НЕРЕЛЯТИВИСТСКАЯ ЭЛЕКТРОНИКА

ELECTRON BEAM PLASMA DIODE WITH CHARGED PARTICLE BACKGROUND

A.Ya. Ender[1], V.I. Kuznetsov[1], and H. Schamel[2]
[1]Ioffe Physical-Technical Institute, St. Petersburg, Russia;
[2]Physikalisches Institut, Universitat Bayreuth, Bayreuth, Germany
E-mail: Victor.Kuznetsov@mail.ioffe.ru

The effect of an immovable charged particle background on the stationary states of a diode with the electron beam is studied. An addition of uniform positive charged particles results in an increase in the space-charge limit of the electron current as well as in the new branches of equilibria arising. On the other hand, extra negative charged particles result in a strong decrease in current cut-off limit, the cut-off being the total one. This effect can be used to perform the fast switches.

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1. INTRODUCTION

90 years ago it was revealed an effect of a sharp current cut-off in the diode. Bursian explained this phenomenon assuming that, in such a diode, two stationary states with the clearly different current levels can exist simultaneously in the given range of the external parameters and, with an excess of the electron beam over certain threshold value of j_{SCL}, named the space-charge limit (SCL) current, retained are only the solution with current's restriction [1].

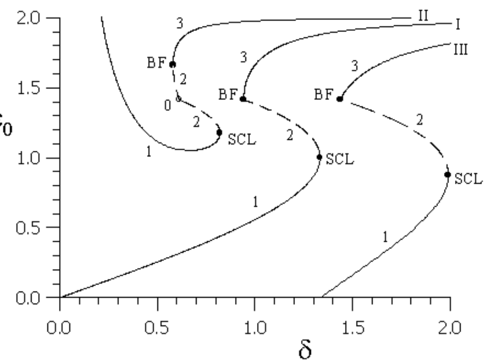

Fig.1. Emitter electric field ε_0 as a function of the diode length δ for the Bursian diode drawn for three values of the voltages V: curve I corresponds to $V = 0$, II to 0.2, III to -0.4. Normal C branch is marked by number 1, overlap C branch – 2, B branch – 3

In Fig.1, an example of $\varepsilon_0(\delta)$ for three values of an external voltage V is given (here $\varepsilon_0 = eE\lambda_D / 2W$, $\delta = d/\lambda_D$ and $V = e\Phi_C / 2W$ are the dimensionless emitter electric field, diode length and external voltage, respectively; the beam energy and Debye length

$$W = m(v_e^0)^2/2, \quad \lambda_D = [W/(2\pi e^2 n_e^0)]^{1/2}, \qquad (1)$$

are chosen as units of energy and length with n_e^0, v_e^0, being the beam density, velocity). The normal *C* branch corresponds to a regime with a total current, and the *B* branch corresponds to a regime with current's restriction, when, in the diode gap, a potential barrier arises (named a virtual cathode), which reflects a part of the electron flow on the emitter backwards. Each curve $\varepsilon_0(\delta)$ has two bifurcation points: SCL point, which determinates a limit current of the Bursian diode, and BF point, which corresponds to Child-Langmuir current. The properties of the Bursian diode are investigated in details in a number of works, see Refs. [2]-[5], and references cited therein. For coordinates of the SCL point, the explicit form is obtained [3]:

$$\delta_{SCL}(V) = (\sqrt{2}/3)(1+\sqrt{1+2V})^{3/2},$$
$$\varepsilon_{0,SCL}(V) = \sqrt{2}(1+\sqrt{1+2V})^{-1/2}. \qquad (2)$$

In this paper, an effect of the background of charged particles on characteristics of the electron beam diode is studied. It may be, e.g., a flow of extra particles injected in parallel along the electrodes or a dust plasma.

2. UNIFORM POSITIVE ION BACKGROUND

In Ref. [6], an effect of uniform background of positive ions on characteristics of the electron beam diode was studied. As a quantitative parameters of an ion background, it is used a nonneutrality parameter γ defined by

$$\gamma = n_i^{bg} / n_e^0, \qquad (3)$$

a value of γ being the same as the parameters of the electron beam are varied. Fig.2 shows in what manner the Bursian branch (*C* branch) develops with an increase in γ. One can see that the presence of the ions permits to increase substantially δ_{SCL}. As $j(\delta) \sim \delta^2$, it makes feasible to say about sensitivity of the true SCL current to an increase in γ. On the other hand, a BF point location is weakly dependent on γ value, and the Child-Langmuir current, defined by the BF point, remains almost the same.

