INFLUENCE OF SECOND ODER ABERRATIONS ON MASS-SEPARATION WITH 270° ANALYZING MAGNET

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In this paper we study nonlinear characteristics of the wide aperture dipole magnet CP-17, which is utilized as an analyzing magnet in the stable isotope separator of the INR (Institute for Nuclear Research) in Kiev. A brief theory of the second order aberration of the magnetic field is outlined. Based on this theory numerical calculations have been performed to study the achievable power resolution depending on nonlinear magnetic field and on beam parameters.

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1. INTRODUCTION

For each isotope separator the goal is to achieve high power resolution and at the same time large transmission as well as large momentum acceptances. However, the achievable power resolution of the isotope separator is limited by the many reasons, including second order aberrations.

Usually the magnetic field of any magnetic-optical devices is not ideal because of some constructive inaccuracies in configuration of poles, some errors in coils installation and etc. Therefore each magnetic optical element produces aberrations – image distortion, which has the effect of increasing image sizes on a focal plane that limits the achievable resolution. This effect is called "emittance blow-up". Uncorrected aberrations may increase the image size by a factor of five or more with a proportional reduction in mass resolution. It is known that in a linear approximation a maximum attainable resolution is determined by the emittance of an ion beam coming out an ion source and dispersion of the isotope separator on the focal plane.

The mass resolution P is defined by formula

$$P = \frac{m}{\Delta m}\frac{\Delta x}{\delta x} = \frac{D}{\delta x}, \qquad (1)$$

where Δx is the separation between two beams of mass m and $m + \Delta m$, δx is the width of full image at the final focus, D is the dispersion.

In this paper we study the influence of aberrations on the resolution of a mass separator, which contains only one large aperture dipole magnet CP-17 [1, 2]. This magnet has an inhomogeneous magnet field with straight pole boundaries. The dispersion of the separator with such magnet varies depending on the optical setup of the system. There is condition, when the maximum value of D can be reached 11 m. For example, if image's width is 1 mm the maximum possible resolution of the separator is 11000. With such power resolution any isotopes can be separated with low contaminations of undesired ions e. g. with high enrichments. But different types of nonlinear field errors reduce resolving power of the separator. To find out the nonlinear features of the large aperture magnet we consider only second order aberrations, which are stronger in comparison with the higher order field imperfections. The numerical algorithm based on the theory [3] and applied in our code is briefly described.

In general, it is not hard to obtain the solution of second-order equation by hand. But this solution is rather complicated to study the particle beam behaviour in optical system, which consists of several magnets. The Monte-Carlo method is one of the tools that gives a simple way to investigate the ion beam characteristics in complicated ion-optical systems.

2. SOURCES OF MAGNETIC FIELD NONLINEARITIES

Particle motion in the magnetic field can be described by equations including second order terms [3]:

$$\begin{aligned} &x'' + (1-n)h^2 x = h\delta + (2n-1-\beta)h^3 x^2 + \\ &h'xx' + 0.5x'^2 + (2-n)h^2 x\delta + \\ &0.5(h'' - nh^3 + 2h^3\beta)y^2 + h'yy' - 0.5hy'^2 - h\delta^2; \end{aligned} \qquad (2)$$

$$\begin{aligned} &y'' + nh^2 y = 2(\beta - n)h^3 xy + h'xy' - \\ &hx'y' + nh^2 y\delta, \end{aligned} \qquad (3)$$

where $\beta = \left[\frac{\rho^2}{2B_0}\frac{\partial^2 B_y}{\partial x^2}\right]_{\substack{x=0\\e=0}}; n = -\rho\left(\frac{1}{B_y}\frac{\partial B_y}{\partial x}\right)_{\substack{x=0\\y=0}}$,

n is the magnetic field index, $\delta = \Delta\rho/\rho$, ρ is the bending radius and $h = 1/\rho$.

The general analytical solution of equations (2) and (3) is presented as combination of the linear and nonlinear second order terms of the Taylor's series:

$$x = \langle x \mid x_0\rangle x_0 + \langle x \mid x'_0\rangle x'_0 + \left(x \mid \delta\right)\delta + \Delta x_{ab}, \qquad (4)$$

$$y = \langle y \mid y_0\rangle y_0 + \langle y \mid y'_0\rangle y'_0 + \Delta y_{ab}, \qquad (5)$$

