STIFFNESS AND DAMPING PROPERTIES OF EMBEDDED MACHINE FOUNDATIONS

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ABSTRACT
In this study, a dynamic analysis of machine foundations under vertical excitations is carried out. The effect of embedment and foundation geometry has been taken into account. The stiffness and damping of soil are considered as frequency dependents. A computer program (**CPESP**) in FORTRAN POWER STATION has been coded to evaluate the stiffness and damping coefficients depending on excitation frequency and embedment depth. Results have shown that increasing the embedment depth leads to increasing the resonant frequency and decreasing the amplitude of vibration.

الخلاصة

في هذه الدراسة تم تنفيذ التحليل الديناميكي لأسس المكائن تحت تأثير الاهتزازات العمودية. تأثيرعمق الطمر (embedment) وشكل الأساس قد أُخِذا في الحسبان. إنّ الجسائة والتخميد للتربة قد اعتبرت معتمد التردد(frequency dependent).تم اعداد برنامجا(**CPESP**) بلغة (FORTRAN POWER STATION) لحساب معامل الجسائة والتخميد للتربة بالإضافة إلى حساب ثابت الجسائة الديناميكي(K) وثابت التخميد الديناميكي(C) المعتمدين على تردد الاهتزاز(ω) وعلى عمق الطمر(embedment). تمت المقارنة بين الدراسة الحالية وطريقة التقريب الدائري المكافئ(equivalent circular approximation).أظهرت النتائج إن زيادة عمق الطمر يؤدي إلى زيادة التردد ألرنيني ويقلل من سعة الاهتزاز.

KEY WORDS
Dynamic, Machine Foundation, Stiffness, Embedment.

INTRODUCTION

Most of the solution methods treat the machine foundation as a block resting on the surface of an elastic soil. The real footings are usually embedded and this considerably affects the dynamic response of footing, Barken.D.D (1962). The rigorous analytical solution of embedded footings has many mathematical difficulties. The most promising way of studying this problem is the finite element analysis as had been used by many researchers such as Lysmer.J(1979) and by kaldjian .M.J (1969) for static analysis.

Nevertheless, there is a need for alternative approximate solutions that would be able to predict the motion and to evaluate the stiffness and damping characteristics of embedded footings.

EQUATION OF MOTION

By applying de Alembert's principle, the equation of motion can be written as; **Fig. (1)**

$$m\ddot{U}_z(t) + C_z(\omega)\dot{U}_z(t) + K_z(\omega)U_z(t) = P_o \exp(i\omega t) \qquad (1)$$

Where:-

m = Total mass.

$m\ddot{U}_z(t)$ = Inertia force.

$C_z(\omega)\dot{U}_z(t)$ =Damping force.

$K_z(\omega)U_z(t)$ = Elastic force.

$K_z(\omega)$ =Frequency dependent stiffness.

$C_z(\omega)$ =Frequency dependent damping.

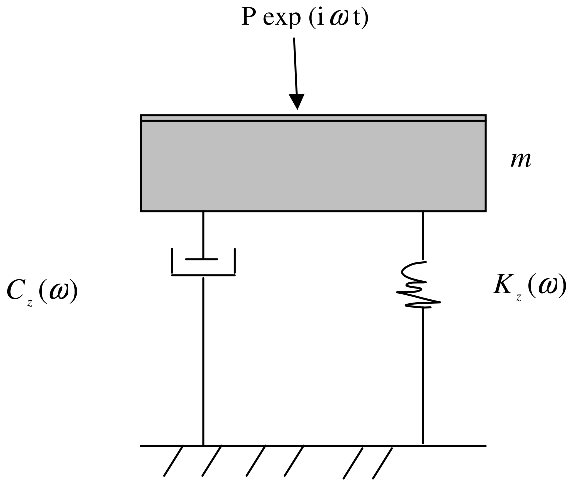


Fig. (1) Foundation resting on spring and dashpot

For harmonic loading with an excitation frequency of ω, the steady state solution can be assumed as:

$$U_z(t) = A_z \exp(i\omega t) \qquad (2)$$

Substituting eq. (2) into eq. (1):-

$$-m\omega^2 A_z \exp(i\omega t) + [K_z(\omega) + i\omega C_z(\omega)]A_z \exp(i\omega t) = P_o \exp(i\omega t)$$

Dividing both sides of the equation by $\exp(i\omega t)$ and separating real and imaginary parts, the amplitude of motion A_z will be:-

$$A_z = \frac{P_o}{[(K_z(\omega) - m\omega^2) + i\omega C_z(\omega)]} \qquad (3)$$

Let $a_1 = K_z(\omega) - m\omega^2$

$a_2 = -\omega C_z(\omega)$

Multiplying the numerator and denominator of eq. (3) by $(a_1 + ia_2)$, the amplitude can be written as:-

$$A_z = \frac{(a_1 + ia_2)P_o}{a_1^{\ 2} + a_2^{\ 2}} = \frac{\exp(i\phi)}{R} P_o \tag{4}$$

Where:-

$R = \sqrt{a_1^{\ 2} + a_2^{\ 2}}$

And

$\phi = \tan^{-1}(a_2/a_1) = \tan^{-1} \dfrac{-\omega_z C_z(\omega)}{K_z(\omega) - m\omega^2}$

Substituting A_z into eq. (2) the steady state solution becomes:-

$$U_z(t) = \frac{\exp(i\phi)}{R} P_o \exp(i\omega t) = \frac{P_o \exp[i(\omega t + \phi)]}{\sqrt{(K_z(\omega) - m\omega^2)^2 + \omega^2 C_z^{\ 2}(\omega)}} \tag{5}$$

The real part of the amplitude of vibration is:-

$A_Z = \dfrac{P_o}{\sqrt{(K_Z(\omega) - m\omega^2)^2 + \omega^2 C_Z^{\ 2}(\omega)}}$

where:-

Eq. (5) gives the dynamic response of the foundation in vertical vibration and for an exciting force of constant amplitude P_o.

