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Review

# Opportunities for quantum computing within net-zero power system optimization

Thomas Morstyn[1,*] and Xiangyue Wang[2]

## SUMMARY

**Optimized power system planning and operation are core to delivering a low-cost and high-reliability transition path to net-zero carbon emissions. The major technological changes associated with net zero, including the rapid adoption of renewables, electrification of transport and heating, and system-wide digitalization, each increase the scope for optimization to create value, but at the cost of greater computational complexity. Although power system optimization problems are now posing challenges for even the largest exa-scale supercomputers, a new avenue for progress has been opened by recent breakthroughs in quantum computing. Quantum computing offers a fundamentally new computational infrastructure with different capabilities and trade-offs and is reaching a level of maturity where, for the first time, a practical advantage over classical computing is available for specific applications. In this review, we identify significant and wide-ranging opportunities for quantum computing to offer value for power system optimization. In addition to reviewing the latest work on quantum computing for simulation-based and combinatorial power system optimization applications, we also review state-of-the-art theoretical work on quantum convex optimization and machine learning and map this to power system optimization applications where quantum computing is underexplored. Based on our review, we analyze challenges for industry implementation and scale-up and propose directions for future research.**

## CONTEXT & SCALE

*Key conclusions:* In this review, we identify significant and wide-ranging opportunities for recent breakthroughs in quantum-accelerated optimization to offer value for the transition to net-zero power systems. These opportunities span a variety of problems across planning and operation, which are key for reliable and affordable decarbonization.

*Seminal discoveries highlighted in the review:* We review the latest work on quantum computing for combinatorial power system optimization applications, including unit commitment, grid-edge flexibility coordination, and network expansion planning. In addition, we map state-of-the-art theoretical work to applications where quantum computing is underexplored, including convex and machine learning-based optimization.

*Implications for research at different scales:* Quantum computing creates opportunities for faster, larger-scale, and higher-fidelity optimization. This is relevant for researchers from engineering, economics, and computer science, as well as policymakers, network planners, system operators, and flexibility aggregators.

## INTRODUCTION

Electric power system decarbonization is a core component of the global transition to net zero, requiring major infrastructure investments in renewable generation, grid energy storage, and transmission infrastructure, along with the rapid adoption of electric transport and heating.[1] At the same time, power systems are being digitalized, with information and communication technology (ICT) for near-real-time sensing and control being extended from the transmission level down to local distribution networks and end customers.[2] Together, these trends significantly increase the scope for optimization across system planning and operation and the value that can be created in terms of improved reliability, affordability, and sustainability.

Mathematical optimization involves the selection of decision variables that maximize an objective function while satisfying constraints.[3] Within this broad field, a wide range of approaches exist for problem classes with different structural properties. Optimization is relevant for various power system applications, from individual sites up to the national transmission scale, and from near-real-time control to years-ahead expansion planning. Figure 1 provides a high-level overview of key applications.

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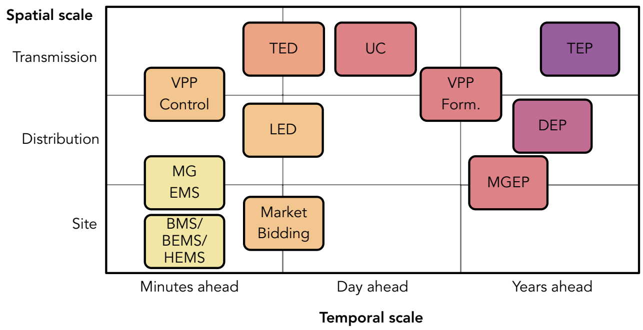


**Figure 1. Overview of key power system optimization applications across different spatial and temporal scales**

"BMS," battery management systems; "HEMS/BEMS," home/building energy management systems; "VPP Control," control of DERs within VPPs; "VPP Form," formation of DERs into VPPs; "TED/LED," transmission/local economic dispatch; "Market Bidding," bidding by flexible assets in electricity markets; *UC*, unit commitment; and "TEP/DEP/MGEP," transmission/distribution/microgrid expansion planning.

*Potential future directions:* To address challenges for industry implementation and scale-up, we propose new research into (1) benchmark problem definitions and performance criteria; (2) domain-specific algorithms and hardware for current noisy intermediate-scale devices; and (3) holistic power industry computing strategies integrating quantum computing with more immediate areas of classical computing innovation.

A core underlying method for power systems is the optimal power flow (OPF) problem, which considers how a set of generators should be dispatched to meet demand at the lowest cost while respecting technical constraints such as generator power ratings, line current flow limits, and bus voltage limits.[4] The basic OPF approach and more advanced variants are widely used throughout the power sector for applications including electricity market economic dispatch,[5] unit commitment (UC) (i.e., deciding which generators to turn on/off ahead of operation),[6] and network expansion planning.[7] Beyond OPF problems, optimization is also relevant for a wider range of power system applications, including battery management,[8] home/building energy management,[9] and the formation of distributed energy resources (DERs) into virtual power plants (VPPs).[10]

For practical implementation, optimization problems present a trade-off between (1) solving problems quickly, (2) at large scale, and (3) with high model fidelity, each of which increase computational requirements. Jointly optimizing the operation of larger groups of resources closer to real-time based on a more accurate model of the power network allows for less conservative utilization, creating the opportunity for demand to be met at lower cost and with less pollution, without compromising system reliability. Particularly when dispatching resources within constrained systems, online optimization will have strict solution quality and timing requirements.[11] For planning, time-to-solution requirements are generally less strict. However, computational burden is still a concern since robust and efficient planning often requires the assessment of a large number of long-term scenarios.[12] In recent years, this has motivated the use of exa-scale supercomputers to enable high-fidelity stochastic power system planning.[13]

An emerging opportunity is presented by quantum computing, which has fundamentally different operating principles and trade-offs compared with classical computing. With the demonstration of quantum supremacy, quantum computing has moved into the noisy intermediate-scale quantum (NISQ) era, where devices with 100 or more qubits and moderate error rates are able to provide speedups for specific applications.[14] Although large general-purpose quantum computers with full error correction are not expected in the next decade, hybrid computing architectures combining NISQ devices with classical high-performance computing (HPC) have the potential to unlock previously intractable computational bottlenecks.

[1]Department of Engineering Science, University of Oxford, Oxford, UK

[2]Center for Data Science, New York University, New York, NY, USA

*Correspondence: thomas.morstyn@eng.ox.ac.uk

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Reviews of early work on quantum computing for power systems discuss optimization alongside analytics, simulation, and communication applications.[15–19] However, for optimization, these reviews have primarily focused on initial work applying quantum combinatorial optimization algorithms to UC problems, while wider potential applications are not covered in detail.

In this review, we identify significant and wide-ranging opportunities for quantum-accelerated optimization to offer value for the transition to net-zero power systems. These opportunities are balanced by challenges for achieving the scale up needed for industry implementation. First, we review emerging sources of computational complexity associated with power system optimization applications, which are of central importance for a low-cost, high-reliability transition to net zero. Beyond UC, we identify a broader set of combinatorial optimization problems where quantum algorithms are relevant, as well as new opportunities for simulation-based, convex, and machine learning-based optimization. We present a mapping from these applications to quantum algorithms offering polynomial and exponential speedups and discuss current limitations and challenges for implementation and scale-up. We conclude by proposing key directions for future research.

