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Flutter and Divergence of Sweptback and Sweptforward Wings

-by-

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SUMMARY

In this note, the equations of the flexural-torsional flutter of a swept wing are established, assuming the wing to be semi-rigid and fixed at the root. The general effect of sweepback, wing stiffness and position of the inertia axis are determined. The critical speeds for flutter and for wing divergence are determined (i) for incompressible flow (ii) for compressible flow, assuming a modified Glauert correction.

The critical flutter speed is in general higher for a sweptback wing having the same wing stiffness as the unswept wing; for a swept forward wing, divergence will occur before flutter.

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NOTATION

Dimensions and Displacements of Wing (see Figure 1)

c = chord at distance y from root chord (parallel to the root chord)

c_o = root chord

c_m = mean chord

c_t = tip chord

d = 0.9 s

f and F define the flexural and torsional modes of oscillation

gc = chordwise distance from leading edge to inertia axis

hc = chordwise distance from leading edge to flexural axis

jc = chordwise distance from flexural axis to inertia axis

ℓ = 0.7 s = perpendicular distance from wing root to flexural centre of reference section

s = perpendicular distance from wing root to tip

s' = distance from wing root to tip, measured along flexural axis

y = perpendicular distance from wing root to a given chordwise element

α = angle of incidence of wing

η = y/ℓ

ϕ = normal displacement of flexural centre at a given chordwise element

ψ = slope of flexural axis at a given chordwise element

θ = angle of twist of a given section perpendicular to the flexural axis.

β = angle of sweepback of flexural axis.

Density

ϵ = air density/wing density = ρ/σ_ω

ρ = air density in slugs per cubic foot

σ_ω = wing density = wing mass per unit area/mean chord, in slugs per cubic foot.

Stiffness coefficients

ℓ_ϕ = elastic moment about perpendicular to flexural axis for unit displacement ϕ_r at the reference section

m_θ = elastic moment about flexural axis for unit displacement θ_r at the reference section

$$B = \frac{V_c\sqrt{\rho}}{\sqrt{\dfrac{m_\theta}{dc_m^2}}}$$

$$r = \frac{\ell_\phi/d^3}{m_\theta/dc_m^2}$$

V = forward speed of aircraft

V_c = critical flutter speed

Introduction

In this note, the equations of the flexural-torsional flutter of a swept wing are established, assuming the wing to be semi-rigid and fixed at the root. The general effects of sweepback, wing stiffness and position of the inertia axis are determined. The critical speeds for flutter and for wing divergence are determined (i) for incompressible flow (ii) for compressible flow, assuming a modified Glauert correction.

Data and Assumptions

General A straight tapered swept wing is considered (Figure 1). The flexural and inertia axes are taken at given constant percentage chord distances behind the leading edge.

Principal Dimensions

s = span (root to tip), perpendicular to root chord

d = perpendicular distance from root to 'equivalent tip section'

= 0.9s

ℓ = perpendicular distance from root to flexural centre of the 'reference section'

= 0.7s

c_o = root chord

c_t = tip chord

c_m = mean chord

hc = distance of flexural axis aft of leading edge (measured parallel to root chord)

gc = distance of inertia axis aft of leading edge (measured parallel to root chord)

$1-\tau$ = taper ratio = c_t/c_o.

β = angle of sweep back of flexural axis. Corresponding distances along the flexural axis are indicated by dashes; thus

s' = span measured along the flexural axis.

Axes Ox, Oy are taken parallel and perpendicular to the root chord through the point O, where the flexural axis meets the root chord. Axes Ox', Oy' are taken perpendicular to and along the flexural axis.

Modes of motion and displacement coordinates

The wing is assumed to be semi-rigid, the modes of displacement in flexure and in torsion being taken to be independent of the speed; all displacements of either kind are taken to be in phase with one another. The modes of displacement are taken to be linear in torsion and parabolic in flexure; this approximates closely to the natural modes of the system.

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The displacement coordinates are defined as follows:-
The flexural coordinate ϕ is the flexural displacement of the flexural centre at a given section divided by y' (positive for downward bending).
The torsional coordinate θ is the angle of twist of a given section perpendicular to the flexural axis measured relative to the corresponding root section Ox', (positive when the trailing edge moves down relative to the leading edge). θ_r and ϕ_r are the flexural and torsional coordinates of the reference section, (the section perpendicular to the flexural axis at 70 per cent of the span, measured along the flexural axis).

The wing is supposed to be placed at a small angle of incidence in a uniform airstream of speed V (Mach number M) and the wing root is supposed to be rigidly fixed.

The displacements θ, ϕ of any point are related to the corresponding displacements at the reference section θ_r, ϕ_r by the equations

$$\frac{\phi}{\phi_r} = \frac{\ell' f(\eta)}{y'} \quad ; \quad \frac{\theta}{\theta_r} = F(\eta)$$

where $\eta = y/\ell = y'/\ell'$.

The symbols used in the equation of motion conform with those in references 1 and 2.

Elastic stiffness coefficients

The flexural and torsional coefficients are denoted by ℓ_ϕ and m_θ respectively. The non-dimensional flutter speed coefficients are plotted against the modified stiffness ratio r defined by

$$r = \frac{\ell_\phi}{d^3} \bigg/ \frac{m_\theta}{dc_m^2} = \frac{\ell_\phi}{m_\theta} \cdot \frac{c_m^2}{d^2}$$

Wing density

The wing density σ_ω is defined to be the total wing mass in slugs divided by the product of the wing area in square feet and the mean chord in feet.

Also $\epsilon = \rho/\sigma_\omega$

where ρ is the air density in slugs per cubic foot.

Let σ_{ω_β}, σ_{ω_o} be the wing densities for a swept and for an unswept wing of the same area and mean chord. The swept wing will have a larger weight due to its larger span, measured along the flexural axis. It can be shown on theoretical grounds

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that the weight of a swept wing should vary approximately as $\sec^2\beta$.

$$\therefore \quad \sigma_{\omega_\beta} = \sigma_{\omega_o} \sec^2\beta$$

$$\text{and} \quad \epsilon = \sec^2\beta . \, \epsilon_o$$

The inertial coefficients

To find the inertial coefficient, we replace the given wing by the wing ABB'A', considering the section AA' to be rigidly fixed to the fuselage.

