The role of asymmetric innovation's sizes in technology licensing under partial vertical integration

Mariola Sánchez* Adrián Nerja†

Abstract

In this paper, we compare the scenarios of exclusive licenses and cross-licenses under the existence of partial vertical integration. To do this, a succesive duopoly model is proposed, with two owners and two firms competing in a differentiated product market. Each technology owner has a share in one of the competing firms, so that competition is also extended to the upstream R&D sector. We propose a novel analysis where differences in the sizes of their process in innovations are allowed, extending the results in Sánchez et al. (2021). We find that the cross-licensing scenario is preferred when the size of the innovation is small; this occurs regardless of the participations in the competing companies and how many innovate. If the innovation is very large, the owners may be better off with exclusive licenses.

Keywords: Patent Licensing, Exclusive licenses, market for technology; asymmetric innovation

JEL Classification: L24, D43, D45.

*Corresponding author: Mariola Sánchez Romero, Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: m.sanchezr@umh.es.

†Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: adrian.nerja@goumh.umh.es

This paper is the Accepted Version of a published paper in the journal Research in Economics with DOI: https://doi.org/10.1016/j.rie.2024.100958
Please, cite as: Sánchez, M., & Nerja, A. (2024). The role of asymmetric innovation's sizes in technology licensing under partial vertical integration. Research in Economics, 100958.

1 Introduction

Technology licenses play a crucial role, from an economical and entrepreneurial point of view: i) for licensee firms, they facilitate collective innovation and ii) for licensor firms, they can provide a low-risk way to leverage intellectual property assets, providing them with a framework that complements and enhance their business goals.[1,2]

According to Mendi et al. (2011), there is a co-existence of different patterns in the transferences of technology in the market. Indeed, most technology transfers are between firms of the same multinational, parent-firm and subsidiaries (affiliated firms). Table 1 shows data from technology transfers in the USA available in the Bureau of Economic Analysis where distinctions based on the nature of the transaction are specified. In concrete, data distinguish transactions between affiliated firms and between non-affiliated firms.

[1] A recent report highlights that the global sales revenue generated by licensed merchandise and services grew to $292.8 billion in 2019, a 4.5 percent increase over the $280.3 billion generated in 2018, a fact that underlines the importance of a growing business. Retrieved October 22, 2020, from Global Licensing survey, available on https://licensinginternational.org/get-survey/

[2] For example, Apple Inc. complements its technical know-how by acquiring core technology from firms like Qualcomm Inc. and Samsung to create its attractive high-performance devices (Hamdan-Livramento, 2012).

Table 1: Technology transfers between affiliated and non-affiliated firms in the USA

		1999	2009	2019
Incomes	Total millions of Dollars	39,913	85,730	117,401
	Affiliated %	71.53	66.80	67.75
	Unaffliated %	28.47	33.20	32.25
Payments	Total millions of Dollars	12,845	29,421	42,733
	Affiliated %	80.21	73.48	73.00
	Unaffliated %	19.79	26.52	27.00
Income-Payment Ratio	Total	3.11	2.92	2.75
	Affiliated	2.77	2.65	2.55
	Unaffliated	4.47	3.64	3.29

Source: Bureau of Economic Analysis.

From Table 1, given the importance of technology transfer between affiliated firms, it seems natural to study the characteristics of such transactions, as well as their implications for the level of competition in the industry. As can be seen, most income and payments come from affiliated firms, that is, vertical connected firms.

Our aim in this paper is to study the strategic decision of patent holders or technology owners about how many competing firms to license in the context of partial vertical integration. In concrete, we consider that technology owners are stakeholders of their clients, so that competition is extended to the upstream R&D sector. We propose a successive duopoly model, with two technology owners and two firms competing in a differentiated product market. Furthermore, we assume that innovations are product-specific and independents, and under a cross-licensing scenario, each duopolist becomes a multiproduct firm. Our novelty resides in that our analysis represents a mixed case to what is normally analyzed in the literature (that focus mainly on full integrated or separated markets), and we propose an extension studied previously in Sánchez et al. (2021). We focus on partial vertical integration and the existence of assymetric sizes of innovation in the market.

We contribute to two strands of the literature: the one that studies partial vertical integration and the one that focuses on the license of technology.

Technology licensing is a topic that has been broadly studied. The literature has focused on two main aspects: the study of the strategic decision by patent holders to whom to assign a license (Badia et al. 2020), and the study of optimal contracts in technology licenses between the licensor and the licensee (Katz and Shapiro 1986). Previous studies about contracts highlight that the optimal mechanism -fixed-fees, royalties, or auctions- to the transference of technology may depend whether the owner of technology is an outsider innovator (e.g., Katz and Shapiro 1986; Kamien and Tauman 1986, Kamien 1992; Stamatopoulos and Tauman 2009; Miao 2013) or, on the contrary, the patent holder is a producer in the market (e.g., Wang 1998; Kamien and Tauman 2002; Sen and Tauman 2007). This literature makes it clear that if the owner of the innovation does not compete, the income from a fixed-fee exceeds that from royalties. On the other hand, incomes from royalties exceed the ones coming from the fixed quota in cases where the patent holder is a producer in the final market. That is so because royalties provide both license income and a competitive advantage in production. Other interesting studies focus on the role that the expected duration of the relationship between technology owners and firms may play in the election of the technology transfer contracts (Mendi 2005; Cebrián 2009). For instance, Mendi (2005) finds that a contract where the time horizon is short is more likely to include fixed payments. Under these facts, we consider technology licensing through a fixed payment.

On the other hand, we aim to study a different approach not considered before in the literature of technology licensing, that is, the incentives of partially vertically integrated firms to license their rivals, given the level of vertical integration. Most theoretical and empirical studies about vertically related markets have focused on two extreme alternatives: full vertical integration and separation. However, in practice is quite common to find partial vertical integrated firms, namely, partial ownership agreements in which a firm acquires less than 100% of shares in a vertically related firm (Gilo and Spiegel 2011; Hunold and Shekhar 2018). Theoretical studies that focus on partial vertical integration have analyzed different perspectives. For example, Fiocco (2016) investigates the strategic incentives for partial vertical integration with two manufacturer–retailer hierarchies or the case when there is backward ownership, i.e., ownership stakes hold of upstream firms by downstream firms (Greenlee and Raskovich 2006). Other interesting pieces of work, explore the (anti-) competitive effects of partial vertical ownership (Levy et al. 2018; Spiegel et al. 2013, Schmalz 2018). Thus, previous studies usually analyze the incentives to partial vertical integrate or the limitation of this phenomena.

However, in our study, the starting point is that the upstream firms (patent holders) are already partially vertical integrated to one competing firm in the downstream market, and the strategic decision revolves around how many firms license their technology. Thus, we analyzed a different angle not considered before in the literature of the license of technology introducing partial vertical integration of patent holders.

To analyze this approach, we propose a model with two technology owners that have to decide to sell one or two licenses, that is, exclusive or non-exclusive licensing. Mendi et al. (2011) evaluate a patent holder in the market and two firms in the downstream market that differ in their level of production costs, where one firm is more efficient than the other. They compare two scenarios, whether the affiliate firm is the more efficient firm or not. Moreover, they analyze the implications that it has on the market. We extend the preliminary results in Mendi et al. (2011) since we contemplate two technology owners and each innovator has a share in one of the firms that compete in the market. Furthermore, we assume a differentiated duopoly (see e.g., Muto 1993; Caballero-Sanz et al. 2002, Mukhopadhyay et al. 1999), where there is cost symmetries through innovations. Due to innovators participate in firms' capital shares, cross-licensing generates a trade-off between raising licensing revenues and increasing competition.