Follow the change in *C* branch when increasing a nonneutrality parameter from 0 to 1, one can see that the Bursian branch ($\gamma = 0$) transforms continuously into a similar branch of the Pierce diode ($\gamma = 1$), a bifurcation point $\delta_{SCL}(0)$ being transforming into a bifurcation point $\delta_{SCL}(1)$. With an increase in current (an increase in a parameter δ), when reaching SCL point, there is an aperiodical instability in the diode and, as a result, a plasma is transforming into a state of electron reflection, strongly different from the initial one. Such scenario does not depend on γ. Thus, one can deduce that the Bursian and Pierce instabilities are the special cases the same aperiodical instability being inherent for the electron beam plasma diode.

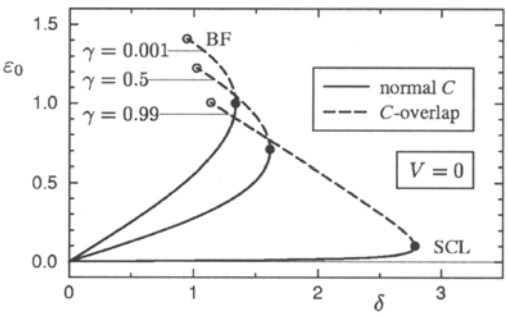

Fig.2. Emitter electric field ε_0 as a function of the diode length δ for three values of $\gamma = 0.001$, 0.5, and 0.99. The diode voltage is $V = 0$. For each curve, closed circles indicate the SCL critical state, whereas open circles indicate the BF state

There are no stationary solutions with $\varepsilon_0 < 0$ at $V \le 0$ in the Bursian diode ($\gamma = 0$). However, when $\gamma > 0$, such solutions exist. It results in arising the new branches which are shown in Fig.3 for a series of V, where a region on a plane $\{\varepsilon_0, \delta\}$ corresponding to $\delta \le 2\pi\gamma^{-3/2}$ is presented.

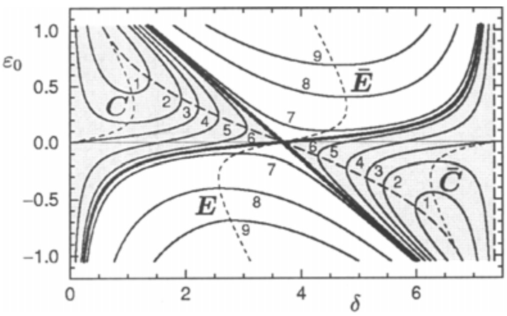

Fig.3. Manifold of equilibrium curves $\varepsilon_0(\delta)$ for $\gamma = 0.9$ drawn for different values of the collector potential V. Curve 1 corresponds to V =-0.3, 2 to –0.1, 3 to 0, 4 to 0.1, 5 to 0.2, 6 to $V = V_{cr} \equiv 0.2469$, 7 to 0.3, 8 to 0.7, and 9 to 1.2

One can see that it is separated in four fields by the bold solid lines which correspond to a voltage being equal to a critical value of $V_{cr}(\gamma)$. At $V < V_{cr}$, there are only C (Bursian) and $\bar{C}$ branches (the upper-left and lower-right corners, respectively). For each voltage, the C branch transforms into a relevant $\bar{C}$ branch if the first is rotated by 180º relative to a point ($0, \pi\gamma^{-3/2}$). In lower-left and upper-right corners, there are two new branches – E and $\bar{E}$ corresponding to $V > V_{cr}$, the solutions with a single maximum on the potential distribution corresponding to the E branch, and all these solutions are stable. A critical value of V determining the boundaries of the sectors (two bold curves) is determined via a formula

$$V_{cr} = 2(1-\gamma)\gamma^{-2}. \quad (4)$$

At $\gamma = 1$ $V_{cr} = 0$, and at $\gamma \to 0$ (a transform to the Bursian diode) $V_{cr} \to \infty$. The bifurcation points (BF and SCL) exist only at $V < V_{cr}$, i.e., on C and $\bar{C}$ branches.

