

SURVEY ON PARAMETERIZED VERIFICATION WITH THRESHOLD AUTOMATA AND THE BYZANTINE MODEL CHECKER

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ABSTRACT. Threshold guards are a basic primitive of many fault-tolerant algorithms that solve classical problems in distributed computing, such as reliable broadcast, two-phase commit, and consensus. Moreover, threshold guards can be found in recent blockchain algorithms such as, e.g., Tendermint consensus. In this article, we give an overview of techniques for automated verification of threshold-guarded fault-tolerant distributed algorithms, implemented in the Byzantine Model Checker (ByMC). These threshold-guarded algorithms have the following features: (1) up to t of processes may crash or behave Byzantine; (2) the correct processes count messages and make progress when they receive sufficiently many messages, e.g., at least $t + 1$; (3) the number n of processes in the system is a parameter, as well as the number t of faults; and (4) the parameters are restricted by a resilience condition, e.g., $n > 3t$. Traditionally, these algorithms were implemented in distributed systems with up to ten participating processes. Nowadays, they are implemented in distributed systems that involve hundreds or thousands of processes. To make sure that these algorithms are still correct for that scale, it is imperative to verify them for all possible values of the parameters.

1. INTRODUCTION

Distributed Systems. The recent advent of blockchain technologies [Nak08, DSW16, AMN⁺16, Buc16, YMR⁺19, But14] has brought fault-tolerant distributed algorithms into the spotlight of computer science and software engineering. Consider a blockchain system, where a blockchain algorithm ensures coordination of the participants (i.e., the processes) in the system. We observe that to achieve coordination, the processes need to solve a coordination problem called *atomic (or, total order) broadcast* [HT93], that is, every process

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has to execute the same transactions in the same order. To achieve that, the algorithm typically relies on a *resilience condition* that restricts the fraction of processes that may be faulty [PSL80]. These classic Byzantine fault tolerance concepts are indeed at the base of several modern blockchain systems, such as Tendermint [Buc16] and HotStuff [YMR⁺19].

In addition to the above mentioned practical importance of distributed systems, the reasons for the long-standing interest [LL77, Lam78, PSL80, FLP85] in this field is that distributed consensus is non-trivial in two aspects:

- (1) Most coordination problems are impossible to solve without imposing constraints on the environment, e.g., an upper bound on the fraction of faulty processes, assumptions on the behavior of faulty processes, or bounds on message delay and processing speed (i.e., restricting interleavings) [PSL80, FLP85, DDS87].
- (2) Designing correct solutions is hard, owing to the huge state and execution space, and the complex interplay of the assumptions about the faulty processes, message delivery, and other environment assumptions mentioned above. Therefore, it is not surprising that even published protocols may contain bugs, as reported, e.g., by [LR93, MS06].

Due to the well-known impossibility of asynchronous fault-tolerant consensus (FLP impossibility) [FLP85], much of the distributed systems research focuses on: (a) what kinds of problems are solvable in asynchronous systems (e.g., some forms of reliable broadcast) or (b) what kinds of systems allow to solve consensus.

Computer-aided Verification. Due to the huge amount of funds managed by blockchains, it is crucial that their software is free of bugs. At the same time, these systems are characterized by a large number of participants, which makes the application of automated verification methods inevitably run into the well-known state space explosion problem. This is why, in such a system, consisting of an a priori unknown number of participants, it is desirable to show the system correctness using parameterized verification techniques. However, the well-known undecidability results for the verification of parameterized systems [AK86, Suz88, EN95, Esp97, BJK⁺15] also apply in this setting. One way to circumvent both the state space explosion problem and the undecidability of parameterized verification is to develop domain specific methods that work for a specific subclass of systems.

In this article, we survey verification techniques for fault-tolerant distributed algorithms, which: (a) for asynchronous systems deal with the concepts of broadcast and atomic broadcast under resilience conditions, and (b) can be used to verify consensus in synchronous and randomized systems. While the benchmarks we used to evaluate these verification techniques predominantly come from the classic fault-tolerant distributed systems literature, we note that they in part address the challenges imposed by distributed systems existing in the real world (such as the two blockchain systems mentioned above). We present several verification methods that have been developed in the threshold-automata framework [KVW17]. This framework has been introduced as a succinct formalization of the transition relations of threshold-based fault-tolerant distributed algorithms.

The remainder of the paper is structured as follows. In Sections 2 to 4, we survey some of the most fundamental system assumptions that allow to solve problems in distributed computing in the presence of faults, and introduce example algorithms. We consider different computational models; namely, synchronous lock-step systems, asynchronous systems, and probabilistic systems. We also discuss how these algorithms (operating in either of the three computation models) can be formalized using threshold automata and how they can be automatically verified. Since threshold automata represent an abstraction of distributed

algorithms, in Section 5 we discuss how this abstraction can be automatically generated from a formalization close to the algorithm descriptions in the literature. Then, in Section 6, we present how our tool ByMC evolved in the last years and which techniques were implemented. In Section 7 we demonstrate how ByMC can be used to analyze Tendermint, a state-of-the-art consensus algorithms used in the Cosmos blockchain ecosystem. And finally, in Section 8, we give an overview of related work.

2. PARAMETERIZED VERIFICATION OF SYNCHRONOUS ALGORITHMS

2.1. Synchronous Algorithms. In a synchronous algorithm, all correct processes execute the algorithm code line-by-line in lock-step. The computation is organized in rounds, which consist of three steps: send-to-all, receive, and local computation. A message sent by a correct process to a correct process is received within the same round. After sending and receiving messages in lock-step, all correct processes continue by evaluating the guards, before they all proceed to the next round. Because this semantics ensures that all processes move together, and all messages sent by correct processes are received within the same round, no additional fairness constraints are needed to ensure liveness (i.e., that something good eventually happens). In practice, this approach is often considered slow and expensive, as it has to be implemented with timeouts that are aligned to worst case message delays (which can be very high in real networks). However, the synchronous semantics offers a high-level abstraction that allows one to design algorithms easier.

We now present an example synchronous algorithm from the literature, which we refer to as `FloodMin` [CHLT00]. Its pseudocode is given in Figure 1 on the left. This algorithm is run by n replicated processes, up to t of which may fail by crashing, that is, by prematurely halting. It solves the k -set agreement problem, that is, out of the n initial values, the goal of each process is to decide on one value, such that the number of different decision values is at most k . By setting $k = 1$, we obtain that there can be exactly one decision value, which coincides with the definition of consensus. For simplicity of presentation, we consider the case where $k = 1$ and where the set of initial values is the set $\{0, 1\}$.

Each process running `FloodMin` has a variable `best`, which stores its input value, coming from the set $\{0, 1\}$ (line 1). One iteration of the loop at line 2 is called a *round*. In each round, each process broadcasts its value `best` in line 3. The variable `best` is updated in each round as the minimum of all received values (line 5). `FloodMin` runs for $\lfloor t/k \rfloor + 1$ rounds (line 2), which in case of $k = 1$ amounts to $t + 1$ rounds. The number $\lfloor t/k \rfloor + 1$ of rounds ensures that there is at least one clean round in which at most $k - 1$ processes crash. When we consider consensus (i.e., when $k = 1$), this means there is a round in which no process crashes, such that all processes receive the same values b_1, \dots, b_ℓ . As a result, during that round, all processes set `best` to the same value. Finally, after the loop line 2 terminates, each process decides on the value of the variable `best` (line 7).

2.2. Synchronous Threshold Automata. Synchronous threshold automata were introduced in [SKWZ19]. A *synchronous threshold automaton (STA)* that models the pseudocode of the algorithm `FloodMin` is given in Figure 1 on the right. It resembles a control flow graph and is used to encode the body of the loop on line 2 in the pseudocode. The nodes of the STA are called *locations*, and encode the value of the process local variables and the program counter. The directed edges of the STA are called *rules*. A rule models one iteration of

the loop, such that the two locations it connects reflect the values of the local variables of a process before and after the execution of one loop iteration. The rules are labeled by *threshold guards*, which encode conditions from the pseudocode (and may be parameterized by the number n of processes, number f of faults, and the upper bound t on the number of faults).

In the STA in Figure 1, the locations $v0$ and $v1$ encode that a correct process has the value `best` set to 0 and 1, respectively. The locations $c0$ and $c1$ analogously encode that a crash-faulty process has the value `best` set to 0 and 1, respectively. The location \star encodes that a crashed process stopped working and does not restart. We discuss how we model crash faults in more detail in Section 5.2.1.

By looking at the pseudocode, we observe that a correct (resp. crash-faulty) process sends a message containing the value 0, if the value of its variable `best` is 0 (line 3). In the STA, a correct (resp. crash-faulty) process sends a message containing the value 0, if it is in location $v0$ (resp. $c0$). Thus, to test how many processes have sent a message containing the value 0, it is enough to *count* how many processes are in one of the locations $v0$ and $c0$. We denote by $\#\{v0, c0\}$ the number of processes in locations $v0$ and $c0$.

However, the pseudocode of `FloodMin` contains conditions over the number of *received* messages (line 4), rather than the number of *sent* messages. As discussed before, in synchronous systems, all messages sent by correct processes in a given round are received by all correct processes in the same round. On the contrary, not all messages sent by faulty processes are received by all correct processes. In the case of `FloodMin`, as the crash-faulty processes may omit to send messages to the correct processes, the number of received messages may deviate from the number of correct processes that sent a message. We will discuss this discrepancy in more detail in Section 5.

To capture this in the STA, we proceed as follows. Observe that in the pseudocode of `FloodMin`, the processes update their value `best` with the smallest received value (line 5). The correct processes that are in location $v0$ can only stay in $v0$, which is captured by the rule r_1 . The correct processes that are in location $v1$ can either:

- (1) move to the location $v0$, if there is at least one message containing the value 0, sent either by a correct or by a faulty process. This is modeled by the rule r_2 , whose guard φ_1 checks if $\#\{v0, c0\} \geq 1$;
- (2) stay in the location $v1$, if there is no message containing the value 0 sent by a correct process. This is modeled by the rule r_3 , whose guard φ_2 checks if $\#\{v0\} < 1$.

Both the guards φ_1 and φ_2 can be satisfied in location $v1$. This is used to model the non-determinism imposed by the crash-faulty processes that manage to send messages only to a subset of the other processes. The rules r_4, r_5 , and r_6 correspond to the rules r_1, r_2 , and r_3 , respectively, and are used to move processes that are crashing to the locations where they are flagged as crash-faulty. The rules r_7 and r_8 move the crash-faulty processes to the location \star , where they stay forever.

2.3. Counter Systems. The semantics of the STA is defined in terms of a *counter system*. For each location in the STA, $\ell \in \{v0, v1, c0, c1, \star\}$, we have a counter $\kappa[\ell]$ that stores the number of processes located in location ℓ . The counter system is parameterized in the number n of processes, number f of faults, and the upper bound t on the number of faults. Every transition in the counter system updates the counters by moving all processes simultaneously; potentially using a different rule for each process, provided that the guards of the applied rules evaluate to true.

```

1 best := input_value;
2 for each round 1 through  $\lfloor t/k \rfloor + 1$  do
3   broadcast best;
4   receive values  $b_1, \dots, b_\ell$  from others;
5   best :=  $\min \{b_1, \dots, b_\ell\}$ ;
6 done;
7 decide best;

```

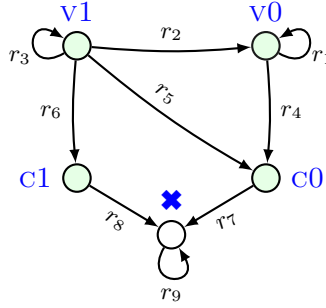


Figure 1: Pseudocode of FloodMin from [CHLT00] and its STA for $k = 1$ and initial values from $\{0, 1\}$

Table 1: The rules of the STA from Figure 1. We omit the rules r_1, r_4, r_7, r_8, r_9 as they have the trivial guard (*true*).

