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Nash Equilibrium Selection Using a Hybrid Two-Player Static Game with Trade-off Ranking Method

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Abstract

In the context of the game theory solution idea, the study seeks to suggest the ranking of an optimal solution when there are several Nash equilibria. The integration of the MCDM approach and game theory is a common strategy for addressing real-world challenges. This study introduces a novel hybrid method, combining a non-cooperative static game from game theory with the trade-off ranking (TOR) method from MCDM. The proposed hybrid method is used to rank multiple Nash equilibria concerning some criteria. The methodology for both static game and TOR method are explained in the paper. The game theory model used is a two-player non-constant-sum static game. The proposed methodology is tested using international cooperation in Iran. The result suggests the ranking of the combined strategies using the proposed method.

Key Words: Game Theory; Multicriteria Decision Making.

Mathematical Subject Classification: 91A05, 91A10

1. Introduction

In real-life situations, some of the solutions to a problem may be practically involving more than one option or a suggestion of several choices. However, a decision-maker (DM) needs to choose only one solution. Such a situation also occurs in a game where the players involved are served with some equilibrium solutions. The conflicting solution in choosing any equilibrium solution can be solved using MCDM.

There are many literatures combining the game theory and MCDM to aid the players or DMs in choosing an equilibrium solution or to rank the solutions concerning some criteria. Nikkhah et. al. (2019) merged the two-person zero-sum game, the technique for order of preference by similarity to ideal solution (TOPSIS) and the analytic hierarchy process (AHP) methods for ranking the tunnel risk management in Mashhad Urban Railway Line 3. Ghannadpour and Zandiyeh (2020) integrated a two-player zero-sum game with the simple additive weighting (SAW) approach. They calculated the risk between the cash transporter and the robber using game theory, and they forecast the robber's success rate using SAW.

The gap of study is filled in this paper by integrating the game theory technique with one of the MCDM method called TOR (Jaini & Utyuzhnikov, 2017, 2018) to rank the best Nash Equilibrium (NE) among several Nash equilibria with respective criteria. A static game of two-player non-constant-sum model in game theory is used. The model is in a normal-form game, represented in a matrix form. This model consists of two conflicting players with several strategies to achieve payoff. The meaning of the non-constant-sum game is a game where the sum of the winning and losing values is not equal to any constant value or can be said as a win-win situation. Both players may win and lose something together.

The ranking of an optimal solution when there exists more than one NE in the game theory solution concept is suggested. A NE consists of strategies that are all best responses to each player whereas none of the players can deviate from a Nash point to get a better result. The problem highlighted in the paper is when the NE is more than one. However, the DMs need to choose only one NE among all the strategy combinations obtained. Therefore, the way to choose is based on their preferences. The Nash equilibria obtained from the game theory solution concept should be chosen based on certain criteria. Thus, the Nash equilibria obtained will be ranked to get the best NE with priority. The Nash equilibria are used as alternatives in MCDM framework.

The Nash equilibria solutions are unable to indicate the chosen one, therefore the TOR method can identify the solution with the fewest compromises and be able to comprehend the DMs' preferences in a setting with competing criteria. Therefore, the preference of the players can be conducted with the help of the TOR method concerning certain criteria. The remainder of this manuscript is structured as follows. The background of the game theory and MCDM is highlighted in the second section. Game theory and MCDM are explained in the two subsequent subsections, respectively. In the third section, static game and trade-off ranking hybridization methodology are proposed. In the fourth section, the numerical example of international cooperation in Iran is provided. Lastly, the end section concludes the paper overall.

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2. Multi-Criteria Decision Making and Game Theory

The study of player interactions is done using game theory (Osborne, 2004). The economists are the ones who are applying it the most. In a situation of strategic interaction, a player acts in the best possible way given what their rivals are likely to do.

Players, strategies, and payoffs are the three fundamental elements in game theory. Common knowledge, the rules of play, and the information set are a few more extra factors that might be present. In general, it is believed that all participants will behave rationally toward one another, who maximizes his interests in a game. Thus, the game theory does not apply to the irrational players. A player counteracts his opponents in the order of play either concurrently or sequentially. The player's knowledge of the opponents' prior actions is being prepared by the information set. A bimatrix game as stated in Definition 1 is used.

