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VERTICAL VIBRATIONS OF BASE ISOLATED MACHINE FOUNDATIONS

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ABSTRACT

Vibration of base isolated machine foundations has been studied using the Scaled Boundary Finite Element Method (SBFEM) and the cone model method. The dynamic stiffness of soil supporting rigid massless foundation was determined. This stiffness is of complex value. The real part represents the reflected energy of the restoring and inertial forces while the imaginary part represents the energy dissipated within the endless extent of the soil as a geometric damping. The effect of geometric and material properties of soil upon the real and imaginary parts of the dynamic stiffness was determined and represented in terms of dimensionless charts for the frequency range of interest. Results have shown that increasing the embedment ratio has a significant effect on the dynamic stiffness, it increases the dynamic stiffness considerably. The effect of stiffness ratio(stiffness of isolator/ stiffness of soil) was demonstrated for isolated machine foundations. The use of soft isolators reduces the dynamic response of foundation and the soil reaction.

الخلاصة

يتناول البحث ألاهتزازات الشاقولية لأسس المكائن المعزولة, مستخدمة طريقة العناصر المحيطية المحددة المقايسة و طريقة النموذج المخروطي لاحتساب مصفوفة الصلابة الديناميكية للتربة تحت الأساس. عناصر مصفوفة الصلابة الديناميكية ذات قيم معقدة يمثل الجزء الحقيقي منها طاقة الانفعال المنعكسة والتي تحاول إرجاع الأساس إلى وضع السكون بينما يمثل الجزء الخيالي الطاقة المتبددة في المدى اللانهائي للتربة.
تم حساب تأثير الشكل الهندسي وعمق الدفن على مصفوفة الصلابة مع تغير التردد, حيث تم احتساب كل من ثابت المرونة ومعامل التخميد للتربة تحت الأساس و رسم تغاير هذه المعاملات مع التردد باستخدام مخططات لا بعدية لمديات التردد العملية. تبين من خلال الدراسة أن الصلابة الديناميكية تزداد بزيادة عمق الأساس. لدراسة تأثير العوازل على الاستجابة الديناميكية للأساس تم استخدام عوازل نابضة وبصلابة معينة (جزء صغير من صلابة التربة) وتبين ان كفاءة العزل تزداد كلما قلت نسبة الصلابة.

KEY WORDS: machine foundations, dynamic stiffness, wave propagation, scaled boundary finite element method, cone model, vibration isolation.

INTRODUCTION

The idealization of soil media by means of a conventional finite element method has a limitation, since it truncates the soil domain at pre specified artificial boundaries and this leads to spurious reflections at the assumed boundaries (Dominguez, 1993). The whole concern is to assemble the **dynamic stiffness matrix** of soil, this matrix is of a complex form , and can be decomposed into real and imaginary parts. The imaginary part corresponds to the absorbed energy dissipated in the endless extent of the infinite domain, while the real part corresponds to the reflected energy at boundaries. The dissipation of energy through soil is called **radiation damping**. Radiation condition states that the soil as an infinite media is an energy sink. (Wolf,1985).

The foundations for machines are usually in the form of reinforced concrete blocks. Brick finite elements may be used to idealize the block foundations. The soil may be idealized by a linear elastic or linear viscoelastic material.

REVIEW OF LITERATURE

Lamb(1904) studied the response of the elastic half space subjected to oscillating vertical forces. Thus, he solved the two-dimensional wave propagation problem. In 1936, Reissner developed the first analytical solution for the vertically loaded cylindrical disk on an elastic half-space. His solution is considered to be the first engineering model. He reached a solution by integrating Lamb's solution over a circular area for the center displacement; he assumed a state of uniform stress under the footing. Barkan (1962) conducted some plate bearing tests to get an equivalent soil spring constant k. From these tests, he prepared tables and empirical formulas to easily estimate the design values of the subgrade reaction for several types of soil for each possible mode of vibration. In 1965, Lysmer in his doctoral thesis, studied the vertical vibration of circular footings by discretizing the circular area into concentric rings (Asik, 2001). Wong and Luco (1976) presented an approximate numerical procedure for calculation of harmonic force-displacement relationship of rigid foundations of arbitrary shape and placed on an elastic half-space.The first Boundary Element Method(BEM) application for soil problems was presented by Dominguez in (1978) who applied the BEM to compute the dynamic stiffness of rectangular foundations resting on, or embedded in, a viscoelastic half-space in frequency domain(Dominguez, 1993). The dynamic stiffness of rigid rectangular foundations on the halfspace was determined by (Triantafyllidis, 1986) for different aspect ratios of *(L/B =1, 2, 5 and 10)* and for Poisson's ratio *(v =0.25, 0.33 and 0.40)*. All modes of vibration were considered and the stiffness and damping coefficients were represent in dimensionless charts for dimensionless frequency up to *3.5*. (Gazetas et al, 1985, 1986a, 1986b, 1987, 1989a, 1989b, 1991a, 1991b) treated the subject in a simple physical manner based on an improved understanding of the physics of the problem. This has been enhanced by the results of the extensive rigorous parametric studies including several analytical results compiled from the literature. Mita and Luco (1989) had tabulated dimensionless impedance functions and effective input motions of square foundation embedded in a uniform half space. Alhussaini (1992) studied the vibration isolation of machine foundation using open and in-filled trenches as a wave barriers using boundary element method (BEM). Meek and Wolf (1992) used the cone model to idealize homogenous soil under base mat, they also used cone model to idealize soil layer on rigid rock. Asik (1993) developed a simplified semi-analytical method, to compute the response of rigid strip and circular machine foundations subjected to harmonic excitation. Wolf and Song (1996) had developed the scaled boundary finite element method for modeling the unbounded media in analysis of DSSI. (Wolf, 1997) developed a spring-dashpot-mass model with frequency-independent coefficients and a few internal degrees of freedom. Spyrakos and Xu (2004) studied the dynamic response of flexible massive strip foundations embedded in a layered soil using coupled FEM/BEM. Chen and Yang (2006) presented a simplified model for simulating unbounded soil in the vertical vibration problems of surface foundations. The model comprises a mass, a spring, and a dashpot without any internal degree of freedom. Kumar and Reddy (2006) investigated experimentally the response of a machine foundation subjected to vertical vibration by sandwiching a spring cushioning system between the machine base and its footing block. Kumar and Boora (2008) also examined experimentally the effect of two different combinations of a spring mounting base and a rubber pad sandwiched between the machine base and its concrete footing block. Using modal analysis (Chen and GangHou, 2008) presented a methodology to evaluate dynamic displacements of a circular flexible foundation on soil media subjected to vertical vibration.

