Pareto and Majority Voting in mCP-nets*

Thomas Lukasiewicz[1,2], Enrico Malizia[3]

[1]Department of Computer Science, University of Oxford, UK
[2]Institute of Logic and Computation, TU Wien, Austria
[3]DISI, University of Bologna, Italy

Abstract

Aggregating preferences over combinatorial domains has several applications in AI. Due to the exponential nature of combinatorial preferences, compact representations are needed, and CP-nets are among the most studied formalisms. mCP-nets are an intuitive formalism based on CP-nets to reason about preferences of groups of agents. The dominance semantics of mCP-nets is based on voting, and different voting schemes give rise to different dominance semantics for the group. Unlike CP-nets, which received an extensive complexity analysis, mCP-nets, as reported multiple times in the literature, lacked such a thorough characterization. In this paper, we start to fill this gap by carrying out a precise computational complexity analysis of Pareto and majority voting on acyclic binary polynomially-connected mCP-nets.

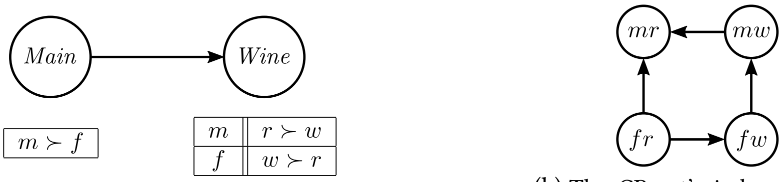
Keywords

Combinatorial preferences, Preference aggregation, CP-nets, Majority voting, Computational complexity

1. Introduction

Managing and aggregating agent preferences have attracted extensive interest for their importance in AI applications, such as recommender systems [2], product configuration [3], planning [4], preference-based constraint satisfaction [5], preference-based query answering/information retrieval [6, 7, 8], and preference-based argumentations [9]. In computer science, preference aggregation has often been based on social choice theory [10]. In this theory, agents' preferences are usually extensively represented. Although this is reasonable with a small set of candidates, this is not feasible with combinatorial domains, i.e., when the set of candidates, or outcomes, is the Cartesian product of finite-value domains for each of a set of features [11, 12]. Combinatorial domains contain an exponential number of outcomes in the number of features, and hence compact representations are needed [11, 12]. CP-nets [13] are among the most studied of these representations, and have also been used in applications even in learning scenarios [14, 15]. In CP-nets, vertices of a graph represent features, and an edge from vertex A to vertex B models the influence of A's value on the choice of B's value. This model captures *conditional ceteris paribus preferences* like "if the rest of the dinner is the same, with a fish dish (A's value), I prefer a white wine (B's value)".

Discussion Papers - 21st International Conference of the Italian Association for Artificial Intelligence (AIxIA 2022)
*This is an abridged version of the AIJ paper [1].
thomas.lukasiewicz@cs.ox.ac.uk (T. Lukasiewicz); enrico.malizia@unibo.it (E. Malizia)
0000-0002-7644-1668 (T. Lukasiewicz); 0000-0002-6780-4711 (E. Malizia)



(a) A CP-net modeling dinner preferences.

(b) The CP-net's induced preference graph.

Figure 1: A CP-net and its preference graph.

Example 1. Assume that we want to model one's preferences for a dinner with a main dish and a wine [13]. In the CP-net in Figure 1a, an edge from vertex $Main$ to vertex $Wine$ models that the value of feature $Main$ influences the choice of the value of feature $Wine$: m and f are the possible values of feature $Main$, denoting "$m$eat" and "$f$ish", respectively, while r and w are the possible values of feature $Wine$, denoting "$r$ed (wine)" and "$w$hite (wine)", respectively. The table associated with feature $Wine$ specifies that when a meat dish is chosen, then a red wine is preferred to a white one, and when a fish main is chosen, then a white wine is preferred to a red wine. The table associated with feature $Main$ indicates that a meat dish is preferred to a fish one. These tables are called *CP tables.* A CP-net like this one can represent the *conditional ceteris paribus preference* "given that the rest of the dinner does not change, with a meat dish ($Main$'s value), I prefer a red wine ($Wine$'s value)".

