relative displacement

111 **2. Disturbed state-based interface model**

112 In current studies, the nonlinear behaviour involving the interface shear stress and relative displacement is  
 113 frequently explained by a hyperbolic relationship. The hyperbolic relationship, however, can be applied to special  
 114 situations, like strain hardening but it is unable to replicate how resistance deteriorates with increasing displacement.  
 115 In addition, the preceding methods neglected the impact of the soil disturbance on the mobilised shear stress at the  
 116 soil-structure interface. Finally, these techniques were mainly developed to study the interaction between  
 117 saturated granular soil and structures and limited research was performed on the unsaturated soil-structure interfaces.

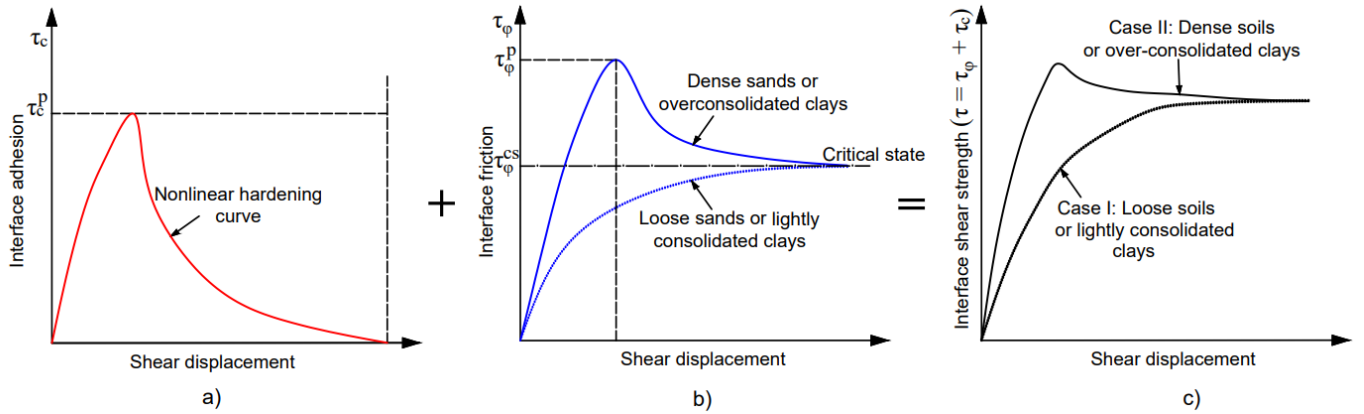
118 In order to address these shortcomings, a more generalised interface model for load-transfer analysis is still needed.  
 119 The following simplifications are employed in the development of the present method:  
 120 i) Adhesion and interface friction coexist and simultaneously contribute to the resistance of the soil-structure interface.  
 121 ii) The soil is thought to remain in a linear elastic state outside of the soil-structure interface.  
 122 iii) The nonlinear displacement mostly happens along the disturbed zones surrounding the structure interface.  
 123 iv) The structures undergo negligible horizontal deformation.

## 124 2.1. Adhesion-friction based interface theory

125 It is well acknowledged that the geomaterial-structure interface shear strength at a specific displacement is  
 126 typically made up of interface adhesion  $\tau_c(s)$  involving the bonding and interface friction  $\tau_\phi(s)$  involving particle  
 127 sliding (Chu and Yin 2005; Mitchell and Soga 2005; Pham 2020b; Chen et al. 2022). According to Figure 2, the  
 128 interface shear strength can be expressed as the sum of the responses as follows:

$$129 \quad \tau(s) = \tau_c(s) + \tau_\phi(s) \quad (1)$$

130 where,  $\tau(s)$ ,  $\tau_c(s)$ ,  $\tau_\phi(s)$  = total, adhesion, and friction components of the interface shear strength, respectively;  $s$   
 131 = relative displacement of the interface.



132 a) b) c)  
 133 **Figure 2.** Mobilization mechanism of interface shear resistance: a) adhesion component; b) friction component; c)  
 134 total interface shear strength

135 In geotechnical engineering, the term adhesion is used to define the cohesive strength between the structure  
 136 surface and soils. Interface adhesion first becomes more mobilized with increasing displacement before approaching  
 137 a constant value as illustrated in Figure 2a. The total adhesion of the soil-structure interface is generally contributed  
 138 by intermolecular forces like electrostatic and Van der Waals, soil suction from water's surface tension on soil particles,  
 139 and cementation from the chemical bonding of soil particles (Figure 3a). The general expression for the involvement  
 140 of several factors in the mobilization of interface adhesion is as follows:

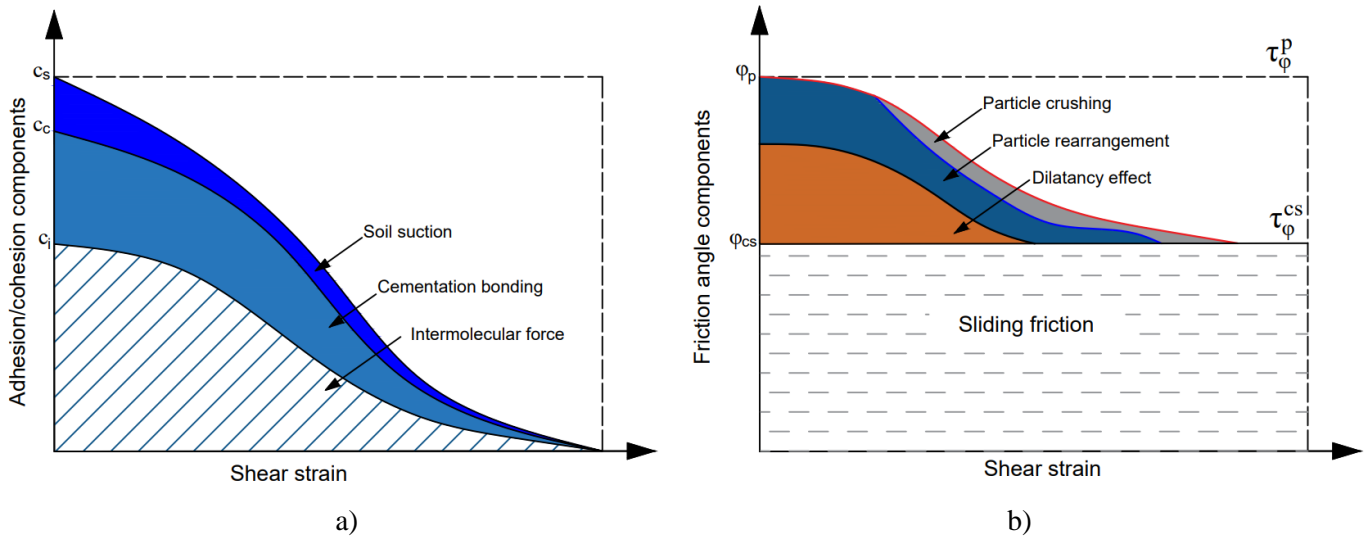
$$141 \quad \tau_c(s) = \tau_c^i + \tau_c^c + \tau_c^s \quad (2)$$

142 where  $\tau_c^i$  = interface adhesion induced by intermolecular force,  $\tau_c^c$  = interface adhesion induced by cementation,  $\tau_c^s$   
 143 = interface adhesion induced by suction

144 Meanwhile, the interface friction is provided by surface-to-surface interaction through a combination of  
 145 sliding, dilatancy effects, particle crushing, and rearrangement of particles as shown in Figure 3b. However, the  
 146 relative contributions of each of these factors to the peak shear strength of sands and clays can vary, and not all  
 147 contributing factors need to appear at the same time for either sands or clays. It is evident that the effects of crushing,  
 148 dilatancy, and particle rearrangement all have a decreasing influence as value  $(\tau_{\phi}^p - \tau_{\phi}^{cs})$  decreases. This tendency is  
 149 appropriate for both sands and clays. Clays, for instance, may have the small effects of crushing and dilatancy but the  
 150 impacts of these factors may increase depending on the over-consolidation ratio or the course-grain amount in clays.  
 151 It should be also noted that the friction component is influenced by several factors such as surface roughness, relative  
 152 density, normal pressure, over-consolidation ratio, and structure material type (Potyondy 1961; Uesugi et al. 1990;  
 153 Martinez and Frost 2017; Wang et al. 2019; Pham 2020c; Ravera et al. 2021; Hashemi et al 2022). Experimental  
 154 results from direct shear and torsional shear tests suggest that interfaces exhibit strain-hardening behaviour when  
 155 loose sand soils, normally consolidated or lightly over-consolidated clays are involved, whereas strain-softening  
 156 behaviour is observed when dense sand or heavily overconsolidated clays are tested (Kishida and Uesugi 1987; Hu  
 157 and Pu 2004; Toufigh et al. 2017). Interface friction is generally mobilized with increasing interface displacements  
 158 as shown in Figure 3b. The general description of how various components contribute to the mobilization of interface  
 159 friction is as follows:

$$\tau_{\phi}(s) = \tau_{\phi}^i + \tau_{\phi}^s + \tau_{\phi}^d + \tau_{\phi}^c \quad (3)$$

160 where  $\tau_{\phi}^i$  = interface friction induced by interlocking,  $\tau_{\phi}^s$  = interface friction induced by sliding,  $\tau_{\phi}^d$  = interface  
 161 friction induced by dilatancy effect,  $\tau_{\phi}^c$  = interface friction induced by particle crushing.



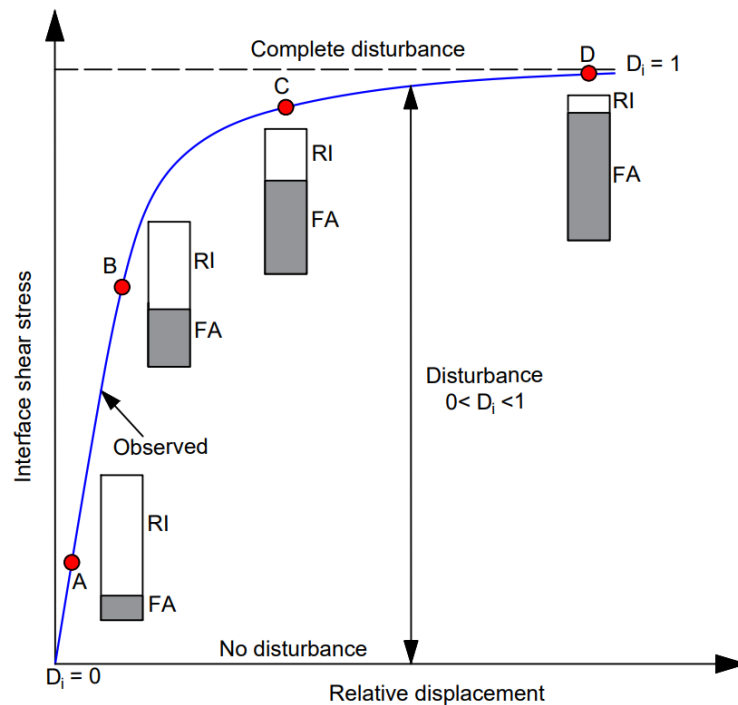
163  
 164 **Figure 3.** Contribution of components on the soil-structure interface resistance: a) Adhesion, b) Friction

165 The interface adhesion concurrently occurs with interface friction as soil-structure relative displacement  
 166 increases. However, the ultimate friction and ultimate adhesion are not mobilized at the same displacement. In most  
 167 cases, the adhesion component is mobilized first, even at relatively small displacements. Following that, the friction  
 168 component starts to be mobilized as the adhesion reaches the ultimate value. The transition between the adhesion and  
 169

170 friction components is represented by the disturbance degree of the interface. If the friction mobilization decrement  
 171 is greater than the adhesion increment, the interface shear stress may then decline and approach the critical state value  
 172 (Figure 2c). An interface model, therefore, must be able to simulate the mutual evolution between adhesion and  
 173 friction components as well as the degree of interface disturbance in order to accurately capture the shear stress-  
 174 displacement relationship. Here, the DSC theory, which offers a suitable approach, is chosen to characterise the  
 175 nonlinear behaviours of the soil-structure interface and is presented in the next sections.

## 176 2.2. Disturbance function for interface

177 Desai (2001) presented the idea of the disturbed state, which proposed that the material interface response  
 178 can be specifically divided into two distinct mechanical states: the relatively intact (RI) state and the fully adjusted  
 179 (FA) state. Parts of a deforming material are said to be in the RI state if they are still in their original (continuum)  
 180 state, whereas parts are said to be in the FA state if they are still in their degraded state (discontinues). The elements  
 181 in the interface zone at the RI and FA states will be randomly distributed and work jointly. With increasing disturbance,  
 182 the response of the two states changes dynamically, reflecting the deteriorations and discontinuities of the materials  
 183 from a micro perspective (Figure 4). The adhesion-friction mobilization theory, which was previously described, and  
 184 the variation between the two RI and FA states are clearly in a logical relationship. Specifically, as friction  
 185 components increase, the number of elements covered by the FA state must rise and disturbance becomes larger. As  
 186 a result, the degree of interface disturbance, which reflects the state of the elements in the interaction zone, has a  
 187 substantial influence on the interface shear strength.



188

189 **Figure 4.** Transition between RI and FA states with disturbance degree

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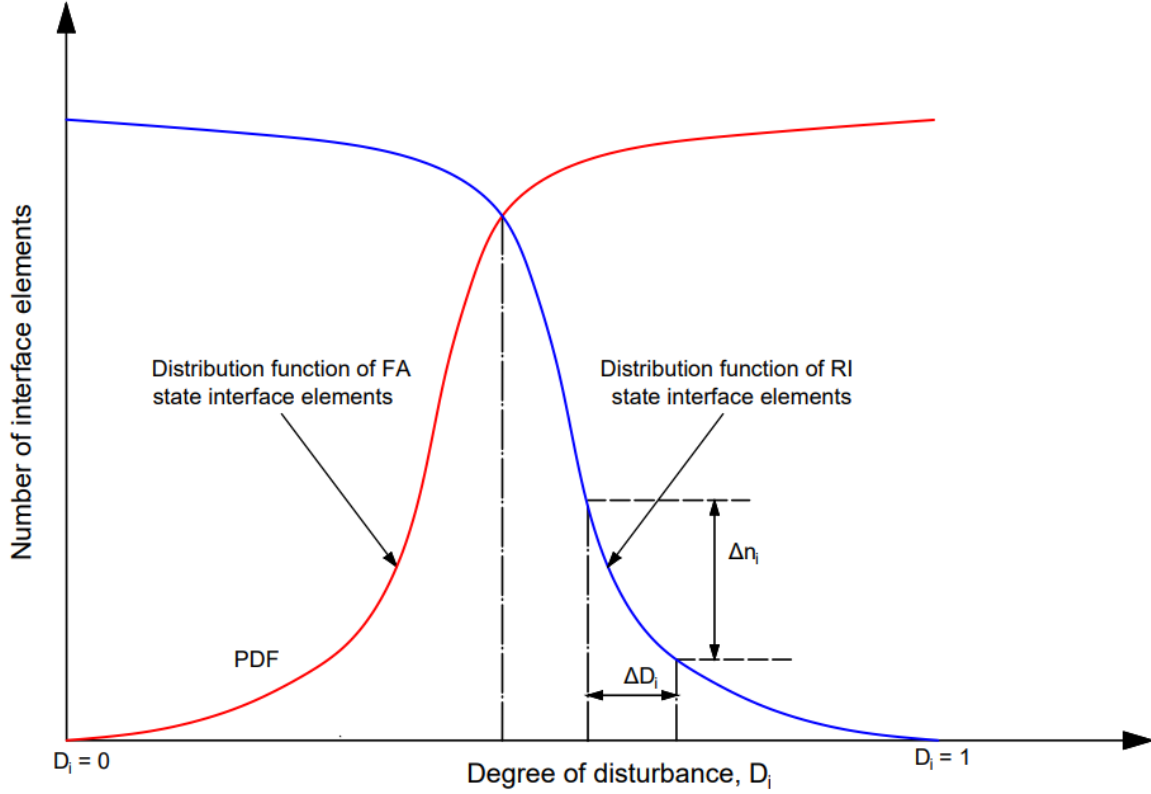
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The disturbance degree of an interface is often described by the disturbance function  $D_i$  ( $0 \leq D_i \leq 1$ ) which practically indicates the degree of interface damage from an external disturbance. The function  $D_i$  will provide a



192 higher value when the interface is subjected to more severe external disturbances. In particular, if  $D_i = 0$ , the  
 193 disturbance will completely disappear, and all interface elements will be in the RI state. But, if  $D_i = 1$ , the interface  
 194 will be completely disturbed, and all interface elements will be in the FA state. In the modelling framework presented  
 195 herein,  $D_i$  is defined as the proportion of FA state interface elements ( $n_{FA}$ ) to total interface elements ( $n_i$ ).

196 
$$D_i = \frac{n_{FA}}{n_i} \quad (4)$$



197

198 **Figure 5.** Probability density function and interface elements

199 The quantity of FA state interface elements and that of RI state interface elements are dynamically changing  
 200 as soil-structure relative displacement increases, as shown in Figure 5. The Weibull distribution can be used to  
 201 mathematically describe the shear strength of all interface elements (Desai 2001; Baghini et al. 2018; Huang et al.  
 202 2020; Wu et al. 2022). The definition of the probability density function (PDF) is as follows:

203 
$$PDF(x) = \frac{\eta}{\xi} \left(\frac{x}{\xi}\right)^{\eta-1} e^{-(x/\xi)^\eta} \quad (5)$$

204 where  $\eta$  and  $\xi$  = two probability parameters,  $x$  = the process variable and is defined as the relative displacement  
 205 between the soil and the structure surface in this work.

206 The number of interface elements in the FA state increases to  $n_{FA}$  when the soil-structure relative  
 207 displacement reaches value  $s$  which can be expressed as follows:

208 
$$n_{FA} = \int_0^s n_i \cdot PDF(x) dx \quad (6)$$

209 Substituting (5) into (6) gives:

$$n_{FA} = \int_0^s n_i \cdot \frac{\eta}{\xi} \left(\frac{x}{\xi}\right)^{\eta-1} e^{-(x/\xi)^\eta} dx = n_i \cdot (1 - e^{-(s/\xi)^\eta}) \quad (7)$$

Replacing Eq. (7) in (4) leads to the following expression:

$$D_i = (1 - e^{-(s/\xi)^\eta}) \quad (8)$$

The disturbance function is rewritten in the simplified form as follows:

$$D_i = 1 - e^{-as} \quad (9)$$

### 2.3. Interface shear stress-displacement model

The disturbance can also be considered as the deviation of the current deforming state with respect to the initial and final states of the material, which can be defined by using the interface shear stress. The disturbance function for the hardening-softening response of the interface generally can be defined as follows:

$$D_i^h = \frac{\tau}{\tau_p} \quad (10)$$

$$D_i^s = \frac{\tau_p - \tau}{\tau_p - \tau_{cs}} \quad (11)$$

where  $D_i^h$  = disturbance function for the hardening phase;  $D_i^s$  = disturbance function for the softening phase which is similar to residual factor;  $\tau$ ,  $\tau_p$ ,  $\tau_{cs}$  = current, peak, and residual shear stress, respectively. It should be noted that the critical state shear stress is defined as a constant stress value even when the interface displacement is continuously increased. When there is no softening phase (only hardening behaviour), the peak and critical state shear stress are identical and are frequently referred to as the ultimate value.

It is necessary to integrate functions  $D_i^h$  and  $D_i^s$  into a unique disturbance function to achieve a smooth transition between the hardening and softening phases on the shear stress-displacement curve. Eq. (11) is substituted into Eq. (10) to produce:

$$\tau = \frac{D_i^h \cdot D_i^s \cdot \tau_{cs}}{D_i^h + D_i^s - 1} = b \cdot D_i - \Delta\tau \quad (12)$$

where  $b$  = model parameter,  $\Delta\tau$  = shear stress variation in the softening region. No shear stress reduction occurs after the post-peak zone in the case of hardening behaviour, so  $\Delta\tau = 0$  and  $b = \tau_p$ . However, if the softening behaviour occurs,  $\Delta\tau \neq 0$ . The method based on the shape of the curve is used to account for the softening behaviour component. The softening curve part is therefore thought to be the result of any hypothetical hardening curve minus the post-peak shear stress decrease zone (coloured area), as illustrated in Figure 6. In this work, the softening portion of the shear stress-displacement curve is assumed to have a hyperbolic shape. The transition curve portion is represented as follows:

$$\Delta\tau = c(S^2 - S_p^2) \quad (13)$$

where,  $c$  = calibration parameter to represent the real form of the curve,  $S_p$  = ultimate displacement corresponding to the peak shear stress.

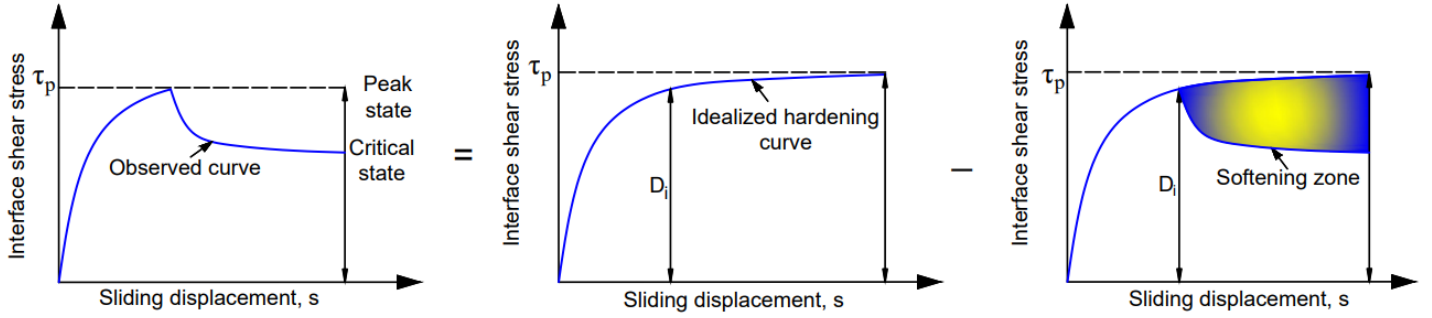


Figure 6. Calculation framework of the hardening-softening curve

Substituting Eqs. (9) and (13) into (12) give interface shear strength:

$$\tau = b(1 - e^{-as}) - c(s^2 - s_p^2) \quad (14)$$

It should be observed that the interface shear stress, however, has a constant value and equals the critical state shear stress when it approaches the critical state condition. This was confirmed by the measurements in reported direct shear tests (Kishida and Uesugi 1987; Mortara et al. 2007; Farhadi and Lashkari 2017; Maghsoodi et al. 2020). Hence, the full form of the proposed equation can be written as follows:

$$\begin{cases} \tau = b(1 - e^{-as}) - c(s^2 - s_p^2) & \text{if } s \leq s_{cs} \\ \tau = \tau_{cs} & \text{if } s > s_{cs} \end{cases} \quad (15)$$

where  $s_{cs}$  = critical state displacement. It is noted that the proposed model comprises three parameters with clear physical meanings, in which  $a$  controls the slope of the hardening curve part,  $b$  controls the peak part of the curve, and  $c$  controls the softening curve portions. However, all three of these parameters must be coupled in a specific way for the convenience of applicability.

It should be noted that in some special cases, interface shear strength may only consist of interface friction due to the absence of interface adhesion. This is true for structures embedded in coarse soils such as sand and gravel. In almost all other cases, frictional resistance and adhesion, which are mobilized concurrently, combine to produce an interface peak shear strength. However, it is difficult to infer from the results of soil tests how much of the shear strength is provided by friction and adhesion. By using the proposed model, the friction and adhesion components can be determined as follows:

$$\tau_c(s) = c_p \cdot \frac{1 - e^{-as}}{1 + e^{-as}} \quad (16a)$$

$$\tau_\phi(s) = \tau(s) - \tau_c(s) = b \cdot (1 - e^{-as}) - c_p \cdot \frac{1 - e^{-as}}{1 + e^{-as}} - c(s^2 - s_p^2) \quad (16b)$$

where  $c_p$  = peak adhesion of the interface.

