E-ISSN: 2717-3453 | P-ISSN: 2783-1442

Journal of Fuzzy Extension and Applications

J. Fuzzy. Ext. Appl

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 3, No. 2 (2022) 126–139.

Paper Type: Research Paper

Intuitive Multiple Centroid Defuzzification of Intuitionistic Z-Numbers

Nik Muhammad Farhan Hakim Nik Badrul Alam[1], Ku Muhammad Naim Ku Khalif [2,*], Nor Izzati Jaini[3], Ahmad Syafadhli Abu Bakar [4], Lazim Abdullah[5]

[1] Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Pahang, Bandar Tun Abdul Razak Jengka, Pahang, Malaysia; farhanhakim@uitm.edu.my.

[2] Centre for Mathematical Sciences, Universiti Malaysia Pahang, Gambang, Malaysia; kunaim@ump.edu.my.

[3] Centre for Mathematical Sciences, Universiti Malaysia Pahang, Gambang, Malaysia; ati@ump.edu.my.

[4] Mathematics Division, Centre for Foundation Studies in Science, University of Malaya, Kuala Lumpur, Malaysia; ahmadsyafadhli@um.edu.my.

[5] Management Science Research Group, Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, Kuala Nerus, 21030 Kuala Terengganu, Terengganu, Malaysia; lazim_m@umt.edu.my.

Citation:

Hakim Nik Badrul Alam, N. M. F., Ku Khalif, K. M. N., Jaini, N. I., Abu Bakar, A. S., & Abdullah, L. (2022). Intuitive multiple centroid defuzzification of intuitionistic Z-numbers. *Journal of fuzzy extension and applications, 3*(2), 126-139.

Received: 15/11/2021 Reviewed: 13/12/2021 Revised: 12/01/2022 Accepted: 15/01/2022

Abstract

In fuzzy decision-making, incomplete information always leads to uncertain and partially reliable judgements. The emergence of fuzzy set theory helps decision-makers in handling uncertainty and vagueness when making judgements. Intuitionistic Fuzzy Numbers (IFN) measure the degree of uncertainty better than classical fuzzy numbers, while Z-numbers help to highlight the reliability of the judgements. Combining these two fuzzy numbers produces Intuitionistic Z-Numbers (IZN). Both restriction and reliability components are characterized by the membership and non-membership functions, exhibiting a degree of uncertainties that arise due to the lack of information when decision-makers are making preferences. Decision information in the form of IZN needs to be defuzzified during the decision-making process before the final preferences can be determined. This paper proposes an Intuitive Multiple Centroid (IMC) defuzzification of IZN. A novel Multi-Criteria Decision-Making (MCDM) model based on IZN is developed. The proposed MCDM model is implemented in a supplier selection problem for an automobile manufacturing company. An arithmetic averaging operator is used to aggregate the preferences of all decision-makers, and a ranking function based on centroid is used to rank the alternatives. The IZN play the role of representing the uncertainty of decision-makers, which finally determine the ranking of alternatives.

Keywords: Defuzzification, Intuitionistic Z-numbers, Intuitive multiple centroid, Ranking function.

1 | Introduction

Decision-making is a cognitive process in selecting the preferred alternatives by gathering information and making an assessment based on the obtained information. The nature of decision-making always deals with the uncertain and partially reliable judgement due to incomplete and vague information [1]. Vague information is not well-defined [1], and this fact may lead to uncertainty of judgements due to the incompetency of decision-makers, psychological biases of opinions and the complexity of alternatives [2]. In fact, decision-making under uncertainty is a tough task faced by many decision-makers [3].

Corresponding Author: kunaim@ump.edu.my

http://dx.doi.org/10.22105/jfea.2022.315297.1173

The classical decision-making methods used crisp numbers 0 or 1 to describe whether a statement is either false or true, respectively. For instance, the importance of a matter is determined by the linguistic terms "important" and "not important", in which the level of importance between these two linguistic terms could not be exactly measured. The emergence of the fuzzy set theory by Zadeh [4] improved the decision-making methods, in which the truth of a statement is justified by a membership function taking any number in the interval [0,1]. From the previous example, the level of importance of a matter can now be measured by many linguistic terms such as "extremely not important", "not important", "moderately important", "important", and "extremely important". Using the knowledge of the fuzzy set, each of these linguistic terms can be represented by a membership value which takes any number in the interval [0,1]. Hence, the vagueness of the opinion on the importance of such a thing can be catered. Moreover, Bellman and Zadeh [5] implemented the fuzzy set theory in a decision-making application.

Additionally, Zadeh [6] and [7] further extended the concept of the fuzzy set to define a fuzzy subset of the real number line whose maximum membership values are clustered around a mean value. The arithmetic properties of fuzzy numbers were further studied by Dubois and Prade [8]. Some commonly used shapes of fuzzy numbers are triangular and trapezoidal. Fuzzy numbers have a better capability of handling vagueness than the classical fuzzy set. Making use of the concept of fuzzy numbers, Chen and Hwang [9] developed fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) based on trapezoidal fuzzy numbers. Apart from that, Chang [10] replaced crisp values with fuzzy triangular numbers to construct pairwise comparison matrices in fuzzy Analytical Hierarchy Process (AHP). Many other Multi-Criteria Decision-Making (MCDM) methods were developed based on fuzzy numbers. However, the classical fuzzy set has its limitation since it does not consider the non-membership function and the hesitation degree [11].

Instead of considering the membership function alone, Atanassov [12] generalized the fuzzy set into an Intuitionistic Fuzzy Set (IFS) by defining the non-membership function. The membership and non-membership values represent the degrees of belongingness and non-belongingness to the fuzzy set, respectively. The IFS has better flexibility in handling uncertainty as compared to the classical fuzzy set [13]. In fact, the IFS is very powerful when human evaluations are needed to solve problems with incomplete information [14]. The IFS has also become an important tool for researchers [15], and it has been implemented in many real-world applications such as pattern recognition, time series forecasting and MCDM. Among the applications of IFSs in MCDM are intuitionistic fuzzy TOPSIS [16], intuitionistic fuzzy preference relations [17] and triangular intuitionistic fuzzy AHP [18].