MATHEMATICAL MODELS FOR MACHINE FOUNDATIONS

The idealization of soil-foundation system is the most important task of the designer, either simple or complex models may be used. It depends on the degree of accuracy required and on the importance of the project. Simple mathematical models are frequently used by office designer as it needs basic knowledge to build and to run the model, these models consist of discrete springs with lumped masses.

A boundary condition capable of eliminating the reflection of waves to the computational domain has to be applied on the artificial boundary. The boundary condition at infinity should be able to irreversibly transfer energy from the bounded domain to the unbounded domain and to eliminate the reflection of waves impinges the boundary. Such a boundary condition is called the radiation condition. Obtaining the radiation condition for large scale engineering problems is the most challenging part of the dynamic soil-structure interaction analysis.

The scaled boundary finite-element method is a powerful semi-analytical computational procedure to calculate the dynamic stiffness of the unbounded soil at the structure–soil interface. This permits the analysis of dynamic soil–structure interaction using the substructure method(Wolf and Song, 1996).

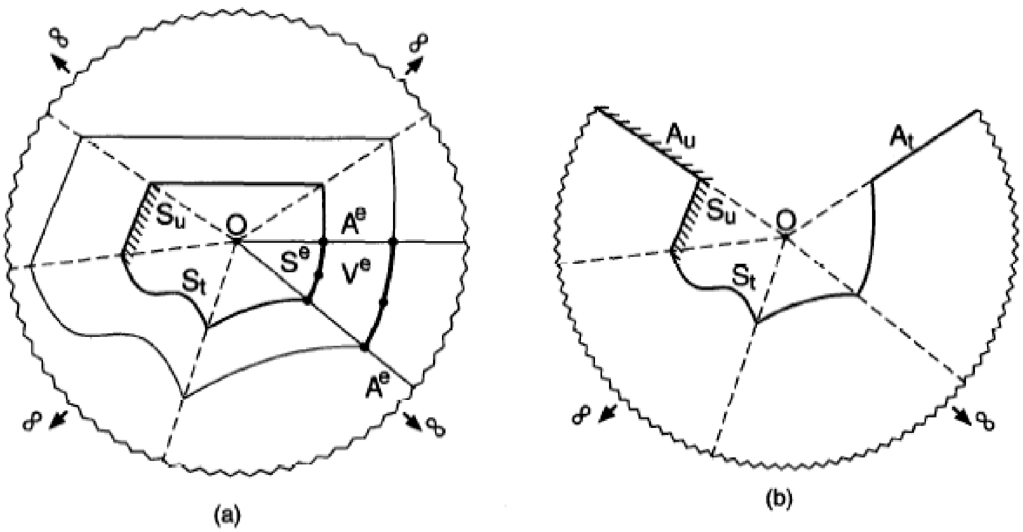


Fig. 1 Modeling of unbounded medium with surface finite elements (section) with: (a) scaling centre outside of medium; (b) extension of boundary passing through scaling centre.

The total boundary must be visible from a point within the medium which is called the scaling center *O*. On the doubly-curved boundary *S* of the medium, the displacements and surface tractions are prescribed on ***Su*** and **S*t*** respectively. The radial direction points from the scaling centre to a point on the boundary, where two circumferential directions tangential to the boundary are identified. The boundary is discretized with doubly-curved surface finite elements with any arrangement of nodes. The dynamic behavior is described by the dynamic-stiffness matrix in the frequency domain [S(w)] relating the displacement amplitudes in the degrees of freedom on the boundary *S* to the corresponding force amplitudes.
By scaling the boundary in the radial direction with respect to the scaling centre *O* with a scaling factor larger than 1, the whole domain is covered. The scaling corresponds to a transformation of the coordinates for each finite element, resulting in the two curvilinear local coordinates in the circumferential directions on the surface and the dimensionless radial coordinate representing the scaling factor. This transformation is unique due to the choice of the scaling centre. This transformation of the geometry involving the discretization of the boundary with finite elements and scaling in the radial direction leads to a system of linear second-order differential equations for the displacements with the dimensionless radial coordinate as the independent variable.
After substituting the definition of the dynamic-stiffness matrix in the differential equations, it is shown that the dynamic-stiffness matrix is a function of the dimensionless frequency which is proportional to the product of the frequency and the dimensionless radial coordinate. This permits the equation for the dynamic-stiffness matrix on the boundary to be expressed as a system of nonlinear first-order ordinary differential equations in the frequency as the independent variable with constant coefficient matrices.

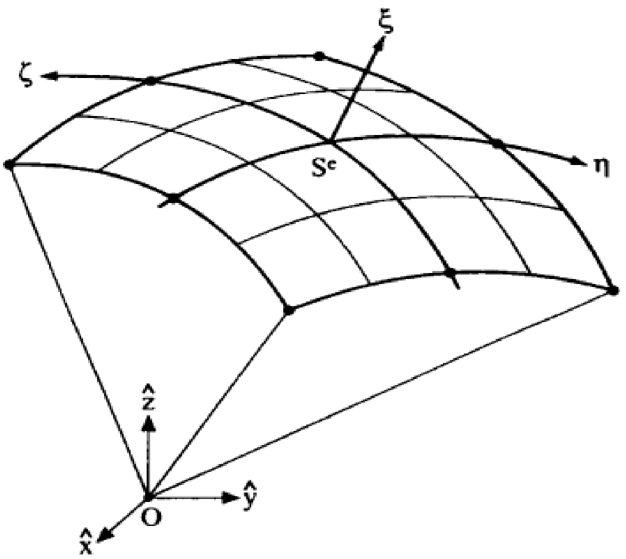


Fig. 2 Scaled boundary transformation of geometry of surface finite element .

Denoting points on the boundary with x, y, z, the geometry is described in the local coordinate system η,ζ

$$x(\eta,\zeta) = [N(\eta,\zeta)]\{x\}$$

$$y(\eta,\zeta) = [N(\eta,\zeta)]\{y\}$$

$$y(\eta,\zeta) = [N(\eta,\zeta)]\{y\} \qquad (1)$$

with the mapping functions ***[N(η,ζ)]*** and the coordinates of the nodes *{x}, {y}, {z}*. The three-dimensional medium is defined by scaling the boundary points with the dimensionless radial coordinate ***x*** measured from the scaling centre with $\xi = 1$ on the boundary and is 0 at the scaling centre The new coordinate system is defined by ξ and the two circumferential coordinates η,ζ, for an unbounded medium $1 < \xi < \infty$

1. Governing Equations in Scaled boundary Coordinates

The differential equations of motion in the frequency domain expressed in displacement amplitudes:

$$\{u\} = \{u(x,y,z)\} = [u_x\ u_y\ u_z]^T \qquad (2)$$

are formulated as

$$L^T\sigma(\omega) + b(w) + w^2\rho u(\omega) = 0 \qquad (3)$$

with the mass density ρ and the amplitudes of the body loads b.