## COMPUTATIONAL CHALLENGES FOR POWER SYSTEM OPTIMIZATION

The transition to net zero is linked to new sources of computational complexity, which can be broadly divided into five main areas. First is the integration of millions of DERs, including small- and medium-scale renewables, home batteries, electric vehicles (EVs), and heat pumps. This will increase the number of controllable resources within power systems by several orders of magnitude.[2] Also, these DERs are "embedded" within local distribution networks, which have more extensive topologies and nonlinear characteristics than transmission networks.[20]

Second is the integration of energy storage technologies to provide reliable flexibility in support of renewable generation. Efficiently dispatching energy storage systems requires a large time-coupled optimization problem to be solved, where previously dispatch problems for different time intervals could be solved separately and in parallel.[21] In addition, although linear storage models are often proposed for system-level optimization, it has been shown that battery storage technologies often have nonlinear characteristics related to efficiency, output power limits, and degradation, which have important implications for optimal decision-making.[8]

Third is the increased need for uncertainty handling, due particularly to the weather-dependence of renewable sources and the behavior-dependence of flexible loads. A range of approaches exist for decision-making under uncertainty, including robust optimization,[22] chance-constrained optimization,[23] and multi-stage recourse models.[24] In general, these all result in larger and/or more complex problems.

Fourth is the increasing global recognition that major reforms to electricity market arrangements are needed to deliver a low-cost path to net zero.[25] This includes the introduction of new local energy/flexibility markets,[26] national mechanisms for aggregated DERs, grid-scale storage, and renewables,[27] and capacity mechanisms supporting investment in firm and clean generation.[28] Modeling strategic interactions and market outcomes often involves computationally intensive game theoretic methods based on combinatorial and multi-level optimization.

Last is the growing importance of long-term planning to achieve net-zero targets. The distributed and variable nature of renewable resources puts added importance on generation technology selection and location, as well as transmission network investment decisions. At the distribution network level, strategic reinforcement linked to local flexibility is critical for the electrification of heating and transport. In general, optimal network expansion planning involves a large number of discrete decision variables due to fixed costs and limited sizing options. This gives these problems a combinatorial search space, which expands exponentially with the number of decisions. Further compounding this is the need to consider the decadal time horizons associated with net-zero transition, rapid technology change,[29] the growing importance of distribution level flexibility for national transmission,[30] and greater coupling between electricity, heating, transport, and water infrastructure.[31]

These sources of complexity have motivated significant work on classical computational scalability. For OPF problems, linear approximations and convex relaxations have been proposed that can be solved in P time while offering near- or exact-accuracy under specific network conditions. However, scalability remains a concern beyond around $10^4$ devices, even using state-of-the-art solvers and taking advantage of inherent sparsity.[32] There has also been work on distributed optimization, where a large problem is decomposed into smaller parallel subproblems and solved iteratively.[33] For convex problems, theoretical guarantees on optimality and constraint satisfaction are generally available. However, these methods rely on hyperparameters, which introduce a trade-off between the initial rate of convergence and later-stage solution quality, and beyond a certain level of parallelization, diminishing speedups are seen due to communication overheads.[34]

For combinatorial optimization, there has been particular progress on the special case of mixed-integer linear programs (MILPs), which are relevant when solving problems with discrete decisions (e.g., UC, expansion planning) combined with linear power flow and resource models. New solvers are still providing algorithmic speedups, but hardware improvements (e.g., more/faster cores) are offering diminishing returns.[35] Mixed integer nonlinear problems (MINLPs) are relevant when problems combine discrete decisions and nonlinear power flow and/or resource models. MINLPs remain computationally challenging, with exact methods relying on problem-specific structural properties,[36] while general-purpose metaheuristic methods scale poorly with increasing problem dimension.[37]

An expanding area of research is the application of machine learning for power system optimization applications. Recent advances in grid metering and widespread roll-outs have propelled the application of machine learning by making granular, near-real-time data available. One branch of work has focused on training neural networks to emulate OPF solvers[38] or the outputs of iterative OPF solvers.[39] Once a neural network has been trained, it can provide an approximate solution two orders of magnitude faster than state-of-the-art solvers. However, a large number of optimization problems must be solved offline as part of the training process. Also, generalizability beyond the system scenarios used for training is a concern, and constraint satisfaction requires penalties or a final remediation step, which may result in a sub-optimal solution.[40]

Another set of work has focused on using machine learning models to speed up subcomponents of optimization algorithms. For example, in Xavier et al.[41] classification models are trained to identify redundant constraints within security-constrained UC (SC-UC) problems, and in Biagioni et al.,[42] neural networks are used to speed up variable updates within a distributed OPF algorithm. An advantage of these approaches is that although the machine learning models provide approximate solutions, the algorithms they are embedded within can often be designed to retain performance guarantees.

Finally, there is work focused on reinforcement learning (RL), which is relevant for problems where a full system model is unavailable or computationally intractable, and thus a mix of exploration/exploitation is valuable. Example applications include battery energy storage optimization with nonlinear efficiency/degradation characteristics and price uncertainty,[43] and multi-agent RL (MARL) for decentralized DER coordination.[44] The trial-and-error nature of RL makes constraint enforcement a particular challenge.

## OPPORTUNITIES FROM QUANTUM COMPUTING

The computational challenges discussed so far have been considered from a classical computing perspective, where the operation of physical computing hardware is designed to support reliable logical operations, enabling higher levels of abstraction and programming. Although individual components may make use of quantum mechanical phenomenon (e.g., quantum tunneling in transistors), components are not designed to maintain quantum coherence, and quantum information is not used for computation.

Quantum computers are designed to make use of quantum information and quantum mechanical phenomenon, such as superposition, entanglement, and adiabatic evolution.[14] Within a quantum computer, the basic unit of information is a qubit (quantum bit). Measuring a qubit yields a single classical bit of information (either a "0" or a "1"). However, although quantum coherence is maintained, a qubit may be in a superposition state of both 0 and 1. There are exponentially more superposition states than classical states, and the ability for a qubit to be in multiple states at once allows it to represent and act on many potential outcomes simultaneously. In addition, qubits may be entangled to make the probability amplitudes associated with their superposition states correlated. As a result, changes applied to one qubit will also affect others it is entangled with, allowing for the creation of highly interconnected systems for fast information processing. The ability to control coherent quantum states underpins quantum algorithms that can provide P or E speedups over the best-known classical alternatives. A range of hardware architectures for physically realizing quantum computers exist, including superconducting qubits, gate-defined quantum dots, trapped ions, silicon carbide color centers, and Majorana zero modes.[45]

There are currently two main models of quantum computing: (1) gate-based and (2) quantum annealing (QA). Gate-based quantum computers rely on quantum gates, which influence qubit probability amplitudes and are analogous to classical logic gates. A limited set of one- and two-qubit gate operations is sufficient to provide a universal computing framework that can implement any quantum state transformation.[14] A range of gate-based quantum computing hardware platforms are being developed by different organizations, with atom computing announcing the milestone of a 1,180 qubit processor in October 2023.[46] Quantum volume is used as a summary metric to compare the capabilities of different devices since performance depends on multiple factors beyond the number of qubits, including gate/measurement error rates and qubit connectivity.[47]

QA provides a more limited form of quantum computing based on adiabatic evolution under gradually changing external conditions. It is specifically relevant for

