How Information Design Shapes Optimal Selling Mechanisms

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Abstract

A monopolistic seller jointly designs allocation rules and (new) information about a payoff relevant state to a buyer with private types. When the new information flips the ranking of willingness to pay across types, a *screening* menu of prices and threshold disclosures is optimal. Conversely, when its impact is marginal, *bunching* via a single posted price and threshold disclosure is (approximately) optimal. While information design expands the scope for random mechanisms to outperform their deterministic counterparts, its presence leads to an equivalence result regarding sequential versus. static screening.

Keywords: mechanism design, information design, sequential screening, random mechanisms, bunching.
JEL classification: D82, D86, L15.

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1 INTRODUCTION

The evolution of informational technology has significantly broadened sellers' ways of selling their products. They are able to design not only *allocation rules* which specify how to allocate products and charge payments to buyers, but also *information policies* which control how much buyers learn about the products, thereby refining their willingness to pay. For instance, they may offer a posted price, associated with full information, to everyone. Alternatively, they could propose a rich menu of allocation rules and information policies.

As an example, many software such as McAfee and various (mobile) apps like Spotify provide users with a *single* free trial version, followed by a *single* subscription fee schedule. The trial version is, therefore, *merely* a learning opportunity for potential buyers to make well-informed purchasing decisions. An opposite example is travel agency platforms such as Priceline and Hotwire which practice so-called "opaque pricing" by which, buyers either book hotels with detailed information at standard prices or opt for limited details at discounted prices. Thus, these travel agencies *screen* their buyers via a menu of prices and information policies.

Price and information discrimination is also in the form of pre-order offers for buyers of not-yet-released products, as exemplified by Google's recent pre-order bonus for the Pixel 8. By contrast, well-known products are typically sold via a *single* posted price, coupled with a *single* timeframe for free return to all buyers.

What leads to these diverse selling strategies? In particular, when is a single posted price and disclosure policy optimal and conversely, when is it necessary to provide a screening menu of prices and information? In addition, is there any benefit from offering random mechanisms? Given that classical mechanism design results (Myerson (1981)) predicts that a posted price is optimal when the informational environment is *fixed*, answering these questions explains how information design shapes optimal selling mechanisms. Regarding the timing, can the seller's revenue be improved by contracting with the buyer at the "*interim*" stage where he knows his type but before the seller's information disclosure? Or equivalently, should she allow the buyer to walk away at the "*posterior*" stage where he observes both his type and the information provided? Answering this question helps understand the impact of consumer protection regulations that grant the consumer a withdrawal right such as the European directive 2011/83/EU.[1] Finally, if the buyer privately observes the information disclosed by the seller, can the buyer enjoy any rent induced from such an *endogenously private* information?

This paper aims to answer these questions. The model, as formally described in Section 2,

[1] For a detailed discussion on such policies, see Krähmer and Strausz (2015*b*).

features a seller (she) who sells an object to a buyer (he) with a privately known initial valuation (*initial type*). The seller controls how much the buyer learns about an *additional component* in his valuation. For example, this additional component represents what the buyer learns via product trials. The seller designs a menu of *information policies* for different types of the buyer, and *allocation rules* for different types and signals. Therefore, she solves a joint mechanism and information design problem in which information plays a dual role. First, it allows the seller to screen the buyer's type through discriminatory disclosure policies. Second, disclosed information serves as input for designing allocation rules. We focus on the case where the buyer privately observes the new information (private signals) and investigate the case with public signals as a benchmark.

1.1 Summary of results.

First, we establish a revenue-equivalence result regarding sequential vs. static screening. Specifically, we show that for any *feasible* and *deterministic* mechanism, there exists a mechanism that generates the same revenue for the seller and non-negative payoff for the buyer at any type and signal realization. As a consequence, there is no revenue loss if contracting at the posterior stage when the buyer knows both his type and signal. This result counters the well-established idea in sequential screening suggesting that the seller's revenue is strictly higher if contracting with the buyer before, rather than after, he learns additional information.[2] The basic intuition is that the seller's ability to flexibly design information can crowd out the advantages of sequential over static screening. A practical implication is that afore-mentioned consumer protections do not necessarily harm the seller, rationalizing the prevalence of free information in many markets.

Second, we investigate the (ir)relevance of signal privacy. In the benchmark problem with public signals, only *expected* allocations and payments (over signals) matter. Hence, this benchmark admits multiple solutions, including $\mathbf{M}^\star$, a *screening* menu of threshold disclosures $\pi^\star$ and prices paid conditional on trade.[3] We provide a simple way to verify the (ir)relevance of signal privacy, which is to check if, under $\mathbf{M}^\star$, the highest type pays the lowest price. If this is true, privacy of signals is irrelevant and $\mathbf{M}^\star$ solves the seller's original problem. We find that this is not always the case and consequently, not observing signals generally hurts the seller. Moreover, *per-signal* allocations and payments matter, which significantly complicates the characterization of optimal mechanisms In particular, it is not *a priori* clear how many signals are needed and which incentive compatibility (IC) constraints are relevant. The seller must also handle double deviations when the buyer lies about both his type and observed signal. Lever-

[2] See Courty and Li (2000) and Krähmer and Strausz (2015*b*).

[3] See Definition 4 for a formal description of $\mathbf{M}^\star$.

aging techniques for mechanisms with non-convex type spaces, we make it *always* possible for the buyer to "correct his lie," facilitating the characterization of optimal double deviations and thereby, optimal mechanisms.

Our main result characterizes optimal mechanisms, starting with binary types. The seller faces a trade-off between maximizing virtual surplus and minimizing the *posterior* rent. A threshold disclosure rule, under which signal realization is either "*good news*" if the state is above some cutoff or "*bad news*" otherwise, is optimal in both targets.[4] Under the optimal mechanism, the seller either *screens* the buyer's types (via a menu of threshold disclosures and posted prices) or *bunches* them (via a single posted price and threshold disclosure), depending on whether the threshold disclosure $\pi^\star$ induces a *threshold flip* of type order: the high type's value after "*bad news*" is *lower* than the low type's after "*good news*." Specifically, screening is optimal when this flip of type order occurs, and bunching otherwise.

To grasp the intuition, note that such a flip of type order occurs when the variation of valuations is mainly driven by the unknown component, leaving some room for the threshold disclosure $\pi^\star$ to reverse the ranking of valuation. Information (about the unknown component) matters, serving as a screening tool. Conversely, if the buyer's type is the main driver, which prevents $\pi^\star$ from flipping the type order, information is not crucial and screening disappears. The optimal mechanism echoes its counterpart in standard mechanism design where the buyer's valuation is his type: a posted price (but associated with threshold disclosure) is optimal.

The significance of this bunching vs. screening result is two-fold. First, it implies that in the above-mentioned scenarios, eliciting signals and random mechanisms are worthless. Second, it rationalizes observed mechanisms in practice. For coming-soon items, the unknown component's impact on the variation of valuations is large and a screening menu is employed. By contrast, its impact is marginal for well-known products where bunching comes into play. The significance of the unknown component also varies across different industries. In the realm of hotels, it matters much more than in software or mobile apps, leading to screening for the former and bunching for the latter.

Having characterized the optimal mechanism for the binary-type setting, we consider larger type spaces. With more than two types, there are also cases where an information policy reverses the ranking of valuations within a group of types but fails to do so for another. Consequently, not only information but also trading probabilities are needed to screen the buyer, leading to a *random* solution. However, the two scenarios of bunching/screening extend to the case with finitely many types, under stronger notions of flip (no flip) of type order. Specifically,

[4] See Definition 1 for our formal definition of a threshold disclosure.

a screening menu is optimal under a *partition* flip by $\pi^\star$ of type order - which generalizes the threshold flip of type order by $\pi^\star$, taking into account medium types and their associated cut-off states. Instead, bunching via a fixed price and threshold disclosure maximizes the seller's revenue when there is *uniformly* no threshold flip of type order under which, the type order is to be preserved between *any* pair of types and after *any* threshold disclosure. This strong requirement of type order preservation helps deal with the challenge of determining the lowest type being served in a rich type space.

As binding (IC) constraints can involve local, global, and upward ones, characterizing optimal random mechanisms becomes difficult. We thus focus on shedding light on how random mechanisms ourperform their deterministic counterparts.[5] We first establish the "*no randomization at the top*" result, extending the well-known "no distortion at the top" to a setting with information design: the highest type receives an efficient (and hence, deterministic) allocation. In turn, this implies an optimal contract for this type, featuring a posted price and no disclosure. While randomization is not needed for the highest type, it can be helpful for the lower types, leading to a better balance of the efficiency vs. rent trade-off.[6] We analyze, by examples, how random mechanisms facilitate screening distant types as well as screening signals.

Finally, we consider a setting with a continuum of types. In this case, the optimality of a screening menu of posted prices and threshold disclosures under a partition flip of type order extends readily. Particularly, in a "continuous" model when valuation shifts smoothly across types and states, this notion corresponds to the ranking of valuations at the zero-virtual-value states by types being reversed. On the other hand, the fact that there are always types whose valuations are close to others' makes it impossible to flip the ranking of willingness to pay across *all* types. We show that when the type order is *almost* preserved, bunching via a posted price is *approximately* optimal.

1.2 Related literature

We contribute to the literature on joint mechanism and information design, comprising two main strands. The first, more related, strand endows the buyer with a private type, initiated by Eső and Szentes (2007) who focus on full disclosure. Most other papers focus on *posted-price*

[5] In the Online Appendix, we solve for the optimal random mechanism in several examples.

[6] While it is natural to expect the two-dimensionality feature of the buyer's valuation to lead to random mechanisms, the seller has another tool for randomization: the distribution of signals, which potentially makes random mechanisms redundant. However, signal misreporting off-path shuts off this additional instrument. Thus, random mechanisms arise to deter double deviations, minimizing the *posterior* rent.

mechanisms,[7] which in turn, makes it without loss of generality to focus on *binary-signal* information structures (Li and Shi (2017), Guo et al. (2022), Wei and Green (2023), Smolin (2023)).[8] Our findings imply that these restrictions are not innocuous in general.

Our model builds on Eső and Szentes (2007) who focus on full disclosure and an environment with (i) the above-mentioned "continuous" model and (ii) certain assumptions on the valuation function. Under such an environment, they show that the upper bound of revenue with public signals can be achieved via full disclosure, associated with a screening menu of prices (for the good) and information fees. However, their optimal mechanism is not incentive compatible and moreover, privacy of signals generally matters outside their environment.[9] Not only do we allow for general information structures, we also characterize a *joint* design of information and allocation rules in a more general environment of type space and valuation functions. This allows us to uncover how information design reshapes the optimal selling mechanism which features not just screening, but also bunching and a random mechanism. At the same time, we strengthen Eső and Szentes (2007)'s finding by showing that the irrelevance of signals extends to other (but not all) environments, with appropriate information design.

Bergemann and Wambach (2015) and Wei and Green (2023) revisit Eső and Szentes (2007)'s continuous model, showing that the latter's *optimal* allocation can be implemented under stronger participation constraints. We show that with deterministic allocations (including Eső and Szentes (2007)'s), this is true for any *feasible* allocations, not just optimal. In turn, this provides an alternative proof for Wei and Green (2023).[10]

In the second, less related, strand of this literature, the buyer's valuation is the unknown component itself. See, for example, Lewis and Sappington (1994), Bergemann and Pesendorfer (2007), Bergemann et al. (2022). Without the buyer's private types, information cannot serve as a screening tool. Moreover, the buyer's private information (about his valuation) arrives only once, making the seller's problem static.[11]

We also contribute to the literature on dynamic mechanism design in which handling off-path

[7]In posted-price mechanisms, each type receives a posted price for the good and in some cases, a posted fee for information.

[8]Exceptions include Zhu (2023) and Krähmer (2020) who establish full surplus extraction results when the seller can correlate information disclosed to multiple buyers, and when randomizing over information structures is allowed and the buyer's type correlates with the unknown component, respectively.

[9]See Krähmer and Strausz (2015*a*) for a detailed discussion

[10]Wei and Green (2023) also shows that information disclosure triggers reverse price discrimination. We show that this can also be derived from the properties of Eső and Szentes (2007)'s optimal mechanism.

[11]If the buyer in our model has no private type, the seller fully extracts the surplus by offering no disclosure and a posted price for the good, which is equal to the expected valuation.

misreporting is a notable issue. Eső and Szentes (2007) explicitly characterize an agent's optimal double deviation, which is to "correct the lie". However, such a lie correction is feasible only if the agent's payoff shares a *common support* across types, which is rather restrictive. We show that by leveraging mechanism design techniques for a non-convex type space, lie correction is feasible even with non-common supports. Moreover, the existing literature (for instance, Battaglini (2005), Eső and Szentes (2007), Pavan et al. (2014)) extensively relies on the first-order approach considering only local incentive compatibility constraints.[12] Instead, we characterize different scenarios of binding constraints, showing that global deviations (associated with double deviation off-path) lead to bunching and random solutions.[13]

Finally, we contribute to the recent literature on Bayesian persuasion following Kamenica and Gentzkow (2011), where a sender designs *only* information disclosure to affect a receiver's action. When the latter has a private type, Kolotilin et al. (2017) show that with binary actions and linear valuation functions, non-discriminatory disclosure is optimal. In our *joint* design problem, the buyer's action space (which is the menu of allocations and payments) is endogenous and can consist of more than two options. We show that the optimality of non-discriminatory disclosure, while not being true in general, holds in some environments even if the seller also designs allocation rules and the valuation function is non-linear.

2 MODEL

2.1 Environment

The principal, a seller (she) sells an object to an agent, the buyer (he). The buyer's valuation for the object, $v(\theta, x) \in \mathbb{R}_+$, depends on two components: (i) the buyer's type $\theta \in \Theta \subset \mathbb{R}$ and (ii) an unknown state $x \in X \subset \mathbb{R}$. There are a finite number of possible types and states, *i.e.*, $|\Theta| < \infty$ and $|X| < \infty$.[14] Random variables θ and x are independent. Let $f(\theta)$ be the probability of each type θ and $\mu(x)$ of each state x. Without loss of generality, assume $f(\theta) > 0$ and $\mu(x) > 0$ for all θ and x.

The realization of $\theta \in \Theta$ is privately known by the buyer. Neither the seller nor the buyer knows the state $x \in X$. The seller commits to a policy of information disclosure about the state, formally defined in Section 2.2.

[12] The validity of this approach usually requires certain regularity conditions, which are not easy to satisfy, see Battaglini and Lamba (2019).

[13] Even with full disclosure, which makes our problem become a standard dynamic screening problem, random mechanisms can outperform their deterministic counterparts. See Example 6.

[14] We study the infinite type and state spaces in Section 7.

To define payoffs, let $q \in [0,1]$ be the trading probability and $p \in \mathbb{R}$ the expected transfer from the buyer to the seller. The seller's *ex post* payoff is then p and the buyer's is $v(\theta,x)q - p$.

For expositional clarity, we use the following notations $\overline{\theta} \equiv \max\Theta$, $\underline{\theta} \equiv \min\Theta$, $\theta^+ \equiv \min_{\theta'\in\Theta}\{\theta' \mid \theta' > \theta\}$ for $\theta < \overline{\theta}$, $\theta^- \equiv \max_{\theta'\in\Theta}\{\theta' \mid \theta' < \theta\}$ for $\theta > \underline{\theta}$, and $\overline{\theta}^+ = \overline{\theta}$, $\underline{\theta}^- = \underline{\theta}$. We define $\overline{x}$ and $\underline{x}$ similarly. Let

$$\phi(\theta,x) \equiv v(\theta,x) - [v(\theta^+,x) - v(\theta,x)]\frac{\sum_{\theta'>\theta} f(\theta)}{f(\theta)}$$

denote the buyer's virtual value. Throughout, assume that both the valuation and virtual valuation increase in the buyer's type and the state.

Assumption 1 (Monotone value)**.** *$v(\theta,x)$ increases in θ and x.*

Assumption 2 (Monotone virtual value)**.** *$\phi(\theta,x)$ increases in θ and x.*

2.2 Selling mechanism

The seller designs, and *ex ante* commits to a *grand* mechanism or a menu of (i) information policies for different types of the buyer and (ii) allocation rules for different types and information received by the buyer.

Information policies: We model information policies as information structures (experiments) $\Pi \equiv (S,\pi)$, which consists of a countable set of signals $S \subset \mathbb{R}$,[15] and a mapping π, which associates to each state θ a distribution over signals $\pi(\cdot \mid x) \in \Delta(S)$. Given a mapping π and a signal realization $s \in S$, the corresponding posterior belief $\Psi(\cdot|s) \in \Delta(X)$ is obtained by Bayes' rule whenever possible, and is given by

$$\mu_{s,\pi}(x) = \frac{\mu(x)\pi(s \mid x)}{\sum_{x'\in x} g(x')\,\pi(s \mid x')}$$

An example of information structures is the threshold rule, defined as follows.

