"I added the numbers, it's math!": how sense-making in "age of the captain" problems differs between a mathematics classroom and a language classroom

Molina, Natalia ; Strohmaier, Anselm R ; Reiss, Kristina M

"I added the numbers, it's math!" How sense-making in "age of the captain" problems differs between a mathematics classroom and a language classroom.

Natalia Molina, Anselm R. Strohmaier and Kristina M. Reiss

Technical University of Munich

Students' solution process of mathematical word problems depends on the situational context, e.g. the school subject (Dewolf, Van Dooren & Verschaffel, 2011). We analysed approaches to "age of the captain problems" (ACP; Verschaffel, Greer & De Corte, 2000) that present a situation that makes no sense, but are nonetheless frequently "solved" by a majority of primary school students. 48 primary school students (age M = 9.4, 54% female) in a mathematics or a language classroom were given five ACP. Afterwards, classroom interviews were conducted in both groups. Quantitative analyses show a non-significant tendency that students in the mathematics class were more likely to provide an arithmetic response to ACP. Interviews revealed that students in both groups experienced a cognitive dissonance regarding the expectation to provide an arithmetic solution, but differed in their approach to resolve it. This suggests that sense-making in ACP is influenced by the classroom context.

Sense-making; age of the captain problems; word problems; context influence

Introduction

Mathematical word problems are documented to exist in the mathematics curriculum for centuries and still play an important role in education nowadays, aiming to bridge mathematics to everyday life (Dewolf et al., 2011). However, when studying how pupils relate word problems to real life experiences, research has shown that the attempt of this connection often fails when students are confronted with word problems in the classroom (Verschaffel, et al., 2000, & Dewolf, et al., 2011). One explanation is that pupils act consequently to the classroom norms by solving word problems according to what they believe is expected from them, and not relying on their real world knowledge or the sense of the problem (Dewolf, et al., 2011).

Therefore, in order to study the processes and influencers of sense-making when solving word problems, the idea of nonsensical problems was introduced (Verschaffel, et al., 2000). Nonsensical mathematical word problems describe a situation in the form of a word problem providing numerical data, but the question posted is completely disconnected from the narrative or the numerical data provided. The idea was introduced from a study performed in 1980 by The Mathematical Research Institute in Grenoble, where researchers presented a nonsensical problem to a group of primary school children; the first example that unfolded this research and gave the name to this type of word problems "Age of the captain problems (ACP)" reads: there are 20 sheep and 6 goats on a ship, how old is the captain? (IREM, 1980). After presenting this word problem to students in the first and second grade, the majority of the participants performed a computational operation by adding the number of goats and sheep and submitting this numerical response as the captain's age (Radatz, 1983).

Following research showed that the percentage of students offering arithmetic answers to nonsensical word problems reaches a peak around third and fourth grade (Radatz, 1983). Moreover, a number of factors have been identified to influence sense-making, including word problem authenticity (Verschaffel et al., 2000), instructional practices when teaching word problems (Verschaffel, De Corte & Borghart, 1997), problem solvers' arithmetical skills (Carraher & Schliemann, 2002), and student arithmetical strategies when solving word problems (Saljo & Wyndhamn, 1993, & Verschaffel et al., 2000). These studies concur that students' expectations on how to solve ACPs largely depends on contextual factors.

Most studies on ACP focus on item or student characteristics, but very few studies have systematically manipulated the external context, e.g. the situation in which the students are asked to solve the problem (Dewolf, et al., 2011). The external context, for example the classroom situation, has shown to influence the process of word problem solving (Strohmaier & Reiss, 2018). Hence, based on this theoretical assumption it is presumed that the classroom context could substantially influence students' sense-making process of nonsensical word problems.

The present study

The present study investigated how primary school students proceed to solve nonsensical mathematical word problems in two different classroom contexts. It therefore aims at the influence of the external context on students' approach about sense making of word problems. A ten item test was developed, including five nonsensical word problems to be solved during the mathematics class as well as during the language class. The main research question under investigation was: would 4th grade students use or not use arithmetic calculations when solving nonsensical word problems in the mathematics and language classes?