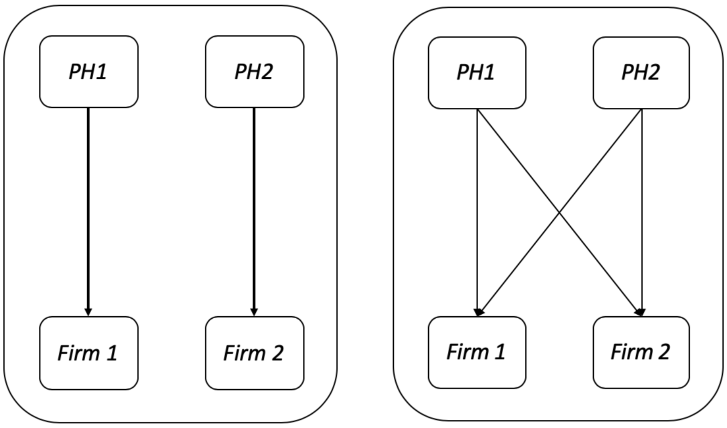
In addition, we consider that the holders of the innovation do not compete in the final product market to which the innovation refers; however, they have interests in the final competition because it is assumed that the holders of the innovation have a share of competing firms in the downstream market. Additionally, the technology licensing is based on a fixed-fee mechanism because although this type of contract does not control reaction curves of competing firms, it allows the patent holder to have more room with its decision on the number of licenses granted, that is our main objective in this work. Furthermore, we do not consider any specific duration of the relationship, so we understand that the relationship is one shot, and following the findings of Mendi (2005), fees are more likely to occur and may fit to center our attention in the role of ownership, and therefore, the existence of technology transference with affiliated firms.

The results, allow us to compare what is the best strategy and the equilibria regarding the number of licenses; exclusive license (one), or cross-licenses (several). We find the main determinants in the decision-making, that may differ between patent holders depending on the cost of production that firms face in the dowstream market. Furthermore, we explore the implications of the asymmetry in the innovation process between patent holders, which might have implications in the diffusion of innovation in the downstream market. As it will be seen, the cross-licensing scenario is best when the size of the innovation is small; this occurs, regardless of the participation in the competing firms and how many innovate (Result 4). Technology owners may be better off in a scenario with exclusive licenses. This is so when the size of the innovation is large, both owners have the same innovation, and the initial cost of production is large enough (Result 1). If a patent holder has a share in one of the competing firms and the innovation size between patent holders is the same, she prefers an exclusive license if the cost of production is large enough and, additionally, she holds a minimum share in the firm (Result 2). Asymmetry in the innovation process, requires further conditions in the differentiation of products (Result 3). The fact that only one of the owners has a stake in a competing firm may lead each innovator to prefer a different scenario (Results 2 and 3).

The rest of the work is organized as follows. Section 2 presents the model and characterizes the balances in the mentioned scenarios: exclusive licenses and cross licenses. The comparison of these scenarios and the main results are introduced in Section 3. The conclusions are presented in section 4.

2 The model

The modeling adopted is that of a successive duopoly, with two technology owners and two firms competing in a differentiated product market. Each technology owner has a stake in one of the competing firms. As indicated above, our purpose is to compare two scenarios: one in which each owner transfers the technology exclusively to her participating firm, figure 1 (a), and another in which the technologies are transferred to the two competing firms, figure 1 (b). We will refer to the first scenario as that of exclusive licenses and the second as that of cross-licenses.



(a) (b)

Figure 1: Exclusive licensing vs. Cross-licensing

2.1 Exclusive licenses

Consider a duopoly in which each firm initially produces a variety of a differentiated product. The system of inverse demands -which is obtained from the problem of maximizing the utility of a representative consumer subject to the budget constraint- is as follows:

$$p_1 = 1 - q_{11} - dq_{22} \tag{1}$$

$$p_2 = 1 - q_{22} - dq_{11} \tag{2}$$

The first subscript refers to the variety and the second refers to the firm. The parameter

d measures the degree of product differentiation $d \in (0, 1)$, where the varieties are more homogeneous the closer to 1 is d. Existing technology allows these varieties to be produced at a constant marginal cost equal to c, with $c < 1$. There are also two technology owners, each of whom has a stake, α_i, where $\alpha_i \in (0, 1)$, of one of the firms that are competing in the market, $i = 1, 2$. The technology owners each have a process innovation. Specifically, owner one, which we denote by PH_1, has a process innovation that reduces the marginal cost of production by magnitude ε. This means that if PH_1 transfer the innovation to her investee firm, she will be able to produce variety one at a marginal cost $c - \varepsilon$. Similarly, PH_2 denotes owner two who has a process innovation of size $\varepsilon - \delta$ thus, the marginal cost of production of variety two, if firm two acquires the innovation, becomes $c - \varepsilon + \delta$. Therefore, competition in the technology market occurs in an asymmetric context, collecting δ this asymmetry. If δ is zero the cost reduction is the same in both cases since they both have the same innovation in size ε. On the other hand, the closer δ to ε the greater the asymmetry between the owners; the extreme case of $\delta = \varepsilon$ means that only the PH_1 has the process innovation; therefore, $\delta \in (0, \varepsilon)$. Table 1 resumes the sizes of innovation. The transfer of the technology is made through a fixed payment, F.

Table 2: Sizes of innovation of both Patent Holders.

Innovation sizes	
$\delta = 0$	Same innovation sizes of both PH
$\delta \longrightarrow \varepsilon$	Greater asymmetry between innovations. Only one PH has the technology
$\varepsilon \longrightarrow 0$	Innovation is small
$\varepsilon \longrightarrow c$	Innovation is very large

Formally we solve a game in several stages. In the first stage, the $PH1$ and $PH2$ owners simultaneously and non-cooperatively choose the fixed payment for the assignment or license of the innovation. In the second stage, each firm decides whether or not to accept the fixed payment contract offered by its respective patent holder. Finally, and given the above, firms compete in quantities.

Solving backward induction, the profit maximization problem when duopolists have both the respective process innovations are

$$\max_{q_{11}} \pi_1 = (p_1 - c + \varepsilon)q_{11} - F_1 \tag{3}$$

$$\max_{q_{22}} \pi_2 = (p_2 - c + \varepsilon - \delta)q_{22} - F_2 \tag{4}$$

The solution of the system formed by the first order conditions, $\partial\pi_1/\partial q_{11} = 0$ and $\partial\pi_2/\partial q_{22} = 0$, is the following:

$$q_{11}^E = \frac{(2-d)(1-c+\varepsilon)+d\delta}{4-d^2} \tag{5}$$

$$q_{22}^E = \frac{(2-d)(1-c+\varepsilon)-2\delta}{4-d^2} \tag{6}$$

where the super index E represents the scenario with exclusive licenses. Given the established assumptions, the second order conditions hold.[3] Noting the numerator of q_{22}^E, can be deduced that if δ is large enough, firm two will not produce. Indeed, a lot of asymmetry would be the equivalent of the definition of large innovation in the present context. The condition is as follows: $\delta > \frac{(2-d)(1-c+\varepsilon)}{2}$.

