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EFFECTS OF DOMINANCE ON OPERATION POLICIES IN A TWO-STAGE SUPPLY CHAIN IN WHICH MARKET DEMANDS FOLLOW THE BASS DIFFUSION MODEL

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Abstract. The Bass model offers several successful applications in forecasting the diffusion process of new products. Due to its potential and flexibilities, the application of this model is not only limited now to forecasting, but also extends to other fields such as analyzing a supply chain's responses, optimizing production plans, and so forth. This study investigates inventory and production policies in a two-stage supply chain with one manufacturer and one retailer, in which the market demand process follows the Bass diffusion model. The model assumes the market parameters and essential information are available and ready for access. This study then applies dynamic programming and heuristic algorithm to find the optimal policies for each stage under different scenarios.

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1. Introduction

Developments in technology and engineering in the last few decades have laid a strong foundation for modern gadgets. However, when a new product is introduced to the market, customers do not immediately accept it, because potential customers have to wait to be sure about the product's effectiveness. Customers' adoption process occurs gradually and is known as the diffusion phenomenon. Real data demonstrating such diffusion can be found through a wide range of products such as color televisions, air conditioners, and refrigerators to name a few. Although the diffusion process depends heavily on potential markets, it is influenced by product advertisement. Deciding an appropriate advertisement rate helps managers optimize their inventory policies, which provide various benefits for the company. Aside from advertising, the dominant property also plays an important role in contributing to the benefit of an optimal inventory policy, especially when the interactions between different stages of the supply chain are complicated.

In this study the dominant property means that, if one stage dominates other stages, then that stage just focuses on maximizing its own profits while ignoring the strategies or actions of other stages. This dominant property can be illustrated through a push or pull system. In such a system, the dominating stage is the

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manufacturer that tries to produce as much as possible and push the items to the retailer without considering the retailer's strategy. On the other hand, the retailer is the dominating stage in a pull system and places orders, ignoring the manufacturer's strategy. This research extends previous studies to consider the impacts of advertising and the dominant relationship in a two-stage supply chain, given that customer demands follow the Bass model. We also develop a mathematical model and a corresponding dynamic programming and genetic heuristic algorithm approach to validate our model.

The rest of the paper is organized as follows. Section 2 is the literature review. Section 3 discusses the development of the proposed model. Section 4 gives an illustrative example of the proposed model. Finally, Section 5 draws conclusions.

2. Literature review

Diffusion of innovation is an important and commonly researched issue in marketing science. Over the years, several studies have modeled this process [2, 5, 13, 16, 17], with Bass [2] suggesting a diffusion of the innovation model and providing extensive empirical evidence to support it. According to Bass, the diffusion of a new product is a combination of two different procedures – innovation and imitation – that describe two adoption processes of potential customers. The standard Bass model can be expressed through the following differential equation.

$$\frac{dD(t)}{\mathrm{d}t} = p\left(M - D\left(t\right)\right) + \frac{q}{M} D\left(t\right)\left(M - D\left(t\right)\right). \tag{2.1}$$

The terms *D(t)*, M, p, and q are cumulative adopters, the potential market, the coefficient of innovation, and the coefficient of imitation, respectively. Following its introduction, the Bass model has been extensively used in modeling the diffusion of a wide range of products and has proved its validation and success well in fitting real data. The success of the Bass model in forecasting a market's adoption rate and its simplicity in application have led to a number of further enhancements by adding realistic constraints and effects in order to achieve better insights and approximation of real marketing issues.

A realistic important issue that has attracted the attention of many scholars when studying the Bass model is the advertisement effect. A large number of studies has targeted the effect of advertisement on this process [3, 7, 11, 15]. Horsky and Simon [7] presented a diffusion model under the effects of advertising. According to them, advertising improves the diffusion process, and an optimal advertisement policy is to advertise intensively in the first stages of the product life cycle and to reduce the advertisement rate when sales increase. Bass *et al.* [3] suggested a general Bass model that is now widely accepted by other scholars. This model poses the effect from a current marketing effort that presents validation through empirical data and that is consistent with diffusion properties. Based on the general Bass model, Krishnan and Jain [11] verified an optimal advertisement policy under different correlations between the discounted advertising coefficient and the advertising-sales ratio. Nikolopoulos and Yannacopoulos [15] studied the optimal advertising for the Bass diffusion model under stochastic dependence of environmental pressure and recommended the optimal stopping rule and the expected length of time for an advertisement campaign.

It is well-known that the type of relationship in a common supply chain is often a dominated relationship where the dominating stages try to maximize their own profit. This issue has been described and studied by many scholars. Lu [12] presented an optimal solution for a one-vendor, one-buyer inventory model, whereby the demand from the buyer has unchanged quantity and frequency. The heuristic approach was also proposed for the case of one vendor as well as multi-players. Almehdawe and Mantin [1] introduced a Stackelberg game between a single manufacturer and multiple retailers. Their study assumed that demand follows an elastic pattern. Changing the role of the dominating player clarifies the efficiency of the supply chain and the benefit for each retailer. Yu *et al.* [19] investigated the vendor inventory management system with the vendor taking up responsibilities for procuring raw materials and supplying the finished product. They found the equilibrium point at which the retailer profit is maximized.