The natural frequency of the undamped free vibration is:-

$$\omega_n = \sqrt{K_z(\omega)/m} \tag{6}$$

In this study a rigid foundation will be studied which is located at depth D below the ground surface. This foundation is subjected to a steady-state vibration by a harmonic vertical force, $P(t) = P_o \exp(i\omega t)$, having an amplitude of P_o and a circular frequency ω, and acting through the centeroid of the base. This dynamic force is resisted by normal soil stresses against the base and by shear stresses along the vertical foundation sides. The rotational oscillations that may occur due to the lack of complete symmetry in the soil reactions at the base and especially at the foundation sides are ignored in this study. The steady-state response of the foundation is thus described by the vertical dynamic settlement $U = U_o \exp(i\omega t)$.

Due to damping the force, $P(t)$ is generally out of phase with the response $U(t)$. The latter can be divided into two components, one in phase [$U_1 \exp(i\omega t)$] and the other 90^o out of phase [$U_2 \exp(i\omega t)$] with P. (11)

The corrected dynamic stiffness, $\overline{K}(\beta)$ and the dynamic damping coefficient, $\overline{C}(\beta)$ are given by:-

$$\overline{K}(\beta) = \overline{K}(\omega) - \omega C \cdot \beta \tag{7.a}$$

$$\overline{C}(\beta) = C + \frac{2\overline{K}(\omega)}{\omega} \cdot \beta \tag{7.b}$$

Where:

β = frequency independent damping ratio. For most soils β ranges typically from 0.02 to 0.05, Richart.F.E. (1970).

Both the effective dynamic stiffness and the radiation damping coefficient of the soil–foundation system are functions of the frequency ω. It is convenient to express $K_{sur})_{dy}$ as a product of the static stiffness, K_{sur} of the system times a dynamic stiffness coefficient $k(\omega)$

$$K_{sur})_{dy} = K_{sur} \cdot k(\omega) \tag{8}$$

STATIC STIFFNESS OF SURFACE FOUNDATIONS

For a surface foundation of an arbitrary shape, the vertical static stiffness K_{sur}, is given by Dominguez,J(1978):

$$K_{sur} = \frac{2LG}{1-\upsilon} S_z \tag{9}$$

Where:-
L =Semi-length of a rectangle circumscribed to base surface.
G =Shear modulus of soil.
υ =Poisson's ratio.
S_z =Vertical static stiffness parameter.
For non-rectangular base, K_{sur} may be obtained as follows, Prakash,S(1988):-

$$K_{sur} = 4GR/(1-\upsilon) \tag{10}$$

Where:-
R= Radius of the equivalent circle = $\sqrt{A_b/\pi}$
The equivalent circle approximation predicts S_z as follows [10]:-

$$S_z = \frac{4}{\sqrt{\pi}} \sqrt{A_b/4L^2} \tag{11}$$

The equivalent circle approximation gives good results for L/B≤2 to 3 as calculated by Dobry and Gazetas (1986). **Fig (2)** shows that:-

$$\begin{aligned} S_z &= 0.8 && for\ A_b/4L^2 < 0.02 \\ S_z &= 0.73 + 1.54(A_b/4L^2)^{0.75} && for\ A_b/4L^2 > 0.02 \end{aligned} \tag{12}$$

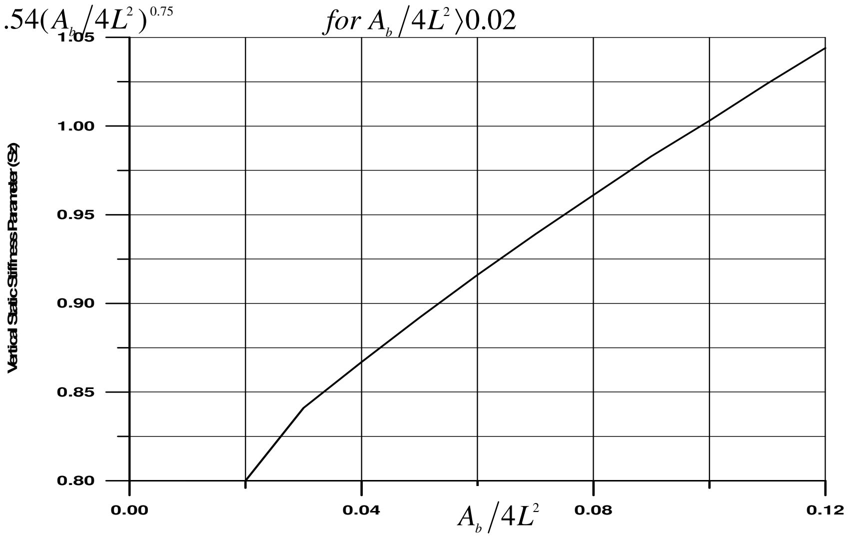


Fig. (2) vertical static stiffness parameter (Sz) versus base shape [10]

EFFECT OF EMBEDMENT ON STATTIC STIFFNESS

In practice, foundations are placed at a specified depth, say D below the ground surface and transmit the load to soil. Usually, increasing the depth D means increasing the foundation stiffness K.

The factors that modify the foundation stiffness are the "trench" and "sidewall contact" effects, that tend to increase the stiffness of the embedded foundation. These two effects are to be explained with the aid of **Fig. (3).**

Trench Effect

Even in perfectly homogenous soil a rigid footing will settle less if it is placed at the bottom of an open trench. The normal and shear stresses resulting from the overlying soil restricts the vertical movement and thus reducing the settlement of the foundation base by increasing its vertical stiffness.

