Investigating formation resistance to fracture during operation of wells prone to plugging

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Abstract. In this paper, a study was conducted to determine the stability of productive formations when selecting liquid hydrocarbon reserves from them. Using mathematical modeling methods and various laws of fluid motion in a porous medium, the optimal conditioned pressure value has been established, at which the least destruction of the rock matrix will be observed. This area of research is relevant due to the fact that for the current situation, one of the most effective methods of increasing well productivity is hydraulic fracturing, which implies a significant impact on the stability of the borehole-rocks system. The obtained conclusions will make it possible to effectively plan geological and technical measures related to the impact on the rock matrix and will allow trouble-free operation of the complicated well stock.

1 Introduction

The largest difference in normal stresses occurs on the borehole wall since the radial stresses are equal to the minimum values, and the tangential stresses are equal to the maximum values [1]. Therefore, depending on the formation pressure drawdown and the strength properties of the fixed zone, the following well operating conditions are possible:

- The fixed near-wellbore part of the formation is in an elastic state throughout the entire volume, that is, the following condition is fulfilled:

$$\sigma_{ra} - \sigma_{\theta a} = \sigma_s; \tag{1}$$

The fixed near-wellbore part of the formation is in a plastic state throughout the entire volume, that is, the following condition is fulfilled:

$$\sigma_{rb} - \sigma_{\theta b} = \sigma_s, \tag{2}$$

Where σ_s is yield point.

It is known from the solution of the elastic problem that the largest difference between the normal stresses in the body of a hollow sphere during fluid filtration through its wall to

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the center takes place on the inner surface ($r = a$) [2]. Therefore, the conditions for the fluidity of the ball material will arise, first of all, on the inner surface of the ball, which can be written as follows:

$$\sigma_{ra} - \sigma_{\theta a} = \sigma_s, \tag{3}$$

Where σ_s is the yield point of the ball material under compressive stress; σ_{ra} and $\sigma_{\theta a}$ are normal stresses, radial and tangential, respectively.

2 Methods and Materials

Using this expression and solving the elastic problem considering the compressibility of the rock, it is possible to determine the value of the pressure drop on the formation, which excludes the possibility of plastic flow of the formation material, that is, the permissible pressure drop P_{ag} [3].

Considering that $\sigma_{ra} = 0$, we can write the following:

$$-\sigma_{\theta a} = \frac{\sigma_s}{z}, \tag{4}$$

Where z is margin of safety; $\sigma_{\theta a}$ is the normal tangential stress determined from equation (5) at $r = a$.

Considering the fact that the stresses caused by uneven hydraulic compression of the elastic component of the material in a granular cemented material with viscous cement can prelax, let us assume that $\alpha = \omega = 0$ ($\chi = -1$). Let us also assume that the unknown external contact compressive stress σ_{rb} is equal to zero. Then the normal tangential compressive stress on the inner surface of the ball will be approximately (assuming that $\frac{b}{a} \geq 1$):

$$\sigma_{ba} = -\frac{1+v}{1-v} \cdot (P_a - P_b), \tag{5}$$

Or, limiting this voltage to an admissible value$\frac{\sigma_s}{z}$, let us write the following:

$$\frac{\sigma_s}{z} = \frac{1+v}{1-v} \cdot (P_a - P_b). \tag{6}$$

Liquid pressure distribution in the wall of a hollow ball during filtration is as follows:

$$P = \left[1 - \left(1 - \frac{a}{r}\right) \cdot (1 - \frac{a}{b} + \frac{naK_1}{hK_2} ln \frac{R_k}{r_c + b})^{-1}\right] \cdot P_a, \tag{7}$$

Where a is radius of the perforated hole, m; b is thickness of the filter zone, m; n is the number of perforations in the column; h is thickness of the formation part entered by perforation, m; $K_1$is filter zone permeability, m^2; K_2 is formation permeability, m^2; R_k is supply contour radius, m; r_c is borehole radius, m; P_a is total pressure drop across the formation and the filter, MPa.