Substituting (5) − (6) in the expressions of profits, we get $\pi_1^E = (q_{11}^E)^2$ y $\pi_2^E = (q_{22}^E)^2$. The $PH1$ will design a license contract so that the firm accepts it. To determine the fixed payment, we calculate the opportunity cost of the license, that is, the difference between having it and not having it. The firm is willing to pay an amount F such that $F \leq \pi_1(c-\varepsilon, c-\varepsilon+\delta) - \pi_1(c, c-\varepsilon+\delta) \equiv F_1$. The first term on the right of the inequality refers to the profits π_1^E we obtained above. To complete the fee payment, F_1, we solve an asymmetric duopoly where the firm 1 produces with the initial marginal cost, c, while the rival does it with the corresponding innovation, $c-\varepsilon+\delta$. Solving we get the following profits:

$$\pi_1(c, c-\varepsilon+\delta) = \frac{((2-d)(1-c)-d(\varepsilon-\delta))^2}{(4-d^2)^2} \tag{7}$$

Thus, the fee is

$$F_1^E = \frac{4\varepsilon[(2-d)(1-c)+\varepsilon-d(\varepsilon-\delta)]}{(4-d^2)^2}. \tag{8}$$

[3] $\frac{\partial^2\pi_1^E}{\partial q_{11}} = \frac{\partial^2\pi_2^E}{\partial q_{22}} = -2 < 0$.

Furthermore, we solve an asymmetric duopoly where firm 2 produces with the initial marginal cost, c, wile the rival does it with the corresponding innovation $c-\varepsilon$. Thus, we get firstly the following profits:

$$\pi_2(c-\varepsilon, c) = \frac{4(\varepsilon+\delta)\phi - 2(1-c)d(2-d)\varepsilon + (1-c)^2(2-d)^2 + d^2\varepsilon^2}{(4-d^2)^2} \tag{9}$$

where $\phi = (c(2-d) - (1-d)\varepsilon + d + \delta - 2)$. Therefore, the fee is:

$$F_2^E = \frac{4(\delta-\varepsilon)\left[c(2-d) - (1-d)\varepsilon + d + \delta - 2\right]}{(4-d^2)^2}. \tag{10}$$

Proposition 1 *In the scenario with exclusive licenses, the fixed payment is higher for the* $PH1$, $F_1^E > F_2^E$.

Proof. See Appendix. ■

It is an intuitive result since the reduction in marginal cost is greater with the technology of the $PH1$. In the market equilibrium, this allows the firm to obtain a greater market share, so the opportunity cost of not having the innovation is greater. Then, $PH1$ is able to charge a greater fix payment to its downstream market.

Thus, due to the fact that $PH1$ has a stake in firm one, her profits are the following:

$$\begin{aligned} \Pi_{PH1}^E &= F_1^E + \alpha_1(\pi_1^E - F_1^E) = \\ &= \frac{4\varepsilon[(2-d)(1-c) + \varepsilon - d(\varepsilon-\delta)] + \alpha_1[(2-d)(1-c) - d(\varepsilon-\delta)]^2}{(4-d^2)^2} \end{aligned} \tag{11}$$

In a similar way, we get the profits for $PH2$:

$$\begin{aligned} \Pi_{PH2}^E &= F_2^E + \alpha_2(\pi_2^E - F_2^E) = \\ &= \frac{4(\varepsilon-\delta)[(2-d)(1-c) + (1-d)\varepsilon - \delta] + \alpha_2[(2-d)(1-c) - d\varepsilon]^2}{(4-d^2)^2} \end{aligned} \tag{12}$$

2.2 Cross-licenses

In this scenario, both patent holders sell their licenses to every firm in the downstream market. This implies that each duopolist becomes a multiproduct firm, that is,they produce variety one at marginal cost $c - \varepsilon$ and variety two at marginal cost $c - \varepsilon + \delta$. That is why now the reverse demand system is defined as follows:

$$p_1 = 1 - (q_{11} + q_{12}) - d(q_{21} + q_{22}) \tag{13}$$

$$p_2 = 1 - (q_{21} + q_{22}) - d(q_{11} + q_{12}) \tag{14}$$

Then, the profit-maximization problem when duopolists have both innovations are given by:

$$\max_{q_{11},q_{21}} \pi_1 = (p_1 - c + \varepsilon)q_{11} + (p_2 - c + \varepsilon - \delta)q_{21} - F_1 - F_2 \tag{15}$$

$$\max_{q_{12},q_{22}} \pi_2 = (p_1 - c + \varepsilon)q_{12} + (p_2 - c + \varepsilon - \delta)q_{22} - F_1 - F_2 \tag{16}$$

The solution of the system formed by the four first-order conditions, where the second-order conditions for maximum are verified[4], yields the following equilibrium quantities for each variety:

$$q_{11}^{NE} = q_{12}^{NE} = \frac{(1-d)(1-c+\varepsilon) + d\delta}{3(1-d^2)} \tag{17}$$

$$q_{21}^{NE} = q_{22}^{NE} = \frac{(1-d)(1-c+\varepsilon) - \delta}{3(1-d^2)} \tag{18}$$

The superscript NE refers to the equilibrium outcomes with cross-licensing or non-exclusive licenses. As in the scenario with exclusive licenses, we write the condition $\delta > (1-d)(1-c+\varepsilon)$ that, if met, would indicate that the process innovation of $PH1$ is large since the quantity variety two would be negative. The definition of large innovation will be taken into account in the analysis in the cases that will be presented below. The condition of the scenario with exclusive licenses is more demanding, $\delta > \frac{(2-d)(1-c+\varepsilon)}{2} > (1-d)(1-c+\varepsilon)$.

The next step is to calculate the fixed payment that firms must pay each patent holder. Let's see how we obtain the fixed payment that the firm one will pay to $PH1$ for the cession of the process innovation. As we have pointed out above, $PH1$ designs the contract for the firm to accept it, that is, the fixed payment cannot exceed the opportunity

[4] $\frac{\partial^2 \pi_1^{NE}}{\partial q_{11}} = \frac{\partial^2 \pi_2^{NE}}{\partial q_{12}} = \frac{\partial^2 \pi_1^{NE}}{\partial q_{21}} = \frac{\partial^2 \pi_2^{NE}}{\partial q_{22}} = -2 < 0$

cost of acquiring the technology. Thus, firm one is willing to pay an amount F such that $F \leq \pi_1(c-\varepsilon, c-\varepsilon+\delta; c-\varepsilon, c-\varepsilon+\delta) - \pi_1(c, c-\varepsilon+\delta; c-\varepsilon, c-\varepsilon+\delta) \equiv F_1^{NE}$. The profits of the first term on the right of the inequality correspond to the profits π_1^{NE}. We need to solve an asymmetric duopoly with a single-product firms (firm one) and another that is a multi-product firm (firm two), that is,

$$\max_{q_{21}} \pi_1 = (p_2 - c + \varepsilon - \delta) q_{21} \tag{19}$$

$$\max_{q_{12}, q_{22}} \pi_2 = (p_1 - c + \varepsilon) q_{12} + (p_2 - c + \varepsilon - \delta) q_{22} \tag{20}$$

taking the inverse demands in (13)-(14) where $q_{11} = 0$. Once the equilibrium quantities has been calculated, it is replaced in profits. Making the difference between profits with and without innovation of size ε we get the fixed fee:

$$F_1^{NE} = \frac{[(1-d)(1-c+\varepsilon) + d\delta]^2}{9(1-d^2)} \tag{21}$$

This same fixed fee is the one that the owner $PH1$ will charge to firm 2. We derive the fixed fee for $PH2$ in the same way, obtaining:

$$F_2^{NE} = \frac{[(1-d)(1-c+\varepsilon) - \delta]^2}{9(1-d^2)} \tag{22}$$

As in Proposition 1, $F_1^{NE} > F_2^{NE}$. This is because the impact of innovation when using $PH1$ technology is higher, then the reduction of costs or the overall downstream profits, and that is why $PH1$ is able to extract a higher fee.

