Predictions of Lunar Phenomena in Babylonian Astronomy

Lis Brack-Bernsen, Regensburg

In this paper I shall mainly be concerned with predictions of the length of the Babylonian lunar month. The reason for this choice is the fact that in the important text TU 11 eight different methods for predicting "full" or "hollow" months are collected. This means that we have in this text a substantial amount of material to investigate in addition to what can be found on the topic in other texts.

The tablet AO 6455 (hereafter referred to as TU 11) is perfectly preserved and was published in 1922 in an excellent copy by F. Thureau-Dangin as No. 11 in *Tablettes d'Uruk*. It contains a mixture of primitive and advanced astronomical rules alongside some astrological passages. The tablet TU 11 is a copy written towards the end of the 3rd century B.C., and contains quite a number of errors. Until now only short sections have been translated and commented on.[1] A complete edition by L. Brack-Bernsen and H. Hunger will soon appear in *SCIAMVS* 3. The reader is referred to that edition for a translation and interpretation of the text, and for a detailed discussion of its significance to the history of Babylonian astronomy.

In the present paper I shall only give an overview of the astronomical content of TU 11 and then present all the rules we know of for predicting the length of the Babylonian month. Most of these rules are written on TU 11, and at first glance some of them seem quite strange. Are they just inventions and speculations by some Seleucid scribe, or are they a collection of rules which had really been used by earlier Babylonian astronomers? The present paper will try to provide an answer to this question. If the text just reflects the speculations of one person, then it only tells us how he thought about the problem, and the kind of ways he thought it might be solved. But if it is a genuine collection of methods that were used then it gives us very fruitful hints and ideas about concepts and methods used in intermediate astronomy.[2] Furthermore, it would provide a solid basis for efforts to reconstruct the development of Babylonian lunar theory.[3] Since a great part of my discussion is based on tablets from the cuneiform collection of the British Museum, I am happy to present them here. At this stage I would like to express my warmest thanks to Irving Finkel and Christopher Walker for their search for parallel texts and for drawing my

---

1 NEUGEBAUER (1947) and VAN DER WAERDEN (1949, 1951).

2 In LBAT Sachs classified some tablets as containing Intermediate Astronomy. He defined the term as follows: "This term refers to stages later than MUL.APIN and earlier than ACT. The boundaries in both directions are not sharp".

3 Another possible link between the non-mathematical astronomical texts and the ACT methods, is provided by John Steele in a tablet published in this volume: "A Simple Function for the Length of the Saros in Babylonian Astronomy".

attention to the texts they identified, and to Hermann Hunger and Christopher Walker for making their translations of the texts available to me. Without these translations I would not have been able to work on this topic in the first place.

Some Useful Preliminaries

The Babylonian month began on the evening after new moon (conjunction) on which the thin crescent was visible for the first time. This event of course also indicated the end of the current (old) month.

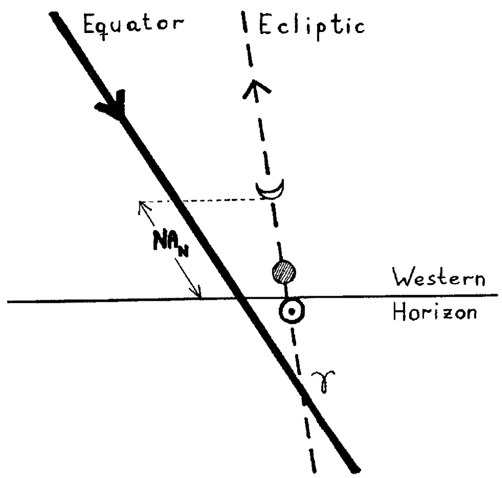

Figure 1: The situation at the western horizon on the evening when the new crescent is visible for the first time after conjunction, announcing the beginning of month I. The dashed line depicts the ecliptic, the path along which sun and moon move. The direction of motion is indicated by the arrow, and j, the sun, shows where the conjunction took place some 1 1/2 days earlier. The moon, moving faster than the sun, has on this evening reached a position so far from the sun, that it will be visible at sunset. On the preceding evening it might have been in Position ● at sunset, still too near to the sun to be seen. The thick line is the equator, it shows the direction along which all luminaries set. The time NA_N from sunset until moonset is measured by the arc of the equator, which sets simultaneously with arc (j 2).

