A note on stock price dynamics and monetary policy in a small open economy

Ida, Daisuke and Okano, Mitsuhiro and Hoshino, Satoshi

Faculty of Economics, Momoyama Gakuin University, Faculty of Economics, Osaka Gakuin University, Faculty of Economics, Okayama Shoka University

24 May 2024

A note on stock price dynamics and monetary policy in a small open economy*

Daisuke Ida[†][‡], Mitsuhiro Okano[§], and Satoshi Hoshino[¶][‖]

May 24, 2024

Abstract

This note examines the role of stock price stabilization in a small open new Keynesian model. We show that stabilizing stock prices is desirable to attain a unique rational expectations equilibrium and that the open economy effect has a significant impact on the determinacy condition.

JEL codes: E52; E58; F41;

Keywords: Stock prices; Monetary policy rules; Terms of trade; Indeterminacy; Taylor principle;

---

*Ida and Okano were supported by JSPS KAKENHI grant number JP24K04971. All remaining errors are our own.

†Faculty of Economics, Momoyama Gakuin University, 1-1, Manabino, Izumi, Osaka 594-1198, Japan. Tel.: +81-725-92-6134. E-mail: ida-dai@andrew.ac.jp

‡Research Fellow, Graduate School of Economics, Kobe University, Japan

§Faculty of Economics, Osaka Gakuin University, 2-36-1 Kishibeminami, Suita-City, Osaka 564-8511, Japan. Tel.: +81-6-6381-8434. E-mail: okano@ogu.ac.jp

¶Faculty of Economics, Okayama Shoka University, 2-10-1 Tsushima-kyomachi, Kita, Okayama-City, Okayama 700-0087, Japan. Tel.: +81-086-230-6087. E-mail: hoshino@po.osu.ac.jp

‖Research Fellow, Graduate School of Economics, Kobe University, Japan

1 Introduction

Should the central bank stabilize stock prices when the exchange rate fluctuates? We have observed several facts that stock prices and exchange rates fluctuate. For instance, in the middle of the 1980s, the Japanese economy faced a sharp yen appreciation, leading to the downward pressure on inflation. The Bank of Japan (BOJ) maintained a lower nominal interest rate to combat deflationary pressure, whereas such a lower interest rate caused a stock price boom. Also, during the 1990s, the stock price boom occurred in the Asian economies because an interest rate differential between the Asian and the US economies led to capital flow into those countries. Moreover, the BOJ has been keeping interest rates at zero in April 2024, whereas the Federal Reserve Board has increased its policy rates to combat inflationary risk. Such an interest rate differential between Japan and the US causes a sharp yen depreciation and a stock price boom in Japan. These facts call for us to pay attention to the role of stock price stabilization when exchange rate dynamics matter.

There has been a long debate on whether stock price stabilization should be included in the central bank's policy objective. While Bernanke and Gertler (2000) argued that the central bank should not stabilize stock prices unless they induce inflationary pressure, Cecchetti, Genberg and Wadhwani (2002) justified the role of stock price stabilization to prevent a substantial recession after a stock price boom. Carlstrom and Fuerst (2007) and Nutahara (2014) argued that targeting firm's stock prices may destabilize the economy in a new Keynesian (NK) model with stock price dynamics. Conversely, Pfajfar and Santoro (2014) showed that stock price stabilization becomes an effective tool to attain the unique rational expectations equilibrium (REE) in the presence of the cost channel. However, they did not investigate the role of stock price stabilization in an open economy framework. Although Di Giorgio and Nisticò (2007) and Ida (2011, 2013) argued the role of stock price stabilization in a two-country model, they did not analytically derive the determinacy condition.

To bridge the gap between observed facts and theoretical debates, we examine the role of stock price stabilization in an open economy from the perspective of equilibrium determinacy. We incorporate the stock price dynamics employed in Carlstrom and Fuerst (2007) into the small-open NK model. Although one may think it is a moderate model extension, we underline significant policy implications for the role of stock price stabilization. We show that stabilizing stock prices can achieve the unique REE under the forward-looking Taylor rule and address that an open economy effect significantly affects the determinacy condition.

