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Pricing Executive Stock Options under Employment Shocks☆

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Abstract

We obtain explicit expressions for the subjective, objective and market value of perpetual executive stock options (ESOs) under exogenous employment shocks driven by an independent Poisson process. Previously, we obtain the executive's optimal exercise policy from the subjective valuation that is necessary for the objective one, or fair value. The perpetual ESO is compared with the true finite maturity ESO finding that the approximation is reasonably good. To illustrate the usefulness of the objective valuation for accounting purposes, we analyze the statistical distribution of the fair value when there is uncertainty about the employment shock intensity. Finally, the role of ESOs in the design of executives' incentives is also discussed.

Keywords: Executive Stock Options, Risk Aversion, Undiversification, Incentives, FAS123R.

JEL Classification: G11, G13, G35, M52

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1. Introduction

The increasing relevance of executive stock options (ESOs) as a component of corporate compensation has led the International Accounting Standard Board (IASB) to issue the International Financial Reporting Standard 2 (IFRS 2) in February 2004. In March 2004, the Financial Accounting Standard Board (FASB) has also revised the Financial Accounting Standard 123 (FAS 123R) with a similar purpose, namely, to provide a fair value method for shared based compensation arrangement.[4]

ESOs are mainly American-style call options that, in contrast with conventional ones, exhibit some features that aim to create the required incentives for aligning executive's goals with shareholders' interests.[5] In concrete, ESOs cannot be sold or transferred, though partial hedge is possible by trading correlated assets. They can only be exercised after ending the vesting period. The executive is also subject to a departure risk or employment shock. If he leaves the firm, either voluntarily or not, he must exercise the ESOs. If the departure occurs during the vesting period, the executive loses his ESO package. The executive typically exercises the option earlier of what it would be optimal for a tradable American option. The related empirical evidence on this fact can be found, among others, in Huddart and Lang (1996), Carpenter (1998) and Bettis et al. (2005). As a result, the standard methods used to price American options are not directly applicable and a growing literature has been searching for the proper valuation of ESOs. In this regard, accounting standards have established that a fair value based method should incorporate, at least, the stylized facts of a vesting period, employment shocks and suboptimal exercise. In this work, we propose a valuation framework that considers all these features for ESO valuation. We also assume that the possible dilution effect is anticipated by the market and it is already reflected in the stock price immediately after the ESO grant.[6]

We obtain the three different ESO valuations that can be found in the literature. In the first place there is the risk-neutral valuation, or market price, corresponding to an unconstrained agent. Next, there is the subjective valuation made by a constrained executive, who has not a fully diversified portfolio since he cannot trade his holdings of ESOs and firm's stocks. This implies a suboptimal exercise rule and hence a lower subjective value. Finally, there is the objective valuation or fair value, which is the cost to the firm of issuing the ESOs. This is the value attached by an agent with a fully diversified portfolio who is restricted to follow the executive's exercise policy. In this work, we will concentrate on the fair value though it is necessary to obtain previously the exercise rule from the subjective valuation. It is satisfied that the fair value lies between the market value and subjective one, see Ingersoll (2006).

It must be remarked that the aforementioned valuations depend crucially on how the exercise policy

[4]We will only concentrate on the FAS 123R since both standards establish rather the same purpose concerning the fair value.

[5]For a detailed discussion about the differences between standard options and ESO grants, see Rubinstein (1995).

[6]See Hull and White (2004), Leung and Sircar (2009) and FASB statement 123R.

is obtained. In this sense, we can distinguish between two different approaches. The first one is based on structural models in which the exercise policy is obtained from the maximization of the executive's expected utility subject to a given set of constraints. Lambert et al. (1991) is an early example. They obtain the subjective ESO value by using the certainty equivalent (CE) principle. Huddart (1994) and Kulatilaka and Marcus (1994) use this framework to provide an estimate of this subjective value determining the exercise rule on a binomial tree. However, they restrict the executive to hold his wealth only in the risk-free asset. Hall and Murphy (2002) allow a more general setting in which the executive's wealth holds the restricted stocks. Cai and Vijh (2005) show a more extended version where the executive's wealth is split in the market portfolio and the risk-free asset. All these studies are characterized to be static since the executive maximizes the expected utility of his terminal wealth. Meanwhile, Kahl et al. (2003) and Ingersoll (2006) use a dynamic approach to solve the constrained executive's consumption-portfolio investment problem.

A second approach is based on reduced form models in which the exercise policy is described either by some exogenous random event or by some exogenous parameter (or both) that forces the early option exercise. An early example can be found in Jennergren and Näslund (1993), who introduce an exogenous and independent Poisson process with constant intensity, or exit rate, whose first arrival forces the early exercise of ESOs. Carpenter (1998) shows that this type of models performs as well as the structural ones. Carr and Linetsky (2000) develop an analytical specification under a stochastic intensity framework. Under the binomial approach, Hull and White (2004) and Ammann and Seiz (2004) calculate the fair value where the early exercise behavior is modeled respectively as a barrier and an adjusted strike price. The continuous version with barrier can be found in Ingersoll (2006). Sircar and Xiong (2007) provide an analytical valuation for a perpetual American ESO considering the resetting and reloading provisions that are features in many option programs. Finally, Civitanic et al. (2008) obtain a closed-form valuation in continuous time such that the exit rate is modeled under the same approach as pricing default bonds and the early exercise is also captured through a barrier.

This paper develops an analytical ESO valuation based on Ingersoll's structural model and it also includes a job termination risk along the lines of Jennergren and Näslund (1993). The Ingersoll's framework assumes a general factor model for the evolution of asset returns. In our work, the risk factors are reduced to just the market risk and the executive can only allocate his wealth across the market portfolio, the firm's stock and the risk-free asset. Since the executive is assumed to be undiversified, he is constrained to hold more of the firm's stock than its corresponding share in the market portfolio. As a result, there are two sources of risk, one from the non diversifiable systematic risk factor and the other from the idiosyncratic component that is not correlated with market risk. Notice that under a well diversified portfolio, the single source of risk would come exclusively from the market portfolio and any other idiosyncratic component would have vanished.

Our analytical expressions are obtained by assuming perpetual options. Moreover, by incorporating an additional source of risk, the job termination risk, we get a valuation model similar to that of Sircar and Xiong (2007) without the reloading and resetting provisions. Note, however, that they just solve the case of a well diversified or risk-neutral agent, so the ESO market value is only obtained. Our model provides closed-form expressions for both the subjective and objective values.

Though this context is unreal, the perpetual ESO prices approach the finite maturity ones reasonably well. As expected, perpetual ESO values become a better approximation the longer the maturity. Interestingly the fair value of an American-style ESO with a maturity of ten years, which is the benchmark case in the literature, turns out to be well captured through our perpetual valuation.

In short, the main contribution of this paper is that we obtain closed-form valuation expressions for perpetual ESOs under employment shocks at the grant date for the three alternative cases mentioned above. We show that these analytical solutions become good approximates for long-maturity ESOs, which are the ones primarily issued by firms as the empirical evidence suggests. This result avoids solving numerically a partial differential equation (PDE) subject to some boundary conditions to price a finite maturity ESO, which is computationally more demanding.[7] We also study the executive's incentives by analyzing both delta and vega measures from the subjective valuation.

The rest of the paper is organized as follows. Section 2 presents the theoretical results for the subjective perpetual ESO valuation. We study, in particular, the price bias incurred by using the perpetual ESO against the finite maturity case. Section 3 includes the theoretical results about the objective perpetual ESO valuation, the corresponding goodness of fit when approximating to the finite maturity case and the implications for the accounting standards. This section finishes with a discussion of the impact that uncertainty on the job termination risk has on the fair value. In Section 4, we show how ESO affects executive's performance through his subjective 'greeks'. Section 5 concludes.

2. Subjective ESO valuation

Our benchmark model will be a perpetual ESO with a stochastic live arising from exogenous employment shocks which force the termination of the employment relationship, as in the model of Jennergren and Näslund (1993). These shocks can arise from either the executive's side, due to voluntary resignation, or from the firm's side, due to the executive dismissal. In any case, the executive is forced to exercise the option if the event occurs after the vesting period, or to forfeit it, if the vesting period has not ended yet. The time at which the employment relationship is terminated is simply modeled as the first event of a Poisson process with hazard rate of λ per unit time. This Poisson process is assumed to be independent of any other stochastic process underlying our asset menu. The hazard rate leads to jumps in the ESO

[7] Recently, Leung and Sircar (2009) have also studied the computation of the firm's ESO cost. They obtain a condition that characterizes the executive's exercise policy using the indifference pricing methodology of Henderson (2005), but it must be solved numerically.

price, but not in the underlying stock price, as in Jennergren and Näslund (1993). We assume that the job termination risk is not priced, so that it can be diversified away. This assumption is very common in the literature. See, for instance, Jennergren and Näslund (1993), Carpenter (1998), Carr and Linetsky (2000), Hull and White (2004), Sircar and Xiong (2007) and Leung and Sircar (2009).[8]

Those employment shocks force exogenous exercise of ESOs. Endogenous exercise results from the executive's behavior of solving a constrained maximization problem. Our asset menu will consist of a risk-free bond, the market portfolio and the company's stock. The equations describing the dynamics of both company's stock and market portfolio prices are given by

$$\begin{aligned} \frac{dS}{S} &= \big(\mu_S - q_S\big)dt + \sigma_S dZ_S, \\ \frac{dM}{M} &= \big(\mu_M - q_M\big)dt + \sigma_M dZ_M, \end{aligned}$$

where μ_S and μ_M denote the growth rate of the stock and the market portfolio respectively, q_S and q_M stand for the corresponding continuous dividend yields, and σ_S and σ_M denote the total volatilities for the respective returns. The firm's stock and the market portfolio are assumed to be imperfectly correlated. Formally, the Wiener processes satisfy the following relationship:

$$\sigma_S dZ_S = \beta\sigma_M dZ_M + \sigma_I dZ_I, \tag{1}$$

where the parameter β is the conventional market beta, and dZ_M and dZ_I are independent standard Wiener processes. Notice that the specific or idiosyncratic risk, σ_I, is not an independent parameter since it must satisfy the restriction $\sigma_I^2 = \sigma_S^2 - \beta^2\sigma_M^2$ as equation (1) makes clear. In short, we can rewrite the equation for the stock price dynamics as

$$\frac{dS}{S} = (\mu_S - q_S)\, dt + \beta\sigma_M dZ_M + \sigma_I dZ_I.$$

As Ingersoll (2006), we assume that the executive is infinitely lived and maximizes an expected lifetime utility of the constant relative risk aversion class. To capture the degree of undiversification, we define the parameter θ as the excess of company's stock holding over the optimal level already incorporated in the market portfolio.[9] So, the higher θ the lower the executive's diversification. Therefore, using $\mathbb{E}_0\big[\,\cdot\,\big]$ to denote the conditional expectation, the executive's problem is:

$$\max_{C,\omega} \mathbb{E}_0 \left\{ \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right\} \tag{2}$$

[8] Leung and Sircar (2009) have considered the case in which λ is a bounded continuous non-negative function of the firm's stock price. However, they state that this generalization does not seem to bring much additional insight to the executive's exercise policy.