The stress amplitudes $\{\sigma\}$ follows from Hooke's law with the elasticity matrix *[D]* as

$$\{\sigma\} = [\sigma_x\ \sigma_y\ \sigma_z\ \tau_{yz}\ \tau_{xz}\ \tau_{xy}]^T = [D]\{\varepsilon\} \qquad (4)$$

The strain amplitudes $\{\varepsilon\}$ are defined from the strain-displacement relationship

$$\{\varepsilon\} = [\varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \gamma_{yz}\ \gamma_{xz}\ \gamma_{xy}]^T = [L]\{u\} \qquad (5)$$

Where ***[L]*** is the differential operator

$$L = \begin{bmatrix} \frac{\partial}{\partial \hat{x}} & 0 & 0 \\ 0 & \frac{\partial}{\partial \hat{y}} & 0 \\ 0 & 0 & \frac{\partial}{\partial \hat{z}} \\ 0 & \frac{\partial}{\partial \hat{z}} & \frac{\partial}{\partial \hat{y}} \\ \frac{\partial}{\partial \hat{z}} & 0 & \frac{\partial}{\partial \hat{x}} \\ \frac{\partial}{\partial \hat{y}} & \frac{\partial}{\partial \hat{x}} & 0 \end{bmatrix} \qquad (6)$$

The derivatives with respect to X,Y,Z are transformed to those with respect to ξ,η,ζ

$$\begin{Bmatrix} \dfrac{\partial}{\partial \hat{X}} \\ \dfrac{\partial}{\partial \hat{y}} \\ \dfrac{\partial}{\partial \hat{z}} \end{Bmatrix} = \left[\hat{J}\right]^{-1} \begin{Bmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \\ \dfrac{\partial}{\partial \zeta} \end{Bmatrix} = \left[J\right]^{-1} \begin{Bmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{1}{\xi}\dfrac{\partial}{\partial \eta} \\ \dfrac{1}{\xi}\dfrac{\partial}{\partial \zeta} \end{Bmatrix} \qquad (7)$$

Applying the weighted residuals method leads to the scaled boundary finite element equations of displacements $\{u(\xi)\}$

$$\begin{aligned} &\left[E^0\right]\xi^2\{u(\xi)\}_{,\xi\xi} + \left(2\left[E^0\right] - \left[E^1\right] + \left[E^1\right]^T\right)\xi\{u(\xi)\}_{,\xi} + \left(\left[E^1\right]^T - \left[E^2\right]\right)\{u(\xi)\} \\ &\omega^2\left[M^0\right]\xi^2\{u(\xi)\} + \{F(\xi)\} = 0 \end{aligned} \qquad (8)$$

Where the coefficient matrices

$$\left[E^0\right] = \int_{S^\xi} \left[B^1\right]^T [D]\left[B^1\right]|J|\, d\eta d\zeta \qquad (9\text{ a})$$

$$\left[E^1\right] = \int_{S^\xi} \left[B^2\right]^T [D]\left[B^1\right]|J|\, d\eta d\zeta \qquad (9\text{ b})$$

$$\left[E^2\right] = \int_{S^\xi} \left[B^2\right]^T [D]\left[B^2\right]|J|\, d\eta d\zeta \qquad (9\text{ c})$$

$$\left[M^0\right] = \int_{S^\xi} [B(\eta,\zeta)]^T \rho [B(\eta,\zeta)]|J|\, d\eta d\zeta \qquad (10)$$

and

$$\{F(\xi)\} = \xi\left\{F^t\right\} + \xi^2\left\{F^b\right\} \qquad (11)$$

Applying the conditions of equilibrium and compatibility at soil-structure interface, getting the scaled boundary equations in dynamic stiffness matrix:

$$\begin{aligned} &\left(\left[S^\infty(\omega)\right] + \left[E^1\right]\right)\left[E^0\right]^{-1}\left(\left[S^\infty(\omega)\right] + \left[E^1\right]^T\right) - \left[S^\infty(\omega)\right] \\ &-\omega\left[S^\infty(\omega)\right]_{,\omega} - \left[E^2\right] + \omega^2\left[M^0\right] = 0 \end{aligned} \qquad (12)$$

The dynamic stiffness matrix $[S\infty(\omega)]$ at high frequency is expanded in a polynomial of $(i\omega)$decreasing order starting at one:-

$$\left[S^\infty(\omega)\right] \approx i\omega[C_\infty] + [K_\infty] + \sum_{j=1}^{m} \frac{1}{(i\omega)^j}\left[A_j\right] \qquad (13)$$

The first two terms on the right hand side represent the constant dashpot matrix $[C_\infty]$ and the constant spring $[K_\infty]$ (subscript ∞ for $\omega \to \infty$). Substituting Equation (13) into Equation (12), and setting the coefficients of terms in descending order of the power of $(i\omega)$ equal to zero

determines analytically the unknown matrices in Equation (13) sequentially.
The scaled boundary finite element equation is solved numerically. To start the algorithm for these nonlinear first order differential equations. The dynamic stiffness matrix *[S∞(ω_h)]* at high but finite ω_h is calculated from the asymptotic expansion polynomials equation as the boundary condition. A standard numerical integration procedure then yields *[S∞(ω)]* for decreasing *ω*. The error introduced through the boundary condition diminishes for decreasing *ω*. The numerical implementation of the aforementioned algorithm is done using the computer program of Wolf (Wolf and Song, 1996) with little modifications.