Definition 1 (Threshold diclosure)**.** *If the information policy follows a threshold rule, each signal realization is classified as either "good news" or "bad news". Moreover,*

$$\pi(\text{"good news"},x) = \begin{cases} 1 & \text{if} \quad x > \hat{x}, \\ \lambda & \text{if} \quad x < \hat{x}, \\ \lambda & \text{if} \quad x = \hat{x}, \end{cases} \quad \text{for some } \hat{x} \in X \text{ and } \lambda \in [0,1].$$

[15] Assuming S is a countable set of $\mathbb{R}$ is without loss.

Thus, a threshold disclosure is represented by a pair $(\hat{x}, \lambda)$ where $\hat{x}$ is the cut-off state and λ the probability with which "*good news*" is sent at the cut-off state. It informs the buyer whether the state is (weakly) higher or lower than $\hat{x}$. To simplify notations, throughout the paper, we use "s^g" to represent "*good news*" and "s^b" for "*bad news*".

A menu of experiments is a set $\{\pi_\theta\}_{\theta\in\Theta}$. The paper focuses on the case in which the buyer privately observes the signal. The benchmark case with public signals is examined in Section 3.2.

Without loss of generality, assume that signals are ordered such that upon observing a higher signal, the buyer's posterior valuation is higher, as follows.

Assumption 3 (Ranking of signals)**.**

$$s > s' \Leftrightarrow \sum_x v(\theta, x)\mu_{s,\pi_\theta}(x) \geq \sum_x v(\theta, x)\mu_{s',\pi_\theta}(x)$$

Allocation rules: An allocation rule specifies the trading probability, q, and the expected transfer from the buyer to the seller, p. Given the information structure, by the revelation principle (see, for example, Myerson (1986)), we focus on direct allocation rules $\{q(\theta, s), p(\theta, s)\}_{\theta,s}$.

Thus, a selling mechanism is a tuple $\mathbf{M} \equiv \left\{\pi_\theta, \left(q(\theta, s), p(\theta, s)\right)\right\}_{\theta,s}$. The formal definitions of a deterministic mechanism and its random counterpart are as follows.

Definition 2. *An mechanism* $\mathbf{M}$ *is deterministic if under* $\mathbf{M}$*,* $q(\theta, s) \in \{0, 1\}$ *for all* $\theta \in \Theta$ *and* $s \in S$*.* $\mathbf{M}$ *is random otherwise.*

Timing: The timing of interactions is as follows:

1. The seller offers a selling mechanism $\mathbf{M}$.
2. The buyer learns his type θ and decides to accept or reject the offer. In case of acceptance, he reports a type $\hat{\theta}$.
3. The buyer *privately* observes a signal s and reports a signal $\hat{s}$.
4. The allocation $(q(\hat{\theta}, \hat{s}), p(\hat{\theta}, \hat{s}))$ is implemented.

According to this timing, the buyer's participation is decided at the *interim* state, as commonly assumed in the mechanism design literature. See our discussion on the timing structure in Section 3.1.

2.3 Seller's problem

An optimal mechanism refers to a revenue-maximizing mechanism. By the revelation principle, it is without loss of generality to focus on direct mechanisms such that the buyer finds it optimal to (i) participate in the mechanism, (ii) truthfully report his type, and (iii) truthfully report his signal conditional on being truthful about his type. Let

$$u(\theta,\theta',s,s') \equiv \sum_x [v(\theta,x)q(\theta',s') - p(\theta',s')]\Psi_\theta(x|s)$$

denote the *ex post* payoff for type-θ buyer, who reports θ', observes s, and reports s'. Note that if the buyer lies about his type, he may want to lie again about the signal. In other words, double deviations from truth-telling may be attractive. Let

$$s^\star(\theta,\theta',s) \in \operatorname*{argmax}_{s'} u(\theta,\theta',s,s')$$

be the optimal signal reporting of type-θ buyer who reports θ' and observes signal s.[16] The *ex ante* payoff for type-θ buyer, who reports θ' and then $s^\star(\theta,\theta',s)$, is then given by

$$U(\theta,\theta') \equiv \sum_x \sum_s u(\theta,\theta',s,s^\star(\theta,\theta',s))\pi(s|x).$$

With abuse of notation, let $u(\theta,s) \equiv u(\theta,\theta,s,s)$, $u(\theta,s,s') \equiv u(\theta,\theta,s,s')$, and $U(\theta) \equiv U(\theta,\theta)$. For the buyer to truthfully report his signal on the equilibrium path (conditional on reporting his type truthfully), it must be that for all θ and s,

$$u(\theta,s) \geq u(\theta,s,s'). \tag{IC-signal}$$

For the buyer to truthfully report his type, it must be that for all θ and θ',

$$U(\theta) \geq U(\theta,\theta'). \tag{IC-type}$$

Finally, the buyer participates in the mechanism if and only if

$$U(\theta) \geq 0. \tag{IR}$$

Definition 3. *A mechanism is feasible if it satisfies all constraints (IR), (IC-type), and (IC-signal),*

Formally, the seller's maximization problem is given by

$$\sup_{\{\pi_\theta,q(\theta,s),p(\theta,s)\}_{s,\theta}} \sum_\theta \sum_x \sum_s p(\theta,s)\pi(s|x)\mu(x)f(\theta)$$

$$s.t. \quad \text{(IR), (IC-type), (IC-signal).}$$

[16] In case the buyer is indifferent between signals off the equilibrium path, fix arbitrarily one of the seller-preferred signals.

3 PRELIMINARY RESULTS

3.1 Sequential vs. static screening

This section establishes an irrelevance result regarding the timing structure of interactions. We show that contracting at the *posterior* stage (after the buyer observes both his type and signal) does not necessarily hurt the seller. Specifically, within the class of deterministic mechanisms, there is no revenue loss if the buyer can walk away after information disclosure.

Proposition 1. *For any deterministic and feasible mechanism, there exists a mechanism which generates the same revenue for the seller and a non-negative ex post pay-off for the buyer.*

The proof is in Appendix A.1. It proceeds by showing that for an arbitrary deterministic and feasible mechanism $\mathbf{M}^d \equiv \{q(\theta, s), p(\theta, s), \pi_\theta\}$, it must be that

$$\big(q(\theta, s), p(\theta, s)\big) = \begin{cases} \big(1, \overline{p}(\theta)\big) & \textit{if } s \geq \hat{s}(\theta), \\ \big(0, \underline{p}(\theta)\big) & \textit{otherwise.} \end{cases}$$

Fix $\theta \in \Theta$, if $\underline{p}(\theta) < 0$. If there exists s such that $\pi(\theta, s) < 0$, then type θ who reveals his type and observes s (strictly) prefers to misreport a signal $s' < \hat{s}(\theta)$ to receive a negative transfer without buying the good. This contradicts with $\mathbf{M}^d$ being feasible. Hence, $\pi(\theta, s) \geq 0$ for all s in this case.

Next, consider $\underline{p}(\theta) \geq 0$. Revise type θ's contract such that type θ now receives (i) a binary-signal experiment $\tilde{\pi}_\theta$ which replaces all $s \geq \hat{s}_\theta$ with "*good news*" and all $s < \hat{s}_\theta$ with "*bad news*", and (ii) a posted price

$$\tilde{p}(\theta) \equiv \overline{p}(\theta) + \underline{p}(\theta) \frac{\sum_{s < \hat{s}_\theta} \pi_\theta(s)}{\sum_{s \geq \hat{s}_\theta} \pi_\theta(s)} > \overline{p}(\theta), \tag{1}$$

where the inequality uses $\underline{p}(\theta) \geq 0$. Call the revised mechanism $\tilde{\mathbf{M}} \equiv \{\tilde{\pi}_\theta, \tilde{p}(\theta)\}_\theta$. As θ pays only if he decides to buy the good, his payoff is non-negative *posterior*. Moreover, the seller's revenue remains unchanged under this revision. See Appendix A.1 for formal arguments.

Proposition 1 has two implications. First, if restricted to deterministic mechanisms, there is no revenue loss for the seller if contracting after knowing both dimensions of private information: his type and disclosed signal. Put differently, there is no benefit from sequentially screening the buyer, despite the sequential arrival of his private information. Consequently, unless the optimal mechanisms are random, the seller's problem is equivalent to a multi-dimensional screening problem. In section 8.2, we leverage this result to solve Wei and Green (2023)'s problem.

Second, by the "no rent at the bottom", the lowest type earns a zero payoff under optimal mechanisms. Therefore, if there exists any type, who does not buy the good and receives a strictly positive payment, the lowest type strictly prefers to mimic such a type (and not buy the good to enjoy the positive transfers) than truth-telling. This implies that only Case 2 in the proof of Proposition 1 is possible if $\mathbf{M}^d$ is optimal. In this case, as argued above, $\mathbf{M}^d$ do no better than "posted-price" mechanisms, which are *signal-independent.* Formally:

Corollary 1. *If restricted to deterministic mechanisms, it is optimal to offer a menu of posted prices and binary-signal experiments under which*

1. *each signal realization is either "good news" or "bad news", and*
2. *the buyer finds it optimal to buy the good if and only if he observes "good news."*

A menu of posted prices and binary-signal experiments, which satisfies the two conditions stated in Corollary 1, is called a *persuasive posted-price mechanism.* This corollary also helps us establish the optimality of random mechanisms which are *signal-dependent* in Section 6.3.

3.2 (Ir)relevance of signal privacy

This section studies an upper bound on the seller's revenue, attained when signals are publicly observed. We show that this upper bound is not always tight and hence, not observing signals generally hurts the seller. First, consider the benchmark problem with public signals. There,

$$U(\theta,\theta') = \sum_x \sum_s [v(\theta,x)q(\theta',s) - p(\theta',s)]\pi_{\theta'}(s|x)\mu(x)$$

depends only on *expected* payments and *expected* allocations over signals, defined as

$$\mathbb{Q}(\theta,x) \equiv \sum_x \sum_s q(\theta,s)\pi_\theta(s|x)\mu(x), \quad \mathbb{P}(\theta) \equiv \sum_x \sum_s p(\theta,s)\pi_\theta(s|x)\mu(x).$$

As a result, the seller's problem reduces to

$$\begin{aligned} (\overline{\mathscr{P}}) \quad & \sup_{\mathbb{Q},\mathbb{P}} \sum_\theta \mathbb{P}(\theta) f(\theta) \\ s.t. \quad & \sum_x v(\theta,x)\mathbb{Q}(\theta,x)\mu(x) - \mathbb{P}(\theta) \geq \sum_x v(\theta,x)\mathbb{Q}(\theta',x)\mu(x) - \mathbb{P}(\theta'), \qquad (\overline{IC}\text{-type}) \\ & \sum_x v(\theta,x)\mathbb{Q}(\theta,x)\mu(x) - \mathbb{P}(\theta) \geq 0. \qquad (\overline{IR}) \end{aligned}$$

Let $V(\mathscr{P})$ denote the value of the seller's original problem, and $V(\overline{\mathscr{P}})$ the seller's benchmark problem with public signals. Obviously, $V(\mathscr{P}) \leq V(\overline{\mathscr{P}})$. Therefore, if there exists a mechanism under which, the "equality" occurs, this mechanism solves the seller's original problem and privacy of signals does not matter. As to be shown, this is not always true.

Note that under Assumption 1 and 2, only local IC constraints bind under $(\overline{\mathscr{P}})$. By standard arguments (omitted), this problem reduces to point-wise maximization w.r.t $\mathbb{Q}$ only:

$$\sup_{\mathbb{Q}} \sum_{\theta} \sum_{x} \phi(\theta,x)\mathbb{Q}(\theta,x)\mu(x)f(\theta). \tag{$\star$}$$

A solution to ($\star$) exists and is generically unique:[17] $\mathbb{Q}(\theta,x) = \mathbb{1}_{\phi(\theta,x)\geq 0}$. *Expected* payment (over signals) is pinned down by $(\overline{IC}_{\theta^+\to\theta})$ and $(\overline{IR}_{\theta_{min}})$. Let $\theta_{min} \equiv \min\{\theta \mid \phi(\theta,\overline{x}) \geq 0\}$ denote the lowest type being served. For any $\theta \geq \theta_{min}$, let

$$x_\theta \equiv \min\{x \mid \phi(\theta,x) \geq 0\} \tag{2}$$

denote the lowest state at which type θ's virtual value is non-negative. Note that $x^\star_\theta$ decreases in θ by Assumption 2.

Lemma 1 (Benchmark problem)**.** *With public signals, the optimal allocation is generically unique, given by*

$$\mathbb{Q}(\theta,x) = \mathbb{1}_{\phi(\theta,x)\geq 0}, \tag{3}$$

and expected transfer is as follows

$$\mathbb{P}(\theta_{min}) = \sum_{x\geq x_{\theta_{min}}} v(\theta_{min},x)\mu(x) \tag{4}$$

$$\mathbb{P}(\theta^+) = \mathbb{P}(\theta) + \sum_{x_{\theta^+}\leq x < x_\theta} v(\theta^+,x)\mu(x) \quad \forall \theta \geq \theta_{min}. \tag{5}$$

The seller retains a certain level of freedom in designing disclosure and *per-signal* allocation rules as long as (i) upon observing any signal, one knows whether the state is above or below the cut-off x_θ and (ii) expected terms are given by equations in Lemma 1. This leads to a multiplicity of solutions to $(\overline{\mathscr{P}})$, including the following menu of threshold disclosures and prices (paid conditional on trade), called $\mathbf{M}^\star$, under which each type of the buyer knows whether the state is above or below some cut-off and pays only if he buys the good. Its formal definition is as follows.

Definition 4. *Under* $\boldsymbol{M}^\star \equiv \{p^\star(\theta,s), q^\star(\theta,s), \pi^\star_\theta\}_{\theta\in\Theta, s\in\{s^g,s^b\}}$ *is a menu of threshold disclosures and prices, in which*

1. $\pi^\star_\theta(s^g|x) = \mathbb{1}_{x\geq x_\theta}$, *where* x_θ *is given by equation (2).*
2. $\left(q^\star,(\theta,s),p^\star(\theta)\right) = \begin{cases} (1, \dfrac{\mathbb{P}(\theta)}{\sum_{x\geq x_\theta}\mu(x)}) & if \quad s = s^g, \\ (0,0) & if \quad s = s^b, \end{cases}$ *where* $\mathbb{P}(\theta)$ *is given by equations (4) and (5).*

[17] When $\phi(\theta,x) = 0$, any $\mathbb{Q}(\theta,x) \in [0,1]$ is optimal.

Assuming $\phi(\theta, x_\theta) > 0$,[18] Proposition 2(a) below shows that if the upperbound of revenue $V(\overline{\mathscr{P}})$ is obtained via some mechanism, it is via $\mathbf{M}^\star$. The basic intuition is that relative to other solutions to the benchmark problem, $\mathbf{M}^\star$ provides less information (just enough to know the sign of virtual values) and a higher price for the good (payments are paid only when "*good news*" is realized). Hence, if there exists a solution that induces truth-telling with private signals, so does $\mathbf{M}^\star$. In addition, Proposition 2(b) shows that this is the case if and only if the highest type pays the lowest price under $\mathbf{M}^\star$. This is because the highest type benefits the most from deviations (from truth-telling). Therefore, whether $\mathbf{M}^\star$ induces truth-telling with private signals depends on whether the highest type pays the lowest price (and thereby, prefers to be truthful).

To formally state Proposition 2, let $R_{\mathbf{M}}$ represent the revenue level obtained with private signals from an arbitrary mechanism $\mathbf{M}$.

Proposition 2.

a) Suppose $\phi(\theta, x_\theta) > 0\ \forall\theta$. *If there exists* $\boldsymbol{M}$ *such that* $R_{\boldsymbol{M}} = V(\overline{\mathscr{P}})$, *then* $R_{\boldsymbol{M}^\star} = V(\overline{\mathscr{P}})$.

b) $R_{\boldsymbol{M}^\star} = V(\overline{\mathscr{P}})$ *if and only if* $p^\star(\overline{\theta}, s^g) = \min_\theta\{p^\star(\theta, s^g)\}$.

It seems counter-intuitive that the highest type pays the lowest price (conditional on buying the good). However, it is worth noting that information disclosure can flip the ranking of (*posterior*) willingness to pay across types, leading to non-monotone price discrimination.[19] As will be shown formally in later sections, this occurs in some, but not all environments. Here, we provide a simple example for illustration.

Example 1. $\Theta = \{l, h\}$ *and* $X = \{B, G\}$. *Types and states are equally likely. With* $\lambda > 0$, *values and virtual values are as follows.*

$v(\theta,x), \phi(\theta,x)$	$x = b$	$x = g$
$\theta = h$	$\lambda, 2$	$5, 5$
$\theta = l$	$0, -\lambda$	$4, 3$

Suppose signals are public. Given that h's virtual value is always positive while l 's is only at state g, the optimal allocation is unique: $\mathbb{Q}(h, x) = 1 \forall x$, $\mathbb{Q}(l, x) = \mathbb{1}_{x=g}$. Hence, by Proposition 2, to investigate the irrelevance of signal privacy, it suffices to verify if under $\mathbf{M}^\star$, h pays a lower price at trade. By Definition 4, under $\mathbf{M}^\star$, h always receives "*good news*" and l only at state g.

[18] This condition ensures that $\mathbb{Q}(\theta, x) = \mathbb{1}_{x \geq x_\theta}$ is the unique solution to $(\overline{\mathscr{P}})$).