3. Model development

In the generalized Bass model, the adoption rate can be controlled through the current marketing effort. This "current marketing effort" describes the impacts of the dynamic marketing decision variables on the adoption rate at time t [3]. Let $x(t)$ be the current marketing effort at time t:

$$x(t) = 1 + \beta_v \frac{v'(t)}{v(t)} + \beta_a \frac{a'(t)}{a(t)}, \tag{3.1}$$

where *v(t)* and *a(t)* are the price and advertisement expenditure at time t, respectively; *v'(t)* and *a'(t)* are the instantaneous changes in price and advertisement at time t, respectively; and $\beta v S$ and βa are the coefficients of price and advertisement, respectively. This study keeps the price constant, and only the advertisement expenditure changes. Using these notations and the initial condition $a(ti) = ai$, the explicit formula for advertisement expenditure given that *x(t)* is unchanged and equal to xi is:

$$a(t) = a_i \exp\left[\frac{(t-t_i)(x_i-1)}{\beta_a}\right]. \tag{3.2}$$

We make a few assumptions in this study. First, the system is transparent, meaning that the data of all stages are available for other stages. Due to a transparent system, the manufacturer can be an observer and obtain parameters of the market. Using these parameters, the manufacturer determines the default value for the production rate, which is preferable to other values. We also assume that the number of replenishments between two stages is predetermined. The lead time is zero, and shortages are not allowed. Fixed costs that occur with orders are identical functions of the order quantity. Under these assumptions, two scenarios can model the interaction within the supply chain system. In the 1st scenario, the manufacturer stage dominates the retailer stage. In the 2nd scenario, the retailer is the dominating stage of the supply chain.

In our annotations the superscripts A and B represent the manufacturer and the retailer, respectively. The following annotations are used in our model formulations.

M Potential market size (ultimate number of adopters)
p Coefficient of innovation
q Coefficient of imitation
k Fixed portion of the market to be satisfied
m Number of replenishments
$D_i^B(t)$ The cumulative demand function appears at the retailer stage between $[t_i,\ t_{i+1}]$
$d_i^B(t)$ The demand rate function appears at the retailer stage between $[t_i,\ t_{i+1}]$
c^B Retailer capacity
x_i Current marketing effort between $[t_i,\ t_{i+1}]$
βa Coefficient of advertisement
a_r Normal advertisement cost rate
a_i Advertisement cost rate at t_i
t_i Time when the ith replenishment arrives at the retailer stage
q_i Quantity of the ith replenishment
Q_i Cumulative quantity up to the ith replenishment
v^A Unit variable cost of raw material for manufacturing one item
v^B Retailer's unit purchasing cost
s^A Manufacturer's predetermined unit selling price
s^B Retailer's predetermined unit selling price
f^A Associated cost per unit time when the production rate is greater than p_d^A
w^A Associated cost per unit time when the production rate is less than p_d^A

h^A Manufacturer's unit holding cost, *i.e.* the cost of having one dollar of the item tied up in inventory for a unit time interval

h^B Retailer's unit holding cost

A_i^B Set-up cost incurred when the retailer receives the ith replenishment from the manufacturer; it is assumed that this function satisfies $A_i^B\,(a+b) \leq A_i^B\,(a) + A_i^B(b)$.

p_d^A Manufacturer's default production rate

p_i^A Manufacturer's production rate between $[t_i, t_{i+1}]$

$\triangle_p^A$ Maximum increases of the production rate from p_d^A

I_i^A Inventory on hand at the manufacturer just before the ith replenishment is delivered

I_i^B Inventory on hand at the retailer just before the ith replenishment is delivered

3.1. Scenario 1: manufacturer stage dominates retailer stage

When dominating the supply chain system, the manufacturer would rather maintain the production rate at the default value than vary it. Consequently, the manufacturer's decision variables are time, quantity of each delivery, and inventory on hand just before each replenishment is delivered. The retailer's decision variables are inventories on hand at each time when the replenishment arrives as well as the final marketing effort.

3.1.1. Manufacturer stage

The manufacturer's profit is the difference between total sales and total operation costs. The total sales of the manufacturer are given by:

$$\text{Total sales} = \sum_{i=1}^{n} (s^A - v^A) q_i. \tag{3.3}$$

Because $\sum_{i=1}^{n} q_i = kM$, or is a constant, the amount of total sales is also a constant. Maximizing profit now becomes an issue of minimizing the manufacturer's holding cost. Given a set of replenishment quantities, $\{q_1, q_2, \dots q_m\}$, let K_i denote the manufacturer's holding cost between ti and t_i+1. Figure 1 illustrates the manufacturer's inventory level.

Since $\frac{I_{i+1}^A - I_i^A + q_i}{p_d^A} = t_{i+1} - t_i$ and the inventory holding cost for one period is equal to the shaded area multiplied by the corresponding scale, in this case we have:

$$K_i = 0.5\left(I_{i+1}^A + I_i^A - q_i\right)\left(t_{i+1} - t_i\right) h^A v^A.$$

Plugging $\frac{I_{i+1}^A - I_i^A + q_i}{p_d^A} = t_{i+1} - t_i$ into the above formula, the final form of the inventory holding cost for a period is given as equation (3.4). Because after the last delivery the manufacturing inventory level is equal to zero, the inventory level of the manufacturer at that time must be equal to the delivery quantity q_m. As a result, the holding cost for the last period is given by equation (3.5).

$$K_i = 0.5\left({I_{i+1}^A}^2 - \left(I_i^A - q_i\right)^2\right) h^A v^A / p_d^A, \tag{3.4}$$

$$K_{m-1} = 0.5\left({q_m}^2 - \left(I_{m-1}^A - q_{m-1}\right)^2\right) h^A v^A / p_d^A. \tag{3.5}$$