2. Cone Model

The soil is idealized as a truncated semi cone of initial radius r_o and apex distance z_o, with an opening angle depends on the static stiffness of half space under rigid disk of radius r_o, which can be exactly determined from theory of elasticity.

$$A(z) = A_o \left(\frac{z}{z_o} \right)^2 \quad (14)$$

$$A_o = \pi r_o^2 \quad (15)$$

From static equilibrium

$$p_o = \frac{E_c \pi r_o^2}{z_o} u_o \quad (16)$$

$$K_o = \frac{E_c \pi r_o^2}{z_o} \quad (17)$$

but $K_{exact} = \frac{4Gr_o}{1-v}$ (18)

comparing Eq.(17) with Eq.(18) leads to,

$$\frac{z_o}{r_o} = \frac{\pi}{2} \frac{(1-v)^2}{1-2v} \quad (19)$$

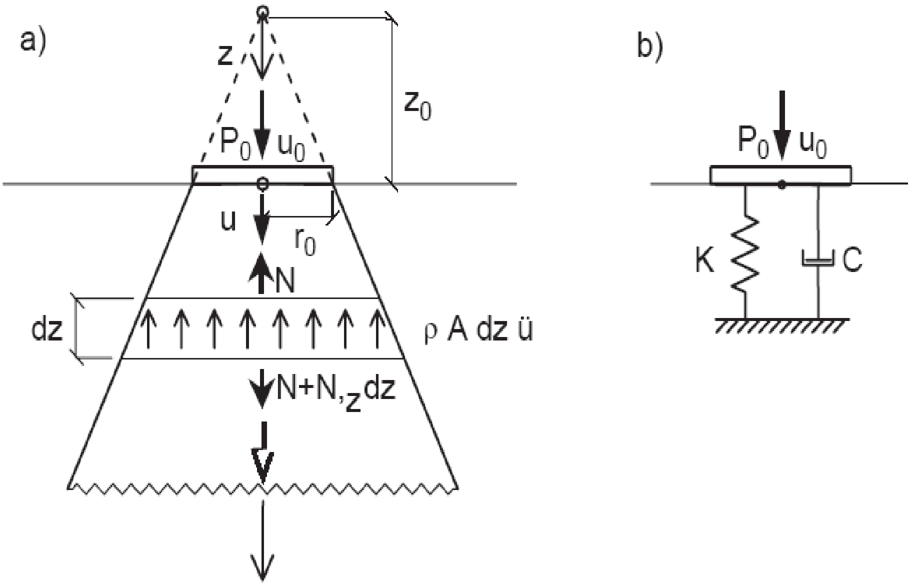


Fig. 3 Disk on surface of homogeneous half-space.
a) Truncated semi-infinite cone. b) Lumped-parameter model

Solving the one dimensional wave equation for out-coming waves only

$$u(z,t) = \frac{z_o}{z} f(t - \frac{z - z_o}{c_p}) \qquad (20)$$

$$p_o = \rho c_p^2 A_o \left[\frac{1}{z_o} u_o(t) + \frac{1}{c_p} \dot{u}_o(t) \right] \qquad (21)$$

$$p_o = K u_o(t) + C\,\dot{u}_o(t) \qquad (22)$$

$$K = \rho c_p^2 A_o \frac{1}{z_o} \qquad (23)$$

$$C = \rho c_p^2 A_o \qquad (24)$$

$$S(\omega) = (K + i\omega C)(u(\omega)) \qquad (25)$$

As $\omega \rightarrow \infty$ the $i\omega C \gg K$ i.e. K can be neglected as compared to ωC . knowing that the C is the same as that of prismatic rod with constant area , it means that wave propagation is perpendicular to the disk. This is the exact wave pattern of disk on a half space in the high frequency limits. Thus, the cone model also yields exact results for $\omega \rightarrow \infty$. As the opening angle of the cone is calculated by matching the static stiffness coefficients, a doubly-asymptotic approximation results for the cone, correct both for zero frequency (the static case) and for the high frequency limit dominated by the radiation dashpot C. Cone model analysis is done making use of the computer program(Conan) provided by Wolf and Deek's (2004).

- Dynamic Stiffness of Soil under Foundation

The vertical dynamic stiffness of soil under machine foundations is addressed here. Initially the dynamic stiffness of flexible foundation by the SBFEM or by the cone model is determined, and then it will pre and post multiplied by the vectors of rigid body motion to retain the dynamic stiffness for the given mode. A complex value stiffness matrix is calculated for each frequency step, starting from a high but finite frequency down to zero frequency i.e. static stiffness. The frequency range of the dynamic stiffness should cover practical range of frequencies for machine foundations. Worked examples, verification problems and parametric studies have been achieved, to show the effects of the geometrical and material parameters on the dynamic stiffness of soil under foundation.

The effects of Poisson's ratio on vertical dynamic stiffness and on damping coefficient of embedded foundation are sown in (Fig. 4).

A verification example of a square foundation with (*v*=0.4) and with different embedment ratios were compiled by SBFEM, the results were compared with the cone model and with boundary element method of (Mita and Luco, 1989). Acceptable agreements are shown (Fig. 4), with some deviations perhaps due to different discretization schemes.

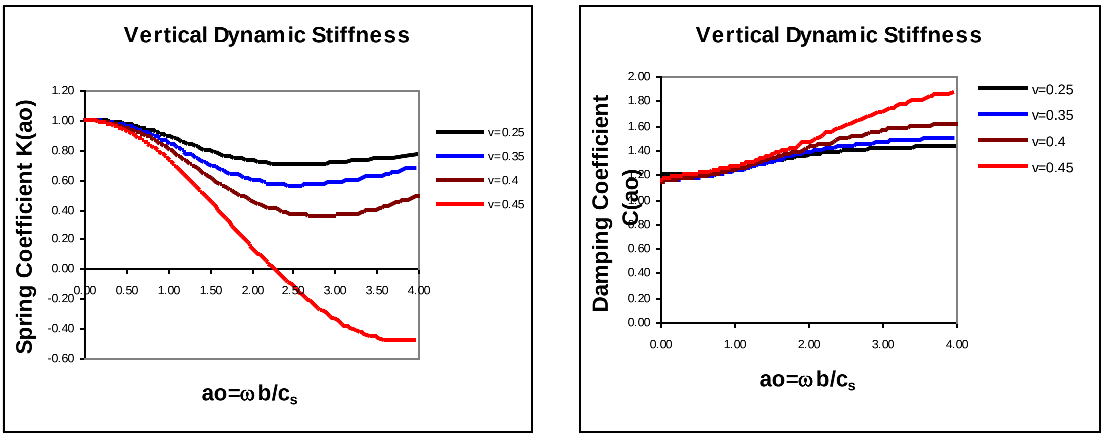


Fig. 4 Effect of Poisson's' Ratio (v) on Vertical Dynamic Stiffness of Rigid Rectangular Foundation .