Zadeh [19] introduced a new type of fuzzy number called a Z-number. The Z-number consists of two components, namely the restriction and reliability components. The first component denotes the degree of values that a variable can take, while the second component measures the reliability of the first component. For simplicity, both components can be considered as trapezoidal fuzzy numbers [19]. For example, the statement "the book is very thick, very surely" is a form of Z-statement in which the first statement determines the thickness of the book. In contrast, the second component indicates the degree of sureness and certainty, which measures the level of reliability of the first component. The application of Z-number in many fields has highlighted its strength in describing the preferences of decision-makers or experts.

Many MCDM methods have been developed by implementing Z-numbers to describe the decision information. Among the early years of this development, Kang et al. [20] proposed a defuzzification method of Z-numbers based on the fuzzy expectation theory. The proposed method was implemented in a vehicle selection problem [21]. Also, Ku Khalif et al. [22] improved the defuzzification method using an Intuitive Multiple Centroid (IMC) approach. Using this method, the trapezoidal fuzzy numbers are partitioned into three areas, and the sub-centroid of each area is calculated. Then, a conversion method of Z-numbers into regular fuzzy numbers was proposed. The proposed defuzzification method was applied in a staff recruitment problem.

Recently, Sari and Kahraman [23] proposed a combination of the Intuitionistic Fuzzy Number (IFN) and Z-number, namely Intuitionistic fuzzy Z-Number (IZN). In the IZN, both restriction and reliability components are characterized by the membership and non-membership functions. Hence the uncertainty and reliability of the preferences are handled. A defuzzification of IZN was given in [23], mimicking the concept of the accuracy function of IFN. Considering the applicability of IMC in [22], this paper aims to propose a defuzzification method for IZN via the IMC approach. The conversion method of IZN into regular IFN is proposed once the IMC index of the membership and non-membership functions of IZN is obtained. An MCDM problem on supplier selection is adopted to illustrate the proposed defuzzification method. The proposed IMC approach has the advantage of representing the fuzzy number together with the height of the membership function. Therefore, converting IZN into IFN results in easy implementation for the application of the MCDM. In fact, the IMC approach considers both the membership and non-membership functions of IFN. Hence the uncertainty in the decision-making process is handled.

This paper is organized as follows: Section 1 presents the introduction. Section 2 reviews some preliminaries related to IFN, Z-numbers and IZN. The methodology in developing the MCDM method is proposed in Section 3. Section 4 illustrates an MCDM problem, and Section 5 concludes the paper.

2 | Preliminaries

This section reviews some preliminaries related to IFS, Z-numbers and IZN. The IFS is a generalization of the classical fuzzy set, defined as follows [12]:

Definition 1. An IFS is a subset of the universe of discourse, U, written in the form of:

$$I=\{x,\mu_I(x),v_I(x)\,|\,x\in U\}. \quad (1)$$

Where $\mu_I : x \to [0,1]$ and $v_I : x \to [0,1]$ are the membership and non-membership functions of the element x, respectively.

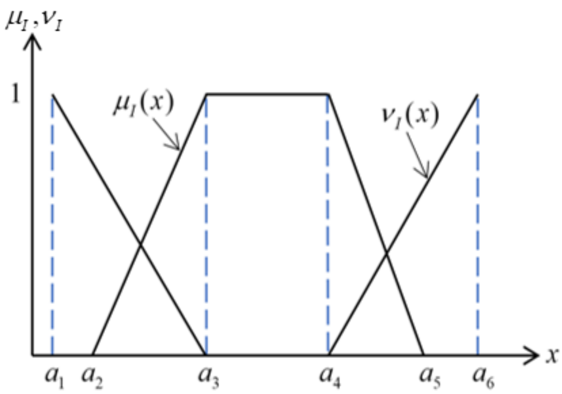

Fig. 1. A trapezoidal IFN.

An extension of the IFS is an IFN. A trapezoidal IFN (TrIFN), $A_I=(a_2,a_3,a_4,a_5;a_1,a_3,a_4,a_6)$ as illustrated in *Fig. 1,* is defined below.

Definition 2. A TrIFN $A_I=(a_2,a_3,a_4,a_5;a_1,a_3,a_4,a_6)$ is characterized by the following membership function.

Hakim Nik Badrul Alam et al.|J. Fuzzy. Ext. Appl. 3(2) (2022) 126-139

$$\mu_{A_I}=\begin{cases}\dfrac{x-a_2}{a_3-a_2}, & x\in[a_2,a_3]\\ 1, & x\in[a_3,a_4]\\ \dfrac{a_5-x}{a_5-a_4}, & x\in[a_4,a_5]\\ 0, & \text{otherwise}\end{cases}. \tag{2}$$

And non-membership function is expressed by

$$\nu_{A_I}=\begin{cases}\dfrac{a_3-x}{a_3-a_1}, & x\in[a_1,a_3]\\ 0, & x\in[a_3,a_4]\\ \dfrac{x-a_4}{a_5-a_4}, & x\in[a_4,a_6]\\ 1, & \text{otherwise.}\end{cases}. \tag{3}$$

Next, the definition of the Z-number is reviewed.

Definition 3. A Z-number, $Z=(A,R)$, consists of two components A and R. A is the fuzzy constraint on the values that a variable can take while R it is a measure of the reliability of A. For simplicity, both components of the Z-number can be represented by trapezoidal fuzzy numbers [19], as shown in *Fig. 2*.

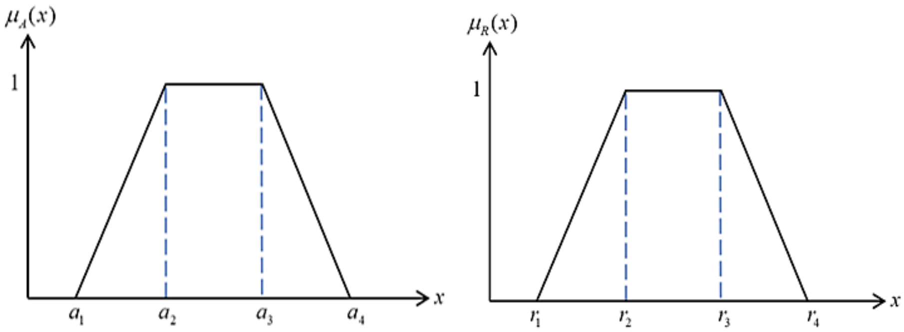

Fig. 2. Representation of a Z-number for trapezoidal fuzzy numbers.