Rule	Guard
r_2	$\varphi_1: \#\{v0, c0\} \geq 1$
r_3	$\varphi_2: \#\{v0\} < 1$
r_5	$\varphi_1: \#\{v0, c0\} \geq 1$
r_6	$\varphi_2: \#\{v0\} < 1$

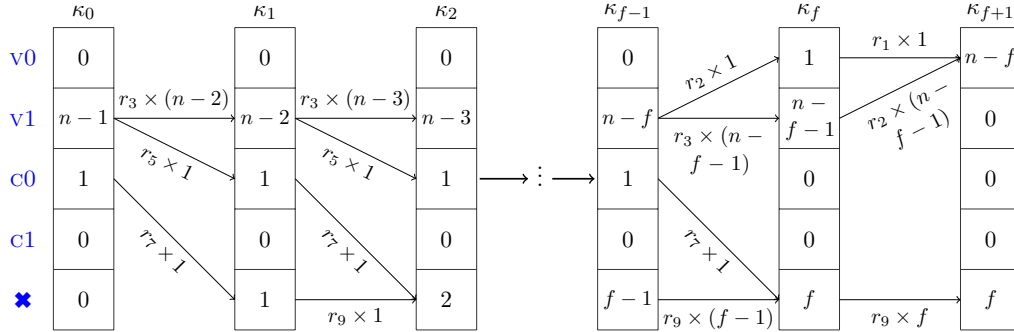


Figure 2: An execution of FloodMin of length $f + 1$.

For the STA of the algorithm FloodMin, presented in Figure 1, in Figure 2 we depict an execution in the counter system for parameter values n, t, f such that $n > t \geq f$ and $f > 3$. Initially, there are $n - 1$ processes in location $v1$, i.e., $\kappa_0[v1] = n - 1$, and one process in location $c0$, i.e., $\kappa_0[c0] = 1$. Observe that these initial values of the counters make all the rules in the STA enabled. This is because all the guards on all rules in the STA are satisfied by these counters, as $\#\{v0, c0\} = \kappa_0[v0] + \kappa_0[c0] = 1 \geq 1$ and $\#\{v0\} = \kappa_0[v0] = 0 < 1$. Thus, in any transition applied to κ_0 , any rule that is outgoing of the locations $v1, c0$ (which are the ones populated in κ_0) can be taken by some process. In particular, the transition outgoing from κ_0 moves: $n - 2$ processes from $v1$ to $v1$ using the self-loop rule r_3 , one process from $v1$ to $c0$ using the rule r_5 , and one process from $c0$ to \times using the rule r_7 .

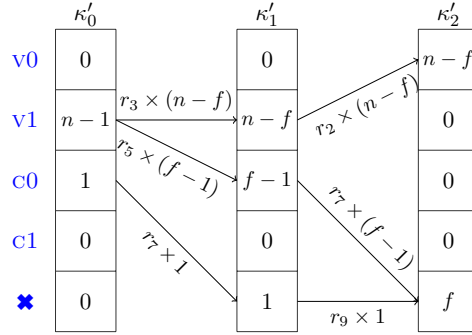


Figure 3: An execution of FloodMin of length 2 that reaches the same configuration as the long execution in Figure 2.

2.4. Bounded Diameter. Consider again the example execution of FloodMin depicted in Figure 2, whose length (i.e., the number of transitions) is $f + 1$. That is, the length of this execution is parameterized in the number f of faults. We see that after the first transition (which we described above) we get a configuration where the values of the counters κ_1 satisfy all guards in the STA again. This is because there is more than one process in the locations v0 and c0 ($\kappa_1[c0] = 1$), and no process in location v0 ($\kappa_1[v0] = 0$). This means that in every round, we can repeat the same transition where one process moves from the location v1 to the location c0, and the other processes in location v1 use the self-loop rule to stay in location v1. Hence, after $f - 1$ transitions, the counters have the following values: there are $n - f$ processes in location v1, one process in c0, and $f - 1$ processes in ✱ (see κ_{f-1} in Figure 2). Since in κ_{f-1} there are in total f processes in the locations reserved for the faulty processes ($\kappa_{f-1}[c0] + \kappa_{f-1}[\text{✱}] = f$), no other processes can be moved from v1 to c0; instead, in the f -th transition, one process moves from the location v1 to v0 using the rule r_2 . This makes the self-loop rule r_3 disabled, as its guard, $\varphi_2 \equiv \#\{v0\} < 1$, evaluates to false. This triggers the move of the $n - f - 1$ process from the location v1 to the location v0 in the $f + 1$ -st (last) transition.

In Figure 3, we see an execution of length two, where in the first transition, $f - 1$ processes move from the location v1 to the location c0 via the rule r_5 , and in the second transition, $n - f$ processes move from the location v1 to the location v0 using the rule r_2 . That is, while a configuration where the counters have values as in κ_{f+1} can be reached by a long execution as depicted in Figure 2, it can also be reached in just two steps as depicted in Figure 3 (observe that $\kappa'_2 = \kappa_{f+1}$). We are interested in whether there is a natural number k (independent of the parameters n , t and f) such that for an arbitrary execution in any counter system, we can always find a shorter execution starting and ending in the same configuration, such that the short execution is of length at most k . If this is the case, we say that the STA has *bounded diameter*. In [SKWZ19], we formalize the concept of bounded diameter for an STA, by adapting the definition of diameter from [BCCZ99]. We also introduce an SMT-based semi-decision procedure for computing a bound on the diameter. The procedure enumerates candidates for the diameter bound, and checks (by calling an SMT solver) if the number is indeed the diameter; if it finds such a bound, it terminates.

2.5. Bounded Model Checking. To verify the safety properties of an algorithm for all values of the parameters, we can solve the dual parameterized reachability problem. Namely, a safety property holds for all values of the parameters iff there do not exist values of the parameters for which a bad configuration, where the safety property is violated, is reachable. The existence of a bound on the diameter motivates the use of bounded model checking, as we can search for violations of the a safety property in executions with length up to the diameter. Crucially, this approach is complete: if an execution reaches a bad configuration, this bad configuration is already reached by an execution of bounded length. Thus, once the diameter is found, we encode the violation of a safety property using a formula in Presburger arithmetic, and use an SMT solver to check for violations. More details on how this encoding is done can be found in [SKWZ19].

The SMT queries that are used for computing the diameter and encoding the violation of the safety properties contain quantifiers for dealing with the parameters symbolically. Surprisingly, the performance of the SMT solvers on these queries is very good, reflecting the recent progress in dealing with quantified queries. We found that the diameter bounds of synchronous algorithms in the literature are tiny (from 1 to 8), which makes our approach applicable in practice. The verified algorithms are given in Section 6.

2.6. Undecidability. In [SKWZ19], we formalized the parameterized reachability problem, which is the question whether a certain configuration can be reached in some counter system for some values of the parameters. We showed that this problem is in general undecidable for STA, by reduction from the halting problem of two-counter machines [Min67]. The detailed undecidability proof can be found in [SKWZ21b]. In particular, the undecidability implies that some STA have unbounded diameters. However, the SMT-based semi-decision procedure for computing the diameter can still automatically compute the diameter bounds for our benchmarks. Remarkably, these bounds are independent of the parameters. In addition, we also theoretically establish the existence of a bound on the diameter, which is independent of the parameters, for a class of STA that satisfy certain conditions.

3. PARAMETERIZED VERIFICATION OF ASYNCHRONOUS ALGORITHMS

3.1. Asynchronous Algorithms. In an asynchronous algorithm, at each time point, exactly one process performs a step. That is, the steps of the processes are interleaved. This is in contrast to the synchronous semantics, where all processes take steps simultaneously. Given the pseudocode description of the algorithm, we can interpret one line of code as an atomic unit of execution of a process. As in the synchronous case, the processes execute send-to-all, receive, and local computation steps. When a process performs a send-to-all step, it places one message in the message buffers of all the other processes. In the receive step, a process takes some messages out of the incoming message buffer. Observe that not necessarily all messages that were sent are in the buffer, and the process may possibly take no message out of it. The local computation step updates the local variables based on the received messages. Often, a step in the asynchronous semantics is more coarse-grained, in the sense that it combines all three steps described above. That is, in this coarse-grained case, a step consists of receiving messages, updating the state, and sending one or more messages.

```

1 int v:=input({0,1});
2 bool accept:=false;
3 while (true) do {
4   if (v = 1) then send <ECHO> to all;
5   receive messages from other processes;
6   if received <ECHO> from  $\geq t + 1$  processes
7     then v:=1;
8   if received <ECHO> from  $\geq n - t$  processes
9     then accept:=true;
10 }

```

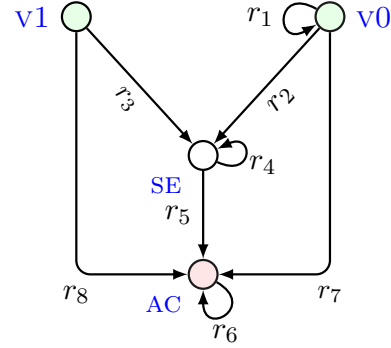


Figure 4: Pseudocode of reliable broadcast à la [ST87b] and its threshold automaton.

Table 2: The rules of the TA from Figure 4. We omit the rules r_1, r_4, r_6 as it has the trivial guard (*true*) and no update.

Rule	Guard	Update
r_2	$\varphi_1: x \geq (t + 1) - f$	$x++$
r_3	<i>true</i>	$x++$
r_5	$\varphi_2: x \geq (n - t) - f$	—
r_7	$\varphi_2: x \geq (n - t) - f$	$x++$
r_8	$\varphi_2: x \geq (n - t) - f$	$x++$

As we do not restrict which messages are taken out of the buffer during a step, we cannot bound the time needed for message transmission. Moreover, we do not restrict the order in which processes take steps, so we cannot bound the time between two steps of a single process. Typically, we are interested in verifying safety (i.e., properties stating that nothing bad ever happens) under these conditions. However, for liveness (i.e., properties stating that something good eventually happens) this is problematic. We need the messages to be delivered eventually, and we need the correct processes to take steps from time to time. That is, liveness is typically preconditioned by fairness guarantees: every correct process takes infinitely many steps and every message sent from a correct process to a correct process is eventually received. These constraints correspond roughly to the asynchronous system defined in [FLP85] in which consensus is not solvable. However, these constraints are sufficient for fault-tolerant broadcast. Intuitively, this is because broadcast has weaker liveness requirements than consensus.

We now look at a classic example of a fault-tolerant distributed algorithm, namely the broadcasting algorithm by Srikanth and Toueg [ST87a], which we refer to as STRB. The description of its code is given in Figure 4. The algorithm tolerates Byzantine failures, where faulty processes may send messages only to a subset of the processes (or even send conflicting data). It solves the reliable broadcast problem, by ensuring that every message sent to a non-faulty process is eventually received.

The algorithm in Figure 4 does this by forwarding message content received from other processes and only accepting a message content if it was received from a quorum of processes. One instance of the algorithm is executed for each message content m . Initially, the variable v captures whether a process has received m , and, if this is the case, it stores the

value 1. A process that has received m (i.e., it has $v = 1$) sends **ECHO** to all (line 4). In an implementation, the message would be of the form (ECHO, m) , that is, it would be tagged with **ECHO**, and carry the content m to distinguish different instances running in parallel; also it would suffice to send the message once instead of sending it in each iteration. If the guard in line 6 evaluates to true at a process p , then p has received $t + 1$ **ECHO** messages. This means that at least one correct process has forwarded the message. This triggers the process p to also forward the message. If a process p receives $n - t$ **ECHO** messages, it finally accepts the message content stored in m due to line 8. The reason this algorithm works is that the combination of the thresholds $n - t$ and $t + 1$, as well as the resilience condition $n > 3t$, ensures that if one correct process has $n - t$ **ECHO** messages, then every other correct process will eventually receive at least $t + 1$ messages (there are $t + 1$ correct processes among any $n - t$ processes). This implies that every correct process will forward the message, and since there are at least $n - t$ correct processes, every correct process will accept. However, manual reasoning about this arithmetics over parameters is subtle and error-prone. Our verification techniques automate this reasoning; they are designed to deal efficiently with threshold expressions and resilience conditions.