The trench effect suggested by Gazetas and Dobry (1986) is:-

$$K_{tre}/K_{sur} = I_{tre} \rangle 1 \qquad (13)$$

Where:-

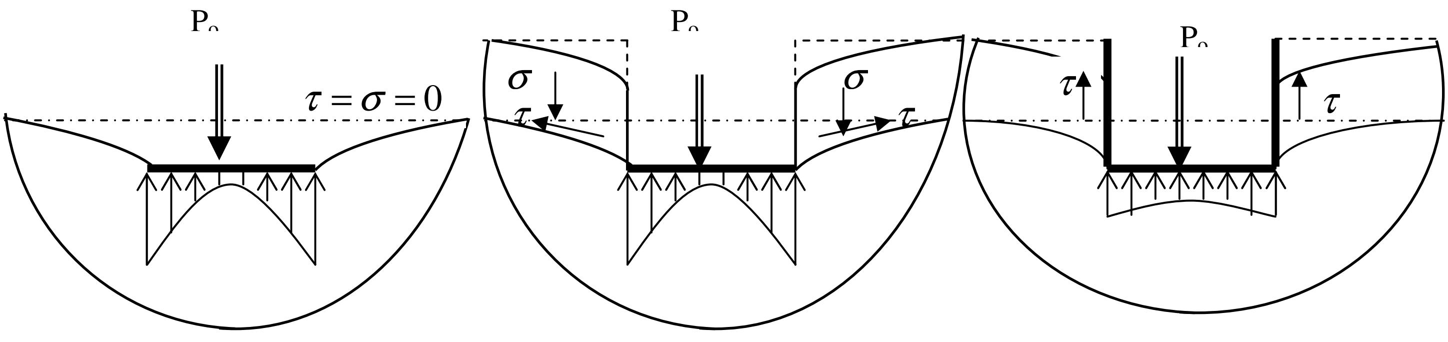
K_{tre} is the vertical static stiffness of an embedded foundation mat with no sidewall contact.

Sidewall Effect

Part of the applied load is transmitted to the ground through shear stresses along the vertical sides of the footing when the sides are in contact with the surrounding soil.

As a result, the overall stiffness of an embedded foundation K_{emb} is larger than K_{tre} stiffness corresponding to a foundation with the same depth of embedment but without side effect , Ricardo.D (1985).

$$\frac{K_{emb}}{K_{tre}} = I_{side} \rangle 1 \qquad (14)$$



$K_{tre}/K_{sur} = I_{tre} \rangle 1$ $K_{emb}/K_{tre} = I_{side} \rangle 1$

(a) (b) (c)

Fig. (3) effects of embedment on vertical static stiffness of foundation
(a) settlement due to surface foundation (b) trench effect
(c) combined trench and sidewall effects.

Experimental studies, such as those of Lysmer.j (1969), offer valuable guidance in this direction. Combining eqs. 13 and 14 lead to:

$K_{emb} = K_{sur} \cdot I_{tre} \cdot I_{side}$

Based on test results the following empirical equations had been derived:-

$$I_{tre} = 1 + \frac{D}{21B}\left(1 + \frac{4}{3}\frac{A_b}{4L^2}\right) \tag{15}$$

$$I_{side} = 1 + 0.19(A_s / A_b)^{0.666} \tag{16}$$

Where:-

I_{tre} =Trench factor.

I_{side} =Sidewall factor.

A_b =Base area of foundation.

A_s =Sides area of foundation.

Fig (4) shows that as (D/B) increases the ratio of (K_{tre}/K_{sur}) also increases. This trend is more pronounced for the case of a square foundation (L/B=1).

The foundation static stiffness (K_{emb}) for a full embedment case is:-

$$K_{emb} = \frac{2LG}{1-\upsilon} S_z \left[1 + \frac{1}{21}\frac{D}{B}(1 + \frac{4}{3}\frac{A_b}{4L^2})\right]\left[1 + 0.19(\frac{A_s}{A_b})^{0.666}\right] \tag{17}$$

Fig.(5) shows that as (D/B) increases the ratio (K_{emb}/K_{sur}) also increases. Again this trend is more pronounced for the case of a square foundation (L/B=1)