Then the expression for determining the permissible pressure drop across the formation will be written as follows:

$$P_{ad}^{y} = \frac{\sigma_s}{z} \cdot \frac{1-v}{1+v} \cdot \left(1 + \frac{nbK_1}{hK_2} \cdot ln\frac{R_K}{r_c+b}\right) \tag{8}$$

3 Results and Discussion

The considered method for calculating the value of the permissible drawdown per formation is based on the results of short-term determinations of the mechanical properties of the filter material, namely, the value of the yield stress σ_s, and, obviously, it does not field the question about the filter durability [4].

It is known that all rocks can deform in time at a stress difference $\sigma_r - \sigma_\theta$ smaller than σ_s. And if the cementing material in the rock flows and loses its properties then the cemented rock will lose properties as plastic deformations develop. Therefore, for another criterion of strength in the meaning of durability of the rock behind the casing in the well, the rate of its plastic flow or the limiting value of the relative deformation can be taken [5].

With long-term observation of the deformation of the rock being elastic in the usual sense, it is possible to observe its deformation in time. Thus, solving the mixed problem of the stress state of a rock is senseless if its outer zone is in an elastic state, and an inner zone, within which the material is in a state of plastic flow. The flow is obviously impossible theoretically due to elastic zone. However, deformation in time is possible in this case as well. Therefore, let us consider the limiting case when a hollow sphere experiences a state of plastic flow over its entire cross section [6].

It is known that in an elastic hollow ball, when a liquid is filtered to its center with a decrease in pressure in its cavity, the stresses in the wall of the ball over the entire section are unambiguous, specifically, negative (compression stresses), and in the ratio $|\sigma_\theta| \geq |\sigma_r|$. In this regard, the following condition of material yield is valid for the plasticity zone:

$$\sigma_r - \sigma_\theta = +\sigma_s \tag{9}$$

A plus sign in front of σ_s in equation (9) is taken from the solution [7], since the difference $\sigma_r - \sigma_\theta$, when the material passes from the elastic state to the plastic flow state, it will obviously not change.

It must be assumed that the yield stress for a granular material decreases with the development of plastic deformations. In the first approximation, this strength parameter is assumed to be linearly decreasing with the development of plastic deformation:

$$\sigma_s = \sigma_{so} \cdot (1 - \eta_\sigma \bar{\varepsilon}), \tag{10}$$

Where

$$\bar{\varepsilon} = \sqrt{\frac{2}{3}} \cdot \sqrt{\left(\varepsilon_r^n - \varepsilon_\theta^n\right)^2 + (\varepsilon_r^n - \varepsilon_z^n)^2}$$

$$\varepsilon_r^n = \frac{du^n}{dr}, \varepsilon_\theta^n = \varepsilon_z^n = \frac{u^n}{r}$$

Undoubtedly, under conditions of plastic flow of a material, the effect of compressibility caused by the action of hydrostatic pressure can be neglected since these stresses relax, and, in addition, elastic deformation makes up an insignificant fraction of deformation in the future [8]. Let us also assume that during plastic flow, the granular cemented material retains a constant volume, that is:

$$\Delta = \frac{du^n}{dr} + 2\frac{u^n}{r} = \frac{1}{r^2} \cdot \frac{d}{dr} \cdot (r^2 u^n) \tag{11}$$

Hence, the following is write:

$$u^n = \frac{c}{r^2}, \varepsilon_r^n = -2\frac{c}{r^3}, \varepsilon_\theta^n = \frac{c}{r^3} \tag{12}$$

From the boundary condition $r = a$, $u^n = u_a^n$ let us find $= a^2 \cdot u_a^n$.
Substituting ε_r^n and ε_θ^n from (12) in (11), we obtain the expression for the intensity of shear deformation during plastic flow:

$$\varepsilon = \varepsilon_a^n \cdot \frac{a^3}{r^3}, \varepsilon_a^n = 2\sqrt{3} \cdot \frac{u_a^n}{a} \tag{13}$$

The equilibrium equation for a stressed porous medium for an isotropic hollow sphere has the following form:

$$\frac{d\sigma_r}{dr} + 2\frac{\sigma_r - \sigma_\theta}{r} = \chi \cdot \frac{dP}{dr}. \tag{14}$$