Every CP-net has an associated *induced preference graph* [13], whose vertices are all the possible outcomes of the domain, and whose edges connect outcomes differing on only one value: there is a directed edge from an outcome to another if the latter is preferred to the former according to the preferences encoded in the CP tables of the CP-net. Figure 1b shows the induced preference graph of the CP-net in Figure 1a, having as vertices all the possible combinations for the dinner, and there is, e.g., an edge from mw to mr, because the combination meat and red wine is preferred to the combination meat and white wine. The preferences encoded in a CP-net are the transitive closure of its induced preference graph. Intuitively, an outcome α is preferred to an outcome β according to the preferences of a CP-net, if there is a directed path from β to α in the induced preference graph, in which case α is said to *dominate* β. $\lhd$

CP-nets were also used to model preferences of *groups*, obtaining the multi-agent model *mCP-nets* [16], which is a profile of CP-nets, one for each agent. The preference semantics of mCP-nets is defined via voting schemes: through its own individual CP-net, every agent votes whether an outcome is preferred to another. Various voting schemes were proposed for mCP-nets [16, 17], and different voting schemes give rise to different dominance semantics for mCP-nets. In this paper, we consider *Pareto* and *majority voting* as they were defined in [16].

Example 2. Consider again the dinner scenario of Example 1, and assume that there are three agents (Alice, Bob, and Chuck), expressing their preferences via CP-nets (see Figure 2). In Pareto voting, an outcome α dominates an outcome β, if *all* agents prefers α to β. In majority voting, an outcome α dominates an outcome β, if the *majority* of agents prefers α to β.

The outcome *fr* is not *Pareto optimal*, because *fr* is Pareto dominated by mr, i.e., the outcome mr is preferred to *fr* by *all* agents. The outcome mw is instead Pareto optimal, as there is no

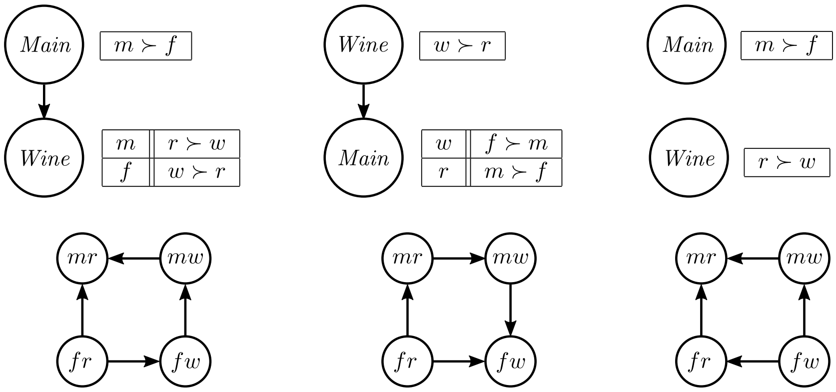


Figure 2: Dinner preferences of Alice, Bob, and Chuck (in this order) modeled via CP-nets (above) and their induced preference graphs (below).

outcome Pareto dominating mw. Hence, from a Pareto perspective, mw is better than fr. The outcome mw, however, is not *majority optimal*, because mr majority dominates mw (Alice and Chuck prefer mr to mw). Conversely, mr is majority optimal, as there is no outcome majority dominating mr. Hence, from a majority perspective, mr is better than mw. Moreover, again according to the majority perspective, mr is a very good outcome, because mr is also *majority optimum*, i.e., mr majority dominates all other outcomes. On the contrary, in this example, there is no *Pareto optimum* outcome, i.e., there is no outcome Pareto dominating all the others.◁

Unlike CP-nets, which were extensively analyzed, a precise complexity analysis of the group dominance semantics in mCP-nets was missing for long time, as explicitly mentioned several times in the literature [18, 19, 20, 17]. Our aim in this paper is to settle these problems in their exact complexity classes, showing, if possible, completeness results.