2 Model description

We incorporate stock price dynamics into the standard small-open economy NK model developed by Galí and Monacelli (2005) and Galí (2015). Except for inflation and the nominal interest rate, the variables are expressed by the log-deviation from the steady state. Also, a variable with a tilde represents the deviation of a variable from its counterpart in the natural level. The online appendix provides a detailed model description.

The log-linearized system is summarized as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_\nu \tilde{y}_t, \tag{1}$$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma_\nu^{-1}(i_t - E_t \pi_{t+1} - r_t^n), \tag{2}$$

$$\tilde{q}_t = [(1-\beta)\Lambda + \gamma\sigma_\nu] E_t \tilde{y}_{t+1} + \beta E_t \tilde{q}_{t+1} - \gamma\sigma_\nu \tilde{y}_t - (i_t - E_t \pi_{t+1} - r_t^n), \tag{3}$$

where

$$\kappa_\nu = \lambda(\sigma_\nu + \varphi);\ \lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta};\ \sigma_\nu = \frac{\sigma}{1+\gamma(\varpi - 1)};\ \varpi = \sigma\eta + (1-\gamma)(\sigma\eta - 1);$$
$$\Lambda = 1 + \frac{\sigma_\nu \gamma(\varpi - 1) - (\sigma + \varphi)}{\mu - 1}.$$

π_t, $\tilde{y}_t$, $\tilde{q}_t$, and i_t denote home inflation, the output gap, the stock price gap, and the nominal interest rate, respectively. r_t^n captures the natural interest rate. Equation (1) denotes the new Keynesian Phillips curve. Equation (2) represents the dynamic IS curve. Equation (3) captures the stock price dynamics. Parameters σ and φ denote the constant relative risk aversion coefficient and the inverse elasticity of labor supply, respectively. β, μ, and γ represent the discount factor, price markup, and the degree of openness, respectively. The parameter η denotes the elasticity of substitution between home and foreign goods.

Equations (1) and (2) are the standard ones derived in a small open NK model (Galí, 2015). In what follows, we concentrate on the role of stock price dynamics (3). As in Carlstrom and Fuerst (2007), this equation depends on the future output gap, the future stock price gap, and the real interest gap. As shown in the online appendix, unlike their study, stock price dynamics are affected by international risk-sharing and terms-of-trade channels. The coefficient Λ hinges on these two channels. $\sigma_\nu\varpi$ denotes the international risk-sharing channel and $\gamma\sigma_\nu$ captures the terms of trade channel. The dominance of the terms of trade channel over the risk-sharing one implies the attenuated impact of the future output gap on current stock prices. Equation (3) corresponds to Carlstrom and Fuerst (2007) if $\gamma = 0$.

To derive the determinacy condition, we consider the forward-looking Taylor rule:

$$i_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t \tilde{y}_{t+1} + \phi_q E_t \tilde{q}_{t+1}, \tag{4}$$

where ϕ_π, ϕ_y, and ϕ_q denote the stabilization terms for inflation, the output gap, and the stock price gap, respectively.

3 Main results

3.1 Determinacy properties: Analytical investigation

First, we analytically derive how the central bank achieves the unique REE when it considers stock price stabilization in its monetary policy rule. To analytically and intuitively derive the determinacy condition, we need the following parameter restriction:

Assumption 1 (Complete stabilization of the output gap)

$$\sigma_\nu = [(1-\beta)\Lambda + \gamma\sigma_\nu] = \phi_y.$$

Due to this assumption, the central bank can completely offset the variations of the future output gap (Muto, 2011). Under the above restriction, we have

Proposition 1 *Under the monetary policy rule (4), the necessary and sufficient condition for a unique REE is given as follows:*

$$(1-\gamma)\phi_q + \kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1) < \beta^{-1}(1+\beta^2), \tag{5}$$

$$\sigma_\nu^{-1}\kappa_\nu(\phi_\pi - 1) + (1-\gamma)\phi_q + (1-\beta) > 0. \tag{6}$$

