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Direct reference and the Goldbach puzzle

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Abstract

So-called Neo-Russellians, such as Salmon, Braun, Crimmins, and Perry, hold that the semantic content of 'n is F' in a context c is the singular proposition $\langle o, P \rangle$, where o is the referent of the name n in c, and P is the property expressed by the predicate F in c. This is also known as the Neo-Russellian theory. Using truth ascriptions with names designating propositions, such as 'Goldbach's conjecture', in this paper, I will argue that, together with highly plausible principles regarding a priori knowledge, the Neo-Russellian theory leads to unacceptable consequences. I will call this 'the Goldbach puzzle'. Since the solution to the Goldbach puzzle cannot be to reject the discussed principles regarding a priori knowledge, the puzzle will undermine the Neo-Russellian theory.

KEYWORDS
direct reference, proper names, propositions

1 | INTRODUCTION

Following the work of Marcus (1961), Donnellan (1970), Perry (1977), Kripke (1980), and Kaplan (1989), so-called *Neo*-Russellians, such as Salmon (1986, 1989), Braun (1998), Crimmins and Perry (1989), and Crimmins (1992), hold that:

(NR_1)   The propositions we say and believe are Russellian propositions, that is, structured propositions whose basic components are the objects, properties and relations our thoughts and speech acts are about.

(NR_2)   Names and other singular terms (pronouns, simple demonstratives, indexicals) function as directly referential terms, that is, the semantic content of 'n is F' in a context c is the singular proposition $\langle o, \Phi \rangle$, where o is the referent of the name n in c, and Φ is the property expressed by the predicate F in c.

This is also known as *the Neo-Russellian theory*.[1]

Using truth ascriptions with names designating propositions, such as 'Goldbach's conjecture', in this paper, I will argue that, together with the following highly plausible principles regarding a priori knowledge, the Neo-Russellian theory leads to unacceptable consequences:

(DP_K) If a normal speaker of a language L knows that a commonplace sentence S of L is true in a context c, then the proposition p expressed by S in L and c is such that, with respect to the speaker's empirical evidence e, proposition p can be known a priori, that is, by ideal reasoning alone.[2]

(K)   With respect to empirical evidence e, a proposition p can be known a priori, that is, by ideal reasoning alone, if and only if p is true in every (relevant) possibility left uneliminated by e and ideal reasoning.[3]

I will call this *the Goldbach puzzle*. Since (DP_K) and (K) seem to be self-evident truths, the Goldbach puzzle will undermine the Neo-Russellian theory. First of all, normal speakers of a language L *by definition* take a commonplace sentence S of L to express exactly the very proposition it in fact expresses in L.[4] Thus, if a normal speaker of a language L knows that a commonplace sentence S of L is true, then the proposition p expressed by S in L is such that, with respect to their empirical evidence, the speaker is in a position to know proposition p by (ideal) reasoning alone. After all, as a normal speaker of L, they are in a position to know by (ideal) reasoning alone that S is true in L if and only if p is true. (K), on the other hand, seems to follow from the fact that to every empirical evidence e, a set of (relevant) possibilities, which cannot be eliminated by e and ideal reasoning, can be assigned. Of particular philosophical interest is here the limiting case with no empirical evidence. For example, in order to account for the fact that there are necessary a posteriori truths, such as 'Kripke is a human being', within the Neo-Russellian theory, Soames (2002) maintains that there are epistemic possibilities, that is, possibilities left uneliminated by ideal reasoning alone, that are not metaphysically possible, such as possibilities in which Kripke is not a human being, but a robot. Accordingly, a proposition is necessarily a posteriori true if and only if it is true in every metaphysical possibility without at the same time being true in every epistemic possibility.[5]

The paper is structured as follows. In Section 2, I will present the Goldbach puzzle. Following this, in Section 3, I will discuss the possibility to reject (K) by employing Adams' (1981) distinction of *truth in a world* and *truth at a world*. I will argue that for Neo-Russellians rejecting (K) in this way is problematic in itself. Concluding, I will compare the Goldbach puzzle with a problem discussed by Richard and Soames in connection with Neo-Russellian accounts of belief ascriptions. I will argue that, unlike the Goldbach puzzle, the Richard–Soames problem does not undermine the Neo-Russellian theory.