As in references 1 and 2, we assume that the mass per unit span (measured along the flexural axis) is $m_\beta \bar{c}^2$ where $\bar{c}$ = local chord perpendicular to the flexural axis and m_β is constant for a given angle of sweepback.

We have approximately $\bar{c} = c \cos\beta$
where c = local chord measured // to the line of flight.

$$\therefore \quad \text{Total wing mass} = 2m_\beta \int_o^{S'} \bar{c}^2 \, dy'$$

$$= 2m_\beta \cos\beta \, c_o^2 \, s \left[1 - \tau + \frac{\tau^2}{3}\right]$$

For the unswept wing, total wing mass $= 2\, m_o \, c_o^2 \, s \left[1 - \tau + \frac{\tau^2}{3}\right]$

Assuming as above that the wing weight varies as $\sec^2\beta$,

$$m_\beta = m_o \sec^3\beta \, .$$

For both swept and unswept wings, total wing area $= 2s\, c_o \left[1 - \frac{\tau}{2}\right]$

and mean chord $= c_o \left[1 - \frac{\tau}{2}\right]$

$$\text{Now} \quad \sigma_\omega = \frac{\text{wing mass}}{\text{wing area x mean chord}}$$

$$\therefore \quad \frac{\sigma_{\omega_\beta}}{m_\beta} = \frac{\sigma_{\omega_o}}{m_o} = \frac{4}{3} \quad \frac{3-3\tau+\tau^2}{4-4\tau+\tau^2}$$

We also assume (as in references 1 and 2) that the radius of gyration $k\bar{c}$ of a chord wise section about a transverse axis through the inertia centre of the section is a constant percentage of the chord. ($k = 0.294$).

Let δm = wt of wing element $\delta x' \, \delta y'$ at point (x', y').

As in references 1 and 2, the inertia coefficients are given by the following formulae:-

$$A_1 = \sum \delta m \, y'^2 \left(\frac{\phi}{\phi_r}\right)^2$$

$$= \int_0^{s'} m_o \sec^3\beta . \, c^2 \cos^2\beta \, \ell'^2 \, f^2 \, dy' = \int_0^{10/7} m_o c^2 \, \ell^3 \, f^2 \sec^4\beta \, d\eta$$

$$= \frac{\rho \, \ell^3 c_o^2 \, a_1}{\epsilon_o}$$

where $$a_1 = \frac{m_o}{\sigma_{\omega_o}} \int_0^{10/7} \left(\frac{c}{c_o}\right)^2 f^2 \sec^4\beta \, d\eta$$

$$\epsilon_o = \frac{\rho}{\sigma_{\omega_o}} \quad \text{and} \quad y' = \eta \, \ell'.$$

Similarly $$A_3 = G_1 = \sum \delta m \, x'y' \left(\frac{\theta}{\theta_r}\right)\left(\frac{\phi}{\phi_r}\right)$$

$$= \int_0^{s'} m_o \sec^3\beta . c^2 \cos^2\beta . \, \ell' fF . \, jc \cos\beta \, dy'$$

$$= \int_0^{10/7} m_o \, c^3 \, \ell^2 \, jfF \sec^2\beta \, d\eta = \frac{\rho \, \ell^2 \, c_o^3 \, a_3}{\epsilon_o}$$

where $$a_3 = g_1 = j \, \frac{m_o}{\sigma_{\omega_o}} \int_0^{10/7} \left(\frac{c}{c_o}\right)^3 fF \sec^2\beta \, d\eta$$

and the centre of inertia of any section is distance $j\bar{c}$ behind the flexural axis.

Also $$G_3 = \sum \delta m \, x'^2 \left(\frac{\theta}{\theta_r}\right)^2$$

$$= \int_0^{s'} m_o \sec^3\beta . \, c^2 \cos^2\beta . F^2 . \, \lambda^2 c^2 \cos^2\beta \, dy'$$

$$= \int_0^{10/7} m_o c^4 \lambda^2 \, \ell F^2 \, d\eta = \frac{\rho \, \ell c_o^4 \, g_3}{\epsilon_o}$$

where $$g_3 = \lambda^2 \, \frac{m_o}{\sigma_{\omega_o}} \int_0^{10/7} \left(\frac{c}{c_o}\right)^4 F^2 \, d\eta$$

and $\lambda^2 = k^2 + j^2.$

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Thus a_1 varies as $\sec^4\beta$, a_3 ($= g_1$) as $\sec^2\beta$ and g_3 is independent of β.

The aerodynamic coefficients

We consider the forces acting on a chordwise strip of the wing (parallel to the line of flight). The geometrical angle of incidence α and the downward displacement of the leading edge of this chordwise strip are given by

$$\alpha = \theta \cos\beta + \psi \sin\beta$$
$$z = \phi \eta \ell' - \theta hc \cos\beta$$

where ψ is the slope of the flexural axis at the section considered, and any chordwise change of camber has been neglected.

$$\phi = \phi_r \; f/\eta$$
$$\psi = \phi_r \; \partial f/\partial\eta = \phi_r f'$$
$$\theta = \theta_r F$$

f and F being functions of $\eta = y/\ell$.

For the aerofoil element, the lift and moment coefficients referred to the leading edge are given by

$$C_L = \alpha \frac{\partial C_L}{\partial \alpha} + \dot{\alpha} \frac{\partial C_L}{\partial \dot{\alpha}} + \dot{z} \frac{\partial C_L}{\partial \dot{z}}$$

$$C_m = \alpha \frac{\partial C_m}{\partial \alpha} + \dot{\alpha} \frac{\partial C_m}{\partial \dot{\alpha}} + \dot{z} \frac{\partial C_m}{\partial \dot{z}}$$

where α is the geometric angle of incidence and $\dot{z}$ is the downward velocity of the leading edge.

