Does Stock Return Predictability Affect ESO Fair Value? *

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Abstract

We study the effects of the predictability in stock returns for the fair value of American Executive Stock Options (ESOs). By assuming a trending Ornstein-Uhlenbeck process for stock returns, we solve for the executive's optimal exercise policy using a methodology based on the least-squares Monte Carlo algorithm. We find that executives tend to wait longer the higher the predictability, independently of the executive's asset menu. We also analyze the implications of following the FAS123R proposals in the computation of the ESO's fair value.

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1 Introduction

Executive Stock Options (ESOs hereafter) are typically an important part of executive compensation packages. Hall and Murphy (2002) report that in 1999, 94% of S&P 500 companies granted ESOs to their top executives, and these grants accounted for 47% of total pay of S&P 500 CEOs. ESOs are American-style stock options modified for incentive reasons. Thus, ESOs cannot be sold or transferred, although partial hedge is possible by trading correlated assets. In addition, they can only be exercised after ending the vesting period. Consequently, standard methods for valuing American options are not directly applicable and a growing literature has been searching for a solution to the issue of ESO valuation.

A common assumption in this literature is that the stochastic process driving the dynamics of the underlying stock price can be represented as a geometric Brownian motion (GBM) process. Under this assumption the stock returns turn out to be independent and identically distributed normal variates. However, there is by now a wide agreement in recognizing that this assumption does not fit well the empirical evidence concerning time series returns. As summarized, for instance, in Taylor (2005), there are three stylized facts characterizing the returns distribution at daily frequencies. First, the distribution of returns is not normal, it has typically fat tails with a high peak (leptokurtosis). Second, the autocorrelation between daily returns (predictability) is extremely low but statistically meaningful and third, there is positive dependence between squared returns. The impact on the ESO valuation of the last issue has been analyzed in Brown and Szimayer (2008) and León and Vaello-Sebastià (2009). Nonetheless, to the best of our knowledge, modeling return autocorrelation in the stock price dynamics equation has been neglected in the literature of ESO valuation. There are two reasons for that. On the one hand, the constant drift term plays no role in the Black-Scholes formula and, on the other hand, the evidence of time dependence in returns is very weak. Nonetheless, as we will show later, very low levels of autocorrelation generate significant biases in pricing ESOs.

Campbell et al. (1997) report autocorrelations for CRSP stock returns for both equally-

weighted and value-weighted indexes. They find a statistically significant positive serial correlation at the first lag, which is robust across subsamples. The weekly and monthly return autocorrelations also exhibit a positive and statistically significant value at first lag over the entire sample and for all subsamples. Poterba and Summers (1988) also find negative autocorrelations for monthly CRSP data. Taylor (2005) surveys the evidence of predictability for several return series such as indexes, equities, futures and currencies for different time horizons and finds that these autocorrelations are very small but significant.[1] For instance, more than 90% of the 600 autocorrelation estimates are between -0.05 and 0.05. This low but statistically significant autocorrelations imply that stock returns are predictable.

This paper aims to analyze the effects of predictability in that restricted sense, namely, return predictability comes only from their time series statistical properties. Therefore, we shall focus on univariate processes although this can be extended to the multivariate case by introducing some additional state variables along the lines, among others, of Lo and Wang (1995). They discuss the effect of predictability on the market value of European options using both univariate and multivariate trending Ornstein-Uhlenbeck (TOU) processes, which are (multivariate) continuous time AR(1) processes. They convincingly argue that the predictability of stock returns may affect the prices of options written on those stocks, even though predictability is typically induced by the drift, which does not enter into the option pricing formula. The rationale is that, unlike the GBM process with a constant drift underlying the standard Black-Scholes formula, the sample variance of discretely-sampled returns is not an appropriate estimator of the instantaneous variance when returns are predictable.

Here we wish to apply their insights to the problem of ESO valuation. As it is well known, we can distinguish among three possible ESO valuations depending on the restrictions faced by the holder. The subjective value by the restricted executive, the objective value (firm's ESO cost) by the issuing firm, which is unrestricted but obliged to follow the executive's exercise policy and the market (risk-neutral) value by an unrestricted holder.

[1] More empirical references about the autocorrelation of returns can be seen in Taylor (2005).

Although we are interested on the implications of predictability in stocks returns for the objective valuation of ESOs, we will also analyze the subjective valuation since this will allow us to infer the executive's exercise policy necessary to obtain the objective value.

The rest of the paper is organized as follows. Section 2 presents the setting for ESO valuation. We distinguish between the case in which the ESO cost is driven by only the stock price, or the one state-variable (S1) model, which corresponds to the framework put forward initially by Hall and Murphy (2002), and the case in which the ESO cost is also affected by the availability of a market portfolio, or the two state-variable (S2) model, introduced by Cai and Vijh (2005). Section 3 presents our results concerning the S1 model. In this section we perform an extensive sensitivity analysis examining the size and sign of the bias that is incurred when the objective ESO value is obtained by assuming a GBM process for the stock price. In Section 4, we include a market portfolio as an additional investment for the executive's wealth and show the differences in the results because of its introduction. In Section 5 we apply our framework to evaluate the bias induced by the computation of the expected firm cost using the FAS123R method. Finally, Section 6 concludes.

2 Model description

Since the purpose of this paper is to analyze how the firm's ESO cost is affected by predictability, we shall previously need to obtain the executive's exercise policy, which comes from solving the ESO subjective valuation. Then, by using the risk-neutral measure and the threshold price from the executive's exercise policy, the ESO objective value follows. See, for instance, Kulatilaka and Marcus (1994), Hall and Murphy (2002) or Ingersoll (2006).

The subjective valuation method is based on the certainty-equivalence principle introduced by Lambert et al. (1991). It identifies the subjective ESO value with the amount of cash that, delivered at the grant date and invested until ESO maturity, reports the same expected utility to the executive than holding the ESO. We assume the executive has an

initial wealth, W_0, of which a proportion α is held in restricted stocks of the company that cannot be sold until the ESO maturity. The larger α, the more related is the executive's wealth to the firm's market value and so, the higher the specific risk supported. The remainder of the initial wealth, $(1-\alpha)W_0$, is distributed between the market portfolio and the risk-free asset in proportions η and $1-\eta$ respectively. Moreover, the executive is granted with an amount of N at-the-money ESOs with an exercise price of K. Thus, the executive wealth at maturity, denoted by T, is conditioned on the ESO exercise's date. Let us denote by $W_{T|t}$ the executive's wealth at maturity conditional on the ESO exercise at time $t \leq T$. Then, if the ESOs are not exercised until maturity, the executive's terminal wealth is

$$W_{T|T} = \alpha W_0 e^{\bar{q}\,T}\frac{S_T}{S_0} + \big(1-\alpha\big)W_0\left(\eta\frac{M_T}{M_0} + \big(1-\eta\big)e^{r_f T}\right) + N\big(S_T - K\big)^+ , \qquad (1)$$

where S_T and M_T denote the firm's stock and market portfolio prices at date T. The yearly stock dividend yield is denoted as $\bar{q}$ and it is assumed to be reinvested in restricted stocks.[2] The main point here is that the executive does not receive any payment from restricted stocks until T. Finally, r_f denotes the yearly risk-free rate.

Alternatively, if the ESOs are exercised at time $t < T$, the executive will invest the ESO payoffs in a certain combination of the market portfolio and the risk-free asset. Then,

$$\begin{aligned} W_{T|t} &= \alpha W_0 e^{\bar{q}\,T}\frac{S_T}{S_0} + \big(1-\alpha\big)W_0\left(\eta\frac{M_T}{M_0} + \big(1-\eta\big)e^{r_f T}\right) + \\ &+ N\big(S_t - K\big)^+\left(\eta\frac{M_T}{M_t} + \big(1-\eta\big)e^{r_f (T-t)}\right), \end{aligned} \qquad (2)$$

where η is the share of market portfolio in the executive's unrestricted wealth. Notice that, despite the ESO exercise, the executive must maintain his restricted stocks until T. We shall assume that M_t follows a GBM process driven by

$$dM_t = \mu_M M_t dt + \sigma_M M_t dZ_{M,t}, \qquad (3)$$

[2]For simplicity, this model assumes that the investment in the market portfolio is done through an index fund, which does not pay dividends since it reinvests all dividend proceeds.

while $p_t \equiv \ln S_t$ follows a trending Ornstein-Uhlenbeck (TOU) process:

$$dp_t = \big(\theta - \kappa\big(p_t - \theta t\big)\big)\, dt + \sigma dZ_{S,t} \ , \tag{4}$$

where $\kappa \geq 0$ and $dZ_{S,t}dZ_{M,t} = \rho dt$. Notice that the TOU process includes the GBM process as a particular case when $\kappa = 0$. We can rewrite equation (4) as

$$d\big(p_t - \theta t\big) = -\kappa\,(p_t - \theta t)\, dt + \sigma dZ_{S,t} \ . \tag{5}$$

Note that, when p_t deviates from its trend θt, it is pulled back at a rate κ, the speed of mean reversion, proportional to its deviation.