The coordinates of SCL point are determined via the formulas

$$\varepsilon_{0,SCL} = (\sqrt{1+2V} - \gamma V - 1)^{1/2} V^{-1/2}, \quad (5)$$

$$\gamma^{3/2}\delta_{SCL} = \begin{cases} \pi - \arcsin y - y, & \varepsilon_{0,SCL} \le \sqrt{\gamma}, \\ \arcsin y - y, & \sqrt{y} < \varepsilon_{0,SCL} \le \sqrt{2-\gamma}, \end{cases} \quad (6)$$

$$y \equiv 2\sqrt{y}\varepsilon_{0,SCL} / (\gamma + \varepsilon_{0,SCL}^2).$$

At $\gamma \to 0$, Eqs (5) and (6) transform into Eqs (2).

At value of V fixed, a value of $\delta_{SCL}(\gamma)$ increases with increase in γ (see, Fig.2). Hence, an addition of the ion background results in an increase in the limit Pierce current density. Each value of V a critical value of γ is corresponded, also, being consequence of V_{cr} existence for every γ. Upper limit in j_{SCL} and δ_{SCL} can be found from Eqs (5) and (6), with substituting the critical value of γ determined from

$$\gamma_{cr} = (\sqrt{1+2V} - 1)V^{-1}. \quad (7)$$

From the very fact that there are new stable E branches at $\gamma > 0$, it follows the very important conclusion: in the stationary regime, the currents above the limit Pierce current corresponding to the well-known Pierce threshold $\delta_{SCL}(\gamma)$, can pass. At any $\gamma \ne 0$, one can continuously increase δ (hence, an electron beam current density), practically, with no restrictions when taking $V > V_{cr}$.

Above, it is supposed that a parameter γ (see, Eq.(3)) is a constant. Of interest is to consider a diode when the density of the background charge is fixed and the electron beam density is varied. The solutions for a problem with the constant ion background can be easily found, having a lot of calculations at different fixed γ values.

Nevertheless, take the equations describing the steady states of a diode with constant background charge. Those are the equations of continuity, of electron motion, and the Poisson's:

$$n(z)v(z) = (1+r)n_0 v_0,$$
$$\frac{d}{dt}v(z) = v(z)\frac{d}{dz}v(z) = -\frac{e}{m}\frac{d}{dz}\Phi(z), \quad (8)$$
$$\frac{d^2}{dz^2}\Phi(z) = 4\pi e[n(z) - n_i^{bg}]$$

and boundary conditions are

$$v(0) = v_0, \quad n(0) = n_0,$$
$$\Phi(0) = 0, \quad \Phi(d) = \Phi_C. \quad (9)$$

In Eqs. (8) and (9) parameter r is a coefficient of electron reflection from a virtual cathode. When considering the steady state in a regime with electron reflection, the problem is as follows: both electron flows – direct and backward – are supplied from the emitter with the weights *1* and *r*, i.e., the electron flow with a weight *1+r* "leaves" the emitter and, to the right of the virtual cathode, an electron flow of a weight *1-r* arises.

Rewrite a parameter n_i^{bg} as

$$n_i^{bg} \equiv \frac{n_i^{bg}}{n_0^*}\frac{n_0^*}{n_0}n_0 = \alpha n_0, \quad \alpha = \frac{\kappa_i}{\delta^2}, \quad \kappa_i = \frac{n_i^{bg}}{n_0^*}. \qquad (10)$$

Here n_0^* is an emitter electron beam density for a diode with no background particles at the SCL point. Now, in the problem under consideration in the dimensionless form, a problem (8) and (9) is of the same form as the previous one with γ fixed, but, everywhere, γ is substituted by α. Contrary to γ, a parameter α depends on a dimensionless diode length δ, so, the main parameters of the problem (ε_0, minimum potential η_m and its position ζ_m, and so on) can be found only with calculating the potential distribution over the entire diode.

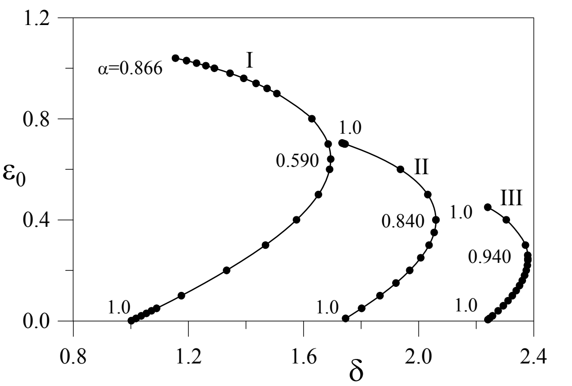

Fig.4. Emitter electric field ε_0 as a function of the diode length δ drawn for three values of κ_i =1.1 (curve I), 3.3 (II), and 5.5 (III). The diode voltage is $V = 0$. For each curve, α values are shown for three points