Making use of Z-numbers in solving a MCDM problem, Ku Khalif et al. [22] proposed an IMC defuzzification approach. In their method, the trapezoidal plane is divided into three parts, where the sub-centroid of each area is found. Then, the sub-centroids are connected to form a triangular plane, and the IMC index is calculated. The IMC index of a trapezoidal fuzzy number $A=(a_1,a_2,a_3,a_4;h)$ is given as follows:

$$\mathrm{IMC}(x,y)=\left(\frac{2(a_1+a_4)+7(a_2+a_3)}{18},\frac{7h}{18}\right). \tag{4}$$

Using the obtained IMC, the Z-number $Z=(A,R)$ is converted into a regular fuzzy number using the following three steps:

Step 1. Convert R into a crisp number σ.

$$\sigma = \frac{2\left(r_1 + r_4\right) + 7\left(r_2 + r_3\right)}{18}. \tag{5}$$

Step 2. Add σ into A.

$$Z^{\sigma} = \left\{\left\langle x, \mu_{R^{\sigma}}(x)\right\rangle \mid \mu_{R^{\sigma}}(x) = \sigma\mu_R(x), x \in [0,1]\right\}. \tag{6}$$

Step 3. Convert Z^{σ} into a regular fuzzy number.

$$Z' = \left\{\left\langle x, \mu_{Z'}(x)\right\rangle \mid \mu_{Z'}(x) = \mu_A\left(\sqrt{\sigma}x\right), x \in [0,1]\right\}. \tag{7}$$

Combining the IFN and Z-number, Sari and Kahraman [23] defined the IZN $Z_I = \left(A_I, R_I\right)$, in which the components A_I and R_I are represented by trapezoidal and triangular fuzzy numbers, respectively, as shown in *Fig. 3*

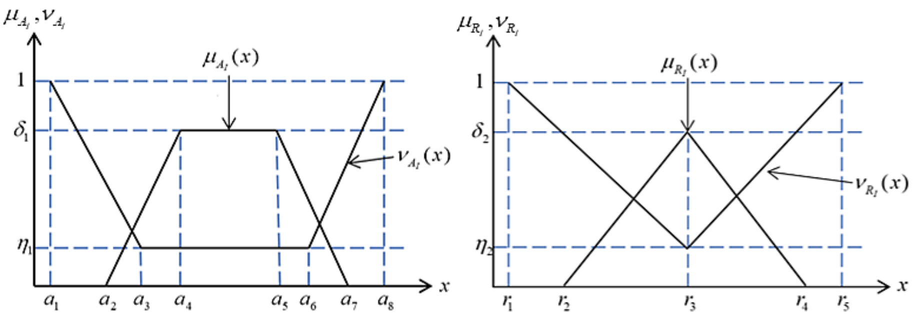

Fig. 3. An Intuitionistic Z-Number (IZN) [14].

In this paper, both components of IZN are represented by trapezoidal fuzzy numbers, following Zadeh's suggestion in [19]. Let $Z_I = \left(A_I, R_I\right)$, where $A_I = \left(a_2, a_3, a_4, a_5, \delta_1; a_1, a_3, a_4, a_6, \eta_1\right)$ and $R_I = \left(r_2, r_3, r_4, r_5, \delta_2; r_1, r_3, r_4, r_6, \eta_2\right)$. Then, A_I and R_I are characterized by the membership and non-membership functions defined in *Eq. (8)* and *Eq. (9)*, respectively.

$$\mu_{A_I} = \begin{cases} \dfrac{x - a_2}{a_3 - a_2}\delta_1 & , x \in \left[a_2, a_3\right] \\ \delta_1 & , x \in \left[a_3, a_4\right] \\ \dfrac{a_5 - x}{a_5 - a_4}\delta_1 & , x \in \left[a_4, a_5\right] \\ 0 & , \text{otherwise} \end{cases}, \quad \nu_{A_I} = \begin{cases} \dfrac{\eta_1 - 1}{a_3 - a_1}x + \dfrac{a_3 - \eta_1 a_1}{a_3 - a_1} & , x \in \left[a_1, a_3\right] \\ \eta_1 & , x \in \left[a_3, a_4\right] \\ \dfrac{1 - \eta_1}{a_6 - a_4}x + \dfrac{\eta_1 a_6 - a_4}{a_6 - a_4} & , x \in \left[a_4, a_6\right] \\ 1 & , \text{otherwise} \end{cases}. \tag{8}$$

$$\mu_{R_I} = \begin{cases} \dfrac{x - r_2}{r_3 - r_2}\delta_2 & , x \in \left[r_2, r_3\right] \\ \delta_2 & , x \in \left[r_3, r_4\right] \\ \dfrac{r_5 - x}{r_5 - r_4}\delta_2 & , x \in \left[r_4, r_5\right] \\ 0 & , \text{otherwise} \end{cases}, \quad \nu_{R_I} = \begin{cases} \dfrac{\eta_2 - 1}{r_3 - r_1}x + \dfrac{r_3 - \eta_2 r_1}{r_3 - r_1} & , x \in \left[r_1, r_3\right] \\ \eta_2 & , x \in \left[r_3, r_4\right] \\ \dfrac{1 - \eta_2}{r_6 - r_4}x + \dfrac{\eta_2 r_6 - r_4}{r_6 - r_4} & , x \in \left[r_4, r_6\right] \\ 1 & , \text{otherwise.} \end{cases}. \tag{9}$$

Hakim Nik Badrul Alam et al.|J. Fuzzy. Ext. Appl. 3(2) (2022) 126-139

3 | Proposed Methodology

This section proposes a methodology used in developing the MCDM model. The methodology consists of three phases, as shown in *Fig. 4*.