**Table 1. Review of quantum computing algorithms applied to power system optimization**

| Cat. | Ref. | Quantum alg. | Power application | Hybrid | NISQ | Speedup |
|---|---|---|---|---|---|---|
| Sim.-based | Eskandarpour et al.[50] | HHL | DC power flow simulation | × | × | E |
| | Gao et al.[51] | hybrid HHL | ——— " ——— | ✓ | ✓ | E |
| | Feng et al.[52] | hybrid HHL | AC power flow simulation | ✓ | ✓ | U |
| | Liu et al.[53] | VQLS | ——— " ——— | ✓ | ✓ | U |
| | Feng et al.[54] | VQLS | ——— " ——— | ✓ | ✓ | U |
| Combinatorial | Koretsky et al.[55] | iterative QAOA | unit commitment (without network constraints) | ✓ | ✓ | U |
| | Ajagekar and You[18] | QA | ——— " ——— | × | ✓ | U |
| | Nikmehr et al.[56] | distributed QAOA | ——— " ——— | ✓ | ✓ | U |
| | Mahroo and Kargarian[57] | distributed trainable QAOA | ——— " ——— | ✓ | ✓ | U |
| | Morstyn[58] | QA | combinatorial linear OPF | × | ✓ | U |
| | Silva et al.[59] | QA | distribution network reconfiguration | × | ✓ | U |
| | Kea et al.[60] | QAOA | EV smart charging | × | ✓ | U |
| | Jing et al.[61] | QAOA | networked microgrid reconfiguration | × | ✓ | U |
| Convex | Amani et al.[62] | hybrid NR | DC OPF | ✓ | × | U |
| | Amani and Kargarian[63] | hybrid IPM | ——— " ——— | ✓ | × | U |

Quantum speedups are specified as polynomial (P), exponential (E), or unproven (U). "— " —" indicates same as above.

quadratic unconstrained binary optimization (QUBO) problems, which are a problem class within combinatorial optimization.[48] A coupled lattice of qubits is first initialized into an easy-to-prepare low-energy ground state, and then the qubit lattice is slowly controlled so that it remains in a low-energy state, which eventually represents the solution of an optimization problem. The specialized nature of QA has enabled the development of devices that are around an order of magnitude larger than gate-based quantum processors, with D-Wave system's quantum advantage processors offering 5,760 qubits.[49]

We have identified opportunities for quantum computing algorithms to offer value for four broad categories of optimization: (1) simulation-based, (2) combinatorial, (3) convex, and (4) machine learning-based. Table 1 presents a summary review of literature applying quantum computing algorithms to power system optimization applications, and Table 2 identifies new opportunities. In these tables, quantum algorithms are categorized and compared based on whether they use hybrid quantum-classical computing (hybrid column), whether they are designed for NISQ-era devices (NISQ column), and whether a polynomial, exponential, or unproven (U) quantum speedup is provided (speedup column). For each category, subsequent sections provide more detailed reviews of the relevant power systems and quantum computing literature.

## Quantum simulation-based optimization

Simulation-based optimization covers a range of approaches where an optimal (or close to optimal) solution is found by simulating a system for many potential decision-variable configurations.[89] This includes methods with theoretical guarantees, such as dynamic programming, as well as metaheuristic approaches, such as evolutionary algorithms and particle swarm optimization. Quantum computing can offer value for these approaches based on its ability to speed up simulations.

Dynamic programming provides a systematic approach for finding the exact solution for multi-time interval discrete optimization problems using Bellman's equation and recursive value calculations for various combinations of state and decision variables. For power system applications, this often involves power flow simulations, i.e., simulations to find network power flows and voltages associated with a particular set of

**Table 2. Mapping of new opportunities for quantum computing algorithms within power system optimization (beyond existing applications from Table 1)**

| Cat. | Ref. | Quantum algorithm | Example power application | Hybrid | NISQ | Speedup |
|---|---|---|---|---|---|---|
| Combin. | Zhao et al.[64] | distributed QA | Benders' decomposition for power system planning[7] | ✓ | ✓ | U |
| | Venkatesh et al.[65] | QA | VPP formation[10] | × | ✓ | U |
| | Okrut et al.[66] | QA | non-cooperative energy market negotiation[67] | × | ✓ | U |
| Convex | Brandão et al.[68] | quantum Arora-Kale-based SDP | exact SDP OPF relaxation[69] | ✓ | × | P |
| | Kerenidis and Prakash[70] | QIPM-based SDP | —— " —— | ✓ | × | P |
| | Bharti et al.[71] | variational QSDP | —— " —— | ✓ | ✓ | U |
| | Kerenidis et al.[72] | QIPM-based SOCP | exact radial SOCP OPF relaxation[73] | ✓ | × | P |
| Machine learning-based | Rebentrost et al.[74] | quantum SVM training | classification of redundant constraints for SC-UC[41] | × | × | E |
| | Park et al.[75] | variational quantum SVM training | —— " —— | ✓ | ✓ | U |
| | Willsch et al.[76] | QA SVM training | —— " —— | × | ✓ | U |
| | Khadiev et al.[77] | quantum DT training | —— " —— | × | × | P |
| | Mannapov[78] | QAOA DT training | —— " —— | ✓ | ✓ | U |
| | Yawata et al.[79] | QA DT training | —— " —— | × | ✓ | U |
| | Wiebe et al.[80] | RBM training | OPF emulation[38] | × | × | U |
| | Allcock and Zhang[81] | BM training | —— " —— | × | × | U |
| | Shingu et al.[82] | variational RBM training | —— " —— | × | × | U |
| | Adachi and Henderson[83] | QA BM training | —— " —— | × | × | U |
| | Gupta and Zia[84] | dissipative QNN | —— " —— | ✓ | ✓ | U |
| | Killoran et al.[85] | continuous variable QNN | —— " —— | ✓ | ✓ | U |
| | Dong et al.[86] | QRL | Battery storage deep RL[43] | × | × | U |
| | Cherrat et al.[87] | policy iteration QRL | —— " —— | × | × | U |
| | Chen et al.[88] | variational QRL | —— " —— | ✓ | ✓ | U |

Quantum speedups are specified as polynomial (P), exponential (E), or unproven (U). "— " —" indicates same as above.

real and reactive power injections. Dynamic programming has been proposed for a range of power system applications, including OPF with energy storage[90] and multi-stage expansion planning.[91] An important benefit of dynamic programming is the ability to directly incorporate nonlinear network and resource characteristics, as long as they can be efficiently simulated. Here, we focus on how quantum computing can speed up these simulations, but it should be noted that this does not resolve the E complexity of dynamic programming with respect to the number of decision variables. A separate strand of work focuses on quantum speedups for specific dynamic programming problems based on Grover's algorithm, which, in certain cases, can provide a quadratic speedup.[92]

Metaheuristic optimization strategies, which are also simulation-based, focus on finding approximate solutions for nonlinear problems that are not amenable to exact methods. A variety of approaches exist, but they broadly involve iterative search based on the evaluation of candidate solutions (i.e., system simulations for specific decision variables). Metaheuristic optimization has been proposed for various power system applications, including optimal distribution network reconfiguration, generation planning, and transmission network planning.[93]

Figure 2 provides an overview of potential quantum simulation-based optimization design patterns for power system applications. One route for quantum computing to speed up power flow simulations is the Harrow-Hassidim-Lloyd (HHL) algorithm, which has an E speedup over the fastest classical algorithms for solving systems of linear equations. In Eskandarpour et al.,[50] HHL is used to solve a power flow problem based on the linear direct current (DC) power flow approximation, which is widely