The Babylonian month had 29 or 30 days: if the moon was already visible at the beginning of day 30 in a month, this day 30 was rejected, which meant that the month only had 29 days. That month was called GUR (rejected) which is normally translated as “hollow”. When the moon was still not visible after sunset on day 30, this day was confirmed as the last of the (long) month. A month of 30 days was called GIN (confirmed), normally translated as “full”.

On the evening when the new crescent indicated the beginning of the new month, the time NA_N between sunset and the setting of the crescent was measured. In the Astronomical Diaries[4] this time interval was recorded together with the length of the month which had just passed as follows: If the crescent was seen (and hence NA_N measured) on day 30 of the last month M-1, Month M would begin with "30 NA_N ...", whereas in the case of the moon being seen only the day after day 30, the new month would start with a statement like: "Month M, 1 NA_N ..." Hence we see that 30 and 1 were also used as an indicator for the hollow and full month. Sometimes, Neugebauer translates 30 and 1 as "post hollow" and "post full" respectively, because, normally, these numbers tell us the length of the past month. Babylonian terminology is, however, not very consistent in that "30" in TU 11 Rev. 22 is used for saying that the current month will have only 29 days.

All numbers on the tablets are given in the Babylonian sexagesimal system (a positional number system with 60 as its basis). For example 2,15 can be read as $2 \cdot 60 + 15$ (or as $(2 \cdot 60 + 15) \cdot 60^n$, since the system did not always specify the absolute value of a number).

A text I shall also refer to is MUL.APIN,[5] an astronomical compendium compiled around the end of the second or the beginning of the first millennium B.C. It is found in several copies, the oldest dating from around 700 B.C. Amongst other things it gives the length of day and night as (a linear zigzag) function of the month; day and night are measured in *mana*. Day plus night equals 6 *mana*, the longest day is 4 *mana* and the shortest 2 *mana* (values which are very inaccurate for the latitude of Babylon). The daily retardation of the moon is also given as a function of the month: it is calculated as 1/15 of the nightlength, but since the retardation is measured in *uš*, while the night is measured in *mana*, it is found as $4 \times$ the night ($4 \times$ N *mana* = $1/15 \times$ N,00 *uš*). It is evident, therefore, that the text must have put 1,00 *uš* = 60 *uš* equal to 1 *mana*.

The Lunar Six, which are more complicated observable phenomena, and the Goal-Year method for their prediction are presented in the Appendix at the end of this paper.

The Astronomical Content of TU 11

TU 11 is divided by horizontal rulings into 29 sections. Sections 9–22 have astronomical content; the remaining sections are astrological. The astronomical sections have brief rules for predicting the time of eclipses, lunar phases and the length of the lunar months.

Sections 9–13 are concerned with the times of (lunar) eclipses.[6] The Babylonians specified the moment of a day by its distance in time to or from sunrise or sunset. Time differences were measured in *uš* which are the same as our time degrees: the daily revolution (by 360°) of the sky takes 24 hours, so that 1° = 1 *uš* ≈ 4 minutes.

Four examples demonstrate through calculations how the time of a future eclipse can be determined by means of the Saros cycle of 18 years. The basis of the calculation is the time T of an eclipse, which took place 1 Saros[7] earlier than the

[4] SACHS and HUNGER (1988).

[5] HUNGER and PINGREE (1989).

[6] Since TU 11 mainly treats the moon, we read these examples as calculating lunar (and not solar) eclipses. Furthermore, the preceding astrological section 8 deals with lunar eclipses.