Finally, profits of the technology owner one, $PH1$, are given by

$$\begin{aligned} \Pi_{PH1}^{NE} &= 2F_1^{NE} + \alpha_1(\pi_1^{NE} - F_1^{NE}) = \\ &= \frac{2(1 - c(1-d) - d(1-\varepsilon+\delta) + \epsilon)^2}{9\,(1-d^2)} \\ &\quad -\frac{1}{9}\alpha_1 \left(\frac{2(1-d)\delta(1+\varepsilon-c) + 2(1-d)\varepsilon(2c-\varepsilon-2) - \delta^2}{1-d^2} \right) \\ &\quad +\frac{1}{9}\alpha_1 \left(\frac{-(1-c(1-d) + d(1+\varepsilon-\delta) + \varepsilon)^2}{1-d^2} + \frac{2(1-c)^2}{1+d} \right). \end{aligned} \tag{23}$$

Proceeding in the same way, we derive the technology owner two's profits:

$$\begin{aligned}
\Pi_{PH2}^{NE} &= 2F_2^{NE} + \alpha_2(\pi_2^E - F_2^{NE}) = \\
&= \frac{2(d+\delta+c-cd-(1-d)\epsilon-1)^2}{9\,(1-d^2)} \\
&\quad -\frac{1}{9}\alpha_2\left(\frac{2(1-d)\delta(1+\varepsilon-c)+2(1-d)\varepsilon(2c-\varepsilon-2)-\delta^2}{1-d^2}\right) \\
&\quad +\frac{1}{9}\alpha_2\left(\frac{-2(d+\delta+c-cd-(1-d)\epsilon-1)^2}{1-d^2}+\frac{2(1-c)^2}{1+d}\right). \qquad (24)
\end{aligned}$$

3 Results

Once we have solved the two scenarios, we analyze the decision of the patent holders on whether to sell an exclusive license or two licenses. In order to answer this main question, we compare patent holders' profits depending on the degree of the innovation process, that is, if innovation is large or small. In concrete, we consider two scenarios related to process innovation: (i) when the innovation does not present cost reduction, that is, $\varepsilon = 0$, and (ii) when there is a large innovation that makes production costs zero and therefore $\epsilon = c$. In the next section, we present the main results and determinants to the strategic decision of the licensing programs for both patent holders.

3.1 Large innovation

Large innovation is considered when the effect in reducing costs in the downstream market firms is very strong. In this case, we consider the extreme case where the cost is reduced to the maximum, that is, $\varepsilon = c$, therefore, $c = 0$.

Furthermore, to present optimal conclusions, we distinguish three scenarios that show different levels of asymmetry in the game between the patent holders:

- *Case I: Symmetry.* In this case, patent holders do not have a share in the downstream firms, that is, $\alpha_i = 0$. Thus, firms in the technology market are not present in the product market. On the other hand, the size of innovation of both patent holders are equal, thus $\delta = 0$.

- *Case II: Stake asymmetry.* We assume that PH_1 has a share in one of the competing firm in the market, $\alpha_1 \epsilon (0, 1)$, but PH_2 has not, $\alpha_2 = 0$. Therefore, PH_1 is present in the product market. We hold the symmetry in the size of innovation for both patent holders.

- *Case III: Full asymmetry.* We add asymmetry in the size of innovation to the scenario in Case II. Thus, $\delta = \varepsilon$, and $PH2$ innovation does not reduce the costs.

3.1.1 Case I

This case is the simplest one and shows a symmetric situation, where both patent holders do not have a stake in the competing firms, and the impact of their technology is the same i.e., the innovation size of both patent holders are equal, $(\delta \longrightarrow 0)$.

In this subcase, PH_1 will prefer a scenario with cross-licensing to one of an exclusive licensing if $2F_1^{NE} > F_1^E$. And since both patent holders have the same size of process innovation, $2F_1^{NE} - F_1^E = 2F_2^{NE} - F_2^E$. In this extreme case where the size of the innovation is very large such that the marginal cost is zero, $(\varepsilon \longrightarrow c)$, we investigate the sign of the difference:

$$2F_1^{NE} - F_1^E\big|_{\varepsilon \longrightarrow c} = \frac{2\,(1-d)}{9\,(1+d)} - \frac{4c(2-c-d)}{(4-d^2)^2} \tag{25}$$

Let c_I represents the threshold that makes $2F_1^{NE} - F_1^E = 0$ given by

$$c_I = \frac{2-d}{2} - \frac{1}{6}\sqrt{\frac{4+32d-11d^2-7d^3-2d^4+2d^5}{1+d}}. \tag{26}$$

We find the following result:

Result 1 *When patent holders i) do not have a share in the firms that compete in the market, $\alpha_i = 0$, ii) they have the same process innovation, $\delta = 0$, and iii) the innovation is very large, $\varepsilon = c$, they prefer a cross-licensing scenario when $c < c_I$, while they prefer a scenario with exclusive licenses when $c_I < c < 1$.*

Proof. See Appendix. ■

When the costs of the firms are high and the impact of the innovation is greater, patent holders would prefer to sell only one license. On the other hand, in the case that the costs are low and the impact of the innovation in the companies is smaller, makes the fees charged much smaller and the benefit of cross-licensing greater. Thus, it suggests that in a symmetrical situation, the greater the impact of the innovation, due to the cost structure of the companies in the downstream market, the more likely that the patent holders will sell exclusive licenses.

As can be seen in the value of c_I above, this cost depends on the relationship between the products of the competing firms in the downstream market, i.e., the level of product substitutability that sets the level of competition in the market, d.

Proposition 2 *Under Case I: Symmetry, the value of c_I decreases as the products are more substitutes ($d \longrightarrow 1$). Therefore, higher competitive level favors patent holders' preference to sell exclusive licenses.*

Proof. See Appendix. ■

Figure 2: Value of c_I under changes in d

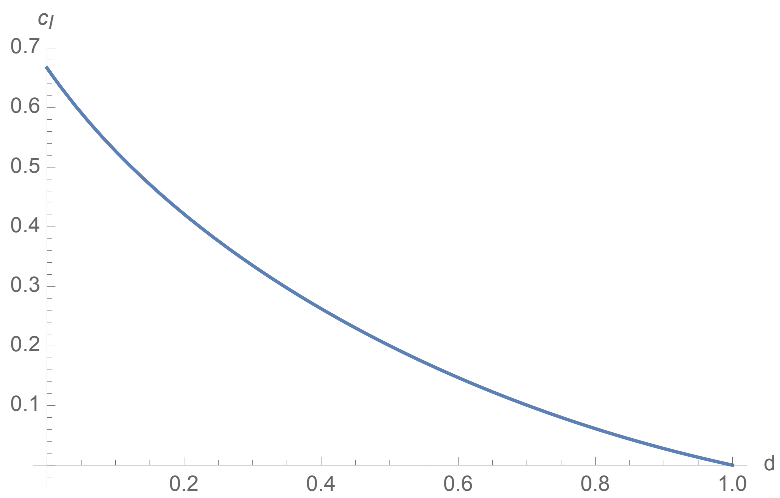


Figure 2 reflects the finding in proposition 2, and can be easily seen that as long as the level of competitiveness increases in the downstream market ($d \longrightarrow 1$), the required value of marginal costs c_I under which patent holders decide to offer exclusive licenses drops. The following table reflects this result numerically:

	d=0	***d=0.5***	***d=0.8***	***d=1***
c_I	0.67	0.20	0.06	0

When products are independents, that is $d = 0$, patent holders decide exclusive licenses as long as $0.67 < c < 1$, and cross-licenses otherwise. However, if $d = 0.8$, patent holders choose exclusive licenses if $0.06 < c < c_I$. Then, it can be seen that as long as products became more homogeneous, it is easier to find a situation with exclusive licenses.