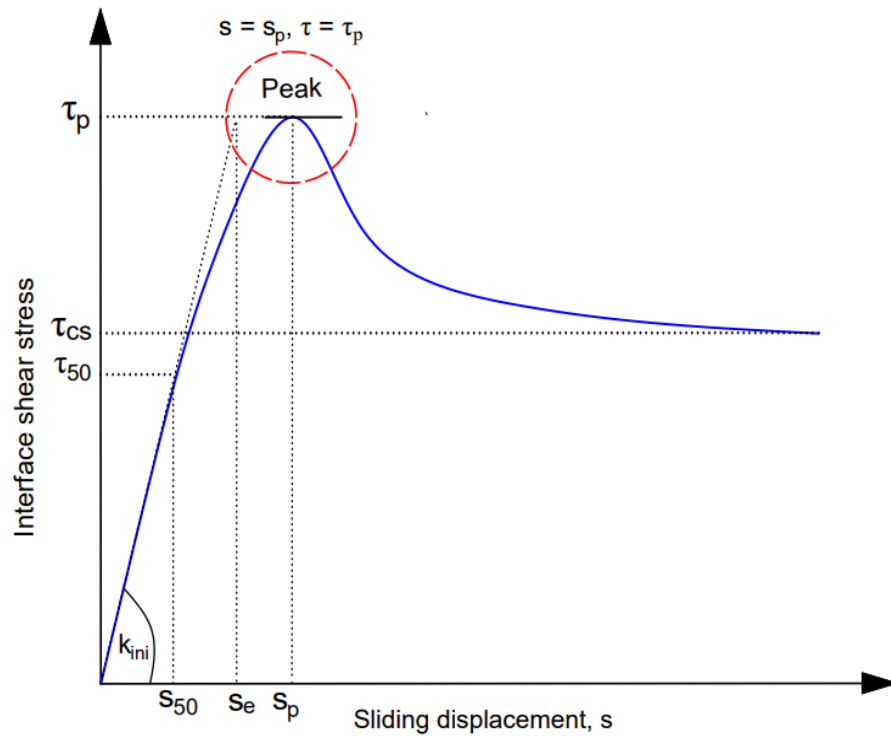
#### 2.4. Determination of model parameters

When calculating the values of the three model parameters ( $a$ ,  $b$ ,  $c$ ), it is acceptable to use information from characteristic points (also known as interface characteristic parameters) in the shear stress-displacement curve. The initial point, ultimate elastic point, peak stress point, and residual stress points are some important locations of the

266 curve that can be considered. These points yield the peak shear stress  $\tau_p$ , the displacement corresponding to the peak  
 267 shear stress  $s_p$ , the critical state shear stress  $\tau_{cs}$ , and the initial shear stiffness  $k_{ini}$ . Figure 7 illustrates the boundary  
 268 conditions to determine these model parameters.

269 The derivative of the interface shear stress-displacement relationship is written as follows:

$$270 \quad \frac{d\tau}{ds} = \frac{\partial[b \cdot (1 - e^{-as})]}{\partial s} + \frac{\partial[(s_p^2 - s^2) \cdot c]}{\partial s} = a \cdot b \cdot e^{-as} + 2c \cdot s \quad (17)$$



271  
 272 **Figure 7.** Determination of model parameters based on the boundary conditions.

273 **Boundary condition 1:** At displacement  $s_p$ , corresponding to the peak shear stress, the disturbance function equals  
 274 the ultimate disturbance parameter  $D_i^p$ .

$$275 \quad D_i^p = 1 - e^{-as_p} \quad (18)$$

276 Experimental results show that  $D_i^p$  varies in a range of 0.90 and 1. The parameter  $a$  is then determined:

$$277 \quad a = -\frac{\ln(1 - D_i^p)}{s_p} \quad (19)$$

278 **Boundary condition 2:** At sliding displacement,  $s = s_p$ , the shear stress equals to peak interface shear stress  $\tau_p$

$$279 \quad \tau_{s=s_p} = b(1 - e^{-a \cdot s_p}) = \tau_p \quad (20)$$

280 The parameter  $b$  can then be implied as follows:

$$281 \quad b = \frac{\tau_p}{1 - e^{-a \cdot s_p}} \quad (21)$$

282 **Boundary condition 3:** At the origin of the stress-displacement curve, the derivative of interface shear stress over  
 283 sliding displacement equals to initial shear stiffness:

$$284 \quad \left. \frac{d\tau}{ds} \right|_{(s=0)} = a \cdot b = k_{ini} \quad (22)$$

285 If  $k_{ini}$  is known, the parameter  $b$  could be also determined by:

$$286 \quad b = -k_{ini} \cdot \frac{s_p}{\ln(1-D_i^p)} \quad (23)$$

287 **Boundary condition 4:** At sliding displacement  $s = s_p$ , the derivative of interface shear stress over displacement is  
 288 zero:

$$289 \quad \left. \frac{d\tau}{ds} \right|_{(s=s_p)} = a \cdot b \cdot e^{-a \cdot s_p} - 2c \cdot s_p = 0 \quad (24)$$

$$290 \quad c = \frac{a \cdot b \cdot e^{-a \cdot s_p}}{2s_p} \quad (25)$$

## 291 2.5. Verification of the interface model

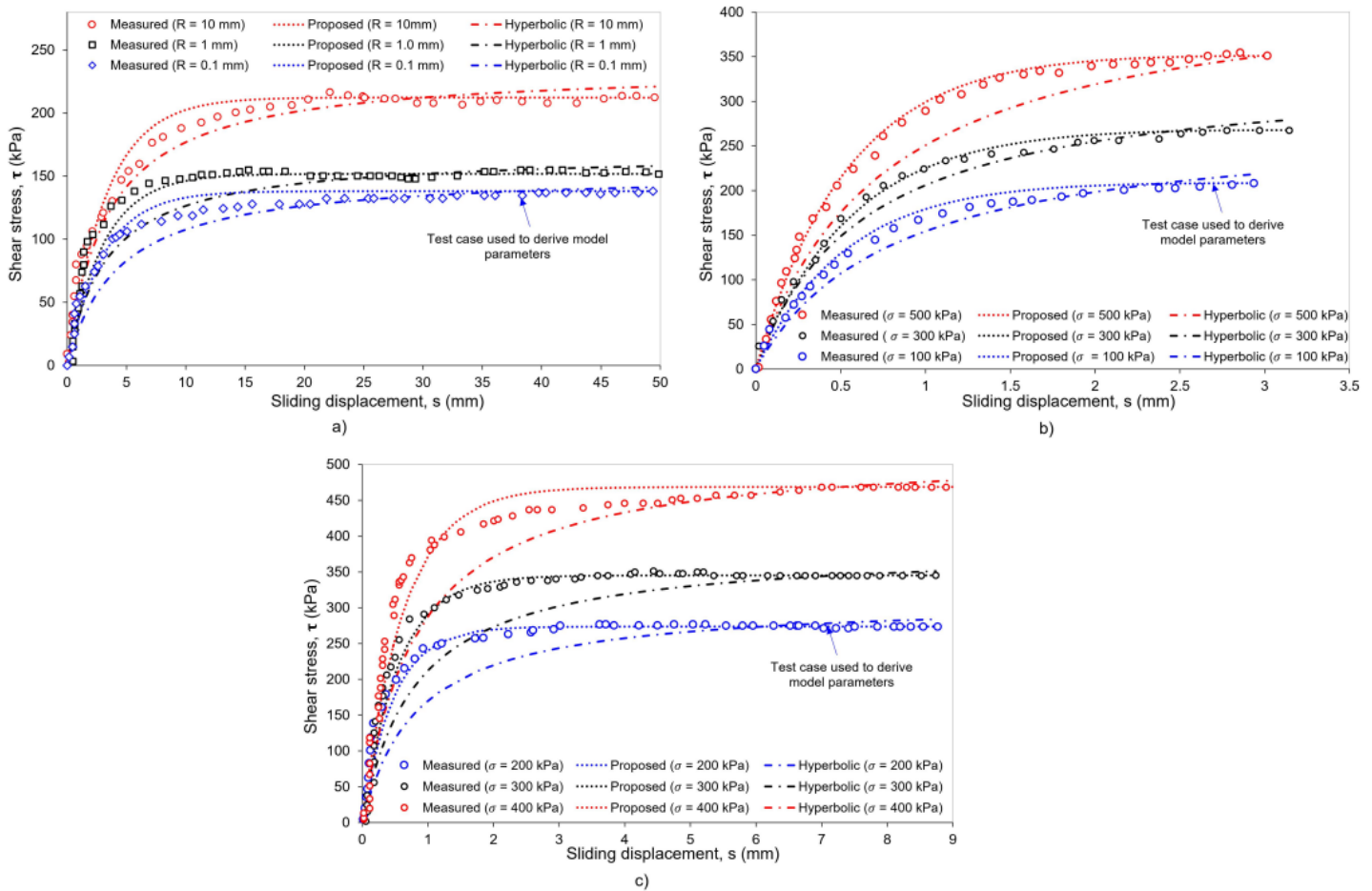
Table 1. Summary of experimental data characteristic parameters

Behaviour type	Test data sets	Soil state	Test case	$D_p$	$s_p$	$\tau_p$	$\tau_{cs}$	$a$	$b$	$c$
				–	mm	kPa	kPa	mm <sup>-1</sup>	kPa	kPa/mm
Hardening behaviour	Zhang and Zhang (2006)	Loose dry gravel	<b>R = 0.1 mm</b>	0.999	20	138.1	138.1	0.35	138.2	0.001
			R = 1.0 mm	0.999	20	151.8	151.8	0.35	152.0	0.001
			R = 10 mm	0.999	20	212.3	212.3	0.35	212.5	0.002
	Evgin and Fakharian (1997)	Loose dry sand	<b><math>\sigma = 100</math> kPa</b>	0.996	2.93	208.3	208.3	1.88	209.1	0.269
			$\sigma = 300$ kPa	0.996	2.93	267.8	267.8	1.88	268.9	0.346
			$\sigma = 500$ kPa	0.996	2.93	350.9	350.9	1.88	352.3	0.453
	Liu et al. (2014)	Unsaturated silty clay	<b><math>\sigma = 200</math> kPa</b>	0.999	8.76	273.5	273.5	0.79	273.8	0.012
			$\sigma = 300$ kPa	0.999	8.76	345.2	345.2	0.79	345.5	0.016
			$\sigma = 400$ kPa	0.999	8.76	468.6	468.6	0.79	469.1	0.021
Hardening-softening behaviour	Hu and Pu (2004)	Dense dry sand	<b><math>\sigma = 100</math> kPa</b>	0.992	0.96	120.9	70.15	5.03	121.9	2.554
			$\sigma = 200$ kPa	0.992	0.96	187.3	1338	5.03	188.8	3.957
			$\sigma = 400$ kPa	0.992	0.96	368.6	2633	5.03	371.6	7.787
	Maghsoodi et al. (2020)	Over consolidated saturated clay	T = 5 <sup>o</sup> C	0.94	1.26	82.5	66.90	2.23	87.8	4.666
			<b>T = 22<sup>o</sup>C</b>	0.94	1.26	84.7	72.52	2.23	90.1	4.790
			T = 60 <sup>o</sup> C	0.94	1.26	93.2	68.38	2.23	99.1	5.271
	Di Donna et al. (2016)	Saturated clay	<b><math>\sigma = 50</math> kPa, T = 20<sup>o</sup></b>	0.95	0.54	32.9	28.7	5.55	34.6	8.895
			$\sigma = 50$ kPa, T = 60 <sup>o</sup>	0.95	0.54	40.0	36.12	5.55	42.1	10.814
			$\sigma = 100$ kPa, T = 20 <sup>o</sup>	0.95	0.54	56.1	49.48	5.55	59.1	15.167
			$\sigma = 100$ kPa, T = 60 <sup>o</sup>	0.95	0.54	80	67.05	5.55	84.2	21.628
	Wang et al. (2019)	Dense dry sand	<b>C<sub>u</sub> = 1.85</b>	0.98	1.20	83.2	44.4	3.26	84.9	2.306
			C <sub>u</sub> = 2.81	0.98	1.20	69.6	58.8	3.26	71.0	1.929
<b>C<sub>u</sub> = 5.20</b>			0.98	1.20	58.4	44.4	3.26	59.6	1.619	

292 **Note:** Bold values are baseline cases that used to determine input parameters. The baseline is selected according to the  
 293 following rules: room temperature, smallest pressure and roughness, largest coefficient of uniformity.

294 The shear test results reported in the literature are used to validate the proposed interface model. The performance of the proposed model is assessed separately based on its applicability to model two scenarios: only

295 hardening behaviour and hardening-softening behaviour. Seven data sets therefore are used to evaluate the model  
 296 performance, in which three datasets are available for loose soils and four data sets are available for dense soils.  
 297 Characteristic reference values of the experimental data, which were taken from these selected studies, are provided  
 298 in Table 1. It should be noted that the values of peak and critical state shear stress are the same in the case of loose  
 299 soils with only a hardening response. However, the values of peak and critical state shear stress differ in dense soils  
 300 with a hardening-softening response. The proposed model was further strengthened by comparison with the  
 301 hyperbolic model in this study.



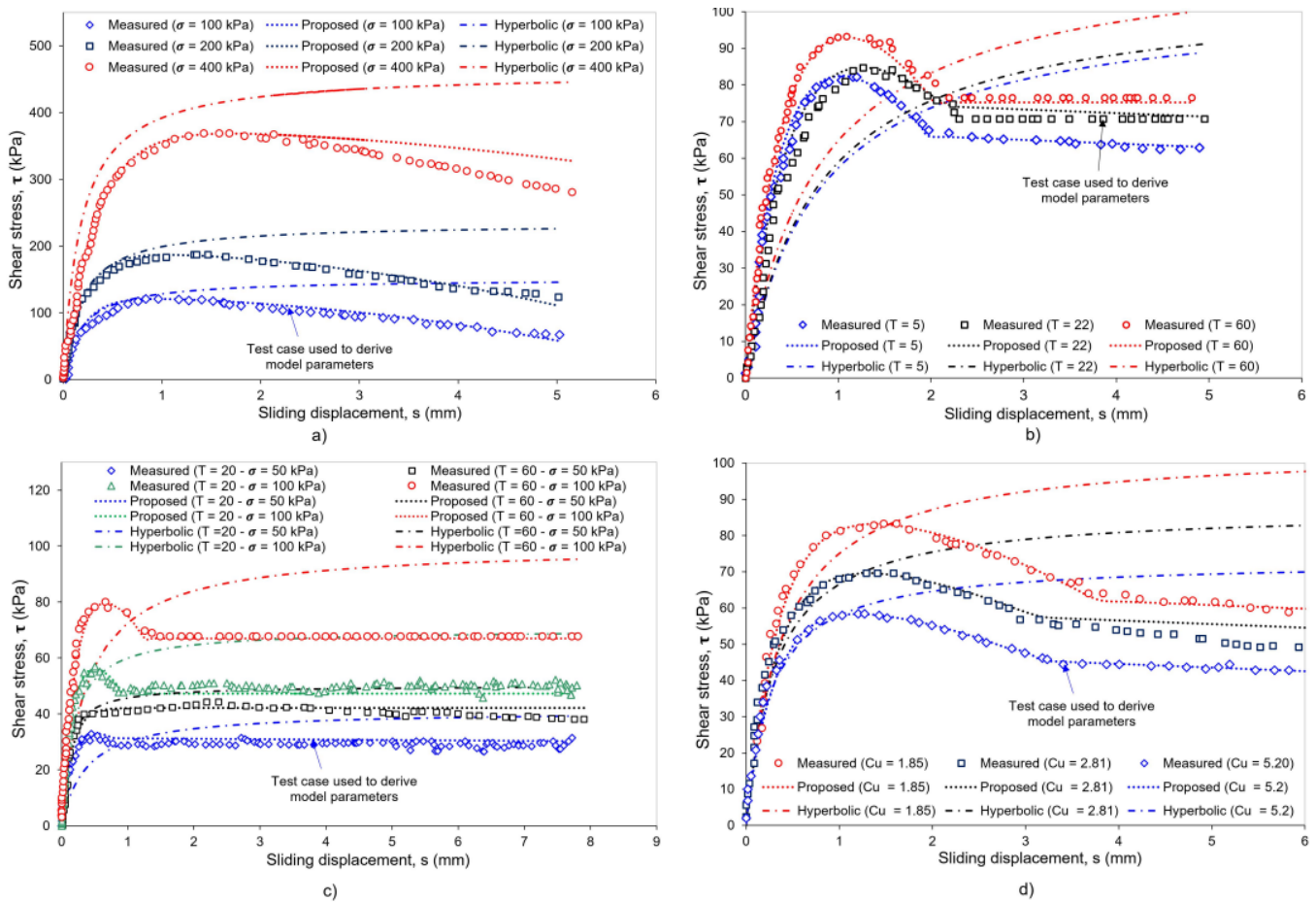
302  
 303 **Figure 8.** Comparison between the predicted and measured outcomes for loose soils: a) test data after Zhang and  
 304 Zhang (2006); b) test data after Evgin and Fakharian (1997); c) test data after Liu et al. (2014)

305 **Figure 8** compares the predicted outcomes from the proposed model with the hyperbolic model and measured  
 306 data for loose soils. It should be noted that each test data set uses the same input parameters for analytical models to  
 307 predict interface shear stress-displacement curves under different scenarios, except for peak shear stress. The results  
 308 predicted by the proposed model matched well with the experimental data, indicating that the model utilized in this  
 309 study is appropriate for describing the interface shear stress mobilized with displacement. It should be noted that the  
 310 proposed model and test results have a better agreement than the hyperbolic model. Moreover, the hyperbolic model  
 311 consistently demonstrates that shear stress rises with increasing displacement, even at high displacement values.



312 Nonetheless, the test results demonstrate that as displacement is increased to a large value, shear stress approaches a  
 313 constant value, which leads to a better performance as compared to the hyperbolic model.

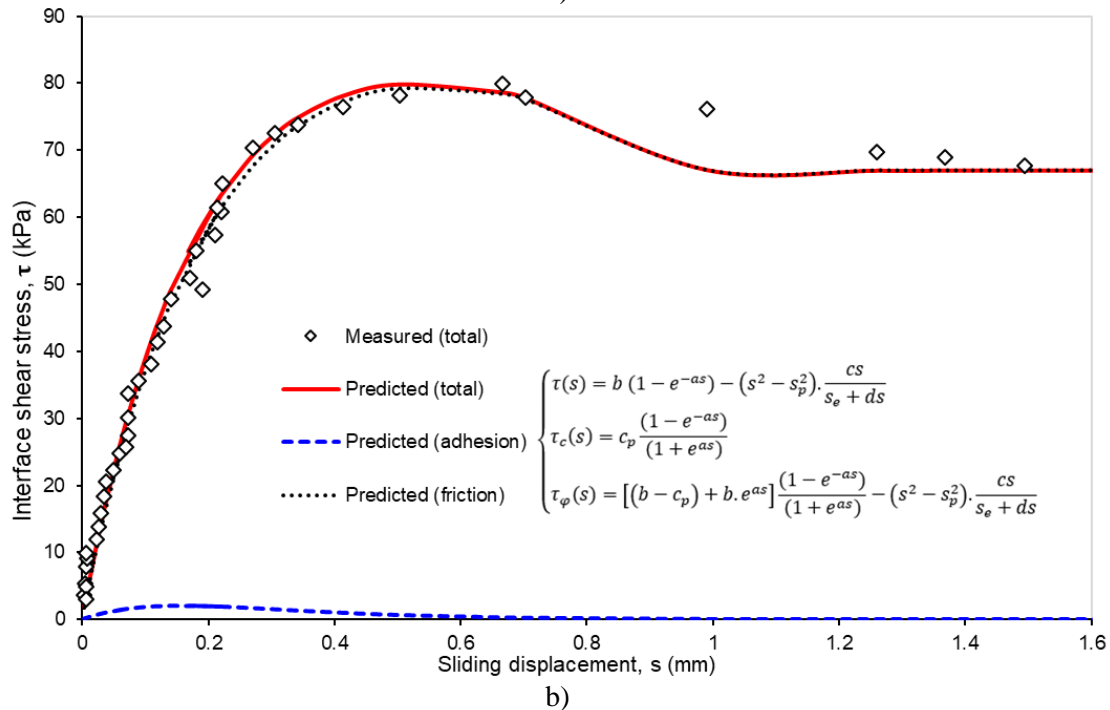
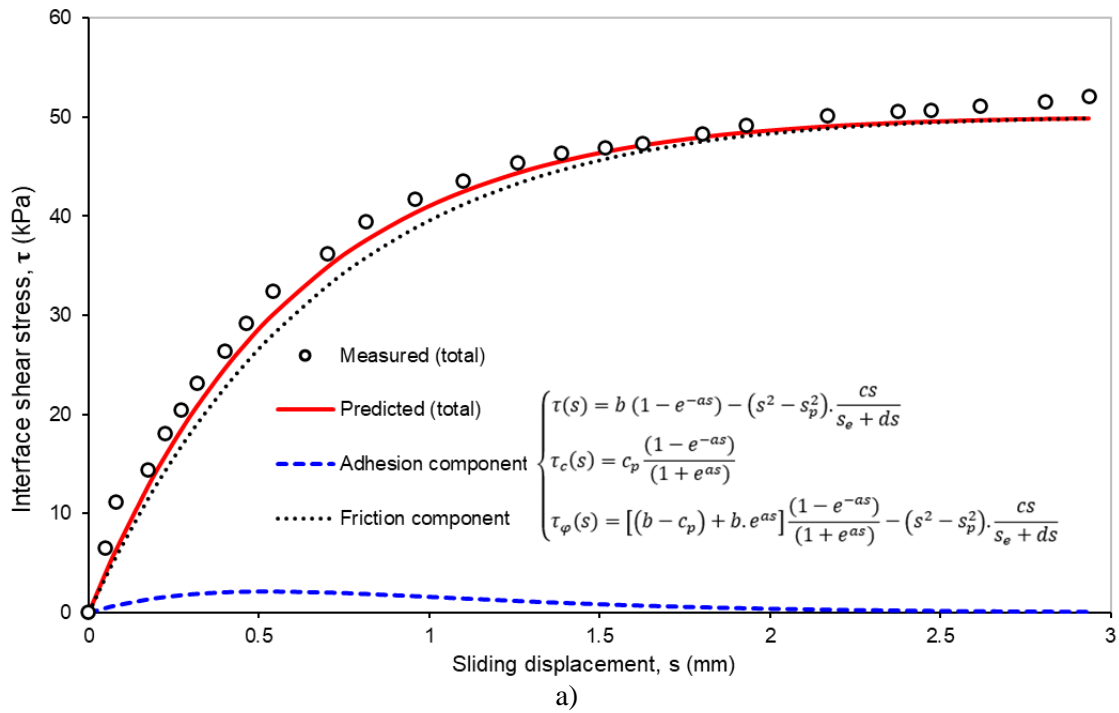
314 The predictions of the proposed model are compared with the hyperbolic model and measured data for dense  
 315 soils or over-consolidated soils in Figure 9. According to the measured data, the shear stress increases with increasing  
 316 sliding displacement to a peak value, then starts to decrease, and finally reaches a constant critical state value. As can  
 317 be seen, the proposed model accurately describes three crucial transition phases of the curve (hardening, softening,  
 318 and critical state zones), indicating that it is a good fit for describing the shear stress-displacement curve, particularly  
 319 for characterizing the softening behaviour. On the other hand, for the hyperbolic model, which only considers the  
 320 hardening behaviour, the predicted curves greatly deviate from the experimental values. The hyperbolic model,  
 321 therefore, fails to reflect the post-peak behaviour of the interface shear stress.



322  
 323 **Figure 9.** Comparison between the predicted and measured outcomes for dense soils: a) test data after Hu and Pu (2004);  
 324 b) test data after Maghsoodi et al. (2020); c) test data after Di Donna et al. (2016); d) test data after Wang et al. (2019)

325 By using the test data from two cases, the proposed model illustrates how adhesion and friction components contribute  
 326 to the interface shear strength, as shown in Fig. 10a for loose soils and Fig. 10b for dense soils. As can be observed,  
 327 the adhesion rises quickly with increasing sliding displacement and approaches a peak value (in this case  $\tau_c^p =$   
 328  $12 \text{ kPa}$ ,  $s_c = 0.47 \text{ mm}$ ) which indicates that the displacement range to mobilize ultimate adhesion is comparatively

329 small. The friction component, however, mobilizes significantly and approaches a limiting value in the case of loose  
 330 soils or later displays softening behaviour in the case of dense soils. It is important to note that the contribution of  
 331 friction to interface shear stress is significantly higher than that of adhesion.



334 **Figure 10.** Evolution contribution of adhesion and friction components to interface shear stress: a) Test data from  
 337 Evgin and Fakharian 1997 ( $\sigma = 50$  kPa); b) Test data from Di Donna et al. 2016 ( $\sigma = 100$  kPa,  $T = 60^\circ\text{C}$ )  
 338

334  
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 340



341 **3. Load-settlement response analysis of piles using DSC model**

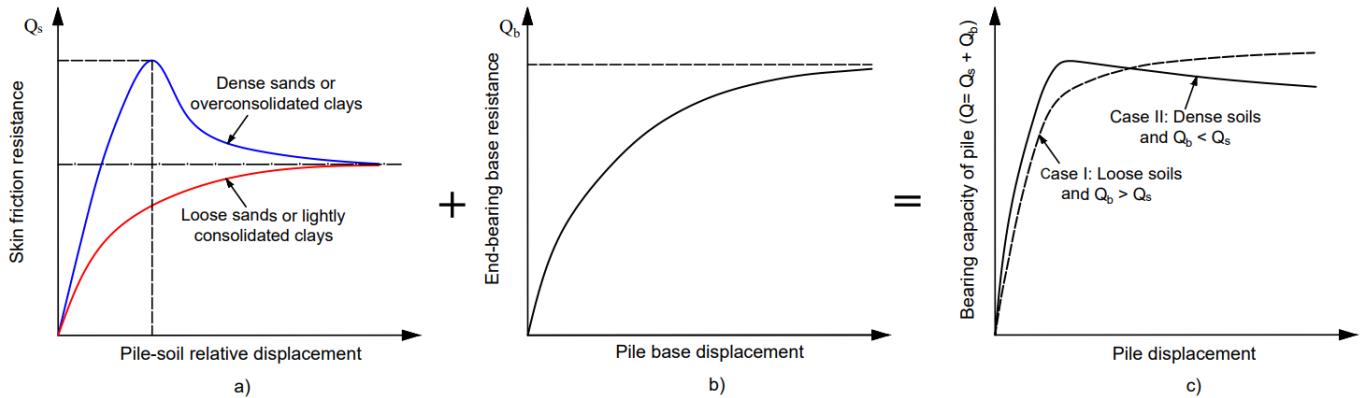
342 The DSC model demonstrated success in predicting the interface shear stress-displacement relationship as  
 343 well as accurately captured the hardening-softening behaviour in the previous section. As a result, the proposed  
 344 interface model is extended and combined with bearing capacity theory to create a more straightforward method for  
 345 analysing the load-settlement response of piles in multi-layered soils.