Definition 1 (Peters, 2015)**:** (Bimatrix Game) A bimatrix game is a pair of $n \times m$ matrices (A, B) where n is the number of rows and m is the number of columns.

Definition 2 (Tadelis, 2013)**:** A player needs to solve a decision problem with a payoff function $v(.)$ over strategies, which is rational if he chooses a strategy $s \in S$ that maximizes his payoff. That is, $s^* \in S$ is chosen if and only if $v(s^*) \geq v(s)$ for all $s \in S$.

Despite the fact that the study of game theory has historically focused on mathematical models of the tactical interactions between rational players, it still places a significant emphasis on the choices made by decision-makers who are conscious of how every action affects the players by deviating from a stable point. In real-life difficult scenarios, players also appear to be more irrational in their decision-making due to pressure or a lack of options.

NE is a solution concept in game theory. NE is the equilibrium where both players' strategies are optimal, every player is conscious of their stable state strategy, and none of the players will unilaterally change their approach in an attempt to increase or decrease their profit (see Definition 2). Each player chooses the optimal reaction to the strategies of the other players from a profile of available strategies (Tadelis, 2013). NE is a condition where the expected payoff cannot be obtained if there is one of the players choose to deviate from selecting the NE (Barron, 2013) and also the optimal condition cannot be achieved by all players to apply all available strategies (Hausken, 2020). The players tend to exclude any available strategy that brings fewer costly strategies even though they prefer all of them.

Moreover, the NE solution concept does not provide enough information on whether the solution is good or bad when there is more than one solution. Furthermore, some Nash equilibria can be earned in one game that will likely use lower payoffs (Leoneti & Prataviera, 2020). Besides, if a player is not using his NE, then the opponents may choose other payoff value that increases their payoff independently by using other strategies. This counteracts the rationality of others being irrational. Kim (1996) and Matsui and Matsuyama (1995) have examined the equilibrium selection problem through theoretical approaches. However, which one of the equilibria should be chosen is still not found (Mailath, 1998) and also it should have an additional criterion to select the specific one since it is not a unique value (Anthropelos & Boonen, 2020).

Occasionally, individuals deviate from the logical framework of rational choice. Myerson (1999) underscores the prevalence of experimental studies actively seeking instances of behavior that challenge the principle of rationality, acknowledging its limitations in interpreting real human behavior. This perspective gains support from the insights of Gilboa and Matsui (1991), who emphasized the absence of compelling reasons to constrain players to select equilibrium solutions exclusively. Rather than adhering strictly to equilibrium choices, people exhibit behavioral patterns that oscillate around Nash equilibria. The inclination to follow these fluctuating behaviors suggests that mixed strategies cannot be defended as stationary points within dynamic processes. Building on this, Rashme et al. (2018) contend that during a crisis, the rationality of choosing Nash equilibrium solutions becomes uncertain and may shift towards compromise solutions. Definition 3 subsequently elucidates the concept of pure strategies for players.

Definition 3 (Tadelis, 2013)**:** A pure strategy for a player i is a deterministic plan of strategy. The set of all pure strategies for the player i is denoted S_i. A profile of pure strategies $s = \{s_1, s_2, \dots, s_k\}, s_i \in S_i$ for all $i = 1,2, \dots, k$ describe a particular combination of pure strategies chosen by all k players in the game.

The pure NE solution concept is focused since all the Nash equilibria's probability to each strategy is one. A pure strategy s of player A happens if there is a row i with probability 1. Similarly, a pure strategy s of player B happens if there is a column j with probability 1. So that all the solutions are importantly equal. Each player with pure strategy has the best response, s^* which is called a saddle point (or equilibrium point). All players have the best response to get the value of the game (see Definition 4).

Definition 4 (Gibbons, 1992)**:** In the k-player normal form game $G = \{s_1, s_2, \dots, s_k; v_1, v_2, \dots, v_k\}$, the strategies $(s_1^*, s_2^*, \dots, s_k^*)$ are Nash equilibria, for each player i, s_i^* is player i's best response to the strategies specified for the $k-1$ other players, $(s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_k^*)$: $v_i(s_1^*, \dots, s_{i-1}^*, s_i^*, s_{i+1}^*, \dots, s_k^*) > v_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_k^*)$ for every feasible strategy $s_i \in S_i$; that is, s_i^* solves $\max_{s_i \in S_i} v_i(s_1^*, \dots, s_{i-1}^*, s_i^*, s_{i+1}^*, \dots, s_k^*)$.