Proposition 3 *Under Case I: Symmetry, when products are perfect substitutes, that is, $d = 1$, patent holders prefer to sell just one license, exclusive licenses.*

Proof. See Appendix. ∎

Proposition 3 shows an extreme case where the products are perfect substitutes. In this case, marginal costs do not affect the decision making of patent holders. Furthermore, as a result of the high level of competition in the product market, firms are willing to pay greater fees for greater innovations with the objective of not losing market share against its competitors. The optimal decision for patent holders is to offer exclusive licenses.

3.1.2 Case II

This subcase analyzes an asymmetric situation where only one patent holder has a stake in one of the competing firm in the downstream market. Supponse that only PH_1 has a share in firm 1, that is, $\alpha_1 > 0$ and $\alpha_2 = 0$. Our aim is to study how this specific asymmetry affects the decision making and licensing strategy of both PH_1 and PH_2. This is a mixed case with respect to what is normally analyzed in the literature, who has studied, on the one hand, the case of an innovator outside the industry and, on the other, the case of an innovative competitor in the industry.[5]

[5]See, for example, Sandonís and Faulí-Oller (2006) or Sandonís and Faulí-Oller (2008) where they study the incentives of an external innovator to merge with an insider firm using other mechanisms for

As before, for PH_1 the sign of (25) minus (11) is undefined. Following the procedure we have just followed when competing firms are not investees (Case I), the difference in profits for $PH1$, when patent holders have the same process innovation ($\delta \longrightarrow 0$) and under a case of large innovation ($\varepsilon \longrightarrow c$), is reduced to:

$$\Pi_{PH1}^{NE} - \Pi_{PH1}^{E} = \frac{(2-d)^2(1-d)\left(\alpha_1(1+d)(5+d) - 2(2+d)^2\right)}{9(1+d)\left(4-d^2\right)^2} + \frac{36(1-\alpha_1)c^2(1+d) - 36(1-\alpha_1)c(2-d)(1+d)}{9(1+d)\left(4-d^2\right)^2}.$$

Similar to Case I, we find two thresholds or conditions that mark the decision of PH_1 in the marginal cost (same threshold c_I of Case I in equation 28) and in the share that PH_1 has in firm 1, α_1. Let $\alpha_{1(II)}$ be

$$\alpha_{1(II)} = \frac{36c(1+d)(2-c-d) - 2(1-d)\left(4-d^2\right)^2}{(1+d)\left(2-6c-3d+d^2\right)\left(6c-(2-d)(5+d)\right)}.$$

On the other hand, the difference between profits of $PH2$ when ($\delta \longrightarrow 0$) and ($\varepsilon \longrightarrow c$) reduces to:

$$\Pi_{PH2}^{NE} - \Pi_{PH2}^{E} = \frac{2(1-d)}{9(1+d)} - \frac{4c(2-c-d)}{\left(4-d^2\right)^2} \equiv 2F_1^{NE} - F_1^{E}|_{\varepsilon \longrightarrow c}.$$

For PH_2, given that there is no interdependence between α_1 and α_2, her decision is the same that the one analyzed in Case I with no share in the competing firms.

The combination of the above conditions leads to the following result.

Result 2 *When i) only the owner $PH1$ has a share in the firm that competes in the market, that is $\alpha_1 > 0$ and $\alpha_2 = 0$, ii) both owners have the same process innovation, $\delta = 0$, and iii) the innovation is very large, $\varepsilon = c$, then if the cost c is high enough, $c > c_I$ and $\alpha_1 < \alpha_{1(II)}$, the PH_1 prefers a scenario with exclusive licenses. Otherwise, PH_1 will prefer cross-licensing.*

Proof. See Appendix. ■

licensing: two-part tariff and auctions.

With stake asymmetry, the optimal licensing program of both patent holders are not lined up and differ. The decision making for PH_2 does not change with respect to the Case I analyzed above. Thus, the optimal strategy for patent holder two remains like under Case I. However, $PH_1's$ decision has changed because under Case II has a stake in a downstream firm. The entrance of PH_1 as owner in firm 1 reveals an additional requirement in order to sell exclusive licenses to her participated firm, which involves specific values in the participation in the firm.

In general, we can appreciate the importance of the cost structure of firms. If, for example, we are dealing with an industry that presents a low-cost structure or less intensive such as the technology industry or IT sector, what is going to be preferred is to carry out a cross-licensing strategy. On the other hand, if the cost structure is large, or higher, technology owners have incentives to sell exclusive licenses, since the impact that innovation can have on it is greater, resulting in higher fixed payments for patent holders. By introducing asymmetry in Case II, the possibility of a participation in the product market by innovative firms hardens the requirement to carry out an exclusive license. This is so because patent holders are now able to derive profits through ownership in the firm in an additional way and accepting to give the technology exclusively requires a minimum profit coming through product market share. The intuition for this is that the more advantage and greater competitiveness than the investee firm has (a large innovation that is exclusively licensing to one firm), the greater the income, not only through fees but also through the share of profits.

3.1.3 Case III

This case introduces an additional asymmetry in the innovation process to the one studied in Case II. Suppose that $\delta = \varepsilon$. This assumption means that only PH_1 has the innovation and therefore, there is asymmetry in the innovation process between both patent holders. The fixed payment would be equal to the profits of having the innovation (monopoly with zero cost) less the profits of not having it (differentiated duopoly with cost c).

If $\delta = \varepsilon$, then for the technology owner one, $PH1$, we have that

$$\Pi^{E}_{PH1} - \Pi^{NE}_{PH1}\big|_{\delta \longrightarrow \varepsilon} = \frac{4c\left(2-c\left(1-d\right)-d\right)}{\left(4-d^2\right)^2} - \frac{2(1-(1-c)d)^2}{9(1-d^2)} + \frac{\left(1-c\right)^2\left(1-d\right)\left(5+d\right)\alpha_1}{9\left(2+d\right)^2}.$$

Following the same procedure as before, the quadratic inequality must be solved in order to obtain under what conditions $PH1$ decides to sell an exclusive license. Let us call the roots of the polynomial c^{-}_{III} and c^{+}_{III} the positive root (see the Appendix for the expressions in (A.11) and (A.12) and proofs). Both are positive, that is, $c^{-}_{III}, c^{+}_{III} > 0$. However, there are conditions under which c^{+}_{III} is fewer than 1, and is when $0.442891 < d < 1$.