3.2. Asynchronous Threshold Automata. Similarly as in *STA*, the nodes in *asynchronous threshold automata (TA)* encode the locations of processes, and the edges (i.e., the rules) represent local transitions. In addition, the TA (i) have shared variables, and (ii) their rules are labeled with expressions of the form $\varphi \mapsto \text{act}$. A process moves along an edge labelled by $\varphi \mapsto \text{act}$ and performs an action act , only if the condition φ , called a *threshold guard*, evaluates to true.

We model the algorithm **STRB** [ST87a], whose pseudocode is in Figure 4 on the left, using the TA shown in Figure 4 on the right. We use a shared variable x to capture the number of **ECHO** messages sent by correct processes. We have two threshold guards: $\varphi_1: x \geq (t + 1) - f$ and $\varphi_2: x \geq (n - t) - f$. Depending on the initial value of a correct process, 0 or 1, the process is initially either in location $v0$ or in $v1$ (which stand for “value 0” and “value 1”, respectively). If its value is 1, then the process broadcasts **ECHO**, and executes the rule $r_3: \text{true} \mapsto x++$. This is modelled by a process moving from location $v1$ to location se (“sent echo”) and increasing the value of x . If its value is 0, then the process has to wait to receive enough messages. That is, it waits for φ_1 to become true, and then it broadcasts the **ECHO** message and moves to location se . This is captured by the rule r_2 , which is labelled by $\varphi_1 \mapsto x++$. Finally, once a process has enough **ECHO** messages for the guard φ_2 to evaluate to true, it sets `accept` to true and moves to ac (“accept”: it encodes that the variable `accept` has been set to true; cf. line 9 in Figure 4). Thus, the rule r_5 is labelled by the guard φ_2 (and no update of the shared variable x), whereas the rules r_7 and r_8 are labeled by $\varphi_2 \mapsto x++$.

3.3. Counter Systems. Similarly to *STA*, the semantics of TA is captured by counter systems. Recall that in a counter system, instead of storing the location of each process, we count the number of processes in each location, as all processes are identical. In the counter systems for the asynchronous semantics, we also store the values of the shared variables, which are incremented as the processes execute the rules. Therefore, a configuration in a counter system comprises: (i) the values of the counters for each location, (ii) the values of the shared variables, and (iii) the parameter values. A configuration is initial if all processes

are in initial locations (in our example, $v0$ or $v1$), and all shared variables have value 0 (in our example, $x = 0$). A transition of a process along a rule from location ℓ to location ℓ' — labelled by $\varphi \mapsto \text{act}$ — is modelled by the following configuration update: (i) the counter of ℓ is decreased by 1, and the counter of ℓ' is increased by 1, (ii) the shared variables are updated according to the action act , and (iii) the parameter values are unchanged. Observe that this transition is possible only if the guard φ of the rule evaluates to true in the original configuration.

As we will see below, the key ingredient of our technique is acceleration of transitions, that is, we allow many processes to move along the same edge simultaneously. In the resulting configuration, the counters and shared variables are updated depending on the number of processes that participate in the accelerated transition. It is important to notice that any accelerated transition can be encoded in SMT.

3.4. Reachability. In [KLVW17a], we determine a finite set of execution “patterns”, and then analyse each pattern separately. These patterns restrict the order in which the threshold guards become true (if ever). Namely, we observe how the set of guards that evaluate to true changes along each execution. In our example TA for the algorithm STRB, given in Figure 4, there are two (non-trivial) guards, φ_1 and φ_2 . Initially, both evaluate to false, as $x = 0$. During an execution, none, one, or both of them become true. Note that once they become true, they can never evaluate to false again, as the number of sent messages, stored in the shared variable x , cannot decrease. Thus, there is a finite set of execution patterns.

For instance, a pattern $\{\} \dots \{\varphi_1\} \dots \{\varphi_1, \varphi_2\}$ captures all finite executions τ that can be represented as $\tau = \tau_1 \cdot t_1 \cdot \tau_2 \cdot t_2 \cdot \tau_3$, where τ_1, τ_2, τ_3 are sub-executions of τ , and t_1 and t_2 are transitions. No threshold guard is satisfied in a configuration visited by τ_1 , and only the guard φ_1 is satisfied in all configurations visited by τ_2 . Both guards are satisfied in all configurations visited by τ_3 . Observe that in this pattern, the transitions t_1 and t_2 change the evaluation of the guards. Another pattern is given by $\{\} \dots \{\varphi_2\} \dots \{\varphi_1, \varphi_2\}$. Here, the guard φ_2 is satisfied before the guard φ_1 . In the pattern $\{\} \dots \{\varphi_1\}$, the guard φ_2 is never satisfied.

To perform verification, we have to analyse all execution patterns. For each pattern, we construct a so-called *schema*, defined as a sequence of accelerated transitions, whose free variables are the number of processes that execute the transitions and the parameter values. Recall that for the TA in Figure 4, the process transitions are modelled by rules denoted with r_i , for $i \in \{1, \dots, 8\}$. For instance, the pattern $\{\} \dots \{\varphi_1\}$ produces the schema:

$$\mathcal{S} = \{\} \underbrace{r_1, r_3, r_3}_{\tau_1} \underbrace{t_1}_{t_1} \{\varphi_1\} \underbrace{r_1, r_2, r_3, r_4}_{\tau_2} \{\varphi_1\}.$$

The above schema has two segments, τ_1 and τ_2 , corresponding to $\{\}$ and $\{\varphi_1\}$, respectively. In each of them we list all the rules that can be executed according to the guards that evaluate to true, in a fixed natural order: only r_1 and r_3 can be executed if no guard evaluates to true, and the rules r_1, r_2, r_3, r_4 can be executed only if the guard φ_1 holds true. Additionally, we have to list all the candidate rules for the transition t_1 that can change the evaluation of the guards. In our example, the guard φ_1 can evaluate to true only if a transition along the rule r_3 is taken.

We say that an execution follows the schema \mathcal{S} if its transitions appear in the same order as in \mathcal{S} , but they are accelerated (every transition is executed by a number of

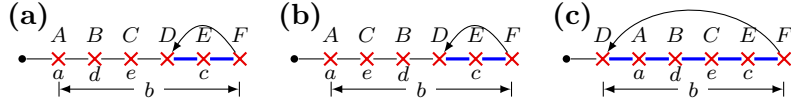


Figure 5: Three out of 18 shapes of lassos that satisfy the formula $\mathbf{F}(a \wedge \mathbf{F}d \wedge \mathbf{F}e \wedge \mathbf{G}b \wedge \mathbf{G}fc)$. The crosses show cut points for: (A) formula $\mathbf{F}(a \wedge \mathbf{F}d \wedge \mathbf{F}e \wedge \mathbf{G}b \wedge \mathbf{G}fc)$, (B) formula $\mathbf{F}d$, (C) formula $\mathbf{F}e$, (D) loop start, (E) formula $\mathbf{F}c$, and (F) loop end.

processes, possibly zero). For example, if (r, k) denotes that k processes execute the rule r simultaneously, then the execution $\rho = (r_1, 2)(r_3, 3)(r_2, 2)(r_4, 1)$ follows the schema \mathcal{S} , where the transitions of the form $(r, 0)$ are omitted. In this case, we prove that for each execution τ of pattern $\{\} \dots \{\varphi_1\}$, there is an execution τ' that follows the schema \mathcal{S} , and the executions τ and τ' reach the same configuration (when executed from the same initial configuration). This is achieved by *mover analysis*: inside any segment in which the set of guards that evaluate to true is fixed, we can swap adjacent transitions (that are not in a natural order). In this way, we gather all transitions of the same rule next to each other, and transform them into a single accelerated transition. For example, the execution $\tau = (r_3, 2)(r_1, 1)(r_3, 1)(r_1, 1)(r_2, 1)(r_4, 1)(r_2, 1)$ can be transformed into the execution $\tau' = \rho$ from above, and they reach the same configurations. Therefore, instead of checking reachability for all executions of the pattern $\{\} \dots \{\varphi_1\}$, it is sufficient to analyse reachability only for the executions that follow the schema \mathcal{S} .

Every schema is encoded as an SMT query over linear integer arithmetic with free variables for acceleration factors, parameters, and counters. An SMT model gives us an execution of the counter system, which typically disproves safety.

For example, consider the following reachability problem: Can the system reach a configuration with at least one process in ℓ_3 ? For each SMT query, we add the constraint that the counter of ℓ_3 is non-zero in the final configuration. If the query is satisfiable, then there is an execution where at least one process reaches ℓ_3 . Otherwise, there is no such execution following the particular schema, where a process reaches ℓ_3 . That is why we have to check all schemas.

3.5. Safety and Liveness. In [KLVW17b] we introduced a fragment of Linear Temporal Logic called ELTL_{FT} . Its atomic propositions test location counters for zero. Moreover, this fragment uses only two temporal operators: \mathbf{F} (eventually) and \mathbf{G} (globally). Our goal is to check whether there exists a counterexample to a temporal property, and thus formulas in this fragment represent negations of safety and liveness properties.

Our technique for verification of safety and liveness properties uses the reachability method from Section 3.4 as its basis. As before, we want to construct schemas that we can translate to SMT queries and check their satisfiability. Note that violations of liveness properties are infinite executions of a lasso shape, that is, $\tau \cdot \rho^\omega$, where τ and ρ are finite executions. Hence, we have to enumerate the patterns of lassos. These shapes depend not only on the values of the thresholds, but also on the evaluations of atomic propositions that appear in temporal properties. We single out configurations in which atomic propositions evaluate to true, and call them *cut points*, as they “cut” an execution into finitely many segments (see Figure 5).

```

1  bool v := input_value({0, 1});
2  int rnd := 1;
3  while (true) do
4    send (R,rnd,v) to all;
5    wait for n - t messages (R,rnd,*);
6    if received (n + t) / 2
7      messages (R,rnd,w)
8    then send (P,rnd,w,D) to all;
9    else send (P,rnd,?) to all;
10   wait for n - t messages (P,rnd,*);
11   if received at least t + 1
12     messages (P,rnd,w,D) then {
13     v := w;
14     if received at least (n + t) / 2
15       messages (P,rnd,w,D)
16     then decide w;
17   } else v := random({0, 1});
18   r := r + 1;
19  od

```

Figure 6: Pseudocode of the Ben-Or algorithm for Byzantine faults, with $n > 5t$

We combine these cut points with those “cuts” in which the threshold guards become enabled (as in the reachability analysis). All the possible orderings in which the evaluations of threshold guards and formulas become true, give us a finite set of lasso patterns.

We construct a schema for each shape by first defining schemas for each of the segments between two adjacent cut points. On one hand, for reachability, it is sufficient to execute all enabled rules of that segment exactly once in the natural order. Thus, each sub-execution τ_i can be transformed into τ'_i that follows the segment’s schema, so that τ_i and τ'_i reach the same final configuration. On the other hand, the safety and liveness properties reason about atomic propositions inside executions. To this end, we introduced a *property-specific mover analysis*: While classic mover analysis [Lip75] generates an alternative trace that leads to the same final configuration as the original trace, in property-specific mover analysis, we need to maintain not only the final configuration but also trace properties, e.g., invariants (that is, properties that hold in all configurations along the trace, and not only in the final one). This new mover analysis allows us to construct schemas by executing all enabled rules a fixed number of times in a specific order. The number of rule repetitions depends on a temporal property; it is typically two or three.

