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Sliding into Multiplicative Thinking: The Power of the 'Marvellous Multiplier'

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Multiplicative thinking is a critical stage in mathematical learning and underpins much of the mathematics learned beyond middle primary years. Its components are complex and an inability to understand them conceptually is likely to undermine students' capacity to develop beyond additive thinking. Of particular importance are the ten times relationship between places in the number system and what happens when numbers are multiplied or divided by powers of ten. Evidence from the research project discussed here suggests that many students have a procedural view of these ideas, and that a conceptual understanding needs to be developed. It is suggested that this may be possible through the use of a device called 'The Marvellous Multiplier'.

Background

Multiplicative thinking is rightly considered to be a 'big idea' of mathematics as it underpins important mathematical ideas such as multiplicative partitioning, proportional reasoning, and algebraic generalisations (Hurst & Hurrell, 2014; Siemon, Bleckley & Neal, 2012). It is well documented that students who do not develop into adequate multiplicative thinkers are likely to struggle with mathematics beyond primary school, or even in the later years of it, yet a large number of students leave primary school without that necessary understanding (Clark & Kamii, 1996; Siemon, Breed, Dole, Izard & Virgona, 2006). Part of the problem may be that students develop a procedural view of mathematics in general and multiplicative thinking in particular and perhaps this is, at least in part, due to the way in which mathematics is taught. Thanheiser, Philipp, Fasteen, Strand & Mills (2013) recently interviewed pre-service teachers about their level of conceptual understanding and uncovered a prevalent and rather confronting attitude that knowing procedures for doing mathematics was all that was required. They developed three principles for pre-service teacher knowledge which could well be applied to teachers in general:

> Underlying concepts serve as the foundation for mathematical procedures, knowing the foundations for the procedures has value including knowing why each procedure yields correct answers, and, until they themselves learn to make sense of mathematics, pre-service teachers (PSTs) will be unprepared to support their future students beyond learning procedures. (Thanheiser et al., 2013, p. 138)

The on-going research project on which this paper is based has shown that there are indeed many primary aged children with a procedural view of aspects of multiplicative thinking. To date over 400 students have participated in the study and whilst the great majority of them can identify the commutative and distributive properties of multiplication, the inverse relationship between multiplication and division, and the extension of basic multiplication and division facts by powers of ten, very few can explain conceptually why those properties and relationships work. Most explanations are along the lines of 'swapping the numbers around' (commutative), 'splitting the numbers' (distributive), 'they're the same family' (inverse), and 'adding or taking a zero' (extension). Similarly, many of the

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students know how to quickly give an answer to an exercise like 74x10 or 74x100 and almost all described and explained the process as a result of 'adding a zero'.

Methodology

The research is built on two instruments – a semi-structured diagnostic one-on-one interview and a written version of the interview (Multiplicative Thinking Quiz – MTQ). The latter was developed in order to gather a large amount of data from a comparatively large sample in a short time. Each interview takes approximately 35-40 minutes whereas the MTQ can be administered to a whole class group in the same time. In general, the MTQ is administered and students are identified for later interviewing in order to probe their understanding. This paper reports only on the use of the interview with a group of 16 Year Five students. The particular sample was chosen as the basis for this discussion because the students' responses to the MTQ indicated that they had or were developing a measure of conceptual understanding of multiplicative thinking. The interviews were audio recorded and later transcribed. The theme considered in this paper encompasses the relationships between the concepts of 'times bigger', extended number facts, and multiplying and dividing by powers of ten, as well as the use of the 'Marvellous Multiplier', a sliding strip device designed to enhance the development of understanding of those concepts. This is explored by considering students' responses to a set of questions from the interview and comparing their thinking before and during the use of the Marvellous Multiplier.

The 'Marvellous Multiplier' (M/M) (Figure 1) is a piece of laminated card showing whole number place value columns into the millions. There is a corresponding row of empty columns where numbers can be written. A second laminated strip showing a single digit number is inserted into two slits at the left and right ends of the place value columns. The M/M is operated by sliding the numbered strip to the left or right and a zero can be written to fill the empty place, or removed, if sliding to the right.

