Implementation of Quasi-Newton Method Based on BFGS Algorithm for Identification and Optimization of Signal Propagation Loss Model Parameters

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***Abstract*—Reliable and precise predictive modelling of signal losses along the communications paths and channels of propagated radio frequency waves is fundamental to the proper design, modelling, operation, and management of mobile broadband cellular networks. As such, the identification and tuning-based estimation of the signal propagation loss parameters has advanced into a recurrent task in the field of radio frequency and telecommunication engineering. Amongst the critical challenges known with identification and predictive estimation signal propagation loss parameters, the generic model-empirical data tuning approach is very vital, yet a most often disregarded and tough optimization problem. Here, a robust and fast computation capacity of Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm Quasi-Newton (QN) method based on the BFGS algorithm is presented for precise identification and optimization of generic log-distance propagation loss model parameters. The proposed QN based BFGS algorithm has been implemented for prognostic analysis of three sets of real-time signal propagation loss data obtained over a Long Term Evolution (LTE) mobile broadband network. When compared with the most popular Levenberg–Marquardt (LM), QN, and Gradient Descent (GD) methods, the proposed method achieved the 30–46% precision accuracies over other methods using three different statistical indicators, particularly in two study locations. The indicators are root mean square error, correlation coefficient and mean absolute error. The awesome precision performance of the proposed method can be explored to overcome premature convergence and poor predictive fitting issues often experienced in the identification and tuning-based estimation of the signal propagation loss parameters during or after cellular network planning processes.**

***Keywords*—numerical optimization method, Qausi-Newton, Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, parametric identification, propagation loss modelling, predictive model tuning, communication**

I. Introduction

Over the past decades, the demand and growth of cellular mobile communication technology has been undergoing different evolutionary phases and deployment in tandem with ever-increasing multimedia user demands. As a result, the earlier analogue voice and digital voice communication-based communication technologies that were rolled out in 80’s and 90’s has long been further developed for enhanced mobile broadband communication since the year 2000 with Third Generation (3G) as the main technology, to the present Fourth Generation Long Term Evolution (4G-LTE) and Fifth Generation New Radio (5G-NR) broadband systems, all which have been empowered with robust wireless internet access.

Generally, before the official deployment or after the commercial rollout of any typical cellular-based communication technology, there exist the crucial planning and optimization phases, where inefficient radio microwave propagation modelling and antenna configuration engineering both play important roles. Particularly, effective radio-microwave propagation predictive modelling and path loss calculation play leading roles in the proper eNode antenna location placement, precise cell coverage area computation, proper intercell interference analysis and correct assignment of the transmission frequencies. Thus, there is a crucial need to fine-tune an existing model or come up with a new one that can provide a precise cell coverage area computation and enhance spatial signal prediction accuracy, particularly in terrestrial radio propagation terrain where various clutter obstructions and environmental conditions are dominant has become very imperative.

But, the frequently asked question that is often difficult to answer is how to conduct a predictive propagation loss modelling and obtain the desired maximum accuracy.

In the remaining sections of the paper, the concise literature review, research methodology and the

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implementation flowchart which reveals how the propagation loss data acquisition is first and the proposed QN method based on BFGS algorithm, including the popular LM, GN and GD methods are unveiled in Sections I and III. Sections IV and V provide the graphical results, the discussion and the conclusion of the paper.

II. Literature Review

In literature, numerous techniques and efforts abound that have been utilized by several researchers toward carrying out effective predictive propagation loss modelling, but not without one or two limitations. The least absolute deviation algorithm has been applied to tune and identify of Ericsson propagation model toward a realistic estimation of propagation losses in cellular networks [1]. The authors realized up to 40% root mean square error reduction in the targeted urban environment. Nathaniel [2–5], Castro *et al.* [6], Castro-Hernandez *et al.* [7] employed the ordinary least square method to estimate and tune the Hata, Erceg and COST-231 model offset parameters to fit in their acquired measured loss data using a representative urban environment.