Δx_{ab} and Δy_{ab} free the non linear terms expressed by

$$\begin{aligned} &\Delta x_{ab} = \langle x \mid x_0^2\rangle x_0^2 + \langle x \mid x_0 x'_0\rangle x_0 x'_0 + \langle x \mid x_0\delta\rangle x\delta \\ &+\langle x \mid x'^2_0\rangle + \langle x \mid x'_0\delta\rangle x'_0\delta + \langle x \mid \delta^2\rangle\delta^2 + \\ &\langle x \mid y_0^2\rangle y_0^2 + \langle x \mid y_0 y'_0\rangle y_0 y'_0 + \langle x \mid y'^2_0\rangle y'^2_0; \end{aligned} \qquad (6)$$

$$\begin{aligned} &\Delta y_{ab} = \langle y \mid y_0 x_0\rangle y_0 x_0 + \langle y \mid x_0 y'_0\rangle y'_0 + \\ &\langle y \mid x'_0 y_0\rangle + \langle y \mid x'_0 y'_0\rangle x'_0 y'_0 + \\ &\langle y \mid y_0\delta\rangle y_0\delta + \langle y \mid y'_0\delta\rangle y'_0\delta. \end{aligned} \qquad (7)$$

The terms noted by bracket <> are unknown coefficients, which have to be defined. For convenience

All of these terms will be marked as q_{ij}. Substitution equations. (4) and (5) in equations. (2) and (3) we can get the differential equations for coefficients q_{ij} :

$$q''_{ij} + \omega^2_{x,y} q_{ij} = f_{ij}(s), \quad \omega^2_x = (1-n)h^2, \omega^2_y = nh^2, \tag{8}$$

$f_{ij}(s)$ are the excitation functions, which are given in Appendix for corresponding coefficients q_{ij}. The solution of equations (8) can be expressed as

$$q(s_k) = S(s_k)\int_0^{s_k} \frac{f(\tau)C(\tau)\partial\tau}{W(\tau)} - C(s_k)\int_0^{s_k} \frac{f(\tau)S(\tau)\partial\tau}{W(\tau)} + C(s_k)q_0 + S(s_k)q'_0, \tag{9}$$

where $W(\tau) = C(\tau)S'(\tau) - C'(\tau)S(\tau) = \det T = 1, T$ is the transfer matrix of an ion-optical system, $C = \cos(k_x \cdot s)$, $S = \sin(k_x \cdot s)$ are the principle solutions, $k_x^2 = (1-n)h.$ Using equation (9) q' and q'' are defined by

$$q' = S'\int_0^{S_k} f(\tau)C(\tau)d\tau - C'\int_0^{S_x} f(\tau)S(\tau)d\tau + C'q_0 + S'q'_0; \tag{10}$$

$$q'' = S''\int_0^{S_k} f(\tau)C(\tau)d\tau - C''\int_0^{S_k} f(\tau)S(\tau)d\tau + (S'C - C'S)f(S_k) + C''q_0 + S''q'_0 = \omega^2 + f. \tag{11}$$

Thus the quadratic terms are expressed through the linear solutions C, S (characteristic functions of the main trajectories) and the corresponding excitation functions $f_{ig}(S)$. Using standard numerical method for integrating of equations (9, 10) one can calculate the particle trajectories in a first and second order field imperfections in any ion-optical system.

3. MASS-SEPARATOR IN FIRST ORDER APPROXIMATION

To study influence of aberrations on the mass separation we consider a separator, where the main component is only one large aperture dipole magnet CP-17 with an inhomogeneous field [1]. Because there are no any additional ion-optical elements, this magnet determines most of the beam parameters in the separator. Such scheme gives us possibility to find out the contribution of nonlinear filed in effect of the power resolution restriction. The CP-17 magnet has a 270^0 bending angle and axial radius ρ of 2 m. This magnet has possibility to produce a double focusing in both horizontal and vertical directions by appropriate adjustment of the quadrupole component of the magnetic field. It provides an adjustable field index n in range from zero to one. This adjustment can be easily obtained by using the special magnet coil. If n is closed to 0.831 the mass dispersion D_m is equal to 11 m in the focal plane, which is placed from the magnet exit by distance 3.36 m. For such value of the dispersion D_m two beams with mass numbers $A = 50$ and $A = 51$ are separated by 23 cm in the focal plane. To have simultaneous focusing in both horizontal and vertical directions the distance between the exit slit of the ion source and magnet entrance must be 3.36 m and in this case the focal plane would be placed from magnet exit by 3.36 m. Fig.1 shows the particles trajectories calculated in a linear approximation throughout the whole system in the horizontal and vertical planes.