If $\delta = \varepsilon$, then for the $PH2$, we have that

$$\Pi^{E}_{PH2} - \Pi^{NE}_{PH2}\big|_{\delta \longrightarrow \varepsilon} = \frac{2(c+d-1)^2}{9\left(d^2-1\right)} < 0.$$

The combination of the above conditions leads to the following result.

Result 3 *When i) only $PH1$ has the innovation, and ii) $PH1$ has a stake in one of the downstream firms, the $PH1$ sells an exclusive license as long as $c^{-}_{III} < c < c^{+}_{III}$ and $0.442891 < d < 1$, or $c^{-}_{III} < c < 1$ if $0 < d < 0.442891$. On the other hand, PH_2 prefers to offer cross-licenses.*

Proof. See Appendix. ■

This result shows the possibility of a vertical integration between the technology provider, PH_1 and firm 1, which operates in the product market. By entering PH_1 as the owner of one of the participating firms in the product market, it can open the possibility of a change in the market structure towards a monopoly vertically integrated. Clearly, it follows from the conditions of Result 3 that this will be the case as long as there is a product differentiation that allows it. Thus, if there is high levels of d, (which implies greater competitiveness and therefore, product substitutability), the requirement in cost levels is lower than if there is low competitive intensity (there is only one condition). However, for high levels of competitive intensity, the cost level requirements are more demanding (two conditions). Once again, the intensity or level of the marginal costs of the firms play an important role in the decision making. In this section, the possibility of evolving into a monopoly entails gaining market share and, therefore, an increase in the possible income of the PH_1, since under monopoly the income rises drastically.

3.2 Small innovation

Small innovation is considered when the impact of the innovation on cost reduction is small. In this case, to carry out the analysis we consider the extreme case where the cost reduction is zero, therefore $\varepsilon = 0$. Thus, the effect that produces such innovation is null or negligible. In fact, the distinction of cases is not necessary because the decision is the same for all of them.

If the innovation is very small, $(\varepsilon \longrightarrow 0)$, then the difference in profits for $PH1$ remains as:

$$\Pi_{PH1}^{NE} - \Pi_{PH1}^{E}|_{\varepsilon \longrightarrow 0} = \frac{(1-c)^2(1-d)[2(2+d)^2 - (1+d)(5+d)\alpha_1]}{9(1+d)(2+d)^2} > 0 \qquad (27)$$

On the other hand, the difference between profits of $PH2$ is the expression in equation (30) in the limiting case of small innovation. Therefore,

$$\Pi_{PH2}^{NE} - \Pi_{PH2}^{E}|_{\varepsilon \longrightarrow 0} = \frac{(1-c)^2(1-d)[2(2+d)^2 - (1+d)(5+d)\alpha_2]}{9(1+d)(2+d)^2} > 0. \qquad (28)$$

Result 4 *With small innovation, both patent holders prefer to sell two licenses, that is, to cross-license both firms in the downstream market.*

In the limiting case of $(\varepsilon \longrightarrow 0)$, if patent holders decide to offer exclusive licenses, no firm would have incentives to acquire the innovation, since the effect is null. Thus, there will be no market for technology. On the other hand, given the existence of cross-licensing, despite the fact that there is no positive effect from innovation, there is a competitive advantage when producing with two different technologies i.e., firms are now able to produce both varieties of products in the downstream market and get profits from them. That is why firms do have incentives to acquire licenses, since if they did not do so, their competitive situation would worsen. This effect has not so much to do with innovation itself, but with multi-production.

4 Conclusions

The objective of this work has been to introduce us in the context of competition in the technology market, with the existence of asymmetry between the process innovations collected with the parameter δ. The fact that the owners of the technology have a stake in one of the companies that operates in the differentiated products market (measured by d) has been studied. As indicated at the beginning, the work is a contribution to the literature of technology license combining a series of elements that, as far as we know, have not been studied before. In addition, the treatment of special cases allows recovering scenarios previously discussed in the literature and which are part of a more comprehensive model here. For example, when one of the α_i is zero and $\delta = \varepsilon$ we have a structure in which a single innovator (who may or may not be present in the market through participation) has to decide whether to sell one or two licenses through a fixed payment.

Our analysis has consisted of comparing two possible scenarios to highlight this possible dilemma that innovative companies face when deciding between exclusive licenses or not. The results reveal that, in a context of exclusive licenses, the fixed payment will be higher for the larger technology, since it supposes less lower production costs for companies; the demand it will be higher, so the opportunity cost of not having it is greater than in the case of a smaller technology.

However, when we introduce cross-licenses, and in this case, the downstream firms become multiproduct, the results show us various options. As the model has a high number of parameters, we have proceeded to study particular cases. At first, we have analyzed what happens when there is no participation by the patent holders as owners, $\alpha_1 = \alpha_2 = 0$, we assume that both have the same process innovation and we leave the costs and degree of substitution as free parameters between varieties. The cross-licensing scenario has been shown to be preferred when the size of the innovation is small. If the innovation is very large, the owners may be better off with exclusive licenses: this occurs when the initial cost of production is large enough - the requirement is less the more differentiated the varieties are. In this way, the firm that competes in duopoly can achieve a greater market share, and therefore, the owner can achieve a higher income. If this is not the case then you prefer to sell to both companies.

The results change when introducing the possibilities of stake by patent holders in downstream firms. Now, if the participating firm is the only one that owns the innovation, it will only sell an exclusive license if the innovation is large and for certain participation shares. This does not happen when innovators are out of competition in the market. Finally, if both owners have the same innovation, the cross-licensing scenario is preferred when the innovation is small; the degree of substitution between varieties plays no role in this decision.

Therefore, our work suggests a series of determinants to explain the observation of scenarios with exclusive licenses and cross licenses. Among these, the ownership positions of innovators in competing companies and the size of process innovations are particularly relevant. From an applied point of view, it will be interesting to identify particular cases of licensing policies that coincide with the predictions of our theoretical model. From a formal point of view, the analysis can be extended to other types of contracts as well as other areas of competition, such as price and levels of investment in R&D.

References

Arora, A. and A. Fosfuri (2003). Licensing the market for technology. *Journal of Economic Behavior & Organization 52*(2), 277–295.

Arrow, K. J. (1962). Economic welfare and the allocation of resources for invention, university of minnesota, the rate and direction of inventive activity: Economic and social factors.

Badia, B. D., Y. Tauman, and B. Tumendemberel (2020). On the diffusion of competing innovations. *Mathematical Social Sciences 108*, 8–13.

Belleflamme, P. and C. Vergari (2011). Incentives to innovate in oligopolies. *The Manchester School 79*(1), 6–28.

Caballero-Sanz, F., R. Moner-Colonques, and J. J. Sempere-Monerris (2002). Optimal licensing in a spatial model. *Annales d'Economie et de Statistique*, 257–279.

Cebrián, M. (2009). The structure of payments as a way to alleviate contractual hazards in international technology licensing. *Industrial and Corporate Change 18*(6), 1135–1160.

Fiocco, R. (2016). The strategic value of partial vertical integration. *European Economic Review 89*, 284–302.

Gilo, D. and Y. Spiegel (2011). Partial vertical integration in telecommunication and media markets in israel. *Israel Economic Review 9*(1).

Greenlee, P. and A. Raskovich (2006). Partial vertical ownership. *European Economic Review 50*(4), 1017–1041.

Hamdan-Livramento, I. (2012). The Evolution of Technology Markets: Separating Fact from Fiction WIPO Magazine. Retrieved August 28, 2020 from url: https://www.wipo.int/wipo_magazine/en/2012/02/article_0005.html.