In the standard notation, the downward normal force is given by

$$\delta Z = - \tfrac{1}{2} \rho V^2 c \, \delta C_L \, \ell d\eta$$
$$= - \rho V c \ell \left(\alpha V \ell_\alpha + \dot{z} \ell_{\dot{z}} + \dot{\alpha} c \ell_{\dot{\alpha}}\right) d\eta$$

$\therefore$ substituting for α, $\dot{\alpha}$ and $\dot{z}$,

$$- \frac{\delta Z}{\rho V \ell c \, d\eta} = (\theta_r F \cos\beta + \phi_r f' \sin\beta) V \ell_\alpha$$
$$+ (\dot{\phi}_r \ell' f - \dot{\theta}_r hc F \cos\beta) \ell_{\dot{z}}$$
$$+ (\dot{\theta}_r F \cos\beta + \dot{\phi}_r f' \sin\beta) \ell_{\dot{\alpha}} c$$
$$= \theta_r F \cos\beta \, V \ell_\alpha + \phi_r f' \sin\beta \, V \ell_\alpha$$
$$+ \dot{\theta}_r \left[F \cos\beta \, \ell_{\dot{\alpha}} c - hc F \ell_{\dot{z}} \cos\beta\right]$$
$$+ \dot{\phi}_r \left[f' \sin\beta \, \ell_{\dot{\alpha}} c + \ell' f \ell_{\dot{z}}\right]$$

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Similarly if δM is the pitching moment on the strip, about the flexural centre,

$$\delta M = \tfrac{1}{2}\rho V^2 \ell c^2 \; (\delta C_m + h\,\delta C_L)\; d\eta'$$

$$= \rho V \ell c^2 \; (\alpha V m_\alpha + \dot z\, m_{\dot z} + \dot\alpha\, c m_{\dot\alpha}$$

$$+ h\alpha V \ell_\alpha + h\dot z\,\ell_{\dot z} + h\dot\alpha\, c\,\ell_{\dot\alpha})\; d\eta'.$$

Substituting for α, $\dot\alpha$ and z,

$$\frac{\delta M}{\rho V \ell c^2\, d\eta'} = (\theta_r F\cos\beta + \phi_r f'\sin\beta)\,(Vm_\alpha + hV\ell_\alpha)$$

$$+ \dot\phi_r \ell' f\,(m_{\dot z} + h\,\ell_{\dot z}) - \dot\theta_r\, hc F\,(m_{\dot z} + h\,\ell_{\dot z})\cos\beta$$

$$+ (\dot\theta_r F\cos\beta + \dot\phi_r f'\sin\beta)\,(m_{\dot\alpha} c + hc\,\ell_{\dot\alpha})$$

Considering the work done in a given displacement,

Let δL_a = increment in the flexural moment

δM_a = increment in the torsional moment

Then $\delta L_a = \ell f\,\delta Z\sec\beta + f'\sin\beta\,\delta M'$

$\delta M_a = F\,\delta M'\cos\beta$.

$$\therefore\; \frac{L_a}{\rho V\ell^2 c_o} = -\sec\beta\int f\,\frac{c}{c_o}\Big\{\theta_r F\cos\beta\, V\ell_\alpha + \phi_r f'\sin\beta\, V\ell_\alpha$$

$$+ \dot\theta_r\left[F\cos\beta\,\ell_{\dot\alpha} c - hc\cos\beta\, F\,\ell_{\dot z}\right]$$

$$+ \dot\phi_r\left[f'\sin\beta\,\ell_{\dot\alpha} c + \ell' f\,\ell_{\dot z}\right]\Big\}\, d\eta'$$

$$+ \frac{c_o}{\ell}\sin\beta\int\left(\frac{c}{c_o}\right)^2 f'\Big\{\theta_r F\cos\beta\,(Vm_\alpha + hV\ell_\alpha) + \phi_r f'\sin\beta\,(Vm_\alpha + hV\ell_\alpha)$$

$$+ \dot\theta_r\left[F\cos\beta\,(m_{\dot\alpha} c + hc\,\ell_{\dot\alpha}) - hcF\cos\beta\,(m_{\dot z} + h\ell_{\dot z})\right]$$

$$+ \dot\phi_r\left[f'\sin\beta\,(m_{\dot\alpha} c + hc\,\ell_{\dot\alpha}) + \ell' f\,(m_{\dot z} + h\ell_{\dot z})\right]\Big\}\, d\eta'$$

and
$$\frac{M_a}{\rho V\ell c_o^2} = \cos\beta\int F\left(\frac{c}{c_o}\right)^2\Big\{\theta_r F\cos\beta\,(Vm_\alpha + hV\ell_\alpha) + \phi_r f'\sin\beta\,(Vm_\alpha + hV\ell_\alpha)$$

$$+ \dot\theta_r\left[F\cos\beta\,(m_{\dot\alpha} c + hc\,\ell_{\dot\alpha}) - hcF\cos\beta\,(m_{\dot z} + h\,\ell_{\dot z})\right]$$

$$+ \dot\phi_r\left[f'\sin\beta\,(m_{\dot\alpha} c + hc\,\ell_{\dot\alpha}) + \ell' f(m_{\dot z} + h\ell_{\dot z})\right]\Big\}\, d\eta'.$$

Now $L_a = \theta_r L_\theta + \phi_r L_\phi + \dot\theta_r L_{\dot\theta} + \dot\phi_r L_{\dot\phi}$

and $M_a = \theta_r M_\theta + \phi_r M_\phi + \dot\theta_r M_{\dot\theta} + \dot\phi_r M_{\dot\phi}$