Theorem 1 states that the NE must exist at least one in the normal-form game includes the mixed strategies. Since this paper only using pure NE, therefore the problem tested must have at least two pure Nash equilibria to fulfill the criteria of this proposed methodology.

Theorem 1 (Nash, 1950)**:** In the n-player normal-form game $G = \{s_1, s_2, \dots, s_k; v_1, v_2, \dots, v_k\}$, if n is finite and s_i is finite for every i then there exists at least one Nash equilibrium, possibly involving mixed strategies.

In contrast, MCDM uses a variety of criteria and options when reaching a decision. In everyday life, people must choose between several options that satisfy competing requirements and find a solution to multiple difficulties at once.

The advancement technology in recent years has shown the merging of MCDM with various methods for the lack of specific methods. The first step before merging is to examine new methods by taking into account the advantages and eliminating the disadvantages (Velasquez & Hester, 2013). Jaini and Utyuzhnikov (2017) developed a TOR method and later expanded the method to a fuzzy environment (Jaini & Utyuzhnikov, 2018). The study suggested the TOR method, which focuses on the criteria that conflict among the alternatives and uses fuzzy sets. The method has been compared with the other two MCDM methods which are TOPSIS and Viekriterijumsko Kompromisno Rangiranje (VIKOR). The TOR method has been tested with several MCDM problems including car selection problem (Jaini & Utyuzhnikov, 2017), personnel selection problem (Jaini & Utyuzhnikov, 2018) and the vehicle routing problem (Jaini et al., 2021). The method is capable of solving problems with the least amount of compromise and understanding the preferences of the decision-maker in conflicting MCDM problems.

Since TOR method is able to give an optimal solution with the least amount of compromise, hence, the TOR method will be used to rank the Nash equilibria. Therefore, this study uses a hybrid methodology between the static game in the non-cooperative game and the TOR method in MCDM. The following section explains the proposed methodology in hybridizing the static game and TOR method.

3. Hybridization Methodology: Static Game and Trade-Off Ranking

Game theory's weakness is that it does not cater to the wants or preferences of the players. Therefore, to address the issue, more research tend to combine game theory with MCDM (Nikkhah et al., 2019). The benefit of combining the two approaches is that the best option is ultimately chosen depending on how well each individual solution performs relative to the others (Mishra et al., 2018; Mishra & Rani, 2019). A DM's choice of options in relation to certain criteria is frequently based on their preference. For the decision-making process to produce more realistic and consistent criteria weights, a mix approach of this kind is required (Mishra et al., 2020).

Furthermore, the hybrid MCDM framework has also been applied more frequently over the past ten years to different decision-making techniques (Abu-taieh et al., 2020; Phochanikorn & Tan, 2019), or within the operations research field. It can manage the DMs degree of trust in the outcomes while implementing a hybrid MCDM with difficult and complex challenges.

The static game of two-player non-constant-sum model is a condition where all players will have their payoff matrix individually. It is easier to show that both players' payoff is not based on the gain and loss of any player in the game by referring to Eq.1. Suppose that the payoff matrices for players A and B are as follows:

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}, B=\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{bmatrix} \qquad 1$$

The representation of the normal-form game in this paper will combine these two payoff matrices into one as follows:

$$\begin{array}{cc} & B \\ A & \boxed{\begin{matrix} (a_{11},b_{11}) & (a_{12},b_{12}) & \dots & (a_{1m},b_{1m}) \\ (a_{21},b_{21}) & (a_{22},b_{22}) & \dots & (a_{2m},b_{2m}) \\ \vdots & \vdots & \vdots & \vdots \\ (a_{n1},b_{n1}) & (a_{n2},b_{n2}) & \dots & (a_{nm},b_{nm}) \end{matrix}} \end{array} \qquad 2$$

Generally, if player A plays row i and player B plays column j, then the pair of numbers is represented as $f_{ij} = (a_{ij}, b_{ij})$, where the first element corresponds to the payoff of player A and the second element represents the payoff of player B. For instance, if player A plays row two and player B plays column one, then the payoff to players A and B are a_{21} and b_{21} respectively.

