Multiobjective Path Planning on Uneven Terrains Based on NAMOA*

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***Abstract*—Existing path planning algorithms are capable of finding physically feasible, shortest, and energy-efficient paths for mobile robots navigating on uneven terrains. However, shortest paths on uneven terrains are often energy inefficient while energy-optimal paths usually take long time to be traversed. Therefore, due to time and energy constraints imposed on mobile robots, these shortest and energy-optimal paths might not be applicable. We propose a multiobjective path planner that can find pareto-optimal solutions in terms of path length and energy consumption. It is based on NAMOA* search algorithm that utilizes a proposed monotone heuristic cost function. The simulation results show that nondominated path options found by the proposed path planner can be very useful in many real-world applications.**

***Index Terms*—Multiobjective, pareto-optimal, path planning, heuristic search, uneven terrains.**

I. Introduction

Path planning algorithms are widely adopted in three dimensional terrain navigation to find feasible paths between two selected points. In an early attempt on finding feasible paths on uneven terrains, Rowe and Ross [1] introduced a physical model which captures terrain properties along with the external forces, such as friction and gravity, imposed on mobile robots. They also introduced anisotropism to their model by considering impermissible traversal directions due to overturn dangers and power limitations. Lanthier et al. [2] proposed an algorithm for computing an approximation to a shortest path on a given terrain based on the physical model proposed in [1]. They introduced a terrain face weight concept, which apprehends the nature of the terrain, slope of each terrain face, and friction. They discretized the terrain by placing Steiner points on boundaries of the terrain faces and connecting them with weighted edges. A path with the minimum total weight in a graph is found using the Dijkstra's algorithm [3].

Shortest paths, however, can be highly energy inefficient on uneven terrains [4]. As a solution, Rowe and Ross used their physical model together with A* search algorithm [5] to find energy-optimal paths on uneven terrains [1]. Based on the terrain face weight concept, Sun and Reif also introduced an energy-efficient path planning algorithm for mobile robots navigating on uneven terrains [6]. However, both these algorithms assume that a terrain surface is a combination of multiple flat surfaces. Hence, energy-efficient paths generated on such approximated terrains may differ from the energy-efficient paths on actual terrain. Recently, Ganganath et at. proposed an efficient heuristic search algorithm to find energy-efficient paths on high-resolution grid-based elevation maps [7]. They also proposed strategies for rapid replanning of energy-efficient paths on partially known terrains [8], [9].

According to our studies in this paper, shortest paths normally travel through both peaks and valleys consuming large amount of energy while energy-optimal paths tend to lie more along valleys on uneven terrains and they are much longer than shortest paths. Mobile robots utilized in outdoor applications are normally powered with portable energy sources with limited capacities. These robots are usually assigned to perform certain tasks within specified time durations. Thus, when it comes to mobile robot navigation on uneven terrains, there is always a trade-off between shortest and energy-optimal paths, rather than biasing toward any of them. The focus of this paper is dedicated for finding nondominated cost paths in terms of path length and energy consumption.

The rest of the paper is organized as follows. Section II presents the problem formulation. Section III explains how to construct a graph for multiobjective path planning using an elevation map of a given terrain. It also briefly discusses impermissible headings on uneven terrains. Multiobjective path planning and heuristic cost estimation are explained in Section IV. Simulation results of the proposed path planner are presented and analyzed in Section V. Concluding remarks are given in Section VI.

II. Problem Formulation

The multiobjective path planning problem considered in this work is to find all physically feasible nondominated cost paths, in terms of path length and associated energy consumption, between two selected points on a given gird-based elevation map of a terrain.

III. Preliminaries

A. Construction of a Graph

High resolution grid-based elevation maps are available for many geographical locations as a result of recent advances in geographical information systems. To facilitate the path planning process, such a map of a terrain is transformed into a weighted digraph $\mathcal{G}$ made of 8-connected neighborhoods, whose nodes represent points on the terrain surface, i.e. each grid center in the map is represented using a node which is connected to nodes that represent up to 8 neighboring grids (grids on edges of the map have less than 8 neighboring grids). Let n be an arbitrary node in $\mathcal{G}$ and n' be a neighboring node of n. The cost of the edge from n to n' is denoted as

$$\vec{c}(n,n') = [c_d(n,n'), c_e(n,n')],$$

where $c_d(n,n')$ and $c_e(n,n')$ respectively represent distance and energy costs associated with nn' traversal. One should note that $\vec{c}(n,n')$ is not always equal to $\vec{c}(n',n)$. In order to determine values of c_d and c_e, first we need to understand certain physical properties of the robot and the terrain.