Proof. See the Appendix A. ■

Equation (6) states that for a given value of ϕ_q, the condition that ϕ_π is above unity is required to attain the unique REE. This condition implies the generalized Taylor principle in our model. In particular, given a value of ϕ_π, the presence of stock price stabilization helps to achieve the unique REE. This result is in stark with that shown in Carlstrom and Fuerst (2007). Carlstrom and Fuerst (2007) employed the current-looking Taylor rule and showed that an increased value of ϕ_q easily destabilizes the economy. In contrast to their results, our study demonstrates that even if ϕ_π is less than unity, an increased value of ϕ_q may retain the unique REE. This result may be due to the introduction of the forward-looking monetary policy rule. However, we address that a larger value of γ attenuates this effect associated with stock price stabilization. To the best of our knowledge, this result has not been observed in the existing literature. Equation (5), however, requires the upper bounds on the coefficients for inflation and stock price stabilization to obtain the unique REE. The necessity of upper bounds on these parameters stems from the introduction of the forward-looking rule (Bullard and Mitra, 2002). In a nutshell, the presence of an open economy effect captured by γ, helps to retain a unique REE even if the central bank puts a larger weight on stock price stabilization.

3.2 Numerical examples

In what follows, we explore the role of stock price stabilization in our model by relaxing the assumption used in the analytical part. Unfortunately, since we cannot analytically and intuitively obtain the determinacy conditions without any model restrictions, we perform the numerical example to overcome this problem.

The calibrated values are those used in the standard NK literature. β, σ, and φ are set to 0.99, 2.0, and 3.0, respectively. We set θ to 0.7. The price markup is 1.1. γ and η are set to 0.4 and 1.5, respectively. Finally, we describe the calibrated values in the Taylor rule. ϕ_π and ϕ_y are set to 1.5 and 0.5, respectively. As a benchmark calibration, we set ϕ_q to 0.1.

Figure 1 plots the determinacy region under several combinations of ϕ_π and ϕ_q. The case of $\gamma = 0$ corresponds to the case where the open economy effect is shut down. Not surprisingly, unlike Carlstrom and Fuerst (2007), under a forward-looking rule, incorporating stock price stabilization helps to achieve the unique REE as long as the central bank does not break upper bounds on the values of ϕ_π and ϕ_q. Importantly, as the degree of openness (γ) increases, a stronger weight on inflation stabilization renders the REE indeterminate, given a value of ϕ_q. This is consistent with Llosa and Tuesta (2008). We document that as long as the central bank adopts an aggressive but moderate response to inflation, an increased value of ϕ_q can make the REE determinate. For instance, as long as ϕ_π ranges from 1.5 to 3.0, the case with $\phi_q = 1.0$ can achieve a unique REE when $\gamma = 0.4$. However, this figure shows that as the value of γ

increases, upper bounds on the inflation reaction are more restricted to attain the unique REE.

[Figure 1 around here]

3.3 Discussion

Why does the degree of openness help to achieve the unique REE when the central bank attempts to stabilize stock prices? Carlstrom and Fuerst (2007) asserted that an inclusion of stock price stabilization in monetary policy rules is likely to destabilize the economy. In their model, an inflationary pressure due to a sunspot shock causes a rise in the real marginal cost. An increase in the real marginal cost decreases the firm's dividend, leading to a decline in stock prices. Therefore, if the central bank considers stock price stabilization as well as inflation stabilization, it cannot react adequately to fluctuations in inflation. This is the source of the equilibrium indeterminacy in their model.

Unlike their model, we have an additional channel associated with an open economy framework. The key to understanding our results is to consider the role of the terms of trade. The terms of trade dynamics are captured by the parameter γ. The terms of trade depreciation induces a rise in CPI inflation, lowering the real interest rate. A fall in the real interest rate boosts the stock prices. Thus, an increase in stock prices associated with a depreciation in the terms of trade can help as a signal for inflation pressure. Therefore, the presence of the terms of trade channel can partially offset the negative effect of inflation on stock prices. However, note that while stock price stabilization may render the REE determinate, the weight on the inflation reaction is more restrictive to attain the unique REE in the forward-looking Taylor rule if the central bank reacts aggressively to a fluctuation in stock prices.