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[1]As the name suggests, the Neo-Russellian theory goes back to Russell (1905, 1910, 1912). However, according to Russell, the only directly referential terms are so-called *logically proper names*, that is, demonstratives, such as 'this' and 'that', referring to sense-data or other objects of immediate acquaintance.

[2]Ideal reasoning is reasoning unrestricted by cognitive limitations. If with respect to empirical evidence e a proposition p can be known by ideal reasoning alone, then this is tantamount to saying that no additional empirical evidence beyond e is required in order for an ideal agent to know p. In other words, with respect to e, p is a priori true.
When I say that a speaker knows a proposition p, I mean that the speaker stands in the relation of propositional knowledge (knowledge-that) to p.

[3]The restriction to *relevant* possibilities should account for potential sceptical scenarios regarding empirical knowledge, which could not be ruled out a priori (see, e.g., Lewis, 1996).

[4]See, for example, Salmon's (2011, 244) definition of *normal speaker*. Note that being a normal speaker neither includes nor precludes being an ideal agent, that is being an agent whose reasoning is unrestricted by cognitive limitations. Hence, when I say that, with respect to a (normal) speaker's evidence e, a proposition p can be known a priori, I mean that, in principle, with respect to e, proposition p can be known by an ideal agent.

[5]Soames no longer advocates a Neo-Russellian theory (see, e.g., Soames, 2015). Nevertheless, his explanation can still be considered the standard account of necessary a posteriori truths within the Neo-Russellian theory.

2 | THE GOLDBACH PUZZLE

Just like 'Napoleon' is the name of a French emperor, 'Goldbach's conjecture' seems to be the name of a proposition, that is, the proposition that every even number greater than two is the sum of two primes.[6] Together with the Neo-Russellian theory, it would follow that the semantic content of 'Goldbach's conjecture is true' is the singular proposition ⟨Goldbach's conjecture, being true⟩, consisting of Goldbach's conjecture and the property of being true. Other examples of names for propositions seem to include 'Frege's theorem', 'Fermat's last theorem' and 'axiom of choice'.[7]

Regardless of whether expressions such as 'Goldbach's conjecture' are actually names designating propositions, according to the Neo-Russellian theory, propositions exist. Thus, it seems that we could simply introduce a (new) name for the (Russellian) proposition that every even number greater than two is the sum of two primes. Against this, it could be objected that since propositions are abstract objects, it is not even possible to name them. After all, the objection goes, we know from Kripke (1980) that the reference of a proper name is usually determined by some kind of causal link. However, we also know from Kripke that the reference of a proper name can be fixed in a number of different ways, a causal link being only one of them. Other possibilities include the use of reference-fixing descriptions and the use of other referring expressions. For example, we can certainly refer to propositions using expressions such as 'that every even number greater than two is the sum of two primes' or 'the proposition that every even number greater than two is the sum of two primes'. Thus, we could fix the reference of 'Goldbach's conjecture' by saying something along the lines of "Let's call the proposition that every even number greater than two is the sum of two primes 'Goldbach's conjecture'". Similarly, we could introduce a name for the proposition that Berlin is the capital of Germany by saying something like "Let's call the proposition that Berlin is the capital of Germany 'Sally'".

Let us take the latter scenario a little further and assume that Harry, a normal English speaker, was subsequently introduced to the name 'Sally'. However, Harry does not know what Sally says. Harry only knows that Sally is a proposition, and that Sally is believed by his friends. Nevertheless, Harry can use the name 'Sally' to refer to the proposition that Berlin is the capital of Germany and to express singular propositions about it.[8] In this regard, Harry can be compared to a speaker who with the name 'Feynman' only associates the information that Feynman is a physicist. As Kripke (1980) points out, as long as the speaker was appropriately introduced to the name 'Feynman', this would not prevent them from referring to Feynman by their uses of the name.[9]

Now, as a normal speaker, Harry could even know that (1) is true without knowing what Sally says, for example, if he has been reliably told that Sally is true.