B. Impermissible Headings

We denote coordinates of a node n in $\mathcal{G}$ as $(n.x, n.y, n.z)$. The projected length of the straight line nn' on the x-y plane can be calculated as

$$d(n,n') = \sqrt{(n'.x - n.x)^2 + (n'.y - n.y)^2}.$$

Then, the Euclidean distance s between n and n' in the 3D space can be calculated as

$$s(n,n') = \sqrt{d(n,n')^2 + \Delta(n,n')^2}$$

Here, the elevation difference between n and n' is given by

$$\Delta(n,n') = n'.z - n.z.$$

The angle of inclination from n to n' (positive for uphilling, negative for downhilling) can be calculated using

$$\phi(n,n') = \tan^{-1}\left[\frac{\Delta(n,n')}{d(n,n')}\right].$$

We adopt a physical model proposed in [1] which assumes a constant velocity v for the mobile robot. Thus, two major external forces applying on the robot are gravity and friction, whose resultant can be given as

$$mg(\mu\ \cos\phi + \sin\phi),$$

where m is the mass of the robot, μ is the friction coefficient, ϕ is the inclination angle, and g is the gravitational field strength.

In its uphill traversal, the maximum inclination angle that the robot can overcome due to power constraints, is defined as

$$\phi_f = \sin^{-1}\left(\frac{P_{\max}}{mgv\sqrt{\mu^2+1}}\right) - \tan^{-1}(\mu),$$

where $P_{\max}$ is the maximum motion power of the robot [1]. Furthermore, the traction depends on the static friction coefficient μ_s at the contact point. An anisotropic traction-loss phenomena can be observed if the inclination angle is greater than ϕ_s [1], which is defined as

$$\phi_s = \tan^{-1}(\mu_s - \mu).$$

Considering aforementioned scenarios, the critical impermissible angle for the uphill traversal can be defined as

$$\phi_m = \min(\phi_f, \phi_s),$$

which is the maximum inclined angle that the mobile robot is capable of overcoming [7].

C. Cost Functions

Based on the impermissible headings explained above, we can obtain the cost of traversing nn' in terms of distance as

$$c_d(n,n') = \begin{cases} \infty, & \text{if } \phi(n,n') > \phi_m, \\ s(n,n'), & \text{otherwise,} \end{cases} \tag{1}$$

and, in terms of energy consumption [7] as

$$c_e(n,n') = \begin{cases} \infty, & \text{if } \phi(n,n') > \phi_m, \\ mgs(n,n')(\mu\cos\phi(n,n') + \sin\phi(n,n')), & \\ & \text{if } \phi_m \geq \phi(n,n') > \phi_b, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The critical breaking angle for downhilling [1] is given by

$$\phi_b = -\tan^{-1}(\mu).$$

It is assumed that the robot has to spend a negligible amount of energy to maintain its constant velocity when the inclination angle is not greater than ϕ_b. In (1) and (2), ∞ indicates that nn' cannot be traversed when the inclination angle is greater than the critical impermissible angle for uphilling.

IV. MULTIOBJECTIVE PATH PLANNING

Let us consider a problem of finding nondominated cost paths from a starting node s to a goal node t on a digraph $\mathcal{G}$. A multiobjective search algorithm is said to be admissible if it can find all such nondominated paths whenever they exist. Here, we employ admissible NAMOA* search algorithm, which was proposed for multiobjective graph search with consistent heuristics [10].