Substituting (9), (10) and (13) in it at χ = -1, we obtain:

$$\frac{d\sigma_r}{dr} + 2\frac{\sigma_{so}}{r} \cdot \left(1 - \eta_\sigma \varepsilon_a^n \cdot \frac{a^3}{r^3}\right) + \frac{dP}{dr} = 0. \tag{15}$$

The magnitude of the pressure gradient is determined by:

$$\frac{dP}{dr} = \frac{q\mu_{zh}}{2\pi K_1} \cdot \frac{1}{K^2}. \tag{16}$$

Substituting this value in (15), after integration under the boundary condition $r = a_1$, $P = P_a$, $\sigma_r = 0$ we obtain:

$$\sigma_r = P_a - P - 2\sigma_{so} \cdot [f(r) - f(a)], \tag{17}$$

$$f(r) = \frac{1}{\sigma_{so}} \cdot \int_0^r \frac{\sigma_s}{r} dr = lnr + \frac{1}{3}\eta_\sigma \varepsilon_a^n \cdot \frac{a^3}{r^3} + c \tag{18}$$

Where $b \geq r \geq a, \sigma_\theta = \sigma_r - \sigma_s$.
The difference $P_a - P$ in equation (17) we find with the help of the following formula:

$$P_a - P = \frac{Q_{zh}\mu_{zh}}{2\pi naK_1} \cdot (1 - \frac{a}{r}). \tag{19}$$

Substituting (19) into (17) and using the boundary condition $r = b,\ P = P_b,\ \sigma_r = \sigma_{rb}$, we obtain the formula for the limiting ratio between the critical oil flow rate and the limiting stress state of the rock material corresponding to its plastic flow over the entire section of the hollow sphere:

$$\frac{Q_{zh}\mu_{zh}}{2\pi naK_1} \cdot \left(1 - \frac{a}{b}\right) = 2\sigma_{so} \cdot [f(b) - f(a)] - \sigma_{rb} \tag{20}$$

The resulting dependence (20) can be considered as a criterion equation. Using this relationship, it is possible to determine the permissible oil production rate or the pressure drop across the formation, which ensures the normal operation of the well and the stability of the formation material behind the casing [9]. To determine the permissible values of the flow rate or pressure drop, we will use a well-known parameter, specifically, the safety factor *z*. Substituting the permissible value for the yield strength

$$|\sigma_{so}| = \frac{\sigma_{so}}{z}. \tag{21}$$

Into formula (20) and replacing the flow rate Q_{zh} with the corresponding value of the pressure drop from (19), we obtain the calculation formula for calculating the permissible pressure drop on the formation:

$$P_{ad}^{n} = \left[2\frac{\sigma_{so}}{z}(f_b - f_a) - \sigma_{rb}\right] \cdot \left(\frac{b}{a} + \frac{nb^2K_1}{(b-a)\cdot hK_2} \cdot ln\frac{R_k}{r+b_c}\right). \tag{22}$$

For approximate calculations, we will take $\sigma_{rb} = 0$, $\eta_\theta \cdot \varepsilon_a^n = 1$ and neglect the value $\frac{b}{a}$ compared to one. Then, formula (22) takes the following form:

$$P_{ad}^{n} \approx 2\frac{\sigma_{so}}{z} \cdot \left(1 + \frac{naK_1}{nK_2} \cdot ln\frac{R_k}{r_c+b}\right) \cdot \frac{b}{a} ln\frac{b}{a}. \tag{23}$$

The condition $\eta_\theta \varepsilon_a^n = 1$ corresponds to the case when the material loses strength on the wall of the hollow sphere ($r = a$) when the flow deformation reaches its limiting values. Further deformation of the rock will be accompanied by its destruction, chipping of granular material in time. In this regard, the question of the durability of the rock behind the casing will be determined by the rate of its outflow through the perforation, which requires special long-term studies of the material flow under conditions close to reality [10].