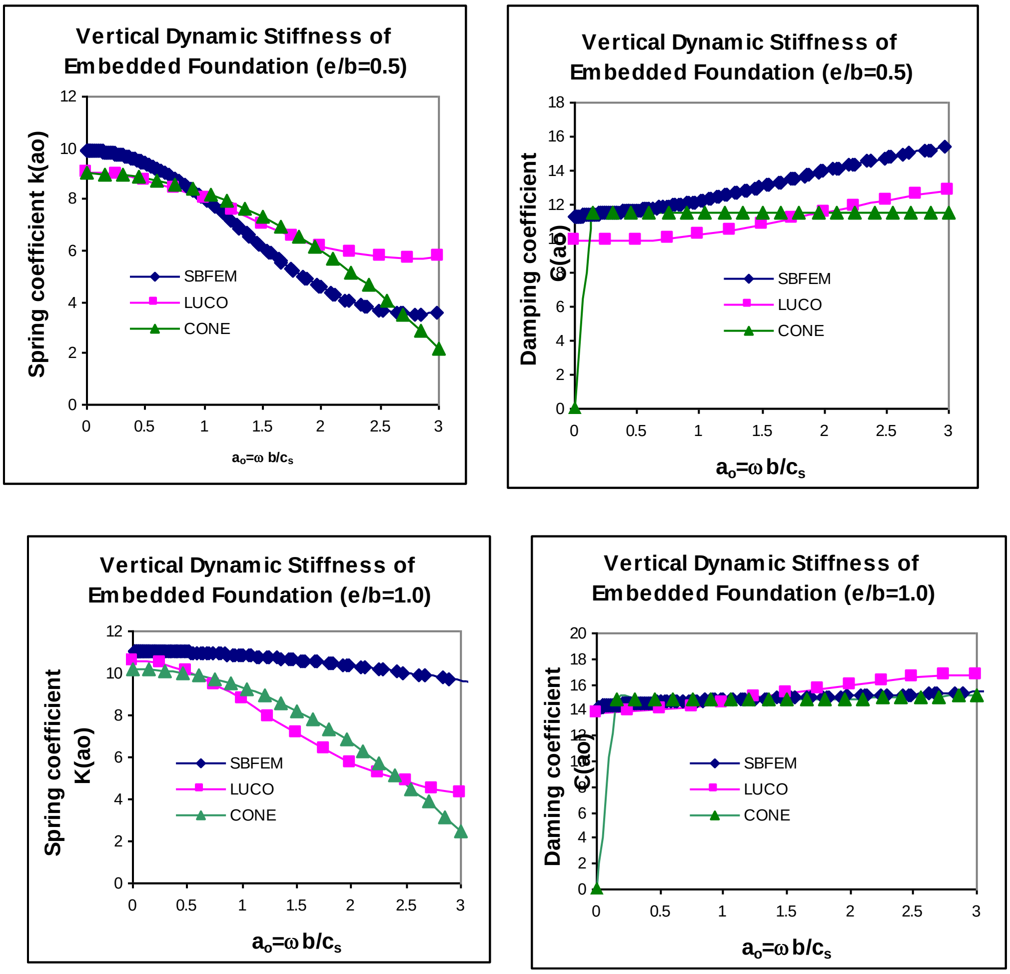


Fig. 5 Vertical Dynamic Stiffness of a Rigid Prism Foundation Resting on/in Half Space (ν=0.40), with Different Embedment Ratios.

- DATA PREPARATION, SPATIAL AND TEMPORAL DISCRETIZATION

The soil foundation interface is discretized in a finite element manner i.e. an eight nodded element for general three dimensional foundations, a tabulated joint coordinates, element incidence and boundary conditions are specified. Material properties for each element are fed. These include the mass density and the elastic constants. Elastic constants may be fed using different options, either two modulus option of elastic shear modulus and the Poison's ratio for isotropic soil or lower triangle of the material constitutive matrix for general anisotropic material.

The spatial discretization of soil foundation interface depends on the wave length [λ]. Extreme minimum value of λ is retained by dividing the shear wave velocity by the highest frequency. The recommended element length or node to node distance is [λ/4 to λ/6] (Wolf, 2003). The solution of the nonlinear differential equation of SBFE in dynamic stiffness is in the form of power series. The solution starts with expansion of dynamic stiffness into power series at high but finite frequency. This high valued frequency should be fed by analyst to start the numerical solution. The dynamic stiffness matrix is calculated for each frequency step from the high frequency down to low or zero frequency value corresponding to the static stiffness matrix. The initial decrement of frequency is fed, and a minimum frequency step should be specified to terminate the solution algorithm.

For cone model, the spatial discretization i.e. the distances between disks should not be more than [λ/5] (Wolf and Deek's, 2004), where λ is the shear wave length. There is no restriction on the selection of the upper or lower frequency, and any frequency may be used.

APPLICATIONS

For a given frequency, the dynamic stiffness of soil supporting a rigid foundation is determined using the scaled boundary finite element method, or cone model. The equivalent springs and dampers coefficients for each nodal degree of freedom are calculated. Each node assumes its share of stiffness and damping due its tributary area. The springs and dampers form the boundary conditions for the finite element model of the foundation mass.

A parametric study shows the effect of mass ratio (b_z) (mass of foundation plus machine/ ρb^3) upon the dynamic response, resonant frequency and damping ratios, for an embedded rectangular foundation of (e/b=0.5) with aspect ratio of length to width of (L/b=3) in (Figs. 6 and 7).

Interpretation of these graphs indicates that as the mass ratio increases the resonant frequency decreases and the damping ratio decreases and the dynamic response increases at distinct peaks. For low mass ratio no peaks seam distinct and the dynamic response always decreases.

The next parametric study shows the effect of stiffness ratio *($K_{(isolator)}$ /$K_{(soil)}$)* of a square embedded foundation on the forces transmitted to soil. This is the dynamic soil reaction as a fraction of the driving dynamic force, or in a conventional form it is the isolation efficiency as seen in (Fig. 8). The effect of stiffness ratio on the fundamental frequency is shown in (Fig. 9). It is clear that isolation effectiveness increases with the decrease in the stiffness ratio i.e. when using more flexible isolator sandwiched between the base and machine. Noting that an increase in the displacement of the machine itself, however, can be controlled by enhancing the isolation system with an energy dissipation devices i.e. dampers. The use of more flexible isolators decreases the fundamental frequency considerably , down to the rigid body mode of lowest frequency when one can get the best isolation effectiveness at the so called isolation frequency.