The steps to find pure Nash equilibria in a bimatrix game are as follows (Peters, 2015):

1. Determine and indicate by marking the pure best replies of player B to every pure strategy of player A.
2. Determine and indicate by marking the pure best replies of player A to every pure strategy of player B.
3. Determine the NE by looking at the pairs of pure strategies that have mutual best replies. In this research, Gambit software is used for this step.

A game can be solved using the Nash equilibrium solution concept with the assumption the game has more than one NE. There exists a set Nash equilibrium, $NE^{\alpha} = (NE^1, NE^2, \dots, NE^a)$ with k Nash equilibria. The selection process of the Nash equilibria is based on the Euclidean distance of each NE point. The calculation is presented using the TOR method (Jaini & Utyuzhnikov, 2017). The TOR method, at its core, revolves around the strategic selection of a solution that minimizes compromises with other alternatives. To enhance clarity, the general steps for employing the TOR method are outlined below:

1. Identify NE Points:
    - Begin by identifying the NE points within the strategic landscape.
2. Select a Reference NE Point:

- Choose a reference NE point from which the trade-off calculations will be conducted.

3. Calculate Distances to Other NE Points:
   - For each identified NE point, calculate the distance from the chosen reference point. This distance serves as a quantitative measure of the trade-off between the solutions.
4. Minimize Trade-Off Distance:
   - The solution with the least distance to other NE points represents a strategic choice that minimizes compromises.

By following these steps, the TOR method provides a systematic approach to decision-making, emphasizing the strategic selection of a solution that optimally balances trade-offs. The general steps using the TOR method to calculate the distance between points (Nash equilibria as alternatives) are as follows:

1. Determination of the extreme solutions, *ES*, from the normalized decision matrix using the formula:

$$ES_k^* = \left\{\min_{1\le j\le q} f_{ij}\right\}, j = 1,2,\dots,c, \text{ for the cost criteria, or}$$
$$ES_k^* = \left\{\max_{1\le j\le q} f_{ij}\right\}, j = 1,2,\dots,c, \text{ for the benefit criteria} \qquad 3$$

2. Calculation of d_{TOR1} and $DT1$.
   - Evaluate the distance between an alternative, *NE* to an extreme solution ES_k^* denoted as $d_{TOR}(ES_k^*, NE^\alpha)$ using the Eq.4 as follows:

$$d_{TOR1}(ES_k^*, NE^\alpha) = \left[\sum_{j=1}^{c}\left(f_{kj}, f_{\alpha j}\right)^2\right]^{1/2}, \alpha = 1,2,\dots,a;\ k = 1,2,\dots,c. \qquad 4$$

   - Evaluate the trade-off degree, *DT* between all extreme solutions with an alternative using the formula as follows:

$$DT1_{NE^\alpha} = \sum_{j=1}^{c}\left[w_j \times d_{TOR1}(ES_k^*, NE^\alpha)\right], \alpha = 1,2,\dots,a; k = 1,2,\dots,c. \qquad 5$$

3. If the degree of trade-off, *DT1*, is the same for some of the alternative, evaluate further for d_{TOR2} and $DT2$.
   - Evaluate the distance between the alternatives denoted as $d_{TOR2}\left(NE^\alpha, NE^\beta\right)$ using the formula

$$d_{TOR2}\left(NE^\alpha, NE^\beta\right) = \left[\sum_{j=1}^{c}\left(\overline{P_{\alpha J}} - \overline{P_{\beta J}}\right)^2\right]^{1/2}, \alpha, \beta = 1,2,\dots,a, \qquad 6$$

where the weighted performance of an alternative *i* in criterion *j* is given as
$\overline{P_{\iota J}} = w_j \times f_{ij}, i = 1,2,\dots,a; j = 1,2,\dots,c$

   - Evaluate the trade-off degree, *DT* between the alternatives using the formula

$$DT2_{NE^\alpha} = \sum_{i=1}^{a}\left[d_{TOR2}(NE^\alpha, NE^i)\right], \alpha = 1,2,..,a. \qquad 7$$

4. Rank the lowest value of *DT1* as the best alternative. If the values of $DT1$ is the same, then rank the lowest value of *DT2* as the best alternative. Thus, calculation of the second level of TOR (step 3) is proceeded if and only if a similar degree of trade-off is obtained in the first level of TOR (step 2).

The sample calculation for Eq.3 to Eq.7 can be reviewed in Jaini and Utyuzhnikov (Jaini & Utyuzhnikov, 2017). By using the TOR method, the solution for the Nash equilibrium is determined by the *DT* value concerning the others. The highest NE position holds the lowest *DT* value.