It is also interesting to describe the population moments of stock returns in discrete time when the stock price follows a continuous TOU process. Let the discrete time returns at time t for a time period of length h be defined as[3]

$$r_{t,h} \equiv \ln\big(S_t\big) - \ln\big(S_{t-h}\big). \tag{6}$$

Then, the mean of $r_{t,h}$ equals θh. When h is measured as a fraction of a year, θ measures the annual mean of log return. Similarly, the variance, σ_r^2, and the autocorrelation at lag one, $\rho_r(1)$, of the discrete time returns are related to the speed of mean reversion according to the following expressions:

$$\sigma_r^2 = \frac{\sigma^2}{\kappa}\big(1 - e^{-\kappa h}\big), \tag{7}$$

$$\rho_r(1) = -\frac{1}{2}\left(1 - e^{-\kappa h}\right), \tag{8}$$

where to simplify notation, the dependence of the former definitions on h has been omitted. Now, as equations (7) and (8) make clear, when κ increases, σ_r^2 decreases but $\rho_r(1)$ increases in absolute value. Finally, observe that the value $\kappa = 0$ covers the case of a GBM process for the stock price, for which autocorrelations of log-returns are zero at all lags.

[3]For a formal proof, see Appendix A.

We consider an executive with a standard power utility function defined over his terminal wealth:

$$U\big(W_T\big) = \begin{cases} \dfrac{W_T^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln\big(W_T\big) & \text{, otherwise} \end{cases} \tag{9}$$

where $\gamma > 0$ is the coefficient of relative-risk aversion.

The executive maximizes the expected utility of his terminal wealth allocating his outside wealth between the market portfolio and the risk-free asset to their optimal sizes, denoted as η^* and $1-\eta^*$ respectively, and selecting his optimal ESO exercise date, t^*. Therefore, the executive's problem may be expressed as

$$\max_{\eta,t} \mathbb{E}_0\left[U\big(W_{T|t}\big)\right]. \tag{10}$$

In short, we assume an asset allocation framework for a buy-and-hold investor (the executive) regarding both risk-free and market asset holdings. This is in the same line of Barberis (2000), who also incorporates the effects of predictability. Of course, a more realistic situation would be that allowing η to vary across the investment horizon period. Nevertheless, this strategy is quite hard to compute since we have to consider at the same time the possibility of exercising the executive's ESO package at any moment after the vesting period. Hence, trying to implement both situations in our model are beyond the scope of our study.

To obtain the subjective valuation of ESO, V^{SUB}, we proceed as follows. Suppose that, at the grant date, a non-restricted amount of cash CE is delivered in place of each ESO. Then, the total executive's wealth at maturity is

$$W_T^{CE} = \alpha W_0 e^{\bar{q}T}\frac{S_T}{S_0} + \big[\big(1-\alpha\big)W_0 + N \times CE\big]\left(\eta\frac{M_T}{M_0} + \big(1-\eta\big)e^{r_f T}\right). \tag{11}$$

Thus, to calculate V^{SUB} we only need to find the amount CE that provides the same expected utility than holding one ESO. This is achieved through the following quadratic

distance minimization:

$$\min_{CE}\left(1-\frac{\mathbb{E}_0\left[U\left(W_T^{CE}\right)\right]}{\mathbb{E}_0\left[U\left(W_{T|t^*}\right)\right]}\right)^2. \tag{12}$$

The methodology used to solve the program described by equation (10) is a modified version of the least-squares Monte Carlo (LSMC) algorithm of Longstaff and Schwartz (2001) for pricing path dependent derivatives. It consists on two stages. In the first stage, equation (12) is solved for a grid of values of the parameter η and then, in a second stage, the optimal value of η is obtained. Finally, once we have obtained the executive's threshold price from his optimal exercise policy, we can solve for the ESO objective value, V^{OBJ}. For a detailed description of this procedure see Appendix B.

3 The one state-variable framework

In this section we focus on V^{OBJ}, that is obtained by using the risk-neutral measure to value an American call option for which the threshold price is defined by the executive's exercise policy. We proceed in two stages. At first, we assume executives will have no availability to a market portfolio to allocate their unrestricted wealth, so that, the parameter η is restricted to be equal to zero. We have denoted this framework as the $S1$ model, since the stock price is the only state-variable to obtain V^{OBJ}. We consider this simplified version of the model because the computational cost is much lower (we do not have to find η^*) and the main results are also valid in the more general case. Nonetheless, in Section 4 we extend our framework to include the availability of the market portfolio, in addition to the risk-free asset and the firm's stock, giving place to the S2 model to get V^{OBJ}.

Following the benchmark case by Hall and Murphy (2002), we consider a representative executive that owns five million dollars in initial wealth and is granted with 150,000 stock options issued at-the-money with $K = \$1$. We maintain as the true data generating process (DGP) for the stock price the TOU process, as described in equation (5). We explore how V^{OBJ} changes as a result of changes in each of the three parameters characterizing the above process. Our yearly benchmark values for these parameters are: $\kappa = 0.50$ for the

speed of adjustment, $\theta = 0.10$ for the long-run drift coefficient, and $\sigma = 0.30$ for the diffusion coefficient. Other parameter values are $r_f = 0.06$ and $\bar{q} = 0$.

According to equation (8), notice that, for our benchmark value of $\kappa = 0.50$, $\rho_r(1)$ is equal to -0.020 in monthly terms ($h = 1/12$), -0.005 in weekly terms ($h = 1/52$) and -0.001 in daily terms ($h = 1/360$). This low value of $\rho_r(1)$ can be found in the empirical evidence, see Table 4.8 in Taylor (2005). Although it may seem low, it will help us to highlight, in a later section, the mispricing that can be incurred in the computation of V^{OBJ} if a standard GBM process, that exhibits independent returns, is postulated erroneously as the true DGP for the stock price.

Now, we are mainly concerned with the consequences of the evidence of stock return predictability on V^{OBJ}. A greater return predictability is, in the present setting, a higher size of $\rho_r(1)$. Since the autocorrelations for stock returns found in empirical studies are generally so low, the GBM process appears to be a good approximation for the DGP of stock prices. However, under the true DGP these small autocorrelations come from a mean reversion process such as the TOU process in equation (5).

All this suggests that we are interested in analyzing how changes in $\rho_r(1)$, and hence changes in κ, affect V^{OBJ}. Therefore, to control for the change in the volatility of the discrete time returns, σ_r, resulting from a change in κ, we have also adjusted the diffusion coefficient, σ, in order to keep unchanged σ_r. To be sure that nothing is lost by imposing this restriction, we have also examined the case in which σ is not adjusted, so that changes in κ also modify σ_r. It has turned out that the differences between both cases, in terms of objective valuation, average exercise dates and other features of interest, have not been significant. Thus, we focus on the case in which σ is adjusted so as to keep σ_r constant.

Considering the above adjustment, a higher κ implies only an increase in the size of $\rho_r(1)$. The range of values for κ, between 0 and 1, implies a range of first-order autocorrelations between 0 and 0.04 in monthly terms, which are representative of the values found in the empirical literature cited previously. For the diffusion coefficient, σ, the values range from $\sigma = 0.25$ to $\sigma = 0.45$, and for the long-run drift coefficient, θ, the values range from $\theta = 0.065$ to $\theta = 0.15$.