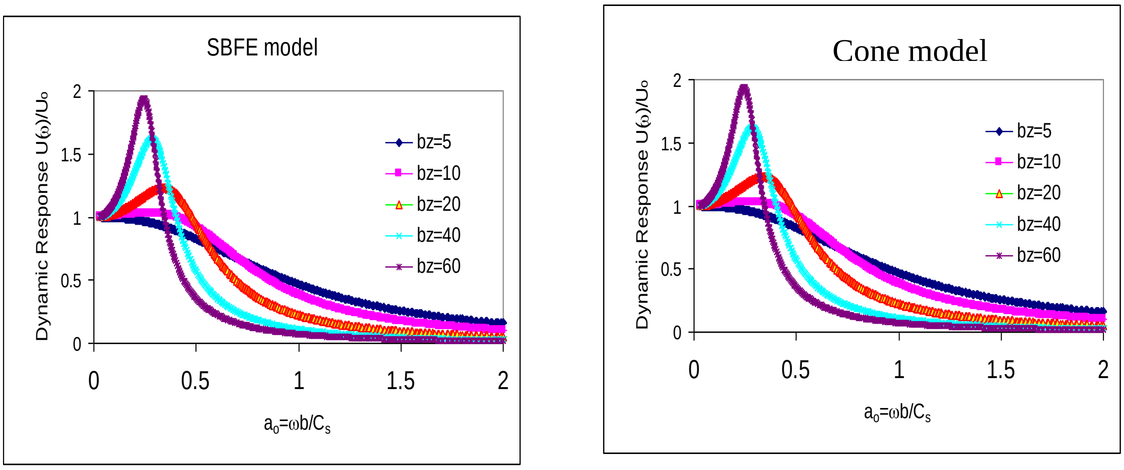


Fig. 6 Effect of Mass Ratio on Dynamic Response of Embedded Rectangular Foundations (E/B=0.5), (L/B=3.0).

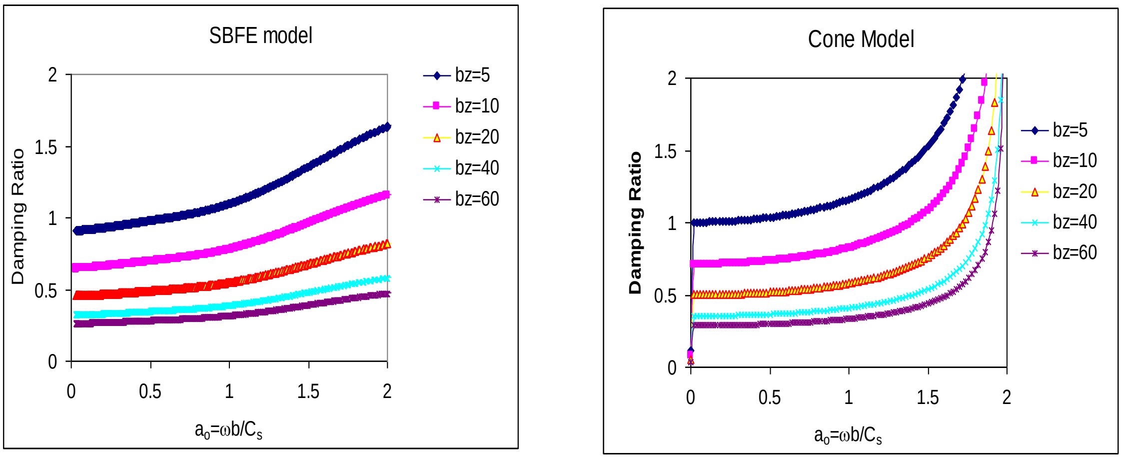
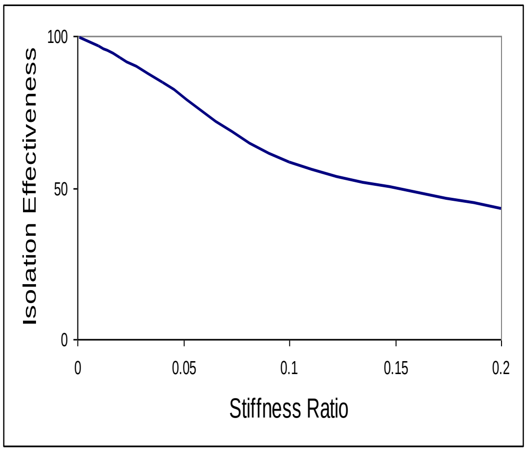


Fig. 7 Effect of Mass Ratio on Damping Ratio of Embedded Rectangular Foundations (E/B=0.5), (L/B=3.0).



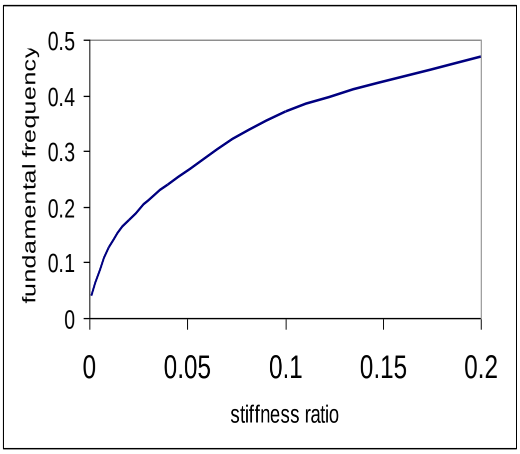


Fig. 8 Effect of Stiffness Ratio on Isolation Efficiency.

Fig. 9 Effect of Stiffness Ratio on Resonant Frequency.