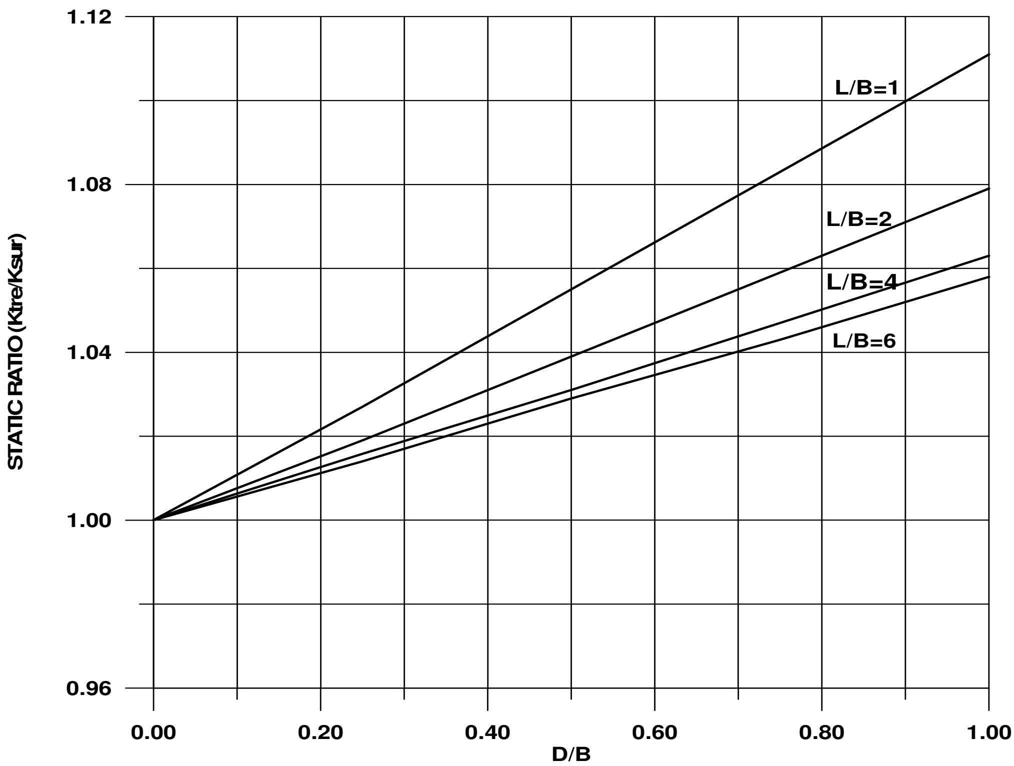


Fig. (4) effect of trench on static stiffness

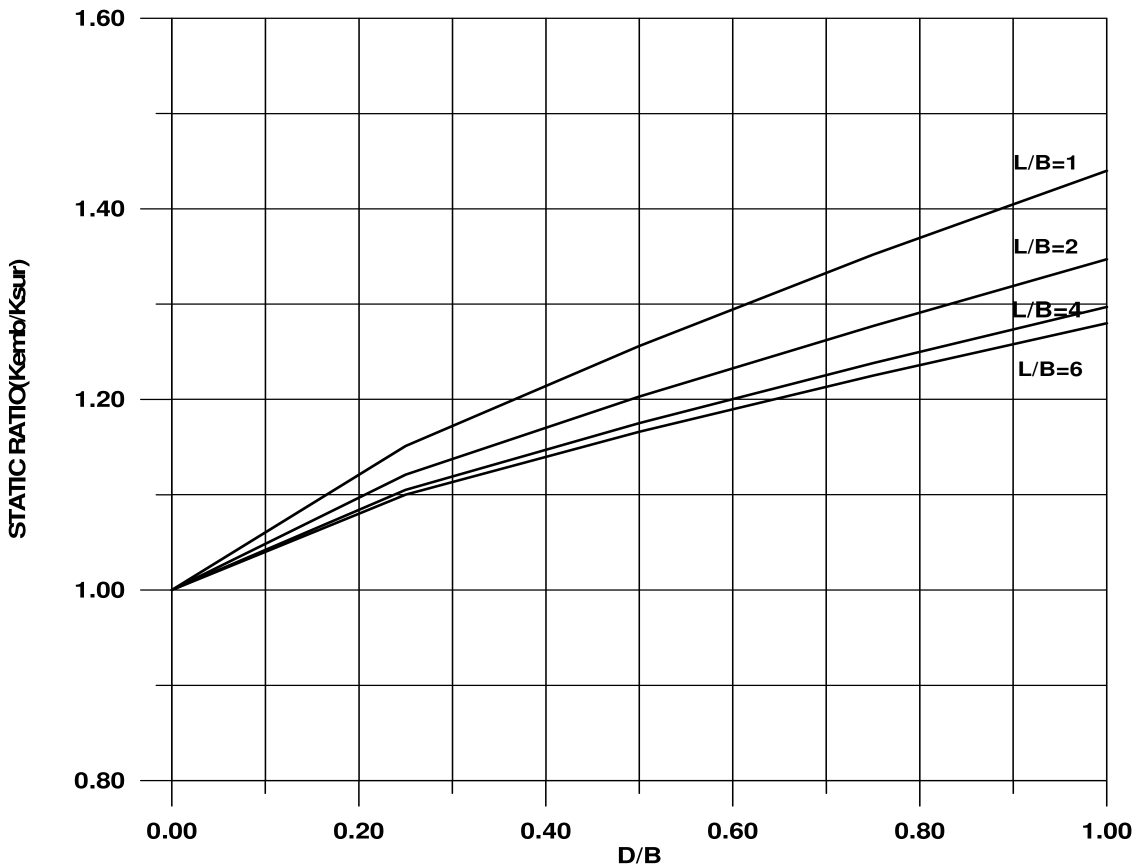


Fig. (5) effect of embedment on static stiffness

DYNAMIC STIFFNESS COEFFICIENT

It had been concluded empirically by George.G(1986) that the vertical stiffness of elastic foundation is frequency dependent. The main parameters affecting the dynamic stiffness are a_o, L/B and υ, where:-

a_o =Normalized frequency $= \omega B/V_s$

Where:-

V_s =Shear wave velocity.

L/B =Foundation aspect ratio.

υ =Poisson's ratio.

The frequency dependent stiffnesses are:-

For Poisson ratio $\upsilon = 0.33$ (unsaturated soil)

$$K_{emb})_{dy} = K_{emb} \cdot k(\omega) \cdot \left[1 - 0.09(a_o)^2 (D/B)^{0.75}\right] \tag{18.a}$$

$$K_{tre})_{dy} = K_{emb} \cdot k(\omega) \cdot \left[1 + 0.09(a_o)^2 (D/B)^{0.75}\right] \tag{18.b}$$

For Poisson ratio $\upsilon = 0.5$ (saturated soil)

$$K_{emb})_{dy} = K_{emb} \cdot k(\omega) \cdot \left[1 - 0.35(a_o)^2 (D/B)^{0.5}\right] \tag{18.c}$$

$$K_{tre})_{dy} = K_{emb} \cdot k(\omega) \cdot \left[1 + 0.35(a_o)^2 (D/B)^{0.5}\right] \quad (18.d)$$

These equations were obtained by Gazetas and Dobry (1986), Where:-
$K(\omega)$:is a dimensionless frequency dependent factor given in **Table (1)**. Hence the dynamic stiffness of an embedded foundation can be written as:-

$$K_{emb})_{dy} = K_{emb} \cdot k(\omega) \cdot F_e = K_{emb} \cdot \overline{F}_e \quad (19)$$

Where:-
$F_e = \left[1 - 0.09(a_o)^2 (D/B)^{0.75}\right]$
or
$F_e = \left[1 - 0.35(a_o)^2 (D/B)^{0.5}\right]$
as given in eq. (18).

The factor $\overline{F}_e$ of eq. (19) is the effective embedment factor. **Fig. (6)** Shows the variation of this factor with the normalized frequency parameter(a_o). The relationships have been obtained in the present study by coding the above equations through a short computer program.

Table (1) dynamic stiffness factor for surface foundation [$K(\omega)$]

Passion ratio	Frequency dependent stiffness factor [$K(\omega)$]	L/B
0.33	1.0035+0.051953 (a_o)-0.123599(a_o)^2	1 and 2
	0.966691+0.55445(a_o)-0.771009(a_o)^2	6
	1.02098+1.10380(a_o)-1.3743(a_o)^2	10
0.50	1.00055-0.0807878(a_o)-0.0362395(a_o)^2	1
	0.95004+0.46544(a_o)-0.35049(a_o)^2	4
	0.841195+1.34818(a_o)-0.823897(a_o)^2	≥6

DAMPING COEFFICIENT

The coefficient of damping $c = c(\omega)$ is a measure of vibration energy transmitted into the soil and carried away by spreading waves. These waves are generated at every point on the soil-foundation interface so that in general $c(\omega)$ increases with increasing area of contact.