The least-square recursive algorithm and Minimax least-square algorithm are engaged in [8, 9] regression to calibrate the Okumura-Hata model and Ericson model for enhanced propagation loss estimation in Code Division Multiple Access (CDMA) networks. The general problem with all these least absolute deviations, least-square recursive algorithm, and Minimax least-square algorithm methods is their poor predictive modeling and handling of high stochastic propagation loss datasets [10]. The problems with above previous approaches suggest a crucial need for a more robust and better propagation loss model that has the capacity to predict pathloss in cell-cluster terrestrial terrain accurately.

But Isabona *et al.* [11–13] engaged the non-linear and numerical based Levenberg-Marquardt (LM) method in comparison with Gauss-Newton (GN) algorithm for empirical-based empirical predictive propagation Loss estimation and tuning. This was done to further tune and adapts the classical theoretical log-distance models for optimal signal attenuation loss data they obtained from stochastic microcellular LTE spatial radio communication channels. The different results revealed that the applied Levenberg-Marquardt method yielded the most precise correlation accuracy on the measured propagation losses compared to using the GN method. A supervised learning approach based on batch Gradient Descent (GD) has been applied to predict signal quality and the authors achieved 0.9 correlation accuracy [14].

A number of numerical analysis and their performance capacities is reviewed in details [15, 16].

These various literatures simply reveals that the Gradient-Newton algorithms are arguably the most popular class of nonlinear numerical optimization methods, used widely in numerical applications not just in machine learning.

Though the aforementioned LM, GN, and GD numerical methods remained the most commonly used classes of nonlinear optimization methods, however their precision are prone to parameter evaporation. Also, if the initial guess parameters of these methods are far from reality, their precision performance would be poor. Moreover, the LM, GN, and GD methods usually have global convergent limitation issues sometimes, since their solutions via residual function or error minimization rely upon their initial starting points [17, 18].

Thus, the limited performance of the above existing models presents a considerable gap in the literature, and the need to fill this gap is not out of place.

This study proposes and applies the Quasi-Newton Method based on BFGS algorithm for the identification and optimization of generic log-distance propagation loss model parameters for a typical built-up terrain in Nigeria. This paper key contribution includes:

- We acquire site specific spatial signal data and applied to obtain the desired measured propagation loss values.
- We proposed an adaptive Quasi-Newton method based on the BFGS algorithm over the commonly used LM, GN, and GD methods.
- We successfully applied the proposed adaptive Quasi-Newton method based on the BFGS algorithm for optimal generic propagation loss model parameter identification and optimization.
- We compared the proposed adaptive Quasi-Newton method based on the BFGS algorithm with other classical method using different statistical indicators.

III. Methodology

Parameter identification is a special process of identifying a model's parametric values from the observed empirical data via least square error tuning. Numerical optimization technique remained key techniques to solving system parameter model tuning and identification problems. The iterative formation and implementation of most numerical optimization process involves accurate selection of their initial estimates. This in turn have controlling effect on their performance and precision results; hence on the entire resultant quality of the identified or tuned system model.

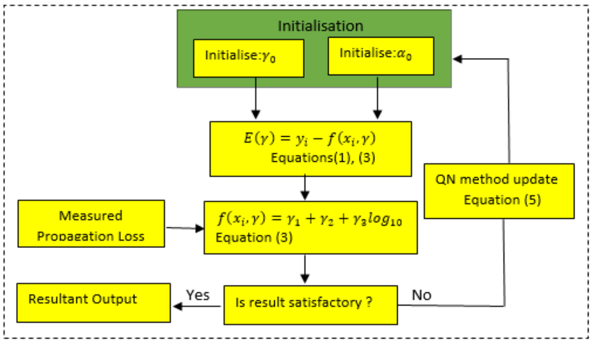
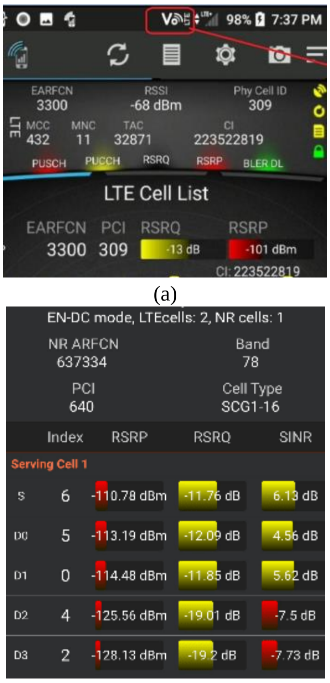


Figure 1. Flowchart of GN-based BFGS Algorithm for Propagation Loss model Parameter Identification.