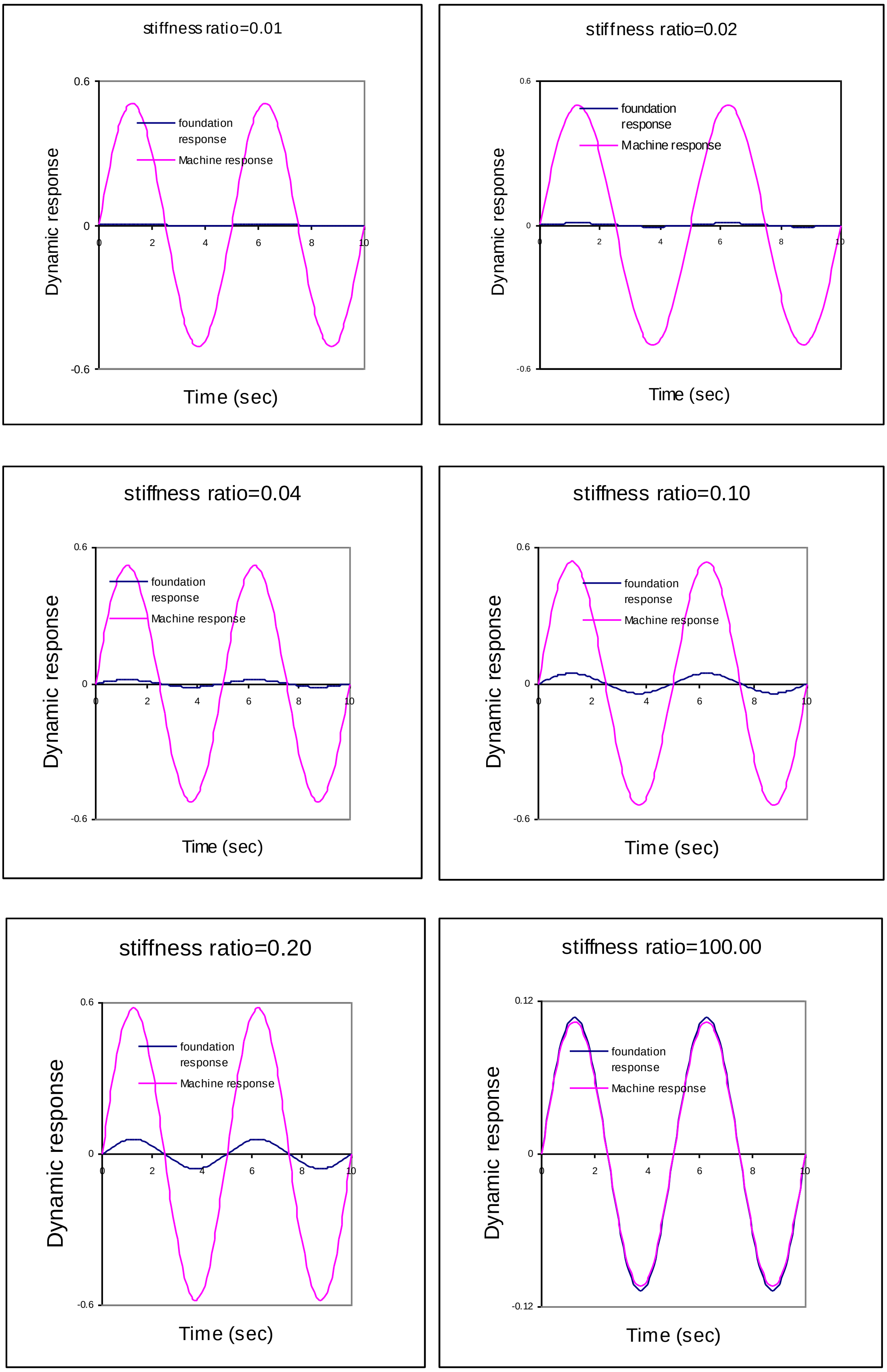


Fig. 10 Effect of Stiffness Ratio on Vertical Dynamic Response of Machine and Foundation.

The presence of isolation between the machine and its base forms a discrete system of two degrees of freedom. The effect of the stiffness ratio on the vertical dynamic displacements of the machine and its supporting foundation is shown in (Fig.10). The uncoupled motion maintained at high isolation effectiveness leads some designers to design the isolated foundation under static loads only. This is a major advantage of using effective base isolation it will not only reduce the dynamic problem into static one but it will make it possible to place the machine even on a tenth floor of a multi story building or in/on soil with uncertain dynamic properties. In addition there is no need for a thorough sophisticated soil dynamics investigations.

- CASE STUDY

A turbine machine of (762,000 kg)mass rests on a reinforced concrete foundation block of dimensions (24.0mx 8.0m x4.0m) and of (1,920,000 kg) mass (Fig.11) The operational frequencies are 50 Hz, 314 rad/sec. The corresponding vertical stiffness and damping of soil obtained from the scaled boundary finite element method are (76 x 10^8 N/m) and (15 x 10^{10} N/m) respectively. The mass of foundation block and machine is (2.592 x 10^6 kg) and the damping ratio is more than 100% , however , 25% of critical damping will be used , according to (DIN 4024, after ACI 351.3R-04) the corresponding damping coefficient is (0.70 x 10^8 N.sec/m). The amplitude of harmonic dynamic force of (1.344 10^6 N) was assumed, which is 20% of the weight of the machine($F(t)=1.344x10^6 sin(314t)$).

For isolated rigid mass model a spring isolator with stiffness of one tenth of that of soil was used, results in two degrees of freedom system. The results of vibration analysis of both isolated and non-isolated foundations are listed in (Table 1)

Several finite element models were used for analyzing this case with different element sizes.

- One element model of 24 x 8 x 4 m element size,
- 12-element model with 4 x 4 x 4 m element size,
- 96-element model with 2 x 2 x 2 m element size,
- 768-element model with 1 x 1 x 1 m element size.
- Two cases of isolated finite element model with stiffness ratio of ten percent, these are one-element and 12-element models were investigated. (Table 2) lists the results of the finite element models of vertical vibration of block foundations with and without isolation.

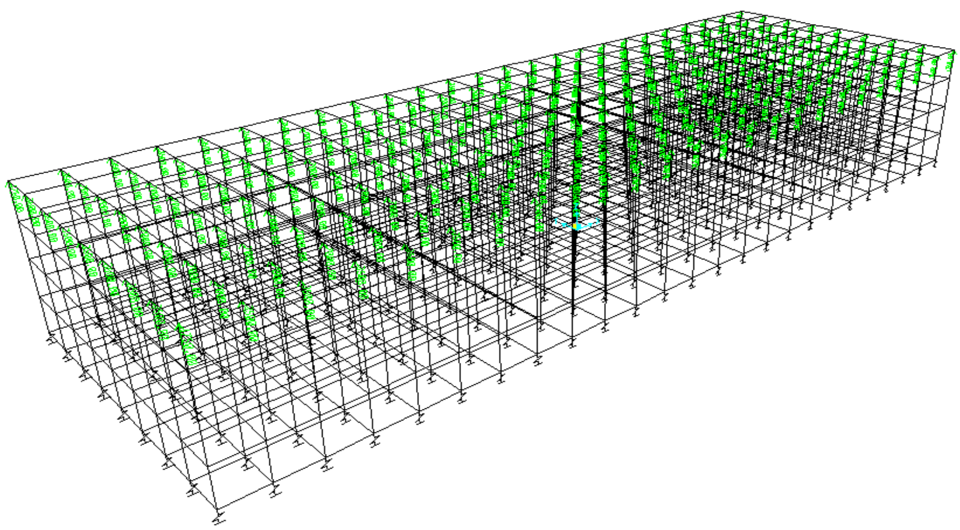


Fig. 11 Finite Element Idealization of Machine Foundations

Table 1 Vertical Dynamic Responses Of Rigid Mass Foundation.