In particular, we focus on acyclic binary polynomially-connected mCP-nets (see Section 2 for these notions) built with standard CP-nets. Unlike what is often assumed in the literature, in this work, we do not restrict the profiles of CP-nets to be $\mathcal{O}$-legal, meaning that we do not require that there is a topological ordering common to all the CP-nets of the profile. We carry out a thorough complexity analysis for the (a) Pareto and (b) majority voting schemes, as defined in [16], of deciding (1) dominance, (2) optimal and (3) optimum outcomes, and (4) the existence of optimal and (5) optimum outcomes. Deciding the dominance for a voting scheme s means deciding, given two outcomes, whether one dominates the other according to s. Deciding whether an outcome is optimal or optimum for a voting scheme s means deciding whether the outcome is not dominated or dominates all others, respectively, according to s. Deciding the existence of optimal and optimum outcomes is the natural extension of the previous problems. A summary of the complexity results obtained is provided in Table 1.

We obtain many intractability results, where the problems are put at various levels of the Polynomial Hierarchy. Although intractability is usually "bad" news, these results are quite interesting, as for most of these tasks only EXP upper bounds were known in the literature to date [16]. Even more interestingly, some of these problems are actually tractable, as they are in P or even LOGSPACE, which is a huge leap from EXP. The complexity results here shown can be extended to wider notions of "compact representations" of preference profiles [1, 21].

Table 1
Summary of the complexity results. *A different proof is provided in [16].

	Problem	Complexity
Pareto	Pareto-Dominance	NP-complete
	Is-Pareto-Optimal	co-NP-complete
	Exists-Pareto-Optimal	$\Theta(1)^*$
	Is-Pareto-Optimum	in LOGSPACE
	Exists-Pareto-Optimum	in P
Majority	Majority-Dominance	NP-complete
	Is-Majority-Optimal	co-NP-complete
	Exists-Majority-Optimal	Σ_2^p-complete
	Is-Majority-Optimum	Π_2^p-complete
	Exists-Majority-Optimum	Π_2^p-hard, in D_2^p

In Section 2, we provide definitions of the framework. In Section 3 and in Section 4 we discuss our results for Pareto and majority voting, respectively. Full proof details are available in [1].

2. Preliminaries

CP-nets. A CP-net N is a triple $\langle \mathcal{G}_N, Dom_N, (CPT_N^F)_{F \in \mathcal{F}_N} \rangle$, where $\mathcal{G}_N = \langle \mathcal{F}_N, \mathcal{E}_N \rangle$ is a directed graph whose vertices $\mathcal{F}_N$ represent the features of the combinatorial domain, Dom_N is a function, and $(CPT_N^F)_{F \in \mathcal{F}_N}$ is a family of functions. For a feature F, Dom_N associates a (value) domain $Dom_N(F)$ with F, while CPT_N^F is the so called "CP table" of F.

Feature F's domain is the set of values that F may have in the outcomes. Here, we assume features to be *binary*, i.e., each feature's domain contains two values. We denote by $\overline{f}$ and f the two values of F. For a feature set $\mathcal{S} \subseteq \mathcal{F}_N$, $Dom_N(\mathcal{S}) = \times_{F \in \mathcal{S}} Dom_N(F)$. An outcome is an element of the set $\mathcal{O}_N = Dom_N(\mathcal{F}_N)$. For a feature $F \in \mathcal{F}_N$ and an outcome α, $\alpha[F]$ is F's value in α. For a feature set $\mathcal{S} \subseteq \mathcal{F}_N$ and an outcome α, $\alpha[\mathcal{S}]$ is the projection of α over $\mathcal{S}$.

CP tables encode preferences over feature values. The CP table of feature F has a row for *any* possible combination of values of *all* the parent features of F in $\mathcal{G}_N$; in each row there is a *total* order over $Dom_N(F)$. This order encodes agent's preferences for F's values when specific values of F's parents are considered: $\overline{f} \succ f$ denotes $\overline{f}$ being preferred to f. If F has no parents, its CP table has only one row with a total order over $Dom_N(F)$. We denote by $\|N\|$ the size of CP-net N, i.e., the space in terms of bits required to represent the whole net N (including, features, links, feature domains, and CP tables).