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[6]It could be objected that there is no particular proposition designated by 'Goldbach's conjecture' as there are a large number of logically equivalent propositions that are true at a circumstance C if and only if in C every even number greater than two is the sum of two primes. This can be compared to Kijania-Placek's (2021, 2022) observations regarding names of places and institutions (see also Viebahn, 2018). Although it is plausible that no particular proposition is designated by a name such as 'Goldbach's conjecture', we will see that it is certainly possible to introduce a new name designating a particular proposition. This is all that is required for the remainder of this paper.

[7]It could be argued that these are examples of partially descriptive names whose semantic content consists of their referent and a property, such as the property of being a conjecture (see, e.g., Soames, 2002). However, we will see that there could, in principle, be proper names that designate a proposition without having any kind of descriptive content. Again, this is all that is required for the remainder of this paper.

[8]Similarly, many normal English speakers do not know what Goldbach's conjecture says. Nevertheless, these speakers can use the name 'Goldbach's conjecture' to refer to Goldbach's conjecture and to express singular propositions about it. For example, it makes perfect sense to utter something along the lines of 'I do not know what Goldbach's conjecture says'. Therefore, it should also make perfect sense to utter something along the lines of 'Sally is a proposition, but I do not know what Sally says'.

[9]According to Kripke, among other things, this requires that the speaker intends to use the name with the same reference as the speaker from whom they heard it. We can assume that this is true of Harry.

(1) Sally is true.

Together with the Neo-Russellian theory and (DP_K), it would follow that, with respect to Harry's empirical evidence, the Russellian proposition expressed by (1), that is, the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$, can be known a priori, that is, by ideal reasoning alone. From this, in turn, it would follow together with (K) that $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ is true in every (relevant) possibility left uneliminated by Harry's evidence and ideal reasoning. Since $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ is true in such a possibility s if and only if in s $\langle$Berlin, being the capital of Germany$\rangle$ is true, it would follow that the singular proposition $\langle$Berlin, being the capital of Germany$\rangle$ is true in every (relevant) possibility left uneliminated by Harry's evidence and ideal reasoning. Thus, according to (K), with respect to Harry's evidence, $\langle$Berlin, being the capital of Germany$\rangle$ would be knowable a priori. However, since Harry does not know what Sally says, we can assume that even as an ideal agent he would require additional empirical evidence in order to rule out relevant possibilities in which Berlin is not the capital of Germany. For example, we could assume that Harry is completely ignorant regarding European capitals. But then, with respect to Harry's evidence, the proposition $\langle$Berlin, being the capital of Germany$\rangle$ would not be knowable priori. This is the Goldbach puzzle.[10]

Note that the solution to the Goldbach puzzle cannot simply be that Harry does not count as a normal English speaker when it comes to 'Sally'. As said above, just like a speaker who was introduced to 'Feynman' by overhearing an utterance of 'Feynman was a physicist' can subsequently refer to Feynman with their uses of the name, Harry can refer to the proposition that Berlin is the capital of Germany with his uses of 'Sally'. In other words, Harry counts as a normal English speaker when it comes to 'Sally', which is why from the fact that Harry knows that (1) is true it follows together with (DP_K) that, with respect to Harry's empirical evidence, the (Russellian) proposition expressed by (1) can be known a priori.

Against this use of (DP_K), Neo-Russellians could object that although Harry can refer to the singular proposition $\langle$Berlin, being the capital of Germany$\rangle$ with his uses of 'Sally', he is not in a position to grasp or entertain the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ expressed by (1). Thus, the objection goes, Harry is not in a position to assign the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ to (1), which is why from the fact that Harry knows that (1) is true we cannot infer that, with respect to Harry's empirical evidence, $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ can be known a priori. However, for a Neo-Russellian, the notion of grasping or entertaining a proposition underlying this objection is too strong. If I am in a position to refer to Feynman with my uses of 'Feynman', then I am in a position to entertain or grasp singular propositions about him. Otherwise, it would be very unclear what it means to refer to an object. Hence, if Harry is in a position to refer to the singular proposition $\langle$Berlin, being the capital of Germany$\rangle$ with his uses of 'Sally', then, among other things, he is in a position to entertain or grasp the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ expressed by (1), consisting of $\langle$Berlin, being the capital of Germany$\rangle$ and the property of being true. This is also why, according to Salmon (2011), disquotational principles such as (DP_K) are virtually analytic.