4 Conclusions

We addressed the role of stock price stabilization in a small-open NK model. We demonstrated the desirability of stabilizing stock prices to attain a unique rational expectations equilibrium and that the open economy effect has a significant impact on the determinacy condition.

References

Bernanke, B., Gertler, M., 2000. Monetary policy and asset price volatility. Technical Report. National bureau of economic research.

Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. Journal of Monetary Economics 49, 1105–1129.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 383–398.

Carlstrom, C.T., Fuerst, T.S., 2007. Asset prices, nominal rigidities, and monetary policy. Review of Economic Dynamics 10, 256–275.

Cecchetti, S.G., Genberg, H., Wadhwani, S., 2002. Asset prices in a flexible inflation targeting framework. Technical Report. National Bureau of Economic Research.

Di Giorgio, G., Nisticò, S., 2007. Monetary policy and stock prices in an open economy. Journal of Money, Credit and Banking 39, 1947–1985.

Galí, J., 2015. Money, Inflation and the Business Cycles. second ed., Princeton University Press, Prinston.

Galí, J., Monacelli, T., 2005. Monetary policy and exchange rate volatility in a small open economy. Reveiw of Economic Studies 72, 707–734.

Ida, D., 2011. Monetary policy and asset prices in an open economy. The North American Journal of Economics and Finance 22, 102–117.

Ida, D., 2013. Tobin's q channel and monetary policy rules under incomplete exchange rate pass-through. Economic Modelling 33, 733–740.

Llosa, L.G., Tuesta, V., 2008. Determinacy and learnability of monetary policy rules in small open economies. Journal of Money, Credit and Banking 40, 1033–1063.

Muto, I., 2011. Monetary policy and learning from the central bank's forecast. Journal of Economic Dynamics and Control 35, 52–66.

Nutahara, K., 2014. What asset prices should be targeted by a central bank? Journal of Money, Credit and Banking 46, 817–836.

Pfajfar, D., Santoro, E., 2014. Credit market distortions, asset prices and monetary policy. Macroeconomic Dynamics 18, 631–650.

A Appendix A: Proof of Proposition 1

Using Equation (4) and Assumption 1 in the main text, we can the log-linearized system as follows:

$$AX_t = BE_tX_{t+1} + Cr_t^n, \tag{A.1}$$

where

$$A = \begin{bmatrix} 1 & 0 \\ \gamma\sigma_\nu\kappa_\nu^{-1} & 1 \end{bmatrix};\ B = \begin{bmatrix} \beta - \sigma_\nu^{-1}\kappa_\nu(\phi_\pi - 1) & -\sigma_\nu^{-1}\kappa_\nu\phi_q \\ \beta\gamma\sigma_\nu\kappa_\nu^{-1} + 1 - \phi_\pi & \beta - \phi_q \end{bmatrix};\ C = \begin{bmatrix} \sigma_\nu^{-1}\kappa_\nu \\ 1 \end{bmatrix}.$$

After several manipulations, the system of two endogenous variables, π_t and $\tilde{q}_t$, can be written as follows:

$$X_t = ME_tX_{t+1} + Nr_t^n, \tag{A.2}$$

where[1]

$$M = \begin{bmatrix} \beta - \sigma_\nu^{-1}\kappa_\nu(\phi_\pi - 1) & -\sigma_\nu^{-1}\kappa_\nu\phi_q \\ (1-\gamma)(\phi_q - 1) & \beta - (1-\gamma)\phi_q \end{bmatrix}.$$

The characteristic polynomial equation is given by $Q(\lambda) = \lambda^2 - \mathrm{Tr}(M)\lambda + \det(M)$, where

$$\mathrm{Tr}(M) = 2\beta - \sigma_\nu^{-1}\kappa_\nu(\phi_\pi - 1) - (1-\gamma)\phi_q,$$
$$\det(M) = \beta\Big[\beta - (1-\gamma)\phi_q - \sigma_\nu^{-1}\kappa_\nu(\phi_\pi - 1)\Big],$$

where $\det(M)$ and $\mathrm{Tr}(M)$ denotes the determinant and the trace of the matrix M, respectively. Both eigenvalues of M are inside the unit circle if both of the following conditions hold:

$$\text{(i) } |\det(M)| < 1,$$
$$\text{(ii) } |\mathrm{Tr}(M)| < 1 + \det(M).$$

[1] We omit the expression for N, which is not related to the following discussion.