In this section, we introduce the Quasi-Newton (QN) numerical method based on the Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithm [13], for the identification and optimization of generic log-distance model parameters. The empirical propagation loss data used for the parametric tuning and identification of the log-distance propagation model parameters were obtained from an all-inclusive signal measurement campaign conducted over LTE mobile broadband networks and the networks transmits at 2.6 GHz in 10MHz frequency band. The implementation flowchart of proposed QN based BFGS algorithm is shown in Fig. 1.

A. RSRP Data Collection and Propagation Loss Calculation



(b)

Figure 2. The general phone-based TEMS pocket (a) login structure and (b) user measurement interface.

With the aid of ASCOM field investigation tools which include a phone-based Test Mobile System (TEMS) pocket, Laptop, and Dongle, accompanied with Global Positioning System (GPS) and compass, all of which when connected together have the power to probe and measure service quality parameters and performance automatically in logfiles, were engaged to measure the Reference Signal Received Power (RSRP) data. The TEMS pocket user login structure and measurements Interface are shown in Fig. 2. The measurements were performed in and around three LTE eNodeB transmitters, with the two located in built-up areas and the last one in open areas of Lokoja, Nigeria. The investigated 4G LTE transmitters' heights range between (28–32 m), transmitting at 2.6 GHz. The measured real-time RSRP data was processed and analyzed in an excel spreadsheet, Mapinfo, and MATLAB user interface. In our investigation, the measured real-time RSRP is employed to calculate the signal propagation losses, PL using Eqs. (1) and (2):

$$PL[dB] = P_{tot} - RSRP \tag{1}$$

$$P_{tot} = g_t + P_t + g_r - f_l - c_l \tag{2}$$

with f_l, c_l, g_r, g_t, P_t and P_{tot} expressing the feeder losses, connector losses, receiver gain, antenna gain, transmit power, and the total radiated power of the eNodeBs.

B. Proposed Quasi-Newton Method Based on BFGS Algorithm

In non-linear regression optimization problems, the parameters of the targeted model are sourced in order for them to be adaptively fitted into the real-time empirical observations. This can be achieved via the minimization of the mean squared function or errors.

Thus, if $Y = (x_i, y_i)$ represents the empirical observations and $Y_f = f(x_i, \gamma)$ defines the target log-distance model function, the mean squared function can be defined as:

$$\gamma \in argmin_{\gamma}\, S(\gamma) = \sum_{i=1}^{n}[y_i - f(x_i,\gamma)]^2 \tag{3}$$

$$f(x_i,\gamma) = \gamma_1 + \gamma_2 + \gamma_3 log_{10}(x_i) \quad i=1,2,\ldots,n \tag{4}$$

where γ_1, γ_2 and γ_3 define the parameters of the targeted log-distance model, Y_f.

Iteratively, γ_1, γ_2 and γ_3 can be determined using the numerical optimization techniques. In this paper, we engaged the Quasi-Newton numerical method and it is given by:

$$QN = \alpha H^{-1} J^T(E) \tag{5}$$

where $E(\gamma) = S(\gamma)$ and H^{-1} defines the approximate inverse, Hessian; the residual error and the Jacobian matrix can be expressed using:

$$E(\gamma) = y_i - f(x_i,\gamma) = (Y - Y_f) \tag{6}$$

$$J^T = \frac{\partial E(\gamma)}{\partial \gamma} \tag{7}$$

The QN iterative update is given by:

$$\gamma_{z+1} = \gamma_z - \alpha_z H_z J^T E(\gamma) \tag{8}$$

$$H_{z+1} = V_z^T H_z V_z + q_z s_z^T s_z \frac{H_z K_z^T}{q_z^T s_z} \tag{9}$$

$$s_z = \gamma_{z+1} - \gamma_z \tag{10}$$

$$q_z = J^T(\gamma_{z+1}) - J^T(\gamma_z) \tag{11}$$

$$q_z = \frac{1}{q_z^T s_z} \tag{12}$$

$$V_z V_z^T = 1_z + q_z s_z^T s_z \tag{13}$$

C. The BFGS Algorithm

To implement the QN method, we employ the BFGS algorithm. The BFGS algorithm is updated iteratively, until a convergence stability condition is reached, thus leading to desired optimal solutions for all γ parameters. At every single iteration z, the QN based BFGS algorithm calculate the approximate Hessian H_z and the Jacobian matrix J at the point γ_z. The α_z indicate the learning rate, which provides a regulation or control at every step size. The QN-BFGS based implementation procedure is given in Algorithm 1:

Algorithm 1: BFGS

1. **Procedure** BFGS
2. choose starting guess parameters γ_0, and $H_0 > 0$
3. $z \leftarrow 0,1,2, \ldots,$
4. **while** true **do**
5. compute the QN search direction $H_z J^T f(\gamma_z)$
6. choose step-size that $\alpha_z > 0$
7. compute $\gamma_{z+1} = \gamma_z + \alpha_z H_z J^T(\gamma_z)$
8. $z \leftarrow z + 1$
9. **end if**
10. **end procedure**

D. The Levenberg-Marguardt Gauss-Newton (GN) and Gradient Descent (GD) Methods

For comparative analysis and benchmarking purposes, the popular Levenberg-Marguardt (LM), Gauss-Newton (GN) and Gradient Descent (GD) are also engaged to determine the γ parameters in the least square error reduction sense. In terms of Jacobian matrix, LM, GN and GD iteration updates can be defined using the expressions in Eqs. (14)–(16):

$$\gamma_{z+1} = \gamma_z - (J_z J_z^T + I\omega)^{-1} J_z^T(E) \tag{14}$$

$$\gamma_{z+1} = \gamma_z - (J_z J_z^T)^{-1} J_z^T(E) \tag{15}$$

$$\gamma_{z+1} = \gamma_z - J^T(E) \tag{16}$$

where ω and I denote the LM damping factor and identity matrix.

IV. Result and Discussion

In this section, the resultant impact of the proposed QN numerical optimization based on the BFGS algorithm is applied to identify and optimize the generic log-distance model parameters in correspondence with the measured propagation loss data. By means of the GN method with the BFGS algorithm, the log-distance model cost function is iteratively minimized via the generation sequence of inverse Hessian matrix approximations. For comparative analysis and bench marking purposes, the popular LM, GN, GD method are also engaged to determine the γ parameters in the least square sense as mentioned earlier.

To quantitatively examine the precision accuracy of the QN method over other popular ones, the Root Mean Square Error, Correlation coefficient, and the Mean Square error, all which are abbreviated as Root Mean Square Error (RMSE), R, and MAE, respectively, are engaged.

Figs. 3–5 display the RMSE predictive precision performances of the QN, LM, GN, and GD methods on measured propagation loss acquired in one open area and two built-up areas of Lokoja town. From the figures, the applied QN-based BFGS method achieved the most preferred precision accuracies of 3.05 dB, 3.46 dB and 4.62 dB in the three locations. This is followed by LM and GD which attained 3.06 dB, 3.02 dB, 6.99 dB and 3.15 dB, 6.44 dB and 6.97 dB, respectively. The worst is the GN method which achieved highest RMSE values of 3.14 dB, 7.72 dB, and 9.95 dB, respectively in the same study locations. The poorest precision performance of the GN method may be due the inability of its Jacobian matrix to correctly approximate the log-distance model error function of Eq. (2).