A. NAMOA Search Algorithm*

NAMOA* can be identified as an extension of A* search algorithm [5] to the multiobjective case. A* algorithm is based on the best-first search which selects most favorable node n for expansion. However, in multiobjective graph search problems, there can be more than one path from s to n which are nondominated by each other. Hence, NAMOA* utilizes path selection and expansion as its basic operations to replace node selection and expansion used in A*. Let Λ_n be a set of all feasible paths from s to n and $\lambda_n \in \Lambda_n$ be one such path. The expected cost of λ_n to reach t can be defined as

$$\vec{f}(\lambda_n) = \vec{g}(\lambda_n) + \vec{h}(n), \tag{3}$$

where $\vec{g}(\lambda_n) = [g_d(\lambda_n), g_e(\lambda_n)]$ is the cost of λ_n, which can be obtained as a summation of $\vec{c}$ along λ_n. In (3), $\vec{h}(n) = [h_d, h_e]$ is a heuristic cost estimation from n to t. The path $\lambda_n \in \Lambda_n$ is said to be dominated by another path $\lambda'_n \in \Lambda_n$ if

$$(f_d(\lambda'_n) \leq f_d(\lambda_n)) \wedge (f_e(\lambda'_n) \leq f_e(\lambda_n)) \wedge (\vec{f}(\lambda'_n) \neq \vec{f}(\lambda_n)),$$

where f_d and f_e are the components of the cost vector $\vec{f}$.

A heuristic cost vector $\vec{h}$ is said to be admissible, if it satisfies

$$g_d(\lambda^*_{n_i}) + h_d(n_i) \leq g_d(\lambda^*_t),$$
$$g_e(\lambda^*_{n_i}) + h_e(n_i) \leq g_e(\lambda^*_t),$$

for all nondominated paths $\lambda_t^* = \langle s, n_1, n_2, \ldots, n_i, \ldots, t \rangle$ and each subpath $\lambda_{n_i}^* = \langle s, n_1, n_2, \ldots, n_i \rangle$. If $\vec{h}$ is admissible, NAMOA* guarantees to find all nondominated paths from s to t given that such paths exist, i.e. NAMOA* is admissible [10].

A heuristic cost vector is said to be monotone, if it satisfies

$$h_d(n) \leq c_d(n, n') + h_d(n'), \tag{4}$$

$$h_e(n) \leq c_e(n, n') + h_e(n'), \tag{5}$$

for all the neighboring nodes (n, n') in $\mathcal{G}$. All monotone heuristic vectors are admissible as well. If $\vec{h}$ is monotone, NAMOA* is proven to expand the least number of paths to find all nondominated paths from s to t in compared with other admissible multiobjective algorithms over a class of problems with monotone heuristics. One may refer to [10] for a detailed explanation of NAMOA* and its properties.

B. Heuristic Cost Estimation

In order to utilize NAMOA* for our problem, we need to formulate heuristic cost functions h_d and h_e which satisfy (4) and (5), respectively. We can simply define

$$h_d(n) = s(n, t),$$

which can easily be proven to satisfy (4) using the triangle inequality. In order to estimate the heuristic energy-cost, we adopt a heuristic cost function introduced in [4] as

$$h_e(n) = \begin{cases} \frac{mg\Delta(n,t)}{\sin\phi_m}(\mu \cos\phi_m + \sin\phi_m), & \\ & \text{if } \phi(n,t) > \phi_m, \\ mgs(n,t)(\mu \cos\phi\ (n,t) + \sin\phi(n,t)), & \\ & \text{if } \phi_m \geq \phi(n,t) > \phi_b, \\ 0, & \text{otherwise}, \end{cases}$$

which has already been proven to satisfy the conditions of monotonicity [7]. Now we have a monotone heuristic cost vector $\vec{h}$ which can be used with NAMOA* to find physically feasible nondominated paths on uneven terrains.

V. Results and Discussion

The proposed multiobjective path planner was evaluated using numerous computer simulations and results of one set of simulations are presented and analyzed in this section.