For each lasso pattern we encode its schema in SMT and check its satisfiability. As ELTL_{FT} formulas are negations of specifications, an SMT model gives us a counterexample. If no schema is satisfiable, the temporal property holds true.

4. PARAMETERIZED VERIFICATION OF ASYNCHRONOUS RANDOMIZED MULTI-ROUND ALGORITHMS

4.1. Randomized Algorithms. The randomized multi-round algorithms circumvent the impossibility of asynchronous consensus [FLP85] by relaxing the termination requirement to almost-sure termination, i.e., termination with probability 1. The processes execute an infinite sequence of asynchronous rounds, and update their local variables based on the messages received only in the current round. Asynchronous algorithms with this feature are called *communication closed* [EF82, DDMW19].

A prominent example of a randomized multi-round algorithm is Ben-Or’s fault-tolerant binary consensus [Ben83] algorithm in Figure 6, which we refer to as Ben-Or. While

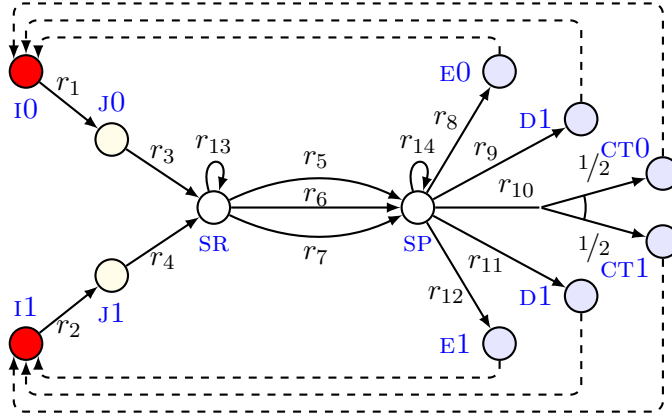


Figure 7: The Ben-Or algorithm as PTA with resilience condition $n > 3t \wedge t > 0 \wedge t \geq f \geq 0$.

Table 3: The rules of the PTA from Figure 7. We omit rules r_1, r_2, r_{13}, r_{14} as they have the trivial guard (*true*) and no update.

Rule	Guard	Update
r_3	<i>true</i>	x_0++
r_4	<i>true</i>	x_1++
r_5	$x_0 + x_1 \geq n - t - f \quad \wedge \quad x_0 \geq (n + t)/2 - f$	y_0++
r_6	$x_0 + x_1 \geq n - t - f \quad \wedge \quad x_1 \geq (n + t)/2 - f$	y_1++
r_7	$x_0 + x_1 \geq n - t - f \quad \wedge \quad x_0 \geq (n - 3t)/2 - f \quad \wedge \quad x_1 \geq (n - 3t)/2 - f$	$y_?++$
r_8	$y_0 + y_1 + y_? \geq n - t - f \quad \wedge \quad y_? \geq (n - 3t)/2 - f \quad \wedge \quad y_0 \geq t + 1 - f$	—
r_9	$y_0 + y_1 + y_? \geq n - t - f \quad \wedge \quad y_0 > (n + t)/2 - f$	—
r_{10}	$y_0 + y_1 + y_? \geq n - t - f \quad \wedge \quad y_? \geq (n - 3t)/2 - f \quad \wedge \quad y_? > n - 2t - f - 1$	—
r_{11}	$y_0 + y_1 + y_? \geq n - t - f \quad \wedge \quad y_1 > (n + t)/2 - f$	—
r_{12}	$y_0 + y_1 + y_? \geq n - t - f \quad \wedge \quad y_? \geq (n - 3t)/2 - f \quad \wedge \quad y_1 \geq t + 1 - f$	—

the algorithm is executed under the asynchronous semantics, the processes have a local variable `rnd` that stores the round number, and use it to tag the messages that they send in round `rnd`. Observe that the guards only take into account messages which are tagged with the round number `rnd`, that is, the algorithm operates only on messages from the current round. In the case of the Ben-Or algorithm, each round consists of two stages: (i) the processes first exchange messages tagged with R, wait until the number of received messages reaches a certain threshold (the expression over the parameters in line 5), (ii) and then exchange messages tagged with P. As in the previous examples, n denotes the number of processes, among which at most t may be faulty. In the Ben-Or algorithm in Figure 6, the processes are Byzantine-faulty. The thresholds $n - t$, $(n + t)/2$ and $t + 1$ that occur in the pseudocode, combined with the resilience condition $n > 5t$, ensure that no two correct processes ever decide on different values. If there is no “strong majority” for a decision value in line 14, a process chooses a new value by tossing a coin in line 17.

4.2. Probabilistic Threshold Automata. As discussed above, randomized algorithms typically have an unbounded number of asynchronous rounds and randomized choices. Probabilistic threshold automata (PTA), introduced in [BKLW19], are extensions of asynchronous TA that allow formalizing these features.

A PTA modelling the Ben-Or algorithm from Figure 6 is shown in Figure 7. The behavior of a process in a single round is modelled by the solid edges. Note that in this case, the threshold guards should be evaluated according to the values of shared variables, e.g., x_0 and x_1 , in the observed round. The dashed edges model round switches: once a process reaches a final location in a round, it moves to an initial location of the next round. The coin toss is modelled by the branching rule r_{10} : a process in location SP can reach either location CT0 or location CT1 by moving along this fork, both with probability $1/2$.

4.3. Unboundedly Many Rounds. In order to overcome the issue of unboundedly many rounds, we prove that we can verify PTA by analysing a one-round automaton that fits in the framework of Section 3. In [BKLW19], we prove that one can reorder the transitions of any fair execution such that their round numbers are in a non-decreasing order. The obtained ordered execution is stutter-equivalent to the original one. Thus, both executions satisfy the same LTL_X properties over the atomic propositions of one round. In other words, the distributed system can be transformed to a sequential composition of one-round systems.

The main problem with isolating a one-round system is that the consensus specifications often talk about at least two different rounds. In this case we need to use round invariants that imply the specifications. For example, if we want to verify agreement, we have to check that no two processes decide different values, possibly in different rounds. We do this in two steps: (i) we check the round invariant that no process changes its decision from round to round, and (ii) we check that within a round no two processes disagree.

4.4. Probabilistic Properties. The semantics of a PTA is an infinite-state Markov decision process (MDP), where the non-determinism is traditionally resolved by an adversary. In [BKLW19], we restrict our attention to so-called *round-rigid adversaries*, that is, fair adversaries that generate executions in which a process enters round $\text{rnd} + 1$ only after all processes finished round rnd . We introduced these adversaries as a first step towards parameterized verification of fault-tolerant randomized distributed algorithms for probabilistic properties. Intuitively, in the probabilistic world, round-rigid adversaries are the correspondence of representative executions in the non-deterministic models that are the result of a reduction for communication-closed algorithms [EF82, CCM09, DDMW19]. In Section 4.5 we discuss the link between round-rigid adversaries and the standard notion of weak adversaries.

Verifying almost-sure termination under round-rigid adversaries calls for distinct arguments. Our methodology follows the lines of the manual proof of Ben-Or’s consensus algorithm by Aguilera and Toueg [AT12]. However, our arguments are not specific to Ben-Or’s algorithm, and apply to other randomized distributed algorithms (see Table 4). Compared to their paper-and-pencil proof, the threshold automata framework required us to provide a more formal setting and a more informative proof, also pinpointing the needed hypotheses. The crucial parts of our proof are automatically checked by the model checker ByMC.

4.5. Weak Adversaries. The approach from Section 4.4 leaves a gap between round-rigid adversaries and the classic adversary definitions we find in the distributed computing literature. This problem is addressed in [BLW21] where the standard notion of a “weak adversary” is considered. Weak adversaries pose a formalization challenge in the counter system semantics of TA. The reason is that these adversaries are defined over individual processes and messages; notions that do not exist in the counter system representation. As a result, a more concrete semantics of TA was introduced, which explicitly captures processes, sets of received messages for each process, and threshold guards over the number of specific messages in these sets. For this semantics, [BLW21] contains a reduction theorem from weak adversaries to round-rigid adversaries. While [BLW21] does not contain a formalization of the abstraction step from the explicit model to TA, we conjecture that such a proof can be done based on the ideas that underlie Section 5.1 below. If this is the case, the verification results from Section 4 and the lower part of Table 4 hold under a wider class of adversaries.

5. MODELING

5.1. From Pseudocode to Threshold Automata. Observe that the parameterized verification approaches, presented in Sections 2, 3, and 4, take as input threshold automata, whose guards are evaluated over the global state (i.e., the messages sent by the processes). When modeling threshold-guarded distributed algorithms with verification in mind, we are faced with a formalization gap between the threshold automata and the algorithm descriptions given in terms of pseudocode, which is supposed to run locally on a node and contains guards over the local state (i.e., the messages received locally by a process). For many cases, this formalization gap is easy to overcome, i.e., the translation from pseudocode to a threshold automaton is immediate, and can easily be done manually. However, for some algorithms this is not the case.

Consider the **Ben-Or** algorithm given in Figure 6. The main challenge we faced in the formalization is to express the path that leads to the coin toss in line 17. Observe that to reach line 17, a process has received at least $n - t$ messages with the label **P** (line 10), as well as less than $t + 1$ messages of type $(\mathbf{P}, \mathbf{rnd}, 0, \mathbf{D})$ and less than $t + 1$ messages of type $(\mathbf{P}, \mathbf{rnd}, 1, \mathbf{D})$, since the placeholder w in line 11 can take both input values from the set $\{0, 1\}$. In other words, if we were to rewrite the pseudocode as a set of guarded commands, the coin toss would be guarded by an expression containing (i) a lower bound condition over the local variables, i.e., the number of received messages labeled by **P** (at least $n - t$ messages), and (ii) two upper bound conditions over the local variables, i.e., the number of received messages of type $(\mathbf{P}, \mathbf{rnd}, w, \mathbf{D})$ (less than $t + 1$ messages), where $w \in \{0, 1\}$. However, as is required by the threshold automata framework, the constraints over the global variables that can lead to the coin toss are captured by the guard of the rule r_{10} , given in Figure 7. It only contains lower bound conditions over the global variables, which are the number of globally sent messages. This translation between lower and upper bound conditions expressed over the local and global variables is non-trivial, and when done manually, requires intuition on the operation of the algorithm. As a result, in writing our benchmarks, we observed that when done manually, this translation is error-prone.

In [SKWZ20, SKWZ21a], we address the problem of automating the translation from pseudocode to a threshold automaton, for both asynchronous and synchronous threshold-guarded distributed algorithms. For randomized algorithms, whose local control flow

motivated this line of work, the same results as for the asynchronous algorithms apply. In order to automate the translation, we need to formalize the local transition relation expressed by the pseudocode. To this end, we introduce a variant of TA, called *receive TA*, whose rules are guarded by expressions over the *local* receive variables.

Let $\text{nr}_i(m)$ denote such a *receive variable*, which encodes how many messages of type m process i has received, and let $\text{ns}(m)$ denote a *send variable*, which stores the number of sent messages of that type. Translating guards over receive variables $\text{nr}_i(m)$ to guards over send variables $\text{ns}(m)$, for each message type m , is based on *quantifier elimination* for Presburger arithmetic [Pre29, Coo72, Pug92]. In order to obtain the most precise guards over the send variables, in the quantifier elimination step, the guards over the receive variables are strengthened by an *environment assumption* Env . As we will see in Section 5.2 below, the environment assumption encodes the relationship between the receive and send variables, which depends on the degree of synchrony and the fault model. Given a guard φ over the receive variables, a guard $\hat{\varphi}$ over the send variables can be computed automatically by applying quantifier elimination to the formula $\varphi' \equiv \exists \text{nr}_i(m_0) \dots \exists \text{nr}_i(m_k) (\varphi \wedge \text{Env})$, where m_0, \dots, m_k are the message types that define the messages exchanged in the execution of the algorithm. This produces a quantifier-free formula $\hat{\varphi}$ over the send variables.