Hoppe, H. C., P. Jehiel, and B. Moldovanu (2006). License auctions and market structure. *Journal of Economics & Management Strategy 15*(2), 371–396.

Hunold, M. and S. Shekhar (2018). *Supply chain innovations and partial ownership.* Number 281. DICE Discussion Paper.

Kamien, M. I. (1992). Patent licensing. *Handbook of game theory with economic applications 1*, 331–354.

Kamien, M. I. and Y. Tauman (1986). Fees versus royalties and the private value of a patent. *The Quarterly Journal of Economics 101*(3), 471–491.

Kamien, M. I. and Y. Tauman (2002). Patent licensing: the inside story. *The Manchester School 70*(1), 7–15.

Katz, M. L. and C. Shapiro (1986). How to license intangible property. *The Quarterly Journal of Economics 101*(3), 567–589.

Kolk, A. and R. Van Tulder (2002). The effectiveness of self-regulation:: Corporate codes of conduct and child labour. *European Management Journal 20*(3), 260–271.

Levy, N., Y. Spiegel, and D. Gilo (2018). Partial vertical integration, ownership structure, and foreclosure. *American Economic Journal: Microeconomics 10*(1), 132–80.

Mendi, P. (2005). The structure of payments in technology transfer contracts: evidence from spain. *Journal of Economics & Management Strategy 14*(2), 403–429.

Mendi, P., R. M. Colonques, and J. J. S. Monerris (2011). Patrones de transferencia de tecnología entre empresas matrices, filiales y rivales: su efecto sobre la competencia. *Economía industrial* (382), 93–100.

Miao, C.-H. (2013). On the superiority of fixed fee over auction in technology licensing. *The Manchester School 81*(3), 324–331.

Mukhopadhyay, S., T. Kabiraj, and A. Mukherjee (1999). Technology transfer in duopoly the role of cost asymmetry. *International Review of Economics & Finance 8*(4), 363–374.

Muto, S. (1993). On licensing policies in bertrand competition. *Games and Economic Behavior 5*(2), 257–267.

Prud'homme, D., M. von Zedtwitz, J. J. Thraen, and M. Bader (2018). "forced technology transfer" policies: Workings in china and strategic implications. *Technological forecasting and social change 134*, 150–168.

Rockett, K. E. (1990). Choosing the competition and patent licensing. *The RAND Journal of Economics*, 161–171.

Sánchez, M., J. A. Belso-Martínez, M. J. López-Sánchez, and A. Nerja (2021). Incentives to exclusive and non-exclusive technology licensing under partial vertical integration. *The Manchester School*.

Sandonís, J. and R. Faulí-Oller (2006). On the competitive effects of vertical integration by a research laboratory. *International Journal of Industrial Organization 24*(4), 715–731.

Sandonís, J. and R. Faulí-Oller (2008). Patent licensing by means of an auction: Internal versus external patentee. *The Manchester School 76*(4), 453–463.

Schmalz, M. C. (2018). Common-ownership concentration and corporate conduct. *Annual Review of Financial Economics 10*, 413–448.

Sen, D. and Y. Tauman (2007). General licensing schemes for a cost-reducing innovation. *Games and Economic Behavior 59*(1), 163–186.

Sohn, S. J. (2006). Choosing the partners in the licensing alliance. *Managerial and Decision Economics 27*(4), 251–260.

Spiegel, Y. et al. (2013). *Backward integration, forward integration, and vertical foreclosure*. Centre for Economic Policy Research.

Stamatopoulos, G. and T. Tauman (2009). On the superiority of fixed fee over auction in asymmetric markets. *Games and Economic Behavior 67*(1), 331–333.

Tauman, Y. and C. Zhao (2018). Patent licensing, entry and the incentive to innovate. *International Journal of Industrial Organization 56*, 229–276.

Wang, X. H. (1998). Fee versus royalty licensing in a cournot duopoly model. *Economics letters 60*(1), 55–62.

WIPO (2005). Exchanging Value: Negotiating Technology Licensing Agreements. Retrieved October 16, 2020 from url https://www.wipo.int/publications/es/details.jsp?id=291.

Yi, S.-S. (1998). Whom to license patented technology. *Managerial and Decision Economics 19*(3), 189–195.

Appendix

Proofs

In order to proof the former results, comparison between patent holders profits in both scenarios, exclusive licenses and cross-licenses, must be made. However, those profits are adjusted to every case made and evaluated in those different situations. All proofs are computed with Wolfram Mathematica software.

Proof of Result 1

For this result, there is a large innovation, $\varepsilon = c$, and symmetry among both patent holders, such as, neither patent holder have an stake in the downstream market, $\alpha_i = 0$, and the effect of innovation is the same, $\delta = 0$. Then, equations (11), (12), (25) and (26) evaluated in those terms are:

$$\Pi^E_{PH1} = \Pi^E_{PH2} = \frac{4c(2-c-d)}{(4-d^2)^2} \tag{A.1}$$

$$\Pi^{NE}_{PH1} = \Pi^{NE}_{PH2} = \frac{2(1-d)}{9(1+d)} \tag{A.2}$$

Then, any patent holder would prefer to sell just one license as long as $(A.1) > (A.2)$. Once is reordered the inequality, we have the next quadratic equation: $c^2 - c(2-d) + \frac{(1-d)(4-d^2)^2}{18(1+d)} < 0$. As expected, once the inequality is solved there are two values of c, however, one of them is above the maximum value of c, because $0 < c < 1$.

Thus, patent holders sell a private license, that is, $A.1 > A.2$, as long as $c > c_I = \frac{1}{6}(3(2-d) - \sqrt{\frac{(2-d)^2(1+d(9+2d(3+d)))}{1+d}})$, and a cross-license otherwise.

Proof of Result 2

For this result, there is a large innovation, $\varepsilon = c$, but an asymmetry among both patent holders is introduced in the sense that the patent holder 1 has an stake in a firm in the downstream market, $\alpha_1 > 0$ and $\alpha_2 = 0$. However, the effect of innovation is the same, $\delta = 0$. Then, equations (11), (12), (25) and (26) evaluated in those terms are:

$$\Pi^{E}_{PH1} = \frac{4c(2-c-d)+\alpha_1(2(1-c)-d)^2}{(4-d^2)^2} \tag{A.3}$$

$$\Pi^{E}_{PH2} = \frac{4c(2-c-d)}{(4-d^2)^2} \tag{A.4}$$

$$\Pi^{NE}_{PH1} = \frac{1}{9}\left(\frac{2(1-d)}{(1+d)}+\alpha_1\right) \tag{A.5}$$

$$\Pi^{NE}_{PH2} = \frac{2(1-d)}{9(1+d)} \tag{A.6}$$

In this case, because of the asymmetry, decisions of both patent holders are different. Patent holder 2 has no stake in a downstream firm, then its result remains as in Case I, also contrasted in Proof of Result 1, where it can be seen that $(A.1) = (A.4)$ and $(A.2) = (A.6)$. Then, it has to be proved when the patent holder 1 prefers exclusive licenses versus cross-licenses, that is, when $(A.3) > (A.5)$.

$$(A.3)-(A.5) = \frac{36c(1+d)(1-\alpha_1)(2-c-d)-(2-d)^2(1-d)\left(2(2+d)^2-(1+d)(5+d)\alpha_1\right)}{9\,(1+d)\,(4-d^2)^2} \tag{A.7}$$

Following the same procedure as before, the following quadratic inequality must be solved in order to obtain under what conditions $PH1$ decides to sell and exclusive license, then, once is reordered:

$$\begin{aligned} 36c^2(1+d)(1-a_1) \\ -36c(2-d)(1+d)(1-a_1) \\ +(2-d)^2(1-d)\left(2(2+d)^2-(1+d)(5+d)\alpha_1\right) < 0 \end{aligned}$$

Once the inequality is solved, two conditions must be fulfilled in order the patent holder 1 decides over an exclusive license.