346 **3.1. Load capacity of a single pile**

347 The ultimate load capacity of a single pile,  $Q_{ult}$ , is typically considered to consist of two components. The  
 348 first component, referred to as skin friction or shaft friction, is caused by interaction between the pile shaft and  
 349 surrounding soils, while the second part, referred to as end-bearing capacity, is caused by ground reaction at the pile  
 350 base. Depending on the strength and stiffness of the soil layers, a specific vertical displacement may be required to  
 351 mobilize the ultimate base and shaft resistance. However, in any conditions, the ultimate load capacity of a pile can  
 352 be written as follows:

353 
$$Q = Q_s + Q_b - W_p = \tau_s \cdot A_s + q_b \cdot A_b - \gamma_p \cdot V_p \quad (26)$$

354 where  $W_p$  = weight of the pile,  $\tau_s$  = unit skin friction,  $q_b$  = unit base resistance,  $A_s$  = pile shaft area,  $A_b$  = pile cross-  
 355 sectional area,  $\gamma_p$  = unit weight of the pile,  $V_p$  = volume of the pile.



356  
 357 **Figure 11.** Schematic of mobilized load-settlement curves for a single pile: (a) skin friction versus displacement; (b)  
 358 base resistance versus displacement; (c) total bearing capacity versus displacement

359 The skin friction, which is mobilized by the relative displacement of the pile and the soil, may exhibit only  
 360 hardening or hardening-softening behaviour depending on the stiffness of the surrounding soils. **Figure 11a**  
 361 demonstrates the relationship between skin friction and pile displacement. The soil below the base of the pile, on the  
 362 other hand, is severely constrained from moving in the lateral and vertical directions and is mainly subjected to  
 363 compressive pressures. Hence, end-bearing resistance at the pile base frequently increases with the increasing  
 364 settlement. Likely, the soil element at the pile base does not respond to applied loads with a peak stress-strain response  
 365 or strain softening because of the significant overburden pressure. Instead, until a critical condition is reached, it

366 exhibits only hardening behaviour. [Figure 11b](#) depicts a model of the relationship between displacement and base  
367 resistance.

368 In general, the load settlement curve of a single pile is frequently impacted by the soil type and density, thus  
369 exhibiting the two following forms: If the soils are dense and the skin friction is greater than the base resistance, the  
370 pile will behave in a hardening-softening way. The pile can display hardening behaviour if there are loose soils present  
371 or if base resistance is larger than the skin friction resistance ([Figure 11c](#)).

### 372 **3.2. Nonlinear model of skin friction resistance**

373 The skin friction resistance over the embedded length of the pile is calculated by multiplying the unit interface  
374 shear strength and surface area of the pile shaft, as indicated in the equation below:

$$375 \quad Q_s = \tau_s \cdot A_s = \begin{cases} [b(1 - e^{-as}) - c(s^2 - s_p^2)] \cdot A_s & \text{if } s \leq s_{cs} \\ \tau_{cs} \cdot A_s & \text{if } s > s_{cs} \end{cases} \quad (27)$$

376 Which, the interface model in the previous section is extended to include the estimation of controlling parameters

$$377 \quad a = -\frac{\ln(1 - D_i^p)}{s_p}$$

$$378 \quad b = \frac{\tau_p}{1 - e^{-a \cdot s_p}}$$

$$379 \quad c = \frac{a \cdot b \cdot e^{-a \cdot s_p}}{2s_p}$$

380 The model parameters must be known in order to determine the skin friction resistance using Equation (27). The  
381 proposed model has three parameters, which can be directly derived based on interface shear experiments or by  
382 applying the effective stress principles.

### 383 **Ultimate and residual skin friction ( $\tau_p$ and $\tau_{cs}$ )**

384 The peak and residual points of the interface shear stress-displacement test curve can be used to determine  
385 the ultimate skin friction  $\tau_p$  and critical state skin friction  $\tau_{cs}$ . In the absence of test results,  $\tau_p$  and  $\tau_{cs}$  can be  
386 estimated using a formula based on soil properties obtained from laboratory or in-situ studies. The drained condition  
387 is predicted using the effective stress approach or so-called  $\beta$ -method ([Pham 2022a](#)). The following equations can be  
388 used to determine the ultimate unit of skin friction based on Coulomb's friction law:

$$389 \quad \tau_p = c' + K \cdot (\sigma'_z)_i \cdot \tan \delta_p = c' + (1 - \sin \varphi_p) \cdot (OCR)^{0.5} \cdot \tan(R_i \cdot \varphi_p) \cdot (\sigma'_z)_i \quad (28a)$$

$$390 \quad \tau_{cs} = c' + K \cdot (\sigma'_z)_i \cdot \tan \delta_{cs} = c' + (1 - \sin \varphi_{cs}) \cdot (OCR)^{0.5} \cdot \tan(R_i \cdot \varphi_{cs}) \cdot (\sigma'_z)_i \quad (28b)$$

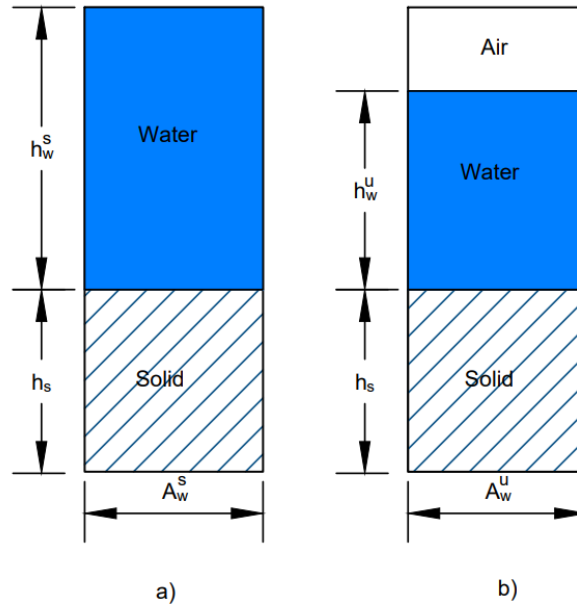
$$391 \quad \varphi_p = \varphi_{cs} + 0.8\psi \quad (29)$$

$$392 \quad (\sigma'_z)_{sat} = \sigma - u_w(1 - D) \quad (30a)$$

$$393 \quad (\sigma'_z)_{unsat} = (\sigma - u_a) + \psi \cdot [\theta + S \cdot (1 - \theta)] + u_w \cdot D \quad (30b)$$

394 where,  $\varphi_p$  and  $\varphi_{cs}$  = peak and critical friction angle respectively,  $\delta_p$  and  $\delta_{cs}$  = friction angle of the pile-soil interface  
 395 at peak and residual state, respectively,  $\psi$  = dilatancy angle,  $R_i$  = interfacial friction coefficient which ranges from  
 396 0.8 to 1.0 depending on the material and roughness, OCR = over-consolidated ratio,  $\sigma = \gamma z$  = normal stress,  $\gamma$  =  
 397 unit weight of soils,  $z$  = depth from the ground surface,  $(\sigma'_z)_i$  = vertical effective stress that is calculated at the centre  
 398 of soil layer  $i$ ,  $(\sigma'_z)_{sat}$  = vertical effective stress of saturated soils,  $(\sigma'_z)_{unsat}$  = vertical effective stress of unsaturated  
 399 soils;  $u_a$  = pore-air pressure,  $u_w$  = pore-water pressure,  $\theta$  = volumetric water content,  $S$  = degree of saturation,  $D$   
 400 = particle contact area ratio,  $\psi$  = soil suction.

401 ***Pore-water pressure of unsaturated soils***



402

403 **Figure 12.** Soil phases in idealized soil model: a) saturated soil and b) unsaturated soil

404 Considering an idealized soil model with saturated and unsaturated conditions as shown in Figure 12, the pore-water  
 405 pressure in unsaturated soil ( $u_w$ ) could be expressed as follows:

406 
$$u_w = \gamma_w \cdot \frac{V_w^u}{A_w^u} = \gamma_w \cdot \frac{V \cdot \theta_w \cdot S}{A_w^u} = \gamma_w \cdot h_w^u \cdot S \quad (31)$$

407 where  $h_w^u$  = water column height;  $S$  = degree of saturation;  $\gamma_w$  = unit weight of water;  $V_w$  = total water volume;  
 408  $A_w$  = cross-section area filled by water.

409 The pore-water pressure under hydrostatic conditions of unsaturated soils in situ is estimated by replacing the water  
 410 column height ( $h_w^u$ ) with the water depth, which is determined as depth calculated from the groundwater table ( $z_w$ ).

411 
$$u_w = \gamma_w \cdot z_w \cdot S \quad (32)$$

412 ***Estimating soil suction with the Soil-Water Characteristic Curve***

413 The concept of soil suction, which refers to the free energy state of soil water, is frequently used to describe the  
 414 behaviour of unsaturated soils (Edlefsen and Anderson 1943; Pham and Sutman 2022a; Pham et al. 2023a). Total soil  
 415 suction is the result of the combined action of matric suction and osmotic suction. The thermodynamic relationship  
 416 between suction and partial pressure of the pore-water vapour can be expressed as follows:

$$417 \quad \psi = (u_a - u_w) + \pi = -\frac{RT\rho_w}{\omega_v} \ln\left(\frac{u_v}{u_{v0}}\right) = -\frac{RT\rho_w}{\omega_v} \ln(RH) \quad (33)$$

418 where  $\psi$  = soil suction or total suction;  $(u_a - u_w)$  = matric suction that is associated with the capillary component;  
 419  $\pi$  = osmotic suction that is associated with the solute component (salt content of pore water);  $R$  = universal gas  
 420 constant [8.31432 J/(mol.K)];  $T$  = absolute temperature [ $T_K = 273 + T_C$ ];  $\omega_v$  = molecular mass of water vapour  
 421 [18.016 kg/kmol];  $\rho_w$  = water density;  $u_v$  = partial pressure of pore-water vapor (kPa);  $u_{v0}$  = saturation pressure of  
 422 water vapour over a flat surface of pure water at the same temperature. The relative vapour pressure in the air  
 423 immediately adjacent to the water,  $u_v/u_{v0}$ , is called relative humidity (RH).

424 There is an ongoing discussion on the role of osmotic suction, despite the fact that the engineering behaviour of  
 425 unsaturated soils is described by the inclusion of soil suction as “effective” stress or suction stress, as seen in equation  
 426 (30b). Tests on compacted soils by Leong and Abuel-Naga (2018) revealed that osmotic suction has a negligible  
 427 impact on the shear strength of soils. As a result, in the following presentation, the soil suction will be compatible  
 428 with the term matric suction, which describes the engineering behaviour of unsaturated soils ( $\psi = u_a - u_w$ ).  
 429 Equation (30b) therefore should be rewritten in the term of matric suction as follows:

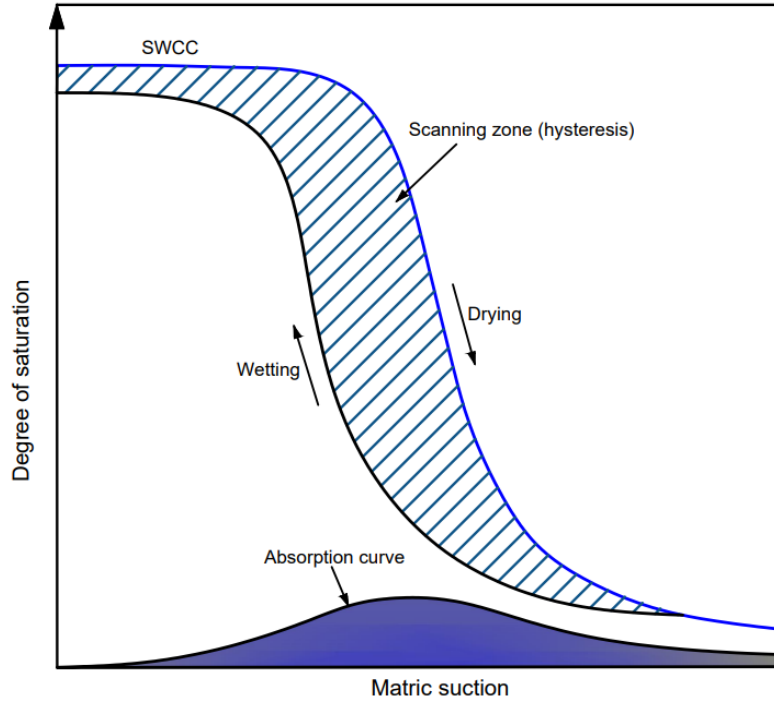
$$430 \quad \left(\sigma'_z\right)_{unsat} = (\sigma - u_a) + (u_a - u_w) \cdot [\theta + S \cdot (1 - \theta)] + u_w \cdot D \quad (34)$$

431 The measurement of the matric suction can be achieved either directly via the use of tensiometers, filter paper (contact  
 432 method), oedometer tests, null-type pressure plate (axis translation technique), or indirectly through the use of thermal  
 433 and electrical conductivity sensors (Pham and Sutman 2022b). These techniques quantify how the moisture content  
 434 varies in relation to the change in matric suction, which is also well-known as the Soil-Water Characteristic Curve  
 435 (SWCC). It is important to note that, as Figure 13 illustrates, SWCC often exhibits hysteresis concerning the wetting  
 436 and drying regions of the curve, where two distinct matric suction pressures may exist for the same volumetric water  
 437 content. A number of researchers (Brooks and Corey 1964, Van Genuchten 1980, and Fredlund and Xing 1994)  
 438 proposed the fitting equations to plot SWCC based on simple regression of only some of the available test points  
 439 because it is time-consuming to measure matric suction at all different moisture levels. The equation by Fredlund and  
 440 Xing (1994) is used in this work to determine SWCC as follows:

$$441 \quad S = \frac{c(\psi)}{\{ \ln[2.7183 + (\psi/a)^n] \}^m} \quad (35)$$

$$442 \quad C(\psi) = 1 - \frac{\ln(1 + \psi/\psi_r)}{\ln(1 + 10^6/\psi_r)} \quad (36)$$

443 where  $a$  = fitting parameter which is primarily a function of the air-entry value of soil;  $n$  = fitting parameter which  
 444 is primarily a function of water extraction rate from the soil once the air-entry value has been exceeded;  $m$  = fitting  
 445 parameter which is primarily a function of residual water content;  $C(\psi)$  = correction factor which is primarily a  
 446 function of residual suction ( $\psi_r$ ) corresponding to residual water content.



447

448

Figure 13. Illustration of SWCC for unsaturated soils

449 It should be mentioned that soil density, which may be impacted by overburden pressure during pile installation, is  
 450 one of the factors that significantly affect matric suction and pore-water pressure in situ. It is therefore crucial to take  
 451 soil density into account while assessing the engineering behaviour of unsaturated soils (Pham 2022b; Pham and  
 452 Sutman 2023). In order to overcome the uncertainty of SWCC related to the overburden pressure, the density-  
 453 dependent SWCC model of Pham et al. (2023c) may be considered a promising option due to its simplicity, which is  
 454 expressed by:

$$455 \begin{cases} S_{e_i} = S_{e_0} = \frac{1}{\ln(2.7127 + [\psi_{e_0}/a]^n)^m} \\ \psi_{e_i} = \psi_{e_0} \times \sqrt{\frac{e_0}{e_i} \cdot \left(\frac{1+e_i}{1+e_0}\right)^{1-\delta}} \end{cases} \quad (37)$$

456 Where  $e_i$  = void ratio corresponds to the current overburden pressure;  $e_0$  = void ratio corresponds to the initial  
 457 overburden pressure (or confining pressure used in laboratory SWCC tests).

458 **Vertical profile of steady-state matric suction**

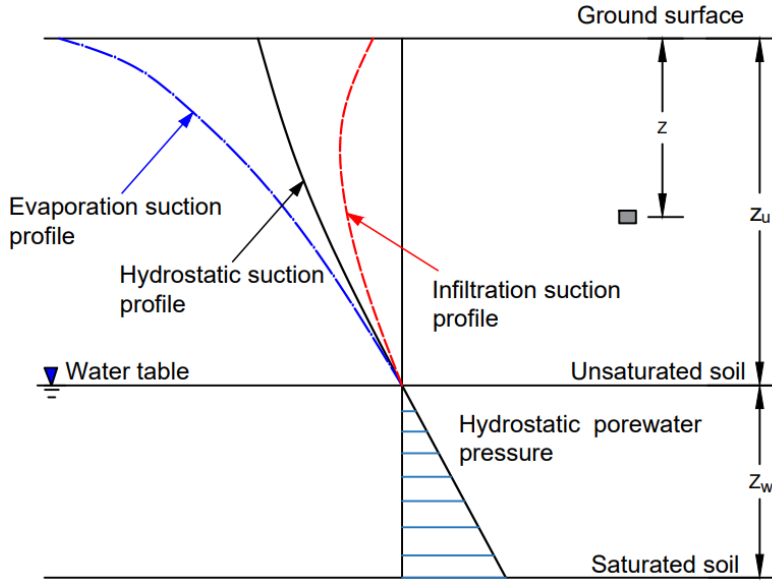


Figure 14. Suction distribution profile with different boundary conditions

In horizontally stratified unsaturated soils, the vertical distribution of matric suction is typically highly dependent on environmental variables such as boundary drainage conditions and infiltration due to precipitation or evaporation rates (Pham et al. 2023d). As a result, the matric suction varies with the depth and permeability conditions, as shown in Figure 14. The theoretical approach presented in this paper can be directly applied to extend the classical limit analysis to unsaturated soil environments. The controlling flow equation can be solved with suitable initial and boundary conditions to create the mathematical prediction of matric suction profiles. The vertical unsaturated soil flow rate for steady-state profiles can be expressed as follows, using Darcy's law:

$$q = -k_u \cdot \left( \frac{d\psi/\psi_0}{dz/z_u} + 1 \right) = -k_u \cdot \left( \frac{d\psi}{\beta \cdot dz} + 1 \right) \quad (38)$$

where  $z_u$  = unsaturated soil thickness or equals the distance from the groundwater table to the ground surface (Fig. 14);  $\psi_0$  = initial matric suction;  $\beta = \psi_0/z_u$  = suction distribution rate;  $k_u$  = unsaturated hydraulic conductivity dependent on suction. To describe the characteristic of unsaturated hydraulic conductivity, Gardner's model (1958) is used:

$$k_u = k_s e^{-\psi/AEV} \quad (39)$$

Where  $AEV$  = air-entry value;  $k_s$  = saturated hydraulic conductivity

Substituting Eq. (39) in Eq. (38) leads to:

$$q = -k_s e^{-\psi/AEV} \cdot \left( \frac{d\psi}{\beta \cdot dz} + 1 \right) \quad (40)$$

$$-\frac{\beta \cdot q}{k_s} dz = -e^{\left(\frac{-\psi}{AEV}\right)} d\psi + \beta \cdot e^{\left(\frac{-\psi}{AEV}\right)} dz \quad (41)$$

$$dz = \frac{e^{\left(\frac{-\psi}{AEV}\right)} d\psi}{\frac{\beta \cdot q}{k_s} + \beta \cdot e^{\left(\frac{-\psi}{AEV}\right)}} = -\frac{AEV}{\beta} \cdot \frac{d\left\{ \frac{q}{k_s} + e^{\left(\frac{-\psi}{AEV}\right)} \right\}}{\frac{q}{k_s} + e^{\left(\frac{-\psi}{AEV}\right)}} \quad (42)$$

479 A steady-state flow rate  $q$  is negative ( $q < 0$ ) for downward infiltration and is positive ( $q > 0$ ) for upward  
 480 evaporation. An analytical solution for the suction profile could be obtained by integrating the above equation and  
 481 imposing the boundary conditions of zero suction at the water table ( $z = 0$ ).

$$482 \int_0^z dz = -\frac{AEV}{\beta} \cdot \int_0^{\psi} \frac{d\left\{\frac{q}{k_s} + e^{\left(-\frac{\psi}{AEV}\right)}\right\}}{\frac{q}{k_s} + e^{\left(-\frac{\psi}{AEV}\right)}} \quad (43)$$

$$483 -\frac{\beta}{AEV} \cdot z = \ln \left[ \frac{\frac{q}{k_s} + e^{\left(-\frac{\psi}{AEV}\right)}}{\frac{q}{k_s} + 1} \right] \quad (44)$$

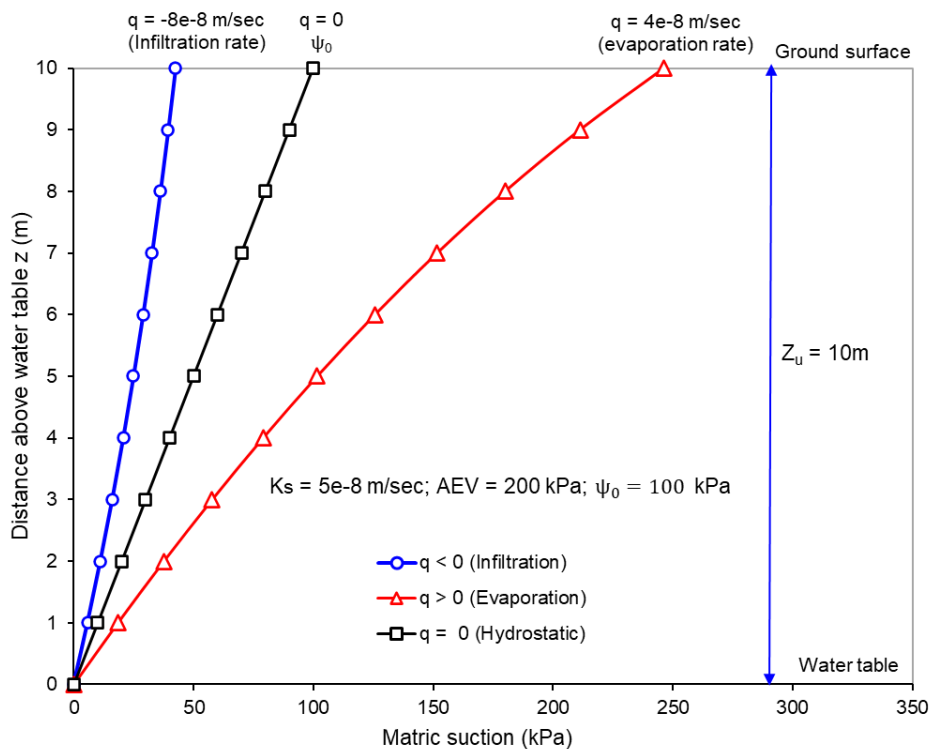
484 Rearranging the above equation could produce the final solution for the suction profile:

$$485 \psi = \left| -AEV \cdot \ln \left[ \left( \frac{q}{k_s} + 1 \right) e^{-\frac{\beta}{AEV} z} - \frac{q}{k_s} \right] \right| = \left| -AEV \cdot \ln \left[ \left( \frac{q}{k_s} + 1 \right) e^{-\frac{\psi_0}{AEV} \frac{z}{z_u} - \frac{q}{k_s}} \right] \right| \quad (45)$$

486 To obtain the soil-water characteristic curve equation integrated flow rate, Eq. (45) could be re-substituted into Eq.  
 487 (35) as follows:

$$488 S = \frac{C(\psi)}{\left\{ \ln \left[ 2.7183 + \left( \left| -AEV \cdot \ln \left[ \left( \frac{q}{k_s} + 1 \right) e^{-\frac{\psi_0}{AEV} \frac{z}{z_u} - \frac{q}{k_s}} \right] \right| / a \right)^n \right] \right\}^m} \quad (46)$$

489 It should be noted that the initial matric suction of soils at the ground surface is represented by  $\psi_0$ . When the  
 490 measured data for  $\psi_0$  is unavailable,  $\psi_0 = \gamma_w \cdot z_u$  might be used as a simplified assumption for hydrostatic conditions  
 491 according to the suggestion of Lu and Griffiths (2004). Figure 15 displays the flow ratio and matric suction profiles  
 492 for clays where the matric suction profile is significantly influenced by the steady flow rate.