Comparison of the value of the permissible pressure drop on the formation obtained from the condition $((-\sigma_\theta = \frac{\sigma_s}{z}))$ at r = a, when the material of the hollow ball is in an elastic state over the entire section with the value of the allowable pressure drop obtained for the case when the material of the hollow ball is in a state of plastic flow across the entire section shows that in the first case the value of the permissible pressure drop across the formation P_{od}^{y} is ten times less than in the second case P_{od}^{n}. This minimum value P_{od}^{y}, apparently, should be taken as acceptable one during well operation, which should ensure the greatest durability of the filter behind the casing, whose material is prone to plastic deformation. In this case, the safety factor *z* can be taken equal to one. In the future, this value can be refined based on the results of field studies and the introduction of methods for securing the bottomhole zone of wells.

It should be born in mind that the value P_{od}^{y} is calculated on the basis of the yield stress σ_s of the rock determined by short-term studies of the material mechanical properties. Therefore, to achieve stability of the formation's downhole part to destruction, it is necessary to match the strength properties of the consolidated rock (namely, the yield point) to the maximum difference in normal stresses acting in this area. However, it is possible that the fixed formation zone near the bottom of the well will begin to collapse some time after the consolidation process. This may be due to such factors of the rock strength reduction as the development of creeping and plastic deformations, as well as a decrease in the value of a_s caused by the dissolving ability of filterable hydrocarbons.

With a rapid load application on the formation (when the well is put into operation), the stressed state of the filter zone will depend on the parameter $\alpha\beta$, specifically, its value. In the future, as plastic deformations develop under conditions of volume conservation, the compressibility effect will play a subordinate value and tend to zero ($\alpha\beta \to 0$). In this case, Poisson's ratio ν will tend to 0.5, and Young's modulus will change over time and determine the relationship between stresses and strain rate.

When determining the stress state of an elastic hollow sphere during filtration through its wall of a liquid, it is necessary to solve the problem of stresses for the case of liquid filtration towards the center of the sphere with a decrease in pressure in its cavity ($\chi = -1$). This case is for us an element of the general problem of the stress state of the annular filter behind the casing during well operation. The problem is solved in the following sequence. First, the problem of liquid filtration is solved by determining the pressure changes in the investigated body during liquid filtration. Then the equilibrium equation for radial deformation is solved.

The change in the sign of the filtration potential leads to a change in the tangential stresses on the borehole wall to a value equal to triple formation pressure drawdown (with the radial stresses equal to zero). This explains the negative effect of well shutdowns, and furthermore the change in the direction of the filtration flow in the near-wellbore part of the formation on the stability of the walls of wells, whose operation is complicated by sand ingress. The maximum difference between the main normal stresses is observed on the borehole wall. Therefore, in order to prevent the destruction of the formation near the bottom hole, it is necessary to comply with the strength properties of rocks with the stresses acting in this zone.

When operating wells prone to plugging, it is necessary to limit the formation pressure drawdown to the maximum permissible value, when the material of the near-filter zone is in an elastic state throughout the entire volume. The change in the sign of the filtration potential leads to a change in the tangential stresses on the borehole wall to a value equal to triple formation pressure drawdown (with the radial stresses equal to zero). This explains the negative effect of well shutdowns, and, furthermore, the change in the direction of the filtration flow in the near-wellbore part of the formation on the stability of the walls of wells, whose operation is complicated by sand ingress.

Thus, the maximum difference in the main normal stresses is observed on the borehole wall. Therefore, in order to prevent the destruction of the formation near the bottom hole, it is necessary to comply with the strength properties of rocks with the stresses acting in this zone. When operating wells prone to plugging, it is necessary to limit the formation pressure drawdown to the maximum permissible value, when the material of the filter zone is in an elastic state throughout the entire volume.

4 Conclusion

Thus, the long-term resistance of the formation to destruction will be determined both by the stresses acting in the rock near the bottom of the well under conditions of fluid inflow, and by the strength and rheological properties of the rock itself depending on the composition of the formation oil, sand and technological conditions of the consolidation process. When operating wells prone to plugging, it is necessary to limit the formation pressure drawdown to the maximum permissible value, when the material of the near-filter zone is in an elastic state throughout the entire volume.

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