Improving dispersive readout of a superconducting qubit by machine learning on path signature

Shuxiang Cao
Clarendon Laboratory
Department of Physics
University of Oxford
Oxford, OX1 1PU
shuxiang.cao@physics.ox.ac.uk

Zhen Shao
Mathematical Institute
University of Oxford
Oxford, OX2 6GG
shaoz@maths.ox.ac.uk

Jian-Qing Zheng
The Kennedy Institute of Rheumatology
University of Oxford
Oxford, UK
jianqing.zheng@{kennedy.ox.ac.uk; outlook.com}

Mustafa Bakr
Clarendon Laboratory
Department of Physics
University of Oxford
Oxford, OX1 1PU
mustafa.bark@physics.ox.ac.uk

Peter Leek
Clarendon Laboratory
Department of Physics
University of Oxford
Oxford, OX1 1PU
peter.leek@physics.ox.ac.uk

Terry Lyons
Mathematical Institute
University of Oxford
Oxford, OX2 6GG
terry.lyons@maths.ox.ac.uk

Abstract

One major challenge that arises from quantum computing is to implement fast, high-accuracy quantum state readout. For superconducting circuits, this problem reduces to a time series classification problem on readout signals. We propose that using path signature methods to extract features can enhance existing techniques for quantum state discrimination. We demonstrate the superior performance of our proposed approach over conventional methods in distinguishing three different quantum states on real experimental data from a superconducting transmon qubit.

1 Introduction

Leveraging quantum mechanics to calculate computational tasks provides substantial advantages over classical computing, particularly in solving complex problems such as computing quantum chemistry properties [3], simulating material attributes [8], and deciphering widely used asymmetric ciphers [27]. To implement these tasks, a fault-tolerant quantum computer with large enough quantum resources is essential, and yet one of the difficulties in building such a machine is to implement a fast, high-accuracy approach to convert quantum information back to classical information, usually

Machine Learning and the Physical Sciences Workshop, NeurIPS 2023.

referred to as readout. The readout of superconducting qubits typically employs the dispersive readout technique [16, 29, 30]. A probing microwave signal is sent to a resonator that is weakly coupled to the qubit, and the qubit state information is then carried by the returned signal. In practice, the returned probing signal is captured as a time series and analyzed on a computer to discriminate the qubit state. Due to the physical limits, the signal-to-noise ratio is typically low. It is therefore worthwhile to explore whether other techniques such as machine learning could enhance state discrimination from the given signal. In general, the qubit state discrimination task is to classify noisy time series.

The signature of a (multi-dimensional) path/time-series is a graded, infinite collection of iterated integrals, where the signature of a path X up to degree N is the collection

$$\mathrm{Sig}^N(X) := \left(\int_{0<t_1<\cdots<t_k<1} \cdots \int \frac{\mathrm{d}X_{i_1}}{\mathrm{d}t}(t_1) \cdot \frac{\mathrm{d}X_{i_2}}{\mathrm{d}t}(t_2) \cdots \frac{\mathrm{d}X_{i_k}}{\mathrm{d}t}(t_k) \mathrm{d}t_1 \cdots \mathrm{d}t_k \right)_{\substack{1\leq i_1,\ldots,i_k\leq d \\ k=0,1,2,\ldots,N}}. \quad (1)$$

The signature describes the paths in a global, geometric, and interacting way. For example, the degree 1 signatures are the increments in different dimensions, and the degree 2 signatures describe the area formed between the graph of a pair of features and the 45-degree line. See [19, 4, 9] for more information about the signature method. In this work, we apply the signature method to the readout time-series data to enhance state discrimination.