Neo-Russellians could try to solve the Goldbach puzzle by claiming that there are (relevant) logical (im)possibilities left uneliminated by Harry's evidence and ideal reasoning in which the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ is true, while its logical consequence $\langle$Berlin, being the capital of Germany$\rangle$ is not. However, this is problematic. If, with respect to his empirical evidence, Harry is in a position to know the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ by (ideal) reasoning alone, then he should

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[10]The problem can be replicated with every sentence of the form 'n is true', where n is a proper name designating an a posteriori true proposition, that is, a proposition whose truth cannot be known by ideal reasoning alone.

be in a position to eliminate the relevant (im)possibilities in which ⟨Berlin, being the capital of Germany⟩ is not true, using his empirical evidence and ideal reasoning. Therefore, the solution to the Goldbach puzzle also cannot be to introduce logical impossibilities in which ⟨⟨Berlin, being the capital of Germany⟩, being true⟩ is true, while its logical consequence ⟨Berlin, being the capital of Germany⟩ is not.

This strongly suggests that Neo-Russellians have to reject (DP_K) or (K). As we have seen above, both (DP_K) and (K) seem to be self-evident truths. However, following Adams' (1981) distinction of *truth in a world* and *truth at a world*, Neo-Russellians could maintain that the notion of apriority is better captured by (K') than by (K):

(K') With respect to empirical evidence e, a proposition p can be known a priori if and only if p describes every (relevant) possibility left uneliminated by e and ideal reasoning correctly.

While (K) corresponds to Adams' notion of *truth in a world*, (K') corresponds to his notion of *truth at a world*.

In fact, the Neo-Russellian theory does not seem to lead to a problem similar to the Goldbach puzzle presented above once we replace (K) with (K'). As we have already seen, from (DP_K) it follows together with the Neo-Russellian theory that ⟨Berlin, being the capital of Germany⟩ is true in every (relevant) possibility left uneliminated by Harry's evidence and ideal reasoning. Neo-Russellians could now argue that from this it does not follow that in every such possibility Berlin is the capital of Germany. After all, the argument goes, even if there are no (relevant) logical impossibilities left uneliminated by Harry's evidence and ideal reasoning in which ⟨⟨Berlin, being the capital of Germany⟩, being true⟩ is true, while its logical consequence ⟨Berlin, being the capital of Germany⟩ is not, there are (relevant) logical impossibilities left uneliminated by Harry's evidence and ideal reasoning in which ⟨Berlin, being the capital of Germany⟩ is true, while Berlin is not the capital of Germany. Since such an impossibility would not be described correctly by the singular proposition ⟨Berlin, being the capital of Germany⟩, together with (K'), this would account for the fact that, with respect to Harry's evidence, ⟨Berlin, being the capital of Germany⟩ cannot be known a priori.

In this way, Neo-Russellians could explain that a normal speaker who knows that (1) is true should be in a position to eliminate the possibility that ⟨Berlin, being the capital of Germany⟩ is not true, using their empirical evidence and ideal reasoning, without necessarily being in a position to eliminate the possibility that Berlin is not the capital of Germany. However, even if we assume that there are logical impossibilities left uneliminated by Harry's evidence and ideal reasoning in which ⟨Berlin, being the capital of Germany⟩ is true, while Berlin is not the capital of Germany, next, I will argue that in connection with the Neo-Russellian theory (K') is nevertheless problematic.

3 | THE PROBLEM OF LOGICAL IMPOSSIBILITIES

According to Neo-Russellians, (2) and (3) express the same proposition, that is, the singular proposition ⟨⟨Karol Wojtyła, Karol Wojtyła⟩, =⟩.

(2) Karol Wojtyła = Karol Wojtyła.
(3) Karol Wojtyła = John Paul II.