Consider the condition (i). When $\beta > (1-\gamma)\phi_q + \kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1)$, we obtain the following:

$$(1-\gamma)\phi_q + \kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1) > -\beta^{-1}(1-\beta^2),$$

which is trivially satisfied. Conversely, if $\beta < (1-\gamma)\phi_q + \kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1)$, we obtain

$$(1-\gamma)\phi_q + \kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1) < \beta^{-1}(1+\beta^2),$$

which leads to the condition (5).

Next, consider the condition (ii). If $2\beta + \sigma_\nu^{-1}\kappa_\nu > \sigma_\nu^{-1}\kappa_\nu\phi_\pi + (1-\gamma)\phi_q$, we obtain the following inequality:

$$\kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1) + (1-\gamma)\phi_q + (1-\beta) > 0,$$

which leads to the Taylor principle (6) in the main text. Finally, when $2\beta + \sigma_\nu^{-1}\kappa_\nu < \sigma_\nu^{-1}\kappa_\nu\phi_\pi + (1-\gamma)\phi_q$, this provides the following condition:

$$\kappa_\nu \sigma_\nu^{-1}(\phi_\pi - 1) + (1-\gamma)\phi_q < 1 + \beta.$$

However, this condition can be included in the condition (5). This completes the proof.

B Appendix B: Model description

Except for the presence of stock price dynamics, the model is the standard small open economy new Keynesian model developed by Galí and Monacelli (2005) and Galí (2015). Representative households in the home country purchase home and foreign goods. Home households can have access to a complete set of state-contingent securities that are traded both domestically and internationally. Firms face both monopolistically competitive environments and nominal staggered-price rigidities as specified by Calvo (1983). We define $g_t = \log(G_t/G)$ as the deviation of a variable G_t from the steady state. G represents the value of the steady state. Unless otherwise noted, a variable with an asterisk denotes the foreign one.

B.1 Households

B.1.1 Preferences

The household's utility function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(C_t) - v(N_t) \Big\} = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \Big\}, \tag{B.1}$$

where C_t and N_t are consumption and labor supply, respectively. β denotes the discount factor. σ and φ denote the constant relative risk aversion coefficient and the inverse elasticity of labor supply. E_0 is the expectations conditional on available information in period 0.

The aggregate consumption is given by

$$C_t = \left[(1-\gamma)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{F,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{B.2}$$

with

$$C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)}, \tag{B.3}$$

where $C_{H,t}$ and $C_{F,t}$ denote home and foreign goods. And, $C_{H,t}(i)$ represents the home goods produced by firm i. η and ϵ are the elasticity of substitution between home and foreign goods and the elasticity of substitution between home individual goods.

The demand function for goods i is given as

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t},$$

with

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right)^{1/(1-\epsilon)}.$$

Household faces the following budget constraints:

$$P_tC_t + P_tQ_tH_t + B_t \leq R_{t-1}B_{t-1} + P_tD_tH_{t-1} + P_tQ_tH_{t-1} + W_tN_t - T_t, \tag{B.4}$$

where H_t is the number of shares issued by firm i and Q_t denotes stock prices. D_t is the dividend and B_t is the nominal bond. R_t is the gross nominal interest rate and P_t is the aggregate price level. W_t and T_t denote nominal wages and lump-sum tax.