$\therefore$ the non-dimensional aerodynamical coefficients are given by

$$b_1 = -\frac{L_{\dot\phi}}{\rho V \ell^3 c_o} = \int_0^{10/7} f \frac{c}{c_o}\left[f\ell_{\dot z} \sec^2\beta + f' \tan\beta\, \ell_{\dot\alpha} \frac{c}{\ell}\right] d\eta'$$

$$- \frac{c_o}{\ell}\int_0^{10/7} f'\left(\frac{c}{c_o}\right)^2 \Big[f \tan\beta\,(m_{\dot z} + h\ell_{\dot z})$$

$$+ \frac{c}{\ell} f' \sin^2\beta (m_{\dot\alpha} + h\ell_{\dot\alpha})\Big] d\eta'$$

$$c_1 = -\frac{L_\phi}{\rho V^2 \ell^3} = \frac{c_o}{\ell}\int_0^{10/7} f f' \frac{c}{c_o} \tan\beta\, \ell_\alpha\, d\eta'$$

$$-\left(\frac{c_o}{\ell}\right)^2 \sin^2\beta \int_0^{10/7} f'^2 \left(\frac{c}{c_o}\right)^2 (m_\alpha + h\ell_\alpha)\, d\eta'$$

$$j_1 = -\frac{L_{\dot\theta}}{\rho V \ell^2 c_o^2} = \int_0^{10/7} f\left(\frac{c}{c_o}\right)^2 (F\ell_{\dot\alpha} - hF\ell_{\dot z})\, d\eta'$$

$$- \frac{c_o}{\ell}\sin\beta\cos\beta \int_0^{10/7}\left(\frac{c}{c_o}\right)^3 F f' \left[(m_{\dot\alpha} + h\ell_{\dot\alpha}) - h(m_{\dot z} + h\ell_{\dot z})\right] d\eta'$$

$$k_1 = -\frac{L_\theta}{\rho V^2 \ell^2 c_o} = \int_0^{10/7} \ell_\alpha F f \frac{c}{c_o}\, d\eta' - \frac{c_o}{\ell}\sin\beta \int_0^{10/7} f'\left(\frac{c}{c_o}\right)^2 F \cos\beta\,(m_\alpha + h\ell_\alpha)\, d\eta'$$

$$b_3 = -\frac{M_{\dot\phi}}{\rho V \ell^2 c_o^2} = -\int_0^{10/7} F\left(\frac{c}{c_o}\right)^2\left[f\,(m_{\dot z} + h\ell_{\dot z}) + f' \sin\beta\cos\beta\,(m_{\dot\alpha} + h\ell_{\dot\alpha})\frac{c}{\ell}\right] d\eta'$$

$$c_3 = -\frac{M_\phi}{\rho V^2 \ell^2 c_o} = -\frac{c_o}{\ell}\sin\beta\cos\beta \int_0^{10/7} F\left(\frac{c}{c_o}\right)^2 f'(m_\alpha + h\ell_\alpha)\, d\eta'$$

$$j_3 = -\frac{M_{\dot\theta}}{\rho V \ell c_o^3} = -\cos^2\beta \int_0^{10/7} F^2\left(\frac{c}{c_o}\right)^3\left[m_{\dot\alpha} + h\ell_\alpha - h\,(m_{\dot z} + h\ell_{\dot z})\right] d\eta'$$

$$k_3 = -\frac{M_\theta}{\rho V^2 \ell c_o^2} = -\cos^2\beta \int_0^{10/7} F^2\left(\frac{c}{c_o}\right)^2 (m_\alpha + h\ell_\alpha)\, d\eta'.$$

It is to be noted that c_1 and c_3 are not in general zero for a swept back wing.

For the assumed modes of flexure and torsion,

$f = \eta^2$, $f' = 2\eta$, $F = \eta$.

Derivation of critical flutter speed

As in references 1 and 2, the equations of motion are

$$A_1\ddot{\phi}_r + B_1\dot{\phi}_r + C_1\phi_r + G_1\ddot{\theta}_r + J_1\dot{\theta}_r + K_1\theta_r = 0$$

$$A_3\ddot{\phi}_r + B_3\dot{\phi}_r + C_3\phi_r + G_3\ddot{\theta}_r + J_3\dot{\theta}_r + K_3\theta_r = 0$$

Let $\phi_r = \Phi e^{\lambda t}$, $\theta_r = \Theta e^{\lambda t}$

Substituting and eliminating Θ, Φ we get

$$\left(a_1\lambda'^2 + b_1\sqrt{\epsilon_o}\,\lambda' + X\right)\left(g_3\lambda'^2 + j_3\sqrt{\epsilon_o}\,\lambda' + Y\right)$$

$$-\left(g_1\lambda'^2 + j_1\sqrt{\epsilon_o}\,\lambda' + k_1\right)\left(a_3\lambda'^2 + b_3\sqrt{\epsilon_o}\,\lambda' + c_3\right) = 0$$

where $\lambda' = \lambda c_o / V\sqrt{\epsilon_o}$

$$X = \frac{C_1}{\rho V^2 \ell^3} = \frac{\ell_\phi}{\rho V^2 \ell^3} + c_1 = X'_c + c_1$$

$$Y = \frac{K_3}{\rho V^2 \ell c_o^2} = \frac{m_\theta}{\rho V^2 \ell c_o^2} + k_3 = Y'_c + k_3$$

i.e. $q_o\lambda'^4 + q_1\lambda'^3 + q_2\lambda'^2 + q_3\lambda' + q_4 = 0$

where
$$q_o = a_1 g_3 - a_3 g_1$$
$$q_1 = (a_1 j_3 - a_3 j_1 + b_1 g_3 - b_3 g_1)\sqrt{\epsilon_o}$$
$$q_2 = \left[a_1 Y - a_3 k_1 + (b_1 j_3 - b_3 j_1)\,\epsilon_o + X g_3 - c_3 g_1\right]$$
$$q_3 = (b_1 Y - b_3 k_1 + X j_3 - c_3 j_1)\sqrt{\epsilon_o}$$
$$q_4 = XY - c_3 k_1$$

The test function is $T_3 = q_1 q_2 q_3 - q_o q_3^2 - q_1^2 q_4$

$T_3 = 0$ at the critical flutter speed V_c.