1.1 Related work: Machine learning for dispersive readout

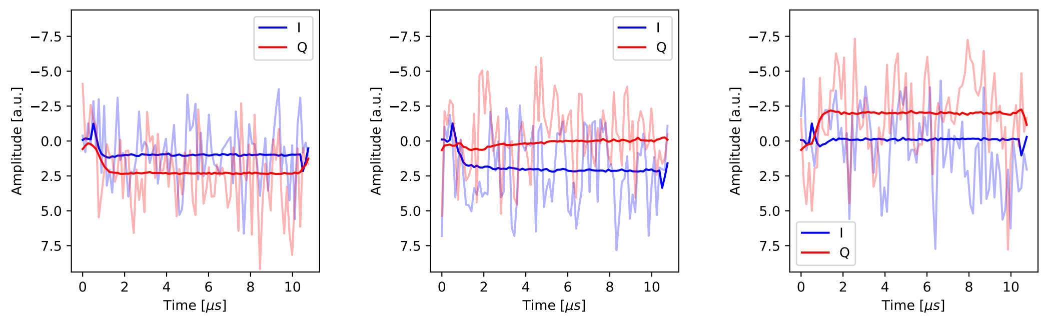
Machine learning approaches have been adopted to implement state discrimination. Traditional machine learning models such as Linear Discriminant Analysis (LDA), Quadratic Discriminant Analysis (QDA), and Support Vector Machines (SVMs) have been benchmarked for discriminating readout signal trajectories [21], and discovered that nonlinear algorithms exhibit higher assignment accuracy. The Hidden Markov Model (HMM) has been employed to track the changes in qubit states throughout the measurement duration [22]. Given that the measurement process occurs over a considerable duration of time compared to the qubit lifetime, there's a possibility for the state of the qubit to change during measurement. Utilization of the HMM has shown superior starting-state assignment fidelity when compared to SVMs and GMMs. Feed-forward neural networks (FFNN) have been proposed for state discrimination and demonstrated their capability to reduce crosstalk noise [17]. Variational autoencoders (VAE) has been employed to distill features from readout signal [18], and have demonstrated an advantage for short and long time measurements.

1.2 Related work: Signature applications

The path signature method have found broad applications. Signature features from time-ordered measurements of the whole brain, ventricles, and hippocampus were used to predict Alzheimer disease [23]. The signature kernel and the tree structure between different processes were used for malware detection [5]. Signature features of EEG traces were used for seizure forecasting [12]. Signatures calculated from financial data streams were used for training a generator that generates data streams indistinguishable from the empirical distribution [2]. Limit order book data generates path signatures for market activity classifications [11]. Online hand-written digit path of Chinese characters can be machine recognized by its signature features [10]. Signature features of the sequence of prices were used to learn the outcome of an investment strategy using the supervised learning setting where historical performances are given [20]. Encrypted network traffic classification also benefits from the path signature method [31]. Signature features were used for the classification of bipolar and borderline personality disorder [25]. Signature features were used for modeling volatility processes, which form a crucial part of stochastic differential equations for financial assets [1, 7, 6]. A generalized signature method that comprehensively considers constructing features from multivariate-time series using the signature was proposed [24].

2 Method

Modeling dispersive readout The most widely used readout technique for superconducting quantum processors utilizes the state-dependent dispersive shift (change in resonance frequency depends on the qubit state) of a resonator coupled to the qubit to discriminate the qubit state. A probing



(a) Qubit state $|0\rangle$ (b) Qubit state $|1\rangle$ (c) Qubit state $|2\rangle$

Figure 1: (a-c) Signal obtained by probing the resonator when the transmon qubit is in states $|0\rangle$, $|1\rangle$, and $|2\rangle$, respectively. The solid line represents the average result over 2000 samples, and the translucent line denotes a single-shot example trace. Blue and red colors correspond to the I and Q channels of the signal, respectively.

signal is send to the resonator; after interaction, the phase and amplitude of the microwave signal are alternated depends on the resonator frequency, encoding the qubit state information. The objective is to improving state assignment accuracy while reduce the readout probing duration, within the constraints of the physical and experimental setup. We model the signal's path [29, 30, 14] as

$$\mathrm{d}X_t^k = \mathrm{d}F_t^k + \mathrm{d}\epsilon_t \tag{2}$$

where X_t^k is a random variable that denotes the received signal sample at time t, and F_t^k is the resonator system response at time t without noise. The signal contains two channels, with $k \in \{I, Q\}$ representing the in-phase and quadrature-phase channel, respectively. Examples of X_t^k and F_t^k in this experiment are presented in Fig.1. The term $\mathrm{d}\epsilon(t)$ is a stochastic differential representing the noise of the system, following a zero-mean distribution, and we model it as a Brownian motion.