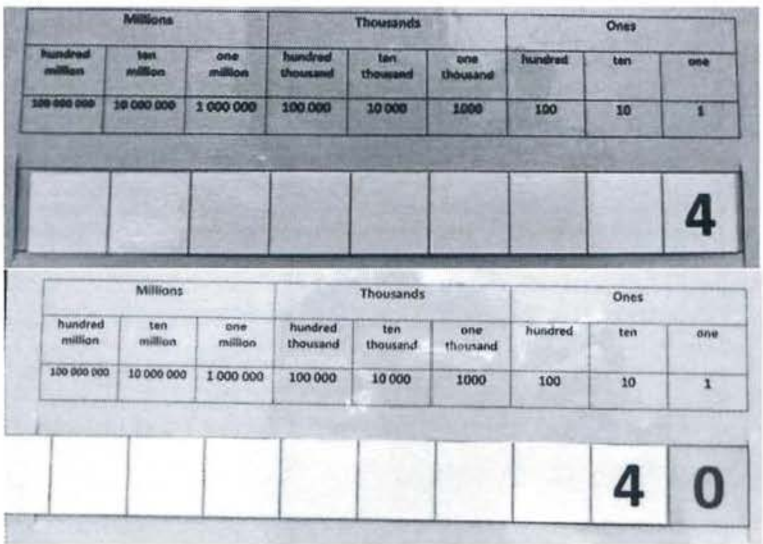

Figure 1: Marvellous Multiplier showing sliding strip at original position and after being slid one place

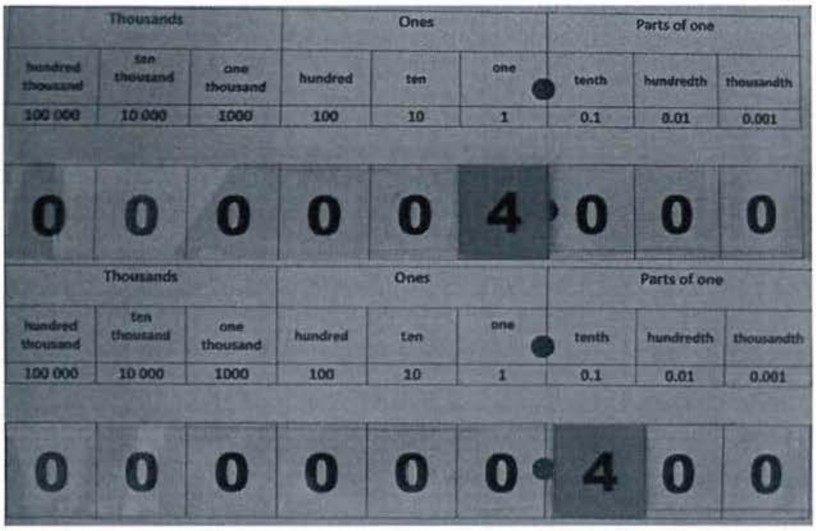

Figure 2: Decimal Marvellous Multiplier showing sliding strip in original and after being slid one place

Figures 1 and 2 shows the M/M with the sliding strip in its initial position, and when the strip had been slid by one place. The purpose of the M/M was to assist students to understand that when numbers are multiplied or divided by a power of ten, all of the digits move one place to the left (for multiplication) or one place to the right (for division) for each power of ten. This equates to a measure of conceptual understanding as opposed to the explanation of 'adding a zero' which is deemed to be procedural in nature. The research team wanted to see if the language used by the students changed when the M/M was introduced and whether or not the students' understanding shifted or was clarified.

Results and Discussion

Theme 1 – 'Times bigger', extended number facts, and powers of ten

The following interview questions were asked in order to generate data about the theme:

- How many times bigger is 40 compared to 4, 400 compared to 40, 4000 compared to 400, and 400 compared to 4 (These questions were asked separately).
- My friend says that if you that 17x6=102, then you must know the answer to 170x6. Is he right? How do you know?
- Write as many other number sentences as you can like 170X6 (with their answers).
- What happens to a number when you multiply by ten, like 74x10? Please explain.
- What happens to a number when you divide it by 10, like 160÷10? Please explain.

The M/M was introduced as and if needed to most students in combination with the fourth and fifth questions, depending on each student's response/s. For some students it was used on several occasions to further probe a point or clarify some point of understanding. Data that were generated from these questions were analysed to see what connections might exist between the embedded ideas and to see whether or not the use of the 'Marvellous

Multiplier' would have any effect on the student's apparent understanding. Table 1 contains a summary of responses to the questions.