[9] Let $\underline{\theta}$ denote the minimum amount of the company's stock that the executive is constrained to hold. If $\xi^\star$ denotes the optimal share of the company's stock in the market portfolio and $\omega^\star$ the optimal share of the market portfolio in the executive's total portfolio, then θ would satisfy the following condition $\theta = \underline{\theta} - \omega^\star\xi^\star \geq 0$.

subject to the following dynamic budget constraint:

$$dW = \{[r + \omega(\mu_M - r) + \theta(\mu_S - r)]W - C\}\,dt + \omega\sigma_M W dZ_M + \theta\sigma_S W dZ_S, \tag{3}$$

with initial condition $W(0) = W_0$. For simplicity, no wage income is assumed. Assuming that CAPM holds, i.e. $\mu_S = r + \beta(\mu_M - r)$, and using the orthogonal decomposition described in equation (1), we can rewrite equation (3) as

$$dW = \{(r + (\omega + \theta\beta)(\mu_M - r))W - C\}\,dt + (\omega + \theta\beta)\sigma_M W dZ_M + \theta\sigma_I W dZ_I. \tag{4}$$

Under the above conditions, we use Ingersoll's result[10] for the case of just one risk factor to show that the pricing kernel or the stochastic discount factor (SDF) that prices the derivative will obey the following stochastic differential equation (SDE):

$$\frac{d\Theta}{\Theta} = -\hat{r}dt - \left(\frac{\mu_M - r}{\sigma_M}\right) dZ_M - \gamma\theta\sigma_I dZ_I, \tag{5}$$

where $\hat{r} = r - \gamma\theta^2\sigma_I^2$.

Notice that, when the executive is either risk-neutral, $\gamma = 0$, or has a well diversified portfolio, $\theta = 0$, the SDF does not include any term reflecting the (diversifiable) idiosyncratic risk of the stock. Hence, the resulting value coincides with the risk-neutral price for marketable options.

The ESO subjective value at time 0 with expiration at date T, denoted by V, will satisfy the martingale condition $\mathbb{E}_0[d(\Theta V)] = 0$. Thus,

$$\begin{aligned} 0 = \mathbb{E}_0\left[d(\Theta V)\right] &= \mathbb{E}_0\left[d(\Theta V)\middle|\text{no employment shock}\right](1 - \lambda dt) \\ &+ \mathbb{E}_0\left[d(\Theta V)\middle|\text{ employment shock}\right]\lambda dt. \end{aligned} \tag{6}$$

Given equation (6), we get the following result:

Lemma 1. *Under the no arbitrage condition of equation (6) and assuming that CAPM holds, the ESO subjective value will satisfy the following fundamental partial differential equation (PDE):*

$$V_t + \left(\frac{\sigma_S^2}{2}\right) V_{SS}S^2 + (\hat{r} - \hat{q}_S)V_S S - (\hat{r} + \lambda)V + \lambda\Psi(S) = 0, \tag{7}$$

where $\hat{r} = r - \gamma\theta^2\sigma_I^2$, $\hat{q}_S = q_S + \gamma\theta(1-\theta)\sigma_I^2$ *and* $\Psi(S)$ *denotes the ESO holder's payoff if there is an employment shock defined as* $\Psi(S) = (S - K)\mathbb{1}_{\{S>K\}}$, *where* $\mathbb{1}_{\{A\}}$ *is the indicator function such that* $\mathbb{1}_{\{A\}} = 1$ *if* A *is true and* $\mathbb{1}_{\{A\}} = 0$, *otherwise.*

Proof.- See Appendix A.

Equation (7) is the PDE defining the executive's valuation of the ESO in the continuation or waiting region. To obtain a solution, a set of terminal and boundary conditions is added, see Kim (1990) for

[10] See equation (7) in Ingersoll (2006).

a detailed discussion of this issue. This set of conditions defines an optimal exercise boundary which is precisely the executive's exercise policy. This boundary defines a threshold price such that it is optimal to exercise the option as soon as the current stock price is above it. This threshold price changes, in particular, with the ESO's term to maturity.

2.1. Pricing without vesting period

There is no known closed-form solution for V in the former equation and typically some numerical approximation is used to find it. However, if we assume that the ESO is a perpetual American call option, the former PDE simplifies to an ordinary differential equation (ODE), as the partial derivative in t disappears. Namely,

$$\left(\frac{\sigma_S^2}{2}\right) V_{SS} S^2 + (\hat{r} - \hat{q}_S) V_S S - (\hat{r} + \lambda) V + \lambda \Psi(S) = 0. \tag{8}$$

Furthermore, in the case of a perpetual option, the threshold price is not time-dependent. We shall denote it by S^*. The value of S^* will be found as part of the solution of the subjective valuation problem by imposing two conditions that must be held at the boundary with the exercise region:

$$V(S^*) = S^* - K, \tag{9}$$

$$V'(S^*) = 1. \tag{10}$$

Now, by considering equations (8), (9) and (10), we obtain the following proposition:

Proposition 2. *Assume that there is no vesting period and the ESO is a perpetual American call option, then the solution to the ODE defined in (8) subject to the following boundary conditions, $V(0) = 0$ and equations (9) and (10), provides the value for the ESO holder, whose explicit solution is given by*

$$V(S) = \begin{cases} \hat{A}_1 K^{1-\hat{\alpha}_1} S^{\hat{\alpha}_1} & \text{if } S \leq K \\ \hat{B}_1 K^{1-\hat{\alpha}_1} S^{\hat{\alpha}_1} + \hat{B}_2 K^{1-\hat{\alpha}_2} S^{\hat{\alpha}_2} + \lambda \left(\frac{S}{\lambda + \hat{q}_S} - \frac{K}{\lambda + \hat{r}} \right) & \text{if } K < S \leq \hat{S}^* \\ S - K & \text{if } S > \hat{S}^* \end{cases} \tag{11}$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the solutions to the quadratic equation $({\sigma^2}_S/2)\hat{\alpha}^2 + (\hat{r} - \hat{q}_S)\hat{\alpha} - (\hat{r} + \lambda)$ and the values of the constants $\hat{A}_1$, $\hat{B}_1$ and $\hat{B}_2$ are defined in Appendix B by equations (28), (27) and (23) respectively. Finally, the threshold price $\hat{S}^*$ is uniquely defined by solving the next equation:

$$\lambda \left(\frac{\hat{S}^*}{K} \right)^{\hat{\alpha}_2} = -(1 - \hat{\alpha}_2)\hat{r} - \hat{\alpha}_2 \hat{q}_S \left(\frac{\hat{S}^*}{K} \right). \tag{12}$$

Proof.- See Appendix B.

The solution, $V(S)$, is homogeneous of degree one in both S and K. The first two rows of equation (11) show the subjective ESO value when the price is below the optimal subjective threshold. Both belong to a situation in which the executive is better-off waiting rather than exercising the option. To understand the gains from this second waiting region, we suggest the following intuitive explanation. Its first component comes from the possible increase in the future price of the underlying stock. The second one concerns the possibility of exercising the ESO if an employment shock occurs at any future time with a probability of λ. Hence, the term $\lambda\left(S/(\lambda+\hat{q}_S)-K/(\lambda+\hat{r})\right)$ denotes the expected ESO present value when it is in-the-money.[11] Of course, in the first waiting region this term does not appear since the option is out-of-the-money.

Some remarks about equation (12) are in order. In contrast to the paper of Sircar and Xiong (2007), we obtain a closed-form expression for the executive's exercise policy under more general risk preferences. The exercise policy is homogeneous of degree one in K, that is, any change in the strike price implies a change in the same proportion in the threshold price. Finally, though it is not easy to find general comparative static results, all numerical simulations have provided results that conform with intuition. For instance, a higher value of the employment shock probability tends to reduce the threshold price.

2.2. Pricing with vesting period

We extend now the results of Proposition 2 to the case of a positive vesting period, ν. The precise result is stated in the following proposition.

Proposition 3. *The subjective ESO value at the granting date, $t=0$, with a vesting period of length ν is given by*

$$V_0^{SUB}=e^{-\lambda\nu}\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}V(S_\nu)\right], \tag{13}$$

where the exponential term is the probability that the executive will remain employed till the end of the vesting period and

$$\begin{aligned}\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}V(S_\nu)\right] &= e^{-\hat{r}\nu}\mathbb{E}_0\left[(S_\nu-K)\mathbb{1}_{\{S_\nu>\hat{S}^*\}}\right]+\\ &+ e^{-\hat{r}\nu}\mathbb{E}_0\left[\left(\hat{B}_1K^{1-\hat{\alpha}_1}S_\nu^{\hat{\alpha}_1}+\hat{B}_2K^{1-\hat{\alpha}_2}S_\nu^{\hat{\alpha}_2}+\lambda\left(\frac{S_\nu}{\lambda+\hat{q}_S}-\frac{K}{\lambda+\hat{r}}\right)\right)\mathbb{1}_{\{K<S_\nu\le\hat{S}^*\}}\right]\\ &+ e^{-\hat{r}\nu}\mathbb{E}_0\left[(\hat{A}_1K^{1-\hat{\alpha}_1}S_\nu^{\hat{\alpha}_1})\mathbb{1}_{\{S_\nu\le K\}}\right],\end{aligned} \tag{14}$$

for any real numbers a, b and c verifying

$$\mathbb{E}_0\left[S_\nu^c\mathbb{1}_{\{a\le S_\nu\le b\}}\right]=\exp\left\{c\mu+c^2\frac{\sigma^2}{2}\right\}\times\left\{\Phi\left(\frac{\ln b-\mu-c\sigma^2}{\sigma}\right)-\Phi\left(\frac{\ln a-\mu-c\sigma^2}{\sigma}\right)\right\},$$

where $\mu=\ln S_0-(\hat{r}-\hat{q}_S-\sigma_S^2/2)\nu$, $\sigma=\sigma_S\sqrt{\nu}$ and $\Phi(\,\cdot\,)$ represents the cumulative standard normal distribution.

[11] This expected present value is obtained by solving the following integral:

$$\int_0^\infty \lambda e^{-\lambda t}\left[e^{-\hat{r}t}\left(Se^{(\hat{r}-\hat{q}_s)t}-K\right)\right]dt$$

where the time for the first employment shock follows an exponential distribution with mean $1/\lambda$.

Proof.- See Appendix C.

Table 1 summarizes the main findings concerning the impact on V^{SUB} by different values of the relevant parameters θ, γ, β, λ and ν. All results conform with intuition. A lower portfolio diversification or a higher relative risk-aversion reduces V^{SUB}. Similarly, a higher value of λ, which means a lower expected job's duration, and a higher vesting period also reduce V^{SUB}. A higher value of β, maintaining total volatility unchanged, implies a lower specific risk and hence a higher value for V^{SUB}. Alternatively, since β measures the correlation of the stock returns with the market portfolio returns, a higher value of β makes the market portfolio a better hedging against the risk of the stock and thus, it increases V^{SUB}. However, the impact of a higher σ_S is ambiguous in ESO valuation as Kulatilaka and Marcus (1994) point out. Though the values of tradable options rise with total volatility, the ESO values can actually falls with it. On the one hand, as long as β remains unchanged, increases in σ_I^2 are one-to-one increases in σ_S^2. This higher total volatility implies a higher V^{SUB}. On the other hand, that higher idiosyncratic volatility also reduces the adjusted interest rate, $\hat{r}$, and increases the adjusted dividend yield, $\hat{q}_S$, and hence a lower V^{SUB}.

[Table 1 is about here]

Finally, the ESO market price, V^{RN}, is obtained under the restriction of $\theta = 0\%$. As expected, V^{RN} increases with the total volatility of the stock return. This feature is independent of the risk decomposition because a well diversified agent does not worry about it. Note that, as expected, V^{RN} acts as an upper boundary for V^{SUB}.

2.3. Perpetual versus finite maturities

It becomes interesting to analyze if ESOs having finite maturities are adequately approximated by perpetual ones. We calculate ESO prices with finite maturities using the least-squares Monte Carlo algorithm (LSMC) of Longstaff and Schwartz (2001). We simulate the risk-neutral price process but replacing r and q_S by $\hat{r}$ and $\hat{q_S}$, respectively, as defined in Lemma 1.