The total holding cost L is:

$$L = \sum_{i=1}^{m} K_i = 0.5 h^A v^A / p_d^A \times \left[\sum_{i=1}^{m-1} (2 I_i^A q_i - {q_i}^2) + {q_m}^2\right]. \tag{3.6}$$

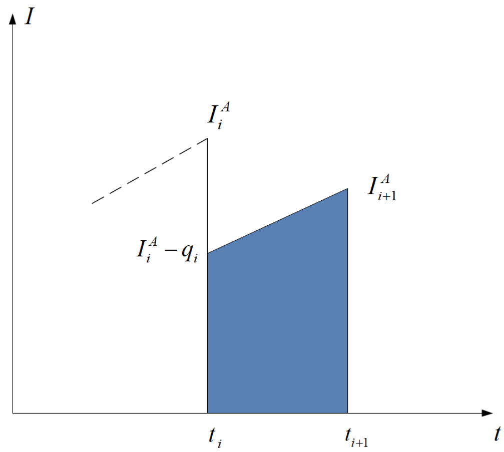

FIGURE 1. Manufacturer's inventory level versus time in the first scenario.

Because $I_i^A \geq q_i$, in order to miminize L, all I_i^A must be equal to q_i. Equation (3.6) is hence:

$$L = \sum_{i=1}^{m} K_i = 0.5h^A v^A / p_d^A \times \left[\sum_{i=1}^{m} {q_i}^2 \right], \tag{3.7}$$

where $0 \leq q_i \leq c^B$ and $\sum_{i=1}^{m} q_i = kM$. To minimize L, we get the following result:

$$q_i = \frac{kM}{m}. \tag{3.8}$$

The optimal replenishment cycle with respect to this manufacturing scheme is:

$$t_{i+1} - t_i = \frac{kM}{p_d^A m}. \tag{3.9}$$

3.1.2. Retailer stage

The customer demand that the retailer has to face is assumed to follow the Bass model [3] and has the form:

$$d_i^B(t) = \left[p\left(M - D_i^B(t)\right) + \frac{q}{M} D_i^B(t) \left(M - D_i^B(t)\right) \right] x_i. \tag{3.10}$$

Because of being the dominated stage, the retailer at each specific time t_i, which is decided by the manufacturer, receives a replenishment quantity q_i from the manufacturer. To maximize profit, the retailer has to decide the inventory on hand at each t_i and I_i^B, where $0 \leq I_i^B \leq c^B - q_i$. In addition, after receiving the last replenishment, q_m, the retailer is able to set up the final marketing effort to obtain optimal profit. Consequently, the retailer's decision variables are $I_1^B, I_2^B, \ldots, I_m^B$, and x_m.

The retailer's profit is the difference between the retailer's total sales and the total relevant cost, where the retailer's total sales are a constant and given by:

$$\text{Total sales} = \sum_{i=1}^{n} (s^B - v^B) q_i = (s^B - v^B) kM. \tag{3.11}$$

In order to maximize profit, the retailer must minimize the total relevant cost. For the time period i, the relevant cost comprises three components: advertisement cost, $\int_{t_i}^{t_{i+1}} a(t)\,\mathrm{d}t$; fixed set-up cost, A_i^B; and inventory holding cost, $h^B v^B \int_{t_i}^{t_{i+1}} \left[\frac{ikM}{m} - D_i^B(t)\right] \mathrm{d}t$. Thus, we have:

$$\text{Total relevant cost} = \sum_{i=1}^{m} \left[\int_{t_i}^{t_{i+1}} a(t)\,\mathrm{d}t + A_i^B + h^B v^B \int_{t_i}^{t_{i+1}} \left[\frac{ikM}{m} - D_i^B(t) \right] \mathrm{d}t \right]. \tag{3.12}$$

Since the retailer's decision variables are $I_1^B, I_2^B, \ldots, I_m^B$, and x_m, it is necessary to convert the total relevant cost into a function that depends on these variables. Due to being dominated, from the retailer's view, the values of t_i in equation (3.12) are constants and determined by the manufacturer *via* equation (3.9). In order to calculate the retailer's total relevant cost, the functions $a(t)$ and $D_i^B(t_i)$ depending on $I_1^B, I_2^B, \ldots, I_m^B$, and x_m must be identified. Since the function $a(t)$ can be easily established when x_i is given through equation (3.2), it is necessary to identify only functions x_i and $D_i^B(t_i)$. We employ equation (3.10), with an appropriate constraint, to determine $D_i^B(t_i)$. The first constraint is the total cumulative demand at each t_i, $D_i^B(t_i)$, which is given by:

$$D_i^B(t_i) = \frac{(i-1)kM}{m} - I_i^B. \tag{3.13}$$

Let $\emptyset_i = \frac{(i-1)kM}{m} - I_i^B$, and then the formula of $D_i^B(t)$ is as follows:

$$D_i^B(t) = M - \frac{M(p+q)}{q + \exp\left[M(p+q)\left[\frac{t}{M}x_i + \frac{1}{M(p+q)}\left(ln\left(\frac{Mp+q\emptyset_i}{M-\emptyset_i}\right) - t_i(p+q)x_i\right)\right]\right]}. \tag{3.14}$$

In addition, equation (3.10) must be satisfied by another constraint – that is, $D_i^B(t_{i+1}) = \frac{ikM}{m} - I_{i+1}^B$. This constraint is applied for equation (3.14), and then the value of x_i, where $i \epsilon [1, m-1]$, is obtained by:

$$x_i = \frac{p_d^A m}{kM(p+q)} \left[ln\left(\frac{Mp+q\emptyset_{i+1}}{M-\emptyset_{i+1}}\right) - ln\left(\frac{Mp+q\emptyset_i}{M-\emptyset_i}\right) \right]. \tag{3.15}$$