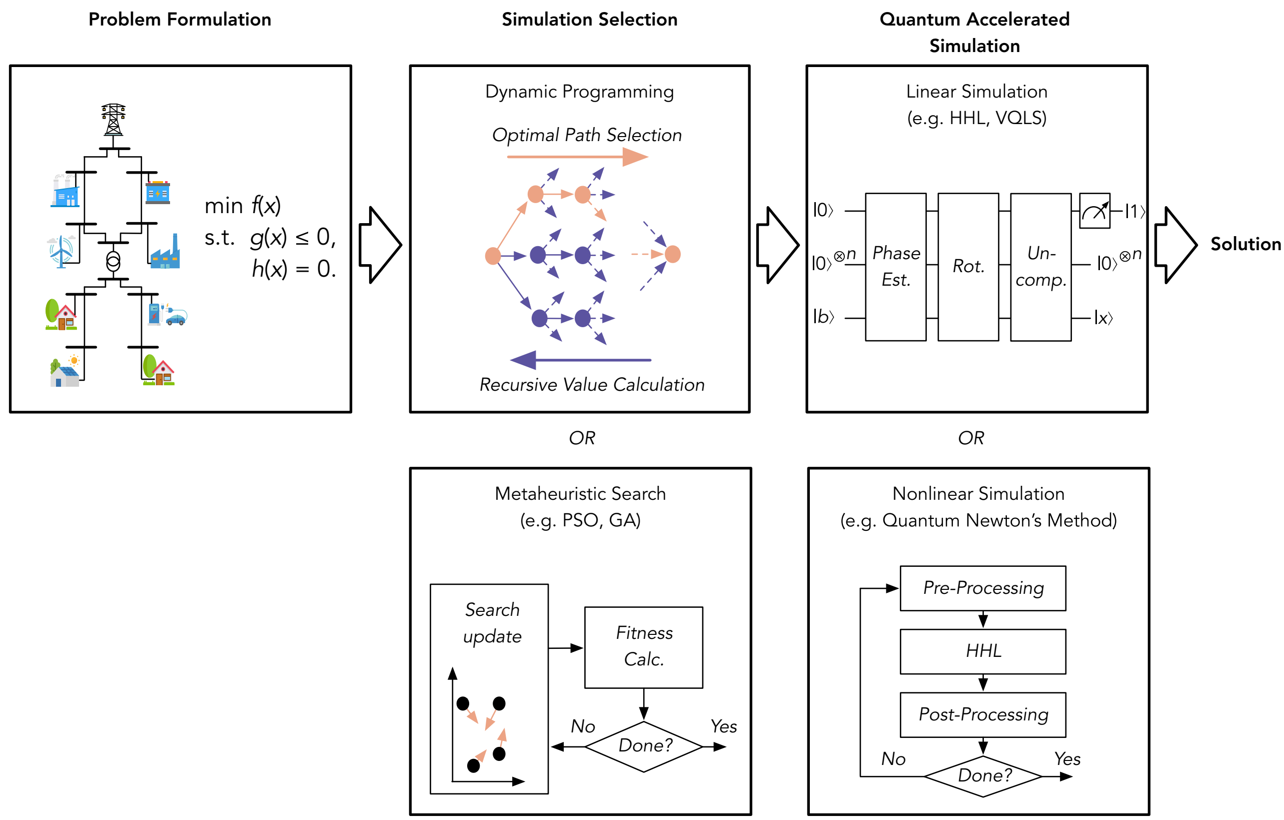


**Figure 2. Overview of potential quantum simulation-based optimization design patterns for power system applications**

Steps common to potential approaches include (1) problem formulation, (2) simulation selection (e.g., dynamic programming, metaheuristic search), and (3) quantum-accelerated simulation (e.g., HHL, quantum Newton's method).

used for transmission systems where small voltage drops and reactive power flows can be assumed. A 4-qubit circuit is designed to solve the power flows for a 3-bus system and simulated using Qiskit. The same approach could also be extended to linear power flow approximations for distribution networks.

Accurate alternating current (AC) power flow simulation requires the solution of nonlinear equations. In Xue et al.,[94] a quantum version of Newton's method is presented, which offers a quantum speedup dependent on the problem size and required accuracy. A quantum algorithm specific to AC power flow equations based on iterative HHL is proposed in Feng et al.[52] This algorithm was simulated using Qiskit for a 5-bus power system and was shown to match the solution quality of the classical fast decoupled load flow method, with each approach converging over 6 iterations. The potential for a quantum speedup here is based on the need to solve linear equations at each iteration. However, the time required to read from and write to the quantum processor at each iteration is not analyzed in detail.

Three key challenges for achieving quantum speedups in practice are (1) efficiently preparing input data for the quantum computer, (2) extracting output data back into a classical computing environment, and (3) implementation on NISQ devices given their limited size and high noise levels.

For efficient preparation of input data, many applications of quantum algorithms, including HHL, rely on quantum random access memory (QRAM) for parallel data access. Various QRAM architectures have been proposed,[95] but these have not yet been practically demonstrated due to limitations of existing hardware.

Another important caveat is that although quantum state tomography can be used to reconstruct the quantum output state as a classical output vector, reconstructing the full output vector counteracts the speedup provided by HHL.[96] HHL is useful when only quantum observables are required or where its quantum output is used by another quantum algorithm (e.g., for quantum machine learning). The use of shadow tomography for efficiently approximating sparse quantum output states is discussed in Liu et al.[53] In Pareek et al.,[97] the end-to-end complexity of HHL-based power flow is investigated, including the complexity of quantum state preparation and tomography. It is shown that for existing approaches, a quantum speedup is not generally possible if the full output vector must be read out.

HHL generally requires a large number of qubits and has strict noise requirements, limiting its application on current NISQ devices. To address this, hybrid algorithms have been proposed, which combine classical computing with small quantum devices to speed up specific parts of the overall calculation. A hybrid quantum-classical algorithm for DC power flow based on HHL is presented in Gao et al.[51] Another hybrid algorithm for solving linear systems is the variational quantum linear solver (VQLS). Here, the quantum computer implements an operation parameterized by classical real values, referred to as an ansatz. These values are iteratively adjusted by a classical computer, until the ansatz approximates a linear solver. Although a general theoretical quantum speedup from variational algorithms is not available, speedups under specific conditions have been shown.[98]

An AC power flow method based on VQLS is presented in Liu et al.[53] At each iteration, classical computing is used to update the Jacobian matrix, which is loaded onto a quantum computer using QRAM, and VQLS is used to update the bus voltage vector. A VQLS step is simulated in Qiskit with a 6-qubit circuit for a 14-bus system. For the noiseless case, convergence is achieved after approximately 30 iterations. Results from noisy simulations and implementation on a real quantum processor are also shown. Another variational method for AC power flow is proposed in Feng et al.,[54] which involves optimizing a variational quantum circuit to approximate the fixed Jacobian updates of the fast decoupled load flow method. This circuit is then used as part of an iterative algorithm, which is shown to converge in 3 iterations for a 118-bus system.

### Quantum combinatorial optimization

So far, most work on quantum computing for power system optimization has focused on combinatorial problems. Figure 3 presents an overview of relevant design patterns.

QA offers the largest quantum computing devices available and can solve combinatorial optimization problems, which can be formulated as QUBO problems. QA directly incorporates binary decision variables, but continuous variables can also be included by expressing these with auxiliary variables arranged in a binary expansion.[99] Also, linear constraints and certain quadratic constraints can be enforced by adding appropriate quadratic penalty terms to the cost function.

QA for optimal UC is demonstrated in Ajagekar et al.[18] with minimum turn-on powers and start up costs but without network constraints. In Morstyn,[58] QA was demonstrated for combinatorial linear power flow optimization, considering