[19] That information disclosure can lead to non-monotone price discrimination has been observed in Bang and Kim (2013) and Wei and Green (2023) where prices decrease in types. Throughout our paper, several examples are presented where under $\mathbf{M}^\star$, prices can be decreasing, increasing and even concave in types (see Example 9).

Moreover, the buyer pays only if "*good news*" is realized with (i) $p(l, s^g) = 4$, such that l 's earns a zero payoff and (ii) $p(h, s^g) = \frac{4+\lambda}{2}$ so that h is indifferent between truth-telling and mimic l . Therefore, $p(l, s^g) \le p(h, s^g) \Leftrightarrow 4 \le \frac{4+\lambda}{2} \Leftrightarrow \lambda \ge 2$.

4 A RESTATEMENT OF THE SELLER'S PROBLEM

Without loss of generality, assume that each signal induces a distinguish (on-path) posterior valuation. Therefore, each signal s observed by type-θ buyer corresponds to his on-path posterior value after observing such a signal, given by

$$\omega^{\pi_\theta}(\theta, s) \equiv \sum_x v(\theta, x)\mu_{s,\pi_\theta}(x)$$

Moreover, that the buyer reveals the realized signal is equivalent to him reporting his posterior valuation. For any type θ, let

$$\Omega_\theta \equiv \{\omega \mid \omega = \omega^{\pi_\theta}(\theta, s) \text{ for some } s \in S\}$$

be the set of all possible on-path posterior values for type θ. Then, requiring signal truth-telling on-path is equivalent to ensuring truth-telling about on-path posterior values, or

$$\omega q(\theta,\omega) - p(\theta,\omega) \ge \omega q(\theta,\omega') - p(\theta,\omega') \quad \forall \theta, \forall \omega, \omega' \in \Omega_\theta$$

As mentioned, the buyer may want to coordinate lies about the realized type and signal. Given that the signal space is endogeneous, this significantly complicates the characterization of truth-telling conditions. To facilitate characterizing the buyer's optimal double deviation, we extend the allocation rule to be defined on the set of all possible on path and off path posterior valuations, denoted by

$$\Omega \equiv [v(\underline{\theta}, \underline{x}), v(\overline{\theta}, \overline{x})].$$

Moreover, it is without loss of generality to require truthful signal reporting on this set Ω, rather than in only $\{\Omega_\theta\}_\theta$,[20] *i.e.*,

$$\omega q(\theta,\omega) - p(\theta,\omega) \ge \omega q(\theta,\omega') - p(\theta,\omega') \quad \forall \theta, \forall \omega, \omega' \in \Omega \tag{IC-value}$$

The characterization of (IC-value) is standard.

Lemma 2 (Myerson, 1981)**.** *An allocation rule* $(q, p) : \Theta \times \Omega \to [0,1] \times \mathbb{R}$ *satisfies (IC-value) if and only if*

[20]See, for example, Skreta (2006), for mechanism design with non-convex type spaces.

1. $\omega q(\theta,\omega) - p(\theta,\omega) = \hat{\omega} q(\theta,\hat{\omega}) - p(\theta,\hat{\omega}) + \int_{\hat{\omega}}^{\omega} q(\theta,z)dz$,
2. $q(\theta,\omega)$ *increases in* ω.

It then follows from Lemma 2 that the buyer, after having lied about his type, reveals his true (off-path) posterior valuation.

Lemma 3 (Optimal double deviations)**.** *Under any allocation rule* $(q,p) : \Theta \times \Omega \to [0,1] \times \mathbb{R}$ *that satisfies (IC-value), it is optimal for type* θ *who mimics* θ' *and observe signal* s *to report his off-path posterior valuation, given by*

$$\omega^{\pi_\theta}(\theta', s) \equiv \sum_x v(\theta, x)\mu_{s,\pi_{\theta'}}(x)$$

The proof (omitted) is similar to what is called "correcting the lie" in the dynamic mechanism design literature. Often, this lie correction is made feasible by assuming that the agent's (new) private information shares a common support across types.[21] This is not applicable in our model as the buyer's new private information, which is his posterior valuation, is endogenous. By extending the allocation rule to be defined in the extended signal space Ω, we make it possible for the buyer to "correct his lie."[22]

Consider $\theta, \theta' \in \Theta$ with $\theta > \theta'$. Then,

$$\begin{aligned}
U(\theta,\theta') &\equiv \sum_x \sum_s [\omega^{\pi_{\theta'}}(\theta, s) q(\theta', \omega^{\pi_{\theta'}}(\theta, s)) - p(\theta', \omega^{\pi_{\theta'}}(\theta, s))]\pi_{\theta'}(s|x)\mu(x) \\
&= \sum_x \sum_s \left[[\omega^{\pi_{\theta'}}(\theta', s) q(\theta', \omega^{\pi_{\theta'}}(\theta', s)) - p(\theta', \omega^{\pi_{\theta'}}(\theta', s))] + \sum_s \int_{\omega^{\pi_{\theta'}}(\theta',s)}^{\omega^{\pi_{\theta'}}(\theta,s)} q(\theta', z)dz \right] \pi_{\theta'}(s|x)\mu(x) \\
&= U(\theta') + \sum_x \sum_s \int_{\omega^{\pi_{\theta'}}(\theta',s)}^{\omega^{\pi_{\theta'}}(\theta,s)} q(\theta', z)dz \pi_{\theta'}(s|x)\mu(x).
\end{aligned}$$

Thus, θ does noe benefit from misreporting θ' if and only if

$$U(\theta) - U(\theta') \geq \sum_x \sum_s \int_{\omega^{\pi_{\theta'}}(\theta',s)}^{\omega^{\pi_{\theta'}}(\theta,s)} q(\theta', z)dz \pi_{\theta'}(s|x)\mu(x).$$

By similar arguments, θ' does not benefit from misreporting θ if and only if

$$U(\theta) - U(\theta') \leq \sum_x \sum_s \int_{\omega^{\pi_\theta}(\theta',s)}^{\omega^{\pi_\theta}(\theta,s)} q(\theta, z)dz \pi_\theta(s|x)\mu(x).$$

[21] See Eső and Szentes (2007) and Krähmer and Strausz (2015*b*) for example.

[22] This trick can also be helpful in other dynamic mechanism design problems where the agent(s)' private information does not share common support across types.

To sum up, the seller's problem can be expressed as follows.

$$(\mathscr{P}) \quad \max_{(\pi,q,U)} \quad \sum_{\theta} f(\theta)\Big[\sum_{x}\sum_{s} v(\theta,x)q(\theta,\omega^{\pi_\theta}(\theta,s))\pi_\theta(s|x)\mu(x) - U(\theta)\Big]$$

$$s.t: \quad \forall\theta, \quad U(\theta)-U(\theta') \geq \sum_{x}\sum_{s}\int_{\omega^{\pi_{\theta'}}(\theta',s)}^{\omega^{\pi_{\theta'}}(\theta,s)} q(\theta',z)dz\pi_{\theta'}(s|x)\mu(x) \quad \forall\theta'<\theta \qquad \text{(dwIC-type)}$$

$$U(\theta)-U(\theta') \leq \sum_{x}\sum_{s}\int_{\omega^{\pi_\theta}(\theta,s)}^{\omega^{\pi_\theta}(\theta',s)} q(\theta,z)dz\pi_\theta(s|x)\mu(x) \quad \forall\theta'>\theta \qquad \text{(uwIC-type)}$$

$$U(\theta)\geq 0 \qquad (IR)$$

$$q(\theta,\omega) \text{ increases in } \omega. \qquad (MON)$$

5 OPTIMAL MECHANISM FOR $|\Theta| = 2$

In this section, we characterize the optimal mechanism for binary types. We derive two findings. First, screening is optimal if and only if the ranking of willingness to pay is flipped under a certain threshold disclosure and bunching is optimal otherwise. Second, eliciting signals and random mechanisms are worthless. Formally, $\Theta = \{h, l\}$ and hence, the seller's problem reduces to $(\mathscr{P}_b)$, given by

$$(\mathscr{P}_b) \quad \max_{(\pi,q,U)} \quad \sum_{\theta} f(\theta)\Big[\sum_{x}\sum_{s} v(\theta,x)q(\theta,\omega^{\pi_\theta}(\theta,s))\pi_\theta(s|x)\mu(x) - U(\theta)\Big]$$

$$s.t: \quad U(h)-U(l) \geq \sum_{x}\sum_{s}\int_{\omega^{\pi_l}(l,s)}^{\omega^{\pi_l}(h,s)} q(l,z)dz\pi_l(s|x)\mu(x) \qquad (IC_{hl})$$

$$U(h)-U(l) \leq \sum_{x}\sum_{s}\int_{\omega^{\pi_h}(l,s)}^{\omega^{\pi_h}(h,s)} q(h,z)dz\pi_h(s|x)\mu(x) \qquad (IC_{lh})$$

$$U(h)\geq 0 \qquad (IR_h)$$

$$U(h)\geq 0 \qquad (IR_l)$$

$$q(\theta,\omega) \text{ increases in } \omega.$$

To state the main result of this section, we introduce the following notion of type order flip, which shapes the optimal mechanism. Recall that $\pi^\star$ is the threshold disclosure associated with $\mathbf{M}^\star$ formally defined in Definition 4, with $\pi_l^\star(s^g|x) = \mathbb{1}_{x\geq x_l}$.

Definition 5 (Threshold flip of type order by $\pi_l^\star$)**.**

If $\pi_l^\star$ induces the threshold flip of type order, $\mathbb{E}[v(h,x) \mid x<x_l] \leq \mathbb{E}[v(l,x) \mid x\geq x_l]$.

By Definition 5, $\pi_l^\star$ induces the threshold flip of type order when $\omega^{\pi_l^\star}(h,s^b) \leq \omega^{\pi_l^\star}(h,s^g)$. In words, this threshold disclosure overturns the ranking of willingness to pay with h's valuation after "*bad news*" being *lower* than l 's after "*good news*". Intuitively, this is the case when the unknown component x causes significant variations of valuations, creating room for $\pi_l^\star$ to flip the type order. By contrast, it does not happen in, for example, an extreme case in which valuation is constant with respect to this component (i.e., $g(\cdot)$ is a degenerate distribution with $\overline{x} = \underline{x}$), as in standard mechanism design problems.

We are now ready to state the main result of this section, assuming that type l 's virtual value is either strictly positive or negative, *i.e.*, $\phi(l, x_l) > 0$. Accordingly, the benchmark allocation is *unique*, given by $\mathbb{Q}(l,x) = \mathbb{1}_{x \geq x_l}$.

Theorem 1 (Binary types)**.** *Fix* $\Theta = \{h, l\}$. *There exists some* $\lambda \in [0,1]$ *and* $\hat{x}_l \in$, *such that in the unique optimal mechanism, the allocation is given by*

$$q(h,x) = 1 \quad \forall x, \quad q(l,x) = \begin{cases} 1 & \text{if} \quad x > \hat{x}_l, \\ 0 & \text{if} \quad x < \hat{x}_l, \\ \lambda & \text{if} \quad x = \hat{x}_l. \end{cases}$$

Moreover,

(a) If $\pi_l^\star$ *induces the threshold flip of type order,* $(\hat{x}_l, \lambda) = (x_l, 1)$. *A menu of posted prices and threshold disclosures is optimal.*

(b) If $\pi_l^\star$ *does not induce the threshold flip of type order,* $(\hat{x}_l, \lambda) \neq (x_l, 1)$. *A posted price, associated with a uniform threshold disclosure, is optimal.*

In short, Theorem 1 states that the optimal mechanism features *screening* whenever $\pi_l^\star$ leads to the threshold flip of type order and *bunching* otherwise. Intuitively, when the unknown component x dominates the buyer's private type θ in triggering the variation of the buyer's valuation (to induce the threshold flip of type order), the new information matters and helps screen the buyer. Conversely, when the ranking of willingness to pay mainly depends on the buyer's type, screening disappears. Then, the optimal mechanism closely resembles its counterpart in standard mechanism design where the state is known: a posted price (but associated with threshold disclosure) is optimal. The optimal mechanism in each case is explicitly characterized in the remainder of this section. To illustrate Theorem 1, consider the following examples.

Example 2 (Binary types and states)**.** $\Theta = \{l, h\}$ *and* $X = \{b, g\}$. *Types and states are equally likely. Assume that* $\phi(l, b) < 0 < \phi(l, g)$ *to make the problem non-trivial.*

In this simple binary-type, binary-state setting, there are two scenarios of optimal mechanisms. If $v(h,b) \geq v(l,g)$, then $\pi^\star$ does not induce the threshold flip of type order. By Theorem 1(a), a fixed price and threshold disclosure is optimal. On the other hand, if $v(h,b) < v(l,g)$, then

$\pi^{\star}$ leads to the threshold flip of type order. By Theorem 1(b), a menu of prices and threshold disclosures is optimal.

Example 3. $\Theta = \{l, h\}$ *and* X *is a finite subset of* $\mathbb{N}$*. Types and states are equally likely. Valuations are given by:* $v(\theta, x) = \theta + x$*.*

Let

$$\Delta_\theta \equiv v(h,x) - v(l,x) = h - l \quad \forall x,$$

$$\Delta_x \equiv v(\theta,\overline{x}) - v(\theta,\underline{x}) = \overline{x} - \underline{x} \quad \forall \theta.$$

Then, Δ_θ represents the variation of valuation due to the buyer's type, whereas Δ_x that due to the state x. For any state $\hat{x} \in \Omega$,

$$\mathbb{E}[v(h,x) \mid x < \hat{x}] - \mathbb{E}[v(l,x) \mid x \geq \hat{x}] = \left(h + \frac{\hat{x} - 1 + \underline{x}}{2}\right) - \left(l + \frac{\hat{x} + \overline{x}}{2}\right) = \Delta_\theta - \frac{\Delta_x + 1}{2},$$

Thus, the threshold flip of type order happens if and only if

$$\Delta_\theta \leq \frac{\Delta_x + 1}{2}, \tag{6}$$

which is the case when the impact of the buyer's type is relatively small, relative to that of the unknown component. By Theorem 1, when (6) holds, it is optimal to offer a menu of threshold disclosures and posted prices. Otherwise, a posted price, coupled with uniform threshold disclosure, maximizes the seller's revenue.[23]

Remark 1. *Theorem* 1 *and its proof extend readily to the case with a continuum of states. As an example, fix* $\Theta = \{l, h\}$ *and* $X = [0, 10]$*, and both* θ *and* x *are uniformly distributed. Then, for any state* $\hat{x} \in \Omega$, $\mathbb{E}[v(h,x) \mid x < \hat{x}] - \mathbb{E}[v(l,x) \mid x \geq \hat{x}] = \Delta_\theta - 5$*. Thus, a menu of prices and information is optimal if* $\Delta \geq 5$ *and a fixed price coupling with a threshold disclosure (for all types) is optimal if* $\Delta < 5$.

Theorem 1 has two important implications:

Corollary 2. *With* $|\Theta| = 2$*, privacy of signals does not matter when the threshold flip of type order happens under* $\pi_l^{\star}$*. It matters otherwise.*

Corollary 3. *With* $|\Theta| = 2$*, the seller does not strictly benefit from using random mechanisms, nor from eliciting signals.*

What leads to the (ir)relevance of signal privacy and the optimality of deterministic mechanisms, signal-independent allocations will be explained when we present the key steps of the proof of Theorem 1, to which we turn next.

[23] In particular, when Δ_θ is too high, the seller does not benefit from information disclosure. In this case, the optimal threshold for type l is the highest state ($\hat{x}_l = \overline{x}$), which means no disclosure is provided.

5.1 *Proof of Theorem* 1

To prove Theorem 1, we solve a relaxed problem, denoted by $(\mathscr{RP}_b)$, ignoring (IC_{lh}) and (IR_h) and provide an implementation. Formally, this relaxed problem is as follows.

$$(\mathscr{RP}_b) \quad \max_{(\pi,q,U)} \quad \sum_{\theta} f(\theta)\Big[\sum_x\sum_s v(l,x)q(\theta,\omega^{\pi_\theta}(\theta,s))\pi_\theta(s|x)\mu(x) - U(\theta)\Big]$$

$$s.t: \quad U(h)-U(l) \geq \sum_x\sum_s \int_{\omega^{\pi_l}(l,s)}^{\omega^{\pi_l}(h,s)} q(l,z)dz\pi_l(s|x)\mu(x) \qquad (IC_{h\to l})$$

$$U(l) \geq 0 \qquad (IR_l)$$

$$q(\theta,\omega) \text{ increases in } \omega. \qquad (MON)$$

The characterization of the solution to $(\mathscr{RP}_b)$ is done via the following steps. First, we prove the optimality of deterministic allocation rules. This step, while standard, is helpful in decomposing the buyer's rent into two components: the *ex ante* rent (due to privacy of types) and the *posterior* rent (due to privacy of signals). Using this rent decomposition, we establish the optimality of binary-signal experiments and furthermore, of threshold disclosures. Finally, we characterize the optimal allocation and implement it.