The contact surface for a vertically oscillating embedded foundation consists of a horizontal base and vertical sides. The base transmits to the underlying ground compression-extension waves in propagation velocity close to the Lymers (1969) analogy·

$$V_{La} = 3.4 \cdot V_s / \left[\pi(1 - \upsilon)\right] \quad (20)$$

Where:
V_s = shear wave velocity
V_{La} = "Lysmer's analog" velocity
On the other hand the sides transmit mainly shear waves through the surrounding soil.
The two types of waves generated at the base and at the sides of an embedded foundation are independent. Summing up the respective radiated energies.

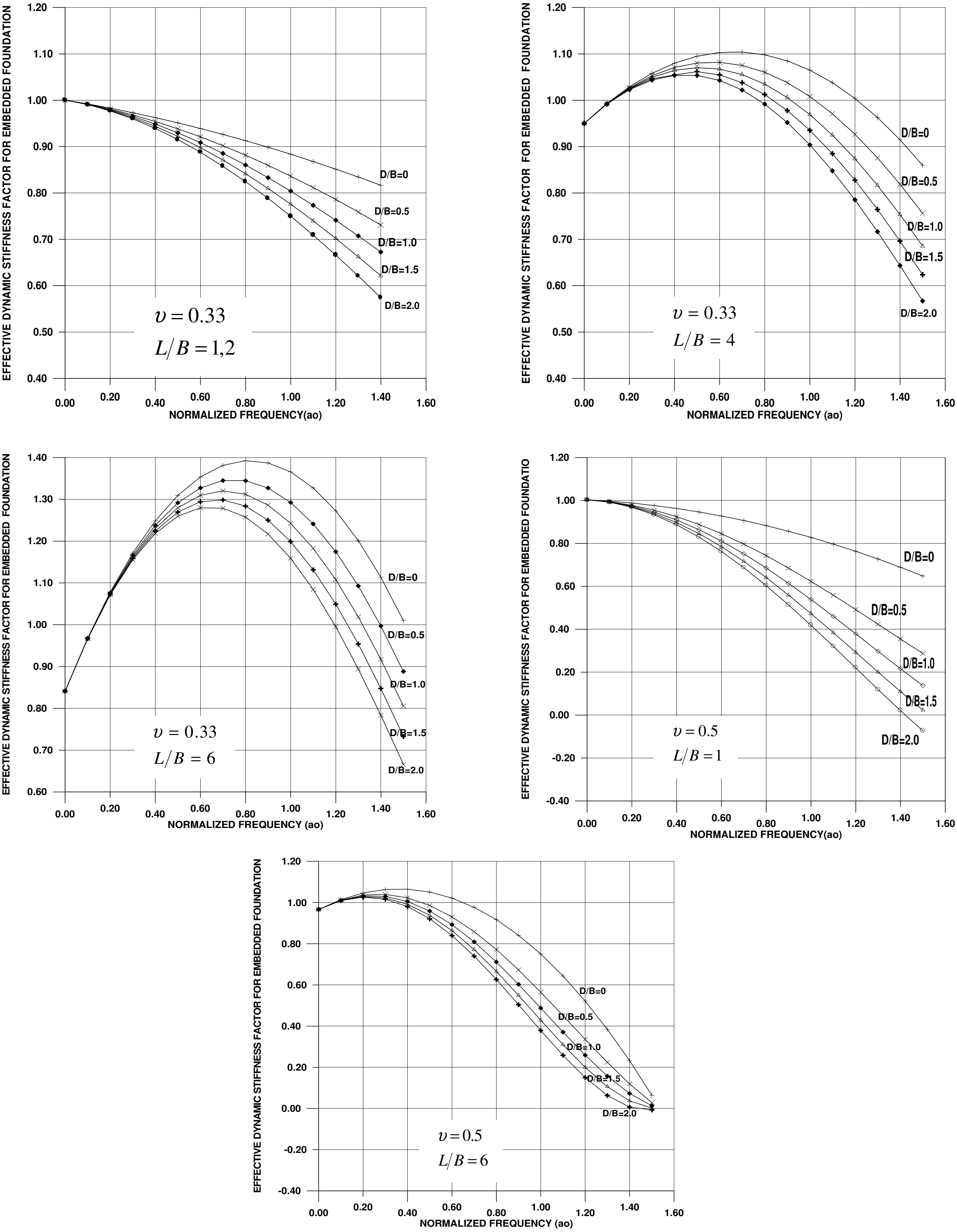


Fig. (6) effective dynamic stiffness factor $\overline{F}_e$ for embedded foundations

$$C = (\rho \cdot V_{La} \cdot A_b) \cdot c(\omega) + \rho \cdot V_s \cdot A_s \qquad (21)$$

Where:-
$c(\omega)$: Coefficient of dynamic damping as given in **Table (2).**

Table (2) dynamic damping coefficient $[c(\omega)]$ (2)

Dynamic Damping coefficient $c(\omega)$	R= L/B
0.9716 -0.0500(Ra_o)^2-0.0660exp (Ra_o)	1
1.2080-0.164(Ra_o)+0.0385(Ra_o)^2+0.2515exp (-Ra_o)	2
1.900-0.0025(Ra_o)+0.0012 (Ra_o)^2	4
1.2285-0.0359(Ra_o)+0.0024(Ra_o)^2+0.1515exp (-Ra_o)	6
1.3112-0.0285(Ra_o)+0.0011(Ra_o)^2+0.4388exp(-Ra_o)	10

COMPUTER PROGRAM

In this study a computer program **(CPESP)** (**C**omputer **P**rogram for **E**valuation of **S**oil **P**roperties) in Fortran Power Station language has been coded for calculating the dynamic stiffness and damping for surface and embedded foundations. In this program the input data are :-

- ❖ Dimensions B and L of the base.
- ❖ Side surface area of foundation A_s.
- ❖ Soil shear modulus G
- ❖ Soil poisson ratio υ
- ❖ Soil density ρ
- ❖ Soil damping factor β

The first step is to compute the Static Stiffness and damping coefficients .
The second step is to compute the dynamic factor for the stiffness and damping .
The effect of embedment was also considered in this program.