The flowchart of the proposed methodology is shown in Fig.1. The two-player non-constant-sum static game model is introduced. The game is solved using the Nash equilibrium solution concept where all the Nash equilibria are adopted as alternatives in the MCDM framework. Next, the TOR method is used to rank the alternatives (Nash equilibria) in choosing the best one. The proposed method is tested using international cooperation in Iran (Hashemkhani Zolfani et al., 2015).

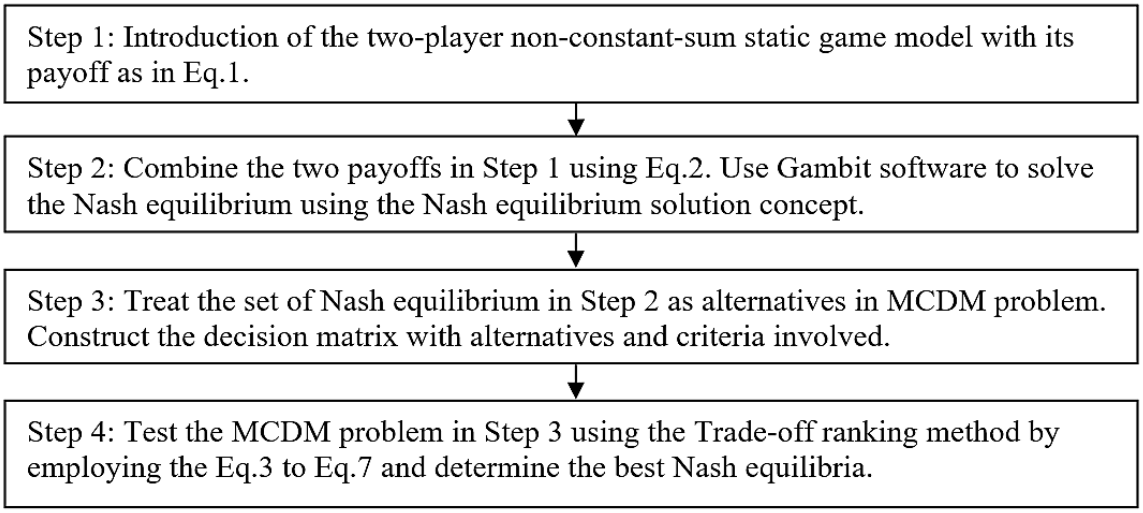

Figure 1: The flowchart of the proposed methodology.

4. Case Study: International Cooperation

Consider a numerical example from (Hashemkhani Zolfani et al., 2015) and literature from (Beske et al., 2014) that proposed a multi-criteria game in supply chain management. There are two players A and B to evaluate the suppliers which are Normal Situation (A) and Dynamic Collaboration (B). For each player, there are five strategies involved. The strategies for players A and B are shown in Table 1.

Table 1: Strategies of Players A and B.

Player	Strategy
Normal Situation (A)	Orientation (S_{A1})
	Supply chain continuity (S_{A2})
	Collaboration level (S_{A3})
	Risk management (S_{A4})
	Pro-activity (S_{A5})
Dynamic Collaboration (B)	Knowledge assessment (S_{B1})
	Partner development (S_{B2})
	Supply chain re-conceptualization (S_{B3})
	Co-evolving (S_{B4})
	Reflexive control (S_{B5})

The payoff matrix of this game is constructed using Eq.2 and shown in Table 2. The suppliers' evaluation with both normal and dynamic conditions to obtain the best combination strategies is the main reason for this model (Hashemkhani Zolfani et al., 2015). The payoff values shown in the payoff matrix is for player A only since both players have the same payoff values, $a_{ij} = b_{ij}$.

Table 2: The payoff matrix game.