3.1 Discussion of results

We study now the impact of changes in κ, σ and θ in the ESO valuation. Figure 1 summarizes our main findings concerning the behavior of V^{OBJ} as a function of these parameters. In all simulations we have considered the case of a 10-year ESO with a vesting period of 3 years, which is common in both practice and literature. The no vesting case is qualitatively similar and it is omitted to save space.

[Insert Figure 1 here]

Several features can be observed in this figure. First, as κ increases, which means a higher predictability, V^{OBJ} seems to converge independently of the size for γ and α (panel A). Second, an increase in either σ (panel B) or θ (panel C) supposes a higher value of V^{OBJ}. Thus, as one would expect, the objective valuation increases with the diffusion coefficient, σ, and this is independent of the values considered for γ and α. Clearly, a higher σ implies a higher σ_r, which leads to higher stock prices in relatively shorter periods of time.

However, the positive relationship between θ and V^{OBJ} may be somewhat surprising. According to the Black-Scholes model, the expected returns are irrelevant for option pricing. Nonetheless, in the present framework θ affects the executive's exercise behavior, and hence the ESO objective valuation. With greater expected returns, a higher θ, it will be more profitable for the executive to hold the ESOs for a longer period. Then, higher expected returns mitigate the executive's suboptimal early exercise.

By definition, V^{OBJ} depends on the executive's exercise policy. This exercise policy can be described in terms of either the threshold price or the expected exercise time, τ^*. We next examine the former results in connection with the obtained values for τ^*. The relevant results are presented in Tables 1 and 2.

As Table 1 shows, higher values of κ imply higher values of τ^* for all considered combinations of γ and α. Of course, either a higher γ or α makes the executive to exercise comparatively earlier but, as κ increases, these differences become very small. Furthermore, the expected exercise time is comparatively lower for the equivalent GBM

process, for which $\kappa = 0$, in all cases.[4]

[Insert Table 1 here]

To understand the role of predictability in the observed behavior for V^{OBJ} and τ^*, we examine Figure 2 in which we have plotted several simulated paths for the firm's stock price under alternative values of κ.[5] We have also plotted the long-run trend of the process by a discontinuous line. As it can be observed, as κ increases from 0.05 to 1.00 ($\rho_r(1)$ changes from -0.002 to -0.04), the simulated paths tend to revert faster to that long-run trend, avoiding the paths become very deep in-the-money. Then, as long as the time to maturity is fixed to 10 years, the average values achieved by the TOU process are comparatively lower when κ is higher, thus the executive has to pospone the ESO exercise, increasing consequently the value of τ^*.

Therefore when prices are weakly mean reverting, so that they resemble a GBM process closely, the executive tends to exercise at relatively high prices. Meanwhile, as the predictability of the process increases, through a higher κ, the executive waits longer on average (see Table 1) but the average value achieved by the firm's stock price turns out to be comparatively lower. This has the effect of reducing V^{OBJ} for a relatively low risk averse executive and increasing V^{OBJ} for higher risk averse executives. Of course, as one would expect, when γ and α are relatively low, the executive tends to wait longer so as to get a higher threshold price which raises V^{OBJ}.

[Insert Figure 2 here]

Table 2 exhibits the values of τ^* for different values of σ and θ. This table also displays the case in which the stock price is driven by an equivalent GBM process.[6] We can appreciate

[4]Notice that this is so because the diffusion coefficient of the TOU process in equation (5) has been adjusted to maintain constant the observed volatility of discrete returns.

[5]To isolate the effects of predictability, the monthly volatility for discrete returns remains constant to 0.0866 in all cases and the four graphics hold the same random numbers. Then, the observed differences only depend on κ. The other relevant parameters have been set to $\theta = 0.10$ and $\sigma = 0.30$ in all simulations.

[6]This GBM process for stock prices has been constructed in such a way that the mean and variance of the discrete time returns are the same than the corresponding ones for the true TOU process. More

that τ^* decreases regarding σ but increases according to θ. Once again, the expected exercise times are comparatively lower for the corresponding equivalent GBM processes.

[Insert Table 2 here]

As Table 2 shows, higher values of σ lead to lower values for τ^*, for every value of γ and α. This can be explained along the lines used to understand its positive impact on V^{OBJ}. On the other hand, a higher value of θ, by increasing the long-run trend of discrete returns, makes executives to be willing to wait longer in the expectation of higher threshold prices for their ESOs.

Finally, we have also explored how the former results change with a different value for κ.[7] In particular, a value of $\kappa = 0.05$ that makes TOU processes to resemble the equivalent GBM processes, produces more variability in V^{OBJ} and τ^* as a result of changes in γ and α.

3.2 Cumulative probabilities

We have shown previously that as the predictability of stock returns increases, executives find optimal to wait longer for exercising the ESOs. This relationship has been inferred from the behavior of the expected exercise time, τ^*. However, this expectation hides some very interesting pieces of information concerning the probability of the ESO exercise. To complete our understanding of the executive behavior, we have computed the probability that ESOs will be exercised before some given date $t \leq T$. This has been done as follows. In our simulations for the sensitivity analysis, we have considered a monthly frequency for the ESO exercise dates. This means that, for a 10-year ESO, there is a total of 120 possible exercise dates. Thus, for each of these exercise dates and using a total number of $200,000$ paths (including antithetics), we have computed the ratio

$$\pi_s = \frac{\#\text{ of paths exercised at time step } s}{\text{total }\#\text{ of paths}}.$$

details are given in Appendix A.

[7] The results are available upon request.

Then, the product

$$(1-\pi_1)\times(1-\pi_2)\times\ldots\times(1-\pi_{t-1})\times\pi_t$$

will provide the probability that ESOs have been exercised at time t. Finally, by straightforward aggregation, we obtain the cumulative probability, namely the probability that the ESO exercise date will be lower than or equal to some given date t. To smooth out the resulting probability distribution, we have replicated the former procedure a total of 50 times using 50 different seeds to generate the random numbers, so that the reported cumulative probabilities are the average of them.

The cumulative probability distributions have been obtained for each of the relevant parameters characterizing the TOU process. Finally, to compare those results with those obtained under no predictability we have also computed the cumulative probabilities of early exercise for each of the corresponding equivalent GBM processes.

[Insert Figure 3 here]

Panel A of Figure 3 represents the probability that the exercise date will be lower than or equal to some given date t as a function of κ. Clearly, as κ increases, this probability is lower for any date before the expiration date. Thus, in comparison with the GBM process, the executive always waits longer for exercising the ESO. Panel B depicts the case for different values of σ when the stock price is driven by either a TOU or its equivalent GBM process. Generally, a higher volatility leads to earlier exercise (lower τ^*) and specifically when the level of predictability is low, as in a GBM process. A larger value of σ means that i) the executive's wealth becomes more volatile due to the ESOs but also because of the restricted stocks, and ii) the option may be deep in-the-money with a higher probability. Boths facts suppose an earlier exercise of the ESO, according to panel B findings. Finally, panel C shows the same situation for several values of θ and it does not deserve further comments.

3.3 The objective bias

In this subsection, we analyze the sign and size of the biases that can occur because of a misspecification of the underlying process for the stock price. As we have mentioned before, the literature has typically postulated a GBM process for the stock price. This assumption implies that the discrete time returns will follow a random walk with drift. But, if the log-price were to follow a TOU process, as we have assumed in our computations, the discrete time returns would follow an ARMA$(1,1)$ process with trend. Thus, the expected firm cost V^{OBJ} would be biased as a result of an erroneous choice for the stochastic process of the price dynamics. We designate this bias as[8]

$$Bias_1 = \frac{V_F^{OBJ} - V^{OBJ}}{V^{OBJ}} \times 100 \ , \tag{13}$$

where V_F^{OBJ} denotes the objective value computed under the false hypothesis that prices are driven by a GBM process. We maintain the notation V^{OBJ} to denote the objective value computed under the true process. We have calculated this bias for different values of κ and the results are displayed in panel A of Figure 4. When $\gamma = 2$ the objective bias is generally positive and increasing with predictability. Note that the size of the bias is not high in this case, meanwhile for $\gamma = 4$ the sign of the bias turns out to be negative and its size becomes quite substantial even for moderate degrees of undiversification.