First, to obtain the optimality of deterministic allocations, note that $(IC_{h\to l})$ and (IR_l) must bind in $(\mathscr{RP}_b)$, *i.e.*,

$$U(l)=0, \quad U(h)=\sum_x\sum_s \int_{\omega^{\pi_l}(l,s)}^{\omega^{\pi_l}(h,s)} q(l,z)dz\pi_l(s|x)\mu(x),$$

Thus, transfers have been eliminated, reducing the seller's relaxed problem to

$$\begin{aligned}\max_{q,\pi} \quad & f(h)\sum_x\sum_s v(h,x)q(h,\omega^{\pi_\theta}(\theta,s))\pi_h(s|x)\mu(x)\\ & +f(l)\sum_x\sum_s \left[v(l,x)q(l,\omega^{\pi_\theta}(\theta,s)) - \int_{\omega^{\pi_l}(l,s)}^{\omega^{\pi_l}(h,s)} q(l,z)dz\right]\pi_l(s|x)\mu(x)\\ s.t \quad & q(\theta,\omega) \text{ increases in } \omega. \qquad (MON)\end{aligned}$$

Fix π. Given that the objective function is linear and the only constraint is (MON), there exists an optimal allocation rule that is deterministic and exhibits a cut-off structure. Moreover, as $v(h,x)$ is always non-negative, h receives an efficient allocation.

Lemma 4 (Deterministic allocations)**.** *In $(\mathscr{RP}_b)$, there exists an optimal allocation rule, given by $q(\theta,\omega)=\mathbb{1}_{\omega\geq\hat{\omega}_\theta}$, where $\hat{\omega}_h = v(l,\underline{x})$.*

Second, we derive the sufficiency of binary-signal experiments. As $q(h,s)=1$ for all s, any π_h is optimal. The relaxed problem reduces to finding the optimal π_l. Let

$$R_l \equiv f(l)\sum_x\sum_s \left[v(l,x)q(l,\omega^{\pi_l}(l,s)) - \int_{\omega^{\pi_l}(l,s)}^{\omega^{\pi_l}(h,s)} q(l,z)dz\right]\pi_l(s|x)\mu(x)$$

denote the term involving π_l in the seller's objective function (revenue) in $(\mathscr{RP}_b)$. Using $q(l,\omega) = \mathbb{1}_{\omega \geq \hat{\omega}_l}$ by Lemma 4, we obtain

$$R_l = f(l) \sum_x \left[\sum_{\hat{s}_l}^{\bar{s}} v(l,x)\pi_l(s|x) - \sum_{\hat{s}_l}^{\bar{s}} \int_{\omega^{\pi_l}(l,s)}^{\omega^{\pi_l}(h,s)} dz \frac{f(h)}{f(l)} \pi_l(s|x) - \sum_{\underline{s}}^{\hat{s}_l} \int_{\omega^{\pi_l}(l,\hat{s}_l)}^{\omega^{\pi_l}(h,s)} dz \frac{f(h)}{f(l)} \pi_l(s|x) \right] \mu(x)$$

$$= f(l) \sum_x \Bigg[\underbrace{\sum_{\hat{s}_l}^{\bar{s}} v(l,x)}_{l\text{'s surplus}} - \underbrace{\sum_{\hat{s}_l}^{\bar{s}} [\omega^{\pi_l}(h,s) - \omega^{\pi_l}(l,s)] \frac{f(h)}{f(l)}}_{h\text{'s } \textit{ex ante} \text{ rent}} - \underbrace{\sum_{\underline{s}}^{\hat{s}_l} [\omega^{\pi_l}(h,s) - \omega^{\pi_l}(l,\hat{s}_l)] \frac{f(h)}{f(l)}}_{h\text{'s } \textit{posterior} \text{ rent}} \Bigg] \pi_l(s|x)\mu(x).$$

R_l depends on (i) the buyer's expected value (on path for l and off path for h), conditional on whether $s \geq \hat{s}_l$ or $s < \hat{s}_l$, and (ii) the cut-off signal, $\hat{s}_l$. By (i), there is no revenue loss in replacing all signals $s \geq \hat{s}_l$ with "*good news*" (s^g) and all $s < \hat{s}_l$ with "*bad news*" (s^b). At the same time, such a binary-signal experiment for type l increases the cut-off signal because $\omega^{\pi_l}(l,s^g) = \mathbb{E}[v(l,x) \mid s \geq \hat{s}_l] \geq \hat{s}_l$. In turn, this improves R_l, which increases in the cut-off signal. We thus obtain the optimality of binary-signal experiments.

Lemma 5 (Binary signals)**.** *In* $(\mathscr{RP}_b)$*, there exists an optimal experiment for type* l *where the signal realization can be either "good news"* (s^g) *or "bad news"* (s^b)*.*

Third, we prove the optimality of threshold disclosures. By replacing all signals $s \geq \hat{s}_l$ (resp., $s < \hat{s}_l$) with "*good news*" (resp., "*bad news*"), R_l becomes

$$f(l) \sum_x \Bigg[\underbrace{v(l,x)\pi_l(s^g|x)}_{l\text{'s surplus}} - \underbrace{[v(h,x) - v(l,x)] \frac{f(h)}{f(l)} \pi_l(s^g|x)}_{h\text{'s } \textit{ex ante} \text{ rent}} - \underbrace{[\omega^{\pi_l}(h,s^b) - \omega^{\pi_l}(l,s^g)] \frac{f(h)}{f(l)} \pi_l(s^b|x)}_{h\text{'s } \textit{posterior} \text{ rent}} \Bigg] \mu(x)$$

$$= f(l) \underbrace{\sum_x \phi(l,x)\pi_l(s^g|x)\mu(x)}_{l\text{'s virtual value}} - f(l) \sum_x \underbrace{\max\left\{ [\omega^{\pi_l}(h,s^b) - \omega^{\pi_l}(l,s^g)] \frac{f(h)}{f(l)}, 0 \right\} \pi_l(s^b|x)\mu(x)}_{h\text{'s } \textit{posterior} \text{ rent}}.$$

Fix $\pi_l(s^b)$. Then, a threshold disclosure minimizes h's posterior rent by simultanenously maximizing $\omega^{\pi_l}(h,s^b)$ and minimizing $\omega^{\pi_l}(h,s^b)$. Moreover, as $\phi(l,x)$ increases in x, a threshold disclosure maximizes l's expected virtual value. Therefore:

Lemma 6 (Threshold structure)**.** *In* $(\mathscr{RP}_b)$*, a threshold disclosure for* l *is optimal.*

Last, we characterize the optimal allocation and provide an implementation. Let $\hat{x}_l \in X$ be the cut-off state associated with the optimal threshold disclosure for l and $\lambda \in [0,1]$ be the probability with which "*good news*" is sent at $\hat{x}_l$. Then, by Lemmas 4, 5, and 6, the optimal allocation is given by

$$q(h,x)=1 \quad \forall x, \quad q(l,x)=\begin{cases}1 & \text{if} \quad x>\hat{x}_l, \\ 0 & \text{if} \quad x<\hat{x}_l, \\ \lambda & \text{if} \quad x=\hat{x}_l.\end{cases}$$

Solving for the optimal allocation reduces to solving for the optimal $(\hat{x}_l,\lambda)$. As will be shown, there are two cases, depending on whether $\pi_l^\star$ triggers the threshold flip of type order. In the first case, when this flip happens, offering $\pi_l^\star$ with $(\hat{x}_l,\lambda)=(x_l,1)$ is optimal. Not only does it induce zero *posterior* rent for h, given that

$$\omega^{\pi_l^\star}(h,s^b)-\omega^{\pi_l^\star}(l,s^g)\leq 0$$

when the threshold flip occurs under $\pi_l^\star$, but it also creates the highest expected virtual value for l 's, given by $f(l)\sum_{x\geq x_l}\phi(l,x)\pi_l(s^g|x)\mu(x)$.

With $(\hat{x}_l,\lambda)=(x_l,1)$, l 's allocation coincides with the benchmark $\mathbb{Q}(l,x)=\mathbb{1}_{x\geq x_l}$. To find out payments, without loss of generality, assume the buyer pays only if "*good news*" is realized (or trade happens). Thus, $p(h,s^b)=p(l,s^b)=0$. Then, $p(l,s^g)=\omega^{\pi_l^\star}(l,s^g)$ by (IR_l), and $p(h,s^g)$ is such that $(IC_{h\to l})$ holds, or $U(h)=U(h,l)$, which implies

$$p(h,s^g)=\mathbb{E}[v(h,x)]-\left[\omega^{\pi_l^\star}(h,s^g)-p(l,s^g)\right]\pi_l^\star(s^g).$$

Now, verify that ignored constraints are satisfied. First,(IR_h) hold because

$$U(h)=\left[\omega^{\pi_l^\star}(h,s^g)-\omega^{\pi_l^\star}(l,s^g)\right]\pi_l^\star(s^g)\geq 0$$

Second, $IC_{l\to h}$ is satisfied given that

$$\begin{aligned}
U(l,h)&=\mathbb{E}[v(l,x)]-p(h,s^g)\\
&=\mathbb{E}[v(l,x)]-\mathbb{E}[v(h,x)]+\left[\omega^{\pi_l^\star}(h,s^g)-p(l,s^g)\right]\pi_l^\star(s^g)\\
&=\mathbb{E}[v(l,x)]-\mathbb{E}[v(h,x)]+\left[\omega^{\pi_l^\star}(h,s^g)-\omega^{\pi_l^\star}(l,s^g)\right]\pi_l^\star(s^g)\\
&=\left[\omega^{\pi_l^\star}(l,s^b)-\omega^{\pi_l^\star}(h,s^b)\right]\pi_l^\star(s^b)<0=U(l)
\end{aligned}$$

Moreover, under no threshold flip of type order by $\pi^\star$, $\omega^{\pi_l^\star}(h,s^b)\leq\omega^{\pi_l^\star}(l,s^g)=p(l,s^g)$. Therefore, if h mimics l, it is optimal for him to report signals truthfully. This deviating behavior is not beneficial for h by the construction of $p(h,s^g)$. We thus obtain Theorem 1(a):

Lemma 7 (With threshold flip by $\pi_l^\star$)**.** *If $\pi_l^\star$ induces the threshold flip of type order, $q(l,x)=\mathbb{Q}(l,x)=\mathbb{1}_{x\geq\hat{x}_l}$, and $\boldsymbol{M}^\star\equiv\{p^\star(\theta),\pi_\theta^\star\}_\theta$ is optimal.*

By contrast, when $\pi_\theta^\star$ preserves the type order, or $\omega^{\pi_l^\star}(h,s^b) > \omega^{\pi_l^\star}(l,s^g)$, offering $\pi_l^\star$ to l induces a strictly positive *posterior* rent for h. Consequently, the seller trades off between l 's expected virtual value and h's *posterior* rent. On the one hand, she wants the threshold to be close to the cut-off x_l, maximizing l 's expected value. On the other hand, she desires to induce a small *posterior* rent for h.

Let $\pi_l^{\star\star}$ be an optimal experiment for l , associated with $\left(x^{\star\star}(l),\lambda^{\star\star}\right)$. Suppose, $\pi_l^{\star\star}$ can flip the type order, *i.e.*, $v^{\pi_l^{\star\star}}(h,s^b) < \omega^{\pi_l^{\star\star}}(l,s^g)$. Then, given that $\omega^{\pi_l^\star}(h,s^b) \le \omega^{\pi_l^\star}(l,s^g)$, we can construct $\tilde{\pi}_l$ associated with $\left(\tilde{\omega},\tilde{\lambda}\right)$ such that (i) $\left(x^{\star\star}(l),\lambda^{\star\star}\right)$ is closer to $\left(x_l^\star,1\right)$ and (ii) $v^{\tilde{\pi}_l}(h,s^b) \le v^{\tilde{\pi}_l}(l,s^g)$. By (i), l 's expected virtual value under $\tilde{\pi}_l$ is higher than that under $\pi_l^\star$, whereas by (ii), h's *poterior* rent is zero under $\tilde{\pi}_l$. This contradicts with $\pi_l^{\star\star}$ being optimal. Therefore, $\pi_l^{\star\star}$ must not affect the type order. Formally:

Claim 1. $\omega^{\pi_l^{\star\star}}(h,s^b) \ge \omega^{\pi_l^{\star\star}}(l,s^g)$.

The detailed proof is in Appendix B.1. By Claim 1, R_l reduces to

$$
\begin{aligned}
&f(l)\sum_x \phi(l,x)\pi_l^{\star\star}(s^g|x)\mu(x) - f(h)\left[\omega^{\pi_l^{\star\star}}(h,s^b) - \omega^{\pi_l^{\star\star}}(l,s^g)\right]\pi_l(s^b)\\
=&\omega^{\pi_l^{\star\star}}(l,s^g)\left[f(l)\pi_l^{\star\star}(s^g|x)\mu(x) + f(h)\right] - \mathbb{E}[v(h,x)].
\end{aligned}
$$

Therefore,

$$
\pi_l^{\star\star} \in \underset{\pi_l}{\operatorname{argmax}}\,\omega^{\pi_l}(l,s^g)\left[f(l)\pi_l^{\star\star}(s^g,x) + f(h)\right]. \tag{7}
$$

To find optimal transfers, note that by Claim 1, h's value after *"bad news"* is higher than l 's after "*good news*." Hence, if h mimics l , he always reports "*good news*," and always buys the good. Consequently, l 's allocation is the same as h 's from the latter's perspective. This leads to a bunching solution. As information is of no value for h, the seller can offer $\pi_l^{\star\star}$ to both types. Moreover, as h always gets the good either on or off-path, by $(IC_{h\to l})$, both types receive the same posted price.[24] Then, by (IR_l),

$$
p^{\star\star}(h) = p^{\star\star}(l) = \omega^{\pi_l^{\star\star}}(l,s^g). \tag{8}
$$

This bunching mechanism satisfies ignored constraints, and hence, is optimal.

Lemma 8 (No threshold flip by $\pi_l^\star$)**.** *If $\pi_l^\star$ does not induce the threshold flip of type order, $\left(\hat{x}_l,\lambda\right) \neq \left(x_l,1\right)$. A single-option menu, $\{\pi_l^{\star\star}, p^{\star\star}(l)\}$ given by (7) and (8), is optimal.*

[24]With deterministic allocations, it is without loss to offer a menu of posted prices. See Corollary 1.

6 OPTIMAL MECHANISM FOR $|\Theta| \geq 3$

With binary types, there are two scenarios for the optimal mechanism, depending on whether $\pi^\star$ induces the threshold flip of type order or not. While these two scenarios extend to richer type sets under different notions of (no) type order flip, a new scenario arises. This is when optimal mechanisms involve random allocations. In what follows, we study each of the three scenarios: screening, bunching, and random mechanisms in turn.

6.1 *Optimality of screening*

Information helps screen the buyer of binary types when it induces a threshold flip of type order. Similarly, information serves as a screening tool in a richer type space under the following notion of type order flip:

Definition 6 (Partition flip of type order)**.**

The partition flip of type order happens if $\mathbb{E}[v(\theta^+, x) \mid x_{\theta^+} \leq x < x_\theta]$ *decreases in* θ.

Under the partition flip of type order, the expected valuations over relevant partitions of states decrease in types. As the relevant partition for a higher type consists of lower states, such a type order flip requires the new information (about the state) to sufficiently dominate the buyer's initial type in causing valuation fluctuations. Indeed, it coincides with the threshold flip notation when there are only two types. In a richer type set, more than one interior threshold is involved under the menu of threshold disclosure $\{\pi^\star_\theta\}_\theta$, leading to relevant partitions of states.

Theorem 2 below establishes that under the partition flip of type order, the seller benefits from screening the buyer's type via discriminatory information and prices.

Theorem 2 (Screening)**.** *Under the partition flip of type order, the optimal allocation is given by* $\mathbb{Q}(\theta, x) = \mathbb{1}_{x \geq x_\theta}$. *A menu of posted prices and threshold disclosures is optimal.*

The proof proceeds by showing that under the partition flip of type order, $\mathbf{M}^\star$ induces truth-telling even if the seller does not observe signals, and thereby solves the seller's problem. Moreover, offering $\mathbf{M}^\star$ with the buyer privately observing signals is equivalent to offering a menu of posted prices and threshold disclosures $\{p^\star(\theta), \pi^\star_\theta\}_\theta$, where the posted price is equal to the payment paid after "*good news*" in $\mathbf{M}^\star$: $p^\star(\theta) = p^\star(\theta, s^g)$.

This result extends Theorem 1(a): under $|\Theta| = 2$, a screening menu is optimal under the *threshold flip* of type order by $\pi^\star$. The only difference is that the *partition flip* of type order is required here, taking into account interior types. The logic is also similar: when the new information disclosed by the seller matters sufficiently, it can be used to screen the buyers' types.

We close this section with an illustrative example.