493  
 494 **Figure 15.** An example of suction profile estimation using analytical equation (Eq. 45)

### 495 *Ultimate displacement ( $s_p$ )*

496 In the proposed model, the slope of the shear stress-displacement curve is controlled by the parameter called ultimate  
497 displacement,  $s_p$ . Reese (1978) reviewed the relationship between skin resistance and displacement in stiff clays and  
498 concluded that the peak displacement,  $s_p$ , ranges from about 0.5% to 5% of the pile shaft diameter ( $d_p$ ). It is noted  
499 that the  $s_p$  range in sands is comparable to that in clays. Later data (Sharma et al. 1986; Hirayama 1990; Zhang et al.  
500 2010; Pham and Dias 2021b) support the above-mentioned ranges. Judging from the previous studies, the ranges for  
501  $s_p$  are as follows:

$$502 \quad s_p = (0.005 \div 0.05) \cdot d_p \quad (47)$$

### 503 *Ultimate disturbance function ( $D_i^p$ )*

504 In the proposed interface model, the ultimate disturbance function has a significant impact on the shape of  
505 the shear stress-displacement curve. The results of the laboratory tests or a numerical simulation can be used to  
506 determine the value  $D_i^p$ . According to the results of the direct shear tests, the value  $D_i^p$  ranges from about 0.8 to 1  
507 depending on the soil density. In the absence of test results or numerical data, the initial value of the ultimate  
508 disturbance function can initially be estimated as follows:

$$509 \quad D_i^p = \frac{\tau_{cs}}{\tau_p} \quad (48)$$

### 510 **3.3. Nonlinear model of end-bearing capacity**

511 During installation, the portion of soil immediately below the pile base will be subjected to a compressive  
512 axial force and will likely compress in addition to shearing. Shearing is caused by the differences in the vertical and  
513 lateral stresses. However, unlike the soil element surrounding the pile shaft, the soil element directly below the base  
514 is severely limited. Specifically, the pile weight and the friction force at the pile–soil interface limit any heaving or  
515 dilatant response. Additionally, movement is further restricted at the base level by the overburden pressure (vertical  
516 effective stress), particularly for long piles. As a result, the pile strain hardens until it reaches the critical state when  
517 structural loads are applied. Besides, the field test results also reveal the hardening function of base load displacement  
518 (Lee and Park 2008; Seo et al. 2013). This explains why the hardening-strain function (such as hyperbolic) is  
519 commonly utilised in analytical approaches and design standards (FHWA-2006, BS8006-2010, Eurocode 7-2013) to  
520 simulate the base load-settlement model. In this study, the base load-displacement relationship is modelled by using  
521 the hardening function of the proposed interface model as follows:

$$522 \quad Q_b = q_b \cdot A_b = \left[ q_{bu} \left( 1 - e^{-\frac{k_{ini}^b \cdot s}{q_{bu}}} \right) \right] \cdot A_b \quad (49)$$

523 where  $q_{bu}$  = ultimate unit base resistance,  $k_{ini}^b$  = initial compressive stiffness of the soil at the pile base

### 524 *Ultimate unit base resistance ( $q_{bu}$ )*



525 There are several methods for calculating the ultimate unit base resistance, one of which makes use of  
 526 empirical correlation and data from in-situ tests (SPT or CPT). An alternative way is to compute using formulas based  
 527 on laboratory-tested soil parameters. In the analytical approach presented here, the  $q_{bu}$  is calculated by analogy with  
 528 the bearing capacity of shallow footings and is determined as follows:

$$529 \quad q_{bu} = \frac{1}{2} \gamma d_p N_\gamma + \sigma'_{zb} \cdot N_q + c' \cdot N_c \quad (50)$$

$$530 \quad N_q = e^{\pi \tan \varphi_b} \cdot \tan^2(45^\circ + \varphi_b/2) \quad (51)$$

$$531 \quad N_c = (N_q - 1) \cdot \cot \varphi_b \quad (52)$$

532 where  $\gamma$  = unit weight of soils,  $\sigma'_{zb}$  = vertical effective stress at the pile base,  $\varphi_b$  and  $c'$  = internal friction angle and  
 533 cohesion of soils below the pile base,  $N_\gamma$ ,  $N_q$  and  $N_c$  = bearing capacity coefficients.

534 For deep foundations with  $L_p/d_p$  greater than 5 ( $L_p$  being the embedded length of the pile), the first term in Eq. (50)  
 535 is small compared with the other two terms. It should be noted that the predicted outcome of end-bearing capacity is  
 536 significantly influenced by the precision of the value  $N_q$ . However, when compared to the field  $N_q$  values, Eq. (51)  
 537 frequently overestimates the value  $N_q$ , according to several researchers (Coyle and Castello 1981; De Nicola and  
 538 Randolph 1993; Alawneh et al. 2001; Budhu 2010). This is primarily because the failure mechanism below the pile  
 539 base might not develop in the same way as that of a shallow foundation due to the considerable overburden pressure  
 540 acting at the pile base level. In light of the adjusted coefficient, this study suggests the following formula for the factor  
 541  $N_q$  of the soils at the pile base as follows:

$$542 \quad (N_q)_{adj} = f_{adj} \cdot e^{\pi \tan \varphi_b} \cdot \tan^2(45^\circ + \varphi_b/2) \quad (53)$$

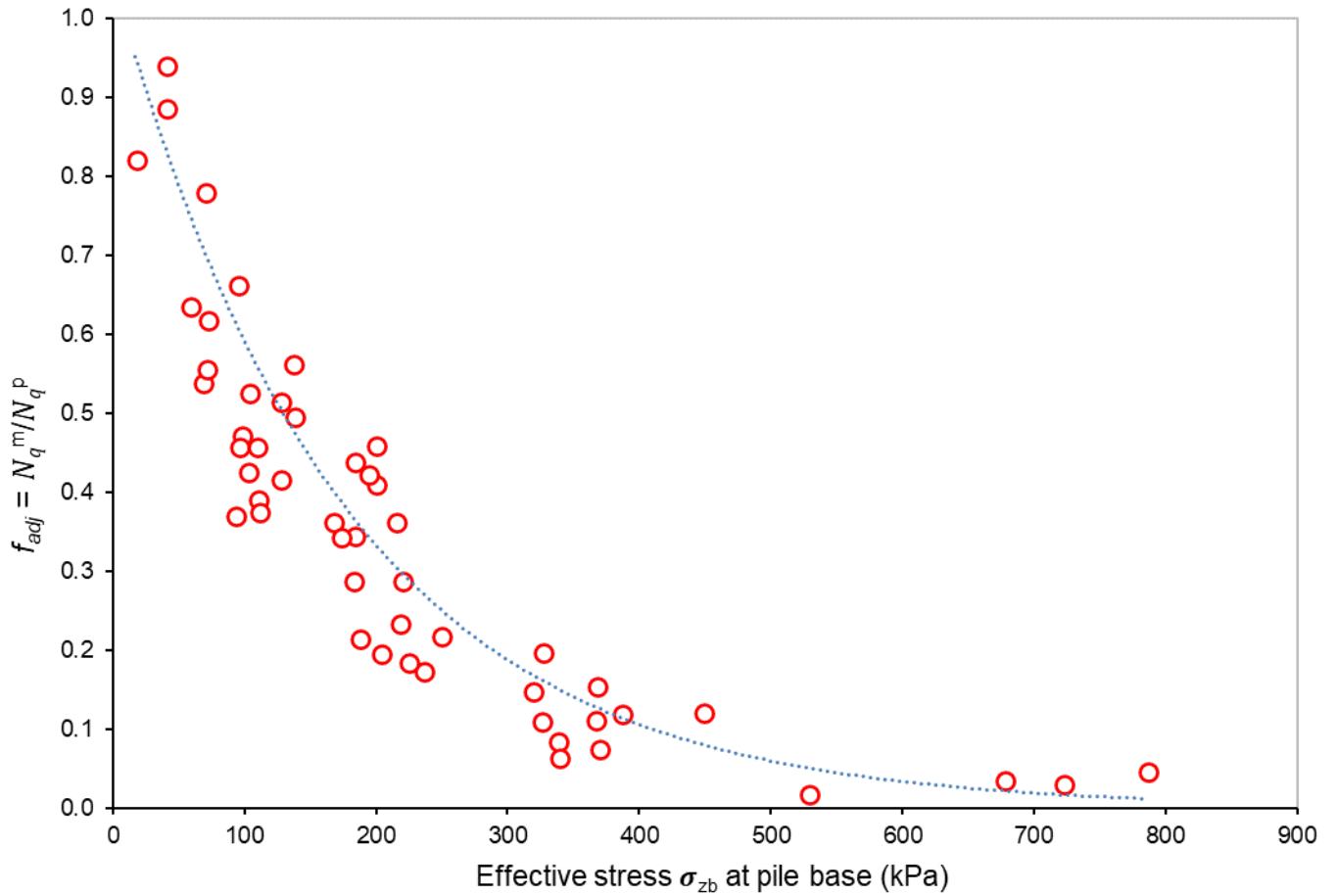
$$543 \quad f_{adj} = \frac{q_{bu}^m}{\sigma'_{zb} + c' \cdot \cot \varphi_b} \cdot \frac{1}{e^{\pi \tan \varphi_b} \cdot \tan^2(45^\circ + \varphi_b/2)} \quad (54)$$

544 where  $f_{adj}$  = adjusted coefficient,  $q_{bu}^m$  = measured ultimate unit base resistance

545 The characteristics of 52 field test projects with measured ultimate unit base resistance are listed in  
 546 [Supplementary Material I](#) along with some brief descriptions. These data sets were collected, which cover a wide  
 547 variety of possible pile geometry, effective stress, friction angle, and soil types. The scatter in the field data is shown  
 548 in [Figure 16](#). It is apparent that as the vertical effective stress is increased, the value of  $f_{adj}$  decreases. Lower values  
 549 of  $f_{adj}$  denote greater overestimation when applying the conventional theoretical formula (Eq. 51). Although the data  
 550 sets have not fully converged, the majority of the points appear to follow an exponential relationship. The following  
 551 formulation is provided for the  $f_{adj}$  as an effective stress-dependent function using the regression analysis technique:

$$552 \quad f_{adj} = e^{-0.006 \times \sigma'_z} = \alpha e^{\beta \times \sigma'_z} \quad (55)$$

553 where  $\alpha$  and  $\beta$  = corrected coefficients. When the measured data are known, the corrected coefficients can be derived.  
 554 Otherwise, the collected data in this study yields the following general statement:  $\alpha = 1$  and  $\beta = -0.006$ .



555

556

Figure 16. The correlation between the adjusted coefficient and vertical effective stress

557

**Initial elastic soil stiffness ( $k_{ini}^b$ )**

558

The initial elastic soil stiffness at the pile base can be determined by relations to ultimate unit base resistance or shear modulus as suggested by [Randolph and Wroth \(1978\)](#) and [Pham et al. \(2023e\)](#):

559

560

$$k_{ini}^b = \frac{q_{bu}}{s_p^b} = \frac{4G_{sb}}{\pi r_p(1-\nu_p)} \tag{56}$$

561

where  $s_p^b$  = ultimate displacement,  $r_p$  = pile radius,  $G_{sb}$  and  $\nu_{sb}$  = shear modulus and Poisson's ratio of the soil below the pile base, respectively.

562

563

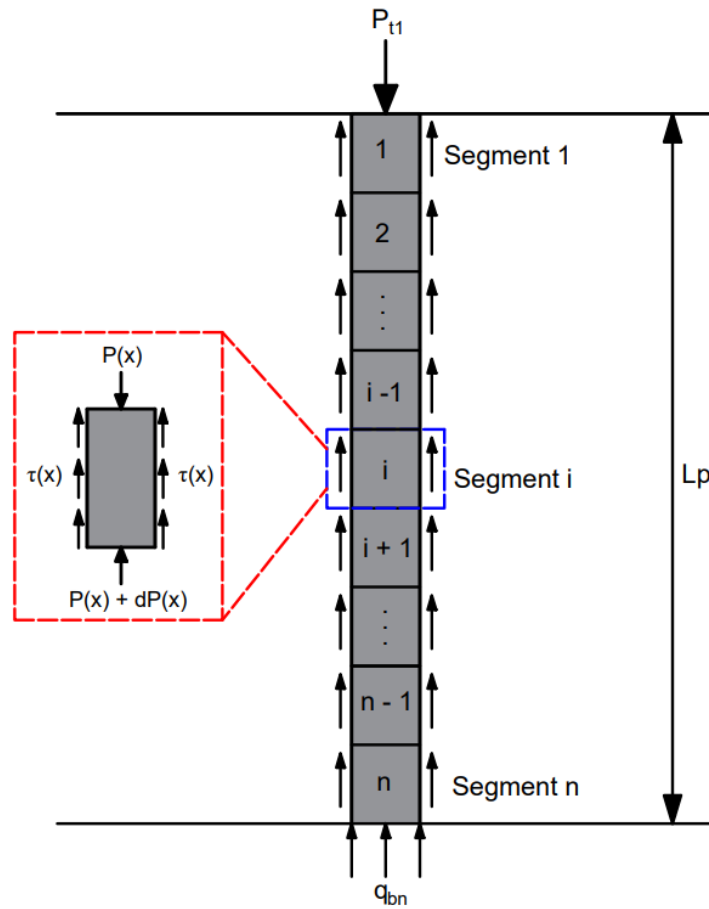
**3.4. Elastic shortening and distribution of axial force.**

564

The elastic shortening of a pile shaft under load undoubtedly contributes to total settlement at the pile head, particularly in the case of semi-columns. The elastic shortening of a pile depends on the relative development of load transfer between the pile and soil along its length, as well as the load being transferred at the pile base. Because the load transfer is nonlinear and decreases with the depth, the elastic compression of the pile is not uniform. The top of the pile generally experiences greater compression than the bottom. To work out the elastic shortening accurately, an

568

569 iterative method is required, whereby the pile is divided into elements, and the compatibility of strains is studied at  
 570 given levels. Figure 17 shows the schematic for force equilibrium analysis on a differential pile element.



571

572 Figure 17. Schematic of typical force equilibrium analysis of pile

573

573 As a result of applying static equilibrium to the forces influencing the differential pile element, it provides:

574

$$dP(x) + C_p \tau(x) dx + A_p q_b(x) dx = 0 \quad (57)$$

575

575 where  $P(x)$  = axial force in the pile at distance  $x$ ;  $C_p$  = cross-section perimeter of a pile.

576

576 It is reasonable to suppose that when loads are applied, the pile is in elastic compression:

577

$$ds = -\frac{P(x)}{E_p A_p} dx \quad (58)$$

578

578 where  $E_p$  = elastic modulus of the pile,

579

579 Therefore,

580

$$dP(x) = -E_p A_p \frac{d^2 s(x)}{dx^2} \quad (59)$$

581

581 Substituting (59) back into (57) gives:

582 
$$\frac{d^2s(x)}{dx^2} - \frac{C_p}{E_p A_p} \tau(x) dx - \frac{1}{E_p} q_b(x) dx = 0 \quad (60)$$

583 By separating the variables, the differential equation can be solved, as

584 
$$\varepsilon(x) \frac{d\varepsilon(x)}{ds} = \frac{d^2s(x)}{dx^2} \quad (61)$$

585 By substituting Eq. (61) into Eq. (60) and integrating

586 
$$\varepsilon(x) = \sqrt{\frac{2C_p \cdot b}{E_p A_p} \cdot \left( s + \frac{e^{-as}}{a} \right) + \frac{2C_p \cdot c}{E_p \cdot A_p} \left[ \frac{s^3}{3} - \frac{s_p^3}{3} \right] + \frac{2q_{bu}}{E_p} \cdot \left( s + \frac{e^{-k_{ini}^b s / q_{bu}}}{k_{ini}^b / q_{bu}} \right) + C} \quad (62)$$

587 The boundary condition at the base of the pile can be expressed as:

588 
$$\begin{cases} s(x = L_p) = s_b \\ \varepsilon(x = L_p) = 0 \end{cases} \quad (63)$$

589 Replacing Eq. (63) into Eq. (62) gives the axial strain over the pile length:

590 
$$\varepsilon = \sqrt{\frac{2C_p \cdot b}{E_p A_p} \cdot \left( s + \frac{e^{-as}}{a} \right) + \frac{2C_p \cdot c}{E_p \cdot A_p} \left[ \frac{s^3}{3} - \frac{s_p^3}{3} \right] + \frac{2q_{bu}}{E_p} \cdot \left( s + \frac{e^{-\frac{k_{ini}^b s}{q_{bu}}}}{\frac{k_{ini}^b}{q_{bu}}} \right) - \frac{2C_p \cdot b}{E_p A_p} \cdot \left( s_p + \frac{e^{-as_p}}{a} \right) - \frac{2q_{bu}}{E_p} \cdot \left( s_p + \frac{e^{-k_{ini}^b s_p / q_{bu}}}{k_{ini}^b / q_{bu}} \right)} \quad (64)$$

### 591 3.5. Algorithm for pile load–settlement analysis

592 Based on the proposed models and the iterative method (Seed and Reese 1957; Lee et al. 2001; Zhang and  
593 Zhang 2012), the pile load-settlement response can be obtained. The calculation details are presented with the  
594 following procedure, and Figure 18 summarises the algorithm for load-settlement analysis of a single pile embedded  
595 in multi-layered soils:

- 596
- Step 1. Assume a single pile divided into  $n$  segments with a length of  $L_i = L_p/n$ . The value of  $n$  can be  
597 specified according to the computational precision demand.
  - Step 2. Estimate the ultimate end-bearing capacity by using Eq. (50)
  - Step 3. Assume a small pile-base movement,  $s = s_{bn}$ . Using Eq. (49), calculate the base end-bearing  
599 resistance,  $Q_b(s)$ , caused by the assumed base displacement.
  - Step 4. From the proposed load-transfer function as given in Eq. (27), obtain the skin friction resistance at the  
601 bottom segment  $n$ ,  $Q_{sn}(s)$ , based on the assumed value.
  - Step 5. Calculate the total bearing capacity along the pile segment  $n$  corresponding to the assumed  
602 displacement by:  $Q_{tn} = Q_{sn} + Q_{bn}$
  - Step 6. Calculate the elastic deformation of the segment as follows, under the assumption that the load in the  
603 segment varies linearly:  
604  
605  
606

607 
$$\Delta_n = \left( \frac{Q_{tn} + Q_{bn}}{2} \right) \frac{L_i}{A_p \cdot E_p}$$

608 • Step 7. Check the elastic deformation of pile segment  $n$  within a specified tolerance such as  $\Delta_n \leq 10^{-6} m$

609 • Step 8. Calculate the updated midpoint ( $s'_{cn}$ ) and top ( $s'_{tn}$ ) displacement of pile segment  $n$  given by:

610 
$$s'_{cn} = s_{bn} + \frac{1}{2} \Delta_{sn} \text{ and } s'_{tn} = s_{bn} + \Delta_{sn}$$

611 • Step 9. Recalculate the skin friction resistance and ultimate bearing capacity of pile segment  $n$  based on the  
612 updated midpoint displacement.

613 • Step 10. Repeat steps (4) – (9) for the next segment, and so on, until the value of the load  $Q_{n1}$  and  
614 displacement  $s_1$  at the pile head are obtained.

615 • Step 11. Calculate the total skin friction or the sum of the total skin friction capacity and mobilized base  
616 capacity.

617 
$$Q_{t1} = \sum_{i=1}^n Q_{si} + Q_{bn}$$

618 • Step 12. Calculate the axial strain of the pile corresponding to the assumed displacement by using [Eq. \(64\)](#)

619 • Step 13. Repeat the procedure from steps (3) through (12) using different assumptions for the settlement at  
620 the pile base in order to get a series of load-displacement values. The proposed method might be used to  
621 determine the load-settlement curve, skin friction distribution along the pile depth, and the distribution of  
622 axial loads along the pile.

623

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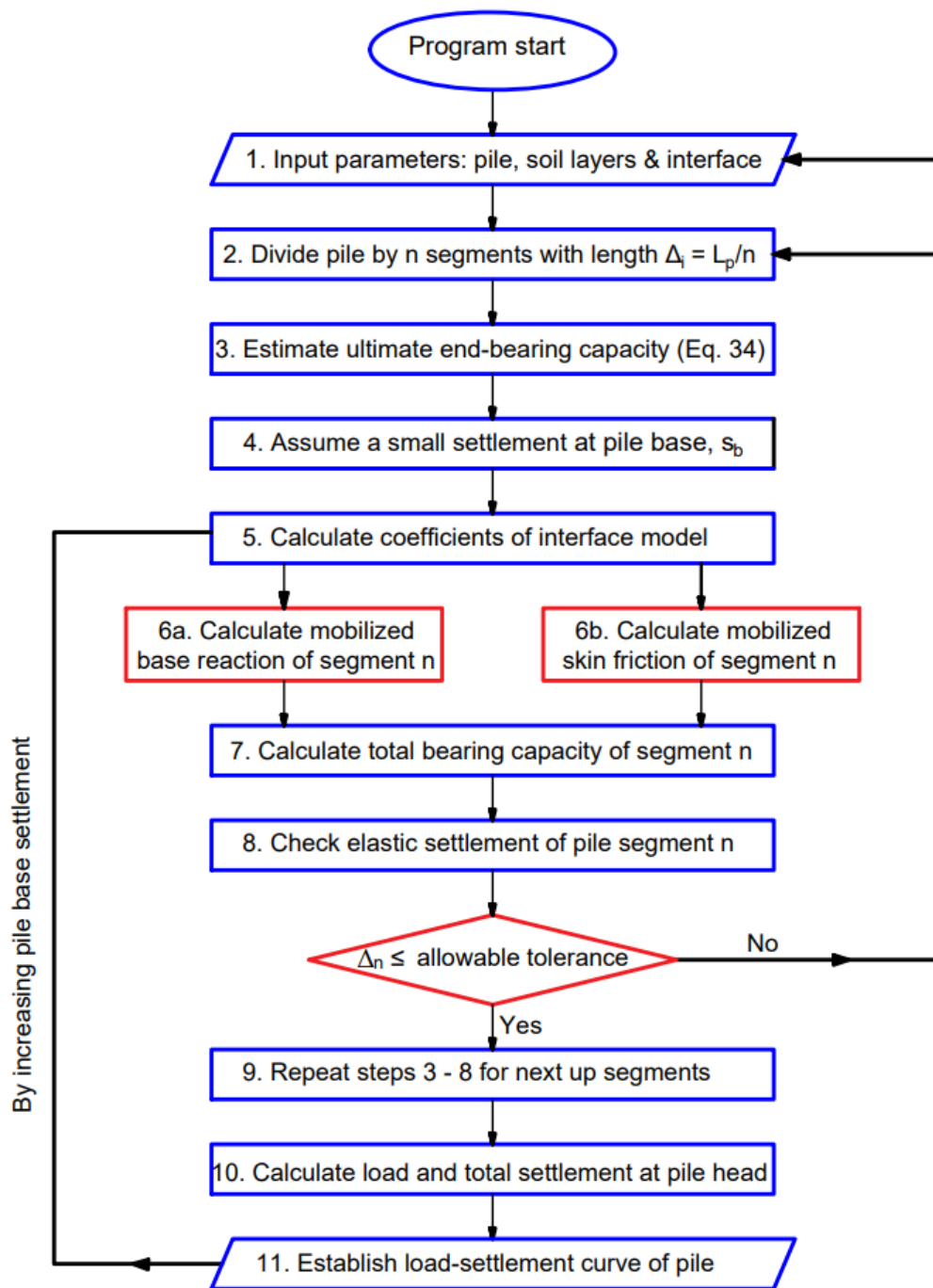


Figure 18. Flow chart for load-settlement response analysis of piles

### 3.6. Discussion of the analytical method

Three factors are crucial for numerical simulations of a geostucture to perform well, which include the input parameter quality, the soil model that was used, and the loading simulation process. First, it should be noted that neither sophisticated nor basic models—such as Mohr-coulomb or elastoplastic—can replicate the deterioration of skin friction at the soil-structure interface. In order to get beyond the limitations of the models that are currently accessible, the DSC-based interface model that was described in this study could be taken into consideration as a

634 choice to be implemented into numerical software. This work also discusses the profile of steady-state in-situ matric  
635 suction, soil-water characteristic curve, and pore-water pressure of unsaturated soils that allows to application of the  
636 proposed method to piles in unsaturated soils.

637 Additionally, it is noted that the proposed analysis method in this study only needs a few basic soil-pile interface  
638 input parameters, similar to current design methods that include friction angle, cohesion, interaction coefficient, and  
639 disturbance degree of soils corresponding to peak shear strength and displacements. Specifically, it is assumed that  
640 the displacement parameter ( $s_p$ ) are proportionate to pile size. Two shear strength parameters ( $\tau_{cs}$ ,  $\tau_p$ ) are derived  
641 from friction angles and cohesion in the same way as other models. And governing coefficients ( $a$ ,  $b$ ,  $c$ ) are computed  
642 directly from the aforementioned parameters. It appears that, in comparison to previous models, the current approach  
643 does not introduce extra parameters while still being able to accurately characterize the hardening-softening behaviour  
644 of the soil-structure interface.