Existing literature and theories around how students make sense of mathematical word problems and the influence of classroom contexts in mathematics education leads us to hypothesize that primary school students would proceed to solve nonsensical problems differently according to the classroom context. It was expected that students would solve nonsensical word problems by using arithmetic calculations during mathematics class (hypothesis 1) and that they would make less use of arithmetic calculations during the language class (hypothesis 2).

In addition, taking into consideration Bruner's (1986) theory on thinking approaches when making sense of mathematical word problems, we further investigated what type of sense making approaches occur when students reflect on nonsensical word problems in the mathematics class and the language class: paradigmatic, narrative, or both. It is expected that students in the mathematics class tend to use more a paradigmatic approach (hypothesis 3) and students in the language class a narrative approach (hypothesis 4).

The focus of the study relies on ACP in relation to classroom context; it thus intends to contribute to the field of mathematics education by uniquely relating these two variables and by exploring possible context effects of mathematical versus non-mathematical classrooms.

Methods

Data Collection

A cross-sectional survey was used to investigate the influence of the context on how pupils proceed to respond to nonsensical problems. Data were collected through a paper based test conducted at an international bilingual school in Berlin during the regular class schedule hours. Both lead teachers of each subject class were present during data collection as well as the researcher. Participants had twenty minutes to solve the problems. Before students were given the test, the researcher showed them each page and instructed them not to leave any of the questions blank. The lead teacher told the students that the test was not graded. When students finished answering the test in both classes, a group interview regarding students' perceptions and feelings about the items was conducted by the researcher.

Participants

48 primary school third- and fourth-graders (26 girls; age M = 9.35) from two different multi-grade classes (mathematics and language) participated in this study. Participants were bilingual. Both classes consisted of twenty four students. They were asked to solve the test during their regular mathematics and language class.

Development of the research instrument and analysis of data

The questionnaire for the present study was developed based on Verschaffel's et al. (2000) definition of nonsensical word problems and on the mathematics curriculum for third and fourth grade. Five regular mathematical word problems and five age of the captain problems similar to those presented by Verschaffel et al. (2000) were alternated in the test (mathematics class $\alpha = .63$ and language class $\alpha = .83$).

In addition, the test included a question regarding gender and age and a cover sheet for each of the two classes. In the mathematics class the cover page presented an illustration of equations and numbers followed by the title "mathematical test for students," while in the language class the cover page presented an illustration of puzzle pieces and the title "problem solving test for students." The decision of a cover page was made in order to be consistent with the class context and the type of test, based on research results stating that the wording and the presence of a cover page in mathematics tests can influence students' approach to it (Strohmaier et al., 2018).

Responses to nonsensical items were coded as 1 for arithmetic responses and 0 for non-arithmetic responses. The responses were coded consistently with the definition of arithmetical operations presented in the literature. Therefore, responses that did not represent the result of a mathematical calculation, as well as written narrative responses were considered "non-arithmetic responses."

An independent sample t-test was performed in order to compare the difference on the mean values from both classes. Moreover, qualitative analysis of the process of sense-making was done based on transcripts of the interviews and categorization of the responses and discussion points into two types of sense making approaches, paradigmatic knowing and narrative knowing (Bruner, 1986).

Results

Even though the number of arithmetic responses in the language class (2.33, SD = 1.85) was slightly lower than in the mathematics class (2.87, SD = 1.45), the difference was not significant $t(46) = 1.12$, $p = .266$.

For the qualitative analysis an excerpt of the group interview is given in table 1. In the mathematics class, a visible example of suspension of sense making is seen

in lines 17 through 19, where one student decided to write an answer using a numerical calculation; explaining that it is what is expected from her in a mathematics test, where she has to answer every question. Moreover, the use of the paradigmatic approach of students' thinking proposed by Bruner (1986) is represented in lines 26 through 32 when a student states that the answer of the nonsensical problem should be zero. In addition, a narrative approach (Bruner, 1986) in the mathematics class is present in lines 21 through 23, when a student realized the nature of the word problem being nonsensical while the problem was being read out loud.