Table 1
Responses of the 16 Year Five students to the Theme One questions

Question	Correct response	Partially correct
Correctly identifies 'how many times bigger' is one number than another.	13	2
Explains conceptually how the answer is obtained in extended number facts	4	4
Gives range of extended multiplication facts based on 170X6 or 102÷6	5	7
Gives range of extended division facts based on 170X6 or 102÷6	5	4
Explains conceptually what occurs when a number is multiplied by ten (digits move)	3	4
Explains conceptually what occurs when a number is multiplied by ten ***(Working with Marvellous Multiplier)***	14	2
Explains conceptually what occurs when a number is divided by ten (digits move)	1	8
Explains conceptually what occurs when a number is divided by ten ***(Working with Marvellous Multiplier)***	14	2

It is evident from Table 1 that most students correctly identified the multiplicative relationship between pairs of numbers which seems to indicate an understanding of the notion of 'times bigger'. However, whilst most of them were able to provide a range of extended number facts, only half were able to conceptually explain (some to a limited extent) what happened when extending number facts. This entailed them expressing that multiplying by ten made the number ten times bigger and for whole numbers, a zero was added. Conversely, some explained that 'adding a zero' means making the number 'ten times bigger'. The four students who did explain the situation well did so in a couple of ways. Three of them used the example of 170x6 and the distributive property to show that (17x10) x6 was the same as (17x6) x10. One other student used the same example (170x6) and explained that "It makes the number a different place value into the thousands" (student Craig). However, only three students were able to say initially that digits in a number moved one place value column for each power of ten by which the number was multiplied, and one student could do that for division.

The following excerpts from interview transcripts provide some insight into the extent to which the M/M helped to clarify students' thinking. The first two students had demonstrated some measure of understanding but had had been limited to the type of explanation in the previous paragraph. With students Jacob and Pete, both the whole number and decimals versions of the M/M were introduced several times for the various examples indicated.

INT: You said that you add a zero when you multiply by ten. What happens if I move this number (4) to here (tens column)? What's it worth now?
JACOB: Forty
INT: What do you need to do?
JACOB: Write a zero
INT: So what actually happened to the number (the 4)?
JACOB: It moved up one place
INT: So what if I move it up to there (to the hundred place) What have I done to it?
JACOB: You've timesed it by ten
INT: What do you need to do?
JACOB: Add another zero
INT: So every time you move it one place, what happens?
JACOB: it keeps going up.
INT: If we move it the other way, what happens to it now?
JACOB: It's been divided by ten (spontaneous answer).
INT: (Using the decimal M/M, moved the 4 to the tenths place) What's it worth now?
JACOB: Zero point four
INT: What's happening to the number when you move it?
JACOB: It's getting smaller by ten each time you move it
INT: So what happens to the number when you multiply it by ten each time?
JACOB: The number moves so the zero fills it. (student Jacob)

INT: (Moved the 4 to the tens place) What happens to the four?
PETE: It gets ten times bigger
INT: What happens when I move it across again?
PETE: Ten times bigger . . . 400
INT: What happens when I move it across again?
PETE: Ten times bigger . . . 4000.
INT: What happens if I move it back one place?
PETE: Ten times smaller . . . 400
INT: And another place?
PETE: Ten times smaller . . . 40
INT: Again
PETE: Ten times smaller . . . 4
INT: What happens to the number four?
PETE: It keeps on moving from say the hundred to the thousand. It gets ten times bigger or smaller every time.
INT: (Using the decimal version) What happened when I moved it (to the tenths column)?
PETE: Decimal point stays and the number moves
INT: So what's happened to the four each time it moves to the right?
PETE: You've divided it by ten
INT: Does this help you to understand what's going on?
PETE: Yes. (student Pete)

It seems that the use of the M/M allowed them to articulate that they understood that the situation involved more than 'adding a zero' when multiplying a number by a power of ten. It also assisted Jacob to understand what happened with the example 16÷10. Initially, Jacob said the answer would be 0.16 and was confused when given a calculator to check the answer (1.6). A blank strip was used with the decimal version of the M/M and Jacob showed that when the number 16 was slid one place to the left, it read as 1.6. He immediately stated, "It's been divided by ten", so clarifying his understanding.