CP-nets' preference semantics is based on "improving flips". Let F be a feature, and let α, β be two outcomes differing *only* on F's value. Flipping F from $\alpha[F]$ to $\beta[F]$ is an *improving flip* iff, in the row of F's CP table associated with the values in α of the parents of F, $\beta[F] \succ \alpha[F]$. Outcome β is preferred to α, or β *dominates* α (in N), denoted $\beta \succ_N \alpha$, iff there is a sequence of improving flips from α to β, otherwise β does not dominate α, denoted $\beta \not\succ_N \alpha$; β and α are *incomparable*, denoted $\beta \bowtie_N \alpha$, iff $\beta \not\succ_N \alpha$ and $\alpha \not\succ_N \beta$.

A CP-net N is *binary* iff all its features are binary; N is *singly connected* iff, for any two features G and F of N, there is at most one path from G to F in $\mathcal{G}_N$. A class $\mathscr{F}$ of CP-nets is *polynomially-connected* iff there exists a polynomial p such that, for any CP-net $N \in \mathscr{F}$ and for any two features G and F of N, there are at most $p(\|N\|)$ distinct paths from G to F in $\mathcal{G}_N$. A CP-net N is *acyclic* iff $\mathcal{G}_N$ is acyclic. Acyclic CP-nets N have a unique optimum outcome o_N, dominating all others, that can be computed in polynomial time [13]. Unless stated otherwise, we consider acyclic binary CP-nets.

***m*CP-nets.** An mCP-net is a tuple of m CP-nets defined over the same set of features having, in turn, the same domain. The "m" of an mCP-net is the agents' number, so a 3CP-net is an mCP-net with $m = 3$. Originally, *partial* CP-nets were allowed to constitute mCP-nets [16]. Here, we assume only standard CP-nets in mCP-nets, and CP-nets do not need to be $\mathcal{O}$-legal (i.e., we de not assume that the CP-nets of an mCP-net have a common topological ordering). mCP-nets' semantics is based on voting. Let $\mathcal{M} = \langle N_1, \dots, N_m \rangle$ be an mCP-net, and let α, β be two outcomes. The sets $S^{\succ}_{\mathcal{M}}(\alpha, \beta) = \{i \mid \alpha \succ_{N_i} \beta\}$, $S^{\prec}_{\mathcal{M}}(\alpha, \beta) = \{i \mid \alpha \prec_{N_i} \beta\}$, and $S^{\bowtie}_{\mathcal{M}}(\alpha, \beta) = \{i \mid \alpha \bowtie_{N_i} \beta\}$, are the sets of agents preferring α to β, preferring β to α, and for which α and β are incomparable, respectively. The dominance semantics considered are [16]:

Pareto: β *Pareto dominates* α, denoted by $\beta \succ^{p}_{\mathcal{M}} \alpha$, iff *all* the agents of $\mathcal{M}$ prefer β to α, i.e., $|S^{\succ}_{\mathcal{M}}(\beta, \alpha)| = m$.

Majority: β *majority dominates* α, denoted by $\beta \succ^{maj}_{\mathcal{M}} \alpha$, iff the *majority* of the agents of $\mathcal{M}$ prefers β to α, i.e., $|S^{\succ}_{\mathcal{M}}(\beta, \alpha)| > |S^{\prec}_{\mathcal{M}}(\beta, \alpha)| + |S^{\bowtie}_{\mathcal{M}}(\beta, \alpha)|$.

For a voting scheme s, an outcome α is s optimal in $\mathcal{M}$ iff $\beta \not\succ^{s}_{\mathcal{M}} \alpha$ for all $\beta \neq \alpha$, whereas α is s optimum in $\mathcal{M}$ iff $\alpha \succ^{s}_{\mathcal{M}} \beta$ for all $\beta \neq \alpha$. Optimum outcomes, if they exist, are unique.