[Insert Figure 4 here]

The results of performing the same analysis but for different values of σ are depicted in panel B of Figure 4. We have found the same pattern described before, namely, when the executive has a low degree of relative risk aversion and is well diversified, V_F^{OBJ} tends to overstate the true cost but not much. As the degree of relative risk aversion increases and the executive becomes worse diversified, V_F^{OBJ} understates the true cost in such a way that the size of the bias turns out to be above 10% for most values of σ.

[8]Notice that the objective value can be computed either using a conventional binomial model or, as we do, by using the LSMC method. We illustrate the size of bias using this latter method, but the results should be similar with other methods of computing V^{OBJ}.

We have also computed the objective bias for several alternative values of θ. The results are shown in panel C of Figure 4. Along the lines of the previous results, the sign of the bias is positive when the degree of risk aversion is low and the executive is relatively well diversified. However, as the degree of risk aversion increases and the executive becomes less well diversified, the sign of the bias becomes negative and its size is also well above 10% for almost all cases considered.

Definitively, we can conclude that for high degrees of relative risk aversion, the objective value assuming a GBM process can understate substantially the true firm's ESO cost. Finally, the size of the bias does not appear to be significantly affected by the length of the vesting period and it has not been reported here.

4 The two state-variable framework

Here we explore how the executive's exercise policy and hence, the ESO objective value is modified by including a market portfolio. This extended set-up is assumed to provide the true objective value, V^{OBJ}. As in the previous section, we consider a representative executive that owns five million dollars in initial wealth and is granted with 150,000 10-year stock options issued at-the-money with $K = \$1$. We maintain the other parameters at their previous values, namely, $\bar{q} = 0$ and $r_f = 0.06$. The parameters characterizing the dynamics of the market portfolio are $\mu_M = 0.10$ and $\sigma_M = 0.20$. The initial value of the market portfolio will be set to one and the correlation between the innovations in the stock and the market portfolio, ρ, will be set to 0.50. Basically, to obtain V_{S2}^{OBJ} we solve the executive's problem for a grid of values of η in order to obtain the value η^* which maximizes the executive's utility.[9] This makes $S2$ model more demanding in computational terms than the $S1$ model, which doesn't need finding η^*.

The results are summarized in a ratio that measures the difference between the objective value in the one state-variable model, V_{S1}^{OBJ}, and the objective value in the two

[9] Appendix B provides more details on the numerical algorithm.

state-variable model, V_{S2}^{OBJ}, as a percentage of the latter. Hence,

$$Bias_2 = \frac{V_{S1}^{OBJ} - V_{S2}^{OBJ}}{V_{S2}^{OBJ}} \times 100 \ . \tag{14}$$

Recall that V_{S2}^{OBJ} measures the true cost of ESOs for firms. Cai and Vijh (2005) have shown that this ratio is positive when the stock price is driven by a GBM process. The reason is that, under model S1, executives can allocate their unrestricted wealth only in the risk-free asset whereas under model S2, they can also invest in the market portfolio. Since this alternative becomes more attractive, the average exercise times will be generally lower in the S2 model.

Our results are presented in Table 3. We have restricted our attention to the impact of changes in predictability, so that we only report the results concerning changes in κ. In this regard, we have maintained our choice of adjusting σ such that the volatility of discrete time stock returns is not affected.

[Insert Table 3 here]

For the case of $\gamma = 4$, V^{OBJ} increases with κ in both S1 and S2 models.[10] For $\kappa \geq 0.25$, which implies a first order autocorrelation higher than 0.0103 in absolute value, the firm's ESO cost for both models becomes indistinguishable. To get some intuition for this result, recall that the market portfolio is driven by a GBM process. Then, when executives observe a higher predictability in the firm's stock returns, they do not see any advantage in exercising earlier to put the proceeds into the market portfolio. As a result, the average exercise time and the threshold price are essentially the same in both the S1 and the S2 models. This result becomes quite relevant to the extent that V_{S1}^{OBJ} can be obtained at a lower computational costs than V_{S2}^{OBJ}.

Finally, we have also studied how the share of the market portfolio is affected by changing the predictability of the firm's stock returns. The results are displayed in Table

[10] ESO cost for firms behaves differently when $\gamma = 2$. In the S1 model, V^{OBJ} decreases with κ. In the S2 model, V^{OBJ} increases with κ. However the behavior of the percentage bias is completely analogous, namely, as predictability increases the bias decreases until being negative.

4, which focuses on the impact of different values of κ and distinguishes between the 3-year vesting and the no vesting cases. It is shown that a greater predictability leads the executive to put more wealth in the market portfolio and less into the risk-free asset. In fact, the executive puts even more weight in the market portfolio for a higher undiversification level, α, when the predictability of the stock returns is above a given threshold (see the last two columns in Table 4). Note that this pattern is independent of the vesting period length.

[Insert Table 4 here]

We have seen that a greater predictability in stock returns leads executives to hold a higher share of their unrestricted wealth in the market portfolio and also to wait longer for exercising the ESOs. From an intertemporal point of view this means that executives are substituting their holdings of the risk-free asset by ESOs, because of their higher expected return which is subject to less uncertainty as predictability increases.

5 The FAS 123R bias

As a result of the increasing relevance of ESOs in managers compensation packages, the International Financial Reporting Standard (IFRS) and the Financial Accounting Standards Board (FASB) have issued several requirements for a fair valuation of those expenses. Since the methods suggested are very similar, we shall focus in the FASB's recommendations. In its Financial Accounting Standard Board (2004) No 123 revised statement, FAS 123R, the FASB requires firms to disclose the method used to estimate the grant-date fair value of their ESO compensation packages. Among the valuation techniques that the FAS 123R considers acceptable, it appears the Black-Scholes (BS) and binomial models.

The BS formula is appealing because of its simplicity. However, there are some features of ESOs that are not well captured by this formula. In particular, the BS model assumes European-style options although ESOs are American options which are typically exercised before maturity. To consider this fact, the FAS 123R (paragraph A26) explicitly requires this fair value be computed by replacing the ESO expiration date, T, by its expected

exercise time or expected life, τ^*. Specifically, the ESO price should be calculated as

$$V^{FAS} = BS(\tau^*). \tag{15}$$

This procedure left firms with the problem of estimating τ^*. We can distinguish two possibilities in this regard. In the first place, the firm could use historical data about the executives' exercise behavior. A second possibility is modeling the early exercise using some binomial model that captures the executives' exercise policy and then using the resulting expected life as an input in the BS model.

Kulatilaka and Marcus (1994) argue against the use of the first method because its dependence on a particular realization of stock prices. So, it seems much better to use the second alternative although it is not free of criticisms either. In particular, firms typically assume that stock prices are driven by a GBM process. If the true model is a TOU process, the implied average exercise date would be biased and this would imply a corresponding bias in the fair value.

Therefore, it is interesting to compare the firm's ESO cost under the TOU process, V^{OBJ}, with the BS value obtained under the erroneous DGP that prices are driven by a GBM process, $BS(\tau_F^*)$, where τ_F^* denotes its corresponding expected life. Specifically, we will analyze the following percentage bias:

$$Bias_3 = \frac{BS(\tau_F^*) - V^{OBJ}}{V^{OBJ}} \times 100. \tag{16}$$

We have focused on the behavior of this bias for different values of the parameter κ. We have taken as τ_F^* the value for the case of $\kappa = 0$, which is the equivalent GBM process for the stock price. Furthermore, the volatility of the discrete period returns, σ_r, has been kept constant by a suitable adjustment of the diffusion coefficient, σ, as in Section 3. As a result, the appropriate value of the firm's stock return volatility to be plugged into the BS formula is 30% for all values of κ. In our analysis we have assumed a 10-year ESO for the cases of no vesting and a 3-year vesting period. The results are depicted in Figure 5

for the S1 model.