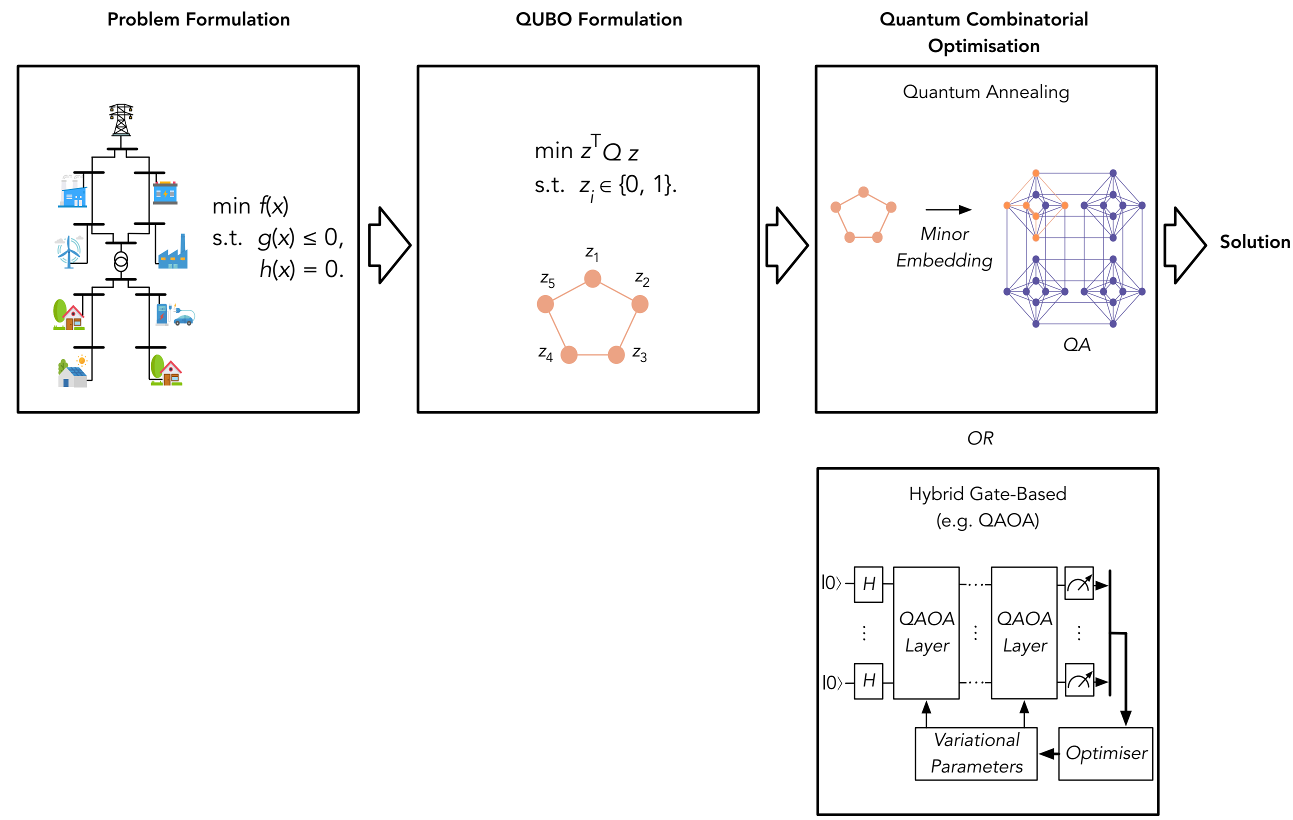


**Figure 3. Overview of design patterns for quantum combinatorial power system optimization**

Steps common to potential approaches include (1) problem formulation, (2) reformulation as a QUBO, and (3) quantum combinatorial optimization (e.g., QA, QAOA).

network upgrades, generator placement/sizing, flexibility from on/off EV charging, and multi-phase linear network constraints. In Silva et al.,[59] a QUBO formulation suitable for QA is presented for distribution network reconfiguration to minimize losses assuming constant current loads. In Zhao et al.,[64] a hybrid quantum-classical version of Benders' decomposition algorithm is proposed for MILPs, which involves iteratively solving integer problems using QA and solving LPs using classical computing. Benders' decomposition is widely used for network expansion planning problems (see, e.g., Shahidehopour et al.[7]).

There are a number important caveats to the broad applicability of QA. Errors are introduced into existing QA hardware due to issues such as flux noise and digital-to-analog quantization error.[100] Due to these errors, in practice, QA is run multiple times and the lowest energy solution is selected. However, the best solution obtained for a large problem may not be optimal, and constraints enforced via penalty terms may not be satisfied. Also, analytic speedups from QA are only available for specific problem types.[101] Empirical work benchmarking QA against state-of-the-art alternatives has shown variable results depending on the problem under study and the hardware used.[102]

Another issue is the limited scale of existing QA hardware, which although much larger than gate-based quantum computers, remains too small for most industrial power system applications. Once a QUBO problem has been formulated, it needs to be embedded on the annealer's qubit lattice, with each binary variable associated with a qubit, and qubit couplings encoding quadratic cost function weights. An added complexity is that current annealers do not have fully connected qubit lattices and thus cannot support arbitrary qubit coupling. To overcome this, a feasible "minor embedding" needs to be found for the qubit lattice, where a single logical qubit may be represented by a chain of multiple coupled physical qubits. This increases the number of required qubits, and finding an optimal minor embedding is itself a challenging problem, although fast heuristic algorithms are available.[103] Using D-Wave's 2,048-qubit 2000Q processor, the UC formulation in Ajagekar et al.[18] can be scaled up to 12 U, with output powers discretized into 10 levels. However, QA's limited precision means that beyond 5 U the time required to obtain high-quality solutions rapidly increases above a classical mixed-integer formulation solved using Gurobi on an i7 CPU. In Morstyn,[58] combinatorial OPF problems solved using D-Wave's 5,760 qubit Advantage processor could scale to 9 EVs in a distribution network, given a 24 h optimization horizon with 1 h intervals, placement/sizing for 3 PV generation sites, and voltage limits imposed on 3 node-phase pairs. The required number of qubits varied by approximately $\pm$10% due to heuristic minor embedding. Although the maximum problem sizes remain small, promising observations for future scaling are that the required number of qubits increases linearly with both the number of EVs and the number of voltage constraints and that QA is able to outperform classical simulated annealing on an i9 CPU.

For gate-based quantum computers, the quantum approximate optimization algorithm (QAOA) is a hybrid quantum-classical variational algorithm that, similar to QA, generates approximate solutions to QUBO problems. The potential for a quantum speedup depends on the particular problem characteristics, as well as the quantum device error rate, number of qubits, and maximum circuit depth. QAOA for UC is proposed in Koretsky et al.[55] To avoid the need to discretize continuous variables, QAOA is combined with an outer-loop classical optimizer. Problems with up to 10 U are simulated using IBM's Qiskit software. The chance of high solution quality increases with the number of QAOA training iterations and when the circuit depth is increased from 1 to 2. However, for 10 U, the chance of a near-optimal solution is still only 6% after 1,500 iterations.

In Nikmehr et al.,[56] alternating direction method of multipliers (ADMM) is used to decompose QAOA-based UC into subproblems that can be solved in parallel on smaller quantum processors as part of an iterative distributed optimization process. Simulation case studies are completed, including for a 9 DER system, which is solved using 3 simulated QAOA circuits. A similar approach combining ADMM with QAOA is proposed in Mahroo et al.,[57] with recurrent neural networks used to speed up the convergence of quantum circuit parameters. QAOA has also been proposed for on/off EV smart charging,[60] and networked microgrid reconfiguration based on its potential to speed up the graph max-cut problem.[61]

In addition to the applications discussed so far, power system game theory is an unexplored area where QA and QAOA-accelerated combinatorial optimization could also be valuable. In particular, a QA-based algorithm for coalition structure generation games is presented in Venkatesh et al.,[65] where the aim is to divide agents into coalitions to maximize overall social welfare. This is relevant for the formation of local energy trading coalitions[104] and VPPs.[10] Non-cooperative game theory is also widely used for energy market analysis and design.[67] In Okrut et al.,[66] QA is demonstrated for finding the Nash equilibrium of stylized two-player games (i.e., the solution where neither player is motivated to unilaterally deviate).

### Quantum convex optimization

Convex optimization involves minimizing a convex objective function (or equivalently maximizing a concave objective) with a convex feasible set for decision variables.[105] Exact and approximate convex formulations for power system optimization applications are of significant interest due to the availability of polynominal time classical algorithms suitable for solving large convex problems. Important convex problem classes include linear programs (LPs), convex quadratic programs (QPs), second-order cone programs (SOCPs), and semi-definite programs (SDPs). These form a hierarchy since the set of LPs is a subset of the set of convex QPs, which is in turn a subset of SOCPs, and SDPs are the most general.