An mCP-net is acyclic, binary, and singly connected, iff all its CP-nets are acyclic, binary, and singly connected, respectively. A class $\mathscr{F}$ of mCP-nets is polynomially-connected iff the set of CP-nets constituting the mCP-nets in $\mathscr{F}$ is polynomially-connected. Unless stated otherwise, mCP-nets here considered are acyclic, binary, and belong to polynomially-connected classes.

3. Complexity of Pareto voting on *m*CP-nets

We now focus on Pareto voting. Being based on the concept of unanimity, an NP witness for an outcome Pareto dominating another outcome is the set of the witnesses of all agents preferring one to the other (for the class of (m)CP-nets here considered, dominance check is known to be NP-complete [13]). For the hardness, on 1CP-nets, $\succ^{p}$ and $\succ$ are equivalent.

Theorem 3. *Let $\mathcal{M}$ be an mCP-net, and let α, β be two outcomes. Deciding whether $\beta \succ^{p}_{\mathcal{M}} \alpha$ is* NP-complete. *Hardness holds even on 1CP-nets.*

Consider the problem of deciding whether a given outcome α is Pareto optimal for a given mCP-net $\mathcal{M}$. We can *dis*prove α being Pareto optimal by guessing an outcome β along with the witness that $\beta \succ^{p}_{\mathcal{M}} \alpha$, and checking the witness (in NP, see Theorem 3). For the hardness, it is possible to provide a reduction from unsatisfiability of 3CNF Boolean formulas.

Theorem 4. *Let $\mathcal{M}$ be an mCP-net, and let α be an outcome. Deciding whether α is Pareto optimal in $\mathcal{M}$ is* co-NP-complete. *Hardness holds even on 2CP-nets.*

Deciding whether an mCP-net has a Pareto optimal outcome is trivial, because there is always one. Indeed, if this were not the case, then the optimal outcomes of the single CP-nets would be dominated by some other outcome, which would be a contradiction. An mCP-net has a Pareto optimum outcome iff all its CP-nets have the very same individual optimal outcome (that is also Pareto optimum). Indeed, to be Pareto optimum, an outcome has to dominate all other outcomes in all the CP-nets of the mCP-net: this property is satisfied only by an outcome that is the optimum in all the CP-nets of the mCP-net. By combining this with the fact that checking the optimality of an outcome in a CP-net is in LOGSPACE [1], we can state the following.

Theorem 5. *Let $\mathcal{M}$ be an mCP-net. Deciding whether an outcome α is Pareto optimum in $\mathcal{M}$ is in* LOGSPACE*; deciding whether $\mathcal{M}$ has a Pareto optimum outcome is in* P.

4. Complexity of majority voting on mCP-nets

In this section, we deal with majority voting. It is possible to design four different acyclic binary singly-connected CP-nets with the following dominance relationships: $\bar{a}\bar{b} \succ_{N_1} \bar{a}b \succ_{N_1} ab \succ_{N_1} a\bar{b}$; $\bar{a}b \succ_{N_2} ab \succ_{N_2} a\bar{b} \succ_{N_2} \bar{a}\bar{b}$; $ab \succ_{N_3} a\bar{b} \succ_{N_3} \bar{a}\bar{b} \succ_{N_3} \bar{a}b$; and $a\bar{b} \succ_{N_4} \bar{a}\bar{b} \succ_{N_4} \bar{a}b \succ_{N_4} ab$. Interestingly, the 4CP-net constituted by the four above nets do not have majority optimal and optimum outcomes. Therefore, we can state the following.

Theorem 6. *There are acyclic binary singly-connected mCP-nets not having majority optimal and optimum outcomes.*

Let us now focus on majority dominance. Observe that $\beta \succ^{maj}_{\mathcal{M}} \alpha$ iff $|S^{\succ}_{\mathcal{M}}(\beta, \alpha)| > \frac{m}{2}$. Hence, a certificate consists in the witnesses of at least $\frac{m}{2}$ agents preferring β to α (which is in NP; see above). On the hardness side, on 1CP-nets, $\succ^{maj}$ equals $\succ$ (which is NP-hard; see above).