[7] The Saros is a period of 223 synodic months = 6585 1/3 day ≈ 18 years: In a good

eclipse to be predicted. To this time T is added one third of the day plus one third of the night ending up with T+2[,00], which is then reduced to give the time in *uš* after sunset or sunrise of the new eclipse. The text here apparently uses the knowledge that eclipses will repeat after one Saros and that the time of full moon will be shifted by about 1/3 of (day plus night) after 223 synodic months. There are many other texts devoted to eclipses, e.g. lists of possible dates for eclipses, arranged in Saros cycles.[8]

This knowledge is also used in the "Goal-Year" method for predicting lunar phases, which is used and briefly described in sections 14 and 16.[9] All the remaining sections (14, 15, and 17 through 22) give rules for determining the length of the Babylonian month.

Duration of the Babylonian Month

Before we consider the different Babylonian methods for predicting full or hollow months, it is necessary to present some background knowledge on how to determine the length of the synodic month. This "empirical" background knowledge has been found by analyzing computer simulated lunar data.

The first crescent announces the new month, and by so doing it also determines the length of the former month. But the first crescent also contains information on the length of the month that has just started: The size of NA_N measured (or calculated) at the beginning of a month is connected to the length of that current month. This is illustrated in Figure 2 below. Here the time between the setting of the sun and the first crescent is depicted for a series of consecutive Babylonian months.[10]

The full months are marked with a black dot. Note: all minima of the curve have a dot, but none of the maxima has one. Hence the figure gives us a first, albeit rather crude, rule: a small NA_N indicates a long (full) month, while a large NA_N announces a short (hollow) month. For intermediate values of NA_N, there is apparently no clear information on months length: in this figure NA_N at lunation 10 is larger than its value at lunation 13, but month 10 is full while month 13 is hollow. We therefore have a simple rule:

$$\text{If } NA_N \begin{cases} \text{is large, then the month will become hollow} \\ \text{is small, then the month will become full} \end{cases}$$

A closer analysis of NA_N reveals a very useful insight: it is the magnitude or size of consecutive NA_N which decides the month length. Where NA_N for a month(M) is smaller than its value for the next month(M+1), month(M) will be full; where it is

approximation it also equals 239 anomalistic months and 242 draconitic months. The term "Saros" is modern; the Babylonians simply called it "18 years".

8 See STEELE (2000a, 2000b) and AABOE et al, (1991), pp. 35–62.

9 For a detailed presentation of the Goal-Year method, see the Appendix, which also introduces the Lunar Six time intervals.

10 For the construction of the figures, I have used Peter Huber's computer file, creslong.dat, which among others gives for each month the magnitude of NA_N and the length of the months. I warmly thank him for providing and allowing me to use his computed lunar files.

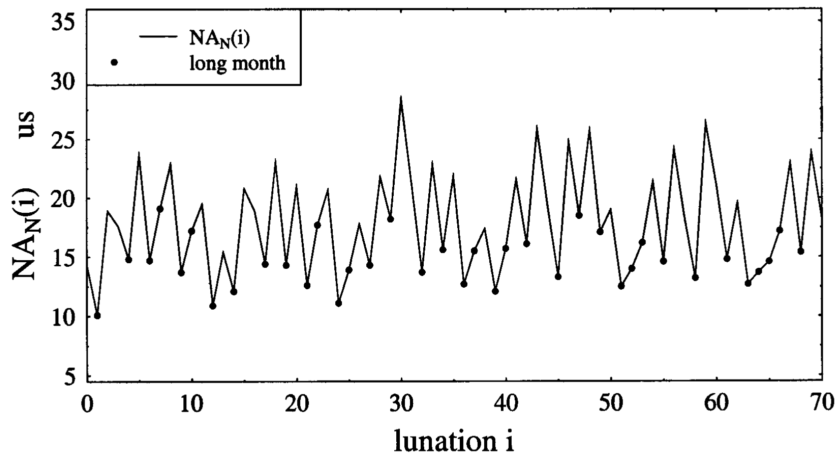

Figure 2: For consecutive months $i = 0, 1, 2,..., 70$, the time $NA_N(i)$ from sunset to the setting of the new crescent is plotted as function of the lunation number *i*. A black circle at a lunation *i* indicates that month(*i*) will have 30 days.