APPLICATIONS

Application (1)

The developed coefficient of dynamic stiffness and damping are applied to obtain the dynamic stiffness and damping using the (**CPESP**) program for the embedded foundation shown in **Fig. (7)**. The results are shown in **Table (3)**

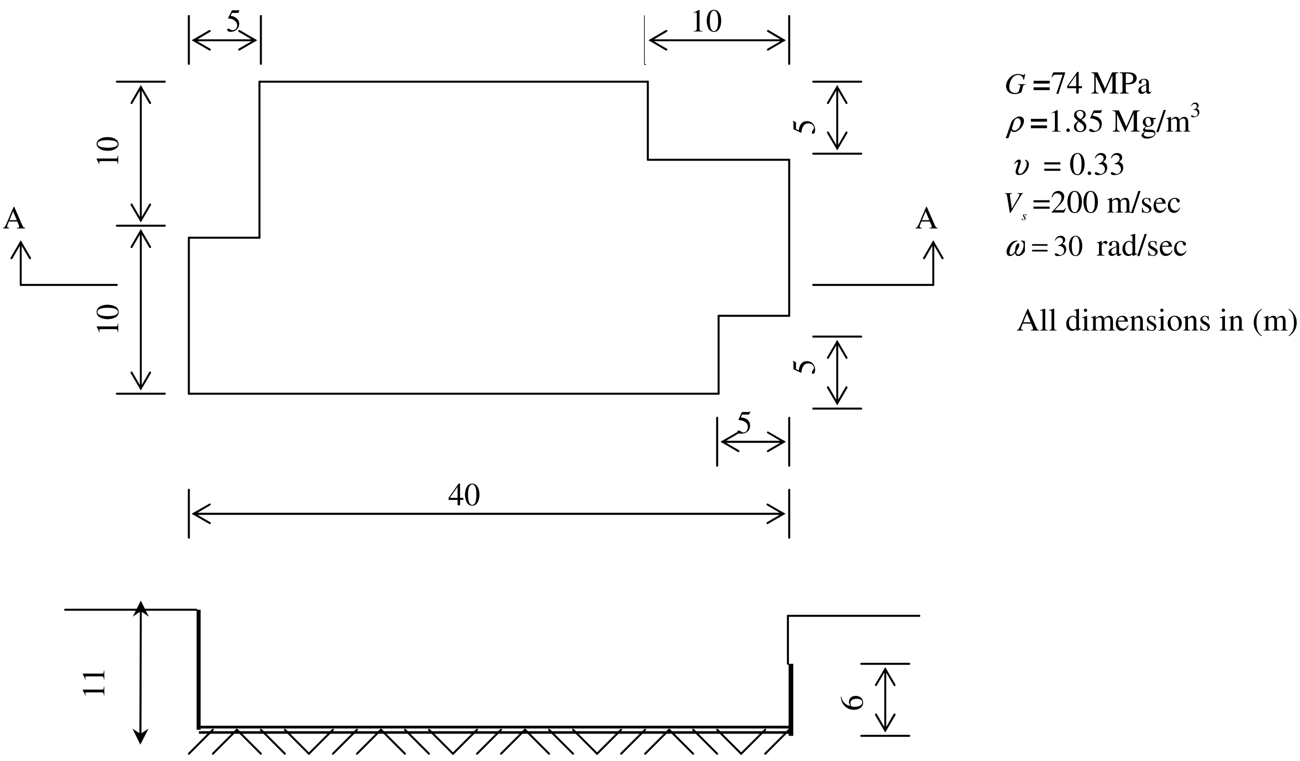


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Fig (7) geometry and material parameters, Gazetas.G(1979),application (1)

Application (2)

The foundation of application (1) has been solved using the equivalent circle approximation.

The effective radius of foundation(R) = $\sqrt{A_b/\pi}$ =14.65

The equivalent static surface stiffness $K_{sur} = \dfrac{4GR}{1-\upsilon}$

$$K_{sur} = 6.472*10^6 \text{ kN/m}$$

Using eqs. (15),(16) the results are :-

I_{tre} =1.055

I_{side} =1.211

The static embedment stiffness will be:-

K_{emb} =8.268*10^6 kN/m

From Table (1) and using eq. (18a) then the effective embedment factor is equal to 0.713.

$K_{emb})_{dy} = K_{emb} * \overline{F}_e$

$K_{emb})_{dy}$ =5.895*10^6 kN/m

Table (3) shows the final results for applications (1) and (2) .

Table (3) dynamic stiffness of embedded foundation using the present study and approximate methods.

Method of analysis	Surface static stiffness (kN/m)	Trench factor	Sidewall factor	Dynamic stiffness coefficient	Dynamic embedded stiffness (kN/m)
Present method (Application 1)	$7.333*10^6$	1.0818	1.211	0.684	$6.08*10^6$
Equivalent Circle Approximation (Application 2)	$8.633*10^6$	1.055	1.211	0.713	$5.895*10^6$

Table (3) compares the results of the present study and the equivalent circle approximation and the maximum discrepancy is about 3%.

The Lysmers analog velocity using eq. (20) is:-
V_{La} =323.06 m/sec

From **Table (2)** and using (a_o =1.5) then the dynamic damping coefficient is :-
$c(\omega)$ =1.075
The dynamic damping of soil using eq. (21) is:-
C = $0.726*10^6$ kN.m^{-1}.sec

The corrected dynamic stiffness and damping using eq. (7) are:-
$\overline{K}(\beta)$ =$4.98*10^6$ kN/m
$\overline{C}(\beta)$ =$0.749*10^6$ kN.m^{-1}.sec

Application (3)
The obtained coefficients in this study **Table (1 and 2)** are used to study the dynamic response of machine foundation under vertical dynamic load by using SAP 2000. The analysis parameters are:-

Foundation Parameters	Soil Parameters	Machine Parameters
L=9.6 m	υ = 0.33	F_v=6.27 sin (ωt)
B=4.8 m	γ =18.725 kN/m^3	ω=61.36 rad/sec
D= 1.55 m	G = 98 Mpa	Machine weight=260.65 kN
Foundation weight=1714 kN		

The result obtained are summarized in **Table (4)** and **Fig. (8)**.