[Insert Figure 5 here]

We can see that the bias is generally negative. Hence, the estimation of the ESO expected life using a GBM process for the stock price, when TOU is the true process, understates the true expected cost systematically. This result is related to the behavior of the expected exercise date already discussed in subsection 3.1. We have shown that executives tend to wait longer for exercising ESOs when predictability increases, so that the lowest τ^* is achieved when $\kappa = 0$. This, in turn, implies a low BS value for the ESO in comparison with the firm's ESO cost under the TOU process. We show that the size of this bias increases with the predictability of the process when $\gamma = 2$, but decreases when $\gamma = 4$. This is explained by the different behavior of V^{OBJ} for each value of the relative risk aversion parameter.

We end this section by reporting briefly the results for the FAS bias when the S2 model is considered. By using a similar procedeure to that described for the S1 model, we find that the inclusion of a market index only increases the undervaluation incurred when the FAS123 recommendations are used.

6 Conclusions

We have shown that predictability matters for valuing American ESOs from the firm's perspective. The objective value, or firm's ESO cost, is biased if one assumes erroneously that stock prices are driven by a geometric Brownian motion (GBM) instead of the true process driven by a trending Ornstein-Uhlenbeck (TOU) process. This bias is significant even for relatively low values of the first order autocorrelations. The executive performs his ESO exercise under two alternative asset menu settings. One of them consists only of the risk-free asset and the other one is extended by also including the market portfolio. When increasing the predictability (κ) the pricing differences between both approaches vanish. Moreover, independently of the executive's wealth composition, he waits longer for the ESO exercise the higher the predictability. Finally, we examine the consequences

of predictability for the FAS123 proposals. When the erroneous GBM process is used for prices, it generates an overvaluation of the firm's ESO cost, even for moderate low levels of predictability.

For further research, it would be interesting the executive's asset allocation problem with the possibility of changing the allocation in the market portfolio each time joint with holding American ESOs.

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Appendix A. *ARMA representation of exact discretization of TOU process.*

We rewrite equation (5) in terms of the detrended process of p_t :

$$dq_t = -\kappa q_t dt + \sigma dZ_{S,t}, \quad \kappa > 0$$

where $q_t \equiv p_t - \theta t$ and initial condition $q_0 = p_0 = \ln S_0$. The exact solution to this univariate Ornstein Uhlenbeck (OU) process reverting to an unconditional mean of zero is, according to Bergstrom (1984), the following discrete-time process:

$$q_t = f_h q_{t-h} + \varepsilon_t, \quad \varepsilon_t \sim \ iid\ N\left(0, \sigma^2_{\varepsilon,h}\right)$$

where $f_h \equiv e^{-\kappa h} < 1$ and $\sigma^2_{\varepsilon,h} = \dfrac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa h}\right)$. Next, we obtain the equation for $\Delta_h q_t \equiv q_t - q_{t-h}$ and rewrite in terms of the stock return, $r_{t,h}$, given in equation (6). Then,

$$r_{t,h} = \theta h\big(1 - f_h\big) + f_h r_{t-h,h} + \eta_t, \quad \eta_t = \varepsilon_t - \varepsilon_{t-h},$$

where η_t follows a MA(1) process verifying that

$$\mathbb{E}\left[\eta_t\right] = 0\ , \quad Var(\eta_t) = 2\sigma^2_{\varepsilon,h} \qquad \text{and} \qquad Cov(\eta_t, \eta_{t-jh}) = \begin{cases} -\sigma^2_{\varepsilon,h} & \text{if } j = 1 \\ 0 & \text{if } j \geq 2 \end{cases}$$

Hence, $r_{t,h}$ is described by a stationary discrete-time ARMA (1,1) process with an unconditional mean of θh, an unconditional variance σ_r^2 shown in equation (7) and a negative first-order autocorrelation $\rho_r(1)$ exhibited in equation (8). Finally, the returns are negatively correlated at all lags. The higher order autocorrelation coefficients are easily obtained as $\rho_r(j) = f_h^{j-1}\rho_r(1)$, $\forall j \geq 2$.

The following Table summarizes the moments of the continuously compounded returns, defined as in equation (6) under the two alternative processes.

	TOU	GBM
Mean ($\bar{r}$)	θh	$\left(\tilde{\mu} - \tilde{\sigma}^2/2\right)h$
Variance ($\sigma^2_{r,h}$)	$\frac{\sigma^2}{\kappa}\left(1 - e^{-\kappa h}\right)$	$\tilde{\sigma}^2 h$
Autocorrelation at lag 1	$-\frac{1}{2}\left(1 - e^{-\kappa h}\right)$	0

$\tilde{\mu}$ and $\tilde{\sigma}$ denote the trend and diffusion coefficients of the GBM process.

By matching both unconditional means and variances for the h-period returns under both processes, we obtain the following constraints in the parameters:

$$\tilde{\mu} = \theta + \frac{\tilde{\sigma}^2}{2} \qquad , \qquad \tilde{\sigma} = \sigma\sqrt{\frac{1 - e^{-\kappa h}}{\kappa h}}.$$

In short, these parameter restrictions lead to a fair comparison across the paper when we aim to analyze exclusively the effects of predictability on the objective ESO valuation.

Appendix B. *ESO pricing algorithm.*

The algorithm used to solve the executive's problem in equation (10) is based on the popular LSMC algorithm of Longstaff and Schwartz (2001) for valuing American options. The modification to consider a risk-averse option holder was introduced by León and Vaello-Sebastià (2010). We adapt this approach by changing the dynamics of the stock price process. Given m simulated paths, the algorithm consists on creating a vector Y of length m with the expected utility for each path obtained from the optimal exercise of the ESOs. For the one state-variable case ($S1$) the algorithm simplifies, since the executive only has to search the optimal exercise time, t^* ($\eta = 0$). Then, we first explain the algorithm for the $S1$ model and later we will extend it for the $S2$ case.

Step 1: Given m simulated paths of the state-variable (S_t), the optimal exercise rule at maturity (T) is obvious, to exercise all paths in-the-money. Vector Y_T is obtained using equations (1) and (9) on all paths at T.

Step 2: At $T-1$ the executive has to choose between exercising or continuing with the ESOs for one period. The utility at T in case of exercise at $T-1$, $U\left(W_{T|T-1}\right)$, is computed using equations (2) and (9). Finally, to estimate the expected utility of exercise conditional on the current values of the state-variables, $\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right]$, we regress $U\left(W_{T|T-1}\right)$ over some basis functions of the state-variable (S_{T-1}). The projected values of that regression are $\hat{\mathbb{E}}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right]$.

Step 3: Compute the expected utility in case of keeping the ESO one period ahead. We only have to regress the one period ahead utility, Y_T, over basis functions of S_{T-1}. Then $\hat{\mathbb{E}}_{T-1}\left[Y_T|S_{T-1}\right]$ is the projected value of the previous regression.

Step 4: Compare the expected utility when exercising and holding the ESOs and then, select the maximum. That is, we compare $\hat{\mathbb{E}}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right]$ (step 2) with $\hat{\mathbb{E}}_{T-1}\left[Y_T|S_{T-1}\right]$ (step 3). For those y paths where $\hat{\mathbb{E}}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right] > \hat{\mathbb{E}}_{T-1}\left[Y_T|S_{T-1}\right]$ the ESOs are exercised and the y elements of Y_T are updated with $\hat{\mathbb{E}}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right]$.

Step 5: Repeat steps 2 to 4 up to t_0, where t_0 can be either the grant date or the first time step after the vesting period, and compute the expected utility as the mean of Y_T.

Step 6: Find the certainty-equivalence using equations (11) and (12).

At every time step t we record the minimum value of S_t from all exrcised paths in order to build the exercise threshold. Then V^{OBJ} is obtained using the exercise threshold.

The algorithm for the $S2$ model consists on repeating the previous steps (1 to 6) for a grid of values of η. Then, we take that value η^* which maximizes the expected utility. Introducing a second state-variable, M_t, also means that the two expectations (steps 2 and 3) have to be computed adding basis functions of M_t and the cross product $S_t \times M_t$ as regressors. Specifically, we have used the second-order complete polynomial of $\log(S_t)$ and $\log(M_t)$.[11]

[11] See Judd (1998) for more details about complete polynomials. Moreno and Navas (2003) and Stentoft (2004) provide numerical details about the robustness of the LSMC method.

Table 1: Average exercise dates (in years) and predictability.