The translation procedure was implemented in a prototype [SKWZ20, SKWZ21a] that automatically generates guards over the send variables, by using Z3 [dMB08] to automate the quantifier elimination step. By applying the translation procedure to the guard of every rule in the receive threshold automaton given as input, a threshold automaton with no receive variables is obtained automatically. In [SKWZ20, SKWZ21a], it was shown that the translation procedure based on quantifier elimination is sound for both the asynchronous and synchronous case. This means that a system of n copies of an automatically generated TA over the send variables is an *overapproximation* of a system of n copies of the receive TA given as input. For a class of distributed algorithms that captures typical distributed algorithms found in the literature, it was also shown to be complete.

The translation procedure based on quantifier elimination thus closes the formalization gap between the original description of an algorithm (using received messages) and the threshold automaton of the algorithm, given as an input to a verification tool. More precisely, parameterized verification of threshold-guarded distributed algorithms, starting with a formal model of the pseudocode given by a receive threshold automaton, can be fully automated by: (i) automatically producing a formal model suitable for verification by applying the translation procedure based on quantifier elimination and (ii) automatically verifying its correctness by applying existing tools.

5.2. Modeling Faults in Threshold Automata. To model the behavior that the faults introduce, when producing a (receive) TA for a given algorithm, we have to capture the semantics of executing the code on a faulty process. To capture the faulty semantics in the automaton, we typically need to introduce additional locations or rules, depending on the fault model. Also, depending on the fault model, we have different constraints on the values of the receive and send variables, which are encoded by an *environment assumption* Env (required to obtain the more precise guards after the quantifier elimination discussed above).

We consider two types of faults in this paper – crash and Byzantine faults. In the case of crash faults, a process may crash in the middle of a send-to-all operation, which results in the message being sent only to a subset of processes. In the case of Byzantine faults, no assumptions are made on the internal behavior of the faulty processes. That is,

Byzantine-faulty processes may send any message in any order to any process or fail to send messages.

5.2.1. *Crash Faults.* Crash-faulty processes stop executing the algorithm prematurely and cannot restart. To model this behavior in a TA, we add so-called “crash” locations to which processes move from the “correct” locations. Processes that move to the “crash” locations remain there forever. In addition, we introduce send variables $\text{ns}_f(m)$, for each message type m , that count the number of messages of type m sent by processes which are *crashing*, i.e., by processes that are moving from a “correct” to a “crashed” location.

The TA thus models the behavior of both correct and faulty processes explicitly. This allows us to express so-called *uniform* properties that also refer to states of faulty processes.¹ The environment assumption Env imposes constraints on the number of processes allowed to populate the “crash” locations, and on the number of received messages of each message type m . In particular, the environment assumption Env requires that there are at most f processes in the “crash” locations, where f is the number of faulty processes. Additionally, for each message type m , the number of received messages of type m , stored in the receive variable $\text{nr}_i(m)$ does not exceed the number of messages of type m sent by the correct and the crash-faulty processes, stored in the send variables $\text{ns}(m)$ and $\text{ns}_f(m)$, respectively.

In the synchronous case, where there exist strict guarantees on the message delivery, the environment assumption Env also bounds the value of the receive variables from below: the constraint $\text{ns}(m) \leq \text{nr}_i(m)$ encodes that all messages sent by correct processes are received in the round in which they are sent.

5.2.2. *Byzantine Faults.* To model the behavior of the Byzantine-faulty processes, which can act arbitrarily, no new locations and rules are introduced in the TA. Instead, the TA is used to model the behavior of the correct processes, and the effect that the Byzantine-faulty processes have on the correct ones is captured in the guards and the environment assumption. The number of messages sent by the Byzantine-faulty processes is overapproximated by the parameter f , which denotes the number of faults. That is, in the environment assumption Env , we have the constraint $\text{nr}_i(m) \leq \text{ns}(m) + f$, which captures that the number of received messages of type m does not exceed $\text{ns}(m) + f$, which is an upper bound on the number of messages sent by the correct and Byzantine-faulty processes.

Since we do not introduce locations that explicitly model the behavior of the Byzantine-faulty processes, the TA is used to model the behavior of the $n - f$ correct processes only.² In addition to the constraint that bounds the number of received messages from above, the environment assumption for the synchronous case also contains the constraint $\text{ns}(m) \leq \text{nr}_i(m)$. It is used to bound the number of received messages from below, and ensure that all messages sent by correct processes are received.

¹For instance, in consensus “uniform agreement states that “no two processes decide on different values”, while (non-uniform) “agreement” states that “no two *correct* processes decide on different values”.

²As classically no assumptions are made on the internals of Byzantine processes, it does not make sense to consider uniform properties. Thus we also do not need Byzantine faults explicit in the model.

6. TOOLS: BYZANTINE MODEL CHECKER AND SYNC τ A

Byzantine Model Checker (ByMC) is an experimental tool for verification of threshold-guarded distributed algorithms. Version 2.4.4 is available from GitHub [ByM19]. It implements the techniques for systems of asynchronous threshold automata and probabilistic threshold automata, which are discussed in Section 3 and Section 4. ByMC is implemented in OCaml and it integrates with SMT solvers via the SMTLIB interface [BFT16] as well as OCaml bindings for Microsoft Z3 [dMB08].

The techniques for synchronous threshold automata—discussed in Section 2—are implemented in another prototype tool [STA]. This prototype is implemented in Python and it integrates with the SMT solvers Microsoft Z3 and CVC4 [BCD⁺11] via the SMTLIB interface. The reason for this prototype not being integrated in ByMC is technical: Synchronous threshold automata required another parser and another internal representation.

6.1. Inputs to ByMC. ByMC accepts two kinds of inputs: (asynchronous) threshold automata written in the tool-specific format and threshold-guarded algorithms written in Parametric Promela [JKS⁺13b]. We do not give complete examples here, as the both formats are quite verbose. A discussion about the both formats can be found in the tool paper [KW18].

To give the reader an idea about the inputs we presents excerpts of the reliable broadcast, which was compactly presented using the graphical notation in Figure 4.

All benchmarks discussed in this paper are available from the open access repository [BB19].

6.2. Overview of the techniques implemented in ByMC. Table 4 shows coverage of various asynchronous and randomized algorithms with these techniques.

The most performant SMT-based techniques in Table 4 are presented in this paper and match its sections as follows:

- The technique for safety verification of asynchronous algorithms is called SMT-S (see Section 3).
- The technique for liveness verification of asynchronous algorithms is called SMT-L (see Section 3).
- The technique for verification of multi-round randomized algorithms is called SMT-MR (see Section 4).

For comparison, we give figures for the older techniques CA+SPIN, CA+BDD, and CA+SAT, which are shown in gray. For a detailed tutorial on these older techniques, we refer the reader to [KVW16]. For the context, we give only a brief introduction to these techniques below. As one can see, the techniques that are highlighted in this survey are the most efficient techniques that are implemented in ByMC. After the brief introduction of the techniques, we give further experimental explanation for the efficiency of the SMT-based techniques.

Technique CA+SPIN. This was the first technique that was implemented in ByMC in 2012 [JKS⁺13a, JKS⁺12, GKS⁺14]. In contrast to the SMT-based approaches highlighted in this survey, this technique abstracts every integer counter $\kappa[\ell]$ to an abstract value that corresponds to a symbolic interval. The set of symbolic intervals is constructed automatically by static analysis of threshold guards and temporal properties. For instance, the reliable

```

1 #define V0 0 /* the initial state */
2 #define V1 1 /* the init message received */
3 #define SE 2 /* the echo message sent */
4 #define AC 3 /* the accepting state */
5 #define PC_SZ 4
6
7 symbolic int N; /* the number of processes: correct + faulty */
8 symbolic int T; /* the threshold */
9 symbolic int F; /* the actual number of faulty processes */
10
11 int nsnt = 0; /* the number of messages sent by the correct processes */
12
13 assume(N > 3 * T && T > 0 && F >= 0 && F <= T);
14
15 atomic prec_unforg = all(Proc:pc == V0);
16 atomic prec_corr = all(Proc:pc == V1);
17 atomic ex_acc = some(Proc:pc == AC);
18 atomic all_acc = all(Proc:pc == AC);
19 atomic in_transit = some(Proc:nrcvd < nsnt);
20
21 active[N - F] proctype Proc() {
22   byte pc = 0; byte next_pc = 0; /* the program counter (and its next value) */
23   int nrcvd = 0; int next_nrcvd = 0; /* the number of received messages (and the next value) */
24   /* non-deterministically choose the initial value between 0 and 1 */
25   if ::pc = V0;
26       ::pc = V1;
27   fi;
28 end:
29   do
30     :: atomic {
31       /* receive messages: pick a value between nrcvd and nsnt + F */
32       havoc(next_nrcvd); assume(nrcvd <= next_nrcvd && next_nrcvd <= nsnt + F);
33       /* a step by FSM */
34       if /* find the next value of the program counter */
35         :: next_nrcvd >= N - T -> next_pc = AC;
36         :: next_nrcvd < N - T && (pc == V1 || next_nrcvd >= T + 1) -> next_pc = SE;
37         :: else -> next_pc = pc;
38       fi;
39       if /* send the echo message */
40         :: (pc == V0 || pc == V1) && (next_pc == SE || next_pc == AC) -> nsnt++;
41         :: else;
42       fi;
43       /* update the variables to hold the next values */
44       pc = next_pc; nrcvd = next_nrcvd;
45     }
46   od;
47 }
48
49 ltl fairness { []<>(!in_transit) }
50 ltl relay { ([(ex_acc -> <>(all_acc))]) }
51 ltl corr { (prec_corr -> <>(ex_acc)) }
52 ltl unforg { (prec_unforg -> [](!ex_acc)) }

```

Figure 8: Excerpt of reliable broadcast in Parametric Promela

Table 4: Asynchronous and randomized fault-tolerant distributed algorithms that are verified by different generations of ByMC. For every technique and algorithm we show, whether the technique could verify the properties: safety (S), liveness (L), almost-sure termination under round-rigid adversaries (RRT), or none of them (-).

Algorithm	SMT-S	SMT-L	SMT+MR	CA+SPIN	CA+BDD	CA+SAT
	[KVV15]	[KLVW17b]	[BKLV19]	[JKS ⁺ 13a]	[KVV17]	[KVV17]
	[JKS ⁺ 13a]	[KVV17]	[KVV17]	[KVV15]	[KLVW17b]	[BKLV19]
FRB [CT96a]	S	S+L	-	S+L	S+L	S
STRB [ST87b]	S	S+L	-	S+L	S+L	S
ABA [BT85]	S	S+L	-	-	S+L	-
NBACG [Gue02]	S	S+L	-	-	-	-
NBACR [Ray97]	S	S+L	-	-	-	-
CBC [MMPR03]	S	S+L	-	-	-	-
CF1S [DS06]	S	S+L	-	-	S+L	-
C1CS [BGMR01]	S	S+L	-	-	-	-
BOSCO [SvR08]	S	S+L	-	-	-	-
Ben-Or [Ben83]	-	-	S+RRT	-	-	-
RABC [Bra87]	-	-	S+RRT	-	-	-
kSet [MMR18]	-	-	S+RRT	-	-	-
RS-BOSCO [SvR08]	-	-	S+RRT	-	-	-

broadcast in Figure 4 would have four abstract values that correspond to the intervals: $[0, 1)$, $[1, t + 1)$, $[t + 1, n - t)$, and $[n - t, \infty)$. This finite-state counter abstraction is automatically constructed from a protocol specification in Parameterized Promela and is automatically checked with the classical finite-state model checker Spin [Hol03]. As this abstraction was typically too coarse for liveness checking, we have implemented a simple counterexample-guided abstraction refinement loop for parameterized systems. This approach extends the classic $\{0, 1, \infty\}$ -counter abstraction, which was designed for parameterized verification of mutual exclusion algorithms [PXZ02]. Although this technique is a nice combination of classical ideas, we could only verify two broadcast algorithms by running it. Not surprisingly, Spin was either running out of memory or by timeout.