Student Daniel's explanation was also initially quite procedural as shown by his explanation of extended number facts (170x6=1020). He said, "Timesing it by ten, you add a zero . . . 170 is ten times bigger than 17 and 1020 is ten times bigger than 102". When using the M/M, he said, "Each time it moves one place to the right, it becomes ten times less". At each stage when the number 4 was moved to the left, he said it was getting ten times bigger or smaller and he would write the zero appropriately. "The four is getting ten times bigger each time, but if you started from the first place, (4) it's getting 100 times bigger. Also, for the example 26÷10, Daniel said, "Dividing it by ten, you put a dot point in there", but when the M/M was used, he said, "The numbers moved a place". Again, the use of the M/M helped Daniel to reason through his initial procedure offering an opportunity to develop a more conceptual understanding.

A similar development seemed to occur with Zeke's thinking. Initially, when working with the examples 74x10 and 3.6x10, he gave correct answers and said, "If you had a hundred times, you would add two zeros because a hundred has two zeros". He also added that "Instead of adding a zero [to the 3.6], you move the decimal point because it is like moving up a place". When working with the M/M, he was quick to say that, at each stage [4 to 40, 40 to 400 etc.) that for each place the four digit moved, it was multiplied by ten. Similarly, with the decimal M/M, he was specifically asked about the decimal point and said, "It doesn't move but the numbers do".

Student Oscar also explained the multiplication by ten in terms of 'adding a zero' [for the example 74x10]. For the example 3.6x10, he said that the point is taken away because "It's no longer a decimal, it's a normal number". However, when the M/M was used, he said that both the 74 and 3.6 "were ten times bigger and moved up one place". He said that the decimal point did not move and when asked what happened when we move the 3.6 by ne place to the left, he said "It goes from the decimals".

Student Lex initially struggled to explain extension of number facts beyond 'adding zeros' and when asked where the zero comes from, he said, "It's equal to a ten". He provided an example of 60X170=10200, and said, "The zeros were on the 60 and the 170". When the M/M was introduced, Lex was able to say that each time the four digit was moved to the left, it became 'bigger by ten' and 'bigger by another ten'. He also said that it 'became ten times smaller' each time it was moved as place to the right. The best example of consolidation of his thinking came with the following comment about 3.6x10 – "The three becomes a bigger number into the tens and the tenths becomes a unit . . . If it were 100 [times] it would be 360 . . . the zero replaces the six there [ones] and the six replaces the three there [tens]".

Of the three students who showed a sound initial understanding of extension and multiplication by powers of ten, Student Christian explained why 170x6=1020 saying that "170 is ten times bigger than 17, so 1020 is ten times bigger than 102 . . . it makes the number a different place value into the thousands". Similarly, for 74x10, he initially said to 'add a zero', but then said "It goes into the hundreds . . . it goes up a place value". The use of the M/M further consolidated his understanding and for 3.6x10, he said "They moved up a place. If it had two sixes [3.66], the answer would be 36.6". He described how the number 4 became ten times bigger or smaller for each place movement to the left or right of the M/M. he also said that "each movement is ten times and two movements would be a hundred times". When asked what happened to the number four when it was moved to the left, he said "A zero comes in". He appeared to be making a connection between his initial procedural explanation based on the zero to a more conceptual level of understanding.

Student Dean struggled to explain the 'times bigger' relationships in the first question and did so in terms of comparing the numbers and taking away zeros. While this would generally work, it is quite procedural in nature, and he had similar difficulty explaining multiplication by ten and also extension of number facts. He initially provided an answer of 1002 for the example 170x6 and checked by using the vertical algorithm to correct his answer. When the M/M was used, Dean was able to say that for each time the four digit was moved a place to the left, it became ten times bigger and was also able to say that moving the digit three places made it a thousand times bigger. As well, he could confidently say that for each move by a place to the right, it became ten times smaller and was being divided by ten. The use of the M/M seemed to enable Dean to display some measure of conceptual understanding that was not evident earlier.

Student George was another who initially struggled to move beyond the idea of 'adding a zero' and was unable to provide answers to the 'times bigger' comparative question. However, the use of the M/M seemed to help him considerably. The following exchange shows how his thinking developed and shows development of conceptual understanding.