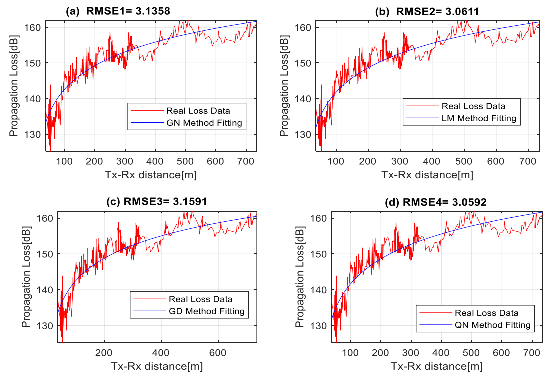


Figure 3. RMSE predictive precision performances of the QN, LM, GN, and the GD methods in Location 1.

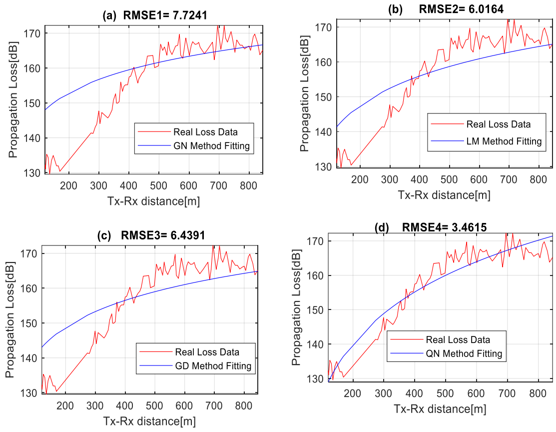


Figure 4. RMSE predictive precision performances of the QN, LM, GN, and the GD methods in Location 2.

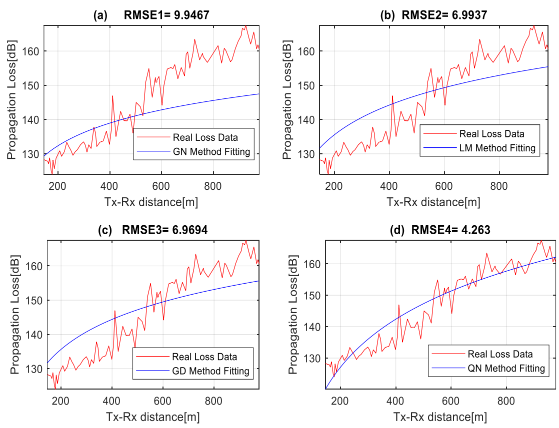


Figure 5. RMSE predictive precision performances of the QN, LM, GN, and the GD methods in Location 3.

But more importantly, the best precision results achieved with the proposed QN-based BFGS method may clearly indicate that it has the ability to deal with stochastic propagation loss data preferably than others. It may also point out that the QN-based BFGS method has better global convergent capacity during log-distance model error function minimization, irrespective of initial chosen starting guess parameters. The LM also achieves some levels of good prediction performance, which is blend of GD and GN methods.

Here, we employ the correlation coefficient fit, R to also measure the strength of prediction performance accuracies of the QN, LM, GN, and the GD methods. The closer the R value is to 1, the healthier the strength of correction fit. Figs. 6–8 display the correction fit performances of the QN, LM, GN, and the GD methods on measured propagation loss acquired one open area and two built-up areas of Lokoja town. Again, from the figures, the applied QN-based BFGS method achieved most preferred R accuracy of 0.999 values, particularly in locations 1 and 2. This is again followed by LM and GN in the same study locations.

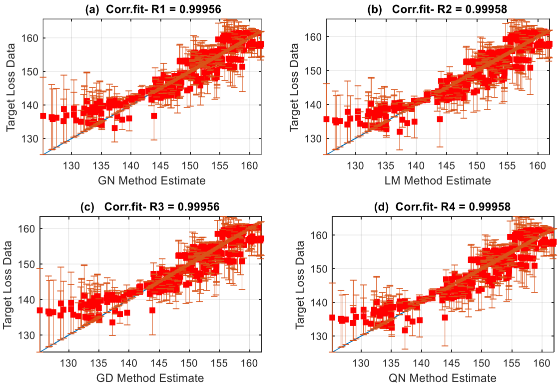


Figure 6: Correlation predictive precision performances of the GN, LM, GN, and the GD methods in Location 1.