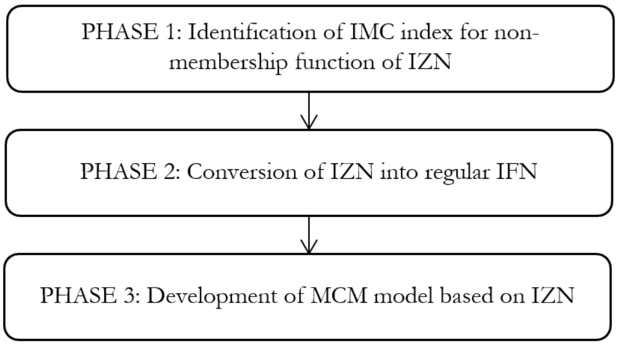

Fig. 4. Proposed methodology.

The IMCs defuzzification of IZN is proposed, considering both the membership and non-membership functions. The IMC defuzzification for the membership function of IZN is given by *Eq. (4)*. Hence, the main contribution of this paper is the development of the IMC defuzzification for the non-membership function of IZN.

3.1 | IMC Index of Non-Membership Function of IZN

Before the conversion method of IZN into a regular fuzzy number could be proposed, the IMC index of the non-membership function should be determined. For this purpose, consider a trapezoidal plane, then the IMC index can be developed using the following steps:

Step 1. Divide the trapezoidal plane into three areas. Hence, the sub-centroid for each area can be calculated using *Eq. (10)* to *(12)*.

$$\alpha(x,y)=\left(r_1+\frac{2}{3}(r_3-r_1),\eta_2+\frac{2}{3}(1-\eta_2)\right). \tag{10}$$

$$\beta(x,y)=\left(\frac{r_3+r_4}{2},\eta_2+\frac{1}{2}(1-\eta_2)\right). \tag{11}$$

$$\gamma(x,y)=\left(r_4+\frac{1}{3}(r_6-r_4),\eta_2+\frac{2}{3}(1-\eta_2)\right). \tag{12}$$

Step 2. Connect the sub-centroids α, β and γ. Hence, a triangular plane is formed, as shown in *Fig. 5*.

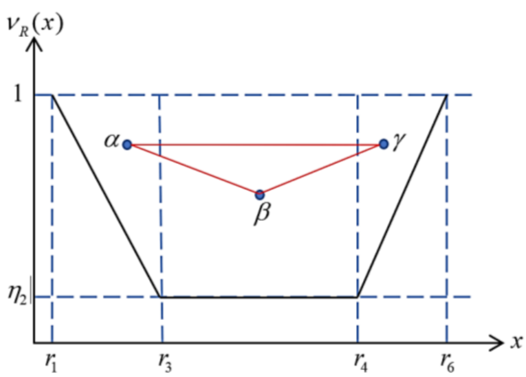

Fig. 5. The triangular plane is formed by connecting the sub-centroids of three areas of non-membership function.

Step 3. The centroid of the triangular plane is calculated by averaging the x- and y-ordinates of the sub-centroids α, β and γ. The IMC index is obtained as follows:

$$\mathrm{IMC}(x,y)=\left(\frac{\alpha(x)+\beta(x)+\gamma(x)}{3},\frac{\alpha(y)+\beta(y)+\gamma(y)}{3}\right)=\left(\frac{2(r_1+r_6)+7(r_3+r_4)}{18},\frac{\eta_2+11}{18}\right). \quad (13)$$

3.2 | Conversion of IZN into Regular IFNs

Using the IMC indices for the membership and non-membership functions as obtained in *Eq. (4)* and *Eq. (13)*, respectively, the reliability components are converted into a crisp number for the purpose of converting the IZN into regular IFN. The conversion of the IZN, $Z_I=(A_I,R_I)$ follows three steps below:

Step 1. Consider *Eq. (4)* and *Eq. (13)*, convert R_I into a crisp number σ.

$$\sigma=\frac{2(r_2+r_5)+7(r_3+r_4)}{18}\times\frac{7\delta_2}{18}+\frac{2(r_1+r_6)+7(r_3+r_4)}{18}\times\frac{\eta_2+11}{18}. \quad (14)$$

Step 2. Add the weight σ to A_I.

$$Z_I^\sigma=\left\{\left\langle x,\mu_{R^\sigma}(x),\nu_{R^\sigma}(x)\right\rangle \mid \mu_{R^\sigma}(x)=\sigma\mu_{R_I}(x),\nu_{R^\sigma}(x)=\sigma\nu_{R_I}(x),x\in[0,1]\right\}. \quad (15)$$

Theorem 1. $E_{A_I^\sigma}(x)=\sigma E_{A_I}(x)$, $x\in X$ subject to $\mu_{A_I^\sigma}(x)=\sigma\mu_{A_I}(x)$ and $\nu_{A_I^\sigma}(x)=\sigma\nu_{A_I}(x)$, $x\in X$.

Proof.

$$\begin{aligned}E_{A_I^\sigma}(x)=&\Bigg(a_2,a_3,a_4,a_5;\frac{2(r_2+r_5)+7(r_3+r_4)}{18}\times\frac{7\delta_2}{18}+\frac{2(r_1+r_6)+7(r_3+r_4)}{18}\times\frac{\eta_2+11}{18};\\ &a_1,a_3,a_4,a_6;\frac{2(r_2+r_5)+7(r_3+r_4)}{18}\times\frac{7\delta_2}{18}+\frac{2(r_1+r_6)+7(r_3+r_4)}{18}\times\frac{\eta_2+11}{18}\Bigg)\\ =&(a_2,a_3,a_4,a_5;\sigma;a_1,a_3,a_4,a_6;\sigma)\\ =&\sigma E_{A_I}(x).\end{aligned}$$

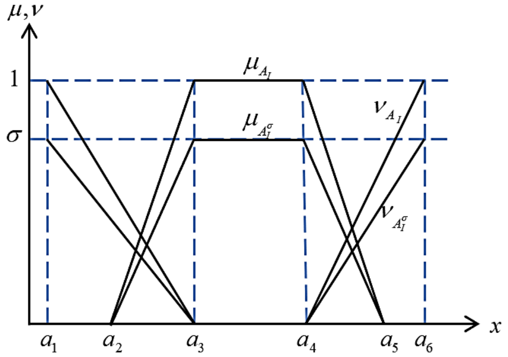

Fig. 6. Membership and non-membership functions of the weighted IZN.

