Automorphisms of Finite Cyclic 3-Groups

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Abstract. A distinct compatible pair of actions will result in distinct nonabelian tensor products. This proves that the number of compatible pairs of action plays a huge role as the greatest variety nonabelian tensor product relies on it. However, the actions of finite cyclic group are defined by automorphisms, thus the number of automorphisms needs to be known before obtaining the number of actions that are compatible with one another. This paper focuses on determining the automorphism finite cyclic 3-groups. The general presentation of the automorphism of such groups is obtained with the help of Groups, Algorithm, and Programming (GAP) Software.

Keywords: *Automorphisms; Cyclic Group; Tensor Product.*

INTRODUCTION

The origin of the nonabelian tensor product was found in the connection of the generalized Van Kampen Theorem by Brown and Loday [1]. A compatible action is crucial before calculating the nonabelian tensor product. According to Brown et. al [2], one of the ways for the actions of finite cyclic group becomes compatible is the actions defined by automorphisms. In this research, the main focus is the cyclic group. *G* is known as cyclic group if and only if it can be generated by element, known as the group generator. Meanwhile, an element, *a* is referred as generator for a group namely *G* if there is an element *a* in *G* where $G = \left\{ a^n \middle| n \in \mathbb{Z} \right\}$. According to Dummit and Foote [3], the automorphism of such group can be illustrated as the direct products, where $\mathrm{Aut}(C_{p^\alpha}) \cong C_{p-1} \times C_{p^{\alpha-1}} \cong C_{(p-1)p^{\alpha-1}}$ where *p* is an odd prime and $\alpha \in \mathbb{N}$. Additionally, they demonstrated that the direct product of two finite cyclic groups and the automorphism of the *p*-power order are isomorphic, which means the generator of finite cyclic group must be found first in the interest of finding the order of automorphisms that have *p*-power order. Emery [4] provided a summary of the automorphism groups of finite groups of small order of cyclic groups. Mohammad [5] determined the generator that give the order of automorphism of *p*-power order isomorphic to the group $C_{p^{\alpha-1}}$ and $\alpha \in \mathbb{N}$. Additionally, he characterized all automorphisms of finite cyclic 2-groups. Shahoodh et. al [6] presented the number of the automorphisms with *p*-power order for any finite cyclic group of the *p*-power order, $C_{p^{\alpha-1}}$. Shahoodh et. al [7] found the compatible pair of nontrivial actions for finite cyclic 3-groups. Although, the compatible pair findings were established based on the order of automorphisms of cyclic group, the automorphisms for such group other than *p*-power order have not been covered yet. For this study, the automorphisms of finite cyclic 3-groups is identified using the Groups, Algorithm and Programming (GAP) software [8]. There are four sections in this paper. The introduction is the first portion, and several definitions used

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