From equations (3.14) and (3.15), x_i and $D_i^B(t)$ depend only on I_i^B and I_{i+1}^B. Consequently, the relevant cost between two replenishments i and $i+1$ is the functions of these two decision variables. However, at the last replenishment, by controlling the final marketing effort, x_m, the retailer can decide the end of the planning horizon. As a result, the last relevant cost is the functions I_m^B and x_m. By replacing x_m in equations (3.2) and (3.14) as well as solving equation (3.14) with the constraint that cumulative demand at the end of the planning horizon is equal to kM, we can find the final formula for the relevant cost of the last replenishment as:

$$V = A_m^B + \frac{\beta_a a_m}{x_m - 1}\left[exp\left(\frac{G(x_m-1)}{x_m(p+q)\beta_a}\right) - 1\right] + \frac{h^B v^B kMG}{x_m(p+q)} + \frac{h^B v^B MpG}{qx_m(p+q)} + \frac{h^B v^B M}{x_m q} ln\left(\frac{M-kM}{M-\emptyset_m}\right), \tag{3.16}$$

where:

$$G = ln\left[\frac{(Mp+qkM)(M-\emptyset_m)}{(Mp+q\emptyset_m)(M-kM)}\right]. \tag{3.17}$$

We apply the gradient method to update and find the optimal value of x_m. In order to find values of I_i^B, various meta-heuristics methods can be employed to solve this issue. Section 4 presents and discusses a numerical example with a detailed algorithm.

3.2. Scenario 2: retailer stage dominates manufacturer stage

Due to being a dominating stage, the retailer's decision variables are the quantity of each replenishment, q_i, and the marketing effort between two consecutive replenishments, k_i, given that replenishment is placed only when the inventory level drops to zero. Under the retailer's domination, the manufacturer's variables are the production rate, p_i^A, between two consecutive replenishments and the initial inventory.

3.2.1. Retailer stage

Since new replenishment is placed only when the inventory level falls to zero, it is like stating that $D_i^B(t_i) = Q_{i-1}$ for every t in the appropriate domain, where $Q_{i-1} = \sum_{j=1}^{i-1} q_j$. Presumably, it is $D_i^B(t_0)$ for some specific time t_0. Solving equation (3.10) with the constraint $D_i^B(t_i) = Q_{i-1}$, the explicit formula for $D_i^B(t)$ is obtained as equation (3.18). Employing the constraint $D_i^B(t_{i+1}) = Q_i$ to equations (3.18) and (3.19) is thus presented as:

$$D_i^B(t) = M - \frac{M(p+q)}{q + \exp\left[M(p+q)\left[\frac{t}{M}x_i + \frac{1}{M(p+q)}\left(ln\left(\frac{Mp+qQ_{i-1}}{M-Q_{i-1}}\right) - t_i(p+q)x_i\right)\right]\right]}, \tag{3.18}$$

$$t_{i+1} = t_i + \frac{1}{x_i(p+q)}\left[ln\left(\frac{Mp+qQ_i}{M-Q_i}\right) - ln\left(\frac{Mp+qQ_{i-1}}{M-Q_{i-1}}\right)\right]. \tag{3.19}$$

As mentioned in the 1st scenario, the retailer needs to minimize the total relevant cost for obtaining the optimal inventory policy. The retailer's total relevant cost in this scenario is:

$$\text{Total relevant cost} = \sum_{\mathrm{i}=1}^{m}\left[\int_{t_i}^{t_{i+1}} a(t)\,\mathrm{d}t + A_i^B + h^B v^B \int_{t_i}^{t_{i+1}} \left[Q_i - D_i^B(t)\right]\mathrm{d}t\right] \tag{3.20}$$

The constraints for equation (3.20) are $q_i \le c^B$, $\sum_{\mathrm{i}=1}^{m_1} q_i = kM$, and $x_i > 0$. Due to the large number of variables in equation (3.20), which includes $\{q_1, q_2, \ldots\ldots, q_m\}$ and $\{x_1, x_2, \ldots\ldots, x_m\}$, finding the exact optimal solution for this function is a difficult task. Therefore, we apply a meta-heuristic algorithm to find optimal or near-optimal solutions. The appendix provides a more detailed description of the algorithm.

3.2.2. Manufacturer stage

Due to transparent data and the retailer's announcement, the manufacturer knows exactly the demand that it has to satisfy. Assuming that the production rate is fixed between time *ti* and *ti+1*, the manufacturer stage is modeled as follows.

Let $H_i(r) =$ minimum operation cost from time t_i to the end of the planning horizon, given that r is the inventory level at time t_i, before replenishment quantity q_i is delivered. Since shortages are not allowed, $r \ge q_i$. The recurrence relationship for this problem is then given by:

$$H_i(r) = \min_{y = q_{i+1}, \ldots, z} \left(S_i(r,y) + H_{i+1}(y)\right), \tag{3.21}$$

where $z = \sum_{j=i+1}^{m} q_j$ and $q_i \le r \le q_i + z$.