Fig.4 represents dependency ε_0 on δ for a diode with $V = 0$ for a regime with no electron reflection for three κ_i values. The branches obtained correspond to *C* branch for $\gamma \le 1$ (see, e.g., curve 3 in Fig.3). Consider, for clarity, curve *I*. At SCL point, $\alpha = 0.590$, and, with δ decreasing, this parameter increases: on the upper branch, up to 0.866 at BF point, on the lower branch – up to 1.0 corresponding to $\varepsilon_0 = 0$.

3. UNIFORM ELECTRON BACKGROUND

When studying an effect of the electron background on the characteristics of an electron beam diode, first, evaluate the values of a series of the parameters at the SCL point for short-circuit diode by a concrete example: a voltage forming the beam, V^*=1V, and a diode length d^*=1mm. Then for minimum potential Φ_m^*, beam velocity v_0^*, density n_0^*, and current density $j_0^* = en_0^* v_0^*$ we obtain

$$\Phi_m^* = -0.75V, \quad v_0^* = 5.9289\cdot 10^7 \frac{cm}{s},$$
$$en_0^* = 3.1438\cdot 10^{-11}\frac{K}{cm^3}, \quad j_0^* = 1.8639\cdot 10^{-3}\frac{A}{cm^2}. \qquad (11)$$

A total charge in a gap relative to the units of a length and square of the electrode for the SCL point equals

$$\frac{Q}{d^*} = \frac{e}{d^*}\int_0^{d^*} dz n(z) = \frac{1}{d^*} en_0^* v_0^* t_d^* = \frac{3}{2} en_0^* = = 4.7157\cdot 10^{-11} C/.cm^3. \qquad (12)$$

For the background electrons would affect the diode states, their total charge should be of the order of Q/d^* Eq. (12).

When studying the steady state, as in the case of the ion background, Eq. (10), represent the electron background density as a form

$$n_e^{bg} = \frac{\kappa_e}{\delta^2} n_0 = \beta n_0. \qquad (13)$$

A coefficient κ_e, determining the background electron density, is related with n_e^{bg} via $n_e^{bg} = (9/16)\kappa_e n_0^*$.

If the beam electrons are absent, it is maintained the potential distribution $\Phi(z) = -\pi e(9/8)\kappa_e n_0^*(d-z)z$, which minimum potential $\Phi_m = -(9\pi/32)e\kappa_e n_0^* d^2$ locates at the center of a diode space and its dimensionless value $\eta_m = -(1/8)\kappa_e$, in the diode. For the Bursian diode with no background $\eta_m^* = -3/8$ at the SCL point. $\kappa_e = 3$ corresponds to such value of η_m. When $\eta_m = -1/2$, an electron beam, left the emitter with the energy $m(v_0^*)^2/2$, stops at the point z_m, and $\kappa_e = 4$ corresponds to this case. Thus, an essential effect of the background is expected at a parameter κ_e within 3 and 4.

Now, involve an electron beam of fixed energy $W_0^* = eV^*$ from the emitter in the vacuum diode with background electrons. Depending on a relation between the beam and background density, there are different regimes within the gap: an entire beam reaches the collector or a portion of electrons (as well as total ones) is reflected by potential barrier and backwards the emitter, resulting in a current cut-off.

A system of equations in the case under consideration coincides with Eq. (8) when substituting α by $-\beta$. Performing similarly to that as in [2] and [7] this system can be transposed to a single ordinary differential equation of the 3rd order:

$$\frac{d}{d\tau}\left(\frac{d^2}{d\tau^2}\zeta - \beta\zeta\right) = 1 + r. \qquad (14)$$

Here τ is a time-of-flight of an electron from the emitter to a point ζ. Integrating Eq. (14) with relevant boundary conditions, one can calculate potential distributions and build up dependencies the emitter electric field ε_0 and a convective current at the collector *j* on a diode length δ.