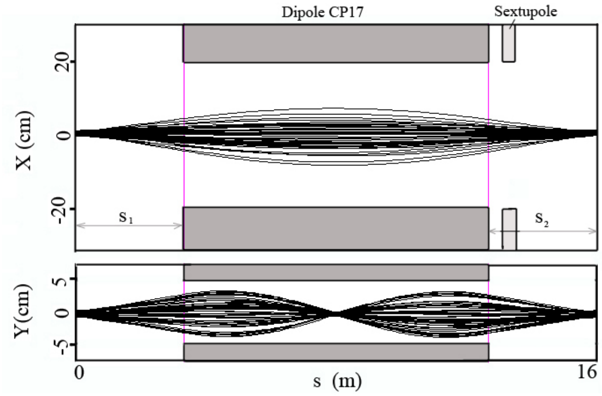

Fig.1. Particle trajectories throughout the separator (a first order approximation, s_1=s_2=3.36 m)

In this case the resolution power is defined by the ion beam emittance coming out from the ion source. Fig.5 shows the curves that correspond to numerically calculated dependence the resolution power P on the beam emittance in a first order approximation. One can note that P can be larger than 1000 for emittance up to $10\, mm \cdot mrad$ and the momentum spread less than 10^4. It should be noted that here and in all following simulations the values of the emittance are statistical rms values with standard deviation 4. It is necessary to ensure situation when the overlap between different isotopes would be less than 0.01%, since the beam contains 99.99% of the particles at $4\delta rms$ (usually emittance calculations use $2\delta rms$, 95% of the beam).

The influence of aberrations brings an additional upper limit on obtaining resolution power. Using theory given in Section 2 series of calculations has been performed in a second order approximation.

4. SECOND ORDER PARTICLE TRACKING

The aberrations of the magnetic field reduce the mass resolution of separator due to increasing of beam spots in the focal plane. Mainly beam distortion is caused by the sextupole components $\partial^2 H / \partial x^2$ of the magnetic field, which create the excitation functions f_{ij}. However, the experimentally measured magnetic field map is not always known. In this case one needs to make some assumptions based on a theoretical calculation. For rough estimation of sextupole gradient $d^2 H / dx^2$ we can use the formula

$$\delta B_S = \frac{1}{2}\frac{\partial^2 H_y}{\partial x^2} a^2, \tag{12}$$

where a is a half-aperture of the magnet, δB_y is the field error, which can be expressed through the parameter of Δ (magnet field accuracy)

$$\delta B_s = H_0 \cdot \Delta, \tag{13}$$

H_0 is the main magnet field in the median plane. Using equations (12) and (13) one can derive the expression for the sextupole gradient:

$$\frac{\partial^2 H_y}{\partial x^2} = \frac{2H_0\Delta}{a^2}. \quad (14)$$

As shown in [1] the theoretically calculated value Δ is about 0.75%. But in practice this value can be larger by factor 2 and more especially for low field, at which magnet should operate. For insurance in our calculations we assume that Δ is 2%.

Under such value Δ aberrations give significant contribution to growth of the beam spot on the focal plane compare to the first order approximation. Fig.2,a shows how the beam spot on the focal is disturbed in case of the second order field errors. The tail of beam intensity distribution (Fig.3) appears due to particles traveling through the magnet with high amplitude, where the field nonlinearity is stronger. One should also note that the influence of the nonlinear field effect is amplified if the particles have relatively large momentum deviations. Therefore the particle amplitude trajectories are increased and the beam spot on focal plane is blow up proportionally to the momentum spread of the beam as shown in Fig.2,b. In this case the reduction of the mass resolution has strong dependence on the momentum spread of the isotope beam.

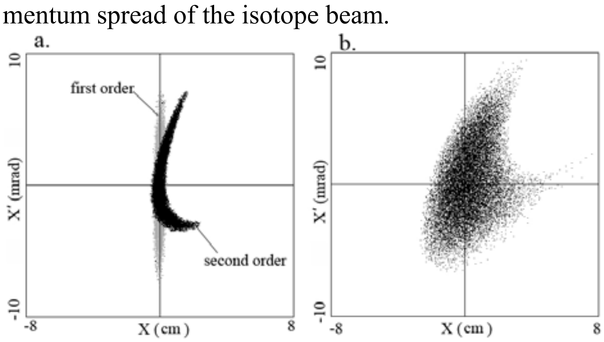

Fig.2. Beam spots at the focal plane a. $d\rho=0$; b. $\partial p/p=0{,}2\%$ $\varepsilon_x=10$ mm·mrad