We compute $S_i(r,y)$ as follows:

$$S_i(r,y) = +\infty, \quad if p_i^A > p_d^A + \triangle_p^A, \tag{3.22}$$

$$S_i(r,y) = (r - d_i)\, h^A v^A (t_{i+1} - t_i) + \frac{h^A v^A p (t_{i+1} - t_i)^2}{2} \ \ldots$$

$$+\begin{cases}\left(p_i^A-p_d^A\right)f^A\left(t_{i+1}-t_i\right), \text{ if } p_d^A+\triangle_p^A > p_i^A > p_d^A \\ \left(p_i^A-p_d^A\right)w^A\left(t_{i+1}-t_i\right), \text{ if } p_i^A \leq p_d^A\end{cases}, \tag{3.23}$$

where $p_i^A = \frac{y+q_i-r}{t_{i+1}-t_i}$. The boundary conditions for equation (3.19) are given by:

$$H_m\left(q_m\right)=0 \tag{3.24}$$

$$H_m\left(x\right)=+\infty \ (x\neq q_m). \tag{3.25}$$

We note that before the last delivery the inventory level of manufacturing is q_m and will immediately drop to zero after the success of last delivery. As a result of this, the manufacturer does not incur any cost, *i.e.* $H_m\left(q_m\right)=0$. Since it is not allowed to manufacture any quantity more than demand in the last replenishment, *i.e.* the redundant items cannot be sold to the retailer, other values of inventory level not equal to q_m are prohibited and incur an infinite cost.

The optimal solution is now given by:

$$\min_{r=d_1,\ldots,z}\left[\frac{h^A v^A r^2}{2p_d^A}+H_1(r)\right], \tag{3.26}$$

where $z=\sum_{i=1}^{m} q_i$. Let $t_{\max}=\max_{i=1,\ldots,m-1}$. The worst-case solution time for this stage is proportional to $z\times\left[\left(p_d^A+\triangle_p^A\right)\times t_{\max}\right]^{m-2}$.

4. Numerical example

Consider a supply chain that includes a manufacturer, a retailer, and a market. The market has its own parameters M, p, and q, corresponding to potential market size, the coefficient of innovation, and the coefficient of imitation, respectively. Both the manufacturer and the retailer agree that they only satisfy k portion of the market, and this is done through m times of replenishments. In addition, so as to maximize profit, the retailer decides on using advertisement to control the demand rate. This advertising process has two parameters, βa and a_r, denoting the coefficient of advertisement and the normal advertisement cost rate, respectively. The retailer incurs a set-up cost, A_i^B, every time it receives replenishment. This set-up cost is under the form of a staircase function given by:

$$A_i^B = A_0 + e_0\left\lceil\frac{q_i}{b_0}\right\rceil. \tag{4.1}$$

The unit holding cost, unit selling price, and unit purchasing cost of the retailer and manufacturer are h^B, s^B, v^B, h^A, s^A, and v^A, respectively. The manufacturer has a default production rate, p_d^A. In order to achieve a more flexible manufacturing system, the manufacturer's production rate can increase to a maximum value of $p_d^A+\triangle_p^A$. However, this flexibility incurs a cost whenever the production rate differs from the default value. This cost can be interpreted as the money paid for the labor due to working overtime or the penalty of not utilizing resources well. Depending on the scenarios, the decision variables of each stage are different. Tables 1–3 give parametric values of this numerical example.

Table 1. Market and advertisement's parameters.

Parameter	M	p	q	k	m	a_r	β_a
Value	800	0.0163	0.0325	0.96	6	10	1.5

TABLE 2. Manufacturer's parameters.

Parameter	s^A	v^A	h^A	p_d^A	$\triangle_p^A$	f^A	w^A
Value	35	22	0.02	60	10	3	1

TABLE 3. Retailer's parameters.

Parameter	s^B	v^B	h^B	c^B	A_0	e_0	b_0
Value	55	35	0.02	160	100	80	50

TABLE 4. Manufacturer's production plans in the 1st scenario.

No.	1	2	3	4	5	6
Time	0	2.133	4.267	6.4	8.533	10.667
q_i	128	128	128	128	128	128

Total relevant cost = 360.5.

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6

FIGURE 2. Retailer's chromosome structure in the 1st scenario.

TABLE 5. Retailer's inventory policies in the 1st scenario.

No.	1	2	3	4	5	6
Time	0	2.133	4.267	6.4	8.533	10.667
I_i^B	0	32	32	32	21	32
x_i	1.853	1.186	0.888	0.927	0.9277	10

Total relevant cost = 2922.5.

4.1. Scenario 1: the manufacturer dominates the retailer

4.1.1. The manufacturer's decision

As we have proposed above, when dominating the system, the manufacturer sets the replenishment cycle and the quantities delivered to the retailer. The cycle and the quantities are identified by equations (3.8)–(3.9). Under this condition, Table 4 lists the manufacturer's production plan. The manufacturer's total relevant cost is computed by equation (3.7).

4.1.2. The retailer's decision

Under the manufacturer's domination, the retailer's only two decision variables are the inventory on hand just before each replenishment arrives and the last marketing effort. To solve the retailer's issues, various meta heuristic algorithms can be applied, but this example adopts the genetic algorithm (GA). Each chromosome represents a feasible solution corresponding to a retailer's inventory policy (see Fig. 2). Figures 3 and 4 show the flowchart for this procedure and the retailer's total relevant cost corresponding to the generations in the 1st scenario by GA. Table 5 gives the retailer's inventory policies in this scenario. Appendix A provides the detailed descriptions of the algorithm.

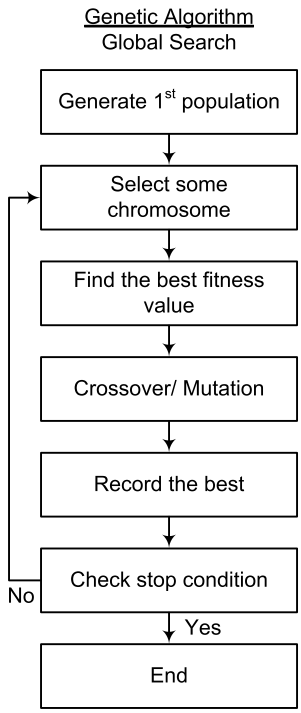

Figure 3. Flowchart of the genetic algorithm.