		κ					
γ	α	0.0	0.05	0.10	0.25	0.50	1.00
2	1/2	7.41	7.42	7.78	8.14	8.31	8.50
	2/3	6.33	6.79	7.27	7.80	8.18	8.43
4	1/2	5.01	5.71	6.30	7.19	7.75	8.23
	2/3	4.58	5.22	5.73	6.94	7.58	8.13

This table collects the results concerning how the average exercise dates, τ^*, of our 10-year ESO are affected by different values of mean reversion, κ, which drives the autocorrelation of the TOU process in equation (5), respecting different levels of γ and α. The column for $\kappa = 0$ presents the values of τ^* for the corresponding equivalent GBM process as described in subsection 3.1. For all cases, the risk-free rate, the dividend yield, the ESO maturity and the length of the vesting period have been set to their benchmark values, namely, $r_f = 0.06$, $\bar{q} = 0$, $T = 10$ and $\nu = 3$.

Table 2: Average exercise dates (in years). The effects of σ and θ.

Panel A: Diffusion coefficient, σ											
		(GBM)					(TOU)				
γ	α	0.25	0.30	0.35	0.40	0.45	0.25	0.30	0.35	0.40	0.45
2	1/2	9.31	7.47	6.67	6.17	5.85	8.63	8.32	8.07	7.87	7.71
	2/3	7.72	6.41	5.68	5.34	5.15	8.53	8.20	7.89	7.66	7.50
4	1/2	5.53	5.04	4.80	4.70	4.64	8.20	7.77	7.48	7.26	7.10
	2/3	4.87	4.57	4.49	4.42	4.42	8.03	7.60	7.28	7.02	6.82
Panel B: Drift coefficient, θ											
		(GBM)					(TOU)				
		0.065	0.075	0.10	0.125	0.15	0.065	0.075	0.10	0.125	0.15
2	1/2	6.71	6.90	7.63	8.31	9.43	7.69	7.87	8.40	8.70	9.00
	2/3	5.77	5.95	6.57	6.94	7.58	7.52	7.72	8.28	8.60	8.91
4	1/2	4.95	4.95	5.07	5.10	5.20	7.24	7.39	7.85	8.16	8.48
	2/3	4.58	4.58	4.61	4.76	5.40	7.02	7.18	7.69	8.03	8.39

The right panels collect the results concerning how the average exercise dates, τ^*, of our 10-year ESO are affected by the volatility and the mean of returns of the TOU process in equation (5). For comparison purposes, the left panels present the values of τ^* for the corresponding equivalent GBM process as described in subsection 3.1. For all cases, the risk-free rate, the dividend yield, the time to maturity of the ESO and the length of the vesting period have been set to their benchmark values, namely, $\kappa = 0.50$, $r_f = 0.06$, $\bar{q} = 0$, $T = 10$ and $\nu = 3$.

Table 3: Predictability and ESO cost: S1 versus S2 models.

		κ				
ν	α	0.05	0.10	0.25	0.50	1.00
0	1/2	13.34	8.16	1.41	-0.24	-0.62
	2/3	19.93	13.29	3.11	0.46	-0.34
3	1/2	8.99	6.80	1.01	-0.07	-0.53
	2/3	10.02	8.14	2.62	0.44	-0.31

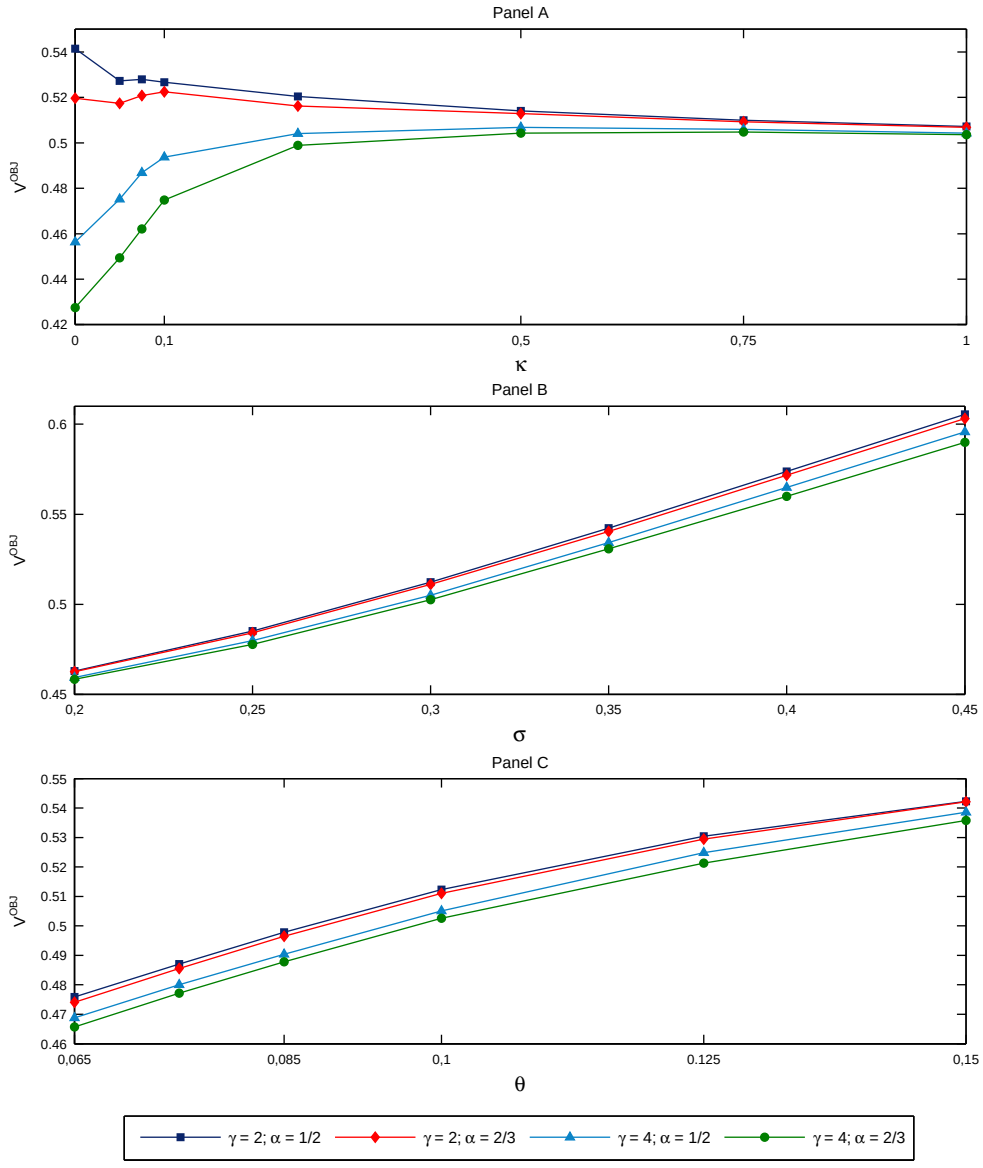
We present here for $\gamma = 4$ the percentage bias defined in equation (14), concerning the firm's ESO cost when executives have available a market portfolio to allocate their unrestricted wealth (S2 model) and when they have not (S1 model). As it is shown, the differences decrease as κ increases to become negligible for relatively high values of this parameter. For the rest of the parameters see Table 2 and Section 4.

Table 4: Optimal share of market portfolio (η^*).