Step 3. Convert Z_I^σ into a regular IFN.

$$Z' = \left\{ \left\langle x, \mu_{Z'}(x), \nu_{Z'}(x) \right\rangle \mid \mu_{Z'}(x) = \mu_{A_I}\left(\sqrt{\sigma x}\right), \nu_{Z'}(x) = \nu_{A_I}\left(\sqrt{\sigma x}\right), x \in [0,1] \right\}. \tag{16}$$

Theorem 2. $E_{Z'}(x) = \sigma E_{A_I}(x)$, $x \in \sqrt{\sigma}X$ subject to $\mu_{Z'}(x) = \mu_{A_I}\left(\sqrt{\sigma x}\right)$ and $\nu_{Z'}(x) = \nu_{A_I}\left(\sqrt{\sigma x}\right)$, $x \in \sqrt{\sigma}X$.

Proof.

$$E_{Z'}(x) = \left(a_2, a_3, a_4, a_5; \sqrt{\frac{2(r_2+r_5)+7(r_3+r_4)}{18} \times \frac{7\delta_2}{18} + \frac{2(r_1+r_6)+7(r_3+r_4)}{18} \times \frac{\eta_2+11}{18}}; \right.$$

$$\left. a_1, a_3, a_4, a_6; \sqrt{\frac{2(r_2+r_5)+7(r_3+r_4)}{18} \times \frac{7\delta_2}{18} + \frac{2(r_1+r_6)+7(r_3+r_4)}{18} \times \frac{\eta_2+110}{18}} \right)$$

$$= \left(a_2, a_3, a_4, a_5; \sqrt{\sigma}; a_1, a_3, a_4, a_6; \sqrt{\sigma} \right)$$

$$= \sigma E_{A_I}(x).$$

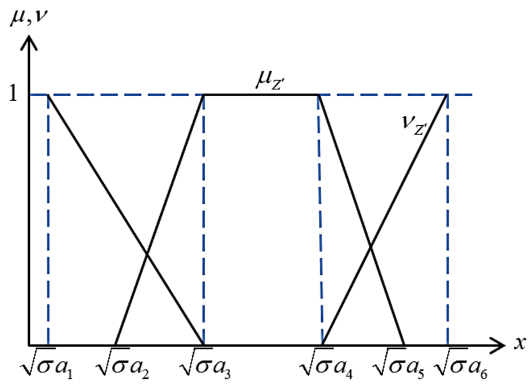

Fig. 7. The IFN converted from IZN.

Theorem 3. $E_{Z'}(x) = E_{A_I}(x)$.

Proof. From *Theorem 1* and *Theorem 2*, $E_{A_I^\sigma}(x) = \sigma E_{A_I}(x)$ and $E_{Z'}(x) = \sigma E_{A_I}(x)$. Therefore, $E_{Z'}(x) = E_{A_I}(x)$.

3.3 | IZN-Based Decision-Making Model

The advantages of IZN in handling uncertainty and vagueness, as discussed in the Introduction section, can be further highlighted in the implementation for solving an MCDM problem. In the previous sub-section, a method of converting the IZN into regular IFN was proposed. Hence, using the proposed defuzzification method, a novel MCDM model can be developed. In this paper, the arithmetic averaging operator is used to aggregate the decision-maker's preferences in the form of IZN. Furthermore, a ranking function based on centroid is used to rank the alternatives. Finally, the MCDM method based on IZN is given as follows:

Step 1. The decision-makers preferences in linguistic variables are converted into trapezoidal IZN. For this model, the linguistic terms of the restriction and reliability components are given in *Table 1* and *Table 2*, respectively.

Table 1. IZN corresponds to linguistic terms for the restriction component.

Linguistic Terms	Trapezoidal IZN
Very Low (VL)	⟨(0.0,0.0,0.0,0.0;1),(0.0,0.0,0.0,0.0;0)⟩
Low (L)	⟨(0.0,0.1,0.2,0.3;1),(0.0,0.1,0.2,0.3;0)⟩
Medium Low (ML)	⟨(0.1,0.2,0.3,0.4;1),(0.0,0.2,0.3,0.5;0)⟩
Medium (M)	⟨(0.3,0.4,0.5,0.6;1),(0.2,0.4,0.5,0.7;0)⟩
Medium High (MH)	⟨(0.5,0.6,0.7,0.8;1),(0.4,0.6,0.7,0.9;0)⟩
High (H)	⟨(0.7,0.8,0.9,1.0;1),(0.7,0.8,0.9,1.0;0)⟩
Very High (VH)	⟨(1.0,1.0,1.0,1.0;1),(1.0,1.0,1.0,1.0;0)⟩

Table 2. IZN corresponds to linguistic terms for the reliability component.

Linguistic Terms	Trapezoidal IZN
Not Sure (NS)	⟨(0.0,0.0,0.0,0.1;1),(0.0,0.0,0.0,0.1;0)⟩
Not Very Sure (NVS)	⟨(0.1,0.2,0.4,0.5;1),(0.1,0.2,0.4,0.5;0)⟩
Sure (S)	⟨(0.5,0.6,0.8,0.9;1),(0.5,0.6,0.8,0.9;0)⟩
Very Sure (VS)	⟨(0.9,1.0,1.0,1.0;1),(0.9,1.0,1.0,1.0;0)⟩

Step 2. Decision-makers' preferences in form of trapezoidal IZN are converted into the trapezoidal IFN using the defuzzification method proposed in Section 3.2.