The LSMC consists on backward induction. At maturity, the ESO is exercised if it is in-the-money, then the subjective value is $V_T^{SUB} = (S_T - K)^+$.[12] One period before, at $T - \Delta t$ where Δt is the length of one time step, on one hand there is a probability equal to $1 - e^{-\lambda \Delta t}$ to abandon the firm and the payoff of the ESO would be $(S_{T-\Delta t} - K)^+$. On the other hand, with probability $e^{-\lambda \Delta t}$ the executive remains in the firm and thus, he must decide either to hold or to exercise voluntarily the ESO. The executive will exercise the ESO if $S_{T-\Delta t} - K > e^{-\hat{r}\Delta t}\mathbb{E}_{T-\Delta t}\left[V_T^{SUB}\right]$, in this case the ESO value will be $S_{T-\Delta t} - K$. Otherwise, the payoff will be the discounted expected one period ahead ESO value. Thus, the ESO value

[12]Note that $(S_T - K)^+$ is the same as $(S_T - K)\mathbb{1}_{\{S_T > K\}}$.

at any time t, such that $T > t > \nu$, is computed as

$$V_t^{SUB} = e^{-\lambda\Delta t}\left[X_t \mathbb{1}_{\{S_t-K<X_t\}} + (S_t-K)^+ \mathbb{1}_{\{S_t-K\geq X_t\}}\right] + \left(1-e^{-\lambda\Delta t}\right)(S_t-K)^+ ,$$

where $X_t = e^{-\hat{r}\Delta t}\mathbb{E}_t\left[V_{t+1}^{SUB}\right]$ is the discounted expectation of the ESO value.[13] The conditional expected ESO value is computed by least-squares such that for those paths in-the-money, the one period ahead ESO value is regressed over some basis functions of the current stock price. We work backwards until the vesting or grant date with this scheme. We use 40,000 paths with antithetics simulated with monthly frequency[14] and we take the average of 50 previous estimations.

[Figure 1 is about here]

The left-hand side of Figure 1 exhibits the relative bias for the subjective perpetual valuation, V^{SUB}, respecting the finite maturity one, V_T^{SUB}, for alternative values of β and λ with maturities ranging from 1 year to 10 years. Thus, $Bias\ _T^{SUB} = \left(V^{SUB} - V_T^{SUB}\right)/V_T^{SUB}$. As expected, for very short maturities the perpetual approximation generates large positive biases. However, this bias converges quickly to zero as T enlarges. For instance, under the setting of $\left(\beta,\, \theta\right) = \left(0,\, 0.1\right)$ and $\lambda = 0.1$, for $T = 1$ it holds that $Bias\ _1^{SUB} = 1.15$, meanwhile $Bias\ _5^{SUB} = 0.2$ and $Bias\ _{10}^{SUB} = 0.06$. Note also that for higher values of λ, the bias decreases. For instance, given the above values of β and θ but $\lambda = 0.2$, we obtain that $Bias\ _1^{SUB} = 0.8$, $Bias\ _5^{SUB} = 0.1$ and $Bias\ _{10}^{SUB} = 0.03$.

3. ESO cost for firms

ESOs are extensively used as a payment scheme that may help firms in retaining and motivating its executives. As a result, an increasing part of the executives' compensation packages have taken the form of ESOs and this has motivated a discussion about the precise way in which this cost should be evaluated. There is not yet a consensus on this issue due to the lack of a general agreement about what should be the right model. Despite this, there is a general agreement of the role played by the subjective value in the computation of the ESO cost for firms. This cost depends on the executive's exercise policy from his subjective ESO valuation. Since the firm is not restricted to hedge the risk associated with the stock options, the objective valuation will be a straightforward modification of the risk-neutral one. That is, the ESO cost for the firm, or fair value, is the amount that would receive if the ESO were sold to a well diversified investor committed to follow the risk-averse executive's exercise policy and facing exogenous employment shocks that may end the employment relationship.

[13] Hull and White (2004) and Ammann and Seiz (2004) also introduce in the same way the exit rate for the backward induction in their binomial tree models.

[14] Stentoft (2004) obtains that the LSMC method with 10 exercise points per year produces very accurate prices compared with the ones obtained using a binomial model with 50,000 time steps. He argues that more accurate prices are obtained when increasing the simulated paths or the number of basis functions used as regressors.

3.1. Objective ESO valuation

We shall denote by V^{OBJ} the objective ESO value. It is immediate that the former definition implies that V^{OBJ} is the solution to the ODE described in equation (8) for the case of well diversified agents. Recall that this case follows from setting either the parameter γ or θ to zero. In this case the adjusted risk free rate, $\hat{r}$, and the adjusted dividend yield, $\hat{q}_S$, become r and q_S respectively. For ease of comparison, the ODE for the risk-neutral valuation is

$$\left(\frac{\sigma_S^2}{2}\right) V_{SS} S^2 + \left(r - q_S\right) V_S S - \left(r + \lambda\right) V + \lambda \Psi\left(S\right) = 0, \tag{15}$$

where the boundary conditions are now given by

$$V(0) = 0, \tag{16}$$

$$V(\hat{S}^*) = \hat{S}^* - K, \tag{17}$$

such that $\hat{S}^*$ is the executive's threshold price obtained in Proposition 2.[15] In short, the expression for V^{OBJ} when there is no vesting is stated in the following corollary, which is a slight modification of that proposition.

Corollary 4. *Assuming that there is no vesting period and the ESO is a perpetual American call option, the objective ESO value is given by the solution to the ODE defined in equation (15) subject to the boundary conditions (16) and (17):*

$$V\left(S\right) = \begin{cases} \tilde{A}_1 K^{1-\alpha_1} S^{\alpha_1} & \text{if} \quad S \leq K \\ \tilde{B}_1 K^{1-\alpha_1} S^{\alpha_1} + \tilde{B}_2 K^{1-\alpha_2} S^{\alpha_2} + \lambda \left(\dfrac{S}{\lambda + q_S} - \dfrac{K}{\lambda + r} \right) & \text{if} \quad K < S \leq \hat{S}^* \\ S - K & \text{if} \quad S > \hat{S}^* \end{cases} \tag{18}$$

where $\hat{S}^*$ is the executive's threshold price obtained in Proposition 2, α_1 and α_2 are the roots for the risk-neutral case and the values of $\tilde{A}_1$, $\tilde{B}_1$ and $\tilde{B}_2$ are defined in Appendix D by equations (35), (34) and (33), respectively.

Proof.- See Appendix D.[16]

This result can also be extended to the case of a positive vesting period. Specifically, by plugging both the SDF under risk-neutrality and the executive's threshold price into Proposition 3, we get the next corollary.

[15] Notice that V^{OBJ} can also be obtained as the solution to a perpetual up-and-in barrier option with $\hat{S}^*$ as the upper barrier. See Ingersoll (2006) for the finite maturity case.

[16] As in the subjective valuation case, $V(S)$ is linearly homogeneous in S and K.

Corollary 5. *The objective ESO value is given by*

$$\begin{aligned} V_0^{OBJ} &= e^{-\lambda\nu}\Big\{ e^{-r\nu}\mathbb{E}_0\left[(S_\nu - K)\mathbb{1}_{\{S_\nu > \hat{S}^*\}}\right] + \\ &+ e^{-r\nu}\mathbb{E}_0\left[\left(\tilde{B}_1 K^{1-\alpha_1}S^{\alpha_1} + \tilde{B}_2 K^{1-\alpha_2}S^{\alpha_2} + \lambda\left(\frac{S_\nu}{\lambda + q_S} - \frac{K}{\lambda + r}\right)\right)\mathbb{1}_{\{K < S_\nu \le \hat{S}^*\}}\right] \\ &+ e^{-r\nu}\mathbb{E}_0\left[(\tilde{A}_1 K^{1-\alpha_1}S_\nu^{\alpha_1})\mathbb{1}_{\{S_\nu \le K\}}\right]\Big\}, \end{aligned} \quad (19)$$

where the constants $\tilde{A}_1$, $\tilde{B}_1$ and $\tilde{B}_2$ are those obtained in Corollary 4.

Proof.- The above equation follows easily from equation (14) of Proposition 3 by setting $\gamma = 0$, or $\theta = 0$, and taking $\hat{S}^*$ in Proposition 2 as the threshold price.

Figure 2 plots the difference between the objective and subjective valuations with respect to the objective one, $\left(V^{OBJ} - V^{SUB}\right)/V^{OBJ}$. This ratio increases as either the relative risk aversion, γ, or the undiversification level, θ, increases. It is also shown that the lower the value of λ the higher this ratio.

[Figure 2 is about here]

3.2. Perpetual versus finite maturity

We perform the same study as in subsection 2.3 but we compare now the objective perpetual ESO price, V^{OBJ}, with the finite maturity case, V_T^{OBJ}. The right-hand side of Figure 1 exhibits the behavior of this new relative bias, i.e. $Bias_T^{OBJ} = \left(V^{OBJ} - V_T^{OBJ}\right)/V_T^{OBJ}$, with respect to the same range of time to maturity, T, for the same values of β and λ as in the subjective case. It is also shown that this bias converges quickly to zero as T enlarges and decreases for higher values of λ. Moreover, it happens that the less diversified the executive, a higher value of θ, the lower the size of this bias.

Table 2 displays different values of V^{OBJ} and V_T^{OBJ}, the latter for just two maturities, 5 and 10 years, and a representative set of values for the remaining parameters. It complements the results shown in Figure 2 by illustrating numerically the prices implied in some relative biases. For instance, for $\lambda = 0.1$, $\beta = 0$, $\gamma = 2$ and $\nu = 1$, the relative biases through these prices for $\theta = 0.1$ are 0.26 and 0.09 for $T = 5$ and $T = 10$ years respectively, while these values for $\theta = 0.4$ are 0.12 and 0.01 respectively.

3.3. Accounting implications

As a result of the increasing relevance of ESOs in the managers' compensation packages and the need to converge with other international standards, the FASB has revised its statement N° 123. The new FAS 123R requires firms to disclose the method used for estimating the grant date fair value of their ESO compensation packages. Among the valuation techniques that the FAS 123R consider acceptable are both lattice and closed-form models, such as the binomial model and the Black-Scholes formula respectively. Since ESOs are typically exercised before maturity, the FAS 123R (paragraph A26) explicitly requires that this fair value be based on its expected term, or expected life, rather than its maturity term. Furthermore, this expected life must be disclosed by firms as part of the shareholders' available information (FAS 123R, paragraph A240). In general, this expected life must be estimated. Kulatilaka and Marcus (1994), among others, have warned against the use of historical data with the purpose of estimating the option's expected term. This is so because the ESO expected life is linked to the stock market performance during the relevant period and this might lead to poor predictions. As an example of an acceptable method for estimating this parameter, the FAS 123R suggests using the estimated ESO fair value, as obtained from a lattice model, as an input of a closed-form model, such as the augmented Black-Scholes model or equation (20) below, from which the implied expected term can be calculated.

Now, we illustrate the usefulness of the perpetual option approximation to the real finite maturity option to estimate the expected term of ESO. Specifically, we take the expected lives of the objective finite maturity ESOs, implied in the corresponding values of Table 2, and compare them with the implied expected lives that one would obtain by using the objective perpetual ESO prices as the values of the FAS 123 adjusted European call prices. These values are given by

$$V^{FAS} = \exp\big(-\lambda\nu\big) \times BS\big(L\big), \tag{20}$$

where $BS(L)$ denotes the Black-Scholes (1973) formula with a time to maturity equals L and a vesting period of length ν.