A. Simulation Parameters

The terrain model used in generating simulation results presented in this paper can be expressed as

$$z(x,y) = 3.79\Bigg[\sin\left(\frac{y}{3\pi} + \frac{1}{2}\right) + 1.3\cos\left(\frac{x}{3\pi}\right) - 2\sin\left(\frac{y}{3\pi}\right) - 0.3\sin\left(3\sqrt{\left(\frac{x}{2\pi}\right)^2 + \left(\frac{y}{2\pi}\right)^2}\right)\Bigg]^2. \tag{6}$$

The base of the terrain is defined as a 100×100 m^2 square shaped grid map with 100 grids on each side, i.e. there are 10,000 nodes in $\mathcal{G}$ that represents (6). The robot model used in the simulation assumes that $m = 22$ kg, $v = 0.35$ ms^{-1}, $P_{\max} = 72$ W. The rest of parameters are defined as $\mu = 0.01$, $\mu_s = 1.00$, and $g = 9.81$ ms^{-2}.

B. Simulation Results

The simulation results given in Fig. 1 illustrate physically feasible shortest and energy-optimal paths from (10,70) m to (90,45) m on the terrain given in (6). The shortest path was obtained by using the Dijkstra's algorithm [3] with the distance-cost function given in (1). The length of the shortest path is 98.47 m and according to the energy-cost function given in (2), the mobile robot consumes 3177.14 J to traverse it in 281.34 s. The energy-optimal path was obtained by using the Z* search algorithm [7]. The length of the energy-optimal path is 111.97 m and the mobile robot consumes only 240.07 J for the traversing which is completed in 319.91 s. The physically feasible nondominated paths between the same start and goal locations were obtained using the proposed multiobjective path planner and they are illustrated in Fig. 2. There are 76 nondominated paths in total. Their path length and corresponding energy consumption are given in Fig. 3.

C. Discussion

According to the results given in Fig. 1, it is obvious that the shortest path has traveled through both peaks and valleys consuming large amount of energy. In contrast, the energy-optimal path has traveled along valleys on the given terrain consuming considerably lower amount of energy. However, due to the increased path length of the energy-optimal path, the mobile robot takes longer time to complete the traversal. Now assume a scenario in which the mobile robot is powered by a battery with a capacity of 1500 J and it is required to reach the goal location within 300 s. In such a situation, the mobile robot cannot reach the goal location using the shortest path obtained because it does not have the enough energy for complete traversal. On the other hand, it cannot utilize the energy-optimal path either due to the time constraint, even though it meets energy requirements of the path.

According to the results given in Fig. 2, the proposed path planner results in multiple nondominated path options, and Fig. 3 illustrates that there are several such path options which satisfy both energy and time constraints given above. In fact, the mobile robot now has possibility to select a shortest path or an energy-optimal path which satisfies predefined constraints. Therefore, the proposed multiobjective path planner can be considered as a more versatile path planner compared to its counterparts. Furthermore, the availability of multiple path options increase the adaptability of mobile robots that are utilized in uncertain environments; if a robot discovers that it cannot traverse the initially selected path on the half way through due to a path blockage, it may still be possible to select another path to continue from the current location as the proposed path planner provides multiple possibilities.

The proposed multiobjective path planner is based on basic operations of path selections and expansions which are inherited to NAMOA*. On the other hand, its counterparts which are single objective, are based on node selections and expansions. Therefore, it is difficult to give a direct and fair comparison of the computational efficiency between the proposed path planner and its counterparts.

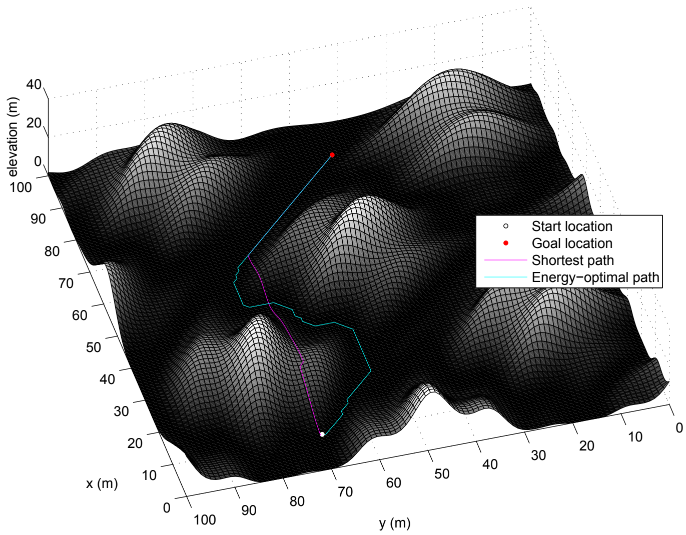

Fig. 1. Shortest and energy-optimal paths from (10,70) m to (90,45) m.