1. $c > c_I$

2. $\alpha_1 < \alpha_{1(II)} = \frac{36c(1+d)(2-c-d)-2(1-d)\left(4-d^2\right)^2}{(1+d)(2-6c-3d+d^2)(6c-(2-d)(5+d))}$

Then $PH1$ sells an exclusive license as long as $c > c_I$, its stake in the downstream market is low enough, $\alpha_1 < \alpha_{1(II)}$. In any other case, $PH1$ would decide to sell cross-licenses.

Proof of Result 3

For this result, a further asymmetry is introduced where the patent holder 2 makes no innovation, such as, $\delta = \varepsilon$. Then, equations (11) and (25) must be reevaluated, such as:

$$\Pi^{E}_{PH1} = \frac{4c\,(2 - c\,(1-d) - d) + (1-c)^2(2-d)^2\alpha_1}{(4-d^2)^2} \tag{A.8}$$

$$\Pi^{NE}_{PH1} = \frac{1}{9}\left(\frac{2(1-(1-c)d)^2}{1-d^2} + (1-c)^2\alpha_1\right) \tag{A.9}$$

We are going to proved when the patent holder 1 prefers exclusive licenses versus cross-licenses, that is, when $(A.8) > (A.9)$

$$(A.8) - (A.9) = \frac{4c\,(2-c\,(1-d)-d)}{(4-d^2)^2} - \frac{2(1-(1-c)d)^2}{9(1-d^2)} + \frac{(1-c)^2\,(1-d)\,(5+d)\,\alpha_1}{9(2+d)^2}.$$

Following the same procedure as before, the following quadratic inequality must be solved in order to obtain under what conditions $PH1$ decides to sell and exclusive license, then, once is reordered:

$$\begin{gathered} 2c^2\left(18 - d\left(18 + d\left(2 - d\left(18 - 8d + d^3\right)\right)\right)\right) \\ -4c\,(2-d)\,(1-d)\,(9 + d\,(1-d)\,(1-d)\,(3+d)) \\ (2-d)^2(1-d)^2\left(2(2+d)^2 - (1-c)^2\,(1+d)\,(5+d)\,\alpha_1\right) < 0 \end{gathered}$$

The previous quadratic inequality throws the following results, where $PH1$ sells an exclusive license as long as $c^-_{III} < c < c^+_{III}$ and $0.442891 < d < 1$, or $c^-_{III} < c < 1$ if $0 < d < 0.442891$.

Then, these are the values of c required:

$$c_{III}^{-} = 1 - \frac{2d\left(7 + d^2 + d^4\right) - \sqrt{2(2-d)^2(1-d)^2\left(1+d\right)\left(18\left(1+d-2d^2\right) - \left(5+d\right)\left(2-10d^2-d^4\right)\alpha_1\right)}}{36 - 2d\left(18 + d\left(2 - d\left(18 - 8d + d^3\right)\right)\right) - (2-d)^2(1-d)^2\left(1+d\right)\left(5+d\right)\alpha_1} \tag{A.10}$$

$$c_{III}^{+} = 1 + \frac{2d\left(7 + d^2 + d^4\right) - \sqrt{2(2-d)^2(1-d)^2\left(1+d\right)\left(18\left(1+d-2d^2\right) - \left(5+d\right)\left(2-10d^2-d^4\right)\alpha_1\right)}}{36 - 2d\left(18 + d\left(2 - d\left(18 - 8d + d^3\right)\right)\right) - (2-d)^2(1-d)^2\left(1+d\right)\left(5+d\right)\alpha_1} \tag{A.11}$$

Both are positive, that is, $c_{III}^{-}, c_{III}^{+} > 0$. However, there are conditions under which c_{III}^{+} is fewer than 1, and is when $0.442891 < d < 1$.

Proof of Result 4

In this case, there is a small innovation, and we analyze the extreme scenario for $\varepsilon = 0$. Then, cross-license is the chosen by both patent holders. To prove that, equations (29) and (30) must be positive. Note that both equations only differ in α_1 and α_2. Generally, we need to check that

$$\frac{(1-c)^2(1-d)[2(2+d)^2 - (1+d)(5+d)\alpha_i]}{9(1+d)(2+d)^2} > 0.$$

Every term is positive by inspection except the last term in the numerator. Considering that $\alpha_i = 1$, we have that in order to be positive, $2(2+d)^2 - (1+d)(5+d) > 0$. Once is computed and reordered, we have that $d^2 - 2d + 3 > 0$. If d is evaluated in its maximum, $d = 1$, it can be easily seen that the inequality is fulfilled because $1 - 2 + 3 = 2 > 0$.

Proof of Proposition 1

For this proposition we want to prove which patent holder's fee is higher in the case of exclusive licenses, that is, if $F_1^E > F_2^E$, or otherwise. Then,

$$F_1^E - F_2^E = \frac{4\delta\left(\left(2-d\right)\left(1-c\right) + 2\varepsilon - \delta\right)}{\left(4-d^2\right)^2} > 0$$

Every term between brackets is positive by definition. The only terms that may be negative is $2\varepsilon - \delta$; however, by definition $\varepsilon \geq \delta$. Thus, proving that $F_1^E > F_2^E$.

Similarly, under cross-licensing scenario, we can derive if $F_1^{NE} > F_2^{NE}$, or otherwise. Then,

$$F_1^{NE} - F_2^{NE} = \frac{\delta\left(2\left(1-c\right)+\left(2\varepsilon-\delta\right)\right)}{9} > 0$$

Every term between brackets is positive by definition. The only term that may be negative is $(2\varepsilon - \delta)$; however, by definition $\varepsilon \geq \delta$. Thus, proving that $F_1^{NE} > F_2^{NE}$.

This proposition shows that the payment (fee) for the best innovation is always higher.

Proof of Proposition 2

In order to prove proposition 2, $\frac{\partial c_I}{\partial d} < 0$. Figure 2 shows this relationship, as well as the table does numerically.

$$\frac{\partial c_I}{\partial d} = \frac{-(4d^5+2d^4-11d^3-16d^2-11d+14)}{6(1+d)\sqrt{(2-d)^2(1+d)(1+d(9+2d(3+d)))}} - \frac{1}{2} < 0$$

To prove that the former inequality holds, can be made pointing out different values of product differentiation, d. The first term is always negative as long as $d < 0,78$. For higher values, the first term is positive, but never higher than $\frac{1}{2}$. For example, when evaluated in the maximum of $d = 1$, the value of the first term is $\frac{1}{9} < \frac{1}{2}$.

Proof of Proposition 3

When products are perfect substitute, equation (A.1) and (A.2) are as follows:

$$\Pi_{PH1}^{E} = \Pi_{PH2}^{E} = \frac{4c(1-c)}{9}$$

$$\Pi_{PH1}^{NE} = \Pi_{PH2}^{NE} = 0$$

Then, it is easily seen that the choice of any patent holder under this situation is always sell just one license.