Table (4) results obtained from the analysis of SAP2000(application 3)

D/B	Resonant Frequency (rad/sec)	Max. displacement (mm)
0.000	84.21	0.896
0.127	92.431	0.649
0.254	97.223	0.426
0.380	101.377	0.214
0.500	105.000	0.184
0.635	108.550	0.137

The same foundation has been analyzed for different embedment ratios (D/B) and the results for the displacement-time output are shown in **Fig.(8)**. It is evident that when the depth ratio increases the vertical displacement decreases.

A convergence in results is obvious when the depth ratio will be about 0.50. This means that the reduction in dynamic displacement will be less pronounced when the depth ratio is to be increased higher than 0.50.

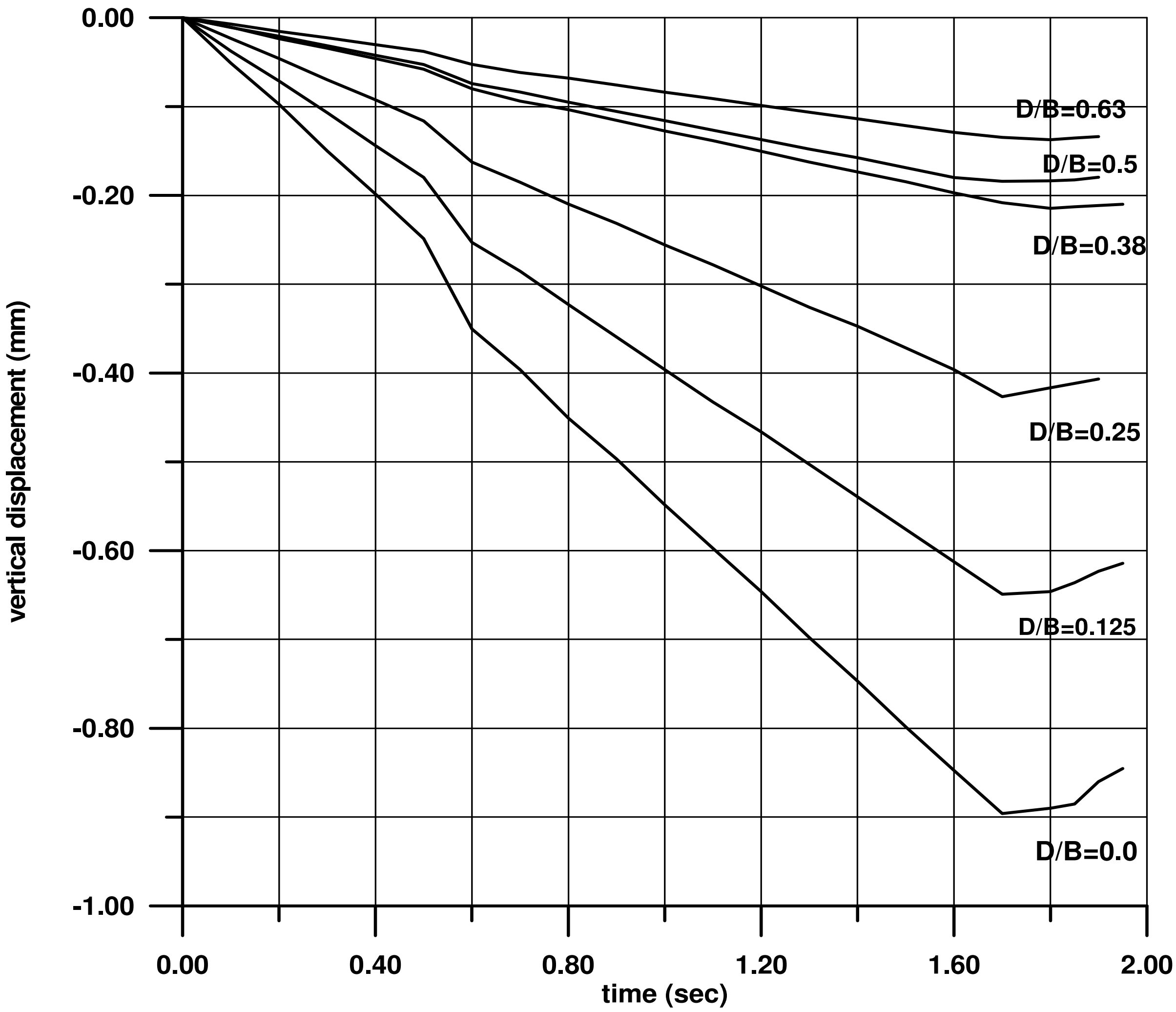

Fig.(8) effect of embedment on vertical response (application 3)

Also the increase in embedment depth leads to an increase in the resonant frequency of machine foundations, **Table (4)** shows this effect. The results show that increasing the embedment depth ratio (D/B) to 0.635 increases the resonant frequency by 22% .

EFFECTS OF USING A SQUARE FOUNDATION ON THE DYNAMIC RESPONSE

It is a matter of interest to study the effect of using a square foundation (L=6.8 m) and (B=6.8 m), i.e. B/L=1.0 instead of the rectangular foundation which has been studied in the previous sections (B/L=0.50). The dimensions of this foundation are based on equal foundation weight and soil pressure as compared to the case of the rectangular foundation.

Fig. (9) shows the vertical displacement-time relationships for different depth ratios (D/B) for the square foundation case. **Table (5)** gives the ratios of displacement amplitudes for the square and rectangular foundations for different depth ratios. The results indicate a reduction in the dynamic displacement in a range of (15%-17%) as compared to those of rectangular foundation.