$q_4 = 0$ is the condition for wing divergence.

Estimation of the aerodynamic coefficients

Using references 1 and 2, the aerodynamic coefficients for incompressible flow over an unswept wing are given by

$$\ell_{\dot{z}} = 1.5, \quad \ell_{\dot{\alpha}} = 1.4, \quad \ell_\alpha = 1.6$$

$$-m_{\dot{z}} = 0.375, \quad -m_{\dot{\alpha}} = 0.7, \quad -m_\alpha = 0.4$$

The values of these coefficients have been derived from experimentally determined derivatives for a wing of finite span.

The aerodynamic acceleration coefficients have been neglected in comparison with the structural inertia coefficients.

For the calculation of wing divergence speeds, quasi static values of the derivatives are used.

There is very little experimental data on the variation of the derivatives with sweepback and with Mach number. For incompressible flow we assume that the coefficients vary as $\cos\beta$; for the swept wing, applying the Glauert correction as for the quasi static condition, the derivatives are multiplied by the factor

$$\frac{1}{(1-M^2)^{1/4}\,(1-M^2\cos^2\beta)^{1/4}}$$

<u>Results</u>

The calculations were performed for a wing of aspect ratio 5 and taper ratio $\frac{1}{2}$. For the unswept wing, ϵ_0 was taken as 0.10, giving a wing density of 0.765 lb/ft.^{3} The flexural axis was taken at 0.4 chord and the inertia axis at (i) 0.5 chord, (ii) 0.4 chord. The sweepback of the flexural axis was varied from + 60° to - 60°.

The non-dimensional critical speed coefficient

$$B = \frac{V_c\sqrt{\rho}}{\sqrt{\dfrac{m_\theta}{dc_m^2}}}$$

is plotted for various angles of sweepback and sweepforward, showing the critical flutter speed and the critical speed for wing divergence.

Curves are drawn for two values of the non-dimensional stiffness ratio

$$r = \frac{\ell_\phi/d^3}{m_\theta/dc_m^2}$$

Figures 2 and 3 are drawn for incompressible flow; figures 4 and 5 for compressible flow ($M = 0.8$).

<u>Conclusions</u>

<u>Critical flutter speed.</u> <u>Effect of sweepback and sweepforward</u>

From figures 2,3,4 and 5 we see that the minimum flutter speed occurs for sweepback angles of 5° to 20°. For highly sweptback or sweptforward wings the flutter speed is double that for unswept wings with the same wing stiffness. (NOTE: In these calculations we have neglected the effect of any rigid body freedoms of the aircraft e.g. pitch and vertical translation. Recent theoretical and experimental work (reference 3) has shown that when these body freedoms are neglected, the calculated flutter speed is liable to be seriously overestimated. The calculations in this report can be applied to an aircraft for which the fuselage is relatively heavy compared with the wings. For such an aircraft, both the inertia effect of the fuselage and damping due to the tailplane tend to suppress the body freedoms in pitch and vertical translation).

Effect of change of flexural stiffness ℓ_ϕ and torsional stiffness m_θ

The curves have been plotted against the non-dimensional parameter B for two values of the non dimensional stiffness ratio r. Thus if the ratio of the stiffnesses is kept constant, the critical flutter speed is proportional to $\sqrt{m_\theta}$, and thus increases as the torsional stiffness increases. Over the range of stiffness ratios considered (r = 1 to 2) the critical flutter speed is increased slightly when the flexural stiffness is decreased.

Effect of variation of the position of the inertia axis

The critical flutter speed increases rapidly as the inertia axis is moved forward. The effect is less beneficial with highly sweptback wings.

Effect of compressibility

In general, at a Mach number of 0.8, the critical flutter speed is some 15 per cent lower than in the incompressible case.

Wing Divergence

Effect of sweepback and sweepforward

Wing divergence is not important for sweptback wings. The reverse is true for swept forward wings, where for angles of sweep greater than 5° to 15° wing divergence will occur at a lower speed than the critical flutter speed.

Effect of change of flexural stiffness ℓ_ϕ and torsional stiffness m_θ

As in the case of flutter, if the ratio of the stiffness is kept constant, the divergence speed is proportional to $\sqrt{m_\theta}$, and thus increases as the stiffness increases. For highly swept forward wings, the wing divergence speed is almost independent of the torsional stiffness, while for unswept wings the

/divergence ...

divergence speed is independent of the flexural stiffness.

Effect of variation of the position of the inertia axis

The wing divergence speed is unaffected by a change in the position of the inertia axis, the flexural axis remaining fixed.

Effect of compressibility

At a Mach number of 0.8, the critical speed for wing divergence is 15 to 20 per cent lower than in the incompressible case.

General conclusions

From the above results it is seen that the critical flutter speed is in general higher for a swept back wing; for a swept forward wing, divergence will occur before flutter.