645 Evidently, the quality of these input parameters has a significant impact on the performance of the method. Within  
646 this framework, the experimental data may improve the input parameter's reliability and hence contribute to the  
647 production of a better prediction. To obtain these parameters, there are a few potential approaches that might be used  
648 such as i) carrying out tests on model-scale piles, ii) carrying out interface shear tests in a laboratory, iii) utilizing on-  
649 site CPT/SPT data, and iv) performing field tests and back analysis approaches. In terms of economy, the interface  
650 shear tests, and model-scale tests may theoretically be the most suitable option. Following that, an analytical method  
651 will be employed to use the input parameters derived from the interface shear tests and model-scale tests to simulate  
652 the outcomes of field tests. The main difficulty in performing model-scale tests or laboratory tests, nevertheless, is  
653 making sure the model pile's condition is precisely the same as the in-situ pile (in terms of scaling effects).

654 On the other hand, the analytical solutions in this work could be applied as an independent analytical approach that  
655 does not necessitate integration with numerical techniques. For design engineers, this offers an efficient and  
656 straightforward option, particularly during the preliminary design stage. For example, the analytical solutions could  
657 likewise be computed automatically by creating spreadsheets or MATLAB code. Compared to performing a  
658 numerical simulation, this saves far too much effort and time. In particular, for rigid pile (elastic settlement), it is  
659 simple to get the solution of the developed method and there is no need to use trial values to get the solution of  
660 unknown parameters.

661 Finally, simulating the impact of pile installation and surrounding soil disturbance in current numerical models is a  
662 significant issue. The DSC-based analytical solutions may offer an additional method to account for this effect. The  
663 proposed approach might be used to anticipate any kind of soil rather than being limited to a specific soil type as an  
664 empirical model. The proposed analytical approach may be helpful not only for forecasting the load-settlement of a  
665 pile using input parameters but also for optimizing design and organizing a suitable loading-test programme.

#### 666 **4. Performance of the proposed method in comparison to field measurements**

667 The effectiveness of the proposed framework is assessed by comparing the predicted results with those  
668 deduced from field experiments on a single pile. To demonstrate the implementation of the proposed approach in a  
669 variety of ground conditions and pile types, the following five projects with well-documented sources have been  
670 chosen from the database of static load tests on piles. As previously mentioned, there are different ways to obtain  
671 model calibration parameters such as using interface shear tests, model tests or on-site CPT/SPT results. However, it  
672 is hard to find a case that offers both results of full-scale field tests and interface shear tests. This could be the result  
673 of the difficulty in preparing soil samples for interface shear experiments that have physical characteristics and  
674 disturbances comparable to the actual soil conditions. A case study was recently reported by Feng et al. (2024) that  
675 presents the results of interface shear and field tests, which are used to demonstrate how the model is calibrated using  
676 interface shear testing. In the case of the remaining four examples, input data points were gathered directly from site  
677 records (SPT or CPT) due to the interface shear test results not being presented in the original papers. The evaluation  
678 of soil shear strength characteristics, including internal friction angle, cohesion and shear modulus, can be achieved  
679 using laboratory test results or empirical correlations with SPT blow counts. A list of these relationships is given in  
680 Appendix II. In this study, the shear strength properties were derived from SPT blow counts in the absence of  
681 laboratory test data using the equation suggested by Hettiarachchi and Brown (2009). The model calibration  
682 parameters for the Feng et al. (2024) example obtained via interface shear testing are shown in Table 2, while the  
683 model calibration parameters for all other cases identified via on-site SPT/CPT are shown in Table 3.

684 It should be mentioned that in the first project (Feng et al. 2024), second project (Caputo et al. 1991) and third  
685 project (Lee et al. 2003), the soil layers were presumed to be saturated as the groundwater table was located near the  
686 ground surface. The last two examples (Fellenius et al. 2004; Lee and Park 2008), however, described that the  
687 groundwater table emerged at a specific depth and the soil layers above became unsaturated. The fundamental steps  
688 for determining the load-settlement curve of the pile using the proposed method could be summarized as follows:  
689 Step 1. Employing the results of interface shear tests for model calibration, if available, or using the SPT or CPT  
690 data to infer the shear strength parameters, such as friction angle, cohesion, undrained shear strength, and modulus.  
691 Step 2. Estimation of peak and critical state skin friction of pile-soil interface (eq. 28) as well as ultimate bearing  
692 capacity of soil at pile base (eq. 50)  
693 Step 3. Determine parameter  $s_p$  by using eq. (47) or by interface shear test results if data is available.  
694 Step 4. Calculate the peak disturbance function by using eq. (48) where  $D_i^p = \tau_{cs}/\tau_p$  or by interface shear test results  
695 if data is available.  
696 Step 5. Calculate the calibration parameters  $a, b, c$  by using eqs. (19), (21), (25)  
697 Step 6. Establish transfer functions for pile shaft (eq. 27) and pile base (eq. 49)  
698 Step 7. Estimate the load-settlement curve.  
699 Step 8. Estimate the axial strain and elastic shortening for semi-columns (eq. 64), or neglect for rigid piles.



Table 2. Summary of model parameters obtained via on-site CPT/SPT for case of Feng et al. (2024).

Parameters	Layer 1 – silty clay	Layer 2 – silty sand	Layer 3 – muddy clay	Layer 4 – Clay	Layer 5 – silty clay	Layer 6 – sandy silt
Soil layer thickness $t_i$ (m)	3.6	5.4	3.0	6.0	12.0	22.0
Unit weight $\gamma$ (kN/m <sup>3</sup> )	18.9	19.9	18.8	17.4	17.3	19.3
Initial shear stiffness $k_{ini}$ (kPa/m)	100	120	180	290	420	-
Ultimate disturbance $D_i^p$ (dimensionless)	0.99	0.96	0.99	0.99	0.94	-
Peak shear stress $\tau_p$ (kPa)	32	61	108	170	300	-
Ultimate displacement $s_p$ (m)	0.003	0.003	0.003	0.003	0.003	-
Internal friction angle $\varphi'$ (°)	28.5	27.4	26.3	23.1	30.1	32.0
Effective cohesion $c'$ (kPa)	4.0	4.6	0	12.0	4.7	2.0

Table 3. Summary of four field test datasets with model parameters obtained via on-site CPT/SPT

Field test projects	Pile geometry	Soil layer 1	Soil layer 2	Soil layer at base
Caputo et al. (1991)	$d_p = 1.5m$ $L_p = 42m$ $E_p = 50 GPa$	Saturated organic silt (32m) $\gamma = 12 kN/m^3$ $\varphi_p = 24$ (preliminary) $\varphi_p = 26.7$ (back analysis) $D_i^p = 0.995$ $S_p = 0.02 m$ $S = 100\%$ (saturation degree)	Saturated silty sand (8m) $\gamma = 16 kN/m^3$ $\varphi_p = 36$ (preliminary) $\varphi_p = 33.4$ (back analysis) $D_i^p = 0.990$ $S_p = 0.02 m$ $S = 100\%$ (saturation degree)	Saturated indurated pozzolana $\gamma = 17.5 kN/m^3$ $\varphi_p = 30.6$ (preliminary) $\varphi_p = 28.5$ (back analysis) $k_{ini}^b = 700$ (preliminary) $k_{ini}^b = 750$ (back analysis)
Lee et al. (2003)	$d_p = 0.356m$ $L_p = 8.24m$ $E_p = 50 GPa$	Saturated loose sand (3m) $\gamma = 17.5 kN/m^3$ $\varphi_p = 33$ (preliminary) $\varphi_p = 33.0$ (back analysis) $D_i^p = 0.991$ $S_p = 0.06 m$ $S = 100\%$ (saturation degree)	Saturated dense sand (5m) $\gamma = 17.5 kN/m^3$ $\varphi_p = 40$ (preliminary) $\varphi_p = 38.6$ (back analysis) $D_i^p = 0.980$ $S_p = 0.06 m$ $S = 100\%$ (saturation degree)	Saturated gravelly sand $\gamma = 17.5 kN/m^3$ $\varphi_p = 46.3$ (preliminary) $\varphi_p = 44.2$ (back analysis) $k_{ini}^b = 300$ (preliminary) $k_{ini}^b = 350$ (back analysis)
Fellenius et al. (2004)	$d_p = 0.406m$ $L_p = 45m$ $E_p = 50 GPa$	Sandy silt (3m was unsaturated soil and 6m was saturated soil) $\gamma = 17.9 kN/m^3$ $\varphi_p = 25.8$ (preliminary) $\varphi_p = 28.2$ (back analysis) $D_i^p = 0.995$ $S_p = 0.01 m$ $S = 60\%$ (saturation degree)	Saturated soft clay (39m) $\gamma = 17.4 kN/m^3$ $\varphi_p = 15$ (preliminary) $\varphi_p = 18.6$ (back analysis) $D_i^p = 0.985$ $S_p = 0.01 m$ $S = 100\%$ (saturation degree)	Saturated soft clay $\gamma = 17.4 kN/m^3$ $\varphi_p = 30.9$ (preliminary) $\varphi_p = 27.5$ (back analysis) $k_{ini}^b = 30000$ (preliminary) $k_{ini}^b = 50000$ (back analysis)
Lee and Park (2008)	$d_p = 1.2m$ $L_p = 37.4m$ $E_p = 50 GPa$	Silty clay (8m was unsaturated and 10.5m was saturated soil) $\gamma = 17.5 kN/m^3$ $\varphi_p = 32.5$ (preliminary) $\varphi_p = 32$ (back analysis) $D_i^p = 0.99$ $S_p = 0.05 m$ $S = 72\%$ (saturation degree)	Saturated clayey silt (18.5m) $\gamma = 15 kN/m^3$ $\varphi_p = 25$ (preliminary) $\varphi_p = 29.2$ (back analysis) $D_i^p = 0.99$ $S_p = 0.05 m$ $S = 100\%$ (saturation degree)	Saturated very stiff clay $\gamma = 16 kN/m^3$ $\varphi_p = 30$ (preliminary) $\varphi_p = 31$ (back analysis) $k_{ini}^b = 40000$ (preliminary) $k_{ini}^b = 100000$ (back analysis)

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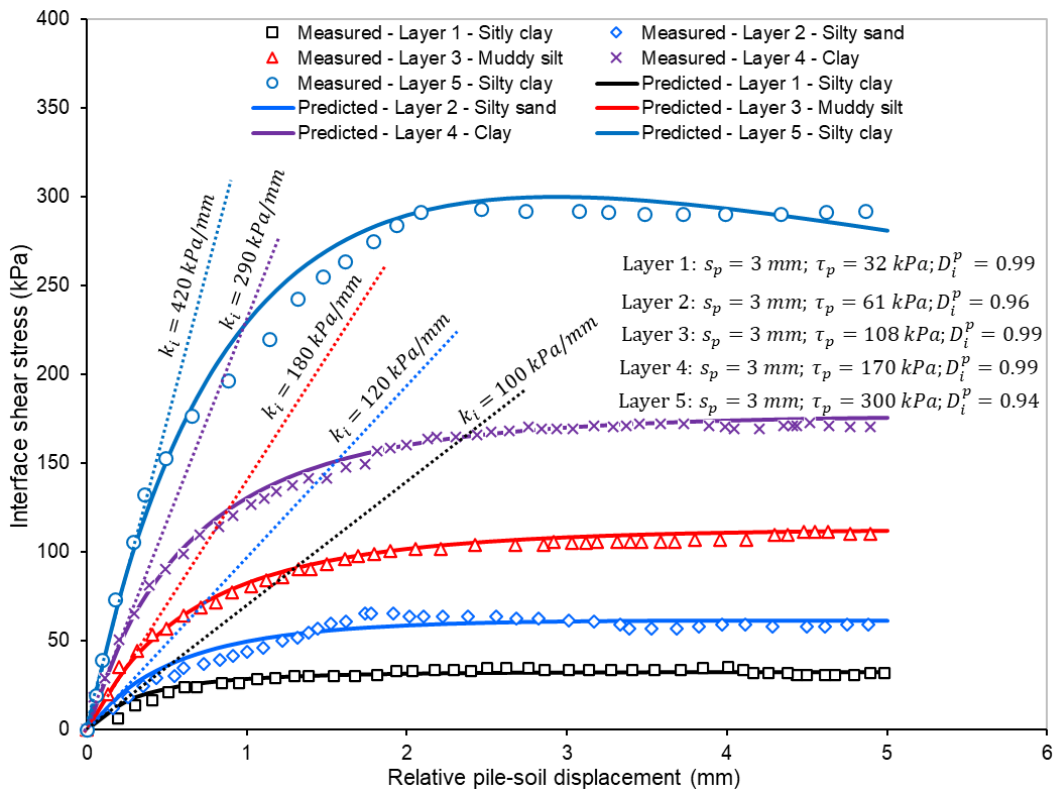
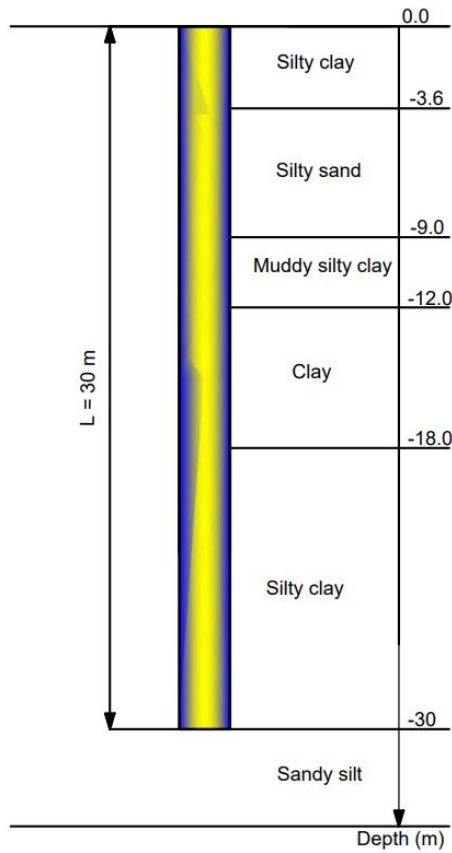
710

It should be highlighted that the effectiveness of the analytical method depends significantly on the quality of the input parameters. Laboratory tests, such as direct shear tests or triaxial tests, can be employed to determine the model parameters. Many complex aspects, including construction techniques, pile types, soil types, and loading techniques, have an impact on the interface behaviour of an actual pile at the site. It is generally known that, after pile installation, the in-situ state of the soil around the pile shaft is significantly altered. It is very difficult to simulate the state of the soil after pile installation. For a driven pile, the radial stress generated around the pile shaft is uncertain, while the interface roughness between the pile shaft and surrounding soil is unknown for a bored pile. Therefore, the magnitude of the resulting input parameters will be significantly impacted by these considerations. A back-analysis methodology is an additional approach to determine these input parameters in order to take into account input

711 parameter uncertainty and support optimization design. As a result, three cases are considered for comparison in this  
712 study: (1) preliminary prediction without using adjusted  $N_q$  factor (Eq. 51); (2) preliminary prediction using adjusted  
713  $N_q$  factor (Eq. 53); (3) prediction using back analysis-based input parameters.

#### 714 4.1. Field measurement of bored piles in Shanghai, China

715 A compelling example, recently described by Feng et al. (2024), was the application of both laboratory  
716 interface shear experiments and full-scale field tests to piles. This enables us to illustrate predicting the load-settlement  
717 response of piles based on the input parameters acquired from interface shear tests. The test bored pile was 30m long  
718 with an outer diameter of 0.6m. The geological conditions of the test pile are typical of a soft soil area, filled with  
719 silty clay and sandy silt, and clay from top to bottom, as shown in Figure 19a. It should be noted that the shear strength  
720 characteristics of each soil layer were obtained from the interface shear tests. Figure 19b shows the relationship  
721 between the shear stress and the relative displacement, which was obtained from the direct shear test results on the  
722 interface between the concrete and the five typical soils at the pile depth. A similar pattern was observed in the changes  
723 in the interface shear stresses for different soil samples during the shear tests. The proposed skin friction model takes  
724 into account four important parameters: ultimate disturbance function ( $D_i^p$ ), displacement corresponding to peak shear  
725 stress ( $s_p$ ), peak shear stress ( $\tau_p$ ), and initial shear stiffness ( $k_{ini}$ ). The initial shear stiffness might be determined by  
726 calculating the slope of the tangential line on the curve during the first segment. The test results (marker) and the  
727 predicted curve (solid line) created by the proposed model agree quite well. This means that the relationship between  
728 interface shear stress and displacement has been accurately captured by the suggested method given in section 2.2.  
729 These calibration model parameters are then used to predict the load-settlement curve of piles in situ. The load-  
730 displacement curves of the test piles under variable mechanical loads are shown in Figure 19c. The success of the  
731 proposed framework was confirmed when it was found that the load-transfer method suggested in this study more  
732 correctly represented the overall trend of the measured values. Additionally, it was noted that the end-bearing  
733 resistance has a nonlinear hardening tendency while the average skin friction resistance exhibits a nonlinear softening  
734 behaviour. At small settlements, the skin friction component is observed to mobilise more quickly than the end-  
735 bearing component, but it reduces after attaining the ultimate value.

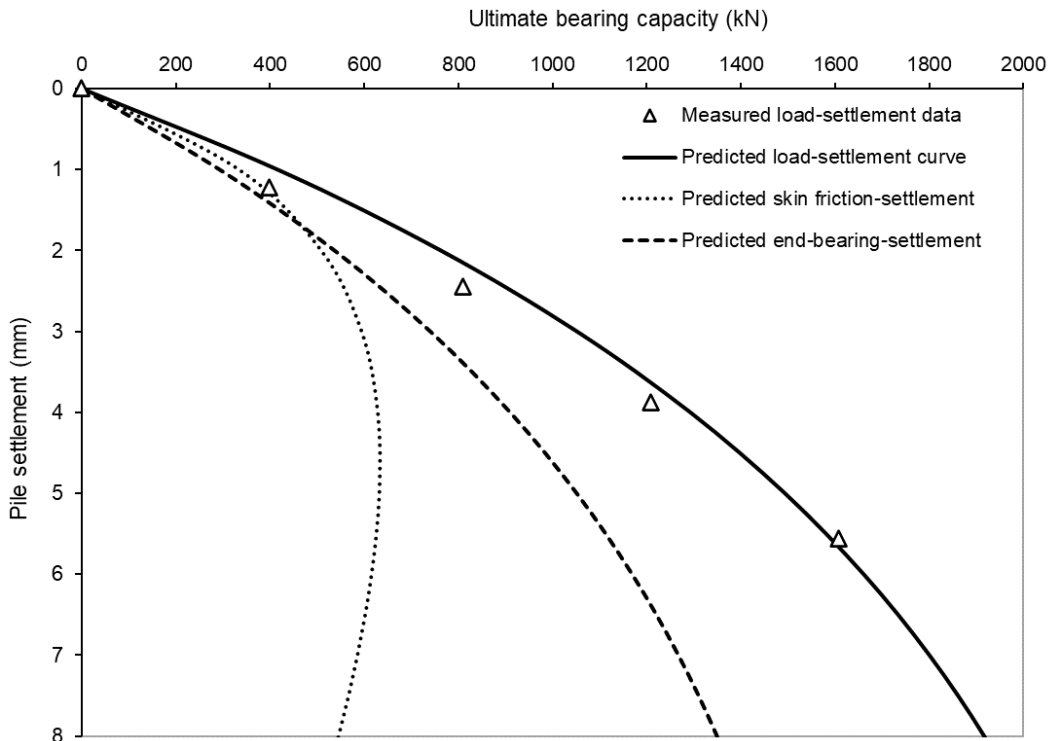


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742 **Figure 19.** Comparison of predicted and measured results for test pile in Shanghai (test data after [Feng](#)  
743 [et al. 2024](#)): a) pile test profile, b) interface shear test results, c) load-settlement curve

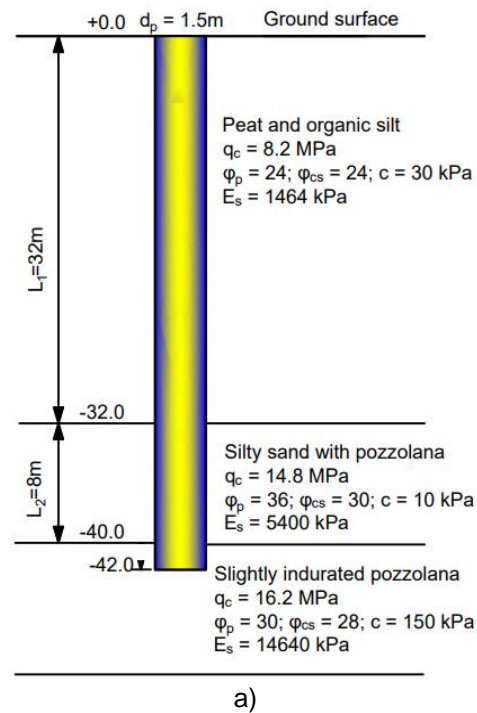
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745 **4.2. Field measurement of bored piles in Naples, Italy**

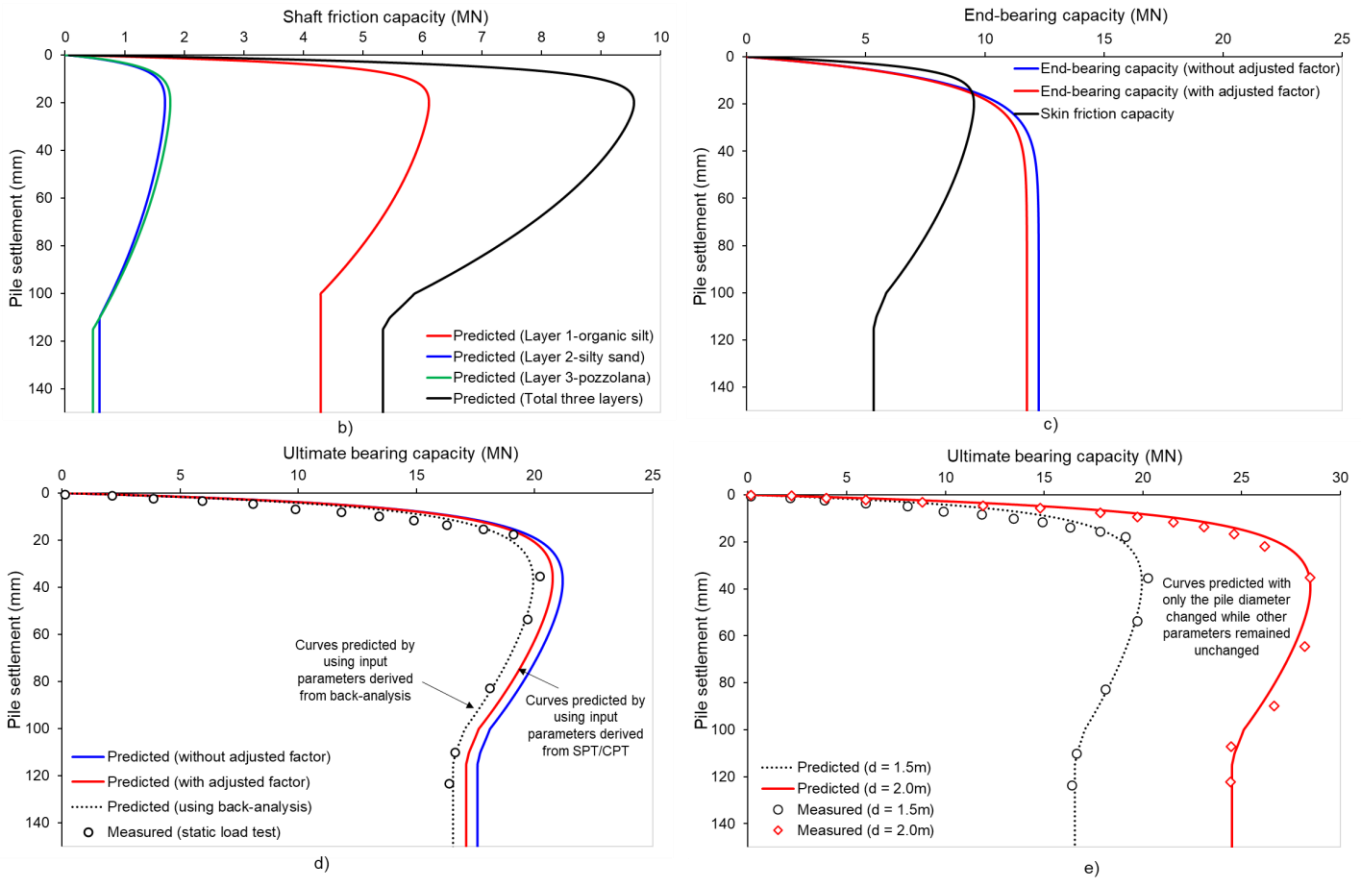
746 [Caputo et al. \(1991\)](#) performed static load tests on two piles with different diameters. Both test piles were 42  
747 m long, but one was 1.5 m, and the other was 2 m in diameter. This project is an interesting illustration of the  
748 advantages of the back analysis procedure, due to having piles with different geometries. [Figure 20a](#) shows the profile  
749 of the test pile and soil conditions. The piles penetrate through the relatively weak soil layers, which are buried in 32  
750 m of organic silt and 8 m of pozzolana. The ultimate resistance is frequently promptly mobilized at a small settlement  
751 due to the characteristic of soft soil, and the degradation then happens quickly after that. [Figure 20b](#) demonstrates the  
752 mobilization of skin friction resistance with pile settlement for various soil layers, where the hardening-softening  
753 behaviour was seen in the curves of three different soil layers. Each skin friction curve in this figure represents the  
754 skin friction capacity of a single layer and the total skin friction capacity of all soil layers where the pile penetrated  
755 through. The ultimate skin friction resistance often peaks at a settlement of 18 mm ( $\approx 0.012 d_p$ ). The comparison  
756 of the end-bearing resistance and skin friction resistance curves is shown in [Figure 20c](#). Although it seems that skin  
757 friction is smaller than the end-bearing resistance, this difference is dependent on the mobilized settlement. For  
758 instance, the skin friction resistance is less than the end-bearing resistance by around 13.6% at the peak state and  
759 45.6% at the residual state.