case	Natural frequency (rad/sec)	Displacements micron (10^{-6}meter)		Transmissibility Ratio %
		foundation	machine	
Non-isolated	54	5.3		3
isolated	32	1.8	20	1.0

Table 2 Vertical Dynamic Responses Of Block Foundation Using Finite Element Method.

case	Natural frequency (rad/sec)	Displacements micron (10^{-6}meter)		Transmissibility Ratio %
		foundation	machine	
One element	53.7	5.25		3.2
12-element	53.69	5.24		3
96-element	53.6	5.2		2.96
768-element	52.8	5.07		2.85
Isolated one-element model	32	2.6	20	1.5
Isolated 12-element model	31	2.32	20	1.3

Examination of (Table 1) and (Table 2) leads to the following non surprising conclusions: Resonant frequency decreases considerably with the use of isolators, the vertical

displacement and the dynamic soil reaction of foundation decrease, while the displacement of machine increases with the use of isolators; Resonant frequency decreases slightly with the use of finite element model and as the number of element increases, this result is not surprising because increasing of element numbers means the use of more flexible (less stiffness) system keeping the mass(s) not changed. The use of isolators in the finite element model results in the same response for foundation and machines with slight reduction in foundation response (displacements and dynamic soil reactions)with the increasing of element number.
Both dynamic responses of machine and foundation in non-isolated and in isolated systems are within acceptable limits(section 3.3). As the non-isolated natural frequency(54 rad/sec) is sufficiently far away from the driving frequency(314 rad/sec). The use of isolators has no practical values if it is not detrimental. However it may be beneficial when the machine assumes unusual frequencies when turn on/off conditions or emergency shutdown.

CONCLUSIONS

Comparing the SBFEM and the cone model results with the published boundary element, the applicability of these models are demonstrated. The important observations of the effect of the geometrical and material properties of soil under foundation upon the dynamic stiffness and dynamic response are presented separately in the following paragraphs. The vertical vibration of rectangular foundations have been studied by the SBFEM and by the cone model, some conclusions may be drawn as follows:-

- The results for the vertical dynamic stiffness of rectangular foundations indicate that the real part for the spring coefficient decreases by (0% to 80%) and may posses negative values and the damping coefficient increases up to (50%) as Poisson's ratio increases from (0.25) to (0.45). The frequency effect on spring and damping coefficients of embedded foundations with different embedment ratios (e/b= 0.5,1.0,1.5,2.0)has been found. Both spring and damping coefficients increase with the increasing of embedment ratio.

- The mass ratio$(M_f/\rho b^3)$ of foundation has a significant role in the dynamic response. As the mass ratio increases from(5) to (20) the resonant frequency decreases by (70%), damping ratio decreases by (50%) and the dynamic response increases by (100%) with a distinct peaks. For low mass ratio (less than 5) no peaks seem distinct and the dynamic response always decreases.
- The effect of stiffness ratio ($K_{(isolator)}/K_{(soil)}$) on the isolation efficiency or on tuning of fundamental frequency. It seems that isolation effectiveness increases from(50%) to (80%) with decreasing in stiffness ratio from(0.20) to (0.05). The use of more flexible isolators decreases the fundamental frequency by(50%) when the stiffness ratio decreases from(0.20) to (0.05).
- The presence of isolation between machine and its base forms a system of two degrees of freedom at each mass of both the machine and the foundation. These two degrees of freedom systems seem to be gradually uncoupled as much as the isolation stiffness being more flexible (stiffness ratio less than 0.05). The major advantage of base isolation, not only reduces the dynamic problem into static one but also it reduces the need for a thorough sophisticated soil dynamic investigations.
- For vertical mode, the use of finite element model affects the response slightly (less than 10%) due to the flexibility of finite element model as compared with rigid mass model. Resonant frequency decreases slightly(less than 1%) with the use of finite element model and as the number of elements increases. This result is not surprising

because increasing of element numbers means the use of more flexible (less stiffness) system keeping the mass(s) not changed.

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NOMENCLATURE

The major symbols used in this paper are listed below; others are indicated with their equations where they first appear.

A	Plan area of foundation
a_o	Dimensionless frequency $a_o = \frac{\omega b}{C_s}$
A_u	Side face with pre-specified displacements
A_t	Side face with pre-specified forces
b	Half width of foundation (characteristic length)
$[b^1],[b^2],[b^3]$	Scaled boundary finite element differential operators
b_z	Mass ratio
C	Wave velocity
C	Damping coefficient
$[C]$	Damping matrix
$[C_\infty]$	Damping matrix of unbounded media
E	Modulus of elasticity
$[E^o],[E^1],[E^2]$	Elastic Matrices of Scaled boundary finite element
$[E^{o*}],[E^{1*}],[E^{2*}]$	Complex Elastic Matrices of Scaled boundary finite element involving material damping
e	Volumetric strain
e/b	Embedment ratio
$f(t)$	Incident wave
G	Shear modulus of elasticity
$[G(\omega)]$	Dynamic flexibility matrix
$g(t)$	reflected wave
$h(t)$	refracted wave
I	Moment of inertia of plan of foundation
i	Imaginary part $=\sqrt{-1}$
J	Jacobean matrix
k	Stiffness coefficient
$[K]$	Stiffness Matrix
$[K_\infty]$	Stiffness Matrix of unbounded media

L	Half length of foundation
$[L]$	Differential operator
m	Lumped Mass
$[M]$	Mass matrix
$[M^o]$	Mass matrix of Scaled boundary finite element
$N(z,t)$	Normal force
O	Similarity centre
r	Radial coordinates
r_o	Characteristic length of foundation
$S(\omega)$	Dynamic stiffness
$[S(\omega)]$	Dynamic stiffness matrix
$\bar{t}_i$	Traction forces
u_i	Component of displacement
$\dot{u}_i$	Component of velocity
$\ddot{u}_i$	Component of acceleration
v	Poisson's ratio
x, y, z	Cartesian's coordinates
α	Reflection coefficient of waves
β_m	Material hysteretic damping
β	Frequency ratio
ε_i	Normal strain component
ε_{ij}	Shear strain component
λ	Wave length
λ, μ	Lame' constants
ρ	Mass density
σ_i	Normal stress component
τ_{ij}	Shear stress component
ω	Frequency
ω_{ij}	Rotational strain
ζ	Damping ratio
ξ, ζ, η	Natural coordinates