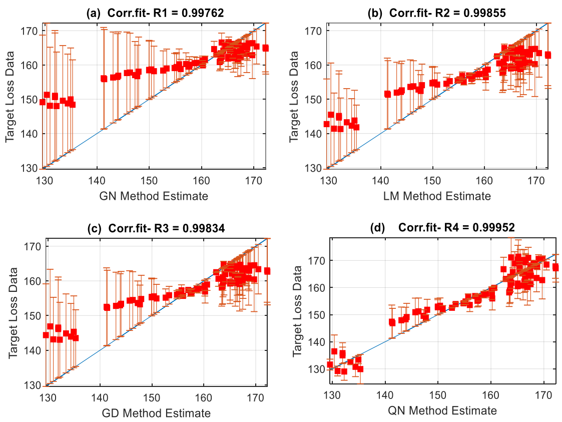


Figure 7. Correlation predictive precision performances of the QN, LM, GN, and the GD methods in Location 2.

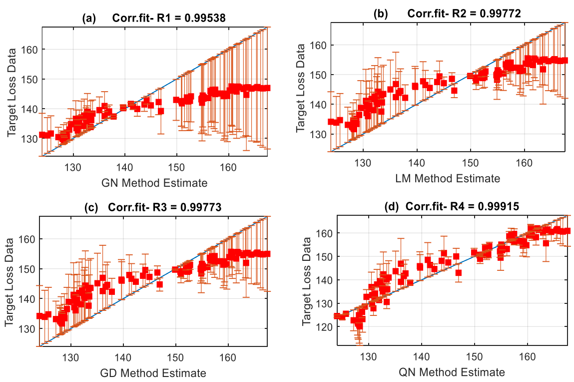


Figure 8. Correlation predictive precision performances of the GN, LM, GN, and the GD methods in Location 3.

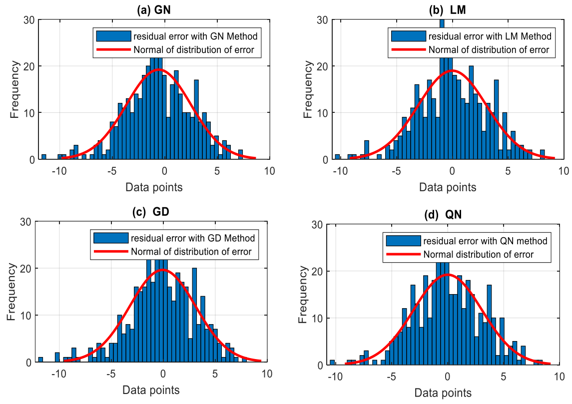


Figure 9. Residual error spread with QN, LM, GN, and the GD methods in Location 1.

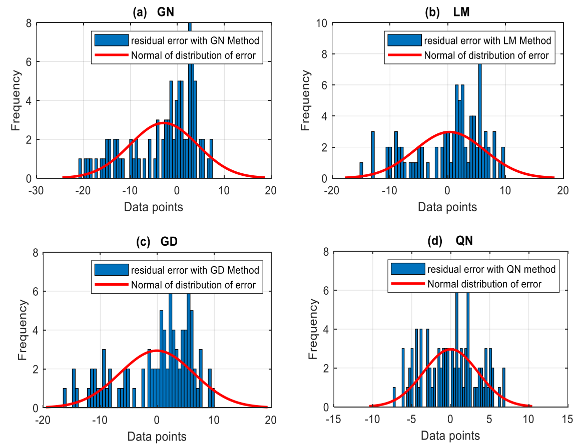


Figure 10. Residual error spread with QN, LM, GN, and the GD methods in Location 2.