Figure 3 displays the results of this comparison for alternative values of both the likelihood of an employment shock and the relative risk aversion level. In all cases, the objective finite ESO has a term to maturity of ten years. As it can be seen, the objective perpetual approximation is good whenever either the likelihood of an employment shock or the relative risk aversion level is high, or whenever the degree of portfolio diversification is low.

[Figure 3 is about here]

3.4. Uncertainty in employment shocks

We study the effects of uncertainty in the likelihood of an employment shock, λ, that is one of the main parameters in this work. We implement a simple exercise just to motivate this situation. Notice that the employment relationship may terminate by the side of either the firm or the executive because of some exogenous event, for instance, the executive finds out a better available job. Each part is more uncertain about the likelihood the other part attaches to this event. Assume that this uncertainty arises only from the executive's side. Further, we also assume the firm's manager ignores the precise value of λ, though he has some a priori probability distribution of λ. Of course, this will also generate a probability distribution for the objective value.

For simplicity, we assume a Triangular distribution[17] for the values of λ. We consider three prior

[17] See chapter 40 in Evans et al. (2000).

distributions for λ, that might be representative of the executive's outside opportunities. For instance, in a highly concentrated industry (a few firms), the executive's employment alternatives would tend to be scarce, so that the probability of leaving his actual job would be lower than in less concentrated industries. These prior distributions are classified according to the skewness behavior capturing alternative industry concentration levels. An asymmetric distribution with positive (negative) skewness represents a higher probability mass for small (large) values of λ because the industry is highly (scarcely) concentrated. And finally, we also consider a symmetric distribution for an intermediate situation.

For each assumed distribution of λ a total of 10,000 values have been simulated. Each distribution has the same support, given by the closed interval $[0\%, 40\%]$. For the cases of positive, zero and negative skewness, the values of λ going from the first to the third quartiles are respectively: $[7.5\%, 21\%]$, $[14.1\%, 25.6\%]$ and $[18.6\%, 32.2\%]$.

Figure 4, containing four pictures denoted from I to IV, displays the box-plots for the objective values, V^{OBJ}, implied by the above three distributions under several combinations of values for the diversification restriction, θ, and the total volatility, σ_S. Clearly, the implied distributions of V^{OBJ} coming from the distributions of λ with positive skewness (boxplots labeled as A) exhibit a higher median value than those distributions of λ with negative skewness (boxplots labeled as C). This is a clear effect of the negative correlation between the size of λ and V^{OBJ}. For instance, these differences go from a median value of \$8 (boxplot A) to \$5 (boxplot C) in picture I. Note also that a higher volatility leads to a higher median for the objective value. This value is higher the higher the executive's diversification.

[Figure 4 is about here]

4. Incentive effects

The literature has paid special attention to the optimal design of the executives' compensation packages as a way of affecting their incentives. In general, these compensation packages consist of a fixed cash component and a certain amount of restricted stocks and ESOs. Typically, the role of ESOs on executives' incentives has been approached by examining the sign and size of the ESO greeks. See, for instance, Ingersoll (2006), Tian (2004) and Chang et al. (2008) among others. Of course, it is understood that the relevant ESO greeks are those coming from the subjective valuation, that is, those related with executives' perception of incentives. In this regard, there are two ways in which ESOs may affect executives' incentives. First, as part of an agency problem, they align executives' and shareholders' interests in raising stock price. The perceived reward from acting this way is measured by the subjective delta greek, that is, the partial derivative of V^{SUB} with respect to the stock price. And second, when a new investment project is taken, the distribution of firm's total risk between the systematic component and the specific one will generally change. Given that they affect V^{SUB} very differently, there might be a moral hazard problem because of executives' risk taking behavior. However, the results of Section 2 show that executives have strong incentives to reduce the firm's specific component and to increase the systematic component of total volatility. This suggests that any analysis of incentives based on ESO greeks would appear to be redundant at first sight.[18]

Nevertheless, there are at least two reasons why option greeks are worth analyzing. Firstly, there is a gap between subjective and objective ESO valuation and hence, there is a difference between the executive's perceived incentives and the cost to the firm of providing those incentives. In particular, Chang et al. (2008) have considered the ratio between the subjective and the objective greeks for finite European ESOs as a measure of the size of the perceived incentives per unit cost of the firm. Since our perpetual American ESOs approximate reasonably the finite case as shown before, we extend their analysis to the case of perpetual American ESOs. Secondly, it turns out that the ratio between subjective and objective greeks might help to design the executive's compensation package. Next, we study both issues.

We begin analyzing the incentives to raise shareholders wealth as measured by the subjective delta, Δ^{SUB}. The firm's cost of providing these incentives are measured by the objective delta, Δ^{OBJ}. The ratio $\Delta^{SUB}/\Delta^{OBJ}$ is then the reward perceived by the executive per unit cost of the firm. A ratio below one suggests that the executive's incentives are lower than those intended by the firm. We show that $\Delta^{SUB}/\Delta^{OBJ}$ is typically below one when plotted as a function of the firm's stock price at the grant date, S_0. With regard to the impact of unemployment shocks, a higher value of λ reduces somewhat the size of this ratio.[19] This is depicted in the left-hand graphic of Figure 5 for at-the-money ESOs.[20] It turns out that this ratio is monotonically decreasing with θ and below one, except for the case of $\theta = 0$ where subjective and objective valuations are equal. This result is not modified by considering different values of λ. Hence, the maximum perceived incentives to increase the firm's market value per unit cost are achieved when the executive does not hold any restricted stock.

The former analysis can also be extended to examine executive's incentives for investments in risky projects. This is measured by the subjective systematic risk vega, denoted by Λ_β^{SUB}, that shows the change in V^{SUB} as a result of a unit change in market beta. The firm's cost of providing those incentives is measured analogously by Λ_β^{OBJ} and the corresponding ratio, $\Lambda_\beta^{SUB}/\Lambda_\beta^{OBJ}$, is represented in the right-hand graphic of Figure 5 .[21] Now, the shape of the curve suggests the existence of a maximum value of the ratio for a certain value of θ, denoted as θ^*. Furthermore, this maximum value turns out to be greater than one.

[18] In all numerical simulations performed, we have found that the subjective vega for firm's specific risk is strongly negative when total risk is held constant, while the subjective vega for systematic risk is positive. Although these results are obtained for perpetual American ESOs, they have not been reported in the main text since they are analogous to those obtained for the finite European ESOs. See, for instance, Tian (2004).

[19] These results are available upon request.

[20] The ratio $\Delta^{SUB}/\Delta^{OBJ}$ has been computed numerically by evaluating both partial derivatives using $S_0 = K$ as a midpoint.

[21] The corresponding numerical partial derivatives have been evaluated using $\beta = 1$ as the midpoint.

This is related with the second issue mentioned above which can be illustrated with the help of Figure 6, in which we plot $\Lambda_\beta^{SUB}/\Lambda_\beta^{OBJ}$ as a function of θ for several values of the idiosyncratic volatility and the likelihood of the employment shock. It is shown that the higher the idiosyncratic component, σ_I, the lower θ^*. The impact of a higher value of λ on θ^* is unclear. For instance, if $\sigma_S = 0.4$ then θ^* falls with λ. However, for the case of $\sigma_S = 0.20$, θ^* increases with λ. In any case, the maximum value of the ratio is clearly decreasing as λ increases.

5. Conclusions

In this work, we get closed-form expressions for the three different ESO valuations that can be found in the literature. The first one is the risk-neutral valuation, corresponding to an unconstrained agent. Second, the subjective valuation made by a constrained executive, who shows diversification restrictions since he is not allowed to trade both his ESOs and firm's stocks. This means a suboptimal exercise rule against the optimal one under risk-neutrality and hence, a lower subjective value. Third, the objective valuation, or fair value, which is the cost to the firm of issuing the ESOs. This value is obtained under the framework of a well-diversified or risk-neutral agent but restricted to follow the executive's exercise policy that has been obtained previously in the subjective valuation.

Our model for ESO pricing is based on a simplified version of the model of Ingersoll (2006) by considering just one risk factor, the market risk, in which a job termination risk is introduced along the lines of Jennergren and Näslund (1993). Our analytical expressions are obtained by assuming perpetual options. Though this framework is unreal, we show how our valuation of perpetual ESOs approaches reasonably well the true case of finite maturity ESOs.

Thus, we can use our objective perpetual ESO valuation as a good approximate to the fair value, which must be calculated as part of the firm's financial statements. We show that our approach can also approximate the ESO expected term reasonably well, which is also part of the information firms must disclose to their shareholders.

Last but not least, we also study the impact of ESOs on executives' incentives highlighting the role of the employment shock likelihood, λ. It is shown that a higher job turnover rate implies lower incentives for both raising the firm's market value and taking on projects with high correlation with the market expected return.

Finally, as suggestions for future research, it might be interesting exploring the effects of endogenous λ along the lines of Leung and Sircar (2009). Another fruitful idea is that since the firm has no knowledge about the executive's diversification restriction, then this value could be inferred from the executive's exercise policy using a Bayesian learning process.

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Appendices

A. Proof of Lemma 1

Using Ito's lemma, the money value of the ESO, V, obeys the following stochastic partial differential equation:

$$dV = \left[\big(\mu_S - q_S\big) V_S S + \frac{\sigma_S^2}{2} V_{SS} S^2 + V_t \right] dt + \beta \sigma_M V_S S dZ_M + \sigma_I V_S S dZ_I .$$

This equation together with equation (5) will lead to our fundamental equation (8). Indeed, by omitting terms of order higher than dt, we obtain:

$$\begin{aligned}
\mathbb{E}_0\big[V d\Theta\big] &= -\big(r - \gamma\theta^2\sigma_I^2\big) V \Theta dt \\
\mathbb{E}_0\big[\Theta dV\big] &= \big(\mu_S - q_S\big) V_S S \Theta dt + \frac{\sigma_S^2}{2} V_{SS} S^2 \Theta dt + V_t \Theta dt \\
\mathbb{E}_0\big[d\Theta dV\big] &= -\big(\mu_S - r\big) V_S S \Theta dt - \gamma\theta\sigma_I^2 V_S S \Theta dt
\end{aligned}$$

where $\mathbb{E}_0$ denotes the conditional expectation operator under the real measure and the CAPM condition, $\mu_S = r + \beta\big(\mu_M - r\big)$, has been used to obtain the third equation. Now, by straightforward substitution, we get:

$$0 = \mathbb{E}_0\left[V d\Theta + \Theta dV + d\Theta dV\right]\big(1 - \lambda dt\big) + \mathbb{E}_0\left[V d\Theta + \Theta\Big(\Psi(S) - V\Big) + d\Theta\Big(\Psi(S) - V\Big)\right] \lambda dt.$$

Hence,

$$-\big(r - \gamma\theta^2\sigma_I^2\big)V + \big(r - q_S - \gamma\theta\sigma_I^2\big)V_S S + \frac{\sigma_S^2}{2} V_{SS} S^2 + V_t + \lambda\Big[\Psi(S) - V\Big] = 0.$$

Finally, by defining $\hat{r} = r - \gamma\theta^2\sigma_I^2$ and $\hat{q}_S = q_S + \gamma\theta\big(1-\theta\big)\sigma_I^2$ we obtain equation (8).