760 The comparison of the load-settlement curves for ultimate bearing capacity for predicted and measured data  
761 is shown in [Figure 20d](#). It should be observed that as pile settlement increases, the ultimate bearing capacity rises and

762 peaks at 35 mm ( $\approx 0.023 d_p$ ) before decreasing. The comparative findings demonstrate that the black dot line, which  
 763 represents the curves predicted using the back-analysis approach has the best fit with the measured curve. Curves  
 764 predicted with input parameters based on SPT/CPT, on the other hand, have a greater error but still exhibit good  
 765 agreement with the measured one (red solid curves). It should be stressed that, as previously indicated, there are a  
 766 number of complex aspects that could affect the production of a prediction curve and it would be very difficult to  
 767 match perfectly with the measured curve while using input parameters obtained from laboratory tests or CPT.  
 768 Specifically, The average relative error is only 2.2% for the prediction case utilizing input parameters based on back  
 769 analysis, 7.9% for the prediction case using CPT-based model calibration as well as an adjusted factor, and 12.7% for  
 770 the prediction case using CPT-based model calibration without an adjusted factor. The ultimate bearing capacity of  
 771 the test pile with a 2 m diameter is then predicted using the input parameters derived from the back analysis technique  
 772 for the test pile with a 1.5 m diameter. It should be highlighted that for the test pile with a diameter of 2 m, an excellent  
 773 match between the analytical and experimental curves is seen with a relative error of approximately 5.3%,  
 774 demonstrating the robust estimation capabilities of the proposed model (Figure 20e).  
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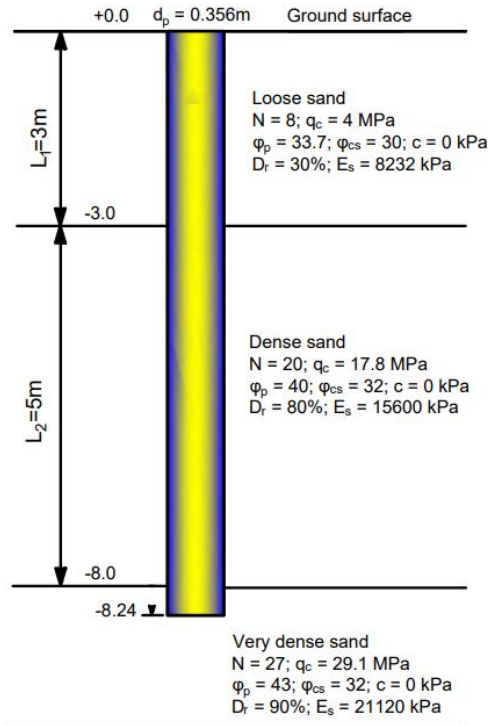


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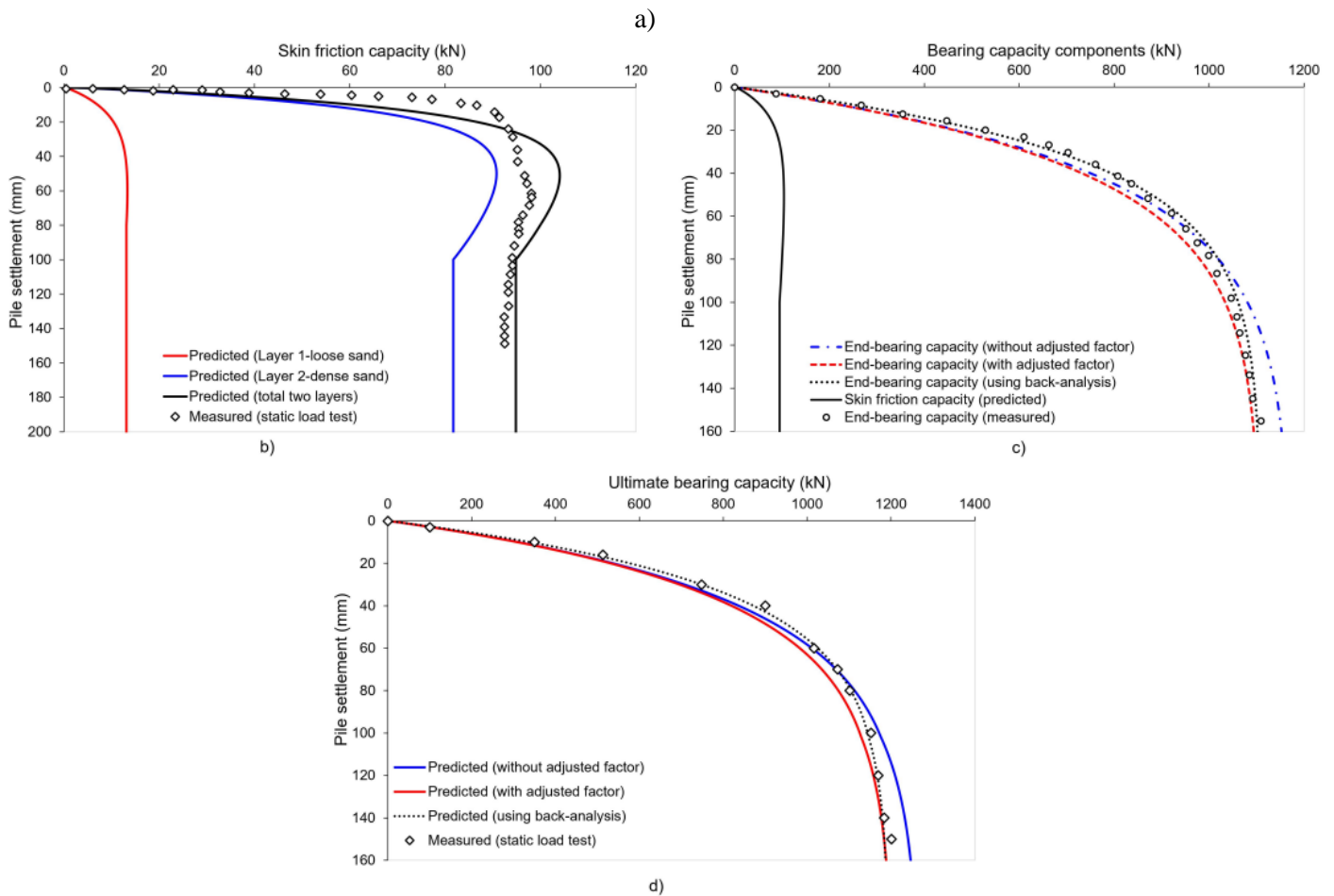
778  
779 **Figure 20.** Comparison of predicted and measured results for test pile in Italy (data after [Caputo et al. 1991](#)): a) pile  
780 test profile, b) skin friction capacity, c) end-bearing capacity, d) ultimate bearing capacity for pile with a diameter  
781 of 1.5m, e) ultimate bearing capacity for pile with a diameter of 2m

782 **4.3. Field measurement of pipe piles in LaGrange, Indiana**



783





785  
786 **Figure 21.** Comparison of predicted and measured results for test pile in Indiana (data after Lee et al. 2003): a) pile  
787 test profile, b) skin friction capacity, c) end-bearing capacity, d) ultimate bearing capacity

788 The load capacity of both closed- and open-ended piles are reported by Lee et al. (2003) through an  
789 experimental programme utilizing calibration chamber model pile load tests and field pile load tests. The test site is  
790 close to the Han River in Indiana. Due to the characteristics of test pipe piles penetrating sand layers of varying  
791 densities, this project is chosen for performance analysis. Moreover, the measured skin friction and end-bearing  
792 capacity of piles are separately provided, which is beneficial for comparison. The profile of the test pile and the soil  
793 conditions are shown in Figure 21a. As can be observed, the soil site is primarily loose, gravelly sand deposit with a  
794  $D_r = 30\%$  up to a depth of 3 meters. Conditions are dense to very dense throughout the rest of the sand deposit, which  
795 is down to a depth of 13–14 m ( $D_r = 80\%$ ). Figures 21b and 21c, respectively, show the results of the resistance  
796 analysis of the components that include skin friction and end-bearing. The measured data reveals that while the end-  
797 bearing resistance only exhibits hardening behaviour, the skin friction resistance exhibits a softening behaviour after  
798 reaching its maximum value at a settlement of 18 mm ( $\approx 0.05 d_p$ ). In this instance, the end-bearing component has  
799 a significantly higher contribution to the ultimate bearing capacity compared to the skin friction. It is important to  
800 note that a variety of complex factors, including the soil characteristics, loading procedure, and construction technique,  
801 can affect how a real pile behaves at the interface, which accounts for the difference between the expected and actual

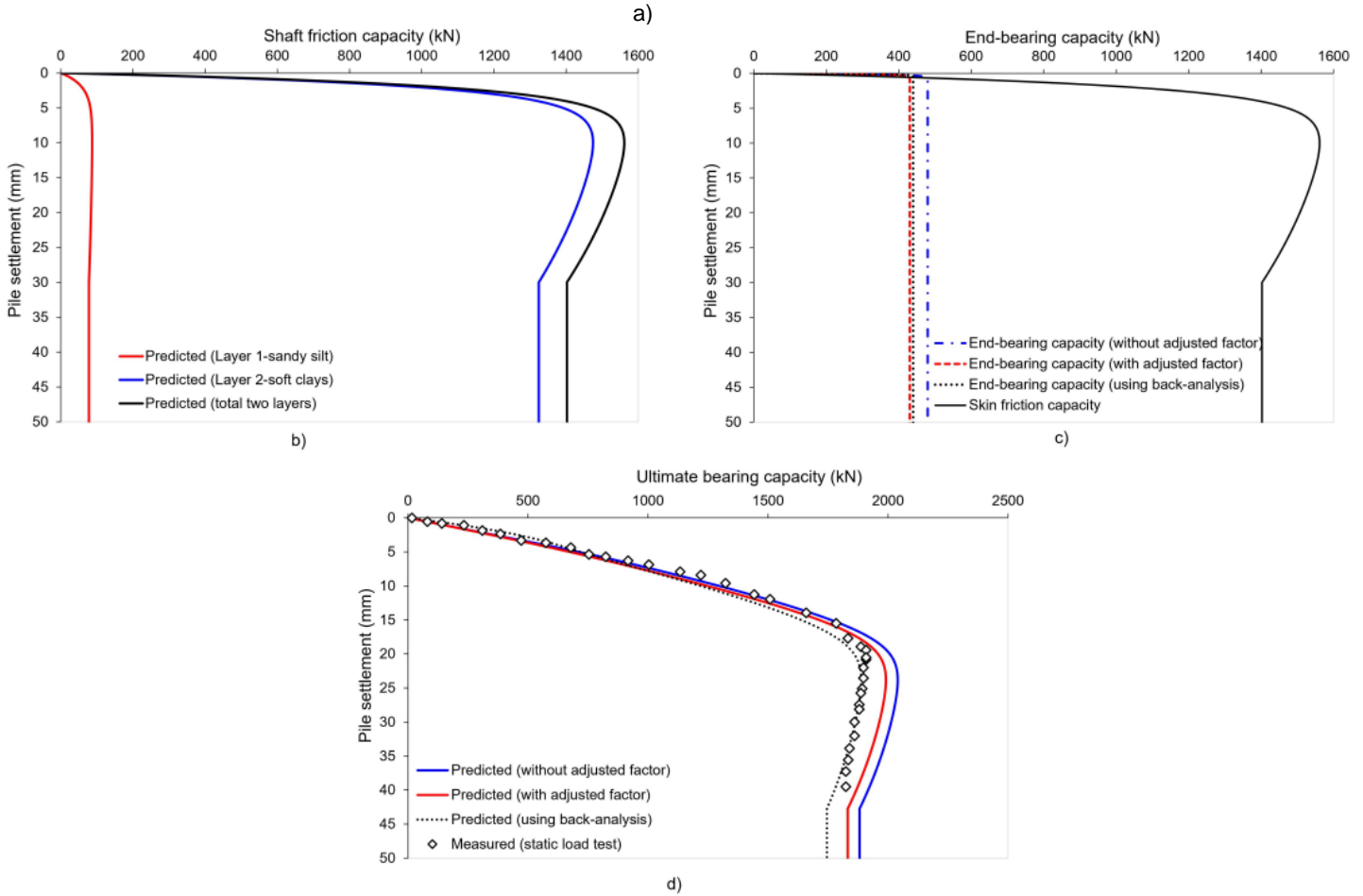
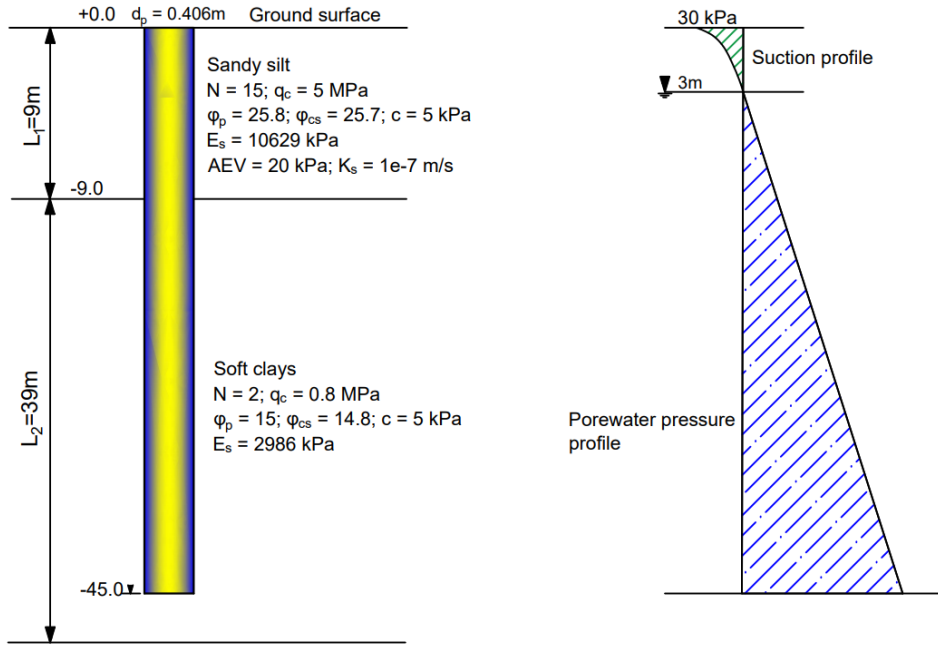
802 curves (Fig. 21b). The pile length in this example was only 8 m, with 3 m in loose sand and 5m in dense sand. Because  
803 of pile construction, soil recovery, and contact damage between the soil and structure for shallow layers, the  
804 mobilisation of shaft friction would be greatly impacted. Consequently, fitting the skin friction-settlement curve  
805 appropriately would be challenging in this scenario. More importantly, though, is that the softening tendency of the  
806 friction shaft components is found to be appropriately predicted by the analytical model. As a result, it could be  
807 concluded that the proposed model still provides a potential choice for the load-settlement analysis of piles.  
808 Concerning modelling the load-settlement response of both resistance components, the predicted curves and the  
809 measured one agree well. The comparison of the predicted and measured curves for the ultimate bearing capacity of  
810 piles is shown in Figure 21d. It is noted that there is good agreement between curves derived using the analytical  
811 model and field measurements. Using the back analysis-based input parameters enhanced the performance of the  
812 proposed model. The average relative error for the prediction case that used input parameters based on back analysis  
813 was 3.4%, for the prediction case with CPT-based model calibration that used an adjusted factor was 8.7%, and for  
814 the prediction case with CPT-based model calibration that did not use an adjusted factor was 15.1%.

#### 815 ***4.4. Field measurement of the driven pile in Sandpoint, Idaho***

816 Fellenius et al. (2004) provide the results of a static load test on a driven pile with a diameter of 0.406 m which was  
817 driven to a depth of 45 m on soft, compressible soil. The profile consists of a layer of sandy and silty soil roughly 6  
818 m thick and a layer of clay measuring roughly 40 meters. Unlike other projects, the pile base in this case is supported  
819 by soft clay. Thus, it is anticipated that the skin friction resistance will significantly contribute to the ultimate bearing  
820 capacity of the pile. In addition, the groundwater table was discovered at a depth of 3 m below the ground surface.  
821 Therefore, the sandy silt layer could be separated into 3m of unsaturated soil with a 60% saturation degree, and the  
822 remaining 6m were accepted to be saturated. Since the measured soil-water characteristic curve was not provided, the  
823 suction profile is predicted using Equation (45) with a flow rate of  $1.15e-9$  m/s as demonstrated in Figure 22a. Figures  
824 22b and 22c show the mobilization of skin friction resistance and end-bearing resistance of the considered pile,  
825 respectively. It should be noted that because the pile has a long length and its base rests on soft soils, skin friction  
826 accounts for around 80% of the total bearing capacity and is more than four times the end-bearing resistance. As a  
827 result, the test pile is classified as a friction pile. The comparison of the predicted and measured load-settlement curves  
828 is shown in Figure 22d. It is noted that the analytical model predicts the load-settlement response with good agreement  
829 to measurements. The average relative error is 5.6% in the prediction case using input parameters based on back  
830 analysis, 14.5% in the prediction case with CPT-based model calibration using an adjusted factor, and 16.1% in the  
831 scenario without an adjusted factor. Moreover, the data from static load tests show that as settlement increases, the  
832 ultimate bearing capacity increases as well, and then exhibits softening behaviour after reaching the ultimate value.  
833 A reason for the softening response in this situation can be attributed to the interface between soft clay and the pile,  
834 which is frequently destroyed at a small displacement, resulting in a reduction in skin friction resistance.



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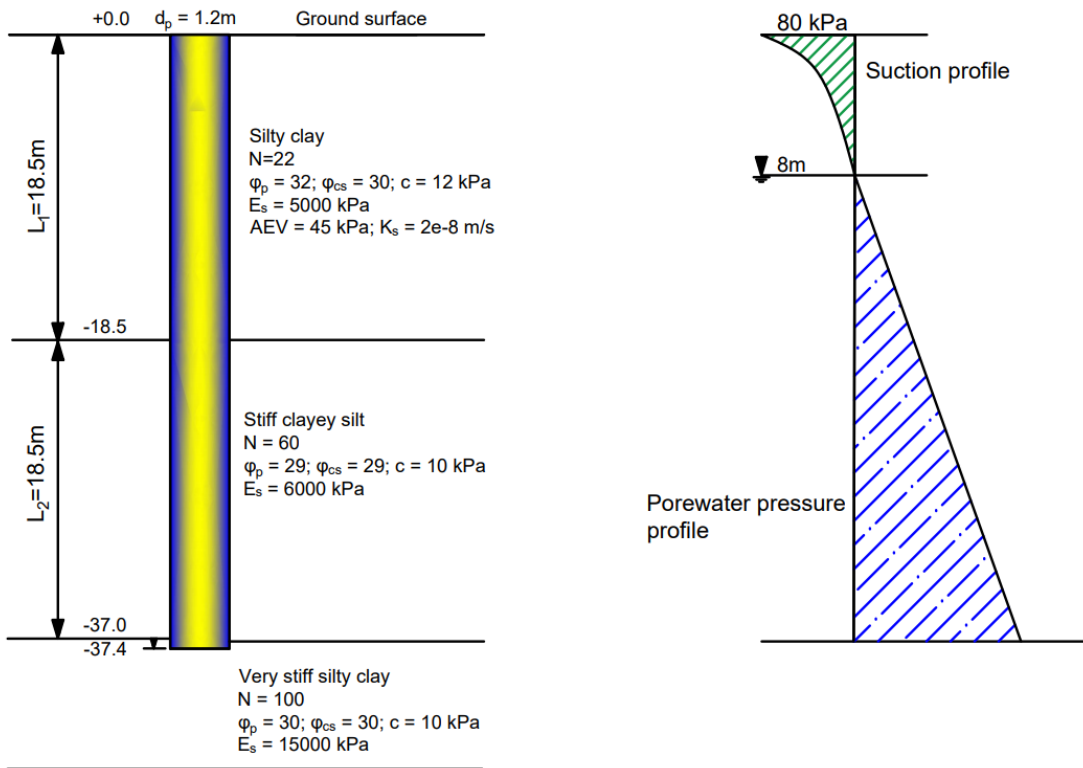
840

Figure 22. Comparison of predicted and measured results for test pile in Idaho (data after Fellenius et al. 2004):  
a) pile test profile, b) skin friction capacity, c) End-bearing capacity, d) ultimate bearing capacity

#### 841 4.5. Field measurement of the bored pile in Singapore

842 The field measurement results on a bored pile with a diameter of 1.2 metres and a length of 37.4 metres were  
843 reported by Lee and Park (2008). Having results from both the O-cell test and the static load test, the project is an  
844 important one for comparison purposes. It should be emphasised that because the pile is regarded as a wholly rigid  
845 entity in the theory of the O-cell test, the elastic settlement of piles is negligible. However, the pile practically suffers  
846 elastic shortening with an applied load, and its strain decreases with depth. Due to this advantage, the results of static  
847 load testing are used as a benchmark for comparison, while the results of O-cell tests are only used to examine the  
848 behaviour of the load-settlement curves of the resistance components. The test piles and soil profile are shown in  
849 Figure 23a. It was found that the test pile penetrated through layers of silty clay and had toe bearing from the very  
850 stiff/stiff silty clay. Because the groundwater table was discovered at a depth of 8m below the ground surface, the  
851 silty clay layer could be separated into 8m of unsaturated soil with a 72% saturation degree, and the remaining 10.5m  
852 were taken to be saturated. It would be ideal to obtain SWCC data at various depths because the flow rate varies with  
853 depth. Unfortunately, the original article did not include the SWCC data. As a result, the authors cite the SWCC data  
854 of silty clay with similar physical properties that were reported by Uchaipichat and Khalili (2009). The SWCC utilised  
855 to determine the AEV, matric suction, and corresponding degree of saturation is shown in Figure 22b. An example  
856 for estimating the confining pressure of unsaturated soils at the ground surface is illustrated here. In this case, the  
857 effective pressure of unsaturated soil at the ground surface would be  $(\sigma'_v)_{unsat} = (u_a - u_w) \cdot S_e = 80 \times 0.72 =$   
858  $57.6 \text{ kPa}$ . The earth pressure coefficient  $K_E = (1 - \sin \phi_{cs}) \cdot (OCR)^{0.5} \cdot \tan(R_i \cdot \phi_{cs}) \approx 0.26$ . The confining pressure  
859 at the ground surface will be  $(\sigma'_h)_{unsat} = (u_a - u_w) \cdot S_e \cdot K_E = 80 \times 0.12 \times 0.26 = 15 \text{ kPa}$

860 Figures 23c and 23d show the prediction outcomes for the skin friction capacity and end-bearing capacity,  
861 respectively. It is discovered that, while the end-bearing capacity constantly increases nonlinearly with settlement,  
862 the skin friction capacity increasingly mobilises with pile settlement and approaches a limiting value at 15.5mm ( $\approx$   
863  $0.013 d_p$ ). Moreover, the skin friction component contributes significantly more to the overall bearing capacity of  
864 the pile than its end-bearing capacity does. Figure 23e compares the results of predictions and measurements for  
865 ultimate bearing capacity. It is evident that the shapes of the predicted and measured curves are quite comparable.  
866 With a relative error of roughly 7.2%, the load-settlement response derived from the analytical model consistently  
867 agrees with that of static load tests. Also, it was discovered that the predicted curve with input parameters based on  
868 back analysis had a greater agreement with the measured curve, indicating that the proposed model's performance  
869 may be enhanced by increasing the accuracy of the input parameters. In comparison to static load tests, the results of  
870 the O-cell test were likewise shown to be overestimated. This is considered to be due to the elastic shortening of the  
871 pile not taken into account by the O-cells, which only assessed the relative displacement between the soils and the  
872 pile.