Theorem 7. *Let $\mathcal{M}$ be an mCP-net, and let α, β be two outcomes. Deciding whether $\beta \succ^{maj}_{\mathcal{M}} \alpha$ is* NP-complete. *Hardness holds even on 1CP-nets.*

For the majority optimal problem, to test that an outcome α is *not* majority optimal, we guess an outcome β and the witness of $\beta \succ^{maj}_{\mathcal{M}} \alpha$ (in NP, see Theorem 7). For the hardness, on 2CP-nets, $\succ^{maj}$ and $\succ^{p}$ are equivalent.

Theorem 8. *Let $\mathcal{M}$ be an mCP-net. Deciding whether an outcome α is a majority optimal in $\mathcal{M}$ is* co-NP-complete. *Hardness holds even on 2CP-nets.*

We now focus on deciding the existence of majority optimal outcomes. We can test in Σ^{P}_2 that $\mathcal{M}$ has a majority optimal outcome by guessing an outcome α (in NP), and checking that α is majority optimal (in co-NP, see Theorem 8). To prove the respective hardness, an involved construction showing a reduction from QBF is required.

Theorem 9. *Let $\mathcal{M}$ be an mCP-net. Deciding whether $\mathcal{M}$ has a majority optimal outcome is* Σ^{P}_2-complete. *Hardness holds even on 6CP-nets.*

Consider the problem of deciding whether an outcome is majority optimum. We can test in Σ^{P}_2 that α is *not* majority optimum by guessing an outcome β (in NP) and checking that $\alpha \not\succ^{maj}_{\mathcal{M}} \beta$ (in co-NP, see Theorem 7). The hardness proof uses similar ideas to those in Theorem 9.

Theorem 10. *Let $\mathcal{M}$ be an mCP-net. Deciding whether an outcome α is majority optimum in $\mathcal{M}$ is* Π_2^{P}-complete. *Hardness holds even on* 3*CP-nets.*

We now deal with the problem of deciding the existence for an mCP-net of a majority optimum outcome. First, observe that an outcome, to be majority optimum, must also be majority optimal. Notice that, when the majority optimum exists, then it is the only majority optimal outcome. For this reason, an mCP-net $\mathcal{M}$ has a majority optimum outcome if and only if $\mathcal{M}$ has majority optimal outcomes and it is not true that $\mathcal{M}$ has majority optimal outcomes that are not also majority optimum. Hence, two tasks need to be solved to decide whether an mCP-net $\mathcal{M}$ has a majority optimum outcome: (1) decide whether $\mathcal{M}$ has majority optimal outcomes; and (2) decided whether $\mathcal{M}$ does *not* have majority optimal outcomes that are not also optimum. We already know that the complexity of task (1) is in Σ_2^{P} (see Theorem 9). The complexity of task (2) can be shown in Π_2^{P}. Indeed, solving the complement of task (2), i.e., deciding whether $\mathcal{M}$ has majority optimal outcomes that are not also optimum, can be shown in Σ_2^{P}. First, we guess two different outcomes α and β (feasible in NP). Then, through a co-NP oracle call, we check that α is actually majority optimal (see Theorem 4), then through another oracle call in co-NP, we check that $\alpha \not\succ_M^{maj} \beta$ (see Theorem 7).

By combining the complexity of these two tasks follows that deciding whether an mCP-net has a majority optimum outcome is in $\mathrm{D}_2^{\mathrm{P}}$. The Π_2^{P}-hardness of the problem can be shown by inspection of the reduction for the hardness in Theorem 10.

Theorem 11. *Let $\mathcal{M}$ be an mCP-net. Deciding whether $\mathcal{M}$ has a majority optimum outcome is in $\mathrm{D}_2^{\mathrm{P}}$ and is* Π_2^{P}-hard. *Hardness holds even on* 3*CP-nets.*

5. Summary and outlook

We have carried out a thorough complexity analysis of the Pareto and majority semantics in mCP-nets. The various problems analyzed here have been put at various levels of the Polynomial Hierarchy, and some of them are even tractable (in P or LOGSPACE). This work was extended to consider also rank voting and relative majority voting [22, 23, 24], where to analyse the complexity of the latter a novel characterization of the complexity class Θ_2^{P} was needed [25]. The semantics intractability of (m)CP-nets has encouraged research to individuate tractable approximate algorithms for CP-nets dominance [26].