Example 4. $\Theta = \{h, m, l\}$. *X is a finite subset of* $\mathbb{N}$. *Types and states are equally likely. Valuations are given by:* $v(h,x) = x + \Delta_\theta$, $v(m,x) = x$, $v(l,x) = x - \Delta_\theta$

In this example, $v(\theta^+, x) - v(\theta, x) = \Delta_\theta\ \forall x$ and $\Delta_x \equiv v(\theta, \overline{x}) - v(\theta, \underline{x}) = \overline{x} - \underline{x}\ \forall \theta$. Besides, virtual values are given by $\phi(h,x) = x + \Delta_\theta$, $\phi(m,x) = x - \Delta_\theta$, $\phi(m,x) = x - 3\Delta_\theta$. Then, $x_h^\star = \underline{x}$, $x_m^\star = \Delta_\theta$, and $x_l^\star = 3\Delta_\theta$. Therefore,

$$\mathbb{E}[v(h,x) \mid x_h^\star \le x < x_m] = \frac{3\Delta_\theta - 1 + \underline{x}}{2},$$
$$\mathbb{E}[v(m,x) \mid x_m \le x < x_l] = \frac{4\Delta_\theta - 1}{2},$$
$$\mathbb{E}[v(l,x) \mid x_l \le x \le \overline{x})] = \frac{\Delta_\theta + \overline{x} - 1}{2}$$

Thus, the partition flip of type order happens if

$$3\Delta_\theta - 1 + \underline{x} \le 4\Delta_\theta - 1 \le \Delta_\theta + \overline{x} - 1 \Leftrightarrow \underline{x} \le \Delta_\theta \le \Delta_x,$$

which requires the impact of the unknown component to be higher than that of the buyer's type (and is of at least $\underline{x}$). If this is the case, by Theorem 2, it is optimal to screen the buyer's type via a menu of threshold disclosures and prices.

6.2 Optimality of bunching

In the binary-type case, the benefit of screening disappears if information disclosure under $\pi_l^\star$ fails to flip the ranking of willingness to pay by types. A similar story holds with more than two types under a stronger notion of (no) threshold flip of type order:

Definition 7 (Uniformly no threshold flip of type order)**.** *Under uniformly no threshold flip of type order,*

$$\mathbb{E}[v(\theta^+, x \mid x < \hat{x}] \ge \mathbb{E}[v(\theta, x) \mid x \ge \hat{x}] \quad \forall \theta \in \Theta, \forall \hat{x} \in X.$$

In words, this condition satisfies if under *any threshold disclosure* and for *any type* θ: θ^+'s value after "*bad news*" must be higher than θ's after "*good news*". This is more likely to hold when valuation heterogeneity is mainly driven by the buyer's type. For instance, when θ^+'s values are always higher regardless of states, *i.e.*, $v(\theta^+, \underline{x}) \ge v(\theta, \overline{x})$, it is impossible to flip their ranking of valuation after *any rule* of information disclosure, not just the threshold ones.

We are now ready to state the main result of this section.

Theorem 3 (Bunching)**.** *Under uniformly no threshold flip of type order, a posted price, associated with a threshold disclosure, is optimal.*

This result extends Theorem 1(b) (if $|\Theta| = 2$ and bunching is optimal under no threshold flip of type order by $\pi_l^\star$), carrying the same intuition. When the buyer's type sufficiently matters for valuation heterogeneity, information becomes inessential for (most types of) the buyer. As a result, its screening function shuts off. The only difference is that no type order flip by *any* threshold disclosure is required here, which is to deal with the challenges of determining the lowest type being served.

The proof proceeds by solving a relaxed problem considering only deviating behaviors under which all types mimic the lowest type being served. This problem mirrors that for the binary-type case $\Theta = \{h, l\}$, with the lowest type being served representing type l and all the other types echoing type h. The optimality of bunching under *uniformly* no threshold flip of type order follows similar arguments for that in the binary-type setting under no threshold flip by $\pi_l^\star$. With no threshold flip being true *uniformly* regardless of pairs of types and threshold rule, the lowest type can be explicitly characterized as well as the optimal posted price and threshold disclosure.

To end this section, revisit Example 4 for an illustration. In this example, for any $\hat{x} \in X$,

$$\mathbb{E}[v(h,x) \mid x < \hat{x}] - \mathbb{E}[v(m,x) \mid x \geq \hat{x}] = \Big(\Delta_\theta + \frac{\hat{x} - 1 + \underline{x}}{2}\Big) - \Big(m + \frac{\overline{x} + \hat{x}}{2}\Big) = \Delta_\theta - \frac{\Delta_x - 1}{2},$$
$$\mathbb{E}[v(m,x) \mid x < \hat{x}] - \mathbb{E}[v(l,x) \mid x \geq \hat{x}] = \Big(m + \frac{\hat{x} - 1 + \underline{x}}{2}\Big) - \Big(l - \Delta_\theta + \frac{\overline{x} + \hat{x}}{2}\Big) = \Delta_\theta - \frac{\Delta_x - 1}{2},$$

where, just to recall, Δ_θ and Δ_x measure the impact of the buyer's private type and the unknown component in valuation variations, respectively. Therefore, uniformly no threshold flip of type order occurs if

$$\Delta_\theta \geq \frac{\Delta_x - 1}{2}, \tag{9}$$

which requires the buyer's type to be significantly impactful, relative to the unknown component. If this is the case, by (Theorem 3), information is not leveraged to screen the buyer. A take-or-leave-it offer with a fixed price and disclosure rule is optimal.

6.3 *Optimality of random mechanisms*

When random mechanisms are necessary, it becomes challenging to characterize optimal mechanisms because binding IC constraints can involve global (Example 5) and upward ones (Example 10). We thus focus on investigating the structure of random mechanisms and their impact on efficiency, mainly via examples.[25] First, we show that randomization is not needed for the highest type. The idea is that whenever this type does not trade with probability 1 (at some

[25] In the Online Appendix, we solve for the optimal random mechanism in several examples.

state), it is possible to improve the seller's revenue by letting him always trade under no disclosure and a posted price being equal to his original expected payment, adding the new surplus.

Proposition 3 (No randomization at the top)**.** *It is optimal to offer the highest type an efficient allocation (i.e., $q(\overline{\theta}, x) = 1 \; \forall \; \omega$), associated with no disclosure and a posted price.*

By contrast, for the lower types, offering efficient allocations is generally sub-optimal and random mechanisms can help balance the efficiency vs. rent trade-off. This is because random mechanisms facilitate (i) screening distant types and (ii) screening signals. In what follows, we study each of these two channels in turn.

Screening distant types: Information alone, sometimes, is not sufficient to effectively screen types whose valuations are far from each other. In such a scenario, trade probabilities can complement information to "separate" those distant types. Thanks to this, random mechanisms allow very low types to be served (with a small probability) while not generating a huge rent for very high types. To illustrate, consider the following example.

Example 5. $\Theta = \{h, m, l\}$ *and* $X = \{b, g\}$*. Types and states are equally likely.*

$v(\theta, x)$	$x = b$	$x = g$
$\theta = h$	6.5	10
$\theta = m$	0	7
$\theta = l$	0	5

Restricting to deterministic mechanisms, Claim 2 below shows that it is optimal to exclude type l.

Claim 2. *Consider Example* 5*. Suppose only deterministic mechanisms are allowed, then it holds*

(a) If l trades, it is optimal to offer full disclosure and a posted price $p = 5$ to all types. The seller's revenue is $\frac{10}{3}$.

(b) If l is excluded, it is optimal to offfer H no disclosure with a posted price $p(h) = 6.75$, and l full disclosure with a posted price $p(m) = 7$. The seller's revenue is $\frac{10.25}{3}$.

The basic intuition for this exclusion of l under deterministic mechanisms is as follows. As h's value is always higher than l 's, it is optimal for h, who mimics l, to always report the highest signal and hence, always buy the good. Then, l 's allocation is the same as h's from the latter's perspective, leading to bunching these types. In turn, this gives too much rent for h, making it optimal to *exclude* l.

Now, we show how the seller can improve her revenue via random allocations for l. The key is that when l trades with a small probability (for any signal), this type's allocation becomes unattractive to H. Specifically, modify the optimal deterministic mechanism by letting l trade with a probability $\varepsilon \in [0, \frac{3}{4}]$ and adjusting transfers such that truth-telling remains satisfied, as follows:

$$q(H,x) = 1 \forall x, \quad q(M,x) = \mathbb{1}_{x=b}, \quad q(L,x) = \begin{cases} \varepsilon & \text{if} \quad x = g, \\ 0 & \text{if} \quad x = b, \end{cases}$$

$$p(h) = 6.5 - \varepsilon, \quad p(m) = 7 - 2\varepsilon, \quad p(L) = 5 \quad \text{paid conditional on trade occurs.}$$

Under this revision, expected payment by H and m reduces by ε; however, that by m increases by $\frac{5\varepsilon}{2}$. Overall, the seller's revenue increases by $f(l)\frac{5\varepsilon}{2} - [f(m) + f(h)]\varepsilon = \frac{3\varepsilon}{2} > 0$.

Screening signals: Now, we consider the second channel via which random mechanisms outperform their deterministic counterpart. By Corollary 1, deterministic mechanisms do not better than persuasive posted-price mechanisms which are signal (state)-*independent* and specifies trade only after "*good news.*" The following example shows how random mechanisms can help elicit realized signals, thereby enabling trade to happen even after "*bad news*".

Example 6. $\Theta = \{h, m, l\}$, $X = \{b, g\}$. *Types and states are equally likely.*

$v(\theta, x)$	$x = b$	$x = g$
$\theta = h$	5	5
$\theta = m$	2	5
$\theta = l$	0	4

Restricting to deterministic mechanisms, Claim 3 below shows that it is optimal to *exclude* m at state b.

Claim 3. *Consider Example* 6. *If only deterministic allocations are allowed, it is optimal to offer full disclosure and a posted price of* 4.

Therefore, h always trades, whereas m and l only trade at state g. While the exclusion of l at state b is trivial, that for m is due to the significant variation in m's values across states which makes it sub-optimal to "pool" the two states.

Now, consider a revised version of the optimal deterministic mechanism which differs from the latter only in letting m trade with probability $\delta \leq \frac{1}{3}$, as follows

$$q(h,x) = 1 \forall x, \quad q(l,x) = \mathbb{1}_{x=b}, \quad p(h) = p(l) = 4,$$

$$\big(q(m,x), p(m,x)\big) = \begin{cases} (1,4) & \text{if} \quad x = g, \\ (\delta, 2\delta) & \text{if} \quad x = b, \end{cases}$$

To see why offering m a new option $(\delta, 2\delta)$ preserves truth-telling with $\delta \leq \frac{1}{3}$, note the following. As h's value is higher than m's regardless of states ($v(h,b) \geq v(m,g)$), it is optimal for him to always report "state g" once mimicking m. As a result, that m receives a new option at state b does not affect h's off-path payoff. As regards l, the new trade creation can potentially increase l 's payoff from mimicking m only if he always reports "state b" off-path. However, as m's unconditional expected value is equal to $v(m,b)$, he gets a zero payoff from this off-path behavior. Finally, with $\delta \leq \frac{1}{3}$, m does not benefit from misreporting "state b" when g is realized.

7 INFINITE-TYPE SETTING

We emphasize that all the proofs of our results extend readily to the infinite-state setting. The extension to the infinite-type case, however, is not trivial. Nevertheless, we find that the key message continues to hold: the optimal mechanism uses information disclosure to screen the buyer's types when the additional information about the unknown component flips the ranking of willingness to pay by types and conversely, features a fixed price and disclosure rule when the impact of information is negligible.

Throughout this section, consider a continnum of types $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$, endowed with the distribution $F(\theta)$. We assume that $F(\theta)$ is differentiable in θ with density $f(\theta)$, and moreover, $v(\theta, x)$ is differentiable in θ. Then, the virtual value in this environment is given by

$$\phi^c(\theta, x) = v(\theta, x) - v_\theta(\theta, x) \frac{1 - F(\theta)}{f(\theta)}.$$

Similar to the finite-type case, assume that $\phi^c(\theta, x)$ increases in θ and x. Then, each state x is associated with a cut-off type θ_x above (*resp.*, below) which the buyer's virtual value is non-negative (*resp.*, negative). Formally,

$$\theta_x \equiv \inf\{\theta \mid \phi^c(\theta, x) \geq 0\}.$$

Moreover, as $\phi^c(\theta, x)$ increases in x, this cut-off type θ_x decreases in x.

7.1 Binary states

To set intuition, we first start with binary states $\Omega = \{b, g\}$ with $b > g$. Proposition 4 below shows that with binary states, the optimal mechanism features either bunching, screening, or a random mechanism, depending on whether a flip of type order occurs between the two cut-off types θ_b and θ_g under full disclosure.

Proposition 4. *Fix* $\Theta = [\underline{\theta}, \overline{\theta}]$ *and* $X = \{b, g\}$. *Then, it holds:*

(a) If $v(\theta_b, b) \leq v(\theta_g, g)$, *a menu of prices and threshold disclosures is optimal.*

(b) If $v(\theta_b, b > v(\theta_g, g)$ *and only deterministic allocations are allowed, a posted price, associated with full disclosure, is optimal.*

The logic of the proof is as follows. With two states $\{b, g\}$, $\mathbf{M}^\star$ comprises two options designed for two intervals of types $[\theta_g, \theta_b)$ and $[\theta_b, \overline{\theta}]$. This mirrors the case when the type space consists of only θ_g and θ_b (and binary states b and g). As a result, whether $\pi^\star$ leads to the partition flip of type order reduces to whether $v(\theta_b, b) \geq v(\theta_g, g)$. Proposition 4(a) corresponds to the situation in which the partition flip of type order occurs. Following the same arguments as Theorem 2, a screening menu of prices and information is optimal. When such a type order flip fails to occur as in Proposition 4(b), we show that the seller adjusts the cut-off types to be $\tilde{\theta}_b, \tilde{\theta}_g$ *just enough* to restore the partition flip of type order: $v(\tilde{\theta}_b, b) = v(\tilde{\theta}_g, g)$. This is when a single option of price and information do no worse (and no better) than a screening menu. In other words, bunching is optimal.

Examples 7 and 8 below illustrate the two cases of Proposition 4.

Example 7. $v(\theta, x) = \theta + x - 1$, $\Theta = [0, 1]$, $X = \{1, 2\}$. *Types and states are likely equally.*

In this example, $\phi(\theta, x) = x + 2\theta - 1$. Thus, $\theta_0 = \frac{1}{2}$ and $\theta_2 = 0$. Hence, $v(\theta_1, 1) = \frac{1}{2}$ and $v(\theta_2, 2) = 1$. As $v(\theta_0, 1) < v(\theta_2, 2)$, a partition flip of type order occurs. By Proposition 4(a), $\mathbf{M}^\star$ is optimal.

Example 8. $v(\theta, x) = 3\theta^2 + 4\theta + x$, $\Theta = [0, 2]$, $X = \{0, 4\}$. *Types and states are likely equally.*

In this example, $\phi(\theta, x) = 6\theta^2 - 4 + x$. Thus, $\theta_4 = 0$ and $\theta_0 = \frac{2}{\sqrt{6}}$. Hence, $v(\theta_0, 0) = 2 + \frac{8}{\sqrt{6}}$ and $v(\theta_4, 4) = 4$. As $v(\theta_0, 1) > v(\theta_4, 2)$, no flip of type order occurs. By Proposition 4(b), either bunching is optimal, or the seller must employ random mechanisms.

7.2 Optimality of screening with infinite types

Theorem 2 establishes that when there are finitely many types, a menu of threshold disclosures and prices is optimal under a partition flip of type order. We now extend this result to the infinite-type case. With a continuum of types, the finiteness of the state space leads to the fact that there are *finitely* many cut-off types $\{\theta_x\}_{x \in X}$. Accordingly, $\mathbf{M}^\star$ comprises $|X|$ options of posted prices and threshold disclosures with each interval of types $[\theta_{x^+}, \theta_x)$ being assigned the same option. Following the proof of Theorem 2 for a type space consisting of only the cut-off types, we obtain the optimality of a screening menu under a partition flip of type order. Let

$$\Theta_x \equiv \{\theta_x\}_{x\in X},$$

represent such a type space of only cut-off types, the result is formally stated as follows.

Proposition 5. *Fix* $\Theta = [\underline{\theta},\overline{\theta}]$ *and* $|X| < \infty$*. If there is a partition flip of type order within* Θ_x*, a menu of threshold disclosures and posted prices is optimal.*

It is worth noting that this result holds even if there is a continuum of states $X = [\underline{x},\overline{x}]$, by approximating an associated model with infinite types and finite states as the distance between states approaches zero. Moreover, with continuums of types and states, the partition flip of type order reduces to the valuation at the cut-off state $v(\theta, x_\theta)$ decreasing in types.[26]

7.3 Approximate optimality of bunching infinite types

When there are finitely many types, Theorem 3 shows that under the uniformly no threshold flip of type order, offering a single posted price and threshold disclosure is optimal. When valuation shifts smoothly across (a continuum of) types, there are always types whose valuations are sufficiently close to others'. This makes it impossible to preserve the ranking of willingness to pay uniformly across the types. Nevertheless, we show that bunching is *approximately* optimal under ε*-uniformly* no threshold flip of type order, formally defined below.