In the regime with no electron reflection we obtain an equation for a relation ε_0 with δ

$$\beta^{3/2}\delta = \sqrt{\beta}(\varepsilon_0 + |\varepsilon_\delta|) - \ln\frac{1+\sqrt{\beta}\,|\varepsilon_\delta| + w\beta}{1-\sqrt{\beta}\varepsilon_0 + \beta}, \quad (15)$$

$$\varepsilon_\delta = -\sqrt{\varepsilon_0^2 + 2(w-1) + (w^2-1)\beta}.$$

Here $w = (1+2V)^{1/2}$. In the regime with partial electron reflection (*0<r<1*) an equation relating ε_0 with δ has a form

$$\beta^{3/2}(\delta - \zeta_r) = \sqrt{\beta}\,|\varepsilon_\delta| - (1-r)\ln\frac{1-r+\sqrt{\beta}\,|\varepsilon_\delta| + w\beta}{1-r},$$

$$\beta^{3/2}\zeta_r = \sqrt{\beta}\varepsilon_0 - (1+r)\ln\frac{1+r+\sqrt{\beta}\varepsilon_0 + \beta}{1+r},$$

$$\varepsilon_0 = \sqrt{2(1+r)+\beta}, \quad \varepsilon_\delta = -\sqrt{2(1-r)w + w^2\beta}. \quad (16)$$

Here ζ_r is a position of a reflection point.

And at last for the regime with total electron reflection for the reflection point and for a potential distribution to the right of this point we obtain:

$$\beta^{3/2}\zeta_r = \sqrt{\beta}(\varepsilon_0 - \varepsilon_r) - 2\ln\frac{2+\sqrt{\beta}\varepsilon_0 + \beta}{2+\sqrt{\beta}\varepsilon_r}, \quad (17)$$

$$\varepsilon_r = \sqrt{\varepsilon_0^2 - 4 - \beta},$$

$$\eta(\zeta) = -\frac{1}{2} - \varepsilon_r(\zeta - \zeta_r) + \frac{1}{2}\beta(\zeta - \zeta_r)^2. \quad (18)$$

Then, in a regime with a total reflection, we have a relation ε_0 with δ as follows

$$\delta = \zeta_r + \beta^{-1}(\varepsilon_r + \sqrt{\varepsilon_r^2 + w^2\beta}). \quad (19)$$

Now, with the background electrons, the solutions with the virtual cathode, giving the solutions with the total reflection within a gap at $V < -1/2$, can exist. At the potential minimum point, we have

$$\zeta_m = \zeta_r + \beta^{-1}\varepsilon_r, \quad \eta_m = -\frac{1}{2}(1+\beta^{-1}\varepsilon_r^2). \quad (20)$$

Using the obtained formulas, the dependencies ε_0 on δ were built up for a number of a background density κ_e values. Fig.5 demonstrates a background electron density effect on the curves $\varepsilon_0(\delta)$.

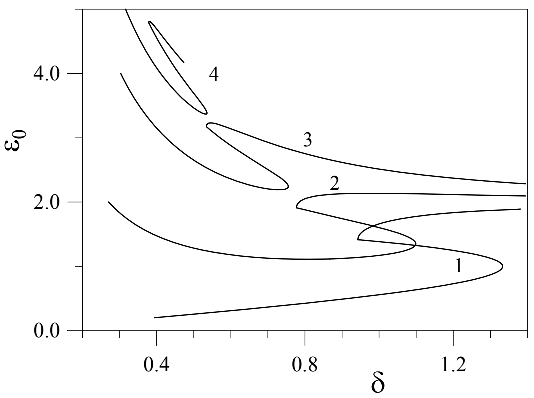

Fig.5. Emitter electric field ε_0 as a function of the diode length δ drawn for four values of κ_e =0 (curve 1), 1 (2), 2.3 (3) and 3 (4). The diode voltage is $V = 0$

As for the conventional Bursian diode, for a diode with a background, these curves show two bifurcation points – SCL and BF. A density of background electrons affects strongly the locations of these points. With an increase in a background density, the bifurcation points shifts to the lower δ, facilitating their using as the memory elements in the information technology.

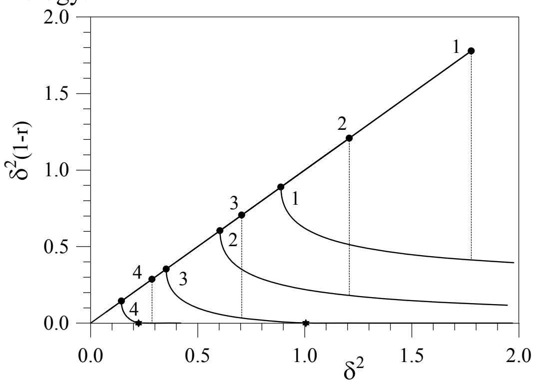

Fig.6. Collector convection current as a function of the beam current diode length δ drawn for 4th values of κ_e. Parameters are the same as in Fig.5

Fig.6 shows in what manner the changes in a dependence of the passing current on the beam current occur. An important point of the background electron diode is that a current cut-off in such a diode can be as low as zero (see, i.e., curve 4).