Nevertheless, a normal English speaker can know a priori, that is, by (ideal) reasoning alone, that (2) is true, without them being in a position to know a priori that (3) is true. In order to explain this, Neo-Russellians reject (SDP_K) and, instead, accept (SDP'_K).[11]

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[11]For similar principles in connection with a priori knowledge see, for example, Soames (2002). Salmon (1986, 1989) and Braun (1998) discuss related principles in connection with belief.

(SDP_K) A normal speaker of a language L can know a priori that a sentence S of L is true in a context c if and only if the proposition p expressed by S in L and c is such that the speaker can know p a priori.

(SDP'_K) A normal speaker of a language L can know a priori that a sentence S of L is true in a context c if and only if the proposition p expressed by S in L and c is such that the speaker can know p a priori when it is presented to them under their S-related mode of presentation, that is, under the way p is presented to them when they take it to be the proposition expressed by S in L and c.

Since a normal speaker can know the singular proposition $\langle\langle$Karol Wojtyła, Karol Wojtyła$\rangle, =\rangle$ a priori when it is presented to them under their (2)-related mode of presentation, without them being in a position to know the proposition a priori when it is presented to them under their (3)-related mode of presentation, the explanation goes, they can know a priori that (2) is true without being in a position to know a priori that (3) is true.

Once Neo-Russellians replace (SDP_K) with (SDP'_K), they have to adjust (K') by claiming that:

(K'_{MoP}) An agent can know a proposition p a priori, that is, by ideal reasoning alone, when it is presented to them under a mode of presentation m if and only if every possibility that is not described correctly by p is such that, when it is presented to the agent under m (or under a mode of presentation consisting, among other things, of m), it can be eliminated by ideal reasoning alone.[12]

Accordingly, just like propositions, epistemic possibilities can be presented to agents under different modes of presentation. This is not surprising. After all, we can conceive of epistemic possibilities as sets of (Russellian) propositions.[13] Neo-Russellians would then maintain that every possibility that is not described correctly by $\langle\langle$Karol Wojtyła, Karol Wojtyła$\rangle, =\rangle$ can be eliminated by an ideal agent (without any empirical evidence) if the possibility is presented to them under their (2)-related mode of presentation, but not if it is presented to them under their (3)-related mode of presentation.[14] Together with (K'_{MoP}) and (SDP'_K), this would provide an explanation of the fact that, as a normal English speaker, such an agent can know a priori that (2) is true without being in a position to know a priori that (3) is true. Again, this presupposes that there are logical impossibilities left uneliminated by ideal reasoning alone, for example, logical impossibilities in which Karol Wojtyła is not identical with Karol Wojtyła. Next, I will argue that even if there are such logical impossibilities, together with (SDP'_K), also (K'_{MoP}) leads to unacceptable consequences for Neo-Russellians.

For example, according to the Neo-Russellian theory, (4) expresses a logical truth.

(4) If Karol Wojtyła is Polish, then John Paul II is Polish.

Nevertheless, even normal English speakers whose reasoning is unrestricted by cognitive limitations need (additional) empirical evidence in order to know that (4) is true.[15] Again,

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[12]Here we only consider the limiting case with no empirical evidence, which is why we can omit the restriction to relevant possibilities as this restriction was intended to exclude sceptical scenarios regarding empirical knowledge.

[13]It is neither required that such an epistemic possibility is total, in the sense of containing, for every proposition p, either p or its negation, nor that it is consistent (see below). A mode of presentation of such an epistemic possibility consists of modes of presentation of the propositions contained by the possibility.

[14]In other words, under their (2)-related mode of presentation (or under a mode of presentation consisting, among other things, of their (2)-related mode of presentation), the agent can eliminate an epistemic (im)possibility containing the negation of $\langle\langle$Karol Wojtyła, Karol Wojtyła$\rangle, =\rangle$ by ideal reasoning alone, whereas under their (3)-related mode of presentation they cannot.

[15]Arguably, an agent already needs empirical evidence in order to assign to (4) exactly the very proposition it in fact expresses in English. However, even a speaker who assigns to (4) exactly the very proposition it in fact expresses in English and whose reasoning is unrestricted by cognitive limitations needs additional empirical evidence in order to know that (4) is true.