Using linear power flow approximations, the OPF problem can be formulated as an LP or QP depending on the cost function.[4] For radial distribution networks, an SOCP power flow relaxation is available, which is exact under broadly applicable conditions,[73] and for mesh networks, an exact SDP relaxation is available.[69] SOCP is also relevant for linear OPF under chance constraints, assuming Gaussian uncertainty.[23]

Also, local optima for non-convex problems can be found using sequential convex optimization. For example, in Sadat et al.[106] nonlinear AC OPF is solved using sequential linear programming. Beyond OPF, convex optimization can be used for energy storage system control[8] and home/building energy management.[9]

The use of quantum computing to speed up convex optimization is an emerging area of theoretical research. Figure 4 presents an overview of potential design patterns for quantum convex power system optimization. An initial application of quantum computing to convex OPF is presented in Amani et al.,[62] where a hybrid approach integrating HHL with a Newton Raphson (NR)-based solver is proposed for linear DC OPF. This algorithm was simulated using Qiskit and applied to a 3-bus case study, where a 90% HHL success probability requires 14 qubits, and convergence takes between 4 and 7 iterations for different power system load levels. The convergence of the NR-based solver is also investigated for a larger case study based on the IEEE 14-bus test system, considering different error-rate bounds on the quantum calculations at each iteration. This work is extended in Amani et al.,[63] where a hybrid interior point method (IPM) is proposed combining HHL-based updates with classical updates to address noise. Qiskit simulations show the potential for a 4.5× speedup for a 300-bus network over classical IPM, and this speedup is shown to be stable across various network load levels.

In Brandão et al.,[68] a quantum algorithm for SDPs is presented, based on the classical Arora-Kale framework, which provides a quadratic speedup over classical algorithms in terms of both the number of decision variables and constraints. Full quantum state reconstruction is a challenge, and the algorithm is instead designed to provide the objective function at optimality. The approach has a strong runtime dependence on the upper bound of the primal variable norm, which has been improved by subsequent work, and shadow tomography has been integrated allowing low-rank decision vectors to be recovered.[107]

In Kerenidis et al.,[70] a quantum IPM (QIPM) is proposed that also provides a P speedup for SDPs. The speedup is not as strong as Brandão et al.[68] in terms of the number of decision variables but has less dependence on other parameters. Another QIPM is proposed in Kerenidis et al.,[72] which provides a P speedup for SOCP problems. These algorithms include efficient approaches for state vector

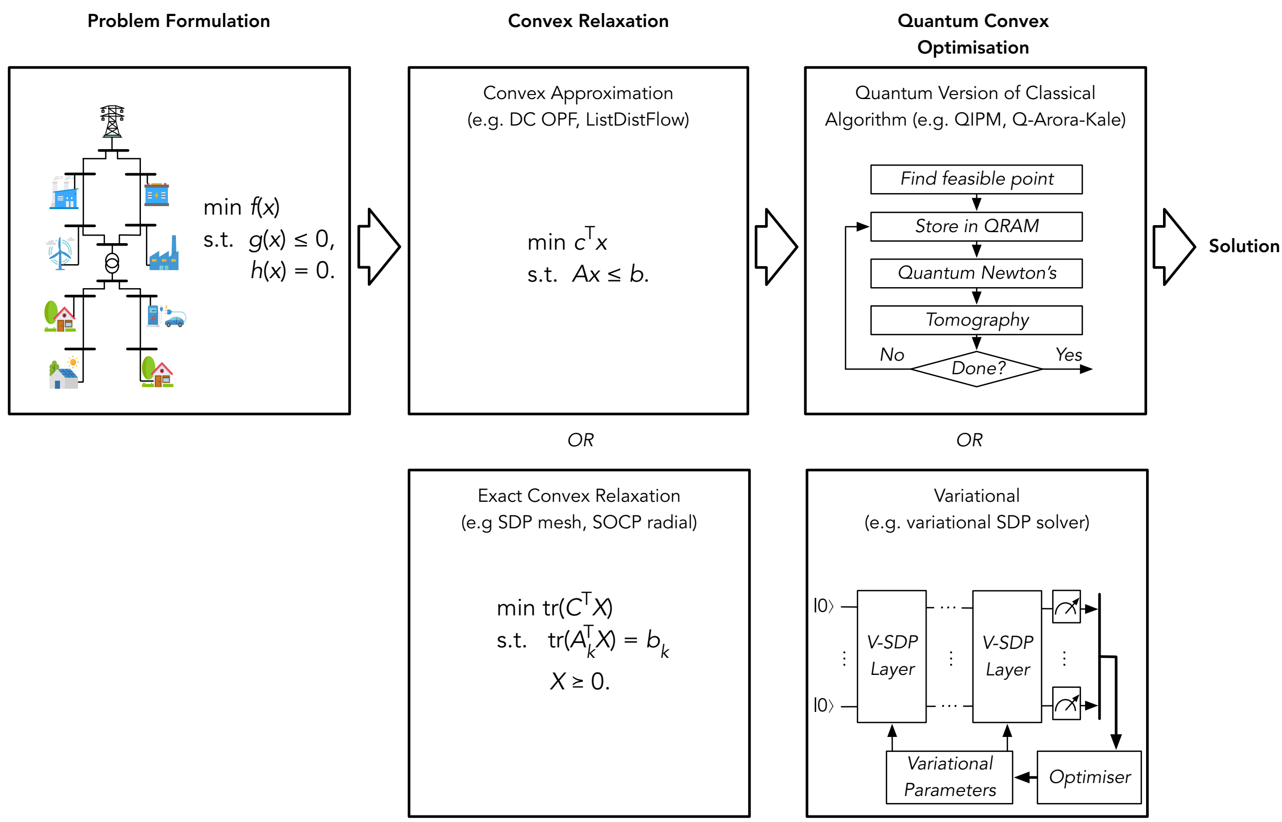


**Figure 4. Overview of potential design patterns for quantum convex power system optimization**

Steps common to potential approaches include (1) problem formulation, (2) convex relaxation (e.g., to an LP, SOCP, and SDP), and (3) quantum convex optimization (e.g., QIPM, variational SDP).

tomography but require QRAM. Another caveat is that the quantum speedups depend on intermediate matrices being well-conditioned, which is difficult to guarantee theoretically, meaning that speedups remain problem dependent and require empirical validation.

The quantum algorithms for convex optimization discussed so far require much larger and lower error-rate gate-based devices than are currently available. In Bharti et al.,[71] a hybrid variational algorithm is presented for solving SDPs using NISQ devices. This method still involves a classical SDP solver, but it is used to solve a series of problems with a much lower dimension than the original problem. A quantum speedup is not proven, but numerical evidence from an eigenvalue calculation problem shows that, for a given number of qubits, the solution error reduces exponentially with the number of ansatz states, supporting the method's potential scalability.