R E F E R E N C E S

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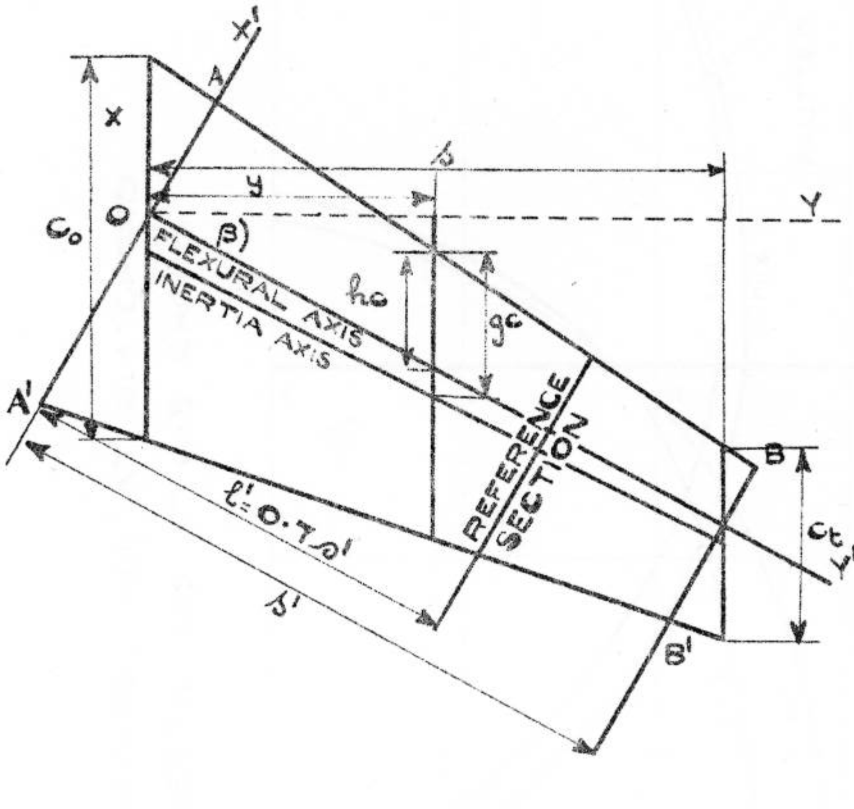
WING PLAN FORM.



ASPECT RATIO 5

TAPER RATIO 2 : 1

FIG 1

VARIATION OF CRITICAL SPEED FOR FLUTTER AND WING DIVERGENCE FOR SWEPTBACK AND SWEPTFORWARD WINGS

(INCOMPRESSIBLE FLOW)

FLEXURAL AXIS AT 0·4 CHORD
INERTIA AXIS AT 0·5 CHORD

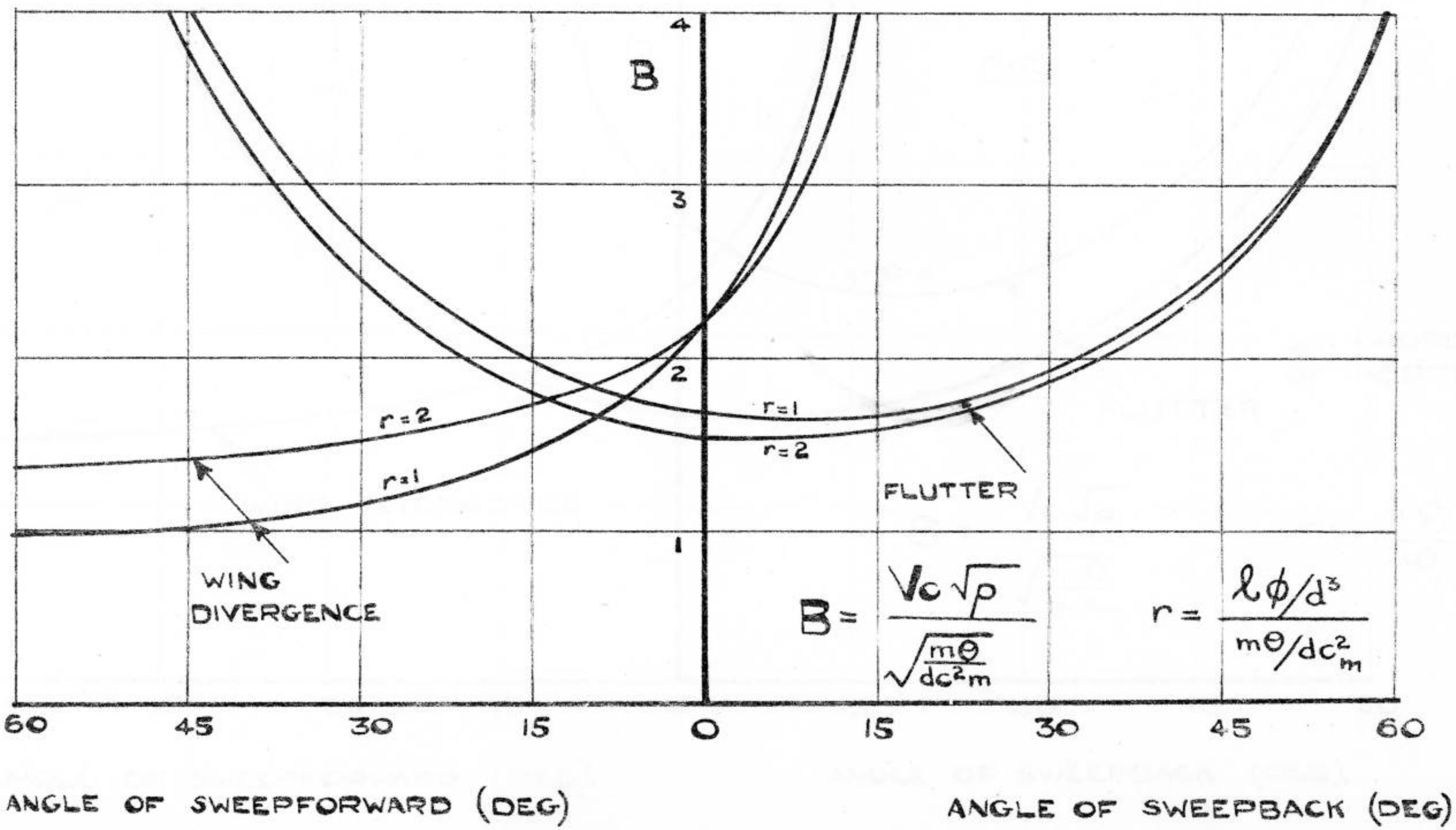


FIG 2

COLLEGE OF AERONAUTICS
REPORT No 39

VARIATION OF CRITICAL SPEED FOR FLUTTER AND WING DIVERGENCE
FOR SWEPTBACK AND SWEPTFORWARD WINGS
(INCOMPRESSIBLE FLOW)