Step 3. If there is more than one decision-maker involved, then all the decision-makers preferences are aggregated using the arithmetic averaging operator. Let n be the number of decision-makers, and then the arithmetic averaging operator is given in *Eq. (17)*.

$$\frac{1}{n}\sum_{k=1}^{n}\left(a_{k2},a_{k3},a_{k4},a_{k5};a_{k1},a_{k2},a_{k3},a_{k6}\right)$$
$$=\left(\frac{\sum_{k=1}^{n}a_{k2}}{n},\frac{\sum_{k=1}^{n}a_{k3}}{n},\frac{\sum_{k=1}^{n}a_{k4}}{n},\frac{\sum_{k=1}^{n}a_{k5}}{n};\frac{\sum_{k=1}^{n}a_{k1}}{n},\frac{\sum_{k=1}^{n}a_{k2}}{n},\frac{\sum_{k=1}^{n}a_{k3}}{n},\frac{\sum_{k=1}^{n}a_{k6}}{n}\right). \tag{17}$$

Step 4. All criteria are aggregated using the arithmetic averaging operator for each alternative (in each row). If there are p criteria, then the aggregated trapezoidal IFN is given by,

$$\frac{1}{p}\sum_{k=1}^{p}\left(a_{k2},a_{k3},a_{k4},a_{k5};a_{k1},a_{k2},a_{k3},a_{k6}\right)$$
$$=\left(\frac{\sum_{k=1}^{p}a_{k2}}{p},\frac{\sum_{k=1}^{p}a_{k3}}{p},\frac{\sum_{k=1}^{p}a_{k4}}{p},\frac{\sum_{k=1}^{p}a_{k5}}{p};\frac{\sum_{k=1}^{p}a_{k1}}{p},\frac{\sum_{k=1}^{p}a_{k2}}{p},\frac{\sum_{k=1}^{p}a_{k3}}{p},\frac{\sum_{k=1}^{p}a_{k6}}{p}\right). \tag{18}$$

Step 5. The ranking of each alternative is calculated. For this purpose, the ranking function for trapezoidal IFN based on centroid [24] is used. Let $I=\left(a_2,a_3,a_4,a_5;a_1,a_3,a_4,a_6\right)$ be a trapezoidal IFN, and then its ranking is given by,

$$R(I)=\sqrt{\frac{1}{2}\left[\tilde{x}_{\mu}(I)-\tilde{y}_{\mu}(I)\right]^2+\frac{1}{2}\left[\tilde{x}_{v}(I)-\tilde{y}_{v}(I)\right]^2}. \tag{19}$$

Hakim Nik Badrul Alam et al. | J. Fuzzy. Ext. Appl. 3(2) (2022) 126-139

Where $\tilde{x}_{\mu}(I)=\frac{1}{3}\left(\frac{a_4^2+a_5^2-a_2^2-a_3^2-a_2a_3+a_4a_5}{a_4+a_5-a_2-a_3}\right)$, $\tilde{x}_{\nu}(I)=\frac{1}{3}\left(\frac{2a_6^2-2a_1^2+2a_3^2+2a_4^2+a_1a_3-a_4a_6}{a_4+a_6-a_1-a_3}\right)$,

$\tilde{y}_{\mu}(I)=\frac{1}{3}\left(\frac{a_2+2a_3-2a_4-a_5}{a_2+a_3-a_4-a_5}\right)$ and $\tilde{y}_{\nu}(I)=\frac{1}{3}\left(\frac{2a_1+a_3-a_4-2a_6}{a_1+a_3-a_4-a_6}\right)$.

4 | Supplier Selection Problem

A supplier selection problem is adopted from [25] to illustrate the implementation and advantages of IZNs in MCDM. The considered problem aims to select the best supplier in an automobile manufacturing company with five criteria: quality (C1), cost (C2), technological capability (C3), partnership (C4), and on-time delivery (C5). Six suppliers are considered as alternatives.

Step 1. Three decision-makers (experts) were requested to give preferences on all the criteria for each alternative. First, their preferences are presented in *Tables 3* to *5*. Then, the linguistic preferences are converted into trapezoidal IZN according to *Tables 1* to *2*.

Table 3. Linguistic preferences of Expert 1 [16].

Suppliers	C1	C2	C3	C4	C5
A1	(VL, NVS)	(ML, VS)	(MH, S)	(H, NVS)	(MH, NS)
A2	(L, S)	(MH, NVS)	(M, NS)	(H, VS)	(M, NVS)
A3	(MH, NVS)	(ML, S)	(H, VS)	(M, NVS)	(VL, NVS)
A4	(H, S)	(H, NS)	(MH, VS)	(ML, NS)	(VH, S)
A5	(M, NVS)	(M, VS)	(MH, NS)	(H, NS)	(VH, S)
A6	(ML, NS)	(ML, NS)	(H, S)	(H, NVS)	(L, S)

Table 4. Linguistic preferences of Expert 2 [16].

Suppliers	C1	C2	C3	C4	C5
A1	(M, NVS)	(ML, NS)	(MH, S)	(MH, NVS)	(H, VS)
A2	(MH, S)	(M, S)	(MH, NVS)	(H, NS)	(M, VS)
A3	(MH, NS)	(M, VS)	(H, NVS)	(M, NVS)	(MH, NVS)
A4	(MH, NVS)	(L, NVS)	(VH, NS)	(M, NS)	(ML, NS)
A5	(MH, VS)	(VH, NS)	(M, VS)	(H, NS)	(VH, NS)
A6	(H, NS)	(H, NVS)	(ML, NVS)	(H, S)	(MH, VS)

Table 5. Linguistic preferences of Expert 3 [16].

Suppliers	C1	C2	C3	C4	C5
A1	(MH, S)	(M, S)	(H, NVS)	(M, NVS)	(M, S)
A2	(H, NVS)	(MH, NVS)	(H, NS)	(M, NS)	(ML, NVS)
A3	(M, VS)	(ML, NS)	(H, S)	(MH, S)	(MH, NS)
A4	(MH, VS)	(VL, NS)	(VH, S)	(MH, S)	(M, NS)
A5	(H, NVS)	(VH, NVS)	(ML, NS)	(H, VS)	(VH, NVS)
A6	(MH, S)	(H, NS)	(ML, NS)	(VH, NVS)	(H, NS)

Step 2. Using the proposed IMC defuzzification of IZN as presented in Section 3.2, trapezoidal IZN is converted into trapezoidal IFN. *Table 6* shows the trapezoidal IFN obtained by defuzzifying Expert 1's preferences in the form of IZN. Then, the rest of the experts' preferences are defuzzified analogously.

Table 6. Converted IFN of preferences for Expert 1.