The most widely used approach to discriminate the qubit state is using the Gaussian Mixture Model (GMM) on the signal $\{\tilde{F}^I, \tilde{F}^Q\}$, where $\tilde{F}^k$ is the integral of X_t^k, given by

$$\mathrm{d}\tilde{F}_t^k = X_t^k \mathrm{d}t = F_t^k \mathrm{d}t + \epsilon_t \mathrm{d}t \tag{3}$$

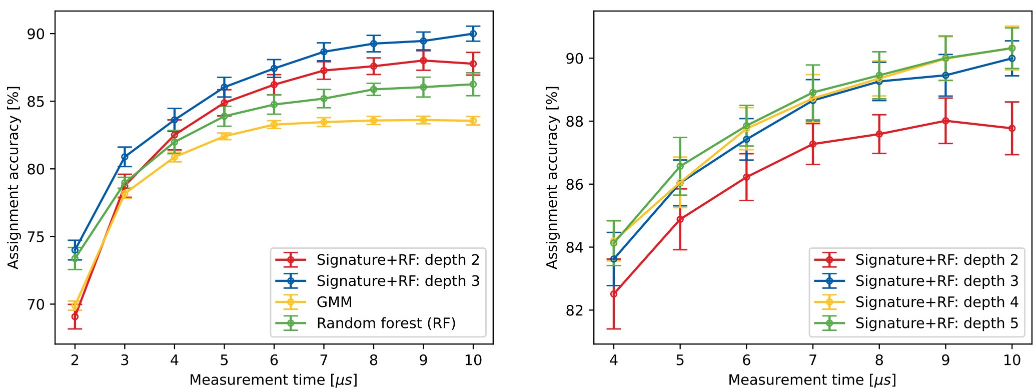
When F^k is carefully chosen by setting the probing microwave signal frequency and envelope, the first term remains meaningful while the noise term $\int \mathrm{d}\epsilon(t)$ will approximate the expectation value 0.

Path signature as a universal feature set Signature features have a strong theoretical basis. For example, we have from [13] that under mild assumptions, the full collection of signatures defined in Eq.(1) uniquely determines the path X up to reparametrizations and translations. Note that in evaluating a second-degree term of the signature, where one of the dimensions is the time argument (specifically, the values of this dimension are $\{t_0, t_1, ..., t_n\}$), this term can be written as:

$$S(X)_t^{k,\tau} = \int_{0<s<t} S(X)_s^k \mathrm{d}s = \int_{0<s<t} (X_s^k - X_0^k)\mathrm{d}s \tag{4}$$

Where τ denotes the time argument dimension. Considering that in the absence of a signal, the measured traces contain the value of zeros, the initial point of the measurement traces X_0^k will consistently be 0. The expression $S(X)_t^{k,\tau}$ then reduces to $\tilde{F}_k(t)$. These lower order signatures have well-known geometric meanings – degree 2 signature generates the Lévy area of the path X projected to two dimensions. The Levy area of a two dimensional path $\{\, X_i(t), X_j(t) \,\}$ is related to its signature by

$$A = \frac{1}{2}\left(S(X)^{i,j} - S(X)^{j,i}\right), \tag{5}$$



(a) (b)

Figure 2: (a-b) Comparison of assignment accuracy between the baselines and feature extraction using path signatures. The parameter depth=k indicates retaining signatures up to the k-th degree. The error bar represents one standard deviation across 10 different experiments with different random seeds.

where $S(X)^{i,j}$ corresponds to the terms in Eq.(1) with $k = 2$, $i_1 = i, i_2 = j$. Therefore, using other terms of the path signature would be a natural extension of the current features for discriminating quantum states.