INT: What happens when I move it across here (to the tens place)?
GEORGE: It would be worth tens . . . forty
INT: What do you need to do to make it 40?
GEORGE: Put a zero in
INT: How many times bigger is it?
GEORGE: Ten times more
INT: What happens if I move it another place?
GEORGE: It becomes 400. . . ten more times bigger
INT: What if I move it another place? What does it become then? . . .
GEORGE: Four thousands . . . ten times bigger again
INT: What if I move it back one place (moved to the hundreds place). What happens to it?
GEORGE: You've taken ten off it . . . no wait, it's not ten off, it's ten times smaller
INT: What if I move it one more time (moved to the tens place)
GEORGE: It just got ten times smaller.

Conclusions

It needs to be noted that the sixteen students on whose responses this paper is based, were all from the same class. It is also worthy of note that their responses to the original written quiz (MTQ) were considerably better than those from any of the other groups of students to whom the MTQ was administered. Even though they were displaying a more conceptual level of initial understanding, the use of the Marvellous Multiplier at least provided an opportunity for them to articulate and/or consolidate their understanding and in most cases extended and developed that understanding. This is demonstrated by Table 1 from which the following points can be made.

- Although 13 of the students could correctly identify the relationships in the 'times bigger' questions, only four of them could explain conceptually the extension of number facts and multiplication and division by powers of ten. This is interesting as the two ideas are closely related.
- Initially, four students could explain multiplication and division by powers of ten in terms of 'moving the digit' (i.e., conceptually). When the M/M was used, this number rose to 14 respectively.
- Students such as Daniel, Dean, Lex and George who showed an initial partial understanding were able to explain situations more clearly and based on a developing level of conceptual understanding when the M/M was used.

To summarise this, it could be said that the use of the Marvellous Multiplier is beneficial in helping students to understand multiplicative concepts in the following ways:

- If their understanding is strong, the M/M helps to clarify and strengthen it.

- If their understanding is lacking, the M/M helps them to develop their thinking and responses beyond procedures.
- Students may choose to explain situations by a procedural method but when prompted with the use of the M/M, they can explain situations in a more conceptual way. At the very least, the M/M offers a mechanism by which students can articulate their understandings. In short, the M/M enables them to explain better what they understand.

It is possible that the 16 students who were interviewed had been exposed to a more conceptual level of thinking by their teacher as there was a higher proportion of them who exhibited that type of understanding than for any of the other student groups who completed the written quiz. Even so and in general, those students who showed a more conceptual level of understanding, initially provided a procedural explanation to the questions.

The following excerpt from the interview with Student Jeremy gives an insight into the extent to which students' thinking can be articulated. Jeremy initially explained 74x10 as "The easiest way is to add a zero . . . it makes the number bigger by ten". His choice of words ('easiest') is interesting as he seems to be alluding to the fact that he knows it is a 'short cut'. He was asked about the example 3.6x10.

> JEREMY: "3.6X10 . . . I can't add a zero on this because that just makes the fraction longer. It doesn't make it any different . . . I just move the six one space up past the decimal point to make it 36 . . . I moved the six onto the other side of the decimal point to make it 36.
> INT: How do you know?
> JEREMY: Timesing by ten is basically like having a number line where you have the thousands, hundreds, tens and ones and then into the fractions. When you times it by ten, you just move all of the numbers up one space on that number line".

It seems that the Marvellous Multiplier can be a useful tool for helping students understand the multiplicative concepts that underpin procedures they might use. As Thanheiser et al. (2013) alluded, it is important that pre-service teachers (and teachers, by inference) are prepared to think and teach from a conceptual standpoint, rather than a procedural one. Students seem to have the potential for thinking that way and it will be interesting to explore the use of the Marvellous Multiplier with a larger cohort of students.

References

Clarke, F. & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1- 5. Journal for Research in Mathematics Education, 27(1), 41-51.

Hurst, C., & Hurrell, D. (2014). Developing the big ideas of number. International Journal of Educational Studies in Mathematics, 1(1), 1-17.

Siemon, D., Breed, M., Dole, S., Izard, J., & Virgona, J. (2006). *Scaffolding Numeracy in the Middle Years – Project Findings, Materials, and Resources*, Final Report submitted to Victorian Department of Education and Training and the Tasmanian Department of Education, Retrieved from http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/snmy.ppt

Thanheiser, E., Philipp, R.A., Fasteen, J., Strand, K., & Mills, B. (2013). Preservice-Teacher Interviews: A Tool for Motivating Mathematics Learning. *Mathematics Teacher Educator(1)*2, 137-147.