Definition 8 (ε-uniformly no threshold flip of type order)**.** ε*-uniformly no threshold flip of type order occurs if for some* $\varepsilon > 0$*,*

$$\mathbb{E}[v(\theta+\varepsilon, x) \mid x \le \hat{x}] \ge \mathbb{E}[v(\theta, x) \mid x \ge \hat{x}] \quad \forall \theta, \hat{x}.$$

As ε vanishes, the seller's maximized revenue can be approximated by offering a fixed price and threshold disclosure. Let R_ε represent the revenue guarantee under the ε-uniformly no threshold flip of type order, the approximate optimality of bunching is formalized as follows.

Proposition 6. $R_\varepsilon \to V(P)$ *as* $\varepsilon \to 0$

8 DISCUSSION

8.1 Posterior rent and privacy of signals

As explained in the binary-type model, not observing signals generally hurts the seller due to the presence of the buyer's *posterior* rent. Specifically, implementing the benchmark allocation

[26]This is the case in, for example, the environments studied in Eső and Szentes (2007) and Wei and Green (2023) under which the valuation function is concave in types and states, and the cross derivative is positive.

requires the seller to pay the buyer's *posterior* rent (apart from his *ex ante* rent), making $V(P) < V(\overline{P})$. When valuation shifts smoothly across (infinite) types, the relevance of signal privacy comes from a different reason. Indeed, any allocations *implementable with private signals* can be implemented without generating *posterior* rent to the buyer.[27] Therefore, if the seller fails to achieve the upper bound of revenue $V(\overline{P})$, it is due to an implemetability issue. In such a scenario, information design can expand the set of implementable allocations. To illustrate, consider the following example where the benchmark allocation is implementable with private signals only if uninformative experiments are possible.

Example 9. $v(\theta, x) = \theta^2 + \theta + x - 2$. *Types and states are uniformly distributed over* $\Theta = [0,1]$ *and* $\Omega = [0,3]$.

In this example, $p^\star(\theta, s^g) = -\theta^2 + \frac{2}{3}\theta + 1$. Moreover, $p^\star(\theta, s^g)$ a concave function in $[0,1]$ with $p(0, s^g) = 1, p(1, s^g) = \frac{2}{3}$. Thus, $p^\star(\overline{\theta}, s^g) = \min_\theta p^\star(\theta, s^g)$. Then by Proposition 2, the seller implements the benchmark allocation via $\mathbf{M}^\star$. Suppose the seller provides full disclosure to all types. To implement the benchmark allocation, it must be that for any θ and x, $q(\theta, x) = \mathbb{1}_{x \geq x_\theta}$. For the buyer to report truthfully their states, it is necessary that

$$p(\theta, x) = \begin{cases} \overline{p}(\theta) & \text{if } x \geq x_\theta, \\ \underline{p}(\theta) & \text{otherwise.} \end{cases}$$

To prevent the lowest type $\underline{\theta}$ from mimicking some type θ and always report $x < x_\theta$, it must be that $\underline{p}(\theta) \geq 0$. Therefore,

$$\int_{x \geq x_\theta} \mu(x) dx p^\star(\theta, s^g) = \int_{x \geq x_\theta} \mu(x) dx \overline{p}(\theta) + \underline{p}(\theta) G(x_\theta) \geq \int_{x \geq x_\theta} \mu(x) dx \overline{p}(\theta)$$

where the equality uses the fact that all mechanisms implementing the benchmark allocation share the same expected payment. Thus, $p^\star(\theta) \geq \overline{p}(\theta)$ for all type θ.

Consider $\theta = \frac{1}{3}$, we have $\overline{p}^\star(\frac{1}{3}, s^g) = \frac{10}{9}$, and $v(\frac{1}{3}, x_{\frac{1}{3}}) = \frac{13}{9}$. Thus, $v(\frac{1}{3}, x(\frac{1}{3})) > p^\star(\frac{1}{3}, s^g) \geq \overline{p}(\frac{1}{3})$. Then, if the buyer observes any state $x \in \left(\overline{p}\left(\frac{1}{3}\right), v(\frac{1}{3}, x(\frac{1}{3}))\right)$, it is optimal for him to misreport state $\overline{x}$, receiving the good at a price lower than his valuation. Thus, the benchmark allocation is not implementable under full disclosure.

8.2 Alternative proof for Wei and Green (2023)

Wei and Green (2023) revisit Eső and Szentes (2007)'s "continuous" model, adding a twist that

[27] We omit the formal proof, which extends the arguments in Krähmer and Strausz (2015*a*) to a setting with information design and possibly finitely many states.

the buyer can walk away after information disclosure. In this section, we solve the former's problem by directly modifying the latter's optimal mechanism.[28]

Under Eső and Szentes (2007)'s optimal mechanism, the seller offers full disclosure and a menu of "information fees" $\hat{c}(\cdot)$ and "strike prices" $\hat{p}(\cdot)$ for the good to implements the benchmark optimal allocation. Thus, $\big(q(\theta), p(\theta)\big) \in \Big\{\big(0, \hat{c}(\theta)\big), \big(1, \hat{c}(\theta) + \hat{p}(\theta)\big)\Big\}$. This menu is a deterministic mechanism. Therefore, following the arguments in the proof of Proposition 1, it is revenue-equivalent to a persuasive-posted price mechanism which offers type θ (i) a binary-signal experiment which sent "*good news*" if $x \geq x_\theta$ and "*bad news*" otherwise, and (ii) a posted price.

$$\tilde{p}(\theta) = \hat{c}(\theta) + \hat{p}(\theta) + \frac{\hat{c}(\theta)\int_{x \leq x_\theta} \mu(x)dx}{\int_{x \geq x_\theta} \mu(x)dx} = \hat{p}(\theta) + \frac{\hat{c}(\theta)}{\int_{x \geq x_\theta} \mu(x)dx}.$$

In addition, Wei and Green (2023) show that information design leads to reverse price discrimination in the continuous model. This feature can also be obtained by leveraging the properties of Eső and Szentes (2007)'s optimal mechanism. Let $\mathbf{X}(\theta) \equiv \frac{1}{1-G(x_\theta)}$ represent the inverted trade probability for θ. Then, $\tilde{p}(\theta) == \hat{p}(\theta) + \hat{c}(\theta)\mathbf{X}(\theta)$, and

$$\tilde{p}'(\theta) = \hat{p}'(\theta) + \hat{c}'(\theta)\mathbf{X}(\theta) + \hat{c}(\theta)\mathbf{X}'(\theta) = \hat{c}(\theta)\mathbf{X}'(\theta) < 0,$$

where the second equality uses the fact that under Eső and Szentes (2007)'s optimal mechanism, $\hat{c}(\theta)$ and $\hat{c}(\theta)$ solves $\hat{c}'(\theta) = \hat{p}'(\theta)[1 - G(x_\theta)] = \hat{p}'(\theta)\frac{1}{\mathbf{X}(\theta)}$, and the last uses $\mathbf{X}'(\theta) < 0$. Thus, $\tilde{p}(\cdot)$ is a decreasing function.

8.3 On the number of signals

As we have seen, it is without loss of generality to offer binary-signal experiments with deterministic allocation. This is no longer true when random mechanisms are necessary. When the variations vary significantly across states, a rich menu is needed to screen the states effectively. As a result, binary-signal experiments are not sufficient. In this section, we illustrate this with a simple example where an optimal experiment sends at least three signals to some type.

Example 10. $\Theta = \{h, m, l\}, X = \{vb, b, g, vg\}$. *Types and states are equally likely.*

$v(\theta, x)$	$x = vb$	$x = b$	$x = g$	$x = vg$
h	7	7	7	7
m	0	3	7	7
l	0	0	0	6

[28] Indeed, this modified mechanism coincides with Wei and Green (2023)'s solution.

In this example, m's valuation varies significantly across states with that at state x_1 being sufficiently low. If restricted to binary-signal experiments, the seller can only separate the state space for type m into two partitions which, under the optimal mechanism, include $\{vb, b\}$ and $\{b, g, vg\}$. Armed with three signals, the seller can distinguish a very unfavorable state vb from a better one b, fine-tuning the design of allocations. The formal proof is in the Online Appendix.

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A PRELIMINARY RESULTS: OMITTED PROOFS

A.1 Proof of Proposition 1

Consider $\mathbf{M}^d \equiv \{q(\theta,s), p(\theta,s), \pi_\theta\}$, a deterministic and feasible mechanism. As $\mathbf{M}^d$ is deterministic, $q(\theta,s) \in \{0,1\}$ for all s. By (MON), to induce the signal truth-telling, it is necessary that $q(\theta,s)$ increases in s. Therefore, it must be that $q(\theta,s) = \mathbb{1}_{s \geq \hat{s}(\theta)}$. Moreover, signal truth-telling also requires

$$p(\theta,s) = \begin{cases} \overline{p}(\theta) & \textit{if } s \geq \hat{s}(\theta), \\ \underline{p}(\theta) & \textit{otherwise.} \end{cases}$$

Hence, for any s, $\big(q(\theta,s),p(\theta,s)\big)=\big(q(\theta),p(\theta)\big)\in\big\{\big(0,\underline{p}(\theta)\big),\big(1,\overline{p}(\theta)\big)\big\}$.

Consider the case with $\underline{p}(\theta)>0$, we complete the arguments in the main text by showing that the seller's revenue remains unchanged under the revised mechanism $\tilde{\mathbf{M}}$ that offers type θ a posted price $\tilde{p}(\theta)$ defined by (1). As trade surplus is not affected, it suffices to show that the buyer of any type obtain the same payoff under the revision. Consider type θ who reports θ'. Note that θ receives weakly less information and moreover, he now either pays weakly more (if $s\geq\hat{s}_\theta$) or pays nothing and earns a zero payoff (if $s\geq\hat{s}_\theta$). Thus, his payoff under $\tilde{\mathbf{M}}$ is weakly lower than that under $\mathbf{M}^d$, regardless of on or off-path. On the other hand, by revealing his type and buying the good only after "*good news*", he secures the payoff level obtained from truth-telling under $\mathbf{M}^d$. Therefore, θ's payoff remains unchanged.

A.2 Proof of Proposition 2

Part (a): Given that $\phi(\theta,x_\theta)>0\ \forall\theta$, the optimal allocation in $(\overline{\mathscr{P}})$ is uniquely given by $\mathbb{Q}(\theta,x)=\mathbb{1}_{x\geq x_\theta}$. If there exists $\mathbf{M}$ such that $R_{\mathbf{M}}=V(\overline{\mathscr{P}})$, $\mathbf{M}$ must be a deterministic mechanism and moreover, its disclosure policy tells the buyer . While the buyer pays the same expected payment under $\mathbf{M}$ and $\mathbf{M}^\star$, he pays more to get the good under $\mathbf{M}^\star$. Thus, $R_{\mathbf{M}^\star}=V(\overline{\mathscr{P}})$.

Part (b):*"Only if":* Suppose $p^\star(\overline{\theta},s^g)=\min_\theta\{p^\star(\theta,s^g)\}$, we now show that $\mathbf{M}^\star$ induces truth-telling even if the seller does not observe signals. Note that $\mathbf{M}^\star$, as a solution to $(\overline{\mathscr{P}})$, induces truth-telling with pbulic signals. Therefore, it suffices to show that under $\mathbf{M}^\star$, with private signals, there is no type θ who prefers to report θ' and then either (i) always report " *bad news*," (ii) always misreporting signals, or (iii) always report a "*good news*," or . By always reporting "*bad news*" off-path, θ obtains a zero payoff and hence, (i) is not beneficial. Now, consider the second deviating behavior. Note that if type θ, who report θ', prefers to misreport "*good news*" (buys the good) rather than truthfully "*bad news*" (and gets a zero payoff), it must be optimal for him to buys he good (or report "*good news*" upon observing this signal. Hence, if (ii) is benficial, so is (iii). However, if (iii) is beneficial, then θ prefer to mimic $\overline{\theta}$ (who receives the lowest price) than truth-telling, which contradicts the fact that the buyer is truthful under $\mathbf{M}^\star$ with public signals. Thus, the buyer does not benefit from either (ii) or (iii).

Part (b): *"If":* Suppose $\exists\theta$ such that $p(\overline{\theta},s^g)>p(\theta,s^g)$. By mimicking θ and always reporting "*good news*", type $\overline{\theta}$ always gets the good at a lower price $p(\theta,s^g)$. Therefore, θ prefers to misreport θ than truth-telling. Consequently, if the seller offers $\mathbf{M}^\star$ with private signals, she obtains $R_{\mathbf{M}^\star}<V(\overline{\mathscr{P}})$.

B SCREENING VS. BUNCHING: OMITTED PROOFS

B.1 Proof of Claim 1

Let

$$\alpha_l \equiv \max\left\{x' \mid x \le \hat{x}_l^\star : \mathbb{E}[v(h,x) \mid x > x'] < \mathbb{E}[v(l,x) \mid x > x']\right\},$$
$$\beta_l \equiv \min\left\{x' \mid x \ge \hat{x}_l^\star : \mathbb{E}[v(h,x) \mid x < x'] < \mathbb{E}[v(l,x) \mid x > x']\right\}.$$

If $x^{\star\star}(l) \in [\underline{x}, \alpha_l]$, the seller can do strictly better by offering a threshold disclosure $\tilde{\pi}(l)$ under which (i) the threshold is α_l^+ and (ii) with probability λ, "*good news*" is sent at α_l^+, such that such that $\omega^{\tilde{\pi}}(h, s^g) = \omega^{\tilde{\pi}}(h, s^b)$. Note that λ exists because by definition of α_l ,

$$\mathbb{E}[v(h,x) \mid x > \alpha_l^+] > \mathbb{E}[v(L,x) \mid x > \alpha_l^+],$$
$$\mathbb{E}[v(h,x) \mid x > \alpha_l] < \mathbb{E}[v(l,x) \mid x > \alpha_l].$$

Thus, it must be that $x^{\star\star}(l) > \alpha_l$. By similar arguments, we also have $x^{\star\star}(l) < \beta_l$. Thus, $x^{\star\star}(l) \in (\alpha_l, \beta_l)$. Hence,

$$\mathbb{E}[v(h,x) \mid x > x^{\star\star}(l)] > \mathbb{E}[v(l,x) \mid x > x^{\star\star}(l)],$$

which implies $\omega^{\pi^{\star\star}(l)}(h, s^b) \ge \omega^{\pi^{\star\star}(l)}(l, s^g)$.

B.2 Proof of Theorem 2

The proof leverages Lemma B.1 below, which provides two expressions of the price gap between two adjacent types under $\mathbf{M}^\star$.

Lemma B.1. *There exist positive functions $\lambda(\theta)$ and $\beta(\theta)$ such that:*

(a) $p^\star(\theta^+, s^g) - p^\star(\theta, s^g) = \Big[\mathbb{E}[v(\theta^+, x) \mid x_{\theta^+} \le x <] - p^\star(\theta, s^g)\Big]\lambda(\theta), \ \forall\ \theta \ge \underline{\theta}.$

(b) $p^\star(\theta, s^g) - p^\star(\theta^-, s^g) = \Big[\mathbb{E}[v(\theta, \omega) \mid x_\theta \le x < x_{\theta^-}] - p^\star(\theta, s^g)\Big]\beta(\theta), \quad \forall\ \theta \ge \underline{\theta}^+.$

Proof of Lemma B.1. To examine the ranking of $p^\star(\cdot)$, we start with the expected payment $\mathbb{P}(\theta) = p^\star(\theta)\sum_{x \ge x_\theta} \mu(x)$. By its definition (see equations (5)),

$$\mathbb{P}(\theta^+) - \mathbb{P}(\theta) = \sum_{x_{\theta^+} \le x < x_\theta} v(\theta^+, x)\mu(x) \quad \forall \theta \ge \underline{\theta},$$
$$\mathbb{P}(\underline{\theta}) = \sum_{x_{\underline{\theta}} \le x < x_{\theta^-}} v(\underline{\theta}, x)\mu(x).$$

Using

$$\sum_{x_{\theta^+}\leq x<x_\theta} v(\theta^+,x)\mu(x) = \mathbb{P}^\star(\theta^+) - \mathbb{P}^\star(\theta) = p^\star(\theta^+,s^g)\sum_{x\geq x_{\theta^+}}\mu(x) - p^\star(\theta,s^g)\sum_{x\geq x_\theta}\mu(x) \tag{10}$$

Part (a). Write the RHS of (10) as $\sum_{x\geq x_{\theta^+}}\mu(x)[p^\star(\theta^+,s^g)-p^\star(\theta,s^g)]+p^\star(\theta,s^g)\sum_{x_{\theta^+}\leq x<x_\theta}\mu(x)$. Then, we obtain

$$\begin{aligned}
p^\star(\theta^+,s^g)-p^\star(\theta,s^g) &= \frac{\sum_{x_{\theta^+}\leq x<x_\theta} v(\theta^+,x)\mu(x) - p^\star(\theta),s^g)\sum_{x_{\theta^+}\leq x<x_\theta}\mu(x)}{\sum_{x\geq x_{\theta^+}}\mu(x)}\\
&= \left[\mathbb{E}[v(\theta^+,x)\mid x_{\theta^+}\leq x<x_\theta] - p^\star(\theta,s^g)\right]\frac{\sum_{x_{\theta^+}\leq x<x_\theta}\mu(x)}{\sum_{x\geq x_{\theta^+}}\mu(x)}\\
&\propto \mathbb{E}[v(\theta^+,x)\mid x_{\theta^+}\leq x<x_\theta] - p^\star(\theta,s^g).
\end{aligned}$$

Part (b). Write the RHS of (10) as $\sum_{x\geq x_\theta}\mu(x)[p^\star(\theta^+,s^g)-p^\star(\theta,s^g)]+p^\star(\theta^+,s^g)\sum_{x_{\theta^+}\leq x<x_\theta}\mu(x)$, and the rest followed by similar arguments. □

Armed with Lemma B.1, we now show that the highest type pays the lowest price under $\mathbf{M}^\star$. It followws from Lemma B.1 that if for all $\theta\geq\underline{\theta}$,

$$\mathbb{E}[v(\theta^+,x)\mid x(\theta^+)\leq x<x_\theta] \leq \mathbb{E}[v(\theta,x)\mid x_\theta\leq x<x_{\theta^-}], \tag{11}$$

then the sign of $[p^\star(\theta^+,s^g)-p^\star(\theta,s^g)]$ is decreasing in θ. Moreover, this sign is non-positive because by (6.1) for $\underline{\theta}$,

$$\mathbb{E}[v(\underline{\theta}^+,x)\mid x(\theta^+)\leq x<x(\underline{\theta})] \leq \mathbb{E}[v(\underline{\theta},x)\mid x(\underline{\theta})\leq x<x(\underline{\theta}^-)] = p^\star(\underline{\theta},s^g),$$

implying $p^\star(\underline{\theta}^+,s^g)-p^\star(\underline{\theta},s^g)\leq 0$, by part (1) of Lemma B.1. Therefore,

$$p^\star(\theta^+,s^g)-p^\star(\theta,s^g)\leq 0 \quad \forall\theta\geq\underline{\theta}. \tag{12}$$

This implies that $p^\star(\theta,s^g)$ is the lowest price. Then by Proposition 2, $R_{\mathbf{M}^\star}=V(\overline{P})$. Moreover, as $\mathbf{M}^\star$ induces truth-telling with private signals, the seller can simply offer a menu of posted prices and threshold disclosure $\{\pi^\star_\theta,p^\star(\theta)\}_\theta$, where $p^\star(\theta)=p^\star(\theta,s^g)$.