## Quantum machine learning-based optimization

Complex statistical relationships can be captured with limited amounts of quantum data, creating the potential for quantum computing to outperform classical computing for a range of machine learning tasks. Quantum machine learning is an active research field and has been applied to power system analytics applications such as transient stability assessment.[108] As discussed, in recent years there has been significant interest in the development of machine learning-based power system optimization strategies, including neural network-based optimization emulation (e.g., Pan et al.[38]), machine learning-assisted optimization (e.g., Xavier et al.[41]), and

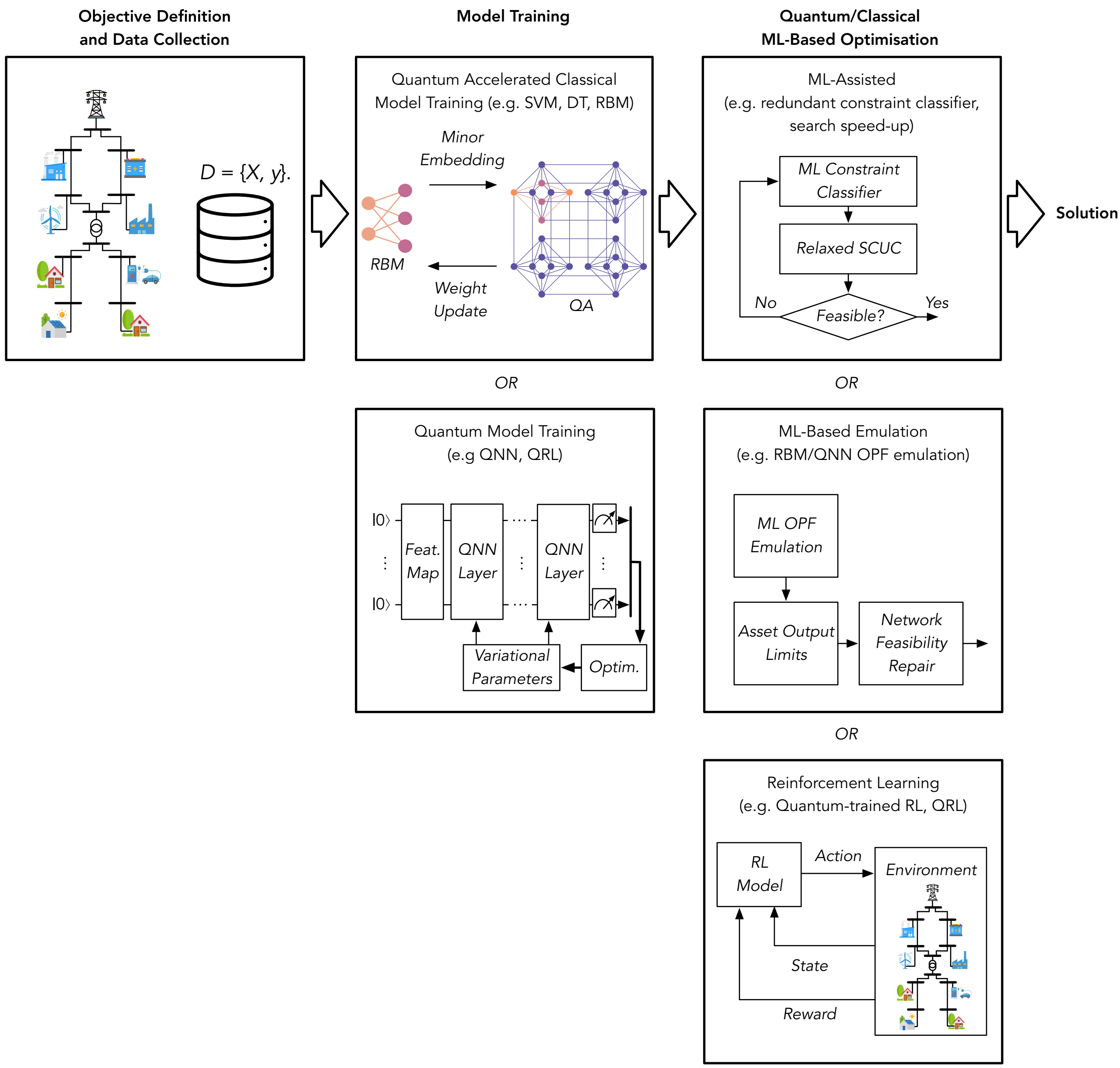


**Figure 5. Overview of potential design patterns for quantum machine learning-based power system optimization**

Steps common to potential approaches include (1) objective definition and data collection, (2) model training (either quantum-accelerated training of a classical model or quantum model training), and (3) machine learning-based optimization (e.g., machine learning-assisted, optimization emulation, and reinforcement learning).

RL (e.g., Cao et al.[43]). In this section, we are interested in the opportunity for quantum computing to offer value for these approaches, either by using quantum computers for training classical machine learning models or by implementing machine learning models on quantum hardware. Figure 5 presents an overview of potential design patterns.

Support vector machines (SVMs) are a popular supervised learning method for classification and regression and are relevant for machine learning-assisted optimization (see, e.g., Xavier et al.[41]). A series of work has focused on using quantum computers to improve SVM training. An E speedup for training SVMs with P kernel functions is available using gate-based quantum computing and QRAM.[74] A variational algorithm for training SVMs using NISQ devices is presented in Park et al.,[75] with empirical results showing a sub-quadratic training time. In Willsch et al.,[76] SVM training is reformulated as a QUBO problem, which can be solved using QA. The stochastic nature of QA means that an ensemble of approximately optimal models is readily produced, which, when operated jointly, can often outperform individual classifiers produced by classical SVM training.

Decision trees (DTs) are another supervised learning method with different trade-offs to SVMs. In Khadiev et al.,[77] a hybrid quantum-classical algorithm for DT training is presented, which provides a near quadratic speedup with respect to the number of attributes. A DT training algorithm making use of QAOA is presented in Mannapov et al.,[78] and in Yawata et al.,[79] a QA formulation for training regression trees with multi-feature splitting conditions is presented.

The literature on quantum computing for neural networks can broadly be divided into work on training acceleration for classical neural networks and work on quantum circuits that are analogous to classical neural networks. Quantum computing is particularly relevant for accelerated training of restricted Boltzmann machines (RBMs), which are a type of generative stochastic neural network. In Wiebe et al.,[80] quantum algorithms with and without QRAM are presented for efficient RBM training. These algorithms are also relevant for full Boltzmann machines (BMs), which offer additional expressive power but are generally considered intractable for classical training.[81] RBMs and BMs can also be trained using NISQ variational algorithms[82] and QA.[83]

A range of different concepts for quantum neural networks (QNNs) have been proposed, each combining different elements of quantum computing and neural computing.[109] For NISQ devices, QNN research has focused on variational algorithms,[110] where the variational parameter updates are done in a manner analogous to training a feed-forward classical neural network. Within the variational QNN framework, there are different implementations, such as dissipative QNNs[84] and continuous variable QNNs.[85] In Du et al.,[111] it is shown that QNNs can provide greater expressive power than classical neural networks for generative tasks. However, QNNs are not yet fully mature, and existing implementations can exhibit untrainability without a good initialization.

Quantum computing has also been proposed for RL. Gate-based quantum RL (QRL) was first proposed in Dong et al.,[86] with superposition enabling simultaneous value function updating across multiple classical states and the inherently probabilistic nature of quantum measurement providing a natural action selection policy balancing exploration and exploitation. A simulation of this approach was demonstrated for a 20×20 Gridworld problem. Quantum policy iteration RL is proposed in Cherrat et al.[87] and demonstrated with the FrozenLake and InvertedPendulum OpenAI Gym environments. For NISQ devices, variational algorithms can replace neural networks as function approximators within classical deep RL approaches.[88]

For all quantum machine learning methods, a major challenge is data loading, i.e., encoding enough data to represent a problem using the limited number of qubits of NISQ devices. To address this, two leading approaches are tensor networks,[112] which compress high-dimensional data into a manageable size, and data reloading,[113] where multiple steps of data uploading and processing are done in series for each qubit in a quantum circuit.