B. Proof of Proposition 2

The solution to equation (8) can be easily shown to be

$$V\big(S\big) = \begin{cases} \hat{a}_1 S^{\hat{\alpha}_1} & \text{if } S < K \\ \hat{b}_1 S^{\hat{\alpha}_1} + \hat{b}_2 S^{\hat{\alpha}_2} + \left(\dfrac{\lambda S}{\lambda + \hat{q}_S} - \dfrac{\lambda K}{\lambda + \hat{r}}\right) & \text{if } S \geq K \end{cases} \tag{21}$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are, respectively, the positive and negative root of the quadratic equation $\hat{\alpha}^2 + \big(\hat{b} - 1\big)\hat{\alpha} - \hat{c}$, for

$$\hat{c} \equiv \frac{2}{\sigma_S^2}\big(\hat{r} + \lambda\big) = -\hat{\alpha}_1\hat{\alpha}_2 \quad \text{and} \quad \hat{b} \equiv \frac{2}{\sigma_S^2}\big(\hat{r} - \hat{q}_S\big) = 1 - \hat{\alpha}_1 - \hat{\alpha}_2 \ . \tag{22}$$

In equation (8) the negative root has been eliminated in the region $S < K$ by imposing the boundary condition $V\big(0\big) = 0$. The constants $\hat{a}_1$ and $\hat{b}_2$ can be solved in terms of $\hat{b}_1$ and K by using the usual conditions of value matching and smooth pasting for $S = K$:

$$\begin{aligned}
\hat{a}_1 K^{\hat{\alpha}_1} &= \hat{b}_1 K^{\hat{\alpha}_1} + \hat{b}_2 K^{\hat{\alpha}_2} + \left(\frac{\lambda}{\lambda + \hat{q}_S}\right)\left(\frac{\hat{r} - \hat{q}_S}{\lambda + \hat{r}}\right) K \\
\hat{\alpha}_1 \hat{a}_1 K^{\hat{\alpha}_1} &= \hat{\alpha}_1 \hat{b}_1 K^{\hat{\alpha}_1} + \hat{\alpha}_2 \hat{b}_2 K^{\hat{\alpha}_2} + \left(\frac{\lambda}{\lambda + \hat{q}_S}\right) K
\end{aligned}$$

The solution for $\hat{b}_2$ is:

$$\hat{b}_2 = \frac{2\lambda}{\sigma_S^2}\left(\frac{1}{\hat{\alpha}_2\big(\hat{\alpha}_2 - 1\big)\big(\hat{\alpha}_1 - \hat{\alpha}_2\big)}\right) K^{1-\hat{\alpha}_2} \equiv \hat{B}_2 K^{1-\hat{\alpha}_2} \tag{23}$$

And for $\hat{a}_1$:

$$\hat{a}_1 = \hat{b}_1 + \frac{2\lambda}{\sigma_S^2}\left(\frac{1}{\hat{\alpha}_1\big(\hat{\alpha}_1 - 1\big)}\right)\frac{K^{1-\hat{\alpha}_1}}{\big(\hat{\alpha}_1 - \hat{\alpha}_2\big)} \tag{24}$$

To determine the remaining constant, $\hat{b}_1$, and the threshold price, $S^\star$, we use equations (9) and (10) to get:

$$\hat{b}_1 {\hat{S}^*}^{\hat{\alpha}_1} + \hat{b}_2 {\hat{S}^*}^{\hat{\alpha}_2} + \left(\frac{\lambda \hat{S}^*}{\lambda + \hat{q}_S} - \frac{\lambda K}{\lambda + \hat{r}}\right) = \hat{S}^* - K \ , \tag{25}$$

$$\hat{\alpha}_1 \hat{b}_1 {\hat{S}^*}^{\hat{\alpha}_1} + \hat{\alpha}_2 \hat{b}_2 {\hat{S}^*}^{\hat{\alpha}_2} + \frac{\lambda \hat{S}^*}{\lambda + \hat{q}_S} = \hat{S}^* \ . \tag{26}$$

Solving first for $\hat{b}_1$ in equation (26) one gets:

$$\hat{b}_1 = \frac{-\hat{\alpha}_2}{\hat{\alpha}_1}\hat{b}_2\big(\hat{S}^*\big)^{\hat{\alpha}_2-\hat{\alpha}_1} + \frac{1}{\hat{\alpha}_1}\left(1-\frac{\lambda}{\lambda+\hat{q}_S}\right)\big(\hat{S}^*\big)^{1-\hat{\alpha}_1} .$$

Or, after substituting for $\hat{b}_2$ and simplifying

$$\hat{b}_1 = \frac{1}{\hat{\alpha}_1}\left\{\left(\frac{2\lambda}{\sigma_S^2}\right)\frac{1}{(1-\hat{\alpha}_2)(\hat{\alpha}_1-\hat{\alpha}_2)}\left(\frac{\hat{S}^*}{K}\right)^{\hat{\alpha}_2-\hat{\alpha}_1} + \left(1-\frac{\lambda}{\lambda+\hat{q}_S}\right)\left(\frac{\hat{S}^*}{K}\right)^{1-\hat{\alpha}_1}\right\}K^{1-\hat{\alpha}_1} \equiv \hat{B}_1 K^{1-\hat{\alpha}_1} . \tag{27}$$

Hence, we can write $\hat{a}_1$ as:

$$\hat{a}_1 = \hat{B}_1 K^{1-\hat{\alpha}_1} + \frac{2\lambda}{\sigma_S^2}\left(\frac{1}{\hat{\alpha}_1(\hat{\alpha}_1-1)}\right)\frac{K^{1-\hat{\alpha}_1}}{(\hat{\alpha}_1-\hat{\alpha}_2)} \equiv \hat{A}_1 K^{1-\hat{\alpha}_1} \tag{28}$$

so that, equation (11) in the main text is obtained.

Finally, by combining equations (25) and (26) we get the implicit equation for solving for $\hat{S}^*$ which, by using the relations given in equation (22), can be written as equation (12) in the main text.

C. Proof of Proposition 3

We want to solve the following conditional expectation:

$$\begin{aligned}
&\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}V(S_\nu)\right] = \hat{A}_1K^{1-\hat{\alpha}_1}E_0\left[\frac{\Theta_\nu}{\Theta_0}S_\nu^{\hat{\alpha}_1}\mathbb{1}_{\{S_\nu\le K\}}\right] + \hat{B}_1K^{1-\hat{\alpha}_1}\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}S_\nu^{\hat{\alpha}_1}\mathbb{1}_{\{K\le S_\nu\le \hat{S}^*\}}\right] + \\
&+\hat{B}_2K^{1-\hat{\alpha}_2}\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}S_\nu^{\hat{\alpha}_2}\mathbb{1}_{\{K\le S_\nu\le \hat{S}^*\}}\right] + \frac{\lambda}{\lambda+\hat{q}_S}\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}S_\nu\mathbb{1}_{\{K\le S_\nu\le \hat{S}^*\}}\right] - \frac{\lambda}{\lambda+\hat{r}}K\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}\mathbb{1}_{\{K\le S_\nu\le \hat{S}^*\}}\right] + \\
&+\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}S_\nu\mathbb{1}_{\{S_\nu\ge \hat{S}^*\}}\right] - K\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}\mathbb{1}_{\{S_\nu\ge \hat{S}^*\}}\right] .
\end{aligned}$$

Thus, all expectations take the general form $\mathbb{E}_0\left[\frac{\Theta_\nu}{\Theta_0}S_\nu^c\mathbb{1}_{\{a\le S_\nu\le b\}}\right]$ for c any given real number. Given the stochastic dynamics driving S_ν^c and (Θ_ν/Θ_0) we have explicit expressions for each one of them, namely:

$$\begin{aligned}
S_\nu^c &= S_0^c\cdot\exp\left\{c\left(\mu_{-}q_S-\frac{\sigma_S^2}{2}\right)\nu + c\sigma_S\sqrt{\nu}\varepsilon\right\}\\
\frac{\Theta_\nu}{\Theta_0} &= \exp\left\{-\left(\hat{r}+\frac{1}{2}\left(\frac{\mu_M-r}{\sigma_M}\right)^2+\frac{\gamma^2\theta^2\sigma_I^2}{2}\right)\nu - \left(\frac{\mu_M-r}{\sigma_M}\right)\sqrt{\nu}\varepsilon_M - \gamma\theta\sigma_I\sqrt{\nu}\varepsilon_I\right\}
\end{aligned}$$

where ε, ε_M and ε_I are *independent standard normal variables* satisfying $\sigma_S\varepsilon = \beta\sigma_M\varepsilon_M + \sigma_I\varepsilon_I$. Then, the expectation we seek to solve is given by a double integral of the form:

$$\begin{aligned}
&\int_{\varepsilon_M}\int_{\varepsilon_I} S_0^c\exp\left\{c\left(\mu_S-q_S-\frac{\sigma_S^2}{2}\right)\nu + c\big(\beta\sigma_M\varepsilon_M+\sigma_I\varepsilon_I\big)\sqrt{\nu}\right\}\times\\
&\exp\left\{-\left(\hat{r}+\frac{1}{2}\left(\frac{\mu_M-r}{\sigma_M}\right)^2+\frac{\gamma^2\theta^2\sigma_I^2}{2}\right)\nu - \left(\frac{\mu_M-r}{\sigma_M}\right)\sqrt{\nu}\varepsilon_M - \gamma\theta\sigma_I\sqrt{\nu}\varepsilon_I\right\}\times\\
&\phi(\varepsilon_M)\phi(\varepsilon_I)d\varepsilon_M d\varepsilon_I
\end{aligned} \tag{29}$$

where $\phi(\,\cdot\,)$ denotes the density function of a standard normal variable. Notice that the range of integration for ε_M and ε_I must be such that $a \le S_\nu \le b$.

Following Cochrane and Saa-Requejo (1999), we define the new variables:

$$\delta_1 = \frac{\beta\sigma_M\varepsilon_M+\sigma_I\varepsilon_I}{\sigma_S} \quad ; \quad \delta_2 = \frac{\sigma_I\varepsilon_M-\beta\sigma_M\varepsilon_I}{\sigma_S} .$$

Notice that δ_1 and δ_2 are too *independent standard normal variables.* By reversing the change we have the following expression in terms of ε_M and ε_I:

$$\varepsilon_M = \frac{\beta\sigma_M\delta_1+\sigma_I\delta_2}{\sigma_S} \quad ; \quad \varepsilon_I = \frac{\sigma_I\delta_1-\beta\sigma_M\delta_2}{\sigma_S} .$$

After substitution into equation (29), we get the following expression:

$$S_0^c \exp\left\{c\left(\mu_S - q_S - \frac{\sigma_S^2}{2}\right)\nu - \left(\hat{r} + \frac{1}{2}\left(\frac{\mu_M - r}{\sigma_M}\right)^2 + \frac{\gamma^2\theta^2\sigma_I^2}{2}\right)\nu\right\} \times$$
$$\int_{\delta_1}\int_{\delta_2} \exp\left\{\left[c\sigma_S - \left(\frac{\beta\big(\mu_M - r\big) + \gamma\theta\sigma_I^2}{\sigma_S}\right)\right]\sqrt{\nu}\delta_1\right\} \times$$
$$\exp\left\{-\left[\left(\frac{\big(\mu_M - r\big)}{\sigma_M}\right) - \beta\sigma_M\gamma\theta\right]\frac{\sigma_I}{\sigma_S}\sqrt{\nu}\delta_2\right\}\phi\big(\delta_1\big)\phi\big(\delta_2\big)d\delta_1 d\delta_2 \ . \tag{30}$$