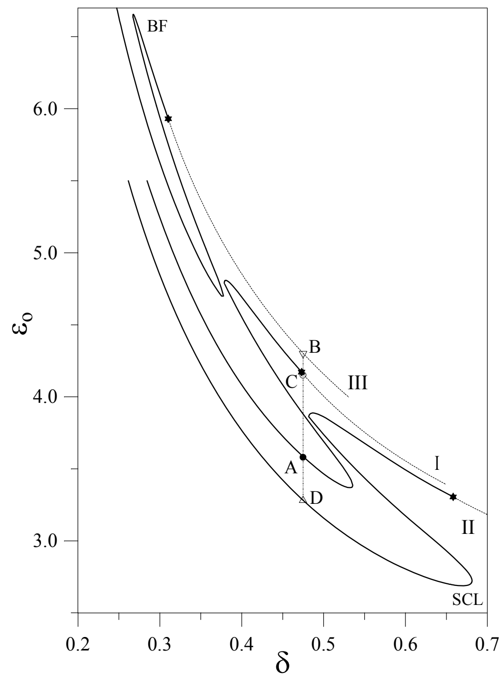

Fig.7. Emitter electric field ε_0 as a function of the diode length δ for $\kappa_e = 3$ drawn for three values of $V = 0$ (curve I), 0.1 (II), and -0.1 (III). Dashed curves correspond to the regime with total reflection, and star indicates left boundary of such region

The dependencies ε_0 on δ are presented in Fig.7, for a diode with the background electrons with $\kappa_e = 3$ drawn for three values of the external voltage *V*. Here, using points A – D and a vertical line drawn through them, an example is shown for presentation of the manner in which a device can be realized with a fast current switches to zero level and vice versa using a series of short voltage pulses. An initial state lies on a curve *I* corresponding to an external voltage $V = 0$ at a point *A*. A flowing current $j = j_A$. When supplying a short negative voltage pulse $\Delta V = -0.1$, the diode turns out to be, initially, on a curve *III* at a point *B* where $j = 0$, then, it returns to a curve *I* but, at a point *C*, corresponding to a regime of total reflection. Now, if a short positive voltage pulse $\Delta V = +0.1$ is supplied, the system is transferred, first, to a curve *II* at a point *D*, where $j = j_A$, and, at last, it turns out to be on a curve *I* again, at a point *A*. Emphasize that the duration of the control pulses can be only several time-of-flights of an electron via the diode gap.

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ЭЛЕКТРОННЫЙ ПУЧОК ПЛАЗМЕННОГО ДИОДА, ОКРУЖЕННЫЙ ЗАРЯЖЕННЫМИ ЧАСТИЦАМИ

А.Я. Эндер, В.И. Кузнецов, Х. Шамель

Изучается влияние эффекта окружения неподвижными заряженными частицами на стационарные состояния диода с электронным пучком. Дополнение однородных положительных заряженных частиц приводит к увеличению предельного ограниченного пространственным зарядом тока электронов, а также к возникновению новых областей равновесия. С другой стороны, избыток отрицательно заряженных частиц приводит к сильному уменьшению предельного тока, а также общего тока. Этот эффект можно использовать для создания быстрых переключателей.

ЕЛЕКТРОННИЙ ПУЧОК ПЛАЗМОВОГО ДІОДА, ЩО ОТОЧЕНИЙ ЗАРЯДЖЕНИМИ ЧАСТИНКАМИ

А.Я. Ендер, В.І. Кузнецов, Х. Шамель

Вивчається вплив ефекту оточення нерухомими зарядженими частинками на стаціонарні стани діода з електронним пучком. Доповнення однорідних позитивних заряджених частинок приводить до збільшення граничного обмеженого просторовим зарядом струму електронів, а також до виникнення нових областей рівноваги. З іншого боку, надлишок негативних заряджених частинок призводить до сильного зменшення граничного струму, а також загального струму. Цей ефект можна застосувати для створення швидких перемикачів.