FLEXURAL AXIS AT 0·4 CHORD

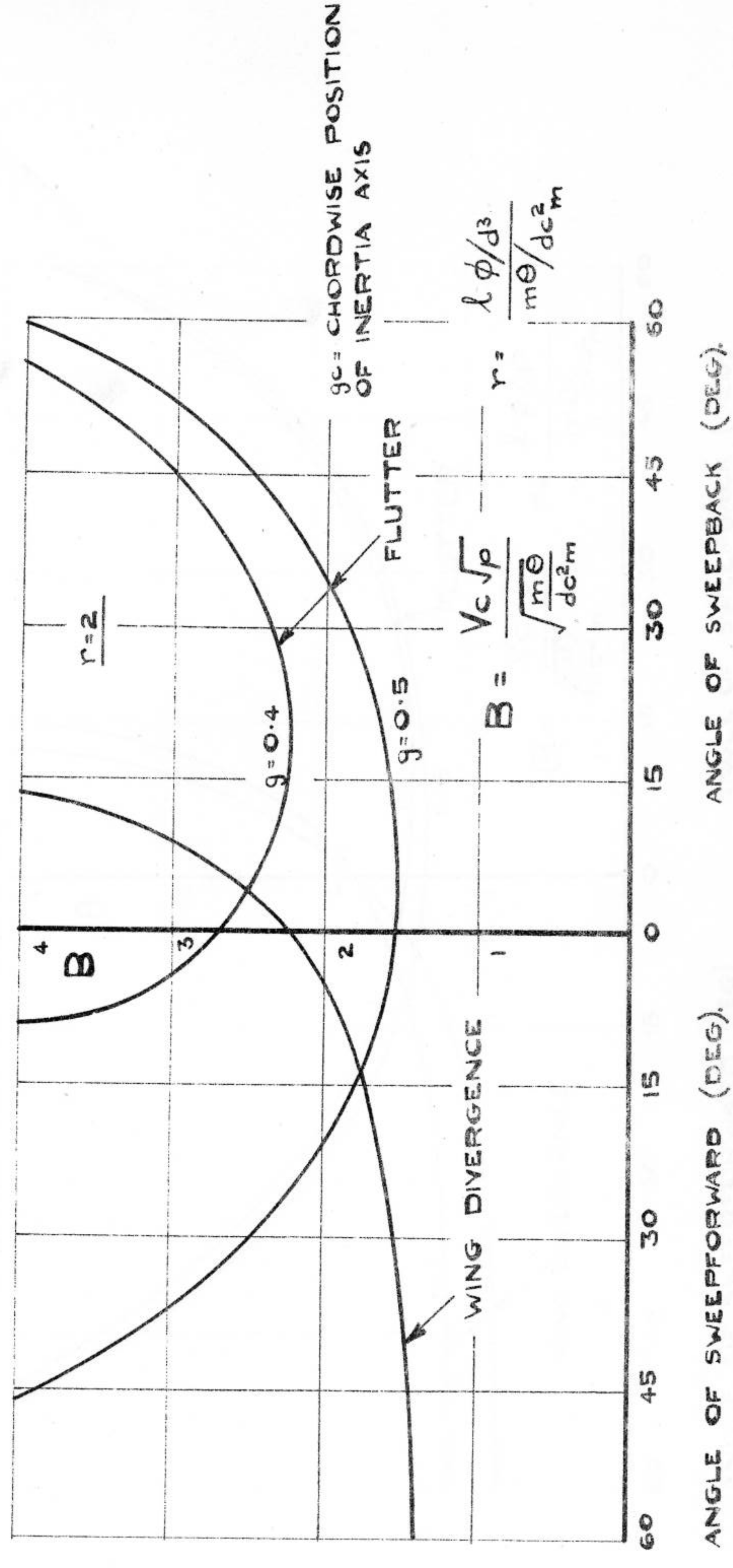


FIG 3

COLLEGE OF AERONAUTICS
REPORT No 39.

VARIATION OF CRITICAL SPEED FOR FLUTTER AND WING DIVERGENCE
FOR SWEPTBACK AND SWEPTFORWARD WING
(COMPRESSIBLE FLOW ; M = 0·8)