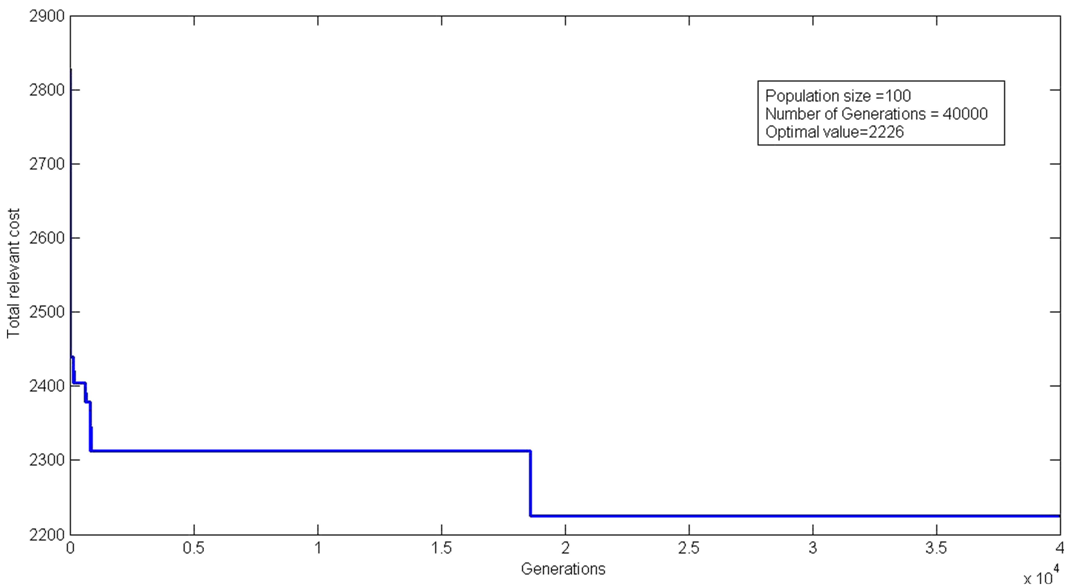

Figure 4. Retailer's total relevant cost corresponding to the generations in the 1st scenario.

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	x_1	x_2	x_3	x_4	x_5	x_6

Figure 5. Retailer's chromosome structure in the 2nd scenario.

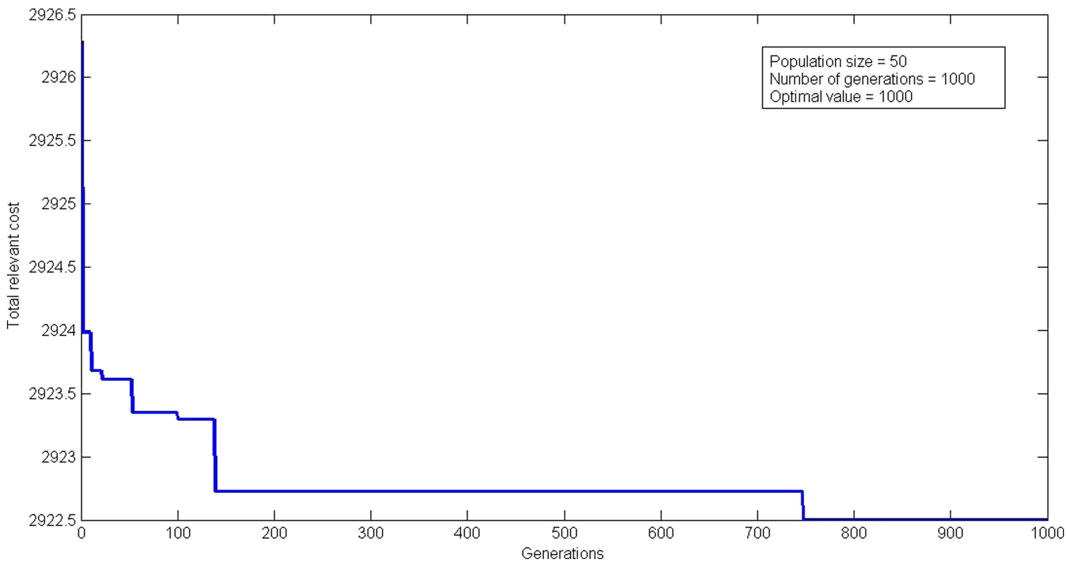

Figure 6. Retailer's total relevant cost corresponding to the generations in the 2nd scenario.

Table 6. Retailer's inventory policies in the 2nd scenario.

No.	1	2	3	4	5	6
Time	0	13.456	14.405	14.664	14.920	15.546
q_i	35	145	147	144	148	149
x_i	0.1463	3.9673	8.8464	7.9318	3.9888	8.3224

Total relevant cost = 2225.6.

4.2. Scenario 2: the retailer dominates the manufacturer

4.2.1. The retailer's decision

Being the dominating stage, the retailer's decision variables are the quantity of each replenishment and the marketing efforts between two consecutive replenishments. Due to the huge amount of decision variables and the complexity of the objective functions, we again adopt GA. In this example, the range of marketing effort is $[0; 10]$, the crossover rate is 0.8, and the mutation rate is updated adaptively. Figure 5 shows the chromosome of this issue. Table 6 gives the best results found by GA, while Figure 6 displays the best value that GA finds corresponding to the generation. Appendix A explains the reason why GA is adopted and represents descriptions of the algorithm.