From the first-order condition of the household's utility maximization problem, we obtain

$$C_t^{-\sigma} = \beta R_t E_t\left(C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}}\right), \tag{B.5}$$

$$\frac{N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}, \tag{B.6}$$

$$C_t^{-\sigma} Q_t = \beta E_t C_{t+1}^{-\sigma}(Q_{t+1} + D_{t+1}), \tag{B.7}$$

Equation (B.5) represents the consumption Euler equation. Equation (B.6) states the relationship that the marginal rate of substitution between consumption and labor supply is equal to the real wages. Equation (B.7) captures the stock price dynamics.

B.1.2 Exchange rate and international risk-sharing

Following Galí and Monacelli (2005) and Galí (2015), we assume the law of one price (LOOP) in this model. Thus, the following relationship holds

$$P_{F,t} = \mathcal{E}_t P_t^*, \tag{B.8}$$

where $\mathcal{E}_t$ is the nominal exchange rate and $P_{F,t}$ is nominal prices for foreign goods.

Under the assumption that households can freely trade state-contingent bonds domestically and internationally, we obtain the following international consumption risk-sharing condition:

$$C_t = \vartheta C_t^* Z_t^{1/\sigma}, \tag{B.9}$$

where $Z_t = \mathcal{E}_t P_t^* / P_t$ is the real exchange rate and C_t^* is foreign consumption. ϑ is the constant parameter.

B.1.3 The terms of trade and the real exchange rate

In this economy, the terms of trade is defined as

$$S_t = \frac{P_{F,t}}{P_{H,t}}. \tag{B.10}$$

Using this relationship, we assume that consumer price index (CPI) inflation can be obtained as follows:

$$\pi_t^c = \pi_t + \gamma(s_t - s_{t-1}), \tag{B.11}$$

where $\pi_t = \log(P_{H,t}/P_{H,t-1})$ denotes producer price inflation and $\pi_t^c = \log(P_t/P_{t-1})$ denotes CPI inflation.

We can also discuss the relationship between the terms of trade and the real exchange rate as follows:

$$z_t = (1 - \gamma)s_t. \tag{B.12}$$

B.2 Firms

The firms use the following production functions:

$$Y_t(i) = A_t N_t(i), \tag{B.13}$$

where $Y_t(i)$ represents output and A_t denotes the exogenous productivity disturbance.

Following Calvo (1983), nominal price rigidity is introduced in the intermediate goods sector. A fraction $1-\theta$ of all firms adjusts their price, whereas the remaining fraction of firms θ does not. Under these circumstances, the first-order condition from the firms' profit maximization problem is linearized around the zero-inflation steady state as follows:

$$\log \bar{P}_{H,t} = \mu + (1-\theta\beta)E_t \sum_{j=0}^{\infty} (\theta\beta)^j E_t \psi_{t+j}, \tag{B.14}$$

where $\bar{P}_{H,t}$ denotes optimal nominal prices and ψ_t is the nominal marginal costs. μ is the price markup, which is defined as $\epsilon/(\epsilon-1)$.[2]

After several manipulations, we can obtain the following new Keynesian Phillips curve expressed by the variable price markup:

$$\pi_t = \beta E_t \pi_{t+1} - \lambda \hat{\mu}_t, \tag{B.15}$$

where $\hat{\mu}_t = \log(P_{H,t}) - \psi_t - \mu$ and

$$\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}. \tag{B.16}$$

B.3 Exports

Following Galí (2015), we introduce the export sector. The export demand for goods i ($X_t(i)$) is given as follows:

$$X_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} X_t, \tag{B.17}$$

where the aggregate export is given by

$$X_t = \left(\int_0^1 X_t(i)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)}. \tag{B.18}$$

Then, the aggregate export can be rewritten as follows:

$$X_t = \gamma S_t^{\eta} Y_t^{*}. \tag{B.19}$$

[2]See Galí (2015) for a detailed derivation of this equation.