3 Experiment

3.1 Dataset

A trace is collected through a two-step measurement process. The qubit state is first determined using the conventional GMM method and then prepared into $|0\rangle$, $|1\rangle$, and $|2\rangle$ states by applying quantum operations $|0\rangle \to |1\rangle$ and $|1\rangle \to |2\rangle$ with fidelity (how accurate the gate performs) of approximately 99.7%. Subsequently, a second measurement is conducted and the traces are recorded. Traces from the initial measurement not yielding the $|0\rangle$ state are discarded, as they imply a subsequent unsuccessful qubit state preparation.

Each trace was acquired using two 1 Gsps (Gigasample per second) sampling rate analogue digital converter devices, providing two (I and Q) distinct dimensions. The recorded signal has a carrier frequency of 125 MHz; a short-term Fourier transformation was applied to segments of 256 samples to demodulate the signal at this frequency.

A database of 5,000 traces for each state was created using the above-mentioned approach. In each experiment, 2,000 traces per state were randomly selected from the database, resulting in 6,000 traces per experiment. These traces then divided into the training set of 4,800 traces and the testing set of 1,200 traces, and the accuracy reported are testing set accuracies. Each hyperparameter configuration was evaluated across 10 repeated experiments with different random seeds for selecting the data from the database and splitting the training and testing dataset to ensure a robust statistical analysis.

3.2 Results and discussion

The metric utilized in this study is assignment accuracy, representing the percentage of the discriminated state that matches the intended prepared state. Each method is benchmarked with variations in the cut-off point of the time-series provided to the classifier. See Fig.2.

GMM The GMM model is used as one of the baselines. Trace data are reduced to two-dimensional points utilizing the method in Eq.(3). Subsequently, a GMM with three components and diagonal covariance is trained to evaluate the accuracy. This is the standard approach to implement state discrimination.

Random forest (RF) The RF model serves as an alternative baseline. The two-dimensional time series is flattened into a vector for RF training. The optimal number of trees is determined through a search among 10, 30, 50, 70, and 100 to yield the best outcome. It is observed that RF consistently outperforms the GMM method in assignment accuracy.

Signature-based RF The signature is computed using both base point and time augmentation. The truncation depth of the signature is adjusted, followed by classification via an RF, maintaining a setup consistent with the baseline. The computed signatures are rescaled so that the last path signature term is roughly $O(1)$. We observed that the path signature method, integrating with RF, consistently outperforms both GMM and simple RF. Increasing the signature depth further improves accuracy, though a depth beyond 3 does not yield statistically significant enhancement.

The accuracy achieved in our experiment is lower that of the results reported in prior studies [15, 30, 28]. This difference can be attributed primarily to our experiment setup which did not incorporate any quantum amplifiers. In addition, the qubit device used in our work was not specifically optimized for implementing readout.

4 Conclusion

Path signature is a powerful method for feature extraction derived from the theory of rough paths in stochastic analysis. This study demonstrates an application of path signature in quantum computing and employs the path signature to distinguish quantum states in superconducting qubits. The experiment demonstrates that the integration of path signature and RF outperforms conventional models in state discrimination tasks. Building on this work, further investigation could compare and integrate this method with other machine learning classifiers and explore the underlying reasons for the improved accuracy of the path signature approach.

Acknowledgments and Disclosure of Funding

This project is supported by the Eric and Wendy Schmidt AI in Science Postdoctoral Fellowship, a Schmidt Future program. S. C. was supported by Schmidt Futures. J.Q. Z. was supported by Kennedy Trust Prize Studentship [AZT00050-AZ04]. Z. S. was supported by the EPSRC [EP/S026347/1]. T. L. was funded in part by the EPSRC [EP/S026347/1], in part by The Alan Turing Institute under the EPSRC [EP/N510129/1], the Data Centric Engineering Programme (under the Lloyd's Register Foundation grant G0095), the Defence and Security Programme (funded by the UK Government) and in part by the Hong Kong Innovation and Technology Commission (InnoHK Project CIMDA). The authors would like to acknowledge the use of the University of Oxford Advanced Research Computing (ARC) facility in carrying out this work [26].