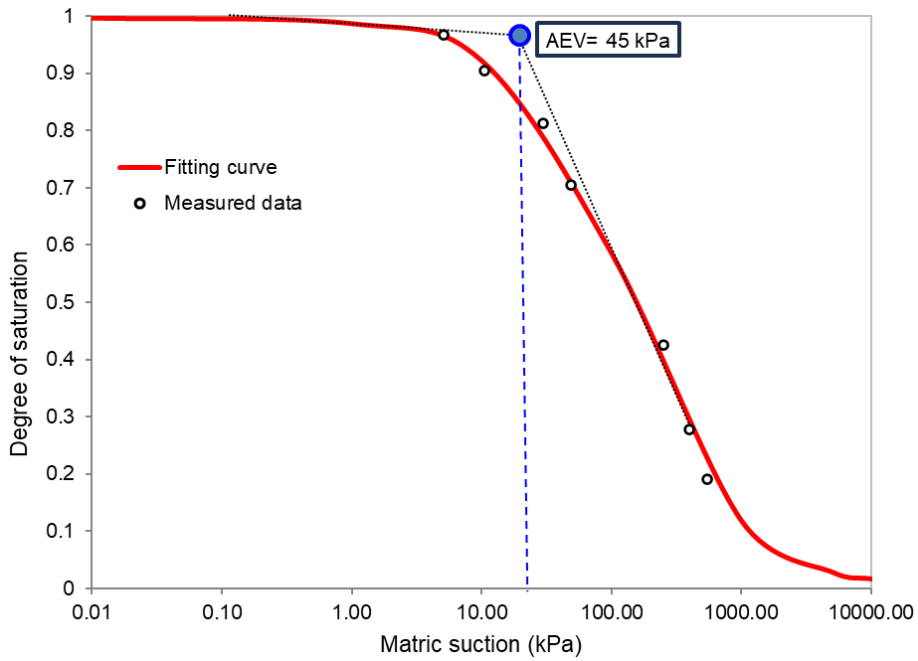


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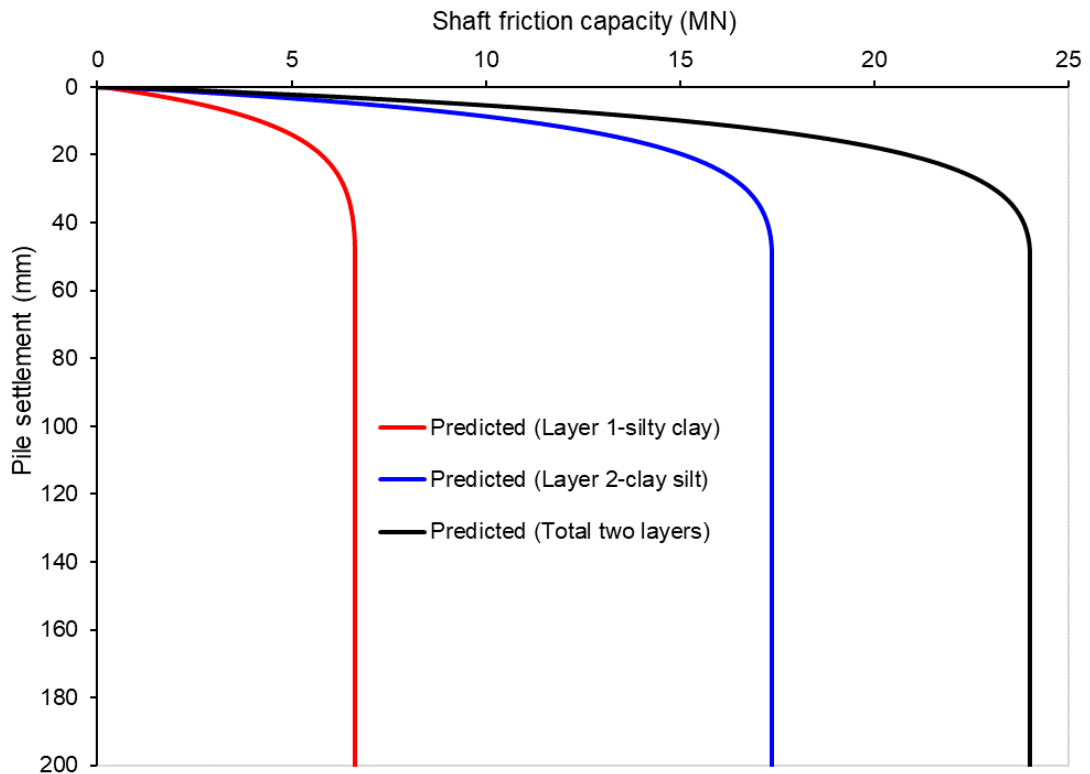
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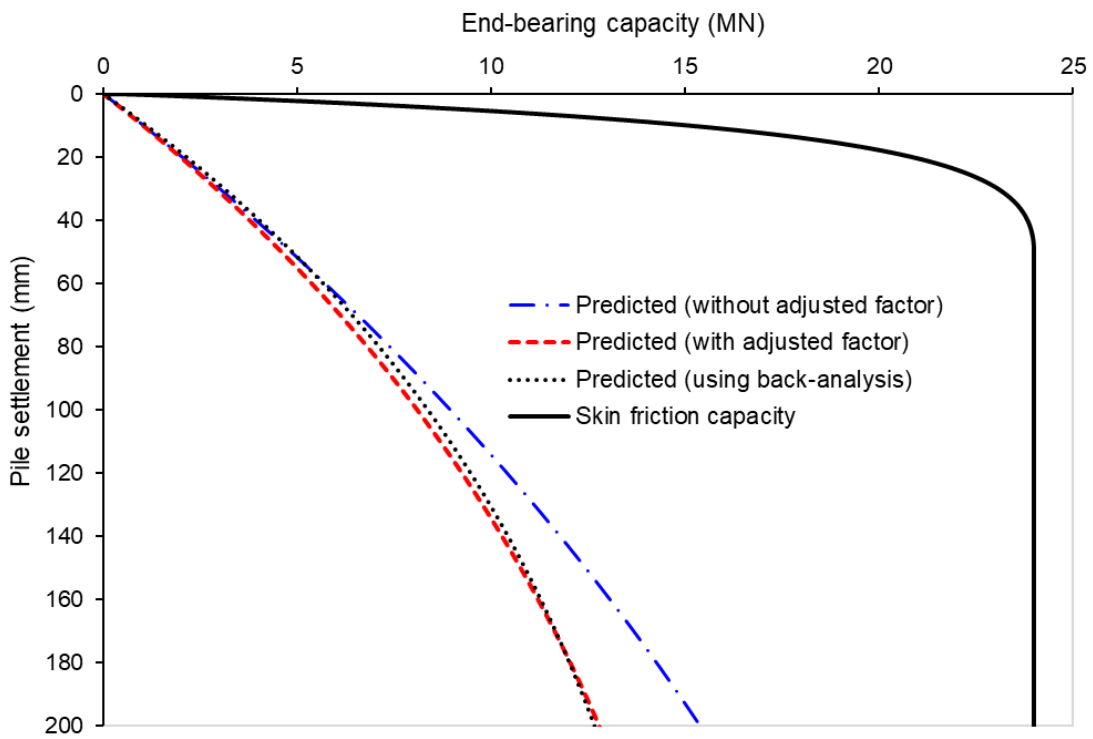
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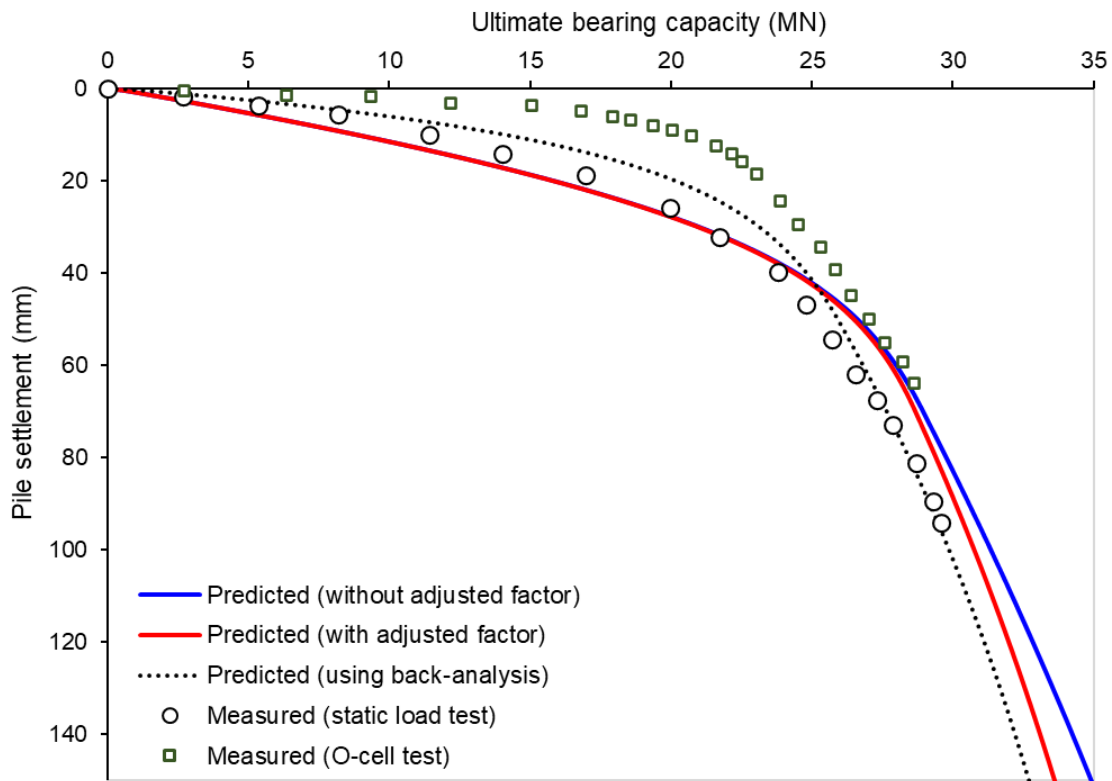
c)



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e)

884 **Figure 23.** Comparison of predicted and measured results for test pile in Singapore (data after [Lee and Park 2008](#)):  
 885 a) pile test profile, b) SWCC data; c) skin friction capacity, d) End-bearing capacity, e) ultimate bearing capacity

886

887 **5. Conclusions**

888 To represent the hardening-softening behaviour, an interface shear stress-displacement model was developed  
 889 using the adhesion-friction integration and disturbed state concept. The only additional parameter in the proposed  
 890 model was the "ultimate disturbance function" which can be easily determined using either conventional direct shear  
 891 tests or soil shear strength parameters. The new model captured the hardening-softening behaviour, which is not  
 892 possible through the use of hyperbolic models.

893 The experimental results from direct shear tests were used to validate the DSC interface model. The  
 894 comparison results demonstrated that the shear stress-displacement curve can be successfully simulated and that the  
 895 proposed model had a better performance compared to the hyperbolic model for both loose and dense soils.

896 The proposed interface model was extended to integrate with bearing capacity theory to develop the load  
 897 transfer framework for pile foundations. In this study, a simplified approach was presented, which takes into account  
 898 the hardening relationship of the base reaction and the nonlinear softening relationship of skin friction. This work  
 899 also developed prediction methods for the profile of steady-state matric suction in situ, soil-water characteristic curve,  
 900 and pore-water pressure of unsaturated soils. The present method was validated with five well-documented field

901 measurement projects. The comparison results showed that the proposed approach and the behaviour observed in the  
902 field were in good agreement.

903 The proposed analytical approach and developed algorithm provided a cost-effective and reliable solution that  
904 was appropriate for the analysis of piles embedded in saturated and unsaturated multi-layered soils.

905 .

906

### **Conflict of Interest**

907 The authors declare that there have no known competing financial interests or personal relationships that could have  
908 appeared to influence the work reported in this paper.

909

### **Data Availability Statement**

910

All data, models, and code generated or used during the study appear in the submitted article.

911

912

### **Appendix A. Static load test dataset**

913

**Table A1.** Static load test dataset for analysis of bearing capacity factor  $N_q$

914

### **Appendix B. Empirical correlation**

915

**Table B1.** Empirical correlation between  $N_{SPT}$  and shear strength parameters

916

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917

[Figure 1.](#) Mobilization of interface shear stress with pile-soil relative displacement

918

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919 total interface shear strength

920

[Figure 3.](#) Contribution of components on the soil-structure interface resistance: a) Adhesion, b) Friction

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926 Zhang (2006); b) test data after Evgin and Fakharian (1997); c) test data after Liu et al. (2014)

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929 al. (2019)

930

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933 base resistance versus displacement; (c) total bearing capacity versus displacement

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951 a) pile test profile, b) SWCC data, c) skin friction capacity, d) End-bearing capacity, e) ultimate bearing capacity

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## References

- 965 Alawneh, A. S., Nusier, O., Husein Malkawi, A. I., & Al-Kateeb, M. (2001). Axial compressive capacity of driven  
966 piles in sand: a method including post-driving residual stresses. *Canadian Geotechnical Journal*, 38(2), 364-377.
- 967 Al-Shafei, K. A., Cox, W. R., & Helfrich, S. C. (1994, May). Pile load tests in dense sand: Analysis of static test  
968 results. In *Offshore Technology Conference* (pp. OTC-7381). OTC.
- 969 Ali, H., Reiffsteck, P., Baguelin, F., van de Graaf, H., Bacconnet, C., & Gourves, R. (2010, May). Settlement of pile  
970 using cone loading test: Load settlement curve approach. In *CPT'10: 2nd Int. Symp. on Cone Penetration Testing*.
- 971 Azzouz, A. S., Baligh, M. M., & Ladd, C. C. (1981). Cone penetration and engineering properties of the soft Orinoco  
972 clay.
- 973 Baldi, G., Bellotti, R., Ghionna, N., & Jamiolkowski, M. (1989). Stiffness of sands from CPT, SPT and DMT—A  
974 critical review. In *Penetration testing in the UK: Proceedings of the geotechnology conference organized by the*  
975 *Institution of Civil Engineers and held in Birmingham on 6–8 July 1988* (pp. 299-305). Thomas Telford Publishing.
- 976 Baghini, E. G., Toufigh, M. M., & Toufigh, V. (2018). Analysis of pile foundations using natural element method  
977 with disturbed state concept. *Computers and Geotechnics*, 96, 178-188.  
978 <https://doi.org/10.1016/j.compgeo.2017.11.005>
- 979 Budhu, M. (2010). *Soil mechanics and foundations*. John Wiley and Sons.
- 980 Bui, T. Y., Li, Y., Tan, S. A., & Leung, C. F. (2005). Back analysis of O-cell pile load test using FEM. In *Proceedings*  
981 *of the 16th International Conference on Soil Mechanics and Geotechnical Engineering* (pp. 1959-1962). IOS Press.
- 982 Bohn, C., Lopes dos Santos, A., & Frank, R. (2017). Development of axial pile load transfer curves based on  
983 instrumented load tests. *Journal of Geotechnical and Geoenvironmental Engineering*, 143(1), 04016081.
- 984 Brown, M. J., Hyde, A. F. L., & Anderson, W. F. (2006). Analysis of a rapid load test on an instrumented bored pile  
985 in clay. *Geotechnique*, 56(9), 627-638.
- 986 Briaud, J. L., Ballouz, M., & Nasr, G. (2000). Static capacity prediction by dynamic methods for three bored  
987 piles. *Journal of Geotechnical and Geoenvironmental engineering*, 126(7), 640-649.  
988 [https://doi.org/10.1061/\(ASCE\)1090-0241\(2000\)126:7\(640\)](https://doi.org/10.1061/(ASCE)1090-0241(2000)126:7(640))
- 989 British Standards Institution (2010). *Code of Practice for Strengthened/reinforced Soils and Other Fills: BS 8006-1:*  
990 *2010*. BSI. London, UK.
- 991 Brooks, R. H., and Corey, A. T. (1964). "Hydraulic properties of porous media." *Hydro. Paper No. 3*, Colorado State  
992 Univ., Fort Collins, Colorado, USA.
- 993 Castelli, F., & Maugeri, M. (2002). Simplified nonlinear analysis for settlement prediction of pile groups. *Journal of*  
994 *geotechnical and geoenvironmental engineering*, 128(1), 76-84. [https://doi.org/10.1061/\(ASCE\)1090-](https://doi.org/10.1061/(ASCE)1090-0241(2002)128:1(76))  
995 [0241\(2002\)128:1\(76\)](https://doi.org/10.1061/(ASCE)1090-0241(2002)128:1(76))



- 996 Caputo, V. (1991). Equivalent elastic analysis of settlement for piled foundations. Proc. 10th ECSMFE, Florence,  
997 Italy. Vol. 4, pp. 1346-1348.
- 998 Chen, C., Zhu, S., Zhang, G., Morsy, A. M., Zornberg, J. G., & Mao, F. (2022). A generalized load-transfer modeling  
999 framework for tensioned anchors integrating adhesion–friction-based interface model. *International Journal of*  
1000 *Geomechanics*, 22(5), 04022036. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0002338](https://doi.org/10.1061/(ASCE)GM.1943-5622.0002338)
- 1001 Cho, H. I., Lee, M. G., Sun, C. G., & Kim, H. S. (2021). Evaluation of engineering site conditions in North  
1002 Gyeongsang Province, South Korea, using a large database of borehole and standard penetration test  
1003 data. *Engineering Geology*, 294, 106306.
- 1004 Chu, L. M., & Yin, J. H. (2005). Comparison of interface shear strength of soil nails measured by both direct shear  
1005 box tests and pullout tests. *Journal of geotechnical and geoenvironmental engineering*, 131(9), 1097-1107.  
1006 [https://doi.org/10.1061/\(ASCE\)1090-0241\(2005\)131:9\(1097\)](https://doi.org/10.1061/(ASCE)1090-0241(2005)131:9(1097))
- 1007 Coyle, H. M., & Castello, R. R. (1981). New design correlations for piles in sand. *Journal of the Geotechnical*  
1008 *Engineering Division*, 107(7), 965-986.
- 1009 Cui, Y. L., Qi, C. G., Zheng, J. H., Wang, X. Q., & Zhang, S. M. (2021). Field test research on post-grouting effect  
1010 for super-long cast-in-place bored pile in thick soft foundation. *Geotechnical and Geological Engineering*, 1-10.
- 1011 Comodromos, E. M., Anagnostopoulos, C. T., & Georgiadis, M. K. (2003). Numerical assessment of axial pile group  
1012 response based on load test. *Computers and Geotechnics*, 30(6), 505-515.
- 1013 Dai, G., Salgado, R., Gong, W., & Zhang, Y. (2012). Load tests on full-scale bored pile groups. *Canadian*  
1014 *geotechnical journal*, 49(11), 1293-1308.
- 1015 Dalai, S., & Patra, C. (2021). Prediction of angle of internal friction based on SPT N values. In *Proceedings of the*  
1016 *Indian Geotechnical Conference 2019: IGC-2019 Volume V* (pp. 471-477). Springer Singapore.
- 1017 Desai, C. S. (2001). *Mechanics of materials and interfaces: The disturbed state concept*. CRC Press, Washington,  
1018 USA, ISBN 0-8493-0248-X
- 1019 De Nicola, A., & Randolph, M. F. (1993). Tensile and compressive shaft capacity of piles in sand. *Journal of*  
1020 *geotechnical engineering*, 119(12), 1952-1973. [https://doi.org/10.1061/\(ASCE\)0733-9410\(1993\)119:12\(1952\)](https://doi.org/10.1061/(ASCE)0733-9410(1993)119:12(1952))
- 1021 Dias, T. G. S., & Bezuijen, A. (2018). Load-transfer method for piles under axial loading and unloading. *Journal of*  
1022 *Geotechnical and Geoenvironmental Engineering*, 144(1), 04017096. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001808](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001808)
- 1024 Dithinde, M., Phoon, K. K., De Wet, M., & Retief, J. V. (2011). Characterization of model uncertainty in the static  
1025 pile design formula. *Journal of Geotechnical and Geoenvironmental Engineering*, 137(1), 70-85.  
1026 [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000401](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000401)
- 1027 Di Donna, A., Ferrari, A., & Laloui, L. (2016). Experimental investigations of the soil–concrete interface: physical  
1028 mechanisms, cyclic mobilization, and behaviour at different temperatures. *Canadian Geotechnical Journal*, 53(4),  
1029 659-672. <https://doi.org/10.1139/cgj-2015-0294@cgj-wgge.issue01>
- 1030 Dunham, J. W. (1954). Pile foundations for buildings. *Proc. ASCE, Soil Mechanics & Foundations Division*, 80(385).

- 1031 Edlefsen, N., & Anderson, A. (1943). Thermodynamics of soil moisture. *Hilgardia*, 15(2), 31-298.  
1032 <https://doi.org/10.3733/hilg.v15n02p031>
- 1033 Eurocode 7: Geotechnical Design. Worked examples. EUR 26227. Luxembourg (Luxembourg): Publications Office  
1034 of the European Union; 2013. JRC85029
- 1035 Evgin, E., & Fakharian, K. (1997). Effect of stress paths on the behaviour of sand steel interfaces. *Canadian*  
1036 *geotechnical journal*, 33(6), 853-865. <https://doi.org/10.1139/t96-116-336>
- 1037 Farhadi, B., & Lashkari, A. (2017). Influence of soil inherent anisotropy on behavior of crushed sand-steel  
1038 interfaces. *Soils and Foundations*, 57(1), 111-125. <https://doi.org/10.1016/j.sandf.2017.01.008>
- 1039 Fakharian, K., Meskar, M., & Mohammadlou, A. S. (2014). Effect of surcharge pressure on pile static axial load test  
1040 results. *International Journal of Geomechanics*, 14(6), 04014024.
- 1041 Feng, S., Fang, J., Zhao, Y., Zhang, Z., & Wang, Y. (2024). Thermomechanical Analysis of Energy Piles Using a  
1042 Load-transfer Approach Considering Soil Coupling Effects. *Computers and Geotechnics*, 168, 106147.  
1043 <https://doi.org/10.1016/j.compgeo.2024.106147>
- 1044 Feng, S. J., Lu, S. F., & Shi, Z. M. (2016). Field investigations of two super-long steel pipe piles in offshore  
1045 areas. *Marine Georesources & Geotechnology*, 34(6), 559-570. <https://doi.org/10.1080/1064119X.2015.1038760>
- 1046 Fellenius, B. H., Harris, D. E., & Anderson, D. G. (2004). Static loading test on a 45 m long pipe pile in Sandpoint,  
1047 Idaho. *Canadian geotechnical journal*, 41(4), 613-628. <https://doi.org/10.1139/t04-012>
- 1048 Fleming, W. G. K. (1992). A new method for single pile settlement prediction and analysis. *Geotechnique*, 42(3),  
1049 411-425. <https://doi.org/10.1680/geot.1992.42.3.411>
- 1050 FHWA NHI-06-088, (2006) Federal highway administration US Department. Transp NHI I:2–14. Washington,  
1051 United state
- 1052 Fioravante, V. (2002). On the shaft friction modelling of non-displacement piles in sand. *Soils and foundations*, 42(2),  
1053 23-33. [https://doi.org/10.3208/sandf.42.2\\_23](https://doi.org/10.3208/sandf.42.2_23)
- 1054 Fredlund, D. G., & Xing, A. (1994). Equations for the soil-water characteristic curve. *Canadian geotechnical*  
1055 *journal*, 31(4), 521-532. <https://doi.org/10.1139/t94-061>
- 1056 Gardner, W. R. (1958). Some steady-state solutions of the unsaturated moisture flow equation with application to  
1057 evaporation from a water table. *Soil science*, 85(4), 228-232.
- 1058 Guo, X., Pham, T. A., & Dias, D. (2022). Probabilistic analysis of geosynthetic-reinforced and pile-supported  
1059 embankments. *Computers and Geotechnics*, 142, 104595. <https://doi.org/10.1016/j.compgeo.2021.104595>
- 1060 Guo, X., Pham, T. A., & Dias, D. (2023). Multi-objective optimization of geosynthetic-reinforced and pile-supported  
1061 embankments. *Acta Geotechnica*, 18(7), 3783-3798. <https://doi.org/10.1007/s11440-022-01782-4>
- 1062 Hashemi, A., Sutman, M. and Abuel-Naga, H., 2022. Thermomechanical response of kaolin clay-concrete interface  
1063 in the context of energy geostructures. *Canadian Geotechnical Journal*, (ja).
- 1064 Hammam, A. H., & Salam, A. A. (2018). Behavior of Bored Piles in Two Soil Layers, Sand Overlaying Compressible  
1065 Clay (Case Study). In *Advances in Analysis and Design of Deep Foundations: Proceedings of the 1st GeoMEast*

- 1066 *International Congress and Exhibition, Egypt 2017 on Sustainable Civil Infrastructures 1* (pp. 15-27). Springer  
1067 International Publishing.
- 1068 Han, F., Prezzi, M., & Salgado, R. (2018). Static and dynamic pile load tests on closed-ended driven pipe pile.  
1069 In *IFCEE 2018* (pp. 496-506).
- 1070 Haustorfer, I., & Plesiotis, S. (1988, August). Instrumented dynamic and static pile load testing at two bridge sites.  
1071 In *Fifth Australia-New Zealand conference on geomechanics: prediction versus performance; preprints of Papers*.
- 1072 Hara, A., Ohta, T., Niwa, M., Tanaka, S., & Banno, T. (1974). Shear modulus and shear strength of cohesive  
1073 soils. *Soils and Foundations*, 14(3), 1-12.
- 1074 Hatanaka, M., Uchida, A., & Oh-oka, H. (1995). Correlation between the liquefaction strengths of saturated sands  
1075 obtained by in-situ freezing method and rotary-type triple tube method. *Soils and Foundations*, 35(2), 67-75..
- 1076 Heerema, E. P. (1979). Pile driving and static load tests on piles in stiff clay. In *Offshore Technology Conference* (pp.  
1077 OTC-3490). OTC.
- 1078 Hirayama, H. (1990). Load-settlement analysis for bored piles using hyperbolic transfer functions. *Soils and*  
1079 *Foundations*, 30(1), 55-64. <https://doi.org/10.3208/sandf1972.30.55>
- 1080 Hettiarachchi, H., & Brown, T. (2009). Use of SPT blow counts to estimate shear strength properties of soils: energy  
1081 balance approach. *Journal of Geotechnical and Geoenvironmental engineering*, 135(6), 830-834.  
1082 [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000016](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000016)
- 1083 Huang, M., Jiang, S., Xu, C., & Xu, D. (2020). A new theoretical settlement model for large step-tapered hollow piles  
1084 based on disturbed state concept theory. *Computers and Geotechnics*, 124, 103626.  
1085 <https://doi.org/10.1016/j.compgeo.2020.103626>
- 1086 Hu, L., & Pu, J. (2004). Testing and modeling of soil-structure interface. *Journal of Geotechnical and*  
1087 *Geoenvironmental Engineering*, 130(8), 851-860. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2004\)130:8\(851\)](https://doi.org/10.1061/(ASCE)1090-0241(2004)130:8(851))
- 1088 Ismael, N. F. (2001). Axial load tests on bored piles and pile groups in cemented sands. *Journal of geotechnical and*  
1089 *geoenvironmental engineering*, 127(9), 766-773.
- 1090 Japan Road Association (1990). Specifications for highway bridges. (*Part 4*), 5, 38.
- 1091 Kulhawy, F. H., & Mayne, P. W. (1990). *Manual on estimating soil properties for foundation design* (No. EPRI-EL-  
1092 6800). Electric Power Research Inst., Palo Alto, CA (USA); Cornell Univ., Ithaca, NY (USA). Geotechnical  
1093 Engineering Group.
- 1094 Kumar, R., Bhargava, K., & Choudhury, D. (2016). Estimation of engineering properties of soils from field SPT using  
1095 random number generation. *INAE Letters*, 1, 77-84.
- 1096 Kishida, H., & Uesugi, M. (1987). Tests of the interface between sand and steel in the simple shear  
1097 apparatus. *Géotechnique*, 37(1), 45-52. <https://doi.org/10.1680/geot.1987.37.1.45>
- 1098 Kim, D., Bica, A. V., Salgado, R., Prezzi, M., & Lee, W. (2009). Load testing of a closed-ended pipe pile driven in  
1099 multilayered soil. *Journal of geotechnical and geoenvironmental engineering*, 135(4), 463-473.