There are various possible directions for further research. Having constraints on the feasibility of outcomes is an interesting direction of investigation. Without any constraint, CP-nets model agents' preferences when it is assumed that all outcomes are attainable. However, this is not always the case. For example, to decide whether an outcome is majority dominated by another, we should check that the latter is feasible. A similar idea characterized the solution concepts in NTU cooperative games defined via constraints [27, 28]. This approach could be merged with the definition of constrained CP-nets [5, 29]. CP-nets offer a rather flexible formalism to express preferences, and in fact it would be interesting to extend the preference model adopted in the AI explanation framework [8] to something capable of leveraging the representational power of CP-nets, and extend this to other explanation frameworks [30, 31, 32]. The model of CP-nets could also be extended to hypergraphs [33, 34] by following the intuitions provided in [35], where ceteribus paribus preferences are put in relationship with hypergraphs transversals.

Acknowledgments

This work was supported by UK EPSRC grants EP/J008346/1, EP/L012138/1, and EP/M025268/1, the Alan Turing Institute under the EPSRC grant EP/N510129/1, and the AXA Research Fund.

References

[1] T. Lukasiewicz, E. Malizia, Complexity results for preference aggregation over (m)CP-nets: Pareto and majority voting, Artif. Intell. 272 (2019) 101–142.
[2] F. Ricci, L. Rokach, B. Shapira (Eds.), Recommender Systems Handbook, 2nd ed., Springer, New York, NY, USA, 2015.
[3] R. I. Brafman, C. Domshlak, TCP-nets for preference-based product configuration, in: Proc. of the ECAI 2002 Workshop on Configuration, 2002, pp. 101–106.
[4] R. I. Brafman, Y. Chernyavsky, Planning with goal preferences and constraints, in: Proc. of ICAPS, 2005, pp. 182–191.
[5] C. Boutilier, R. I. Brafman, C. Domshlak, H. H. Hoos, D. Poole, Preference-based constrained optimization with CP-nets, Comput. Intell. 20 (2004) 137–157.
[6] T. Di Noia, T. Lukasiewicz, M. V. Martinez, G. I. Simari, O. Tifrea-Marciuska, Combining existential rules with the power of CP-theories, in: Proc. of IJCAI, 2015, pp. 2918–2925.
[7] M. Calautti, S. Greco, C. Molinaro, I. Trubitsyna, Preference-based inconsistency-tolerant query answering under existential rules, Artif. Intell. 312 (2022) art. no. 103772.
[8] İ. İ. Ceylan, T. Lukasiewicz, E. Malizia, C. Molinaro, A. Vaicenavičius, Preferred explanations for ontology-mediated queries under existential rules, in: Proc. of AAAI, 2021, pp. 6262–6270.
[9] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, On preferences and priority rules in abstract argumentation, in: Proc. of IJCAI, 2022, pp. 2517–2524.
[10] F. Brandt, V. Conitzer, U. Endriss, J. Lang, A. D. Procaccia (Eds.), Handbook of Computational Social Choice, Cambridge University Press, New York, NY, USA, 2016.
[11] J. Lang, Logical preference representation and combinatorial vote, Ann. Math Artif. Intel. 42 (2004) 37–71.
[12] J. Lang, L. Xia, Voting in combinatorial domains, in: F. Brandt, V. Conitzer, U. Endriss, J. Lang, A. D. Procaccia (Eds.), Handbook of Computational Social Choice, Cambridge University Press, New York, NY, USA, 2016, pp. 197–222.
[13] C. Boutilier, R. I. Brafman, C. Domshlak, H. H. Hoos, D. Poole, CP-nets: A tool for representing and reasoning with conditional ceteris paribus preference statements, J. Artif. Intell. Res. 21 (2004) 135–191.
[14] J. T. Guerin, T. E. Allen, J. Goldsmith, Learning CP-net preferences online from user queries, in: Proc. of ADT, 2013, pp. 208–220.
[15] T. E. Allen, C. Siler, J. Goldsmith, Learning tree-structured CP-nets with local search, in: Proc. of FLAIRS, 2017, pp. 8–13.
[16] F. Rossi, K. B. Venable, T. Walsh, mCP Nets: Representing and reasoning with preferences of multiple agents, in: Proc. of AAAI, 2004, pp. 729–734.