S_A/S_B	(S_{B1})	(S_{B2})	(S_{B3})	(S_{B4})	(S_{B5})
(S_{A1})	6.4	6.5	6.8	6.8	6.7
(S_{A2})	7.5	7.4	7.5	7.7	7.5
(S_{A3})	7.4	7.6	7.6	7.8	7.7
(S_{A4})	7.6	7.5	7.8	7.8	7.6
(S_{A5})	6.5	6.6	6.7	6.7	6.5

The payoff matrix in Table 2 is also obtained using the Gambit software (see Fig.2). By using the Gambit software, the Nash equilibria obtained from this game are $S_{A4} - S_{B3}$, $S_{A4} - S_{B4}$ and $S_{A3} - S_{B4}$ with the payoff value of 7.8 (the highest existing value). Refer to Fig. 2, the extensive notations for pure strategies are $\{(0,0,0,1,0), (0,0,1,0,0)\}$, $\{(0,0,0,1,0), (0,0,0,1,0)\}$ and $\{(0,0,1,0,0), (0,0,0,1,0)\}$.

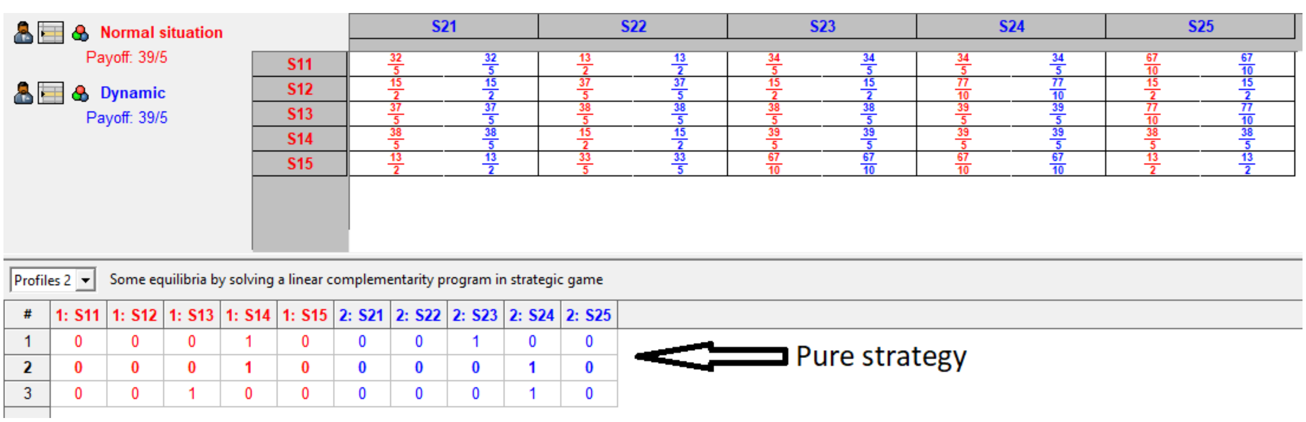

Figure 2: Gambit software result for pure strategy.

Four criteria have been considered in the international cooperation in Iran. The criteria considered are Empowerment Level (C1), Supplying Risk (C2), Strategic Relation (C3) and Future Opportunities (C4). The three Nash equilibria obtained are used as the alternatives. Tables 3 and 4 are obtained from (Hashemkhani Zolfani et al., 2015) before the TOR method steps are calculated.

Table 3: The decision matrix (Hashemkhani Zolfani et al., 2015).

	C1 (Max)	C2 (Min)	C3 (Max)	C4 (Max)
Weight	0.2277	0.3094	0.2674	0.1956
$S_{A3} - S_{B4}$	7.000	6.143	6.857	6.143
$S_{A4} - S_{B3}$	6.143	5.286	6.286	7.000
$S_{A4} - S_{B4}$	6.286	6.571	6.857	6.143

Referring to Table 3, the problem aims to maximize C1, C3 and C4 but minimizes C2. Note that, $S_{A3} - S_{B4}$ and $S_{A4} - S_{B4}$ have the same values for C3 and C4. The $S_{A4} - S_{B3}$ is ranked the best in C2 for minimization and C4 for maximization. Meanwhile, for the other two criteria for maximization, which are C1 and C3, the $S_{A4} - S_{B3}$ is ranked the worst.

Table 4: The normalized decision matrix (Hashemkhani Zolfani et al., 2015).

	C1 (Max)	C2 (Min)	C3 (Max)	C4 (Max)
Weight	0.2277	0.3094	0.2674	0.1956
$S_{A3} - S_{B4}$	1.000	0.935	1.000	0.878
$S_{A4} - S_{B3}$	0.878	0.804	0.917	1.000
$S_{A4} - S_{B4}$	0.878	1.000	1.000	0.878

Referring to Table 4, the TOR method starts with the calculation of the extreme solutions, obtained using Eq.3. At first, only the first level of the TOR method is involved. By using Eq.4, the distance of an alternative to an extreme solution is calculated. Finally, for the first level, the degree of trade-off is calculated using Eq.5. Note that, since there are two same maximum values which are 1 in C3, therefore, the calculation of the extreme solution is calculated for both respective alternatives.