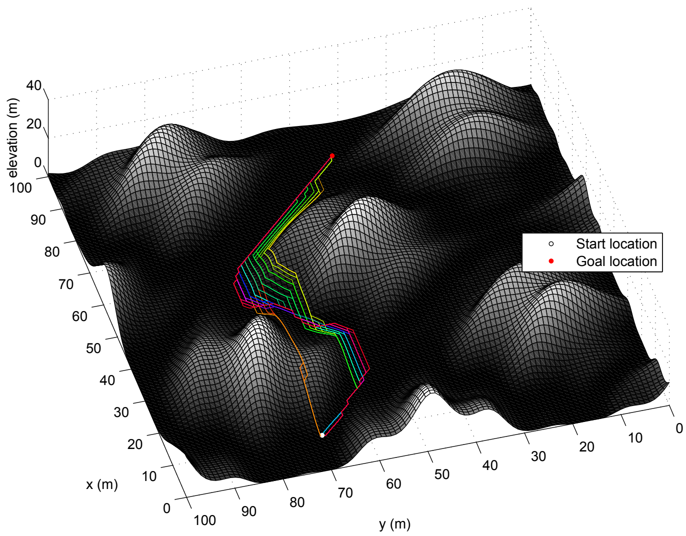

Fig. 2. All nondominated paths from (10,70) m to (90,45) m that are obtained using the proposed multiobjective path planner. Two of these nondominated paths coincide with the shortest and energy-optimal paths given in Fig. 1.

VI. Conclusion

This paper proposed an NAMOA* based multiobjective path planner for uneven terrain navigation of mobile robots. The proposed path planner is capable of finding all nondominated paths between two points. Such nondominated paths provide practical options for mobile robots when traversing a shortest or an energy-optimal path is not feasible. In addition, nondominated paths can be handy in dynamic and uncertain environments since they allow mobile robots to select another path without replanning if the current path is not realizable anymore. Further experiments need to be carried out using mobile robots to verify the applicability of the proposed path planner in real-world applications.

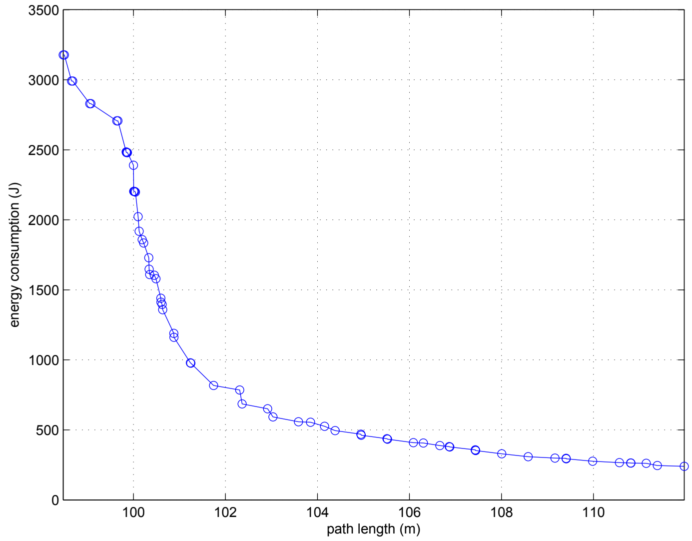

Fig. 3. A pareto frontier obtained using the proposed multiobjective path planner. It represents path lengths and energy consumptions of the nondominated paths given in Fig. 2.

Acknowledgment

This work is supported by the Department of Electronic and Information Engineering, the Hong Kong Polytechnic University (Project G-YBKH) and the Hong Kong PhD Fellowship Scheme (Project RTKL).

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