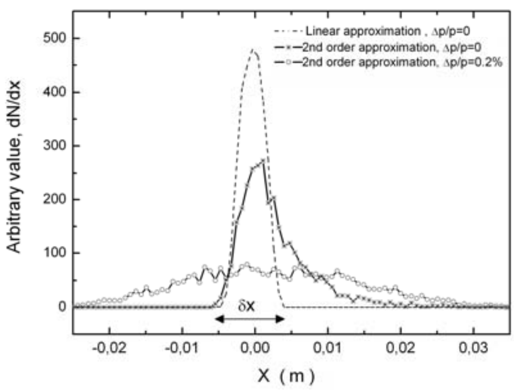

Fig.3. Horizontal beam profiles on the focal plane. These distributions correspond to the particles beam spots shown in Fig.2

In Fig.4 we present the numerically simulated dependence of the mass resolution power on the momentum spread in cases of both the first and second order approximation. We see that the momentum spread gives main contribution to the reduction of the mass resolution of the separator. For momentum spread less than 0.1% the second order aberrations is significant. The results outcomes in Fig.4 eligible only in case the emittance value of 10 mm·mrad.

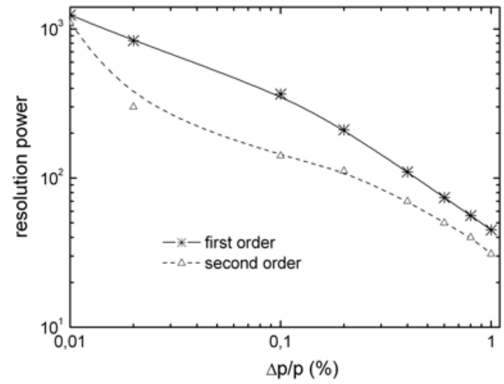

Fig.4. Dependence of the resolution power on the momentum spread $\varepsilon_x = 10$ mm·mrad

To see how the resolution power depends only on emittance a series simulations have been done, where momentum spread of beam is fixed at value of 0.1%. The results of simulations are shown in Fig.5. Here we see that second order aberrations have very strong effect for emittances larger 10 mm·mrad. The power resolution catastrophically is dropped up to several units for relatively large emittances 20…100 mm·mrad. However situation can essentially be improved by means of a sextupolar correction. Using a sextupole magnet position of which is shown in Fig.1 one can significantly compensate the influence of second order field imperfection but not completely (Fig.5).

Finally in Fig.6 we present achievable parameters of ion beam, which should be supplied at obtaining designed power resolutions. The results shown in Fig.6 were obtained after the particle tracking throughout the second order magnet system, where the parameter Δ has moderate lower value 0.5%.

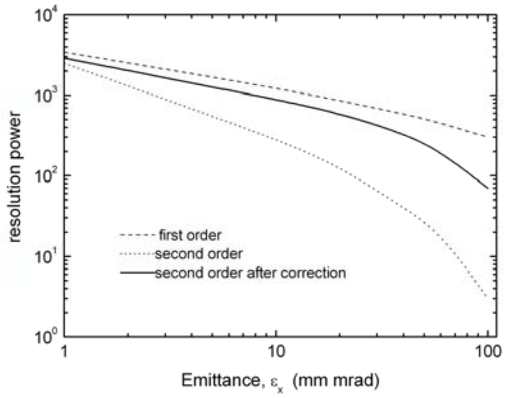

Fig.5. Resolution power versus horizontal emittance obtained with first and second order calculations $\Delta\rho/\rho = 0.1\%$

CONCLUSIONS

The ion optic of a mass separator with single dipole magnet CP-017 was studied by taking into account second order aberrations. It was shown that the image aberration due to the dipole magnet limits the power resolution of the separator if the relative magnet error field is larger than 0.5%. The energy spread and the ion beam emittance coming out of the ion source are another limitations.

The results of the second order simulations are that the resolving power is reduced by factor larger than 5 for emittance larger 20 mm·mrad if momentum spread of beam does not exceed 0.1%. The main effect, which appears due to second order aberrations, is long tail formed in the particle distribution on the focal plane. The way to prove the resolving power close to designed values is to correct the particle trajectories coming out the exit of the magnet by sextupole magnet. Then the resolving power of 150 could be ensured for emittances up to 70 mm·mrad at the momentum spread less 0.1%.