	C1	C2	C3	C4	C5
A1	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.099,0.199,0.298,0.398;0.000,0.199,0.298,0.497)	(0.418,0.502,0.586,0.669;0.335,0.502,0.586,0.753)	(0.383,0.438,0.493,0.548;0.383,0.438,0.493,0.548)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)
A2	(0.000,0.084,0.167,0.251;0.000,0.084,0.167,0.251)	(0.274,0.329,0.383,0.438;0.219,0.329,0.383,0.493)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.696,0.796,0.895,0.994;0.696,0.796,0.895,0.994)	(0.274,0.329,0.383,0.438;0.219,0.329,0.383,0.493)
A3	(0.274,0.329,0.383,0.438;0.219,0.329,0.383,0.493)	(0.084,0.167,0.251,0.335;0.000,0.167,0.251,0.418)	(0.696,0.796,0.895,0.994;0.696,0.796,0.895,0.994)	(0.164,0.219,0.274,0.329;0.110,0.219,0.274,0.383)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)
A4	(0.586,0.669,0.753,0.837;0.586,0.669,0.753,0.837)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.497,0.597,0.696,0.796;0.398,0.597,0.696,0.895)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.837,0.837,0.837,0.837;0.837,0.837,0.837,0.837)
A5	(0.164,0.219,0.274,0.329;0.110,0.219,0.274,0.383)	(0.298,0.398,0.497,0.597;0.199,0.398,0.497,0.696)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.837,0.837,0.837,0.837;0.837,0.837,0.837,0.837)
A6	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.000,0.000,0.000,0.000;0.000,0.000,0.000,0.000)	(0.586,0.669,0.753,0.837;0.586,0.669,0.753,0.837)	(0.383,0.438,0.493,0.548;0.383,0.438,0.493,0.548)	(0.000,0.084,0.167,0.251;0.000,0.084,0.167,0.251)

Step 3. Once the decision-maker's preferences have been converted into trapezoidal IFN, aggregation can be done to combine the preferences of all involved experts (3 experts). The aggregated trapezoidal IFN representing all the three experts are presented in *Table 7*.

Table 7. Aggregated IFN of preferences for all experts.

	C1	C2	C3	C4	C5
A1	(0.194,0.240,0.287,0.333;0.148,0.240,0.287,0.379)	(0.117,0.178,0.239,0.300;0.056,0.178,0.239,0.361)	(0.407,0.481,0.555,0.629;0.351,0.481,0.555,0.685)	(0.274,0.329,0.383,0.438;0.237,0.329,0.383,0.475)	(0.316,0.377,0.438,0.499;0.288,0.377,0.438,0.527)
A2	(0.267,0.341,0.415,0.489;0.239,0.341,0.415,0.517)	(0.266,0.331,0.395,0.459;0.202,0.331,0.395,0.459)	(0.091,0.110,0.128,0.146;0.073,0.110,0.128,0.164)	(0.232,0.265,0.298,0.331;0.232,0.265,0.298,0.331)	(0.209,0.279,0.348,0.418;0.139,0.279,0.348,0.488)
A3	(0.191,0.242,0.294,0.345;0.139,0.242,0.294,0.396)	(0.127,0.188,0.249,0.310;0.066,0.188,0.249,0.371)	(0.555,0.634,0.714,0.793;0.555,0.634,0.714,0.793)	(0.249,0.313,0.378,0.442;0.185,0.313,0.378,0.507)	(0.091,0.110,0.128,0.146;0.073,0.110,0.128,0.164)
A4	(0.452,0.532,0.611,0.690;0.401,0.532,0.611,0.742)	(0.000,0.018,0.037,0.055;0.000,0.018,0.037,0.055)	(0.445,0.478,0.511,0.544;0.411,0.478,0.511,0.577)	(0.139,0.167,0.195,0.223;0.112,0.167,0.195,0.251)	(0.279,0.279,0.279,0.279;0.279,0.279,0.279,0.279)
A5	(0.348,0.418,0.488,0.557;0.297,0.418,0.488,0.609)	(0.282,0.315,0.348,0.381;0.249,0.315,0.348,0.415)	(0.099,0.133,0.166,0.199;0.066,0.133,0.166,0.232)	(0.232,0.265,0.298,0.331;0.232,0.265,0.298,0.331)	(0.461,0.461,0.461,0.461;0.461,0.461,0.461,0.461)
A6	(0.139,0.167,0.195,0.223;0.112,0.167,0.195,0.251)	(0.128,0.146,0.164,0.183;0.128,0.146,0.164,0.183)	(0.213,0.260,0.306,0.351;0.195,0.260,0.306,0.370)	(0.506,0.552,0.598,0.644;0.506,0.552,0.598,0.644)	(0.166,0.227,0.288,0.349;0.133,0.227,0.288,0.382)

Step 4. Based on the aggregated IFN of preferences in *Table 7*, all the criteria preferences are aggregated for each alternative (in each row). Hence, the final aggregation result is shown in *Table 8* below.

Hakim Nik Badrul Alam et al. | J. Fuzzy. Ext. Appl. 3(2) (2022) 126-139

Table 8. Final aggregation of experts' preferences for each alternative.

Suppliers	Aggregated IFN
A1	(0.261,0.321,0.380,0.440;0.216,0.321,0.380,0.485)
A2	(0.213,0.265,0.317,0.369;0.177,0.265,0.317,0.405)
A3	(0.243,0.298,0.352,0.407;0.204,0.298,0.352,0.446)
A4	(0.263,0.295,0.326,0.358;0.241,0.295,0.326,0.381)
A5	(0.285,0.318,0.352,0.386;0.261,0.318,0.352,0.410)
A6	(0.230,0.270,0.310,0.350;0.215,0.270,0.310,0.366)

Step 5. Based on the final aggregation result in *Table 8*, the ranking function is evaluated using *Eq. (19)*. The ranking function value for each alternative is listed in *Table 9* below.

Table 9. Final aggregation of experts' preferences for each alternative.

Suppliers	Ranking Function	Ranking Order
A1	0.123685	4
A2	0.090099	6
A3	0.104851	5
A4	0.278681	2
A5	0.338737	1
A6	0.168720	3

From the ranking function evaluated in the final step of the MCDM model, the Supplier A_5 is ranked first while the supplier A_2 is ranked last. The obtained ranking is $A_2 \prec A_3 \prec A_1 \prec A_6 \prec A_4 \prec A_5$, which $A_i \prec A_j$ denotes that the ranking of alternative A_i is lower than A_j. Comparing this ranking order to the one obtained in [25] $A_2 \prec A_1 \prec A_3 \prec A_6 \prec A_5 \prec A_4$, there is a slight change in their order. This paper used IZN to describe the decision information while Z-numbers were used in [25]. IZN is believed to be able to better handle the uncertainty of the judgement as compared to the regular Z-numbers. Hence, this fact has become one of the factors which determine the final ranking order of alternatives.