We turn next to the specification of the integration region for each of the new variables. Clearly, for $a \leq S_\nu \leq b$ we have the following boundaries in terms of the δ_1 variable:

$$\frac{\ln\big(a/S_0\big) - \big(\mu_S - q_S - \sigma_S^2/2\big)\nu}{\sigma_S\sqrt{\nu}} \leq \delta_1 \leq \frac{\ln\big(b/S_0\big) - \big(\mu_S - q_S - \sigma_S^2/2\big)\nu}{\sigma_S\sqrt{\nu}}$$

or more compactly $A \leq \delta_1 \leq B$. By the other hand the range of integration for δ_2 is unrestricted. Hence, by omitting the exponential term that appears outside the double integral (30), we are left with

$$\frac{1}{\sqrt{2\pi}}\int_{\delta_2} \exp\left\{-\left[\left(\frac{\mu_M - r}{\sigma_S}\right) - \beta\sigma_M\gamma\theta\right]\left(\frac{\sigma_I}{\sigma_S}\right)\sqrt{\nu}\delta_2 - \frac{1}{2}\delta_2^2\right\}d\delta_2 \times$$
$$\frac{1}{\sqrt{2\pi}}\int_A^B \exp\left\{\left[c\sigma_S - \left(\frac{\beta\big(\mu_M - r\big) + \gamma\theta\sigma_I^2}{\sigma_S}\right)\right]\sqrt{\nu}\delta_1 - \frac{1}{2}\delta_1^2\right\}d\delta_1 \ .$$

Each integral is solved by completing the square. Thus, for the first integral, we get

$$\exp\left\{\frac{1}{2}\left[\left(\frac{\mu_M - r}{\sigma_M}\right) - \beta\sigma_M\gamma\theta\right]^2\left(\frac{\sigma_I}{\sigma_S}\right)^2\nu\right\},$$

whereas for the second integral, we obtain:

$$\exp\left\{\frac{1}{2}\left[c\sigma_S - \left(\frac{\beta\big(\mu_M - r\big) + \gamma\theta\sigma_I^2}{\sigma_S}\right)\right]^2\nu\right\} \times \left\{\Phi\left[\frac{\ln\big(b\big) - \mu - c\sigma^2}{\sigma}\right] - \Phi\left[\frac{\ln\big(a\big) - \mu - c\sigma^2}{\sigma}\right]\right\},$$

for $\mu = \ln(S_0) + \big(\hat{r} - \hat{q}_S - (\sigma_S^2/2)\big)\nu$ and $\sigma = \sigma_S\sqrt{\nu}$. In the computation of this integral we have made use of the relationship $\hat{r} - \hat{q}_S = r - q_S - \gamma\theta\sigma_I^2$ and the CAPM condition $\big(\mu_S - r\big) = \beta\big(\mu_M - r\big)$.

Finally and after some algebra, the product of the three remaining exponentials can be greatly simplified to

$$\exp\big\{-\hat{r}\nu\big\}\exp\left\{c\mu + c^2\frac{\sigma^2}{2}\right\} \ .$$

Summing up, we obtain:

$$\mathbb{E}_0\left[\left(\frac{\Theta_\nu}{\Theta_0}S_\nu^c\right)\mathbb{1}_{\{a \leq S_\nu \leq b\}}\right] = \exp\big\{-\hat{r}\nu\big\}\exp\left\{c\mu + c^2\frac{\sigma^2}{2}\right\} \times \left\{\Phi\left(\frac{\ln b - \mu - c\sigma^2}{\sigma}\right) - \Phi\left(\frac{\ln a - \mu - c\sigma^2}{\sigma}\right)\right\} \ .$$

D. Proof of Corollary 5

Following the same steps as in Appendix C, we find that the solution to equation (15) is given by

$$V\big(S\big) = \begin{cases} \tilde{a}_1 S^{\alpha_1} & \text{if } S < K \\ \tilde{b}_1 S^{\alpha_1} + \tilde{b}_2 S^{\alpha_2} + \left(\dfrac{\lambda S}{\lambda + q_S} - \dfrac{\lambda K}{\lambda + r}\right) & \text{if } K \leq S \leq \hat{S}^* \\ S - K & \text{if } S \geq \hat{S}^* \end{cases} \tag{31}$$

where α_1 and α_2 are, respectively, the positive and negative root of the quadratic equation $\alpha^2 + \big(b-1\big)\alpha - c$, for

$$c \equiv \frac{2}{\sigma_S^2}(r + \lambda) = -\alpha_1\alpha_2 \quad \text{and} \quad b \equiv \frac{2}{\sigma_S^2}\big(r - q_S\big) = 1 - \alpha_1 - \alpha_2 \ . \tag{32}$$

Again, a similar procedure leads to the following values for the constants $\tilde{a}_1$, $\tilde{b}_2$ and $\tilde{b}_1$:

$$\tilde{b}_2 = \frac{2\lambda}{\sigma_S^2}\left(\frac{1}{\alpha_2\big(\alpha_2-1\big)\big(\alpha_1-\alpha_2\big)}\right)K^{1-\alpha_2} := \tilde{B}_2 K^{1-\alpha_2} \tag{33}$$

$$\begin{aligned}
\tilde{b}_1 &= \left(1-\frac{(2\lambda/\sigma_S^2)}{(\alpha_1-1)(1-\alpha_2)}\right)\big(\hat{S}^*\big)^{1-\alpha_1}-\left(1+\frac{(2\lambda/\sigma_S^2)}{\alpha_1\alpha_2}\right)K\big(\hat{S}^*\big)^{\alpha_1}-\tilde{b}_2\big(\hat{S}^*\big)^{\alpha_2-\alpha_1} = \\
&= \left\{\left(1-\frac{(2\lambda/\sigma_S^2)}{(\alpha_1-1)(1-\alpha_2)}\right)\left(\frac{\hat{S}^*}{K}\right)^{1-\alpha_1}-\left(1+\frac{(2\lambda/\sigma_S^2)}{\alpha_1\alpha_2}\right)\left(\frac{\hat{S}^*}{K}\right)^{\alpha_1}-\tilde{B}_2\left(\frac{\hat{S}^*}{K}\right)^{\alpha_2-\alpha_1}\right\}K^{1-\alpha_1} \\
&\equiv \tilde{B}_1 K^{1-\alpha_1}
\end{aligned} \tag{34}$$

$$\tilde{a}_1 = \tilde{b}_1+\frac{2\lambda}{\sigma_S^2}\left(\frac{1}{\alpha_1\big(\alpha_1-1\big)}\right)\frac{K^{1-\alpha_1}}{(\alpha_1-\alpha_2)} \equiv \tilde{A}_1 K^{1-\alpha_1}\ . \tag{35}$$

Table 1: Subjective perpetual ESO valuation

		$\beta = 0$					$\beta = 1$				
	θ	0.00	0.10	0.20	0.30	0.40	0.00	0.10	0.20	0.30	0.40
γ	σ_S	Panel A: $\lambda = 0.1$, $\nu = 0$ years									
2	0.30	11.000	8.802	7.404	6.420	5.683	11.000	9.673	8.714	7.987	7.416
	0.40	12.859	9.617	7.813	6.624	5.768	12.859	10.266	8.694	7.611	6.810
	0.60	16.248	10.866	8.410	6.923	5.911	16.248	11.273	8.899	7.432	6.419
4	0.30	11.000	7.273	5.415	4.281	3.516	11.000	8.605	7.127	6.105	5.348
	0.40	12.859	7.644	5.456	4.215	3.416	12.859	8.527	6.453	5.190	4.332
	0.60	16.248	8.174	5.538	4.185	3.345	16.248	8.656	6.007	4.596	3.719
		Panel B: $\lambda = 0.1$, $\nu = 3$ years									
2	0.30	9.778	7.692	6.365	5.415	4.689	9.778	8.516	7.613	6.928	6.392
	0.40	11.324	8.232	6.450	5.221	4.310	11.324	8.858	7.337	6.259	5.446
	0.60	14.095	8.756	6.054	4.356	3.216	14.095	9.187	6.616	4.961	3.823
4	0.30	9.778	6.233	4.382	3.169	2.311	9.778	7.505	6.098	5.101	4.343
	0.40	11.324	6.259	3.892	2.466	1.560	11.324	7.159	5.012	3.617	2.645
	0.60	14.095	5.733	2.636	1.205	0.533	14.095	6.296	3.193	1.635	0.828
		Panel C: $\lambda = 0.2$, $\nu = 0$ years									
2	0.30	8.296	6.930	5.994	5.306	4.774	8.296	7.485	6.863	6.370	5.969
	0.40	9.951	7.863	6.597	5.720	5.066	9.951	8.297	7.216	6.434	5.837
	0.60	13.080	9.380	7.507	6.311	5.468	13.080	9.677	7.884	6.718	5.885
4	0.30	8.296	5.920	4.612	3.766	3.170	8.296	6.803	5.807	5.086	4.536
	0.40	9.951	6.492	4.856	3.865	3.197	9.951	7.116	5.610	4.639	3.952
	0.60	13.080	7.340	5.179	3.990	3.238	13.080	7.716	5.573	4.360	3.574
		Panel D: $\lambda = 0.2$, $\nu = 3$ years									
2	0.30	6.240	5.062	4.263	3.668	3.199	6.240	5.538	5.009	4.593	4.258
	0.40	7.365	5.560	4.433	3.625	3.011	7.365	5.939	4.996	4.300	3.761
	0.60	9.450	6.120	4.290	3.108	2.304	9.450	6.404	4.674	3.528	2.729
4	0.30	6.240	4.192	3.019	2.215	1.630	6.240	4.954	4.100	3.470	2.978
	0.40	7.365	4.320	2.743	1.758	1.120	7.365	4.894	3.495	2.550	1.878
	0.60	9.450	4.077	1.901	0.874	0.389	9.450	4.464	2.296	1.184	0.602

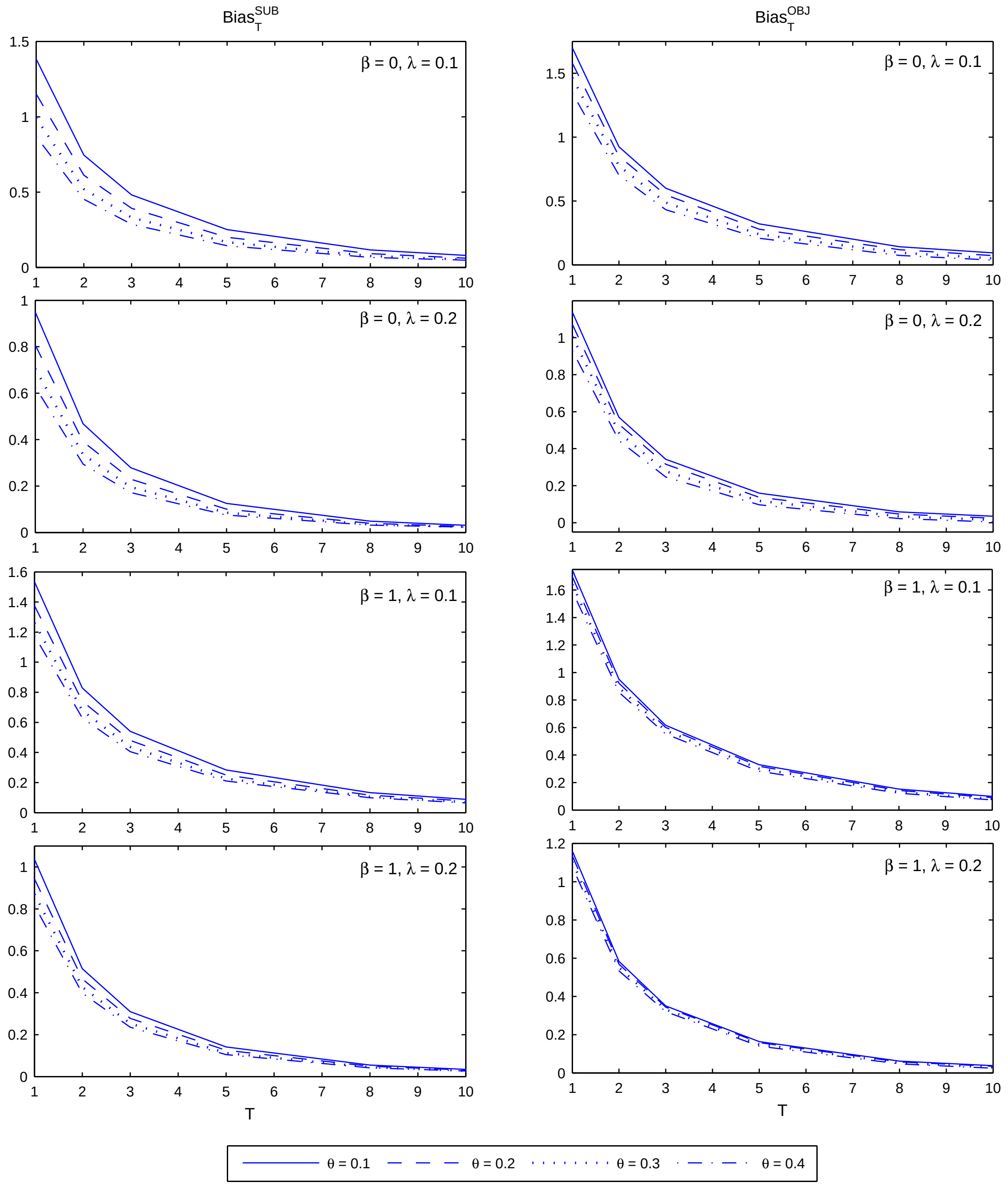
This table shows the subjective ESO value obtained by using either equation (11) or (14). The first column, γ, contains different risk-aversion coefficients. The second column, σ_S, contains the different levels of firm's stock volatility. The next five columns are obtained with $\beta = 0$ and the remaining ones with $\beta = 1$. We also consider different values of θ ranging from 0% to 40%. The table is divided in four panels for different combinations of the employment shock intensity, λ, and the vesting period in years, ν. Other parameters for ESO valuation are: $S_0 = K = \$30$, corresponding to the initial firm's stock price and the option strike price respectively, the annual risk-free rate, $r = 6\%$, both the annual stock and market portfolio continuously compounded dividend rates, $q_S = 1.5\%$ and $q_M = 0\%$, and the annual market portfolio volatility, $\sigma_M = 20\%$.