Technique CA+BDD. This technique implements the same counter abstraction as in CA+SPIN, but runs the symbolic model checker nuXmv instead of Spin, which implements model checking with BDDs and SAT solvers [CCD⁺14]. Although this extension scaled better than CA+SPIN, we could only check two additional benchmarks by running it. Detailed discussions of the techniques CA+SPIN and CA+BDD can be found in [GKS⁺14, KVV16].

Technique CA+SAT. By running the abstraction/checking loop in nuXmv, we found that the bounded model checking algorithms of nuXmv could check long executions of our benchmarks. However, bounded model checking in general does not have completeness guarantees. In [KVV14, KVV17], we have shown that the counter systems of (asynchronous) threshold automata have computable bounded diameters, which gave us a way to use bounded model checking as a complete verification approach for reachability properties. Still, the computed upper bounds were too high for achieving complete verification.

Table 5: The relative sizes of the benchmarks introduced in [KW18].

#	Input	Case	Threshold Automaton			
			$ \mathcal{L} $	$ \mathcal{R} $	$ \Phi^{\text{rise}} $	$ \Phi^{\text{fall}} $
1	frb	-	7	14	1	0
2	frb	<i>hand-coded TA</i>	4	9	1	1
3	strb	-	7	21	3	0
4	strb	<i>hand-coded TA</i>	4	8	2	0
5	nbacg	-	24	64	4	0
6	nbacg	<i>hand-coded TA</i>	8	16	0	1
7	nbacr	-	77	1031	6	0
8	nbacr	<i>hand-coded TA</i>	7	16	0	1
9	aba	$\frac{n+t}{2} = 2t + 1$	37	202	6	0
10	aba	$\frac{n+t}{2} > 2t + 1$	61	425	8	0
11	aba	<i>hand-coded TA</i>	5	10	2	2
12	cbc	$\lfloor \frac{n}{2} \rfloor < n - t \wedge f = 0$	164	2064	0	0
13	cbc	$\lfloor \frac{n}{2} \rfloor = n - t \wedge f = 0$	73	470	0	0
14	cbc	$\lfloor \frac{n}{2} \rfloor < n - t \wedge f > 0$	165	2072	0	1
15	cbc	$\lfloor \frac{n}{2} \rfloor = n - t \wedge f > 0$	74	476	0	1
16	cbc	<i>hand-coded TA</i>	7	14	0	1
17	cf1s	$f = 0$	41	280	4	0
18	cf1s	$f = 1$	41	280	4	1
19	cf1s	$f > 1$	68	696	6	1
20	cf1s	<i>hand-coded TA</i>	9	26	3	3
21	c1cs	$f = 0$	101	1285	8	0
22	c1cs	$f = 1$	70	650	6	1
23	c1cs	$f > 1$	101	1333	8	1
24	c1cs	<i>hand-coded TA</i>	9	30	7	3
25	bosco	$\lfloor \frac{n+3t}{2} \rfloor + 1 = n - t$	28	152	6	0
26	bosco	$\lfloor \frac{n+3t}{2} \rfloor + 1 > n - t$	40	242	8	0
27	bosco	$\lfloor \frac{n+3t}{2} \rfloor + 1 < n - t$	32	188	6	0
28	bosco	$n > 5t \wedge f = 0$	82	1372	12	0
29	bosco	$n > 7t$	90	1744	12	0
30	bosco	<i>hand-coded TA</i>	8	20	3	4

6.3. Comparing the benchmark sizes. To explain the performance of the techniques that belong to the SMT and CA groups, we present the sizes of the benchmarks in Table 5. Plots 9 and 10 visualize the number of locations and rules in threshold automata and the result of the counter abstraction of Parametric Promela, respectively. As one can see, the threshold automata produced by counter abstraction are significantly larger in the number of locations and rules than the threshold automata over integer counters. The techniques based on counter abstraction require finite domains. Hence, the explosion in the number of locations and rules is a natural result of applying abstraction. In contrast, by reasoning over integer counters in the SMT-based techniques we keep the size of the input relatively small. In our understanding, this should explain better efficiency of the SMT-based approaches.

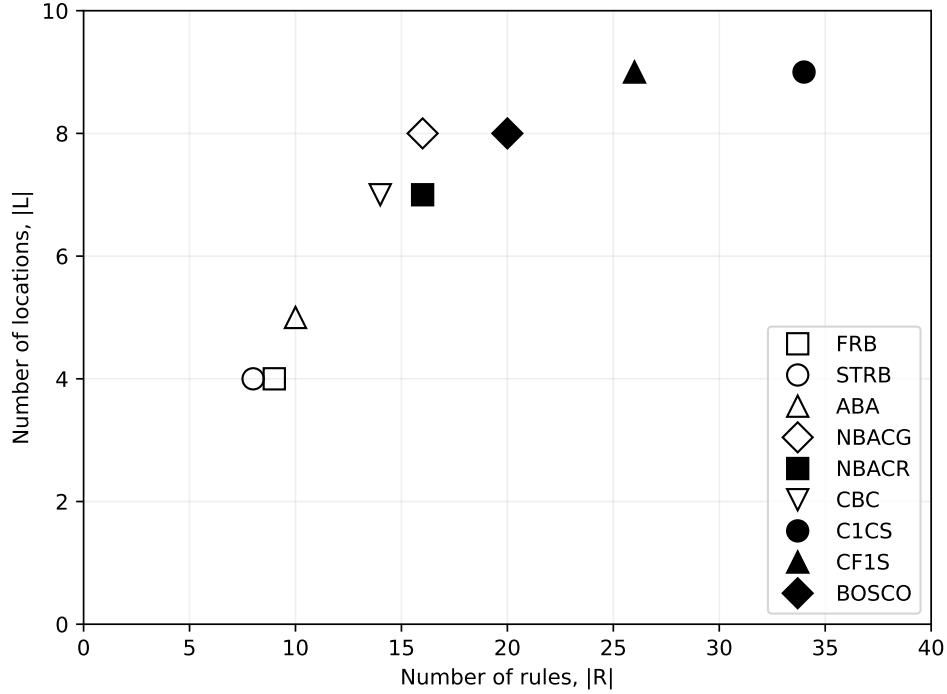


Figure 9: Relative sizes of the threshold automata used as input

6.4. Model checking synchronous threshold automata. The bounded model checking approach for STA introduced in Section 2 is not yet integrated into ByMC. It is implemented as a stand-alone tool, available at [STA]. In [SKWZ19], we encoded multiple synchronous algorithms from the literature, such as consensus [Lyn96, Ray10, BGP89b, BGP89a, BSW11], k -set agreement (from [Lyn96], whose pseudocode is given in Figure 1 and [Ray10]), and reliable broadcast (from [ST87b, BSW11]) algorithms. We use Z3 [dMB08] and CVC4 [BCD⁺11] as back-end SMT solvers. Table 6 gives an overview of the verified synchronous algorithms. For further details on the experimental results, see [SKWZ19].

7. TOWARDS VERIFICATION OF TENDERMINT CONSENSUS

Tendermint consensus is a fault-tolerant distributed algorithm for proof-of-stake blockchains introduced in [BKM18]. Tendermint can handle Byzantine faults under the assumption of partial synchrony. It is running in the Cosmos network, where currently over 100 validator nodes are committing transactions and are managing the ATOM cryptocurrency [BK18].

7.1. Challenges of verifying Tendermint. Tendermint consensus heavily relies on threshold guards, as can be seen from its pseudocode in [BKM18][Algorithm 1]. For instance, one of the Tendermint rules has the following precondition:

$$\begin{aligned}
 & \mathbf{upon} \langle \text{PROPOSAL}, h_p, \text{round}_p, v, * \rangle \mathbf{from} \text{proposer}(h_p, \text{round}_p) \\
 & \quad \mathbf{AND} \ 2f + 1 \langle \text{PREVOTE}, h_p, \text{round}_p, \text{id}(v) \rangle \\
 & \quad \mathbf{while} \ \text{valid}(v) \wedge \text{step}_p \geq \text{prevote for the first time}
 \end{aligned} \tag{7.1}$$

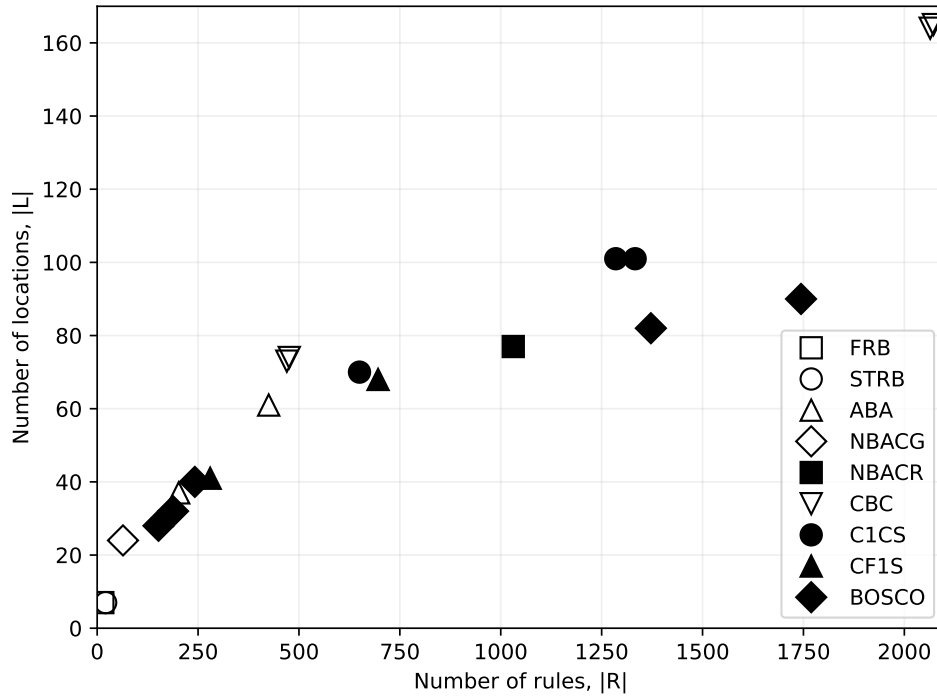


Figure 10: Relative size of the benchmarks in parametric Promela, after applying counter abstraction

The rule (7.1) requires two kinds of messages: (1) a single message of type `PROPOSAL` carrying a proposal v from the process $proposer(h_p, round_p)$ that is identified by the current round $round_p$ and consensus instance h_p , and (2) messages of type `PREVOTE` from several nodes. Here the term $2F + 1$ (taken from the original paper) in fact does not refer to a number of processes. Rather, each process has a voting power (an integer that expresses how many votes a process has), and $2F + 1$ (in combination with $N = 3T + 1$) expresses that nodes that have sent `PREVOTE` have more than two-thirds of the voting power. Although this rule bears similarity with the rules of threshold automata, Tendermint consensus has the following features that cannot be directly modelled with threshold automata:

- (1) In every consensus instance h_p and round $round_p$, a single proposer sends a value that the nodes vote on. The identity of the proposer can be accessed with the function $proposer(h_p, round_p)$. *This feature breaks symmetry among individual nodes*, which is required by our modelling with counter systems. Moreover, the proposer function should be fairly distributed among the nodes, e.g., it can be implemented with round robin.
- (2) Whereas the classical example algorithms in this paper count messages, Tendermint evaluates the voting power of the nodes from which messages were received. This adds an additional layer of complexity.
- (3) Liveness of Tendermint requires the distributed system to reach a global stabilization period, when every message could be delivered not later than after a bounded delay. This model of partial synchrony lies between synchronous and asynchronous computations and requires novel techniques for parameterized verification.