5 | Conclusion

A defuzzification method of IZN via the IMC method was proposed. The IZN has a better capability of handling uncertainty and vagueness as compared to the classical Z-numbers. This is because the restriction and reliability components of Z-numbers are characterized by both the membership and non-membership functions to highlight the uncertainties which arise due to the lack of information when preferences are made by decision-makers. The IMC method was used to defuzzify IZN into regular IFN due to its applicability and practicality in converting Z-numbers into regular fuzzy numbers, as discussed in the previous literature. A supplier selection problem was implemented to illustrate the proposed defuzzification approach. The limitation of this research is that the defuzzification of IZN may lead to a great loss of information since the preferences of decision-makers, which are initially in the form of IZN, are converted into regular IFN. Hence, a method of solving the MCDM problem without converting the IZN into any other form of fuzzy numbers should be developed in the future. The original decision information should be kept as IZN throughout the decision-making process, and a magnitude-based ranking method is suggested to be used for ranking the alternatives. This will avoid the loss of information. Hence, the advantages of IZN in handling uncertainty can be better highlighted.

Acknowledgements

The authors would like to thank Universiti Malaysia Pahang and the Ministry of Higher Education Malaysia for supporting this research.

Funding

This research paper is supported financially by Fundamental Research Grant Scheme under the Ministry of Higher Education Malaysia FRGS/1/2019/STG06/UMP/02/9.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript, and there is no financial interest to report. In addition, the authors certified that the submission is original work and is not under review at any other publication.

References

[1] Aliev, R. A. (2013). Uncertain preferences and imperfect information in decision making. In *fundamentals of the fuzzy logic-based generalized theory of decisions* (pp. 89-125). Springer, Berlin, Heidelberg.
[2] Aliev, R. A., Guirimov, B. G., Huseynov, O. H., & Aliyev, R. R. (2021). A consistency-driven approach to construction of Z-number-valued pairwise comparison matrices. *Iranian journal of fuzzy systems*, *18*(4), 37-49.
[3] Sirbiladze, G. (2021). New view of fuzzy aggregations. part I: general information structure for decision-making models. *Journal of fuzzy extension and applications*, *2*(2), 130-143.
[4] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353.
[5] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141. https://doi.org/10.1287/mnsc.17.4.B141
[6] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information sciences*, *8*(3), 199-249.
[7] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-III. *Information sciences*, *9*(1), 43-80.
[8] Dubois, D., & Prade, H. (1978). Operations on fuzzy numbers. *International journal of systems science*, *9*(6), 613-626.
[9] Chen, S. J., & Hwang, C. L. (1992). Fuzzy multiple attribute decision making methods. In *fuzzy multiple attribute decision making* (pp. 289-486). Springer, Berlin, Heidelberg.
[10] Chang, D. Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European journal of operational research*, *95*(3), 649-655.
[11] Bhattacharya, J. (2021). Some results on certain properties of intuitionistic fuzzy sets. *Journal of fuzzy extension and applications*, *2*(4), 377-387.
[12] Atanassov, K. T. (1986). Intuitionistic fuzzy set. *Fuzzy sets and systems*, *20*, 87–97.
[13] Xue, Y., & Deng, Y. (2021). Decision making under measure-based granular uncertainty with intuitionistic fuzzy sets. *Applied intelligence*, *51*(8), 6224-6233.
[14] Husain, S., Ahmad, Y., & Alam, M. A. (2012). A study on the role of intuitionistic fuzzy set in decision making problems. *International journal of computer applications*, *48*(0975-888), 35-41.
[15] Rahman, A. U., Ahmad, M. R., Saeed, M., Ahsan, M., Arshad, M., & Ihsan, M. (2020). A study on fundamentals of refined intuitionistic fuzzy set with some properties. *Journal of fuzzy extension and applications*, *1*(4), 279-292.
[16] Gautam, S. S., & Singh, S. R. (2016). TOPSIS for multi criteria decision making in intuitionistic fuzzy environment. *International journal of computer applications*, *156*(8), 42-49.
[17] Wang, C. H., & Wang, J. Q. (2016). A multi-criteria decision-making method based on triangular intuitionistic fuzzy preference information. *Intelligent automation & soft computing*, *22*(3), 473-482.
[18] Kaur, P. (2014). Selection of vendor based on intuitionistic fuzzy analytical hierarchy process. *Advances in operations research*, *2014*.
[19] Zadeh, L. A. (2011). A note on Z-numbers. *Information sciences*, *181*(14), 2923-2932.
[20] Kang, B., Wei, D., Li, Y., & Deng, Y. (2012). A method of converting Z-number to classical fuzzy number. *Journal of information &computational science*, *9*(3), 703-709.

[21] Kang, B., Wei, D., Li, Y., & Deng, Y. (2012). Decision making using Z-numbers under uncertain environment. *Journal of computational information systems*, *8*(7), 2807-2814.

[22] Ku Khalif, K. M. N., Gegov, A., & Abu Bakar, A. S. (2017). Hybrid fuzzy MCDM model for Z-numbers using intuitive vectorial centroid. *Journal of intelligent & fuzzy systems*, *33*(2), 791-805.

[23] Sari, I. U., & Kahraman, C. (2020, July). Intuitionistic fuzzy Z-numbers. *International conference on intelligent and fuzzy systems* (pp. 1316-1324). Istanbul, Turkey. Springer, Cham. https://link.springer.com/chapter/10.1007/978-3-030-51156-2_154

[24] Arun Prakash, K., Suresh, M., & Vengataasalam, S. (2016). A new approach for ranking of intuitionistic fuzzy numbers using a centroid concept. *Mathematical sciences*, *10*(4), 177-184.

[25] Wang, F., & Mao, J. (2019). Approach to multicriteria group decision making with Z-numbers based on TOPSIS and power aggregation operators. *Mathematical problems in engineering*, *2019*.