Neo-Russellians could try to explain this by claiming that even such ideal speakers need empirical evidence in order to eliminate logical impossibilities that are not described correctly by the logical truth expressed by (4) when these (im)possibilities are presented to them under their (4)-related mode of presentation. However, even if there are logical impossibilities in which the singular proposition ⟨Karol Wojtyła, being Polish⟩ is both true and false (or both true and not true), there is no logical impossibility that is not described correctly by the Russellian proposition expressed by (4). After all, in such an impossibility the singular proposition ⟨Karol Wojtyła, being Polish⟩ would have to be true, and, thus, the impossibility would be described correctly by the Russellian proposition expressed by (4) as this proposition simply says that if ⟨Karol Wojtyła, being Polish⟩ is true, then so is ⟨Karol Wojtyła, being Polish⟩. Hence, according to the Neo-Russellian theory, even under their (4)-related mode of presentation an ideal agent does not need empirical evidence in order to eliminate every (im)possibility that is not described correctly by the logical truth expressed by (4), for the simple reason that there is no such (im)possibility. Together with (K'_{MoP}) and (SDP'_K), it would follow that ideal speakers do not need (additional) empirical evidence in order to know that (4) is true. Since even ideal speakers need (additional) empirical evidence in order to know that (4) is true, this strongly suggests that Neo-Russellians have to reject (K'_{MoP}) or (SDP'_K). I will call this *the problem of logical impossibilities*.

Neo-Russellians could try to solve the problem of logical impossibilities by rejecting (SDP'_K). However, if Neo-Russellians reject (SDP'_K), then they have to provide an alternative explanation of the fact that normal speakers can know a priori that (2) is true without being in a position to know a priori that (3) is true. Since there is currently no such explanation on the market, it is very unclear what such an explanation could look like. Moreover, it is very likely that together with (K') every such explanation would lead to a problem similar to the problem of logical impossibilities presented above, simply because there are no (im)possibilities that are not described correctly by the Russellian proposition expressed by (4). Therefore, the problem of logical impossibilities strongly suggests that Neo-Russellians have to reject (K') (or (K'_{MoP}), respectively), which is why, for these philosophers, the solution to the Goldbach puzzle cannot simply be to replace (K) with (K') (or (K'_{MoP}), respectively).

Note that for Neo-Russellians it is also not an option to reject (K'_{MoP}) and, instead, accept its *truth in a world* reading, that is, (K_{MoP}):

(K_{MoP}) With respect to empirical evidence e, an agent can know a proposition p a priori when it is presented to them under a mode of presentation m if and only if every (relevant) possibility in which p is not true is such that, when it is presented to the agent under m (or under a mode of presentation consisting, among other things, of m), it can be eliminated by e and ideal reasoning.

Although there are logical impossibilities in which the proposition expressed by (4) is not true and, thus, (K_{MoP}) potentially provides a solution to the problem of logical impossibilities, just like (K), (K_{MoP}) leads to the Goldbach puzzle.

If a normal speaker knows that (1) is true, then, together with the Neo-Russellian theory and the following variant of (DP_K), that is, (DP'_K), it follows that, with respect to their empirical evidence, the speaker is in a position to know the singular proposition ⟨⟨Berlin, being the capital of Germany⟩, being true⟩ a priori when it is presented to them under their (1)-related mode of presentation.

(DP'_K) If a normal speaker of a language L knows that a commonplace sentence S of L is true in a context c, then the proposition p expressed by S in L and c is such that, with respect to their empirical evidence, the speaker can know proposition p a priori when it is presented to them under their S-related mode of presentation.