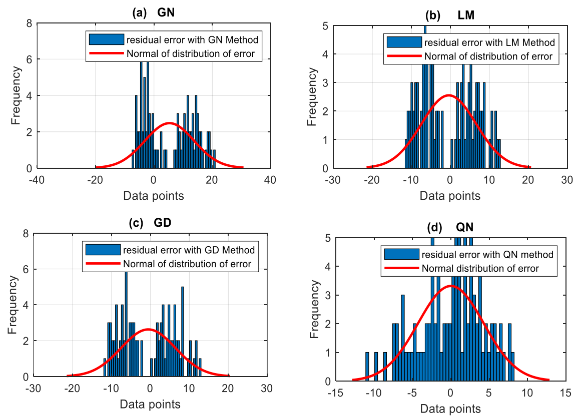


Figure 11. Residual error spread with QN, LM, GN, and the GD methods in Location 3.

Furthermore, Figs. 9–11 are plotted to reveal the propagation prediction absolute error spreads along measurement points using the QN method in correspondence with aforementioned popular ones. A lower error spreads with the QN over others shows that it yielded the best prediction accuracies. To quantify the levels with each method, we use the Mean Absolute Error (MAE) values. Like with the RMSE indicator, the applied QN-based BFGS method achieves most preferred precision MAE accuracies of 2.41 dB, 2.89 dB and

3.47 dB in the three locations. This is followed by LM and GD which attained 2.41 dB, 4. 98 dB, 6.40 dB, and 2.42 dB, 5. 24 dB, 6.37 dB, respectively. The worst is the GN method which achieved highest MAE values of 22.47 dB, 5.52 dB, and 8.15 dB, respectively in the same study locations. In summary, we can see that the proposed method achieved up to 30–46% precision accuracies over other methods, with RMSE and MAE in one open and built-up areas.

Table I shows are the identified γ_1, γ_2, and γ_3 parameters of the generic log-distance model of Eq. (2), using the QN, LM, GN, and GD methods, respectively.

TABLE I. THE IDENTIFIED γ PARAMETERS OF THE GENERIC LOG-DISTANCE MODEL, USING THE QN, LM, GN, AND GD METHODS

Locations	Parameters	GN	LM	GD	QN
1	γ_1	8.410	8.41	9.51	11.87
	γ_2	25.48	25.48	25.15	24.46
	γ_3	23.10	23.10	23.10	23.10
2	γ_1	−28.58	−28.58	−20.81	6.310
	γ_2	15.71	15.710	13.44	5.490
	γ_3	49.97	49.97	49.97	49.97
3	γ_1	−25.90	−25.90	−19.69	5.110
	γ_2	10.48	10.48	8.660	1.400
	γ_3	50.90	50.90	50.90	50.90

V. CONCLUSION

Effective radio-microwave propagation predictive modelling and path loss calculation play leading roles in the proper eNode antenna location placement, precise cell coverage area computation, proper intercell interference analysis and correct assignment of the transmission frequencies in cellular communications.

In this paper, the Quasi-Newton Method based on BFGS algorithm has been proposed for robust identification and optimization of generic log-distance propagation loss model parameters. Particularly, we have engaged the proposed Quasi-Newton Method based on BFGS algorithm for robust identification and optimization of generic log-distance propagation loss model parameters, in correspondence with the field data taken typical open and built-up terrains in Nigeria.

We also compared the proposed adaptive Quasi-Newton method based on the BFGS algorithm with other classical methods using different statistical indicators. In terms of RMSE in the three locations, the applied QN-based BFGS method achieves most preferred precision accuracies, then followed by LM and GD, and the worst is the GN method. The poorest precision performance of the GN method may be due the inability of its Jacobian matrix to correctly approximate the log-distance model error function.

The best precision results achieved with the proposed QN-based BFGS method over aforementioned popular ones may clearly indicate that it has the ability to deal with stochastic propagation loss data preferably than others. More importantly, it may also point out that the GN-based BFGLS method has better global convergent capacity during log-distance model error function minimization, irrespective of initial chosen starting guesses. The LM also achieved some levels of good prediction performance, which is blend of GD and GN methods.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Isabona Joseph, carried out the data acquisition, analysis, evaluation and also supervision of research activity and mentorship. Ituabhor Odesanya implemented the Algorithm used in the MatLab programming language. Arijaje T. E. was responsible for the Statistics. Akinwumi S. A responsible for preparation of the work for publication from the original research group. All authors had approved the final version

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