		κ				
ν	α	0.05	0.10	0.25	0.50	1.00
0	1/2	0.17	0.18	0.26	0.43	0.55
	2/3	0.09	0.08	0.25	0.58	0.76
3	1/2	0.18	0.18	0.26	0.43	0.55
	2/3	0.09	0.09	0.26	0.58	0.76

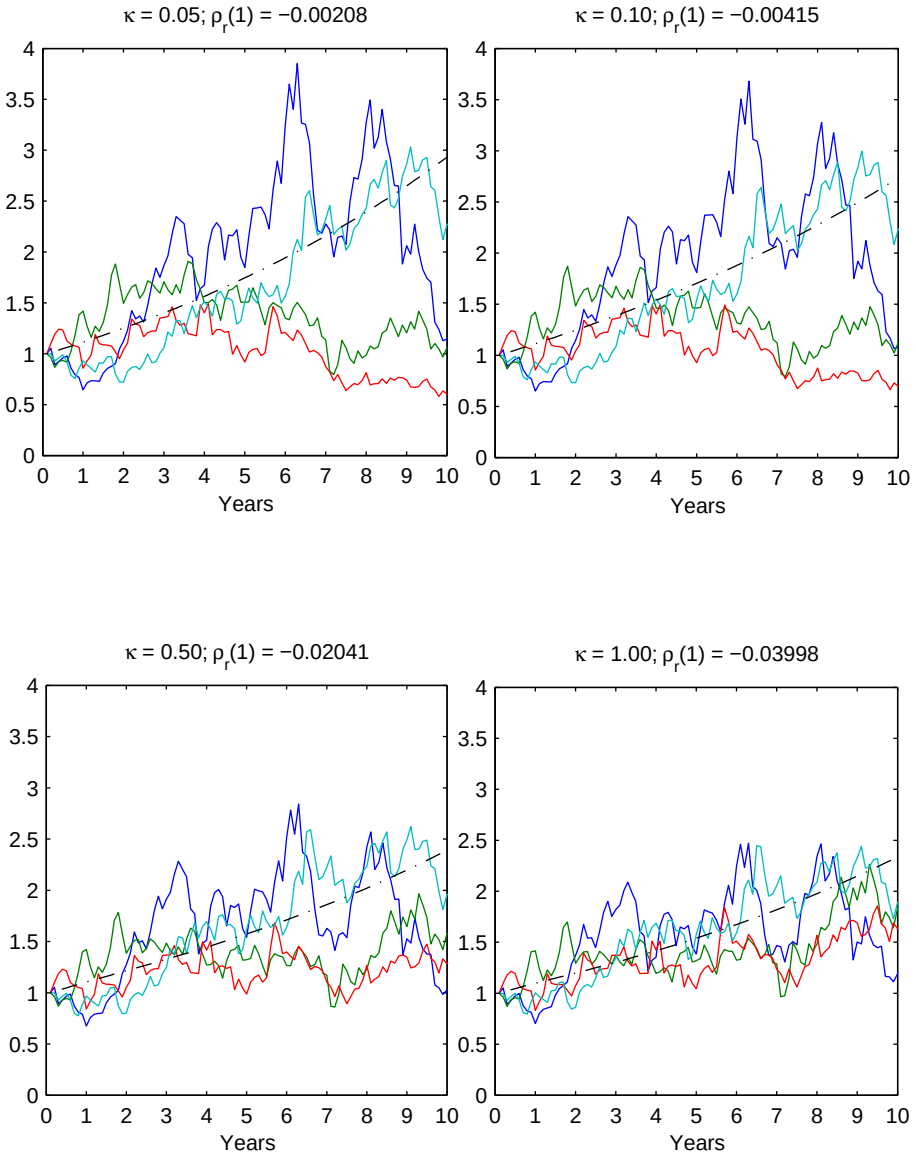
The table displays, for $\gamma = 4$, how η^* is affected by a higher predictability of the firm's stock returns. The volatility of stock returns in discrete time is kept constant.

Figure 1: Behaviour of the objective ESO's value in the S1 framework.



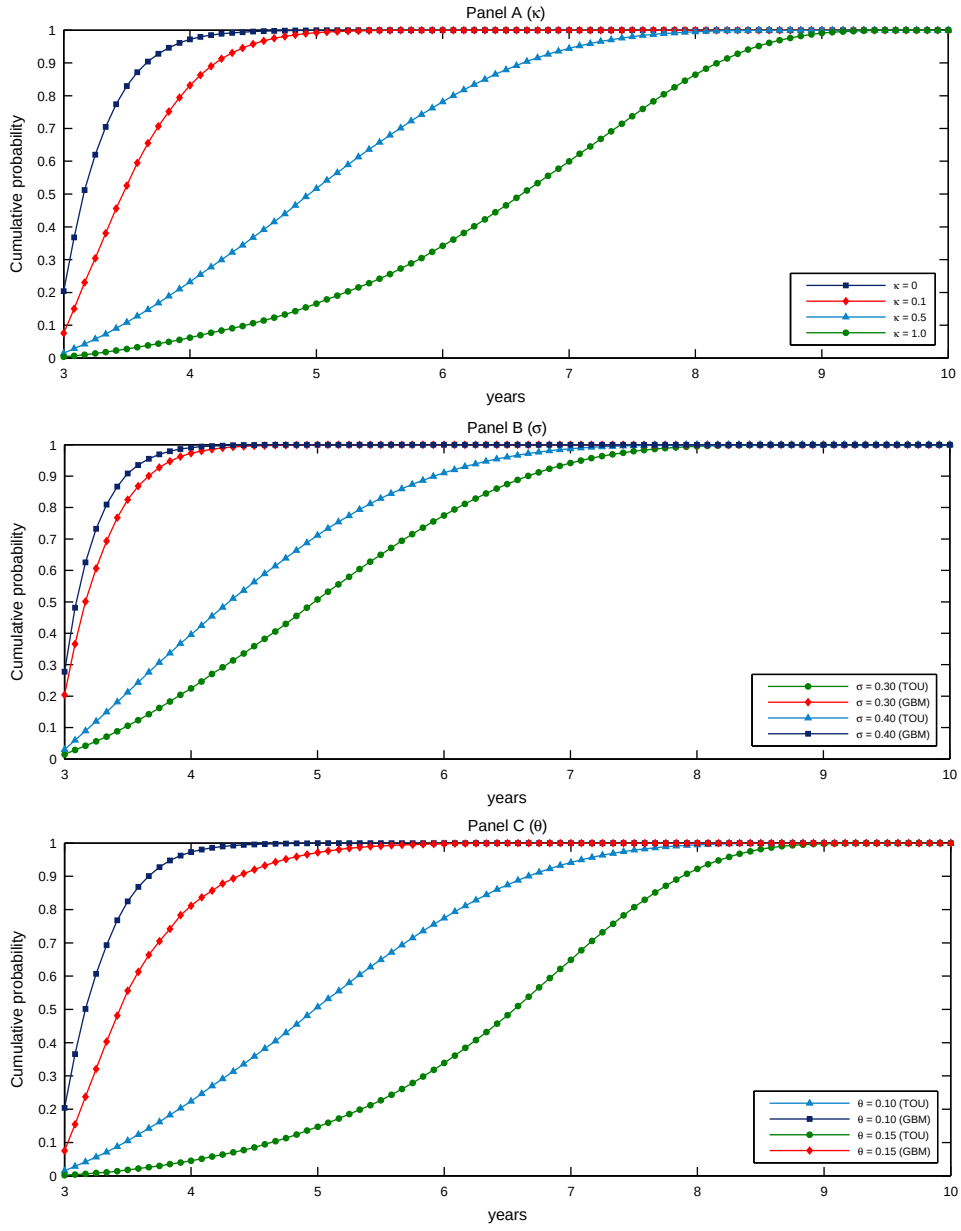
We show the behavior of the objective ESO's value for a 10-year ESO with a vesting period of 3 years, V^{OBJ}, is represented as a function of the speed of adjustment, κ, in panel A; the diffusion coefficient, σ, in panel B and the long-run drift, θ, in panel C. In each panel we have considered two values for the degree of relative risk aversion, $\gamma = \{2, 4\}$, and for the degree of undiversification, $\alpha = \{1/2, 2/3\}$. The other values for the parameters of the TOU process in equation (5) are: $\theta = 0.10$ and $\sigma = 0.30$ for panel A; $\kappa = 0.50$ and $\theta = 0.10$ for panel B, and $\kappa = 0.50$ and $\sigma = 0.30$ for panel C. In all cases, we have set $r_f = 0.06$ and $\bar{q} = 0$.

Figure 2: Alternative paths for different TOU processes.