Table 6: Synchronous fault-tolerant distributed algorithms verified with the bounded model checking approach from [SKWZ19]. With \checkmark we show that: the SMT based procedure finds a diameter bound with Z3 (**DIAM+Z3**) and CVC4 (**DIAM+CVC4**); there is a theoretical bound on the diameter (**DIAM+THM**). We verify safety (S) by bounded model checking with Z3 (**BMC+Z3**) and CVC4 (**BMC+CVC4**).

Algorithm	DIAM+Z3	DIAM+CVC4	DIAM+THM	BMC+Z3	BMC+CVC4
FloodSet [Lyn96]	\checkmark	\checkmark	–	S	S
FairCons [Ray10]	\checkmark	\checkmark	–	S	S
PhaseKing [BGP89b]	\checkmark	\checkmark	–	S	S
PhaseQueen [BGP89a]	\checkmark	\checkmark	–	S	S
HybridKing [BSW11]	\checkmark	\checkmark	–	S	S
ByzKing [BSW11]	\checkmark	\checkmark	–	S	S
OmitKing [BSW11]	\checkmark	\checkmark	–	S	S
HybridQueen [BSW11]	–	–	–	–	–
ByzQueen [BSW11]	\checkmark	\checkmark	–	S	S
OmitQueen [BSW11]	\checkmark	\checkmark	–	S	S
FloodMin [Lyn96]	\checkmark	\checkmark	–	S	S
FloodMinOmit [BSW11]	\checkmark	\checkmark	–	S	S
kSetOmit [Ray10]	–	–	–	–	–
STRB [ST87b]	\checkmark	\checkmark	\checkmark	S	S
HybridRB [BSW11]	\checkmark	\checkmark	\checkmark	S	S
OmitRB [BSW11]	\checkmark	\checkmark	\checkmark	S	S

7.2. Checking parameterized one-round safety with ByMC. While we are not able to verify the complete Tendermint consensus algorithm in ByMC, we use ByMC to verify its one-round safety in the parameterized case. We do this, first we manually construct a threshold automaton from the pseudo code. We translate the above rule into the following rules of a threshold automaton (an explanation of the syntax can found in [KW18]):

```

3: locPrevote -> locPrecommit
  when (nprop0 >= 1 && nprevote0 >= 2 * T + 1 - F)
  do {
    nprecommit0' == nprecommit0 + 1;
    nprecommitAll' == nprecommitAll + 1;
    unchanged(nprop0, nprop1,
              nprevote0, nprevote1, nprevoteNil, nprevoteAll,
              nprecommit1, nprecommitNil);
  };
4: locPrevote -> locPrecommit
  when (nprop1 >= 1 && nprevote1 >= 2 * T + 1 - F)
  do {
    nprecommit1' == nprecommit1 + 1;
    nprecommitAll' == nprecommitAll + 1;
    unchanged(nprop0, nprop1,
              nprevote0, nprevote1, nprevoteNil, nprevoteAll,
              nprecommit0, nprecommitNil);

```


};

We encoded the complete threshold automaton. The encoding is publicly available at: <https://github.com/konnov/fault-tolerant-benchmarks/tree/master/lmcs20>.

Before discussing ByMC’s performance on this benchmark, we quickly discuss an alternative encoding of this benchmark in TLA^+ for comparison with a standard model checker.

7.3. Tendermint in TLA^+ . TLA^+ is a general specification language [Lam02], so it is much easier to write the first specification in TLA^+ , rather than to write down a threshold automaton right away. Additionally, we first debugged our TLA^+ specification with the TLC model checker [YML99b]. The TLA^+ specification can be found in Appendix A. The following snippet encodes the rule (7.1) discussed above, and indeed is a threshold-guarded rule that encodes a transition from a process in location “prevote” to location “precommit”, provided enough “propose” and “prevote” messages are received:

$$\begin{aligned} \text{Line36}(v) &\triangleq \\ &\wedge \text{nproposals}[v] > 0 \\ &\wedge \text{nprevotes}[v] + F \geq 2 * T + 1 \\ &\wedge \text{counters}[\text{“prevote”}] > 0 \\ &\wedge \text{nprecommits}' = [\text{nprecommits} \text{ EXCEPT } ![v] = @ + 1] \\ &\wedge \text{counters}' = [\text{counters} \text{ EXCEPT } ![\text{“prevote”}] = @ - 1, ![\text{“precommit”}] = @ + 1] \\ &\wedge \text{UNCHANGED } \langle \text{nproposals}, \text{nprevotes} \rangle \end{aligned}$$

7.4. Comparison of verification performance. We produced two artifacts, a threshold automaton and a TLA^+ model³.

The standard TLA^+ model checker TLC can check the specification for $N \leq 20$ in reasonable time. We checked the safety of consensus in one round: No two correct processes decide differently in the same round. As TLC is an explicit-state model checker, it did not scale to the value of $N = 125$, which is the current number of validators in the Cosmos main chain. In contrast, we have verified this property with ByMC in the parameterized case in 2 seconds.

In this model of the algorithm, the one-round safety is satisfied under the assumption of $N = 3 \cdot T + 1 \wedge T \geq F$. When we change the assumption to either $N \geq 3 \cdot T + 1 \wedge T \geq F$, or $N = 3 \cdot T + 1$, ByMC produces counterexamples to the property. The actual implementation of Tendermint is different, as it dynamically recomputes the voting powers of the validators and thus it recomputes the thresholds. However, we follow the assumptions of [BKM18] in our modeling.

We observe that our manual abstraction of the TLA^+ specification into a threshold automaton is rather mechanical (while presented in reverse order in this paper, we actually did the TLA^+ encoding first). This abstraction could be done in a fully automatic pipeline: From TLA^+ to a receive threshold automaton, and from a receive threshold automaton to a threshold automaton (see Section 5.1). We are planning to use automatic abstractions to build a bridge between ByMC and Apache — a symbolic model checker for TLA^+ [KKT19].

³The TLA^+ specification and the threshold automaton are publicly available at: <https://github.com/konnov/fault-tolerant-benchmarks/tree/master/lmcs20>.

7.5. Open problems for parameterized verification of multi-round safety. To verify multi-round safety of Tendermint, we would like to invoke a reduction argument similar to the one explained in Section 4.3. However, Tendermint contains the following rule that prevents us from directly applying the reduction result:

$$\begin{aligned}
 & \text{upon } \langle \text{PROPOSAL}, h_p, \text{round}_p, v, vr \rangle \text{ from } \text{proposer}(h_p, \text{round}_p) \\
 & \quad \text{AND } 2f + 1 \langle \text{PREVOTE}, h_p, vr, id(v) \rangle \\
 & \quad \text{while } \text{step}_p = \text{propose} \wedge (vr \geq 0 \wedge vr < \text{round}_p) \\
 & \quad \dots
 \end{aligned} \tag{7.2}$$

The rule in Equation (7.2) allows a process to make a step by using messages from a past round vr . As a result, Tendermint is not communication-closed [EF82, CCM09, DDMW19]. Extending the reduction argument to multi-round systems that are not communication closed is subject to our ongoing work.

8. DISCUSSIONS AND RELATED WORK

In our work, we follow the idea of identifying fragments of automata and logic that are sufficiently expressive for capturing interesting distributed algorithms and specifications, as well as amenable for completely automated verification. We introduced threshold automata for that and implemented our verification techniques in the open source tool ByMC [KW18]. By doing so, we verified several challenging distributed algorithms; most of them were verified for the first time.

Our work is in the context of computer-aided verification of distributed algorithms and systems, which is a lively research area. Parameterized verification of fault-tolerant distributed algorithms has recently been addressed with a wide range of techniques, most of which focus on asynchronous distributed algorithms. In Section 8.1, we survey both existing parameterized model checking techniques and deductive verification techniques, based on producing mechanized proofs using proof assistants.

Further, systems that implement fault-tolerant distributed algorithms are very complex and increasingly hard to get right. With increasing numbers of systems that implement fault-tolerant distributed algorithms [Bur06, JRS11, MAK13], there is an interest in developing tool support for eliminating flaws in distributed algorithms and their implementations by means of automated verification. We survey some of the approaches in this area in Section 8.2.

Finally, the threshold automata framework has proved to be both of practical relevance as well as of theoretical interest. There are several ongoing projects that consider complexity theoretic analysis of verification problems, as well as more refined probabilistic reasoning. In Section 8.3, we give brief overview of these approaches.

8.1. Verification of Distributed Algorithms. In this section, we survey existing model checking techniques, as well as deductive verification techniques, based on producing mechanized proofs using proof assistants.

Several consensus algorithms were verified for small system sizes in [TS11, NTK12, DTT14], by applying model checking to the fixed size instances of up to, e.g., six processes. In the parameterized case, a framework for verifying fault-tolerance of distributed protocols

based on regular model checking was proposed in [FKL08]. In this framework, the fault model specification is separate from the process specification, which is similar our idea of separating the process and environment specifications, and keeping the system specification modular. The approach was manually applied to verify the correctness of an authenticated broadcast protocol that tolerates crash faults in the parameterized case. Parameterized model checking of safety properties for fault-tolerant distributed algorithms using counter abstractions and SMT solvers was proposed in [AGOP16]. Using this approach, the authors automatically verified two authenticated broadcast protocols, operating under different fault models, namely crash, send-omission, general omission, and Byzantine faults.

The Heard-Of model [CBS09] was proposed as a formalization framework for round-based, message passing distributed algorithms, where the computation and fault models are captured by so-called communication predicates. This framework enables a systematic encoding of (asynchronous, synchronous, or partially synchronous) round-based algorithms and facilitates the comparison between different algorithms. Several parameterized verification techniques are designed for algorithms formalized in the heard-of model. For partially synchronous consensus algorithms, expressed in an extension of the heard-of model, [DHV⁺14] introduced a consensus logic and (semi-)decision procedures for verifying user-provided invariants. In [MSB17], a characterization of partially synchronous consensus algorithms in the heard-of model was given. Based on this characterization, the authors proved cut-off theorems specialized to the properties of consensus: agreement, validity, and termination. More recently, [GREP20] proposed an approach for parameterized verification of safety properties of round-based algorithms, that can be expressed in the heard-of model, by combining overapproximation and backward reachability analysis.

Many fault-tolerant distributed algorithms have been formalized using the specification language TLA+ [Lam02], e.g., [GL03, Lam11, MAK13]. As TLA+ is equipped with an explicit state model checker, TLC [YML99a], and a proof system, TLAPS [CDLM10], often TLC is used to debug the TLA+ specification of a given algorithm, for small and fixed system sizes, and show the correctness of the algorithm in the parameterized case by writing a machine-checkable proof in TLAPS. In [TKW20], a cutoff result for failure-detection algorithms was presented, the algorithms from [CT96b] was specified using TLA+, and TLC and APALACHE [KKT19], a new symbolic model checker for TLA+, were used to verify its correctness for the cutoff size. Other theorem provers have also been used to certify the correctness of fault-tolerant distributed algorithms. PVS [ORS92] was used in [LR93], where a bug in an already published synchronous consensus algorithm tolerating hybrid faults was reported, and in [SWR02] to verify Byzantine agreement with link failures, in addition to process failures. Isabelle/HOL [NPW02] was used to certify the correctness of several protocols encoded in the heard-of model in [CBS09, DM12].