## CONCLUSIONS AND FUTURE DIRECTIONS

Our review has found a rich variety of quantum computing algorithms with potential applications for net-zero power system operation and planning. The most developed area is the use of quantum computing for combinatorial problems, which include UC, dispatch for grid-edge devices with discrete flexibility, and network expansion planning. Here, QA and variational algorithms are directly relevant and suitable for NISQ-era devices. Promising empirical results have been obtained for small-scale test cases, but so far, a practical quantum advantage has not been demonstrated for industrial-scale problems, and it is challenging to prove theoretical quantum speedups for these methods. However, this could change quickly given that companies including D-Wave and IBM are rapidly improving their industrial quantum computing offerings. There has also been substantial progress on quantum-accelerated linear and nonlinear power flow simulation. These methods have not yet been leveraged for power system optimization, but there are opportunities for strategies based on dynamic programming and metaheuristics that rely on simulating large numbers of scenarios.

Convex and machine-learning-based power system optimization have been identified as two underexplored areas where quantum computing could offer value in future. For convex OPF in particular, quantum algorithms offer a potential counter to new sources of computational complexity, including the growing number of DERs, the use of chance constraints to handle uncertainty, and the transition from linear approximations to more accurate SOCP and SDP formulations. Convex optimization is still an emerging area of quantum computing research, with new algorithms regularly being developed offering different trade-offs and speedups. Within the power systems literature, machine learning-based optimization is an area of rapid recent development. Here, quantum computing has the potential to speed up training and increase the expressive power of models used for machine learning-assisted optimization, optimization emulation, and RL.

It is important to caveat these potential opportunities with the recognition that quantum computing is at an early stage of development and most theoretical quantum speedups rely on large error-corrected devices that are likely a decade or more away. Despite this, a number of implementation-focused industry initiatives are already underway. The National Renewable Energy Laboratory (NREL) and Atom Computing are building a quantum smart grid control testbed, linking a quantum computer in-the-loop with a real-time digital grid simulator.[114] Also, E.ON and IBM are collaborating on quantum computing for several applications, including vehicle-to-grid optimization,[115] and Phasecraft is conducting a feasibility study with the UK Department for Energy Security and Net Zero on quantum computing for power system planning.[116]

The pressing need to address net zero and the rapid technological development of quantum computing create significant opportunities, but also the risk for hype cycles and misdirected investment. To address this, three key future research directions are proposed: (1) benchmarks and performance criteria, (2) domain-specific algorithms and hardware, and (3) holistic power sector computing strategies.

### Benchmarks and performance criteria

An important enabler for future progress would be a set of standardized power system optimization benchmark problem definitions with clearly specified performance criteria. This is important for directing research into quantum algorithm design and for understanding how far quantum hardware performance metrics (e.g., quantum volume, circuit layer operations per second) need to improve before practical quantum advantage can be achieved for specific applications. Understanding when new market opportunities would be realized could in turn help unlock and efficiently allocate additional government and industry investment.

A major challenge for developing standardized optimization benchmarks for power systems is their heterogeneity between regions in terms of network topology, size, generation/demand characteristics, control/market mechanisms, and planning processes. For OPF problems, important progress has been made with the release of the Power Grid Library by the IEEE Power & Energy Society.[117] This brings together various network models and standardized formulations for AC OPF, OPF with HVDC lines, and UC. Other sources of benchmarks include Sass et al.,[118] which focus on sector-coupled energy system optimization, and CityLearn[119] and GridLearn,[120] which provide standardized software environments for demand-response RL training and testing.

Important applications where standardized benchmarks have yet to be developed include the coordination of aggregated DERs, energy storage optimization, and OPF under uncertainty. Also, although existing benchmarks are primarily focused on system models and algorithms, there is a lack of linked benchmarks and design patterns for computing hardware architectures. To properly assess whether quantum computers can provide real value, it is critical to understand the computational boundaries and trade-offs faced by the power sector at the state-of-the-art of classical computing, which involves interdependencies between data, algorithms, and hardware.

Energy consumption is another important performance criteria for quantum computing, particularly when considering its use for power system optimization. However, it is challenging to assess this in the current NISQ era due to the variety of hardware candidates and the rapid pace of development. The Quantum Energy Initiative was established in 2022 to explore sustainable quantum computing architectures.[121] For large instances of problems where there is a quantum advantage, it is expected that this will also translate into an energy consumption advantage.[122] As an example, analysis of Google's quantum supremacy experiment showed a 557,000× reduction in energy usage compared with classical computing.[123] However, the potential for NISQ energy advantages is less clear and will depend on the specific problem, hardware platform, and supporting energy and cooling infrastructure.[124] In addition to energy efficiency, another important area to explore for sustainable quantum computing is dynamic resource management, so that computing demand can be flexibly matched to variable renewable generation.[125]

## Domain-specific algorithms and hardware

During the current NISQ era, quantum speedups are expected to be heavily dependent on the details of the application, algorithm, and hardware platform.[14] This creates a role for domain-specific algorithm and hardware design analogous to the use of application-specific integrated circuits for classical computing. For NISQ circuits, noise is a key limiting factor for problem size and solution quality. Noise modeling is challenging due to the complicated dependence on the circuit topology and underlying hardware, but can be made tractable using approximations.[126] This provides an important line of inquiry for noise-aware quantum circuit design and hardware selection.

Initially, practical implementation will likely involve hybrid quantum-classical computing, with small quantum processors used for subroutines that are particularly amenable to quantum computation. From our review, an early example of this is Nikmehr et al.,[56] where a NISQ circuit is used to solve mixed-integer subproblems for distributed UC. In general, power system optimization problems are often amenable to decomposition, parallelization, and hierarchical approaches since they involve network-based decisions across multiple spatial and temporal scales.

Even in the longer term, when large error-corrected quantum computers are available, domain-specific design will still be important for achieving practical quantum advantage. It has been argued that quantum gate operations will always take longer than classical operations due to their inherently higher complexity.[127] This would mean that classical computing remains superior for small instances of any particular optimization application, particularly when the theoretical quantum advantage is polynomial rather than exponential. Another issue is the input/output bandwidth limits of quantum computing, which means it is most competitive for problems where intensive computation is done on a relatively small dataset. Quantum advantage is often lost if a large amount of input data needs to be loaded onto the quantum device or where a quantum observable (e.g., the objective function value) is an insufficient output and instead a full decision vector needs to be exported.[96] This motivates the design of problem-specific solutions that directly account for these limitations.

## Holistic power sector computing strategies

Computing is a key component of the trend toward power system digitalization. Transmission system operators need computing architectures that can support increasingly intensive offline analytics and planning and online dispatch and control. These functions are also of growing importance at the local level, as part of the distribution network operator (DNO) to distribution system operator (DSO) transition. Related to this are the growing computing requirements of platforms that manage fleets of DERs, including local energy/flexibility markets, VPPs, EV charging networks, and building/microgrid energy management systems.

When assessing the opportunities offered by quantum computing, system operators should not consider the technology in isolation but instead as part of an ongoing strategic roadmap for computing research and development, accounting for the full range of technology readiness levels. Alongside quantum computing, key areas for innovation in classical computing include graphical processor unit computing[128] and cloud-to-edge computing.[129] For new computing technologies to be of practical value, architectures are needed that address the domain's stringent performance, timing, reliability, and security requirements.

Policymakers and regulators will also have important roles to play as quantum computing is adopted by the power sector. The cost required to develop quantum computing infrastructure creates the risk of market concentration, which could limit competition and innovation.[130] Policymakers can help address this by supporting open research and investing in widely accessible national quantum computing infrastructure.[131] There is also a need for careful regulation and controls that balance risks posed by quantum computing, particularly for cybersecurity, against the potential benefits of open and collaborative science and innovation for the net-zero transition.[132]

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## AUTHOR CONTRIBUTIONS

T.M.: Conceptualization, investigation, data curation, funding acquisition, visualization, supervision, writing - original draft. X.W.: Investigation, writing - review & editing, data curation.

## DECLARATION OF INTERESTS

The authors declare no competing interests.

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