B.4 Equilibrium

The goods market equilibrium is given by

$$Y_t(i) = C_{H,t}(i) + X_t(i). \tag{B.20}$$

Here, aggregate output is given by

$$Y_t = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}. \tag{B.21}$$

Substituting the demand function for consumption and export into aggregate output leads to the following goods market condition:

$$Y_t = (1-\gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \gamma S_t^{\eta} Y_t^*. \tag{B.22}$$

Log-linearization of this equation is given by

$$y_t = (1-\gamma)c_t + \gamma(2-\gamma)\eta s_t + \gamma y_t^*. \tag{B.23}$$

Then, the log-linearization of the terms of trade is proportional to the ratio of home output to foreign output.

$$s_t = \sigma_\nu (y_t - y_t^*), \tag{B.24}$$

where

$$\sigma_\nu = \frac{\sigma}{1+\gamma(\varpi-1)}; \ \varpi = \sigma\eta + (1-\gamma)(\sigma\eta - 1). \tag{B.25}$$

B.5 Log-linerization

B.5.1 Dividend

The dividend is given as follows:

$$D_t = (1 - MC_t)Y_t, \tag{B.26}$$

where MC_t is the real marginal cost. Log-linearized this equation leads to

$$d_t = y_t - \frac{1}{\mu - 1} mc_t. \tag{B.27}$$

Here, the real marginal cost is rewritten as follows:

$$mc_t = \sigma c_t + \varphi y_t + \gamma s_t - (1+\varphi)a_t. \tag{B.28}$$

Using this equation, we can rewrite the dividend equation (B.27) as follows:

$$d_t = \left(1 - \frac{\sigma + \varphi}{\mu - 1} \right) y_t + \frac{\gamma(\varpi - 1)}{\mu - 1} s_t + \frac{1+\varphi}{\mu - 1} a_t.$$

B.5.2 Stock price dynamics

Log-linearizing Equation (B.7) leads to the following:

$$q_t = (1-\beta)E_t d_{t+1} + \beta E_t q_{t+1} - (i_t - E_t \pi^c_{t+1}),$$
$$= (1-\beta)E_t d_{t+1} + \beta E_t q_{t+1} - (i_t - E_t \pi_{t+1}) + \gamma E_t \Delta s_{t+1}, \tag{B.29}$$

where $i_t = \log R_t$ denotes the nominal interest rate. Unlike Carlstrom and Fuerst (2007), stock price dynamics are affected by a change in the terms of trade dynamics.

B.5.3 NKPC

The variable price markup can be rewritten as follows:

$$\hat{\mu}_t = \log(P_{H,t}) - \psi_t,$$
$$= -(\sigma c_t + \varphi y_t) - \gamma s_t + (1+\varphi)a_t, \tag{B.30}$$

where $a_t = \log A_t$. As shown in Galí (2015), the real marginal cost is connected with the variable price markup ($\hat{\mu}_t$).

$$\hat{\mu}_t = -(\sigma_\nu + \varphi)(y_t - y^*_t). \tag{B.31}$$

Substituting this into the NKPC expressed in the variable price markup, we can obtain the following NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda(\sigma_\nu + \varphi)(y_t - y^*_t).$$

B.5.4 Flexible prices

- Aggregate output

$$y^n_t = \frac{1+\varphi}{\sigma + \varphi + \gamma\sigma_\nu(1-\varpi\sigma^2)} a_t - \frac{\gamma\sigma_\nu(1-\varpi\sigma^2)}{\sigma + \varphi + \gamma\sigma_\nu(1-\varpi\sigma^2)} y^*_t \tag{B.32}$$

- Terms of trade

$$s^n_t = \sigma_\nu(y^n_t - y^*_t) \tag{B.33}$$

- Dividend

$$d^n_t = y^n_t \tag{B.34}$$

B.6 Gap expression

A variable with a tilde represents the deviation of a variable from its counterpart at the natural level.