4.2.2. The manufacturer's decision

Due to the transparent system, the retailer's demands are revealed to the manufacturer in order to help it to optimize its production plan. Based on equation (3.24), the boundary value for the manufacturer stage is $H_6(149) = 0$. $S_i(x, y)$ in equation (3.23) and can be interpreted as the total operation cost between the ith demand and the $(i+1)$th demand, where the inventory levels are x and y, respectively. Due to constraints and

TABLE 7. Manufacturer's production plans in the 2nd scenario.

No.	1	2	3	4	5	6
Time	0	13.456	14.405	14.664	14.920	15.546
I_i^A	35	589	510	381	254	149
p_i^A	43.773	69.535	69.486	66.317	68.719	0

Total relevant cost = 1889.9.

assumptions, the value of x in $H_5(x)$ must be in the range $[q_5, q_5 + q_6]$. The calculation of $H_5\,(254)$ is used as an illustrative example. In order to calculate $H_5\,(254) = S_5\,(254, 149) + H_6(149)$, $S_5\,(200, 149)$ is required. Using equation (3.23) with:

$$p_i^A = \frac{y + q_i - r}{t_{i+1} - t_i} = \frac{148 + 149 - 254}{15.55 - 14.92} = \frac{43}{0.63} = 68.7 < 70,$$

we have:

$$S_5\,(254, 149) = (254 - 148) \times 0.02 \times 22 \times 0.63 \ldots$$

$$+ \frac{22 \times 0.02 \times 0.63^2}{2} \times 68.7 + (68.7 - 60) \times 3 \times 0.63.$$

Thus, we arrive at:

$$S_5\,(254, 149) = 289.4 \Rightarrow H_6\,(400) = 289.4$$

Table 7 provides the final manufacturer's production plans.

5. Conclusion

From the results of this research, it is clear that when in the dominant position, either the retailer or the manufacturer spends less money on the total relevant cost, which is extremely important. The retailer reduces the cost by utilizing the batch size in the set-up cost and controlling the demand rate. The manufacturer decreases the cost by avoiding penalties from fluctuating the production rate and holding the inventory too long. In addition, the total relevant costs of the retailer and manufacturer in the two scenarios are unequal. If the objective function is the profit of the entire supply chain, then collaboration and power sharing are really required to ensure satisfaction for both the manufacturer and retailer.

This study proposes a model for considering different dominant relationships in the supply chain system under the Bass model's market effects and the advertisement effect. The proposed model is consistent with the diffusion process and intuitions. Using the model, each stage of the supply chain may respond to the impacts from the dominating stage in order for the firm to maximize its profit. Through the example and the model, we can analyze and verify which stage is better to dominate the supply chain.

This model still has a few limitations. First, when the size of the potential market increases, the computational time required for the dynamic programming technique will grow considerably. Thus, the solution method may not be suitable for solving large-scale problems. Second, the model assumes that the required information is available and accessible for all stages of the supply chain system. As a result of this assumption, each stage can easily identify the best policies, but this assumption may not apply to all types of supply chains. Subsequent studies may consider the degree of information transparency in the model. Third and lastly, this paper does not reflect on some important phenomena, such as delivery backlog or lost sales. Future studies may extend the proposed model to take these issues into consideration. This will certainly bring the current model closer to reality.

Appendix A

Genetic algorithm

Genetic algorithm (GA) is a meta heuristic algorithm inspired by the process of natural selection. Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by mimic natural selection activities, such as mutation, crossover, and selection. In GA, each chromosome is presented for a feasible solution. In the first and the second scenarios, the chromosomes are encoded as $[Q_1 - Q_2 \cdots - Q_m]$ and $[Q_1 - Q_2 \cdots - Q_m - x_1 - x_2 \cdots - x_m]$, respectively. After the fitness values are computed, each chromosome is picked in accordance with the probability distribution of the fitness value. To conduct a crossover, each pair of parents is selected based on the parents' position in the population. The next step randomly creates a crossover point. If there is no capacity violation, then two chromosomes are mated to produce the next generation.

In the traditional GA approach, values of parameters, *i.e.* population size, crossover rate, mutation rate, and number of crossover points, are fixed throughout the whole evolution process. However, several studies pointed out that the performance of GA is enhanced by adjusting values of the genetic parameters [6, 8, 14]. As a result of this, adaptive genetic algorithms, in which the genetic parameters vary throughout the evolution, have been developed [4, 14]. In this paper each chromosome represents a feasible solution corresponding to the replenishment policies. This study applies the adaptive mutation rate in order to increase the diversity of the chromosomes at initial generations and to strengthen the local optimal search at the last generations. In this study the mutation rate at the zth generation is given by:

$$\text{mutation rate}\,(z) = \frac{\rho}{\varepsilon^{\gamma}}, \tag{A.1}$$

where:

$$\gamma = \left\lceil \frac{\lambda z}{\text{Number of generations}} \right\rceil. \tag{A.2}$$

If the mutation occurs at the genes representing the cumulative quantities in the chromosome, then in order to ensure the feasibility of new chromosomes the new value of Q_i after mutating must be in the range of y_1 and y_2, where $y_1 = \max(Q_{i-1}, Q_{i+1} - c^B)$ and $y_2 = \min(Q_{i-1} + c^B, Q_{i+1})$. In both of the two scenarios, the values of ρ, ε, and λ are set at 0.4, 2, and 5, respectively.