Table 2: Objective Valuation: perpetual vs. finite maturity ESO

			$\theta = 0.0$			$\theta = 0.1$			$\theta = 0.2$			$\theta = 0.3$			$\theta = 0.4$		
λ	β	γ	$T = \infty$	$T = 10$	$T = 5$	$T = \infty$	$T = 10$	$T = 5$	$T = \infty$	$T = 10$	$T = 5$	$T = \infty$	$T = 10$	$T = 5$	$T = \infty$	$T = 10$	$T = 5$
Panel A: $\nu = 1$ year																	
0.1	0	2	12.534	11.424	9.659	11.907	10.932	9.441	11.007	10.377	9.096	10.217	9.892	8.790	9.572	9.479	8.541
		4	12.534	11.424	9.659	10.893	10.328	9.053	9.310	9.332	8.462	8.280	8.568	7.970	7.595	7.960	7.551
	1	2	12.534	11.424	9.659	12.136	11.082	9.504	11.500	10.680	9.301	10.892	10.293	9.039	10.361	9.960	8.832
		4	12.534	11.424	9.659	11.404	10.620	9.250	10.084	9.825	8.755	9.099	9.175	8.363	8.383	8.627	8.015
0.2	0	2	9.339	8.429	8.071	9.002	8.609	7.900	8.481	8.249	7.619	8.011	7.938	7.397	7.622	7.659	7.215
		4	9.339	8.429	8.071	8.420	8.207	7.595	7.479	7.573	7.158	6.840	7.070	6.790	6.398	6.672	6.478
	1	2	9.339	8.429	8.071	9.130	8.702	7.950	8.766	8.446	7.772	8.407	8.190	7.575	8.087	7.977	7.425
		4	9.339	8.429	8.071	8.716	8.407	7.755	7.940	7.899	7.372	7.345	7.475	7.081	6.900	7.116	6.823
Panel B: $\nu = 3$ years																	
0.1	0	2	11.324	10.217	8.442	10.795	9.919	8.365	10.175	9.664	8.300	9.701	9.452	8.240	9.342	9.253	8.178
		4	11.324	10.217	8.442	10.103	9.645	8.296	9.201	9.197	8.160	8.665	8.794	8.021	8.318	8.460	7.882
	1	2	11.324	10.217	8.442	10.975	9.981	8.377	10.500	9.780	8.335	10.102	9.623	8.290	9.784	9.478	8.247
		4	11.324	10.217	8.442	10.435	9.778	8.332	9.625	9.432	8.234	9.089	9.110	8.134	8.718	8.821	8.033
0.2	0	2	7.365	6.986	6.105	7.120	6.821	6.052	6.830	6.690	6.008	6.606	6.572	5.968	6.433	6.454	5.927
		4	7.365	6.986	6.105	6.800	6.681	6.006	6.372	6.421	5.915	6.108	6.199	5.822	5.929	6.010	5.729
	1	2	7.365	6.986	6.105	7.204	6.854	6.060	6.981	6.757	6.031	6.793	6.664	6.001	6.640	6.584	5.972
		4	7.365	6.986	6.105	6.953	6.745	6.028	6.573	6.557	5.963	6.316	6.379	5.896	6.132	6.219	5.829

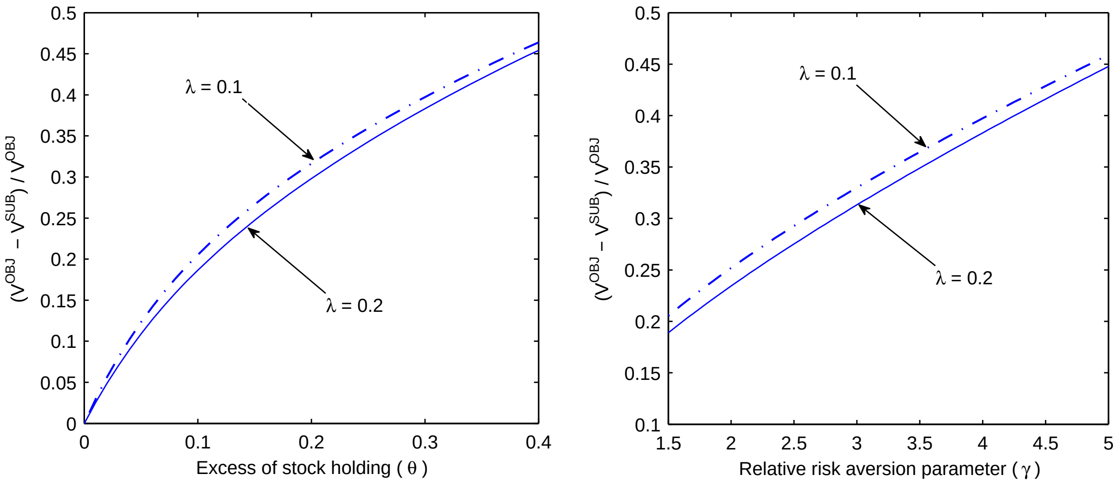
This table compares the objective value of perpetual ESOs to finite ones for maturities of 5 and 10 years. The first column, λ, denotes the intensity of the Poisson process which drives the employment shocks. The second column, β, contains the different levels of the market-β. The third column, γ, represents different levels of executive's risk aversion. The following 5 blocks represents the objective value of ESO for several values of θ ranging from 0 to 40%. We also consider two different vesting period lengths in Panel A ($\nu = 1$ year) and Panel B ($\nu = 3$ years). The remaining parameters are: $S_0 = K = \$30$, $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\sigma_S = 30\%$, $\sigma_M = 20\%$ and $\lambda = 10\%$. For the finite maturity cases, we display the average of 50 previous estimations obtained with 40,000 paths.

Figure 1: ESO price relative biases and time to maturity



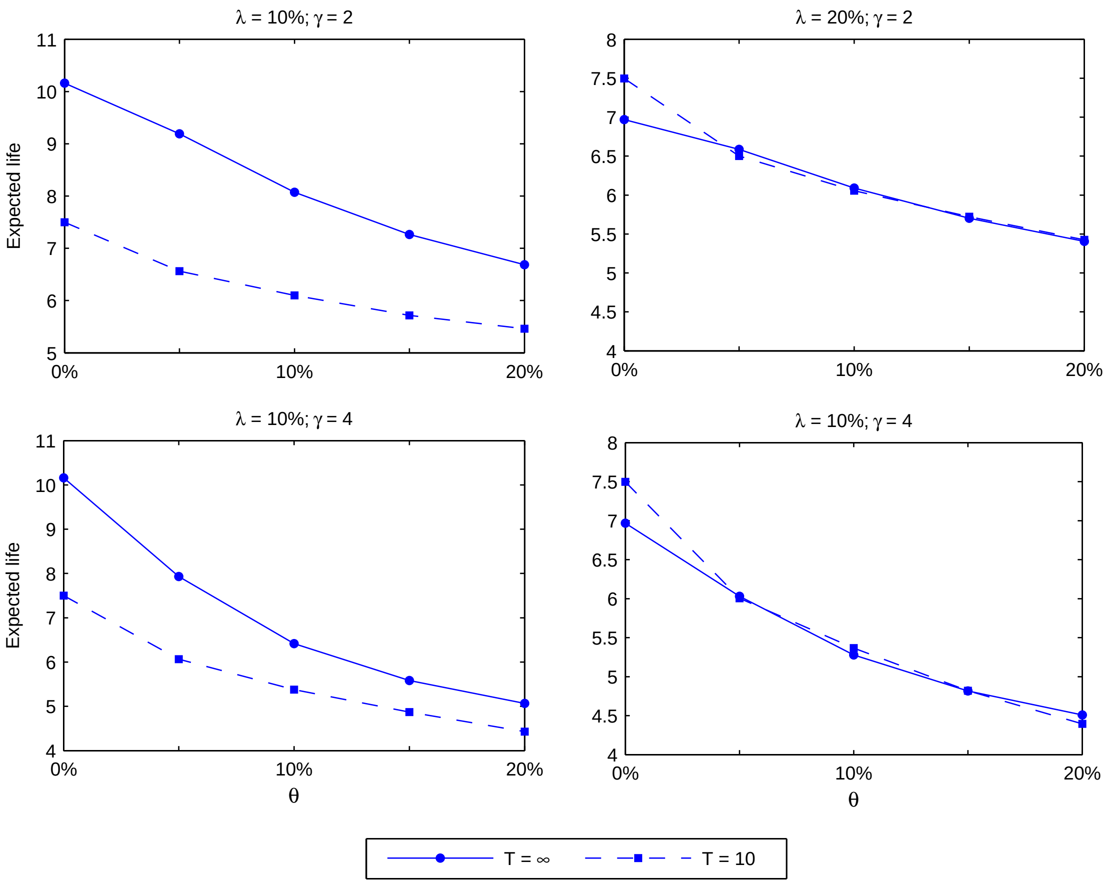
This figure shows the relative biases of subjective and objective perpetual ESOs for different maturities (in x-axis) ranging from $T = 1$ to $T = 10$ years. Each graphic exhibits a different pair of values for (β, λ) and each line, inside each graphic, corresponds to a different value of θ. The left-hand graphics display the relative bias of the subjective perpetual ESO computed as $Bias\,_T^{SUB} = (V^{SUB} - V_T^{SUB})/V_T^{SUB}$, where V^{SUB} is the subjective perpetual ESO price from Table 1 and V_T^{SUB} is the corresponding subjective ESO price with maturity of T years. Similarly, the right-hand graphics display the relative bias of the objective perpetual ESO computed as $Bias\,_T^{OBJ} = (V^{OBJ} - V_T^{OBJ})/V_T^{OBJ}$, where V^{OBJ} is the objective perpetual ESO price from Table 2 and V_T^{OBJ} is the corresponding objective ESO price with maturity of T years. The procedure to obtain V_T^{SUB} and V_T^{OBJ} are described in Sections 2 and 3, respectively. The values for the remaining parameters are: $S_0 = K = \$30$, $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\lambda = 10\%$, $\sigma_S = 40\%$, $\sigma_M = 20\%$ and $\nu = 0$.