- Dynamic IS curve

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma_\nu^{-1}(i_t - E_t\pi_{t+1} - r_t^n), \tag{B.35}$$

where r_t^n denotes the natural rate of interest, which is as follows:

$$r_t^n = \sigma_\nu E_t\Delta y_{t+1}^n + \sigma_\nu(\varpi - 1)\gamma E_t\Delta y_{t+1}^*.$$

- NKPC expressed in the output gap

$$\pi_t = \beta E_t\pi_{t+1} + \kappa_\nu\tilde{y}_t, \tag{B.36}$$

where

$$\kappa_\nu = \lambda(\sigma_\nu + \varphi).$$

- Stock price dynamics

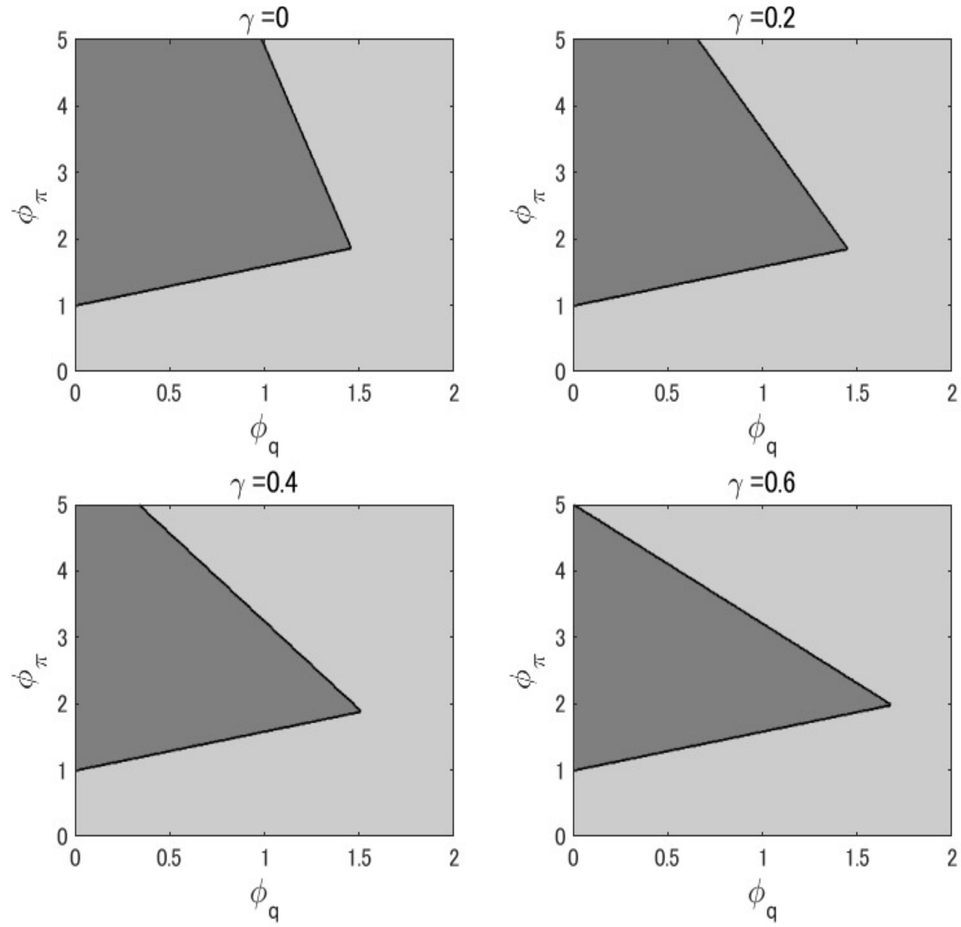
$$\tilde{q}_t = [(1-\beta)\Lambda + \gamma\sigma_\nu]E_t\tilde{y}_{t+1} + \beta E_t\tilde{q}_{t+1} - \gamma\sigma_\nu\tilde{y}_t - (i_t - E_t\pi_{t+1} - r_t^n), \tag{B.37}$$

where

$$\Lambda = 1 + \frac{\sigma_\nu\gamma(\varpi - 1) - (\sigma + \varphi)}{\mu - 1}.$$

When $\gamma = 0$, this equation corresponds to that derived by Carlstrom and Fuerst (2007).

Figure 1: Equilibrium determinacy when ϕ_π and ϕ_q change



Note: Dark and light shading indicate determinate and indeterminate regions, respectively.