Table 5 shows the ranking of the Nash equilibrium using the TOR method and also the comparison with the ranking from (Hashemkhani Zolfani et al., 2015) that using a method called weighted aggregates sum product assessment (WASPAS). The difference results between these two methods are the weightage that represents the DMs preferences in the TOR method and the relative importance value calculation used in (Hashemkhani Zolfani et al., 2015) where $\gamma = 0.5$. By using the TOR method, the result is obtained from the best ranking from the highest criteria weightage. However, a ranking based on the highest value to the lowest value of final score calculation is obtained by using the WASPAS method in (Hashemkhani Zolfani et al., 2015) with a constant γ value. Therefore, the TOR method is suitable to cater to the problem involving the preferences of all DMs together.

Table 5: The comparison results of evaluating the Nash equilibria between TOR and WASPAS methods.

Nash equilibria	TOR Ranking	WASPAS Ranking
$S_{A3} - S_{B4}$	1	2
$S_{A4} - S_{B3}$	2	1
$S_{A4} - S_{B4}$	3	3

In Table 5, the best supplier cooperation combination strategy by using the TOR method is $S_{A3} - S_{B4}$ (Collaboration level – Co-evolving) while the best supplier cooperation combination strategies by using the WASPAS method is $S_{A4} - S_{B3}$ (Risk management – Supply chain re-conceptualization). Besides that, the second rank supplier cooperation combination strategy by using the TOR method is $S_{A4} - S_{B3}$ (Risk management – Supply chain re-conceptualization) while the second rank supplier cooperation combination strategies by using the WASPAS method is $S_{A3} - S_{B4}$ (Collaboration level – Co-evolving). The worst supplier cooperation combination strategy by using both TOR and WASPAS methods is $S_{A4} - S_{B4}$ (Risk management – Co-evolving).

The TOR method suggests that the collaboration level strategy in a normal situation is important when combining with co-evolving in dynamic collaboration. Meanwhile, the WASPAS method suggests the best strategy from dynamic collaboration to be combined with risk management in a normal situation is supply chain re-conceptualization. However, both methods suggest the same worst rank when the co-evolving strategy in the dynamic collaboration is combined with the risk management strategy in the normal situation. Therefore, to answer the aim of this paper that using the TOR method, the best Nash equilibrium is collaboration level and co-evolving from the normal situation and the dynamic collaboration, respectively. Also, as can be seen from Table 4, the strategy $S_{A3} - S_{B4}$ holds the least trade-off in criteria ranking (it ranks first in C1, ranks second in C2, ranks first in C3, and ranks second in C4), while two other strategies have the worst ranking in some of the criteria. For example, strategy $S_{A4} - S_{B3}$ ranks third in C1, while the strategy $S_{A4} - S_{B4}$ ranks third in C2. Since none of the criteria weights are dominant in this case, the TOR method gives the best solution with the least trade-off among all available choices.

5. Conclusion

In the non-cooperative game, a novel hybrid static game with the TOR approach in MCDM has been proposed. Using the suggested methods, a ranking of Nash equilibria has been obtained using the TOR method in the international cooperation problem in Iran. The game theory part is used to evaluate the Nash equilibria. Due to the uncertainty to play with which NE, the TOR method in the MCDM framework is used, involves some criteria and alternatives. The model used is a two-player non-constant-sum static game. The result suggests the ranking of the supplier cooperation combination strategies using the proposed methodology. The proposed methodology is tested using international cooperation in Iran, then the result has been compared with the WASPAS method, one of the newest methods in MCDM. The extended suggestion for the data is to change the fuzzy system to real data set for more practical problems and the TOR method may be modified to cater to the clash of choosing the extreme solution in the TOR method. In addition, this model may add more players and use other types of game models such as dynamic games in the extensive-form game and several fuzzy number types could be utilised to depict the evaluation of the DMs.

6. Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Besides, the Figures and images, which are not ours, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University Malaysia Pahang Al-Sultan Abdullah.

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