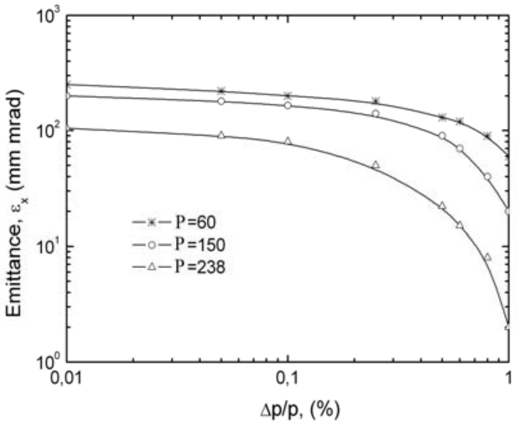

Fig.6. Emittance versus momentum spread for different resolution power calculated in second order approximation, where Δ = 0.5%

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APPENDIX

The excitation functions $f_{ij}(S)$ corresponding to the second order coefficients q_{ij}

q_{ij}	$f_{ij}(S)$
$\psi = \langle x \mid \delta \rangle$	H_0
$\langle x \mid x_0^2 \rangle$	$bh^3 C_x^2(s) + 0.5hC'^2_x(s)$
$\langle x \mid x_0 x'_0 \rangle$	$2bh^3 C_x(s) S_x(s) + hC'^2_x(s) S'_x(s)$
$\langle x \mid x_0 \delta \rangle$	$-(n-2)h^2 C_x(s) + 2bh^3 C_x(s)\psi_x(\mathrm{s}) + hC'_x(s)\psi'_x(s)$
$\langle x \mid x'^2_0 \rangle$	$bh^3 S_x^2(s) + 0.5hS'^2_x(s)$
$\langle x \mid x'_0 \delta \rangle$	$-(n-2)h^2 S_x(s) + 2bh^3 S_x(s)\psi_x(s) + hS'_x(s)\psi'_x(s)$
$\langle x \mid \delta^2 \rangle$	$-(n-2)h^2 \psi_x(s) + bh^3 \psi_x^2(s) + 0.5h\psi'^2_x(s)$
$\langle x \mid y_0^2 \rangle$	$0.5(2\eta - n)h^3 C_y^2(s) S_y(s) - 0.5hC'^2_x(s)$
$\langle x \mid y_0 y'_0 \rangle$	$(2\eta - n)h^3 C_y(s) S_y(s) - 0.5hS'^2_x(s)$
$\langle x \mid y'^2_0 \rangle$	$0.5(2\eta - n)h^3 S_y^2(s) - 0.5hS'^2_x(s)$
$\langle y \mid x_0 y_0 \rangle$	$tC_x(s) C_y(s) + hC'_x(s) C'_y(s)$
$\langle y \mid y_0 x'_0 \rangle$	$tC_x(s) S_y(s) + hC'_x(s) S'_y(s)$
$\langle y \mid x'_0 y'_0 \rangle$	$tS_x(s) C_y(s) + hS'_x(s) C'_y(s)$
$\langle y \mid y_0 \delta \rangle$	$tS_x(s) S_y(s) + hS'_x(s) S'_y(s)$
$\langle y \mid y'_0 \delta \rangle$	$nh^2 C_y(s) + tC_y(s)\psi_x(s) + hC'_y(s)\psi'_x(s)$
$\langle y \mid y'_0 \delta \rangle$	$nh^2 S_y(s) + tS_y(s)\psi_x(s) + hS'_y(s)\psi'_x(s)$

$$b = 2n - 1 - \eta; t = -2(n-\eta)h^3; \eta = (R^2_{magn} / H_0)\cdot(\partial^2 H_y / \partial x^2)$$

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ВЛИЯНИЕ АБЕРРАЦИЙ ВТОРОГО ПОРЯДКА НА МАСС-СЕПАРАЦИЮ С ПОМОЩЬЮ 270°АНАЛИЗИРУЮЩЕГО МАГНИТА

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Исследуются характеристики широко-апертурного дипольного магнита СП-17, который планируется использовать в качестве анализирующего магнита масс-сепаратора ИЯИ (Институт ядерных исследований) в Киеве. Приведена краткая теория аберраций второго порядка. На основе описанной теории проведена серия численных расчетов для исследования разрешающей способности сепаратора в зависимости от нелинейности магнитного поля, а также от параметров пучка.

ВПЛИВ АБЕРАЦІЙ ДРУГОГО ПОРЯДКУ НА МАС-СЕПАРАЦІЮ ЗА ДОПОМОГОЮ 270°АНАЛІЗУЮЧОГО МАГНІТУ

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Досліджуються характеристики широко-апертурного магніту СП-17, який планується використовувати як аналізуючий магніт мас-сепаратора ІЯД (Інститут ядерних досліджень) у Києві. Наведена коротка теорія аберацій другого порядку. На основі описаної теорії проведено цілий ряд чисельних розрахунків для дослідження розподільної здатності сепаратора в залежності від нелінійності магнітного поля, а також від параметрів пучка.