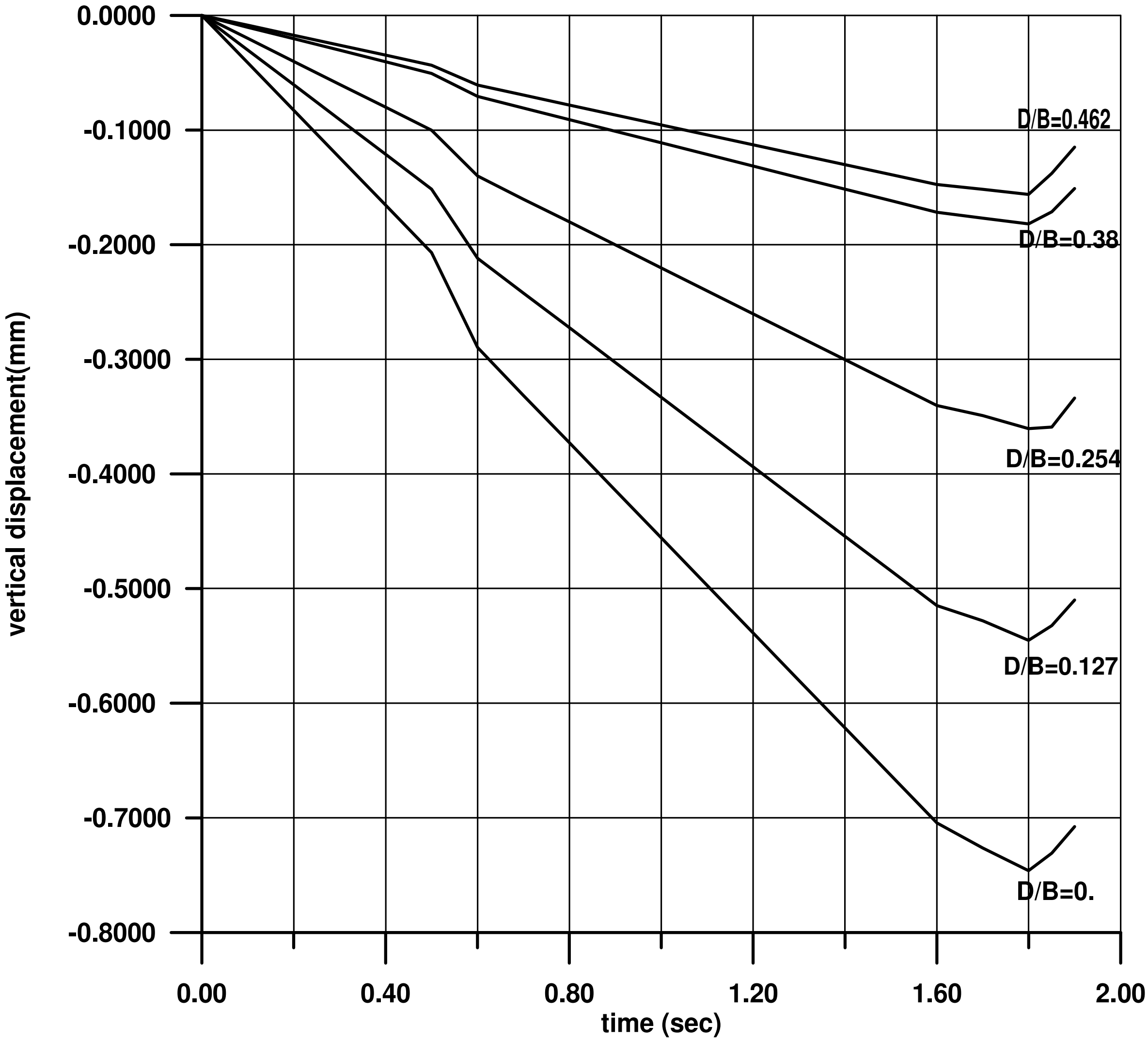

Fig. (9) effect of embedment on vertical displacement for different depth Ratios for a square foundation

Table (5) vertical displacement amplitudes (Δ)

	Depth Ratios (D/B)				
	0.00	0.127	0.254	0.380	Full embedded
Rectangular (B/L)=0.5	0.896	0.649	0.426	0.214	0.184
Square (B/L)=1.0	0.746	0.545	0.360	0.181	0.156
ΔS/ΔR*	0.833	0.840	0.845	0.848	0.85

*ΔS=Vertical displacement amplitude for a square foundation.
ΔR=Vertical displacement amplitude for a rectangular foundation

CONCUSIONS

The effect of embedment upon vertical forced vibration of a rigid footing was investigated theoretically.

The conclusions can be summarized as follows:

1- The use of equivalent circle approach to estimate the dynamic stiffness and damping factors can cause errors as the aspect ratio of the foundation (L/B)and the soil Poisson's ratio (υ) being increased. The error will generally be increased at higher frequencies.

2- Embedment of foundations has a significant effect on the dynamic response. It causes an increase in the dynamic stiffness and damping coefficients and leads to increase the resonant frequency and to decrease the dynamic response of foundation. A convergence in results is obvious when the depth ratio will be about 0.50.This means that the reduction in dynamic displacement will be less pronounced when the depth ratio is to be increased higher than 0.50.

3- The dynamic displacement in the vertical direction is smaller for the case of square foundations as compared to those of rectangular foundations for the same weight and contact soil pressure. The results indicate a reduction in the dynamic displacement in a range of (15% - 17%)as compared to those of the rectangular foundation.

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NOTATIONS

The following symbols are used in this paper:

A_b= base area of foundation.
A_s= sides area of foundation.
a_o= normalized frequency.
B= semi-width of rectangle circumscribed to base surface.
C= dynamic damping of soil.
$\overline{C}_z$ =coefficient of dynamic damping.
D= trench depth.
G=shear modulus of soil.
I_{tre}=trench factor.
I_{wall} =sidewall factor.
K_{emb}= static embedment stiffness of soil.
$K_{emb})_{dy}$= coefficient of dynamic embedment stiffness of soil for trench effect only.
$K_{tre})_{dy}$=coefficient of dynamic stiffness for soil for trench effect only.
$K_{sur})_{dy}$=coefficient of dynamic stiffness for surface foundation.
L= Semi- length of rectangle circumscribed to base surface.
S_z= vertical static stiffness parameters.
V_{La}= "Lysmer's analog" velocity.
V_s=velocity of shear waves.
υ = Poisson's ratio.
ρ = mass soil density.
ω =circular frequency.