Figure 2: Comparing objective and subjective ESO valuation



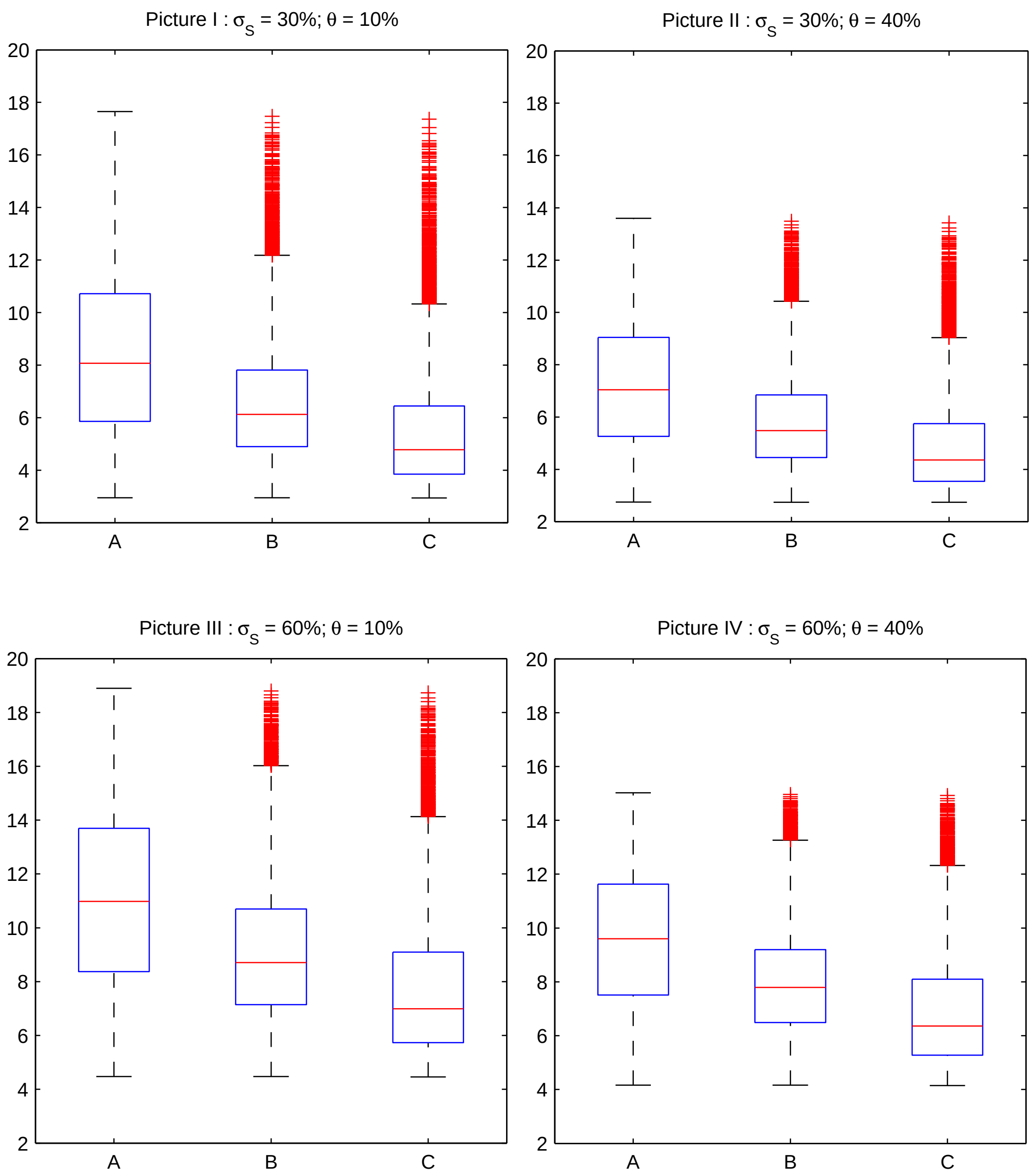
This figure displays how the difference between the objective and subjective perpetual values with respect to the objective perpetual one, $(V^{OBJ} - V^{SUB})/V^{OBJ}$, changes for different values of θ (left-hand graphic) and γ (right-hand graphic) for two alternative values of the intensity rate, $\lambda = 10\%$ and $\lambda = 20\%$. The values for the remaining parameters are: $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\sigma_S = 30\%$, $\sigma_M = 20\%$, $S_0 = K = \$30$ and $\nu = 3$ years.

Figure 3: Expected term and FAS 123R



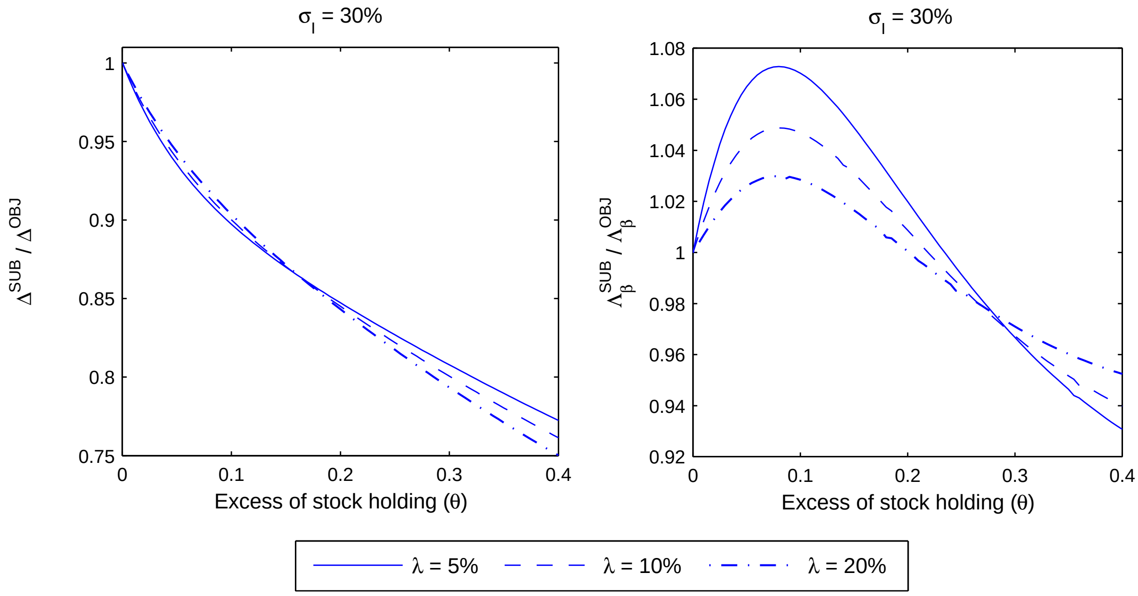
This figure compares the expected term from an objective ESO with maturity of 10 years and a vesting period of 3 years (discontinuous line) with that obtained by calibrating equation (20) using the objective perpetual ESO value (solid line) for different values of θ on the x-axis. The left-hand (right-hand) graphics correspond to $\lambda = 10\%$ ($\lambda = 20\%$) for different values of γ. The values for the remaining parameters are: $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\sigma_S = 40\%$ and $S_0 = K = \$30$.

Figure 4: Box-plots for objective ESO prices



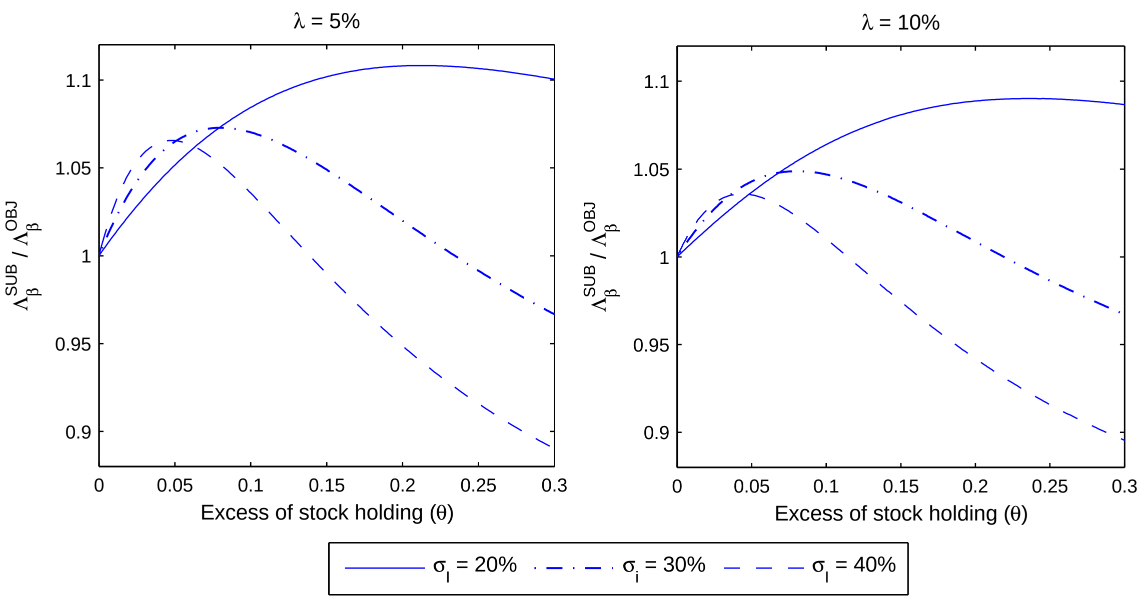
This figure shows the distribution for objective perpetual ESO values implied by the corresponding random samples drawn from alternative Triangular distributions of λ denoted as A, B and C, described in subsection 3.4. All distributions have a common support given by the closed interval $[0\%, 40\%]$. The values for the remaining parameters are: $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\gamma = 4$, $\beta = 1$, $\sigma_S = 40\%$, $\beta = 1$, $S_0 = K = \$30$ and $\nu = 3$ years.

Figure 5: Greek ratios and employment shocks



The left-hand picture displays the delta ratio, $\Delta^{SUB}/\Delta^{OBJ}$, as a function of θ taking $S_0 = K$ as a midpoint in the computation of the numerical partial derivatives. The right-hand picture displays the systematic risk ratio, $\Lambda_{\beta}^{SUB}/\Lambda_{\beta}^{OBJ}$, as a function of θ using $\beta = 1$ as a midpoint. Each picture depicts several plots for different values of the likelihood of employment shocks, λ. The values for the remaining parameters are: $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\gamma = 2$, $\beta = 1$, $\sigma_M = 20\%$, $S_0 = K = \$30$ and $\nu = 3$ years.

Figure 6: Greek ratios and idiosyncratic risk



The figure displays the systematic vega ratio, $\Lambda_{\beta}^{SUB}/\Lambda_{\beta}^{OBJ}$, as a function of θ for $\lambda = 5\%$ (right-hand side) and $\lambda = 10\%$ (left-hand side). In both cases, the ratio is computed for at-the-money ESOs and $\gamma = 2$. Each picture depicts several plots for different idiosyncratic risk values, σ_I. The values for the remaining parameters are: $r = 6\%$, $q_S = 1.5\%$, $q_M = 0\%$, $\beta = 1$, $\sigma_M = 20\%$, $S_0 = K = \$30$ and $\nu = 3$ years.