Furthermore, in the language class suspension of sense making (Verschaffel, et al., 2000) was also exemplified by a student arguing that even though he had an understanding that the problem made no sense, he needed to make up numbers to answer the question. Additionally, in the language class, a narrative approach is represented in line 18 and lines 29 through 35 where three students acknowledge the context of the word problem and its nonsensical nature, and responded accordingly by crossing out the problem or writing a question mark. Moreover, one student makes a distinction on how to solve problems that make sense and nonsensical problems.

In the mathematics classroom, the paradigmatic approach was present in the statements of 4 out of 5 students who participated in this conversation, whereas the narrative approach was only present in the statements of 2 out of 5 students. In addition, in the language classroom the narrative approach was present in the statements of 4 out of 6 students who participated in the conversation; however, there is no representation of the paradigmatic approach.

Table 1.Sample of highlighted discussion statements with participants after taking the test

	Mathematics Class	**Language Class**
Dialogue	(L17) Student (8): I just wrote the answer. (L18) Researcher: with numbers? (L19) Student (8): Yeah, because you said is math test and we have to answer every question. (L20) Researcher: ok. Thank you, anybody else want to share their experience. What about this one (*researcher reads an item), what do you think. (L21) Student (7): oh god! I really wasn't thinking (L22) Researcher: How did you answer that? (L23) Student (7): yes, I added the numbers, its math! But when you read it to me then of course, I get it. (…) (L26) Student (9): if they each sing a song, then you have (writing on a paper the calculation) 173 songs. (L27) Student (7): No! Because it doesn't talk about singing songs, is just saying that they have friends. (L28) Student (9): yeah, but they could sing a song in the party (L29) Student (10): what was the answer? (L30) Student (7): I think it was cero. (L31) Researcher: why zero? (L32) Student (7): because it doesn't say about songs. (L35) Researcher: anybody had time to review their answers? (L36) Yes, I always check (…) I check that the numbers are correct (…) I check my numbers for mistakes	(L17) Researcher: So how did you answer? (L18) Student (2): I crossed out the question. I was first doing the math, but then I read it again and it didn't make sense, so I crossed out the question. (…) (L29) Student (5): well, I read the problem like a lot! They ask about pears, but there are no pears in the problem so you just don't answer it. (L30) Researcher: So you left it blank? (L31) Student (5): no, I mean I wrote a question mark, like why pears? (…) (L34) Researcher: oh, so you added the numbers but then put a question mark, when did you realized you wanted to put a question mark? (L35) Student (6): I was not sure how to answer, but I know my answer was wrong, so I put a question mark.

In addition, a sample of students' answers to three of the age of the captain problems provided in the test during the mathematics class as well as during the language class is represented in Table 2.

Table 2. Sample of students' responses to age of the captain problems

Mathematics Class	Language Class
Marko has 150 friends and Lukas has 23 friends. Marko and Lukas decide to give a party together. How many songs would they sing?	
On the party they would sing 173 songs.	If everyone sings one song then they would sing 173 songs.
They will sing 173 songs.	what the heck?
Giulia's father brought 170 slices of pizza to the table, Giulia's mother brought 113 slices of pizza to the table. How many cake slices would they eat?	
They would eat 283 slices.	? 1 because they would be to full
They would eat 0. They would eat 283 slices.	They eat 0 cake slices because they don't have any cake.
Melissa had 176 cupcakes, she gave 28 to her best friend. How old is the birthday girl?	
She is probably 148.	28 / Wich birthdaygirl?
The birthday girl is 28 years old.	How should I know?

Discussion

The findings of this study revealed that students in the mathematics class as well as in the language class use both arithmetical and non-arithmetical operations when solving nonsensical problems, and that there was no significant difference among the classes.