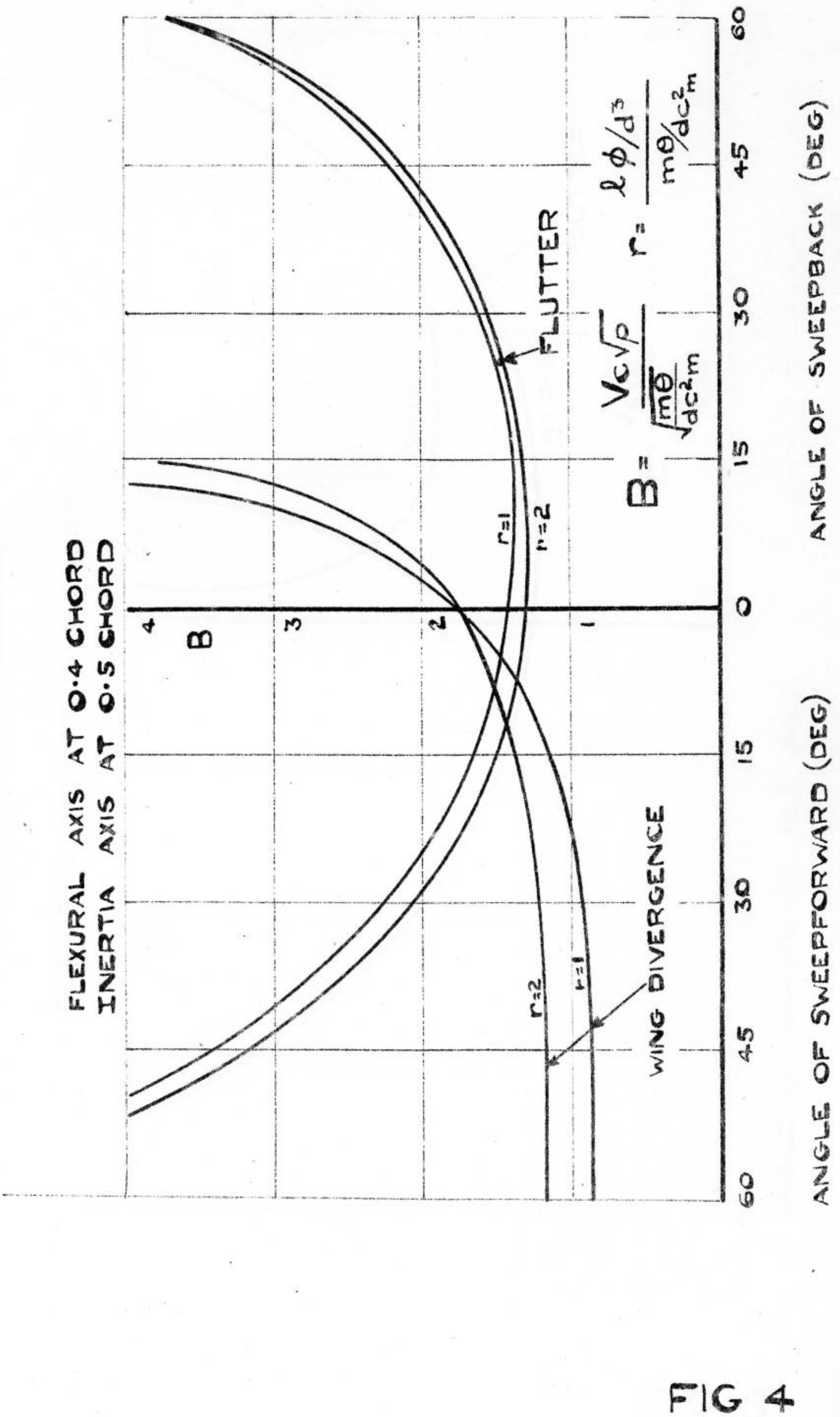


FIG 4

VARIATION OF CRITICAL SPEED FOR FLUTTER AND WING DIVERGENCE
FOR SWEPTBACK AND SWEPTFORWARD WINGS.
(COMPRESSIBLE FLOW ; M = 0·8)

FLEXURAL AXIS AT 0·4 CHORD

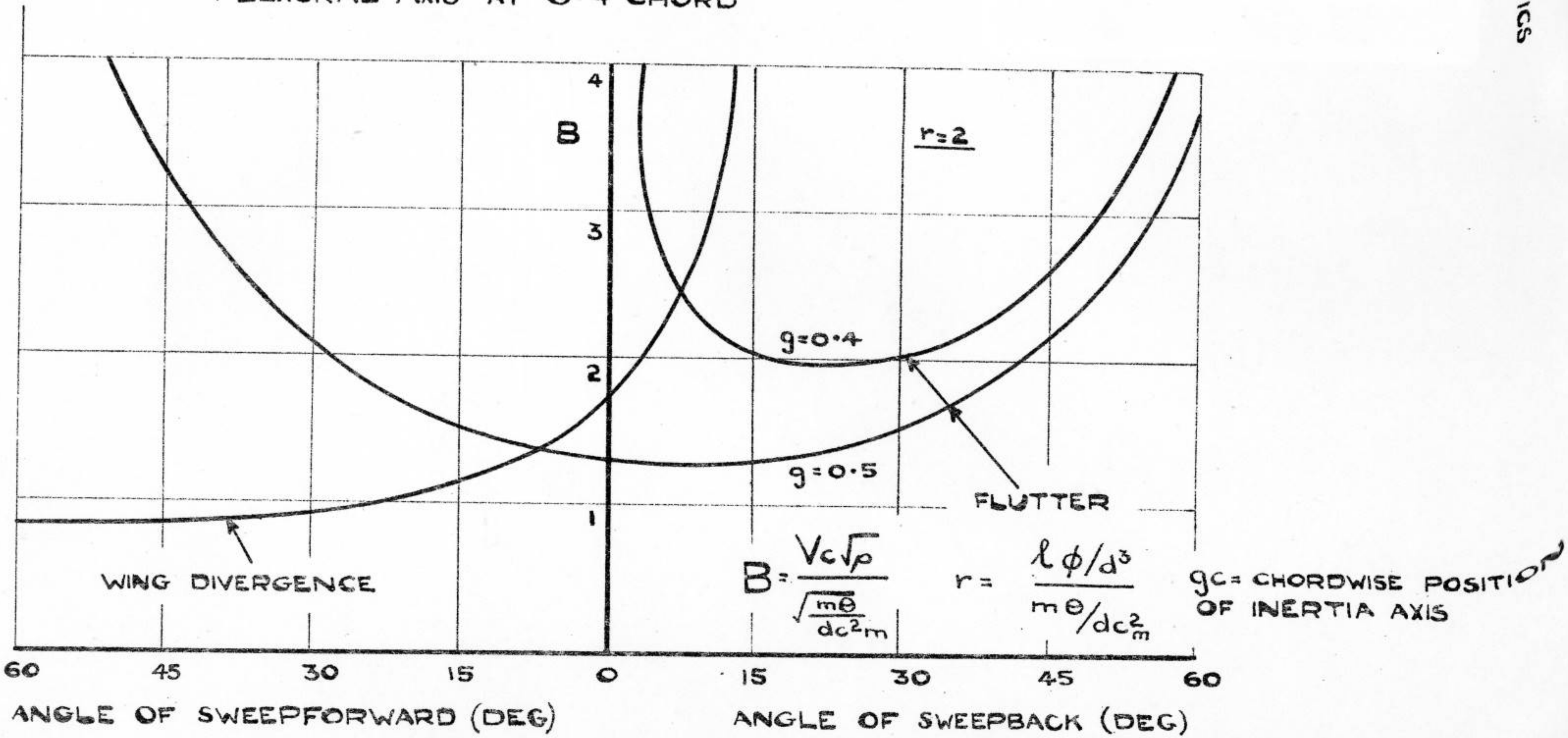


FIG 5