Back probagation

In the 2nd scenario, the total relevant cost has a stage-wise structure. It seems appealing that the chromosomes should be encoded as $[Q_1 - Q_2 \cdots - Q_m]$, and back propagation is adopted to update x_i by using the gradient method. If this approach is applicable, then it will simplify the chromosomes and give a more accurate solution than coding chromosomes as $[Q_1 - Q_2 \cdots - Q_m - x_1 - x_2 \cdots - x_m]$. However, due to the function's sensitivity and the probability of trapping in the local optimal, this approach is not promising.

In order to solve the problem, we apply the gradient descent method combined with the back propagation approach. The gradient method has the advantage of a solid mathematical background and has been applied successfully in solving several optimization problems in engineering fields [9, 10, 18, 20]. The prominent feature of the gradient method is its reliation on the derivative, which is sometimes difficult to compute. However, when the problem has a special structure, like it can be presented with a recurrence relation form, then the back probagation method is highly appropriate to quickly compute the gradient.

Given the set of $[Q_1, Q_2, \ldots\ldots, Q_m]$, the total relevant cost is the function of $[x_1, x_2, \ldots\ldots, x_m]$. From the initial values of marketing efforts $\left[x_1^0, x_2^0, \ldots\ldots x_m^0\right]$, via the number of iterations, the appropriate values of $[x_1, x_2, \ldots\ldots, x_m]$ are determined. Let $Z_i\left(a_i, x_i^j\right)$ be the total relevant cost to go from the time the retailer receives the ith replenishment to the end, given that the advertisement rate at that time is a_i and the retailer uses marketing efforts x_i^j – the value of x_i at iteration j.

The recurrence relation between Z_i and Z_{i+1} is:

$$R_i\left(a_i, x_i^j\right) = P_i\left(a_i, x_i^j\right) + R_{i+1}\left(a_{i+1}, x_{i+1}^j\right), \tag{A.3}$$

where a_{i+1}, t_{i+1}, and $P_i\left(t_i, a_i, x_i^j\right)$ are given by equations (3.2), (3.19), and (A.4), respectively.

$$P_i\left(a_i, x_i^j\right) = \int_{t_i}^{t_{i+1}} a\left(t\right)\mathrm{d}t + A_i^B + h^B v^B \int_{t_i}^{t_{i+1}} \left[Q_i - D_i^B(t)\right]\mathrm{d}t. \tag{A.4}$$

If $x_i^j = 1$, then:

$$P_i\left(a_i, x_i^j\right) = A_i^B + \frac{a_i F_i}{x_i^j(p+q)} + \frac{h^B v^B Q_i F_i}{x_i^j(p+q)} + \frac{h^B v^B M p F_i}{q x_i^j(p+q)} + \frac{h^B v^B M}{x_i^j q} ln\left(\frac{M-Q_i}{M-Q_{i-1}}\right), \tag{A.5}$$

else:

$$P_i\left(a_i, x_i^j\right) = A_i + \frac{\beta_a a_i}{x_i^j - 1}\left[\exp\left(\frac{F_i(x_i^j-1)}{x_i^j\left(p+q\right)\beta_a}\right) - 1\right] + \frac{h^B v^B Q_i F_i}{x_i^j(p+q)} + \frac{h^B v^B M p F_i}{q x_i^j(p+q)} + \frac{h^B v^B M}{x_i^j q} ln\left(\frac{M-Q_i}{M-Q_{i-1}}\right), \tag{A.6}$$

where:

$$F_i = ln\left[\frac{\left(Mp + qQ_i\right)\left(M - Q_i - 1\right)}{\left(Mp + qQ_{i-1}\right)\left(M - Q_i\right)}\right]. \tag{A.7}$$

The boundary condition for equation (A.3) is:

$$Z_{m+1}\left(a_{m+1}, \_\right) = 0. \tag{A.8}$$

Partial differentiation of equation (A.3) with respect to x_i^j is given by:

$$\frac{\partial Z_i}{\partial x_i^j} = \frac{\partial P_i}{\partial x_i^j} + \frac{\partial Z_{i+1}}{\partial a_{i+1}} \times \frac{\partial a_{i+1}}{\partial x_i^j}. \tag{A.9}$$

In order to calculate $\partial Z_{i+1}/\partial a_{i+1}$ and $\partial Z_{i+1}/\partial t_{i+1}$, the partial derivatives of equation (A.3) with respect to a_i and t_i are taken as:

$$\frac{\partial Z_i}{\partial a_i} = \frac{\partial P_i}{\partial a_i} + \frac{\partial Z_{i+1}}{\partial a_{i+1}} \times \frac{\partial a_{i+1}}{\partial a_i}. \tag{A.10}$$

The boundary equation is:

$$\frac{\partial Z_{m+1}}{\partial a_{m+1}} = 0. \tag{A.11}$$

These recurrent relations determine the gradients of decision variables x_i. The value of x_i is updated by using the following equation:

$$x_i^{j+1} = x_i^j + r\frac{\partial Z_i}{\partial x_i^j}. \tag{A.12}$$

The process stops when the stopping condition is met.

This procedure has some limitations. The first is that the ratio between $\max_i \left[\frac{\partial Z_i}{\partial x_i^j}\right] / \min_i \left[\frac{\partial Z_i}{\partial x_i^j}\right]$ is large. This makes the function very sensitive when updating the value by the gradient method. In addition, due to the existence of many local optimal points, the procedure described above may be trapped in local optimums.

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