B.3 Proof of Theorem 3

Let L be the lowest type being served under an optimal mechanism. Consider the following relaxed problem $(\mathscr{RP}_L)$, under which all types mimics L off-path:

$$(\mathscr{RP}_L) \quad \max_{(\pi,q,U)} \quad \sum_{\theta\geq L}\Big[\sum_x\sum_s v(\theta,x)q(\theta,\omega^{\pi_\theta}(\theta,s))\pi_\theta(s|x)\mu(x)-U(\theta)\Big]f(\theta)$$

$$s.t. \quad U(\theta)-U(L)\geq\sum_s\int_{\omega^{\pi_L}(L,s)}^{\omega^{\pi_L}(\theta,s)}q(L,z)dz\pi_L(s) \quad \forall\theta>L \tag{IC}$$

$$U(L)\geq 0 \tag{IR_L}$$

$$q(\theta,\omega) \text{ increases in } \omega. \tag{MON}$$

We will show that the solution to this relaxed problem, which features a posted price and a threshold disclosure, solves the original problem. Obviously, (IR_L) and $(IC_{\theta\to L})$ bind for all $\theta > L$ under $(\mathscr{RP}_L)$, reducing the seller's relaxed problem to

$$\max_{q,\pi} \quad \sum_\theta\sum_x v(\theta,x)q(\theta,\omega^{\pi_\theta}(\theta,s))\pi_\theta(s|x)\mu(x)f(\theta)-\sum_\theta\sum_x\sum_s\int_{\omega^{\pi_L}(L,s)}^{\omega^{\pi_L}(\theta,s)}q(L,z)dz\pi_L(s|x)\mu(x)f(\theta)$$
$$s.t. \quad q(\theta,\omega) \text{ increases in } \omega.$$

Fix π, it is a linear problem in q with (MON) being the only constraint. Thus, the optimal allocation is generally unique, given by

$$q(L,\omega)=\mathbb{1}_{s\geq\hat{s}_l}, \quad q(\theta,x)=1\forall x\forall\theta>L.$$

Fix $q(L,s)=\mathbb{1}_{s\geq\hat{s}_l}$. The term involving π_L in the seller's objective (revenue) is given by

$$\begin{aligned}
\mathbf{R}(\pi_L)\equiv&\sum_x\sum_{\hat{s}_L}^{\bar{s}}v(L,x)\pi_L(s|x)f(L)g(x)\\
&-\sum_{\theta\geq L^+}\Big[\sum_{\hat{s}_L}^{\bar{s}}\big[\omega^{\pi_L}(\theta,s)-\omega^{\pi_L}(L,\hat{s}_L)\big]-\sum_{\theta\geq L^+}\sum_{\underline{s}}^{\hat{s}_L}\max\big\{\omega^{\pi_L}(\theta,s)-\omega^{\pi_L}(L,\hat{s}_L),0\big\}\Big]\pi_L(s|x)f(\theta)\\
=&\sum_x\sum_{\hat{s}_L}^{\bar{s}}v(L,x)\pi_L(s|x)f(L)g(x)\\
&-\sum_{\theta\geq L^+}\Big[\sum_{\hat{s}_L}^{\bar{s}}\big[\omega^{\pi_L}(\theta,s)-\omega^{\pi_L}(L,\hat{s}_L)\big]-\sum_{\theta\geq L^+}\sum_{\underline{s}}^{\hat{s}_L}[\omega^{\pi_L}(\theta,s)-\omega^{\pi_L}(L,\hat{s}_L),0]\Big]\pi_L(s|x)f(\theta)\\
=&\sum_x\sum_{\hat{s}_L}^{\bar{s}}v(L,x)\pi_L(s|x)f(L)g(x)-\sum_{\theta\geq L^+}\sum_{\underline{s}}^{\bar{s}}\big[\omega^{\pi_L}(\theta,s)-\omega^{\pi_L}(L,\hat{s}_L)\big]\pi_L(s|x)f(\theta)\\
\equiv&\,\overline{\mathbf{R}}(\pi_L).
\end{aligned}$$

$\overline{\mathbf{R}}(\pi_L)$ is an upper bound of $\overline{\mathbf{R}}(\pi_L)$. We now show that this bound is tight. By replacing all signals $s \geq \hat{s}_L$ with "*good news*" and all signals $s < \hat{s}_L$ with "*bad news*," $\overline{\mathbf{R}}(\pi_L)$ weakly increases, and $\overline{\mathbf{R}}(\pi_L)$ reduces to

$$\overline{\mathbf{R}}(\pi_L) = \omega^{\pi_L}(L, s^g)\Big[\sum_{\theta \geq L^+} f(\theta) + f(L)\pi_L(s^g)\Big] - \sum_{\theta \geq L^+} \mathbb{E}[v(\theta, x)] f(\theta),$$

Let

$$\pi_L^{\star\star} \equiv \underset{\pi_L}{\mathrm{argmax}}\, \omega^{\pi_L}(L, s^g)\Big[\sum_{\theta \geq L^+} f(\theta) + f(L)\pi_L(s^g)\Big] - \sum_{\theta \geq L^+} \mathbb{E}[v(\theta, x)] f(\theta)$$

By the same arguments used for the binary-type case, $\pi_L^{\star\star}$ features a disclosure rule. Next, we find an optimal payment schedule. As optimal allocation is deterministic, without loss of generality to focus on posted-price mechanisms. By (IR_L), $p^{\star\star}(L) = \omega^{\pi_L^{\star\star}}(L, s^g)$. Consider type θ who mimics L. Under no uniformly no threshold flip of type order, $\omega^{\pi_L^{\star\star}}(\theta, s^g) \geq \omega^{\pi_L^{\star\star}}(\theta, s^g) \geq \omega^{\pi_L^{\star\star}}(L, s^g)$. Hence, it is optimal for θ to always buy the good after mimicking L. Hence, by (IR) $p^{\star\star}(\theta) = p^{\star\star}(L)$ for all $\theta > L$. Obviously, this single option of price and information $\{\pi_L^{\star\star}, p^{\star\star}(L)\}$ satisfies ignored constraints and hence, solves the original problem.

Remark 2. *Let $V(\mathscr{RP}_\theta)$ denote the value of program $\mathscr{RP}_\theta$ in which θ is the lowest type being served. Under no threshold flip of type order, it is optimal to serve only types above (including) L, where L solves $L \in \mathrm{argmax}_\theta\, V(\mathscr{RP}_2(\theta))$.*

C RANDOM MECHANISMS: OMMITTED PROOFS

If only deterministic mechanisms are allowed, by Corollary 1, it is without loss of generality to focus on persuasive posted-price mechanisms, under which the seller offers a menu $\{p(\theta), \alpha(\theta, x)\}$, where $p(\theta)$ is the price for the good and $\alpha(\theta, x)$ represents the probability with which "*good news*" is sent to θ at state x. By Proposition 3, h receives an efficient allocation, i.e., $\alpha(h, x) = 1$ for all x. It remains to solve $p(h)$ and other types' contracts.

C.1 Proof of Claim 2

The proof for Part (b) follows directly from Theorem 1. It thus remains to prove Part (a). To solve for $p(h)$ and other types' contracts, consider the following relaxed problem in which (i) only IR condition for l is kept, and (ii) off the equilibrium path, m mimics l and buys the good only after "*good news*" whereas h either mimics m and buys the good only after "*good news*" or mimics l

and always buys the good.

$$\max_{\{p(\theta),\alpha(m,x),\alpha(l,x)\}_{\theta,x}} \quad f(h)p(h)+\sum_x \left[f(m)p(m)\alpha(m,x)+f(l)p(l)\alpha(l,x)\right]\mu(x)$$

$$s.t. \quad \mathbb{E}[v(h,x)]-p(h)\geq \sum_x [v(h,x)-p(m)]\alpha(m,x)\mu(x) \qquad (IC_{hm})$$

$$\sum_x [v(m,x)-p(m)]\alpha(m,x)\mu(x)\geq \sum_x [v(m,x)-p(l)]\alpha(l,x)\mu(x) \qquad (IC_{ml})$$

$$\mathbb{E}[v(h,x)]-p(h)\geq \mathbb{E}[v(h,x)]-p(l) \qquad (IC_{hl})$$

$$\sum_x [v(l,x)-p(l)]\alpha(l,x)\mu(x)\geq 0 \qquad (IR_l)$$

If $\alpha(m,b)>0$, reduce $\alpha(m,b)$ and increase $p(m)$ such that $\sum_x p(m)\alpha(m,x)\mu(x)$ remains unchanged. By doing so, the seller's revenue increases. Moreover, no constraints are violated because (i) the right-hand side of (IC_{hm}) decreases (as $v(h,b)>0$) and (ii) the left-hand side of (IC_{ml}) remains unchanged (as $v(m,b)=0$). Thus,

$$\alpha(m,b)=0. \tag{13}$$

If $\alpha(l,b)>0$, reduce $\alpha(l,b)$ and increase $p(l)$ such that $\sum_x p(l)\alpha(l,x)\mu(x)$ remains unchanged. By doing so, the seller's revenue increases. Moreover, no constraints are violated because (i) the right-hand side of (IC_{hl}) decreases (as $v(h,b)>0$) and (ii) the right-hand side of (IC_{ml}) and left-hand side of (IR_l) remains unchanged (as $v(m,b)=v(l,b)=0$).Thus,

$$\alpha(l,b)=0. \tag{14}$$

If $\alpha(l,g)<1$, then reduces $\alpha(l,g)$ by ε and increases $p(h)$ by $[v(h,g)-p(m)]\varepsilon$ and $p(m)$ by $\frac{[v(h,g)-p(m)]\varepsilon}{\alpha(l,g)\mu(g)}$. By doing so, no constraint is affected while the seller's revenue increases by

$$[f(h)+f(m)][v(h,g)-p(m)]\mu(g)\varepsilon-f(l)v(l,g)\mu(g)\varepsilon=\phi(l,g)f(l)\mu(g)<0$$

Thus,

$$\alpha(l,g)=1. \tag{15}$$

If (IR_l) does not bind, increase $p(l)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IR_l) binds and hence,

$$p(l)=v(l,g)=5. \tag{16}$$

If (IC_{ml}) does not bind, increase $p(m)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IC_{ml}) binds, or

$$[v(m,x)-p(m)]\alpha(m,x)\mu(x)\geq \sum_x [v(m,x)-p(l)]\alpha(l,x)\mu(x). \tag{17}$$

Using (13), (14), (15), (16), and (25), the relaxed problem becomes

$$\max_{\{p(h),p(m),\alpha(m,g)\}} \quad f(h)p(h)+f(m)v(m,g)\alpha(m,g)\mu(g)$$

$$s.t. \quad \mathbb{E}[v(h,x)]-p(h)\geq[v(h,g)-v(m,g)]\alpha(m,g)\mu(g)+[v(m,g)-v(l,g)]\mu(g) \tag{IC_{hm}}$$

$$\mathbb{E}[v(h,x)]-p(h)\geq\mathbb{E}[v(h,x)]-v(l,g). \tag{IC_{hl}}$$

If (IC_{hm}) does not bind, then it is optimal to increase $\alpha(m,g)$ until $\alpha(m,g)=1$. and increase $p(h)$ until (IC_{hl}) bind. Then by binding (IC_{hl}), implies $p(h)=v(l,g)=5$. By $\alpha(m,g)=1$ and binding (IC_{ml}), obtain $p(m)=p(l)=5$.

If (IC_{hm}) binds, or $\mathbb{E}[v(h,x)]-p(h)=[v(h,g)-v(m,g)]\alpha(m,g)\mu(g)+[v(m,g)-v(l,g)]\mu(g)$. This reduces the relaxed problem to

$$\max_{\alpha(m,g)} \quad f(m)v(m,g)\alpha(m,g)\mu(g)-f(h)[v(h,g)-v(m,g)]\alpha(m,g)\mu(g)$$

$$s.t. \quad [v(h,g)-v(m,g)]\alpha(m,g)\mu(g)+[v(m,g)-v(l,g)]\mu(g)\geq\mathbb{E}[v(h,x)]-p(l). \tag{IC_{hl}}$$

As the objective function and the left-hand side of the constraint both increase in $\alpha(m,g)$, it is optimal to set $\alpha(m,g)=1$. Then, by binding (IC_{ml}) and (IC_{hm}), $p(h)=p(m)=p(l)=5$.

Therefore, in any case, we obtain

$$\alpha(h,g)=\alpha(h,b)=1, \quad \alpha(m,x)=\alpha(l,x)=\mathbb{1}_{x=g}, \tag{18}$$

$$p(h)=p(m)=p(l)=5. \tag{19}$$

The seller's revenue is $4\cdot\Big[f(h)+[f(m)+f(l)]\mu(g)\Big]=\frac{10}{3}$. Note that with $p(h)=5$, type h buys the good at any state. Therefore, this revenue level is obtained by offering a posted price of 4 and full disclosure to all types.