References

[1] Imanol Perez Arribas, Cristopher Salvi, and Lukasz Szpruch. Sig-sdes model for quantitative finance. In *Proceedings of the First ACM International Conference on AI in Finance*, pages 1–8, 2020.

[2] Hans Buehler, Blanka Horvath, Terry Lyons, Imanol Perez Arribas, and Ben Wood. Generating financial markets with signatures. *Available at SSRN 3657366*, 2020.

[3] Yudong Cao, Jonathan Romero, Jonathan P. Olson, Matthias Degroote, Peter D. Johnson, Mária Kieferová, Ian D. Kivlichan, Tim Menke, Borja Peropadre, Nicolas P. D. Sawaya, Sukin Sim, Libor Veis, and Alán Aspuru-Guzik. Quantum chemistry in the age of quantum computing. *Chemical Reviews*, 119(19):10856–10915, Oct 2019.

[4] Ilya Chevyrev and Andrey Kormilitzin. A primer on the signature method in machine learning. *arXiv preprint arXiv:1603.03788*, 2016.

[5] Thomas Cochrane, Peter Foster, Varun Chhabra, Maud Lemercier, Terry Lyons, and Cristopher Salvi. Sk-tree: a systematic malware detection algorithm on streaming trees via the signature kernel. In *2021 IEEE international conference on cyber security and resilience (CSR)*, pages 35–40. IEEE, 2021.

[6] Christa Cuchiero, Guido Gazzani, Janka Möller, and Sara Svaluto-Ferro. Joint calibration to spx and vix options with signature-based models. *arXiv preprint arXiv:2301.13235*, 2023.

[7] Christa Cuchiero, Guido Gazzani, and Sara Svaluto-Ferro. Signature-based models: theory and calibration. *SIAM Journal on Financial Mathematics*, 14(3):910–957, 2023.

[8] Nathalie P. de Leon, Kohei M. Itoh, Dohun Kim, Karan K. Mehta, Tracy E. Northup, Hanhee Paik, B. S. Palmer, N. Samarth, Sorawis Sangtawesin, and D. W. Steuerman. Materials challenges and opportunities for quantum computing hardware. *Science*, 372(6539):eabb2823, 2021.

[9] Adeline Fermanian, Terry Lyons, James Morrill, and Cristopher Salvi. New directions in the applications of rough path theory. *IEEE BITS the Information Theory Magazine*, pages 1–18, 2023.

[10] Benjamin Graham. Sparse arrays of signatures for online character recognition. *arXiv preprint arXiv:1308.0371*, 2013.

[11] Lajos Gergely Gyurkó, Terry Lyons, Mark Kontkowski, and Jonathan Field. Extracting information from the signature of a financial data stream. *arXiv preprint arXiv:1307.7244*, 2013.

[12] Jonas F Haderlein, Andre DH Peterson, Parvin Zarei Eskikand, Mark J Cook, Anthony N Burkitt, Iven MY Mareels, and David B Grayden. Path signatures for seizure forecasting. *arXiv preprint arXiv:2308.09312*, 2023.

[13] Ben Hambly and Terry Lyons. Uniqueness for the signature of a path of bounded variation and the reduced path group. *Annals of Mathematics*, pages 109–167, 2010.

[14] Johannes Heinsoo, Christian Kraglund Andersen, Ants Remm, Sebastian Krinner, Theodore Walter, Yves Salathé, Simone Gasparinetti, Jean-Claude Besse, Anton Potočnik, Andreas Wallraff, and Christopher Eichler. Rapid high-fidelity multiplexed readout of superconducting qubits. *Phys. Rev. Appl.*, 10:034040, Sep 2018.

[15] Evan Jeffrey, Daniel Sank, J. Y. Mutus, T. C. White, J. Kelly, R. Barends, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, A. Megrant, P. J. J. O'Malley, C. Neill, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and John M. Martinis. Fast accurate state measurement with superconducting qubits. *Phys. Rev. Lett.*, 112:190504, May 2014.