[17] M. Li, Q. B. Vo, R. Kowalczyk, Aggregating multi-valued CP-nets: A CSP-based approach, J. Heuristics 21 (2015) 107–140.
[18] J. Lang, Vote and aggregation in combinatorial domains with structured preferences, in: Proc. of IJCAI, 2007, pp. 1366–1371.
[19] M. Li, Q. B. Vo, R. Kowalczyk, An effcient majority-rule-based approach for collective decision making with CP-nets, in: Proc. of KR, 2010, pp. 578–580.
[20] M. Li, Q. B. Vo, R. Kowalczyk, An efficient procedure for collective decision-making with CP-nets, in: Proc. of ECAI, 2010, pp. 375–380.
[21] G. Greco, E. Malizia, L. Palopoli, F. Scarcello, The complexity of the nucleolus in compact games, ACM Trans. Comput. Theory 7 (2014) 3:1–3:52.
[22] T. Lukasiewicz, E. Malizia, On the complexity of mCP-nets, in: Proc. of AAAI, 2016, pp. 558–564.
[23] E. Malizia, More complexity results about reasoning over (m)CP-nets, in: Proc. of AAMAS, 2018, pp. 1540–1548.
[24] T. Lukasiewicz, E. Malizia, Complexity results for preference aggregation over (m)CP-nets: Max and rank voting, Artif. Intell. 303 (2022) art. no. 103636.
[25] T. Lukasiewicz, E. Malizia, A novel characterization of the complexity class Θ_k^P based on counting and comparison, Theor. Comput. Sci. 694 (2017) 21–33.
[26] T. E. Allen, J. Goldsmith, H. E. Justice, N. Mattei, K. Raines, Uniform random generation and dominance testing for CP-nets, J. Artif. Intell. Res. 59 (2017) 771–813.
[27] E. Malizia, L. Palopoli, F. Scarcello, Infeasibility certificates and the complexity of the core in coalitional games, in: Proc. of IJCAI, 2007, pp. 1402–1407.
[28] G. Greco, E. Malizia, L. Palopoli, F. Scarcello, Non-transferable utility coalitional games via mixed-integer linear constraints, J. Artif. Intell. Res. 38 (2010) 633–685.
[29] S. D. Prestwich, F. Rossi, K. B. Venable, T. Walsh, Constraint-based preferential optimization, in: Proc. of AAAI, 2005, pp. 461–466.
[30] İ. İ. Ceylan, T. Lukasiewicz, E. Malizia, A. Vaicenavičius, Explanations for ontology-mediated query answering in description logics, in: Proc. of ECAI, 2020, pp. 672–679.
[31] T. Lukasiewicz, E. Malizia, C. Molinaro, Explanations for inconsistency-tolerant query answering under existential rules, in: Proc. of AAAI, 2020, pp. 2909–2916.
[32] T. Lukasiewicz, E. Malizia, C. Molinaro, Explanations for negative query answers under inconsistency-tolerant semantics, in: Proc. of IJCAI, 2022, pp. 2705–2711.
[33] G. Gottlob, E. Malizia, Achieving new upper bounds for the hypergraph duality problem through logic, in: Proc. of LICS, 2014, pp. 43:1–43:10.
[34] G. Gottlob, E. Malizia, Achieving new upper bounds for the hypergraph duality problem through logic, SIAM J. Comput. 47 (2018) 456–492.
[35] S. Obiedkov, Parameterized ceteris paribus preferences over atomic conjunctions under conservative semantics, Theor. Comput. Sci. 658 (2017) 375–390.