These semi-automated proofs require a great amount of human intervention and understanding of the distributed algorithms. IVy [PMP⁺16, MP20] is an interactive verification tool, whose goal is achieving higher automation when producing a formal proof of a distributed algorithm, by reducing the amount of human guidance as much as possible. The main idea of the IVy methodology is to encode the algorithms in the effectively-propositional fragment (EPR) of first-order logic. It is not always straightforward to encode distributed algorithms and their verification conditions in EPR, but once it is done, the verification condition check is fully automatic.

8.2. Verification of Distributed Systems. IronFleet [HHK⁺17] implements a variant of MultiPaxos [Lam98] in Dafny [Lei10], which allows for Hoare-logic style program verification. Verdi [WWP⁺15] verifies an implementation of the Raft protocol [OO14] using the Coq proof assistant [dt04], and translates the Coq proof into a verified implementation in OCaml. Chapar [LBC16] also uses the Coq to OCaml translation to obtain a verified implementation of a distributed key-value store. When verifying implementations of distributed systems, one has to be very careful about the assumptions, about the calls to unverified external libraries, and about the correctness of the specifications themselves. An empirical study [FZWK17] analyzed the three verified implementations produced by IronFleet, Verdi, and Chapar, and reported existence of bugs in the interfaces between the verified code and the unverified external libraries or operating system.

Recently, it was observed that in order to automatically verify asynchronous distributed programs, one can define reductions [EF82, CCM09] in order to reduce reasoning about an asynchronous system to reasoning about a synchronous system, equivalent to the original one [DDMW19]. Due to the non-determinism that comes from the faults and the asynchronous computation model, these synchronized versions of the asynchronous programs have different fault and computation semantics to those considered in this thesis. We list some of the approaches based on the idea of reductions. PSync [DHZ16] was introduced as a domain-specific language for specifying and implementing fault-tolerant distributed algorithms, which is based on the heard-of model and can be translated to the consensus logic of [DHV⁺14], and thus the same invariant checking techniques can be applied. A decision procedure for invariant checking of a given asynchronous message-passing program, whose computations can be reduced to computations of an equivalent round-based program with a bounded number of send operations per round, is presented in [BEJQ18]. Several methods based on Lipton reduction [Lip75] were introduced in order to reduce asynchronous programs to other programs, for which reasoning is easier. For example, [BvGKJ17] introduces canonical sequentialization, which is a sequential program, equivalent to a given asynchronous message-passing program, and [KEH⁺20] defines inductive sequentialization, a sequential program to which a given asynchronous program is reduced by combining reduction, abstraction, and inductive reasoning. Asynchronous programs are reduced to equivalent synchronous programs in [KQH18], and their invariants checked using a dedicated model checker. Another technique for reducing asynchronous to synchronous programs was given in [vGKB⁺19], where the correctness of the obtained synchronous program is established using Hoare-style verification conditions and SMT solvers.

8.3. Extending Threshold Automata. We introduced threshold automata as a concise representation and a precise semantics for fault-tolerant distributed algorithms. The verification results presented in this thesis typically apply only to threshold automata of specific shape, e.g., without increment of shared variables in a rule that appears in a loop. The restrictions that we imposed on the form of allowed threshold guards and the corresponding actions are inspired by the typical patterns seen in the benchmarks. That is, these restrictions still allow us to model a class fault-tolerant distributed algorithms. From a theoretical viewpoint there remained the question, whether lifting one or the other restriction maintains or breaks decidability of the verification problem.

The work presented in [KKW18] explores various relaxations of the standard restrictions, such as non-linear threshold guards, guards that compare shared variables, decrementing shared variables, or incrementing them inside self-loops. For each of these theoretical

extensions, the authors investigate the existence of a bounded diameter and decidability of reachability properties. For the standard setting, a systematic analysis of computational complexity of verification and synthesis in threshold automata has been conducted in [BEL20]. The authors express the reachability relation as a formula in existential Presburger arithmetic, and therefore prove that coverability and reachability problems, as well as model checking of ELTL_{FT} properties are NP-complete, while synthesizing threshold guards is Σ_p^2 -complete.

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APPENDIX A. ABSTRACTION OF TENDERMINT CONSENSUS ROUND IN TLA+

MODULE *Tendermint_1round_safety*

```

*
* This is a very simplified version of Tendermint that is tuned for safety checking
* with counter systems :
*
* The simplifications are as follows :
* - the model is completely asynchronous, as we are only concerned with safety
* - we only consider binary consensus
* - we only consider one height and one round
* - as we are modeling only one round, there is no locking mechanism in the specification
* - we use symmetry arguments to replace sets of messages with message counters
* - the proposer is modeled as a non-deterministic assignment in the initial state
* - there are no hashes and no validity checks
*
* A more complete specification for multiple rounds can be found at :
*
*   https://github.com/informalsystems/tendermint-rs/blob/master/docs/spec/tendermint-fork-cases/TendermintAcc3.tla
*
* The original specification of Tendermint can be found at :
*
* https://arxiv.org/abs/1807.04938
*
* Igor Konnov, Marijana Lazic, Iliana Stoilkovska, Josef Widder, 2020

```

EXTENDS *Integers*

CONSTANTS N , the number of processes in the system
 T , the threshold on the number of faults
 F the number of actual *Byzantine* faults

the *Tendermint* algorithm is specified under this assumptionASSUME $(N = 3 * T + 1 \wedge F \leq T \wedge F \geq 0)$

$Locs \triangleq \{$
 “propose”, “prevote”, “precommit”, “decide0”, “decide1”, “nodecision”
 $\}$

$Nil \triangleq$ “nil”
 $Values \triangleq \{“0”, “1”\}$ values 0 and 1
 $ValuesExt \triangleq Values \cup \{Nil\}$

VARIABLES

$counters$, the counter for every location: a function from $Locs$ to Naturals
 $nproposals$, the numbers of proposals sent around, for values 0 and 1
 $nprevotes$, the numbers of prevotes sent around, for values 0, 1, and Nil
 $nprecommits$ the numbers of prevotes sent around, for value 0, 1, and Nil

$Init \triangleq$
 initially, all but *Byzantine* processes are in the “propose” state
 $\wedge counters = [l \in Locs \mapsto \text{IF } l = \text{“propose” THEN } N - F \text{ ELSE } 0]$

the proposer can send 0 or 1 (or both, when it is faulty)

$$\wedge nproposals \in [Values \rightarrow \{0, 1\}]$$

$$\wedge nprevotes = [v \in ValuesExt \mapsto 0]$$

$$\wedge nprecommits = [v \in ValuesExt \mapsto 0]$$

The action in line 22.

Line 28 is the same as *Line22* in our safety abstraction.

Line22(v) \triangleq

$$\wedge nproposals[v] > 0$$

$$\wedge counters["propose"] > 0$$

$$\wedge \vee nprevotes' = [nprevotes \text{ EXCEPT } ![v] = @ + 1] \quad \text{line 24}$$

$$\vee nprevotes' = [nprevotes \text{ EXCEPT } ![Nil] = @ + 1] \quad \text{line 26}$$

$$\wedge counters' = [counters \text{ EXCEPT } !["propose"] = @ - 1, \\ \quad \quad \quad !["prevote"] = @ + 1]$$

$$\wedge \text{UNCHANGED } \langle nproposals, nprecommits \rangle$$

Line36(v) \triangleq

note that line 36 also works for $step_p = precommit$,

but it effectively does nothing in our safety abstraction.

$$\wedge nproposals[v] > 0$$

$$\wedge nprevotes[v] + F \geq 2 * T + 1$$

$$\wedge counters["prevote"] > 0$$

$$\wedge nprecommits' = [nprecommits \text{ EXCEPT } ![v] = @ + 1]$$

$$\wedge counters' = [counters \text{ EXCEPT } !["prevote"] = @ - 1, \\ \quad \quad \quad !["precommit"] = @ + 1]$$

$$\wedge \text{UNCHANGED } \langle nproposals, nprevotes \rangle$$

Line44 \triangleq

$$\wedge nprevotes[Nil] + F \geq 2 * T + 1$$

$$\wedge counters["prevote"] > 0$$

$$\wedge nprecommits' = [nprecommits \text{ EXCEPT } ![Nil] = @ + 1]$$

$$\wedge counters' = [counters \text{ EXCEPT } !["prevote"] = @ - 1, \\ \quad \quad \quad !["precommit"] = @ + 1]$$

$$\wedge \text{UNCHANGED } \langle nproposals, nprevotes \rangle$$

Line49(v) \triangleq

$$\wedge nproposals[v] > 0$$

$$\wedge nprecommits[v] + F \geq 2 * T + 1$$

$$\wedge \exists loc \in \{\text{"propose"}, \text{"prevote"}, \text{"precommit"}\} :$$

$$\text{LET } decision \triangleq$$

$$\text{IF } v = \text{"0"} \text{ THEN } \text{"decide0"} \text{ ELSE } \text{"decide1"}$$

$$\text{IN}$$

$$\wedge counters[loc] > 0$$

$$\wedge counters' = [counters \text{ EXCEPT } ![loc] = @ - 1, \\ \quad \quad \quad ![decision] = @ + 1]$$

$$\wedge \text{UNCHANGED } \langle nproposals, nprevotes, nprecommits \rangle$$

OnTimeoutPropose \triangleq

$$\wedge counters["propose"] > 0$$

$$\begin{aligned} \wedge nprevotes' &= [nprevotes \text{ EXCEPT } ![Nil] = @ + 1] \\ \wedge counters' &= [counters \text{ EXCEPT } !["propose"] = @ - 1, \\ &\quad !["prevote"] = @ + 1] \\ \wedge \text{UNCHANGED } &\langle nproposals, nprecommits \rangle \end{aligned}$$

Line34OnTimeoutPrevote \triangleq

$$\begin{aligned} \wedge nprevotes["0"] + nprevotes["1"] + nprevotes[Nil] + F &\geq 2 * T + 1 \\ \wedge counters["prevote"] &> 0 \\ \wedge nprecommits' &= [nprecommits \text{ EXCEPT } ![Nil] = @ + 1] \\ \wedge counters' &= [counters \text{ EXCEPT } !["prevote"] = @ - 1, \\ &\quad !["precommit"] = @ + 1] \\ \wedge \text{UNCHANGED } &\langle nproposals, nprevotes \rangle \end{aligned}$$

Line47OnTimeoutPrecommit \triangleq

$$\begin{aligned} \wedge nprecommits["0"] + nprecommits["1"] + nprecommits[Nil] + F &\geq 2 * T + 1 \\ \wedge counters["precommit"] &> 0 \\ \wedge counters' &= [counters \text{ EXCEPT } !["precommit"] = @ - 1, \\ &\quad !["nodecision"] = @ + 1] \\ \wedge \text{UNCHANGED } &\langle nproposals, nprevotes, nprecommits \rangle \end{aligned}$$

Next \triangleq

$$\begin{aligned} \vee \exists v \in \text{Values} : & \text{Line22}(v) \\ \vee \exists v \in \text{Values} : & \text{Line36}(v) \\ \vee \exists v \in \text{Values} : & \text{Line44} \\ \vee \exists v \in \text{Values} : & \text{Line49}(v) \\ \vee & \text{OnTimeoutPropose} \\ \vee & \text{Line34OnTimeoutPrevote} \\ \vee & \text{Line47OnTimeoutPrecommit} \\ \vee \text{UNCHANGED } &\langle counters, nproposals, nprevotes, nprecommits \rangle \end{aligned}$$

RoundAgreementInv \triangleq

$$counters["decide0"] = 0 \vee counters["decide1"] = 0$$