[16] Sigmund Kohler. Dispersive readout: Universal theory beyond the rotating-wave approximation. *Phys. Rev. A*, 98:023849, Aug 2018.

[17] Benjamin Lienhard, Antti Vepsäläinen, Luke C.G. Govia, Cole R. Hoffer, Jack Y. Qiu, Diego Ristè, Matthew Ware, David Kim, Roni Winik, Alexander Melville, Bethany Niedzielski, Jonilyn Yoder, Guilhem J. Ribeill, Thomas A. Ohki, Hari K. Krovi, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. Deep-neural-network discrimination of multiplexed superconducting-qubit states. *Phys. Rev. Appl.*, 17:014024, Jan 2022.

[18] Piero Luchi, Paolo E. Trevisanutto, Alessandro Roggero, Jonathan L. DuBois, Yaniv J. Rosen, Francesco Turro, Valentina Amitrano, and Francesco Pederiva. Enhancing qubit readout with autoencoders. *Phys. Rev. Appl.*, 20:014045, Jul 2023.

[19] Terry Lyons. Rough paths, signatures and the modelling of functions on streams. In *Proceedings of the International Congress of Mathematicians—Seoul 2014. Vol. IV*, pages 163–184. Kyung Moon Sa, Seoul, 2014.

[20] Terry Lyons, Hao Ni, and Harald Oberhauser. A feature set for streams and an application to high-frequency financial tick data. In *Proceedings of the 2014 International Conference on Big Data Science and Computing*, BigDataScience '14, New York, NY, USA, 2014. Association for Computing Machinery.

[21] Easwar Magesan, Jay M. Gambetta, A. D. Córcoles, and Jerry M. Chow. Machine learning for discriminating quantum measurement trajectories and improving readout. *Phys. Rev. Lett.*, 114:200501, May 2015.

[22] Luis A. Martinez, Yaniv J. Rosen, and Jonathan L. DuBois. Improving qubit readout with hidden markov models. *Phys. Rev. A*, 102:062426, Dec 2020.

[23] PJ Moore, TJ Lyons, John Gallacher, and Alzheimer's Disease Neuroimaging Initiative. Using path signatures to predict a diagnosis of alzheimer's disease. *PloS one*, 14(9):e0222212, 2019.

[24] James Morrill, Adeline Fermanian, Patrick Kidger, and Terry Lyons. A generalised signature method for multivariate time series feature extraction. *arXiv preprint arXiv:2006.00873*, 2020.

[25] Imanol Perez Arribas, Guy M Goodwin, John R Geddes, Terry Lyons, and Kate EA Saunders. A signature-based machine learning model for distinguishing bipolar disorder and borderline personality disorder. *Translational psychiatry*, 8(1):274, 2018.

[26] Andrew Richards. University of oxford advanced research computing. 2015.

[27] Peter W. Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM Journal on Computing*, 26(5):1484–1509, 1997.

[28] François Swiadek, Ross Shillito, Paul Magnard, Ants Remm, Christoph Hellings, Nathan Lacroix, Quentin Ficheux, Dante Colao Zanuz, Graham J. Norris, Alexandre Blais, Sebastian Krinner, and Andreas Wallraff. Enhancing dispersive readout of superconducting qubits through dynamic control of the dispersive shift: Experiment and theory, 2023.

[29] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Approaching unit visibility for control of a superconducting qubit with dispersive readout. *Phys. Rev. Lett.*, 95:060501, Aug 2005.

[30] T. Walter, P. Kurpiers, S. Gasparinetti, P. Magnard, A. Potočnik, Y. Salathé, M. Pechal, M. Mondal, M. Oppliger, C. Eichler, and A. Wallraff. Rapid high-fidelity single-shot dispersive readout of superconducting qubits. *Phys. Rev. Appl.*, 7:054020, May 2017.

[31] Shi-Jie Xu, Guang-Gang Geng, Xiao-Bo Jin, Dong-Jie Liu, and Jian Weng. Seeing traffic paths: encrypted traffic classification with path signature features. *IEEE Transactions on Information Forensics and Security*, 17:2166–2181, 2022.