C.2 Proof of Claim 3

To solve for $p(h)$ and other types' contracts, onsider the following relaxed problem:

$$\max_{\{p(\theta)\alpha(\theta,x)\}_x} \quad f(h)p(h)+\sum_x\big[f(m)p(m)\alpha(m,x)+f(l)p(l)\alpha(l,x)\big]\mu(x)$$

$$s.t. \quad \mathbb{E}[v(h,x)]-p(h)\geq\mathbb{E}[v(h,x)]-p(m) \tag{IC_{hm}}$$

$$\sum_x[v(m,x)-p(m)]\alpha(m,x)\mu(x)\geq\sum_x[v(m,x)-p(l)]\alpha(l,x)\mu(x) \tag{IC_{ml}}$$

$$\sum_x[v(l,x)-p(l)]\alpha(l,x)\mu(x)\geq 0 \tag{IR_l}$$

If $\alpha(l,b) > 0$, reduce $\alpha(m,b)$ and and increase $p(l)$ such that $p(l)\sum_x \alpha(l,x)\mu(x)$ remains unchanged, and increase $p(m)$ such that (IC_{ml}) remains satisfied. By doing so, no constraints are affected, while the seller's revenue strictly increases. Thus,

$$\alpha(l,b) = 0. \tag{20}$$

Note that to ensure that type m's on-path payoff is non-negative, it is necessary that $v(m,g) \geq p(m)$. Hence, if $\alpha(m,g) < 1$, by increasing $\alpha(m,g)$, we strictly improve the seller's revenue while not violating any constraints. Thus

$$\alpha(m,g) = 1. \tag{21}$$

If $\alpha(l,g) < 1$. Then, set $p(l) = v(l,g) = 4$, increase $\alpha(l,g)$ by ε and reduce $p(m)$ and $p(h)$ by $\frac{\varepsilon}{\sum_x \alpha(m,x)\mu(x)}$. Under this change, no constraint are violated. Morever, the seller's revenue increases by $f(L)4\varepsilon - f(H)\frac{\varepsilon}{\sum_x \alpha(m,x)\mu(x)} - f(M)\varepsilon > 0$. Thus

$$\alpha(l,g) = 1. \tag{22}$$

If (IR_l) does not bind, we can increase $p(l)$ upto it becoming binding, thereby increasing the seller's revenue without violating any constraints. Thus, (IR_l) binds. Given that $\alpha(l,b) = 0$, we thus have

$$p(l) = v(l,g) = 4. \tag{23}$$

If (IC_{hm}) does not bind, increase $p(h)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IC_{ml}) binds, and hence,

$$p(h) = p(m) \tag{24}$$

If (IC_{ml}) does not bind, increase $p(m)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IC_{ml}) binds. Given that $\alpha(m,g) = \alpha(l,g) = 1$, $\alpha(l,b) = 0$ and $p(l) = v(l,g)$, this implies

$$\begin{aligned}
[v(m,b) - p(m)]\alpha(m,b)\mu(b) + [v(m,g) - p(m)]\mu(g) &= [v(m,g) - v(l,g)]\mu(g) \\
\Leftrightarrow p(m) &= \frac{v(m,b)\alpha(m,b)\mu(b) + v(m,g)\mu(g) - [v(m,g) - v(l,g)]\mu(g)}{\alpha(m,b)\mu(b) + \mu(g)}.
\end{aligned} \tag{25}$$

Using (13), (14), (15), (16), and (25), the relaxed problem becomes

$$\begin{aligned}
&\max_{\alpha(m,b)} H(\alpha(m,b)) \\
&\equiv \Big[f(h) + f(m)[\alpha(m,b)\mu(b) + \mu(g)]\Big] \frac{v(m,b)\alpha(m,b)\mu(b) + v(m,g)\mu(g) - [v(m,g) - v(l,g)]\mu(g)}{\alpha(m,b)\mu(b) + \mu(g)}
\end{aligned}$$

Under the specification in Example 6, this problem is given by Using (13), (14), (15), (16), and (25), this problem is given by

$$\max_{\alpha(m,b)} H(\alpha(m,b)) \equiv (\alpha(m,b)+3)\frac{(\alpha(m,b)+2)}{\alpha(m,b)+1} = \alpha(m,b)+2+\frac{2(\alpha(m,b)+2)}{\alpha(m,b)+1}$$

Thus, $H'(\alpha(m,b)) = 1-\frac{1}{(\alpha(m,b)+1)^2}$ and $H"(\alpha(m,b)) = \frac{2}{(\alpha(m,b)+1)^3} > 0$. Therefore, $H(\alpha(m,b))$ is a convex function. Moreover, $R(0) = R(1) = 6$. Thus, $\alpha(m,b) = 0$ is optimal. This implies that $p(h) = p(m) = p(l) = 4$. Hence, a posted price $p = 4$ and full disclosure is an optimal deterministic mechanism.

D INFINITE TYPES: OMITTED PROOFS

D.1 Proof of for Proposition 4

Proof of for Proposition 4, Part (a). By Proposition 5 for binary states $X = \{b, g\}$, under the partition flip of type order within the cut-off types, which reduces to $v(\theta_g, g) \le v(\theta_g, g)$ with binary states, a menu of prices and threshold disclosures is optimal. □

Proof of for Proposition 4, Part (b). Let $\tilde{\mathbf{M}}$ be an arbitrary optimal (deterministic) mechanism. By Corollary 1, it is without loss to assume that $\tilde{\mathbf{M}}$ is a persuasive posted-price mechanism, or a menu $\{p(\theta), \alpha(x,\theta)$ where $\alpha(x,\theta)$ is the probability that θ receives "*good news*" at state x and $p(\theta)$ is the price for the good. Let $\tilde{\theta}_b \equiv \inf\{\theta \mid \alpha(\theta, x) = 1 \forall x\}$ represents the lowest type who receives an efficient allocation under $\tilde{\mathbf{M}}$, and $\tilde{\theta}_g \equiv \inf\{\theta \mid \alpha(\theta, x) > 0$ for some $x\}$ be the lowest type being served. With $\tilde{\Theta} \equiv \{\theta \mid \theta \ge \tilde{\theta}_b\}$, $\tilde{\mathbf{M}}$ must solve the following problem:

$$(\mathscr{P}) \sup_{p,\alpha} \quad \int_\theta p(\theta) dF(\theta)$$

$$s.t. \quad \alpha(\theta, x) = 1 \quad \forall x, \theta \ge \tilde{\theta}_b$$

$$\sum_x [v(\theta,x) - p(\theta)]\alpha(\theta,x)\mu(x) \ge \sum_x [v(\theta,x) - p(\theta')]\alpha(\theta',x)\mu(x) \quad \forall \theta,\theta' \in \tilde{\Theta}$$

$$\sum_x [v(\theta,x) - p(\theta)]\alpha(\theta,x)\mu(x) \ge 0 \quad \forall \theta \in \tilde{\Theta}.$$

By IR condition for $\tilde{\theta}_g$, $p(\tilde{\theta}_g) \le v(\tilde{\theta}_g, g)$. Consider $\theta \in [\tilde{\theta}_b, \overline{\theta}]$. If $p(\theta) > v(\tilde{\theta}_g, g)$, then θ prefers to mimic $\tilde{\theta}_g$ and always buy the good at a lower price. Thus, to incentivize θ to reveal his type, it must be that $p(\theta) \le v(\tilde{\theta}_g, g)$. Suppose $v(\tilde{\theta}_b, b) > v(\tilde{\theta}_g, g)$. Then, it is optimal for such a type θ' to mimic $\overline{\theta}$ to always buy the good (and hence, enjoy a higher expected surplus) at a lower price. Therefore, it must be that

$$v(\tilde{\theta}_b, b) \le v(\tilde{\theta}_g, g).$$

By Envelope condition, for the buyer to report truthfully his type, it is necessary that $U'(\theta) = \sum_x v_\theta(\theta, x)\alpha(\theta, x)\mu(x)$. Then, by integration by parts,

$$U(\theta) = U(\tilde{\theta}_g) + \int_{\tilde{\theta}_g}^{\overline{\theta}} \sum_x v_\theta(\theta, x)\alpha(\theta, x)\mu(x)d\theta \tag{26}$$

Consider a relaxed problem that keeps only the IR condition for $\tilde{\theta}_g$ and the necessary envelope condition for truth-telling. Using (26) and the fact that $U(\tilde{\theta}_g) = 0$ at optimum, this relaxed problem reduces to

$$\sup_q \quad \int_\theta \phi^c(\theta, x)q(\theta, x)\mu(x)]f(\theta) \quad s.t. \quad q(\theta, x) = 1 \quad \forall x, \forall \theta \geq \tilde{\theta}_b,$$

where $\phi^c(\theta, x) \equiv v(\theta, x) - v_\theta(\theta, x)\frac{1-F(\theta)}{f(\theta)}$. Solving this point-wise maximization problem yields

$$q(\theta, x) = \begin{cases} 1 & \text{if} \quad \theta \geq \min\{\theta_b, \tilde{\theta}_b\} \\ \mathbb{1}_{x=g} & \text{if} \quad \max\{\theta_g, \tilde{\theta}_g\} \leq \theta \leq \min\{\theta_b, \tilde{\theta}_b\} \end{cases}$$

Prices are pinned down using binding constraints, given by

$$p(\theta) = \begin{cases} v(\min\{\theta_b, \tilde{\theta}_b\}, b)\mu(b) + v(\max\{\theta_g, \tilde{\theta}_g\}, g)\mu(g) & \text{if} \quad \theta \geq \min\{\theta_b, \tilde{\theta}_b\} \\ v(\max\{\theta_g, \tilde{\theta}_g\}, g) & \text{if} \quad \max\{\theta_g, \tilde{\theta}_g\} \leq \theta \leq \min\{\theta_b, \tilde{\theta}_b\} \end{cases}$$

Consider $\theta \geq \min\{\theta_b, \tilde{\theta}_b\}$ and $\theta' \in [\max\{\theta_g, \tilde{\theta}_g\} \leq \theta, \min\{\theta_b, \tilde{\theta}_b\}]$. As $v(\tilde{\theta}_b, b) \leq v(\tilde{\theta}_g, g)$, we have

$$v(\min\{\theta_b, \tilde{\theta}_b\}, b) \leq v(\max\{\theta_g, \tilde{\theta}_g\}, g),$$

which implies $p(\theta) \leq p(\theta')$. Thus, this two-option menu of prices and threshold disclosure induces participation and truth-telling. As $\tilde{\mathbf{M}}$ solves the original problem, by definition of $\tilde{\theta}_b$ and $\tilde{\theta}_g$, it must be that

$$\min\{\theta_b, \tilde{\theta}_b\} = \tilde{\theta}_b, \quad \max\{\theta_g, \tilde{\theta}_g\} = \tilde{\theta}_g.$$

Suppose $v(\min\{\theta_b, \tilde{\theta}_b\}, b) < v(\max\{\theta_g, \tilde{\theta}_g\}, g)$, then $p(\theta) < p(\theta')$. Let $\hat{\theta} \equiv \inf\{\theta \geq \theta_g \mid v(\min\{\theta_b, \tilde{\theta}_b\}, b) \leq v(\max\{\hat{\theta}, \tilde{\theta}_g\}, g)\}$. Then, it is optimal to set $\hat{\theta}$ as the lowest type being served, a contradiction. Therefore, $v(\min\{\theta_b, \tilde{\theta}_b\}, b) = v(\max\{\theta_g, \tilde{\theta}_g\}, g)$. Hence, all types receive the same price and information does not matter for the purchasing decision of any type $\theta \geq \min\{\theta_b, \tilde{\theta}_b\}$. Thus, it is optimal to offer full disclosure for all types. □

D.2 Proof of for Proposition 5

The proof by first solving the seller's benchmark problem with public signals and a continuum of types and then showing that under the partition flip of type order in Θ_x, privacy of signals does not matter. With $\mathbb{P}(\theta)$ and $\mathbb{Q}(\theta,x)$ representing the expected payment and allocation over signals, the benchmark problem writes:

$$\begin{aligned} &\sup_{\mathbb{P},\mathbb{Q}} \quad \int_\theta \mathbb{P}(\theta)dF(\theta) \\ s.t. \quad \forall\theta,\theta': \quad &\sum_x v(\theta,x)\mathbb{Q}(\theta,x)\mu(x)-\mathbb{P}(\theta) \geq \sum_x v(\theta,x)\mathbb{Q}(\theta',x)\mu(x)-\mathbb{P}(\theta') \\ &\sum_x v(\theta,x)\mathbb{Q}(\theta,x)\mu(x)-\mathbb{P}(\theta) \geq 0. \end{aligned}$$

By Envelope condition, truth-telling implies $U'(\theta)=\sum_x v_\theta(\theta,x)\mathbb{Q}(\theta,x)\mu(x)\ \forall\theta\geq\tilde{\theta}_g$. Then by integration by parts, we have

$$U(\theta)=U(\tilde{\theta}_g)+\int_{\tilde{\theta}_g}^{\theta}\sum_x v_\theta(\theta',x)\mathbb{Q}(\theta',x)\mu(x)d\theta'. \tag{27}$$

Consider a relaxed problem which keeps only the Envelope condition and the partition constraint for the lowest type. Using $U(\underline{\theta})=0$ at optimum, this relaxed problem reduces to

$$\sup_{\pi,q} \int_\theta \phi^c(\theta,x)\mathbb{Q}(\theta,x)\mu(x)]dF(\theta),$$

where $\phi^c(\theta,x)\equiv v(\theta,x)-v_\theta(\theta,x)\frac{1-F(\theta)}{f(\theta)}$. As $\phi^c(\theta,x)$ increases in θ and x, it is optimal to se $\mathbb{Q}(\theta,x)=\mathbb{1}_{\theta\geq\theta_x}$ or equivalently, $\mathbb{Q}(\theta,x)=\mathbb{1}_{x\geq x_\theta}$. Fix an arbitrary $x\in X$. For any $\theta\in[\theta_x,\theta_{x^-}]$, $\mathbb{Q}(\theta)=\mathbb{Q}(\theta_x)$ and $\mathbb{P}(\theta)=\mathbb{P}(\theta_x)$. Payments are backed out using $U(\tilde{\theta})=0$ and (27), given by:

$$\mathbb{P}(\theta)=\sum_{x\geq x_\theta} v(\theta,x)\mu(x)-\int_\theta\sum_{x\geq x_{\theta'}} v_\theta(\theta',x)\mu(x)d\theta'$$

Moreover,

$$\int_{\theta_x}^{\theta_x^-}\sum_{x\geq x_{\theta'}} v_\theta(\theta',x)\mu(x)d\theta'=\int_{\theta_x}^{\theta_x^-}\sum_{x\geq x_{\theta_x}} v_\theta(\theta',x)\mu(x)d\theta'=\sum_{x\geq x_{\theta_x}}\int_{\theta_x}^{\theta_x^-} v_\theta(\theta',x)\mu(x)=\sum_{x\geq x_{\theta_x}}[v(\theta_x^-,x)-v(\theta_x,x)]\mu(x),$$

which implies

$$\mathbb{P}(\theta)=\sum_{x\geq x_\theta} v(\theta,x)\mu(x)-\sum_{\theta_x\leq\theta}\sum_{x\geq x_{\theta_x}}[v(\theta_{x^-},x)-v(\theta_x,x)]\mu(x).$$

Following similar arguments in the proof of Theorem 2, under the partition flip of type order within Θ_x, $p^\star(\overline{\theta})=p^\star(\theta_{\underline{x}})=\min_{\theta\in\Theta_x}p^\star(\theta_{\underline{x}},s^g)=\min_{\theta\in\Theta}p^\star(\theta,s^g)$. Therefore, the seller can achieve the benchmark revenue $V(\overline{P})$ via a menu of prices and threshold disclosures.

D.3 Proof of for Proposition 6

Suppose it is optimal to exclude all types below L, or $q(\theta,x)=1$ for all x and $\theta<L$. Then, the seller's revenue must be weakly lower than that obtained from selling to the buyer whose types is distributed by $\hat{f}$ over Θ, where $\hat{f}(\theta)=f(\theta)\ \forall\theta\notin[L,L+\varepsilon]$, $\hat{f}(\theta)=0\ \forall\theta\in[L,L+\varepsilon)$, and $\hat{f}(L+\varepsilon)=\int_{\theta=L}^{\theta=L+\varepsilon}f(\theta)d\theta$. Let $(\hat{P})$ represent the seller's problem when $\theta\sim\hat{f}$ and $V(\hat{P})$ the corresponding value. Consider the following relaxed problem of $(\hat{P})$ where all types mimic $L+\varepsilon$ off the equilibrium path:

$$(\mathscr{RP}_{L+\varepsilon})\quad \max_{(\pi,q,U)}\ \sum_{\theta\geq L+\varepsilon}\sum_{x}\sum_{s}p(\theta,\omega^{\pi_\theta}(\theta,s))\pi_\theta(s|x)\mu(x)\hat{f}(\theta)$$

$$s.t.\quad U(\theta)-U(L+\varepsilon)\geq\sum_{s}\int_{\omega^{\pi_L}(L+\varepsilon,s)}^{\omega^{\pi_L}(\theta,s)}q(L+\varepsilon,z)dz\pi_{L+\varepsilon}(s)\quad\forall\theta>L+\varepsilon\quad(IC_{\theta\to L+\varepsilon})$$

$$U(L+\varepsilon)\geq 0\qquad (IR_{L+\varepsilon})$$

$$q(\theta,\omega)\text{ increases in }\omega.\qquad\text{(MON)}$$

By the same arguments as the proof of Theorem 3, a posted price $\hat{p}_{L+\varepsilon}$, associated with a threshold disclosure $\hat{\pi}_{L+\varepsilon}$, solves this relaxed problem. Note that $(\hat{\pi}_{L+\varepsilon},\hat{p}_{L+\varepsilon})$ does not necessary solve the original problem. In case it does, the seller's revenue is $V(\mathscr{RP}_{L+\varepsilon})$. Let R_ε represent the seller's revenue if she offers $(\tilde{\pi}_{L+\varepsilon},\tilde{p}_{L+\varepsilon})$ (regardless of whether it solves the original problem or not). Then,

$$\begin{aligned}R_\varepsilon&\geq V(\mathscr{RP}_{L+\varepsilon})-\int_{L+\varepsilon}^{L+2\varepsilon}\hat{f}(\theta)d\theta\mathbb{E}[v(L+2\varepsilon,x)]\\&\geq V(\hat{P})-\int_{L+\varepsilon}^{L+2\varepsilon}\hat{f}(\theta)d\theta\mathbb{E}[v(L+2\varepsilon,x)]\end{aligned}$$

Therefore,

$$\lim_{\varepsilon\to0}R_\varepsilon\geq V(\hat{P})-\lim_{\varepsilon\to0}\int_{L+\varepsilon}^{L+2\varepsilon}\hat{f}(\theta)d\theta\mathbb{E}[v(L+2\varepsilon,x)]=V(\hat{P})$$

On the other hand, $\lim_{\varepsilon\to0}R_\varepsilon\leq V(\hat{P})$. Therefore, $\lim_{\varepsilon\to0}R_\varepsilon=V(\hat{P})$.