- 1100 Kitiyodom, P., Matsumoto, T., & Kanefusa, N. (2004). Influence of reaction piles on the behaviour of a test pile in  
1101 static load testing. *Canadian geotechnical journal*, 41(3), 408-420.
- 1102 Krasinowski, A., & Wiszniewski, M. (2017). Static load test on instrumented pile-field data and numerical  
1103 simulations. *Studia Geotechnica et Mechanica*, 39(3), 17-25.
- 1104 Lee, J., Salgado, R., & Paik, K. (2003). Estimation of load capacity of pipe piles in sand based on cone penetration  
1105 test results. *Journal of geotechnical and geoenvironmental engineering*, 129(5), 391-403.  
1106 [https://doi.org/10.1061/\(ASCE\)1090-0241\(2003\)129:6\(391\)](https://doi.org/10.1061/(ASCE)1090-0241(2003)129:6(391))
- 1107 Lehane, B. M., Schneider, J. A., Lim, J. K., & Mortara, G. (2012). Shaft friction from instrumented displacement  
1108 piles in an uncemented calcareous sand. *Journal of geotechnical and geoenvironmental engineering*, 138(11), 1357-  
1109 1368. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000712](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000712)
- 1110 Leong, E. C., & Abuel-Naga, H. (2018). Contribution of osmotic suction to shear strength of unsaturated high  
1111 plasticity silty soil. *Geomechanics for Energy and the Environment*, 15, 65-73.  
1112 <https://doi.org/10.1016/j.gete.2017.11.002>
- 1113 Lee, J. S., & Park, Y. H. (2008). Equivalent pile load-head settlement curve using a bi-directional pile load  
1114 test. *Computers and Geotechnics*, 35(2), 124-133. <https://doi.org/10.1016/j.compgeo.2007.06.008>
- 1115 Lee, J., Salgado, R., & Paik, K. (2003). Estimation of load capacity of pipe piles in sand based on cone penetration  
1116 test results. *Journal of geotechnical and geoenvironmental engineering*, 129(5), 391-403.
- 1117 Long, J. H. (2016). *Static pile load tests on driven piles into Intermediate-Geo Materials* (No. WHRP 0092-12-08).  
1118 University of Illinois at Urbana-Champaign. Dept. of Civil Engineering.
- 1119 Lu, N., & Griffiths, D. V. (2004). Profiles of steady-state suction stress in unsaturated soils. *Journal of Geotechnical  
1120 and Geoenvironmental Engineering*, 130(10), 1063-1076. [https://doi.org/10.1061/\(ASCE\)1090-  
1121 0241\(2004\)130:10\(1063\)](https://doi.org/10.1061/(ASCE)1090-0241(2004)130:10(1063))
- 1122 Lu, Y. C., Liu, L. W., Khoshnevisan, S., Ku, C. S., Juang, C. H., & Xiao, S. H. (2023). A new approach to constructing  
1123 SPT-CPT correlation for sandy soils. *Georisk: Assessment and Management of Risk for Engineered Systems and  
1124 Geohazards*, 17(2), 406-422.
- 1125 Liu, J., Lv, P., Cui, Y., & Liu, J. (2014). Experimental study on direct shear behavior of frozen soil-concrete  
1126 interface. *Cold regions science and technology*, 104, 1-6. <https://doi.org/10.1016/j.coldregions.2014.04.007>
- 1127 Lv, Y., Liu, H., Ding, X., & Kong, G. (2012). Field tests on bearing characteristics of X-section pile composite  
1128 foundation. *Journal of Performance of Constructed Facilities*, 26(2), 180-189.
- 1129 Martinez, A., & Frost, J. D. (2017). The influence of surface roughness form on the strength of sand-structure  
1130 interfaces. *Géotechnique Letters*, 7(1), 104-111. <https://doi.org/10.1680/jgele.16.00169>
- 1131 Maghsoodi, S., Cuisinier, O., & Masrouri, F. (2020). Thermal effects on mechanical behaviour of soil-structure  
1132 interface. *Canadian geotechnical journal*, 57(1), 32-47. <https://doi.org/10.1139/cgj-2018-0583>
- 1133 Mortara, G., Mangiola, A., & Ghionna, V. N. (2007). Cyclic shear stress degradation and post-cyclic behaviour from  
1134 sand-steel interface direct shear tests. *Canadian Geotechnical Journal*, 44(7), 739-752. [https://doi.org/10.1139/t07-  
1135 019](https://doi.org/10.1139/t07-019).

- 1136 Mandolini, A., & Viggiani, C. (1997). Settlement of piled foundations. *Géotechnique*, 47(4), 791-816.
- 1137 Mitchell, J. K., & Soga, K. (2005). *Fundamentals of soil behavior* (Vol. 3, p. USA). New York: John Wiley & Sons.  
1138 ISBN-13: 978-0-471-46302-7
- 1139 Mullins, G., Winters, D., & Dapp, S. (2006). Predicting end bearing capacity of post-grouted drilled shaft in  
1140 cohesionless soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 132(4), 478-487.
- 1141 Mujtaba, H., Farooq, K., Sivakugan, N., & Das, B. M. (2018). Evaluation of relative density and friction angle based  
1142 on SPT-N values. *KSCE Journal of Civil Engineering*, 22, 572-581.
- 1143 Nanda, S., & Patra, N. R. (2014). Theoretical load-transfer curves along piles considering soil nonlinearity. *Journal*  
1144 *of Geotechnical and Geoenvironmental Engineering*, 140(1), 91-101.
- 1145 Nguyen, H. M., Fellenius, B. H., Puppala, A. J., Nguyen, O. C., & Bheemasetti, T. V. (2018). Results of static loading  
1146 tests on single piles and on pile-supported LPG tanks. *Geotechnical and Geological Engineering*, 36, 3811-3822.
- 1147 O'Neill, M. W. (2001). Side resistance in piles and drilled shafts. *Journal of Geotechnical and Geoenvironmental*  
1148 *Engineering*, 127(1), 3-16. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2001\)127:1\(3\)](https://doi.org/10.1061/(ASCE)1090-0241(2001)127:1(3))
- 1149 Uchaipichat, A., & Khalili, N. (2009). Experimental investigation of thermo-hydro-mechanical behaviour of an  
1150 unsaturated silt. *Géotechnique*, 59(4), 339-353. <https://doi.org/10.1680/geot.2009.59.4.339>
- 1151 Uesugi, M., Kishida, H., & Uchikawa, Y. (1990). Friction between dry sand and concrete under monotonic and  
1152 repeated loading. *Soils and Foundations*, 30(1), 115-128. <https://doi.org/10.3208/sandf1972.30.115>
- 1153 Park, D., & Lee, J. (2015). Comparative analysis of various interaction effects for piled rafts in sands using centrifuge  
1154 tests. *Journal of Geotechnical and Geoenvironmental Engineering*, 141(1), 04014082.
- 1155 Paik, K., & Salgado, R. (2003). Determination of bearing capacity of open-ended piles in sand. *Journal of*  
1156 *Geotechnical and Geoenvironmental Engineering*, 129(1), 46-57.
- 1157 Park, S., Roberts, L. A., & Misra, A. (2012). Design methodology for axially loaded auger cast-in-place and drilled  
1158 displacement piles. *Journal of geotechnical and geoenvironmental engineering*, 138(12), 1431-1441.
- 1159 Park, J. (2017). A comparative study on the bearing capacity of dynamic load test and static load test of PHC bored  
1160 Pile. *Journal of the Korean geo-environmental society*, 18(9), 19-31.
- 1161 Peck, R. B., Hanson, W. E., & Thornburn, T. H. (1991). *Foundation engineering*. John Wiley & Sons.
- 1162 Poulos, H. G., & Davids, A. J. (2005). Foundation design for the emirates twin towers, Dubai. *Canadian Geotechnical*  
1163 *Journal*, 42(3), 716-730.
- 1164 Potyondy, J. G. (1961). Skin friction between various soils and construction materials. *Geotechnique*, 11(4), 339-353.  
1165 <https://doi.org/10.1680/geot.1961.11.4.339>
- 1166 Pham, T. A. (2020a). Analysis of geosynthetic-reinforced pile-supported embankment with soil-structure interaction  
1167 models. *Computers and Geotechnics*, 121, 103438. <https://doi.org/10.1016/j.compgeo.2020.103438>
- 1168 Pham, T. A. (2020b). Behaviour of piled embankment with multi-interaction arching model. *Géotechnique*  
1169 *Letters*, 10(4), 582-588. <https://doi.org/10.1680/jgele.20.00084>



- 1170 Pham, T. A. (2020c). Load-deformation of piled embankments considering geosynthetic membrane effect and  
1171 interface friction. *Geosynthetics International*, 27(3), 275-300. <https://doi.org/10.1680/jgein.19.00030>
- 1172 Pham, T. A. (2022a). Design and analysis of geosynthetic-reinforced and floating column-supported  
1173 embankments. *International Journal of Geotechnical Engineering*, 16(10), 1276-1292.  
1174 <https://doi.org/10.1080/19386362.2021.1997209>
- 1175 Pham, T. A. (2022b). Micromechanical-based shear strength equation considering the stress-state effect for  
1176 unsaturated soils. *International Journal of Geomechanics*, 22(9), 06022022.  
1177 [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0002495](https://doi.org/10.1061/(ASCE)GM.1943-5622.0002495)
- 1178 Pham, T. A., & Dias, D. (2021a). Comparison and evaluation of analytical models for the design of geosynthetic-  
1179 reinforced and pile-supported embankments. *Geotextiles and Geomembranes*, 49(3), 528-549.  
1180 <https://doi.org/10.1016/j.geotexmem.2020.11.001>
- 1181 Pham, T. A., & Dias, D. (2021b). 3D numerical study of the performance of geosynthetic-reinforced and pile-  
1182 supported embankments. *Soils and Foundations*, 61(5), 1319-1342. <https://doi.org/10.1016/j.sandf.2021.07.002>
- 1183 Pham, T. A., & Dias, D. (2022). A simplified model for the analysis of piled embankments considering arching and  
1184 subsoil consolidation. *Geotextiles and Geomembranes*, 50(3), 408-431.  
1185 <https://doi.org/10.1016/j.geotexmem.2021.12.003>
- 1186 Pham, T. A., Wijesuriya, K., & Dias, D. (2022). Analytical model for the design of piled embankments considering  
1187 cohesive soils. *Geosynthetics International*, 29(4), 369-388. <https://doi.org/10.1680/jgein.21.00034>
- 1188 Pham, T. A., & Sutman, M. (2022a). An analytical model for predicting the shear strength of unsaturated  
1189 soils. *Proceedings of the institution of civil engineers-geotechnical engineering*, 176(4), 369-387.  
1190 <https://doi.org/10.1680/jgeen.21.00135>
- 1191 Pham, T. A., & Sutman, M. (2022b). Disturbed state concept and non-isothermal shear strength model for unsaturated  
1192 soils. *Bulletin of Engineering Geology and the Environment*, 81(5), 202. [https://doi.org/10.1007/s10064-022-02688-  
1193 x](https://doi.org/10.1007/s10064-022-02688-x)
- 1194 Pham, T. A., & Sutman, M. (2023). Modeling the combined effect of initial density and temperature on the soil–water  
1195 characteristic curve of unsaturated soils. *Acta Geotechnica*, 18(12), 6427-6455. [https://doi.org/10.1007/s11440-023-  
1196 01920-6](https://doi.org/10.1007/s11440-023-01920-6)
- 1197 Pham, T. A., Hashemi, A., Sutman, M., & Medero, G. M. (2023a). Effect of temperature on the soil–water retention  
1198 characteristics in unsaturated soils: Analytical and experimental approaches. *Soils and Foundations*, 63(3), 101301.  
1199 <https://doi.org/10.1016/j.sandf.2023.101301>
- 1200 Pham, T. A., Sutman, M., & Medero, G. M. (2023b). Validation, reliability, and performance of shear strength models  
1201 for unsaturated soils. *Geotechnical and Geological Engineering*, 41(7), 4271-4309. [https://doi.org/10.1007/s10706-  
1202 023-02520-7](https://doi.org/10.1007/s10706-023-02520-7)
- 1203 Pham, T. A., Sutman, M., & Medero, G. M. (2023c). Density-dependent model of soil–water characteristic curves  
1204 and application in predicting unsaturated soil–structure bearing resistance. *International Journal of  
1205 Geomechanics*, 23(4), 04023017. <https://doi.org/10.1061/IJGNAI.GMENG-7504>

- 1206 Pham, T. A., Medero, G. M., & Sutman, M. (2023d). Thermo-hydro-mechanical coupling model of elastic modulus  
1207 characteristic curve for unsaturated soils. *Computers and Geotechnics*, 162, 105704.  
1208 <https://doi.org/10.1016/j.compgeo.2023.105704>
- 1209 Pham, T. A., Tran, Q. A., Villard, P., & Dias, D. (2023e). Numerical analysis of geosynthetic-reinforced and pile-  
1210 supported embankments considering integrated soil-structure interactions. *Geotechnical and Geological Engineering*,  
1211 1-22. <https://doi.org/10.1007/s10706-023-02564-9>
- 1212 Potyondy, J. G. (1961). Skin friction between various soils and construction materials. *Geotechnique*, 11(4), 339-353.  
1213 <https://doi.org/10.1680/geot.1961.11.4.339>
- 1214 Puri, N., Prasad, H. D., & Jain, A. (2018). Prediction of geotechnical parameters using machine learning  
1215 techniques. *Procedia Computer Science*, 125, 509-517.
- 1216 Randolph, M. F., & Wroth, C. P. (1978). Analysis of deformation of vertically loaded piles. *Journal of the*  
1217 *geotechnical engineering division*, 104(12), 1465-1488. <https://doi.org/10.1061/AJGEB6.0000729>
- 1218 Ravera, E., Sutman, M. and Laloui, L., 2021. Cyclic thermomechanical response of fine-grained soil– concrete  
1219 interface for energy piles applications. *Canadian Geotechnical Journal*, 58(8), pp.1216-1230.
- 1220 Rajagopal, C., Solanki, C. H., & Tandel, Y. K. (2012). Comparison of static and dynamic load test of pile. *Electron.*  
1221 *J. Geotech. Eng*, 17, 1905-1914.
- 1222 Reese, L. C. (1978). Design and construction of drilled shafts. *Journal of the Geotechnical Engineering*  
1223 *Division*, 104(1), 95-116. <https://doi.org/10.1061/AJGEB6.0000579>
- 1224 Reese, L. C., and O'Neill, M. W. (1988). *Drilled Shafts: Construction Procedures and Design Methods*. Federal  
1225 Highway Administration, Washington, DC.
- 1226 Reiffsteck, P., van de Graaf, H., & Jacquard, C. (2018). Bearing Capacity and Settlement of Pile Based on Cone  
1227 Loading Test. In *Advances in Analysis and Design of Deep Foundations: Proceedings of the 1st GeoMEast*  
1228 *International Congress and Exhibition, Egypt 2017 on Sustainable Civil Infrastructures 1* (pp. 44-51). Springer  
1229 International Publishing.
- 1230 Robertson, P. K. (2010, May). Soil behaviour type from the CPT: an update. In *2nd International symposium on cone*  
1231 *penetration testing* (Vol. 2, No. 56, p. 8). Huntington Beach: Cone Penetration Testing Organizing Committee.
- 1232 Russo, G. (2013). Experimental investigations and analysis on different pile load testing procedures. *Acta*  
1233 *Geotechnica*, 8, 17-31.
- 1234 Salari, P., Lashkaripour, G., & Ghafouri, M. (2015). Presentation of empirical equations for estimating internal  
1235 friction angle of SP and SC soils in Mashhad, Iran using standard penetration and direct shear tests and comparison  
1236 with previous equations. *International Journal of Geography and Geology*, 4(5), 89.
- 1237 Seed, H. B., & Reese, L. C. (1957). The action of soft clay along friction piles. *Transactions of the American Society*  
1238 *of Civil Engineers*, 122(1), 731-754.
- 1239 Seo, H., Basu, D., Prezzi, M., & Salgado, R. (2009). Load-settlement response of rectangular and circular piles in  
1240 multilayered soil. *Journal of geotechnical and geoenvironmental engineering*, 135(3), 420-430.  
1241 [https://doi.org/10.1061/\(ASCE\)1090-0241\(2009\)135:3\(420\)](https://doi.org/10.1061/(ASCE)1090-0241(2009)135:3(420))

- 1242 Seo, H., Prezzi, M., & Salgado, R. (2013). Instrumented static load test on rock-socketed micropile. *Journal of*  
1243 *Geotechnical and Geoenvironmental Engineering*, 139(12), 2037-2047.
- 1244 Seol, H., Jeong, S., & Cho, S. (2009). Analytical method for load-transfer characteristics of rock-socketed drilled  
1245 shafts. *Journal of geotechnical and geoenvironmental engineering*, 135(6), 778-789.
- 1246 Shioi, Y., & Fukui, J. (2021). Application of N-value to design of foundations in Japan. In *Penetration Testing, volume*  
1247 *1* (pp. 159-164). Routledge.
- 1248 Sharma, H. D., Harris, M. C., Scott, J. D., & McAllister, K. W. (1986). Bearing capacity of bored cast-in-place  
1249 concrete piles on oil sand. *Journal of Geotechnical Engineering*, 112(12), 1101-1116.
- 1250 Shahri, A. A., Malehmir, A., & Juhlin, C. (2015). Soil classification analysis based on piezocone penetration test  
1251 data—a case study from a quick-clay landslide site in southwestern Sweden. *Engineering Geology*, 189, 32-47.
- 1252 Skempton, A. W. (1986). Standard penetration test procedures and the effects in sands of overburden pressure, relative  
1253 density, particle size, ageing and overconsolidation. *Geotechnique*, 36(3), 425-447.
- 1254 Suzuki, Y., Goto, S., Hatanaka, M., & Tokimatsu, K. (1993). Correlation between strengths and penetration  
1255 resistances for gravelly soils. *Soils and Foundations*, 33(1), 92-101.
- 1256 Tang, C., & Phoon, K. K. (2018). Statistics of model factors and consideration in reliability-based design of axially  
1257 loaded helical piles. *Journal of geotechnical and geoenvironmental engineering*, 144(8), 04018050.
- 1258 Tiwari, B., & Al-Adhahd, A. R. (2014). Influence of relative density on static soil–structure frictional resistance of  
1259 dry and saturated sand. *Geotechnical and Geological Engineering*, 32, 411-427. <https://doi.org/10.1007/s10706-013-9723-6>
- 1261 Toufigh, V., Masoud Shirshorshidi, S., & Hosseinali, M. (2017). Experimental investigation and constitutive  
1262 modeling of polymer concrete and sand interface. *International Journal of Geomechanics*, 17(1), 04016043.  
1263 [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000695](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000695)
- 1264 Van Genuchten, M. T. (1980). A closed-form equation for predicting the hydraulic conductivity of unsaturated  
1265 soils. *Soil science society of America journal*, 44(5), 892-  
1266 898. <https://doi.org/10.2136/sssaj1980.03615995004400050002x>
- 1267 Wang, H. L., Zhou, W. H., Yin, Z. Y., & Jie, X. X. (2019). Effect of grain size distribution of sandy soil on shearing  
1268 behaviors at soil–structure interface. *Journal of Materials in Civil Engineering*, 31(10), 04019238.  
1269 [https://doi.org/10.1061/\(ASCE\)MT.1943-5533.0002880](https://doi.org/10.1061/(ASCE)MT.1943-5533.0002880)
- 1270 Waite, C.A., Narkiewicz, S.A., and Jones, W.V. 1980. Offshore vertical and lateral pile tests on composite friction  
1271 piles, Pend Oreille Lake, Idaho. In American Society of Civil Engineers (ASCE), Geotechnical Engineering Division,  
1272 Convention and Exposition, Portland, Oreg., 14–18 April 1980. ASCE, New York
- 1273 Wolff, T. F. (1989, June). Pile capacity prediction using parameter functions. In *Predicted and observed axial*  
1274 *behavior of piles: results of a pile prediction symposium* (pp. 96-106). ASCE.
- 1275 Wu, Z., Xu, J., Li, Y., & Wang, S. (2022). Disturbed state concept–based model for the uniaxial strain-softening  
1276 behavior of fiber-reinforced soil. *International Journal of Geomechanics*, 22(7), 04022092.  
1277 [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0002415](https://doi.org/10.1061/(ASCE)GM.1943-5622.0002415).



- 1278 Xiao, D., Wu, C., & Wu, H. (2018). Experimental study on ultimate capacity of large screw piles in Beijing.  
1279 In *Advances in Analysis and Design of Deep Foundations: Proceedings of the 1st GeoMEast International Congress*  
1280 *and Exhibition, Egypt 2017 on Sustainable Civil Infrastructures 1* (pp. 52-58). Springer International Publishing.
- 1281 Zhang, Q. Q., Zhang, Z. M., Yu, F., & Liu, J. W. (2010). Field performance of long bored piles within piled  
1282 rafts. *Proceedings of the Institution of Civil Engineers-Geotechnical Engineering*, 163(6), 293-305.  
1283 <https://doi.org/10.1680/geng.2010.163.6.293>
- 1284 Zhang, Q. Q., & Zhang, Z. M. (2012). A simplified nonlinear approach for single pile settlement analysis. *Canadian*  
1285 *Geotechnical Journal*, 49(11), 1256-1266. <https://doi.org/10.1139/t11-110>
- 1286 Zhang, G. A., & Zhang, J. M. (2006). Monotonic and cyclic tests of interface between structure and gravelly soil. *Soils*  
1287 *and Foundations*, 46(4), 505-518. <https://doi.org/10.3208/sandf.46.505>
- 1288 Zhu, S., Chen, C., Mao, F., & Cai, H. (2021). Application of disturbed state concept for load-transfer modeling of  
1289 recoverable anchors in layer soils. *Computers and Geotechnics*, 137, 104292.  
1290 <https://doi.org/10.1016/j.compgeo.2021.104292>
- 1291 Zhu, H., & Chang, M. F. (2002). Load transfer curves along bored piles considering modulus degradation. *Journal of*  
1292 *Geotechnical and Geoenvironmental Engineering*, 128(9), 764-77. [https://doi.org/10.1061/\(ASCE\)1090-](https://doi.org/10.1061/(ASCE)1090-0241(2002)128:9(764))  
1293 [0241\(2002\)128:9\(764\)](https://doi.org/10.1061/(ASCE)1090-0241(2002)128:9(764))