From this, in turn, it would follow together with (K_{MoP}) that every (relevant) possibility in which $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ is not true is such that, when it is presented to the speaker under their (1)-related mode of presentation (or under a mode of presentation consisting, among other things, of their (1)-related mode of presentation), it can be eliminated by the speaker's evidence and ideal reasoning. Since, as we have seen above, the solution to the Goldbach puzzle cannot be to resort to logical impossibilities in which $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ is true, while its logical consequence $\langle$Berlin, being the capital of Germany$\rangle$ is not, it would follow that every (relevant) possibility in which $\langle$Berlin, being the capital of Germany$\rangle$ is not true is such that, when it is presented to the speaker under their (1)-related mode of presentation, it can be eliminated by the speaker's evidence and ideal reasoning. From this, in turn, it would follow together with (K_{MoP}) that, with respect to the speaker's empirical evidence, $\langle$Berlin, being the capital of Germany$\rangle$ can be known a priori when it is presented to them under their (1)-related mode of presentation.[16] However, since we can assume that the speaker does not know what Sally says, we can again assume that this is not the case.

4 | CONCLUDING REMARKS

We see that independent of whether Neo-Russellians accept (K) or (K') (or (K_{MoP}) or (K'_{MoP}), respectively) the Neo-Russellian theory leads to unacceptable consequences, that is, the Goldbach puzzle or the problem of logical impossibilities. Since Neo-Russellians seem to be committed to either (K) or (K'), this undermines the Neo-Russellian theory.

An alternative to the Neo-Russellian theory would be to claim that the propositions we say and believe are Fregean propositions, that is, structured propositions whose basic components are ways the objects, properties and relations our thoughts and speech acts are about are presented to the speaker or agent (see, e.g., Frege, 1892, 1918). Accordingly, the semantic content of a proper name is a way its referent is presented to the speaker. Unlike the Neo-Russellian theory, such a Fregean theory of propositions is not committed to the claim that the proposition expressed by (1) is true in an epistemic possibility s if and only if in s the proposition that Berlin is the capital of Germany is true. After all, a Fregean theory is not committed to the claim that the mode of presentation expressed by a proper name, such as 'Goldbach's conjecture' or 'Sally', picks out the actual referent of the name in every possibility left uneliminated by the speaker's evidence and ideal reasoning (see, e.g., Chalmers, 2002, 2011).[17] Thus, from the fact that a normal speaker knows that (1) is true, it does not follow that the proposition that Berlin is the capital of Germany is true in every possibility left uneliminated by the speaker's evidence and ideal reasoning. This would then provide an explanation of the fact that such a speaker needs additional empirical evidence in order to know the proposition that Berlin is the capital of Germany, without rejecting (DP_K) or (K). However, a more detailed discussion of the Fregean theory goes beyond the scope of this paper.

The Goldbach puzzle has some similarities with a problem that has been discussed by Richard (1993) and Soames (2015) in connection with belief ascriptions. Accordingly, unlike (5), (6) implies that Peter knows the content of Ralph's belief.

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[16]Assuming that the speaker's (1)-related mode of presentation of the singular proposition $\langle\langle$Berlin, being the capital of Germany$\rangle$, being true$\rangle$ consists of a mode of presentation m of the singular proposition $\langle$Berlin, being the capital of Germany$\rangle$ and a mode of presentation m' of the property of being true, it would be more accurate to say that, according to (K_{MoP}), with respect to the speaker's empirical evidence, $\langle$Berlin, being the capital of Germany$\rangle$ can be known a priori when it is presented to them under m. After all, every (relevant) possibility in which $\langle$Berlin, being the capital of Germany$\rangle$ is not true is such that, when it is presented to the speaker under a mode of presentation consisting, among other things, of m, that is, the speaker's (1)-related mode of presentation, it can be eliminated by the speaker's evidence and ideal reasoning.

[17]This is why, in order to explain the fact that there are necessary a posteriori true sentences, such as 'Kripke is a human being', Fregeans are not committed to the claim that there are epistemic possibilities that aren't metaphysically possible, such as possibilities in which Kripke isn't a human being.

(5) Peter believes that Ralph believes Goldbach's conjecture.

(6) Peter believes that Ralph believes that every even number greater than two is the sum of two primes.

However, as Bonardi (2017) points out, the Neo-Russellian theory provides an explanation of this fact, since it is not committed to the claim that, just like proper names, that-clauses are directly referential terms. Hence, unlike the problems presented in this paper, the Richard–Soames problem does not undermine the Neo-Russellian theory.

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