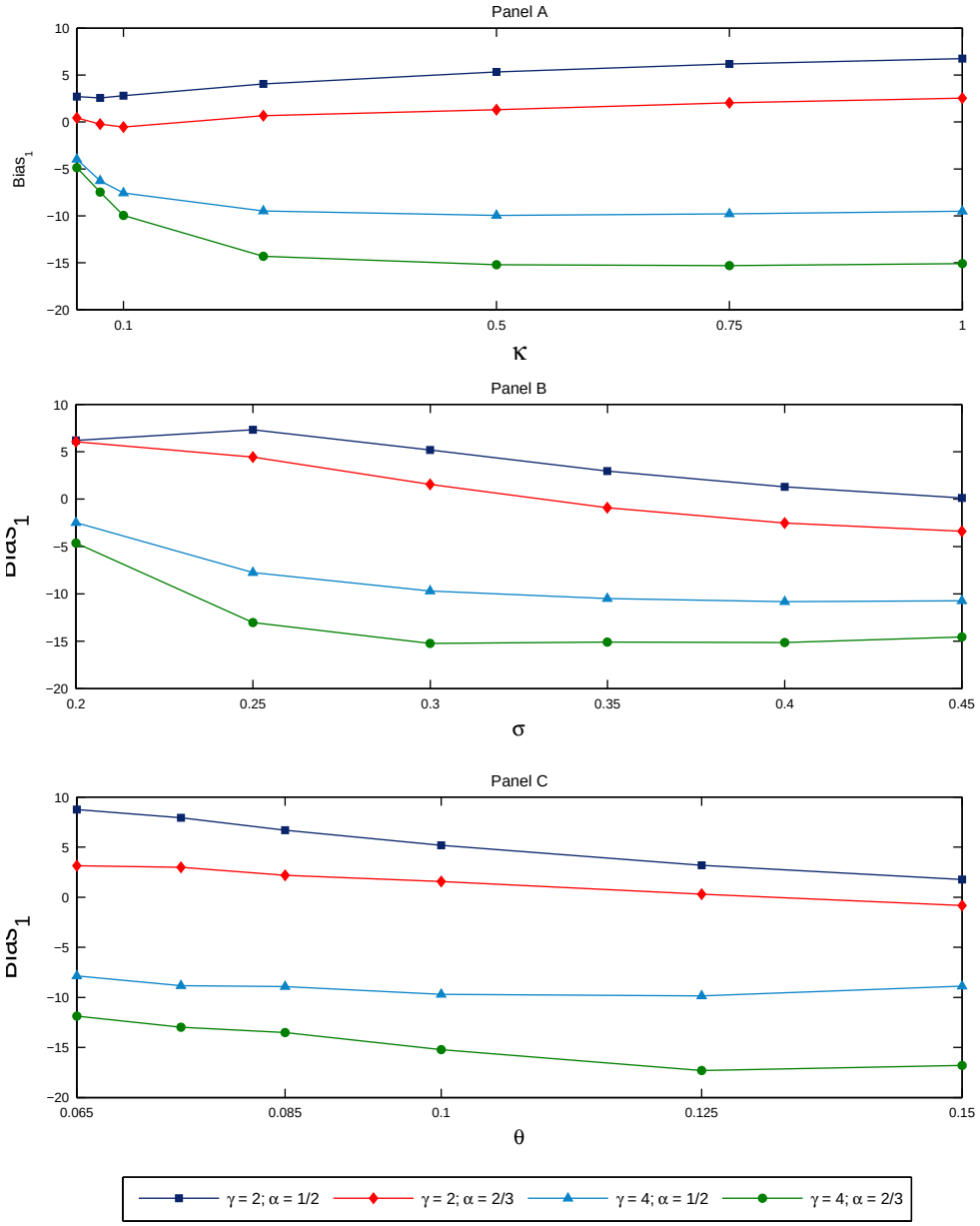
This figure plots alternative paths for the TOU process in equation (5) as a function of the speed of adjustment parameter, κ. The diffusion coefficient, σ, has been adjusted so as to keep constant the volatility of the discrete returns, σ_r, in all cases. The value for this volatility is 0.30 in yearly terms. The long-run drift is equal to its benchmark value, $\theta = 10\%$. In the four panels the discontinuous line represents the long-run trend as given by $\theta t + p_0 e^{-\kappa t}$. As κ increases, the simulated paths tend to revert faster to this long-run trend, although their volatilities are not altered.

Figure 3: Cumulative probabilities of early exercise.



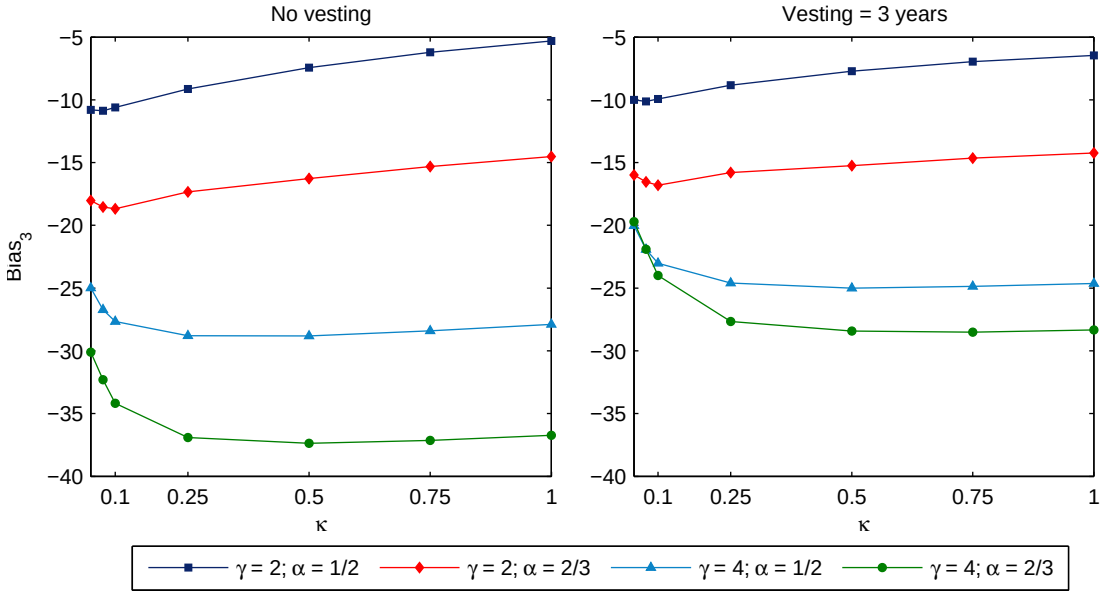
We plot the probability that the average exercise time is not greater than some given date for some values of κ, σ and θ. Panel A shows the curves corresponding to $\kappa = 0$, $\kappa = 0.10$, $\kappa = 0.50$ and $\kappa = 1.0$. The remaining parameters have been set to $\theta = 0.10$ and $\sigma = 0.30$. In panel B, the curves represent the cases $\sigma = 0.30$ and $\sigma = 0.40$, where the other parameters have been set to $\kappa = 0.50$ and $\theta = 0.10$. Finally, in panel C the curves represent the cases $\theta = 0.10$ and $\theta = 0.15$ with the other parameters set to $\kappa = 0.50$ and $\sigma = 0.30$. We have also plotted, for comparison purposes, the same probability for the corresponding equivalent GBM process, as described in subsection 3.1. The general finding is that, executives will tend to wait longer to exercise ESOs as the predictability of the process increases, that is, a higher value of $\left|\rho_r(1)\right|$. In all cases, we have focused on an executive with $\gamma = 4$ and $\alpha = 2/3$.

Figure 4: Objective ESO valuation bias under the S1 framework (S1), $Bias_1$



We plot the percentage bias in equation (13), which is incurred when the expected firm cost of a 10-year ESO with a vesting period of 3 years is evaluated using a GBM process for the stock price when the true one is a TOU process. Panels A to C depict this bias as a function of, respectively, κ, σ and θ, for four combinations of the relative degree of risk aversion and the executive's degree of undiversification. In all cases, we have set $r_f = 0.06$ and $\bar{q} = 0$.

Figure 5: FAS123 bias. The one state-variable framework.



We plot the percentage bias defined in equation (16), which is incurred when the FAS123 procedure is used and the expected life of the ESO is computed using erroneously the GBM process instead of the TOU process for stock returns. Both TOU and GBM have been made comparable by equalizing their means and volatilities. The procedure is described in detail in Section 5. The values of the remaining parameters are: $r_f = 0.06$, $\bar{q} = 0$ and $T = 10$.