Furthermore, the narrative approach (Bruner, 1986) appear to be present during the conversation in both the mathematics and the language classes. A plausible explanation for this finding could be that in the context of an open conversation, students can scaffold their ideas through each other's opinions, making it easier to go deeper in their questioning and critical thinking about the nature and the context of the word problems that are being presented to them.

Moreover, in our study students seemed to refer to a paradigmatic approach (Bruner, 1986) more often in the mathematics class than in the language class, and to the narrative approach (Bruner, 1986) more often in the language class than in the mathematics class. Even though these are merely first descriptive results, they fit our expectations about approaches to sense-making in different classroom contexts (hypothesis 3 and 4).

During the discussion on the language class, participants mostly focused on questioning the meaning and the nature of the mathematical word problems: is it a tricky problem? Does the question make sense? Even more, students reported that throughout the test they were spending time trying to figure out how to respond to the problem. For example, one student stated that a mathematics test should be answered with numbers; however, she decided to post a question mark in addition to the result of her calculation to represent the nonsensical nature of the item; while another student reported that during the test she was debating on whether to respond by using an arithmetical calculation or not, but after re-reading the problem she decided to cross out the question. Taking these students' thinking processes examples into consideration, we can assume that they were being influenced by the classroom context, in which the rules encourage them to focus on the use of reading strategies (such as re-reading) above others.

On the other hand, during the conversation held in the mathematics classroom students were mostly looking for numerical explanations to represent their thinking about non-sensical problems. For instance, one discussion led pupils to conclude that since the word problem did not provide the information needed to answer the question, the answer should have been zero. In addition, students showed an awareness of the problems' nonsensical nature by stating that some of them were a "mistake," but they still decided to present a numerical calculation as an answer to the word problem. This type of thinking processes that students experience during mathematics class, could also be influenced by the rules of the classroom which calls pupils to give experimental proofs, in this case, a mathematical computation or a numerical representation. As one of the participants explained: "to solve the math with numbers."

Conclusion

The present study aimed at investigating how ACP are solved in different classroom contexts. While there was no significant difference regarding the number of arithmetic responses in the two classes, qualitative analyses provided insights into how students react differently to nonsensical word problems. This indicates that the external context can influence not only how students solve word problems, but also how they *not* solve them.

References

Bruner, J. (1986). *Actual minds, possible worlds*. Cambridge, MA: Harvard University Press.

Carraher, D.W., & Schliemann, A.D. (2002). Is everyday mathematics truly relevant to mathematics education? In M.E. Brenner, & J.N. Moschkovich (Eds.). *Every day and academic mathematics in the classroom* (pp. 131-153). Reston, VA: National Council of Teachers of Mathematics.

Dewolf, T., Van Dooren, W., &Verschaffel, L. (2011). Upper elementary school children's understanding and solution of a quantitative problem inside and outside the mathematics class. *Learning and Instruction, 21,* 770-780.

Institut de Recherche sur l'Enseignement des Mathematiques de Grenoble [IREM]. (1980). *Bulletin de l'Association des professeurs de Matématique de l' Enseignement Public, 323*, 235-243.

Radatz, H. (1983). Untersuchungen zum Lösen eingekleideter Aufgaben. *Journal für Mathematik-Didaktik 4*(3) 205-217.

Saljo, R., & Wyndhamn, J. (1993). Solving everyday problems in the formal setting: an empirical study of the school as context for thought. In S. Chaiklin, & J. Lave (Eds.), *Understanding practice. Perspectives on activity and context* (pp. 327-342). Cambridge: Cambridge University Press.

Strohmaier, A. R., & Reiss, K. M. (2018). Mathematics in disguise: Effects of the external context of mathematical word problems. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 251-258). Umeå, Sweden: PME.

Verschaffel, L., De Corte, E, & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modeling of school word problems. *Learning and Instruction, 4,* 339-359.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse, Netherlands: Sweits and Zeitlinger.