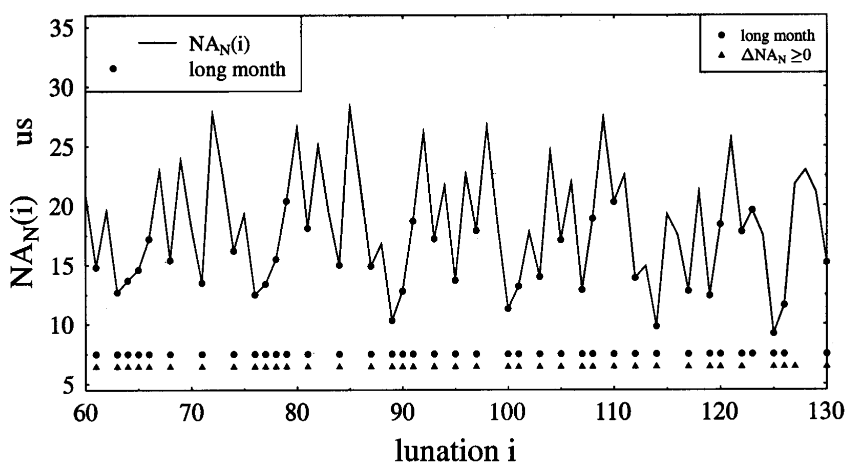

Figure 3: For consecutive months $i = 60, 61, 62,..., 130$, the time $NA_N(i)$ from sunset to the setting of the new crescent is plotted as a function of the lunation number *i*. A black circle at a lunation marks a long month, while a triangle tells that at the first day of the next month, the new crescent will be visible for a longer time before setting. The dots and triangles occur at the same lunations, except for $i = 120$ which has a dot but no triangle, and for $i = 132$, which has a triangle but no dot.

larger than the next, month (M) will become hollow. This is formulated in the following Rule R, which works in 96 cases out of hundred:

$$\text{Rule R}: \begin{cases} \text{If } NA_N\,(M) < NA_N\,(M+1), \text{then month } (M) \text{ is full} \\ \text{If } NA_N\,(M) > NA_N\,(M+1), \text{then month } (M) \text{ is hollow} \end{cases}$$

The rule was found through analyzing figures like Figure 3. Here the size of a NA_N is compared graphically to its value for the next month. With very few exceptions it is true that a month is long (marked by a dot) whenever its NA_N is smaller than the NA_N of the next month (each month(M) for which $NA_N(M) < NA_N(M+1)$ is marked by a triangle). In Figure 3 the dots and triangles occur almost always at the same lunations.

With this "empirical" knowledge we shall now return to the Babylonian texts.

Rules for predicting month lengths found in cuneiform texts

Most of the rules that have been uncovered in texts begin by finding in some way or other the magnitude of NA_N, and use it for predicting the month length. But two very easy and rather primitive rules also exist, and we shall start with these rules.

In section 15 of TU 11 the altitude of the new crescent is used to foretell the length of the new month which has just started:

$$\text{If the first crescent} \begin{cases} \text{is high over the horizon, then the month will become hollow} \\ \text{is low above the horizon, then the month will become full} \end{cases}$$

In the *Reports*[11] another very primitive rule seems to have been used: The month length was connected to the day at which the moon set for the first time after sunrise, which means that the (full) "moon could be seen with the sun". This event takes place in the middle of a month, shortly after opposition: on the day before, the moon (in its full phase) sets before sunrise, or in the terminology of the Reports: "The moon does not wait for the sun, but sets".

$$\begin{matrix}\text{If the moon is seen} \\ \text{with the sun}\end{matrix} \begin{cases} \text{early in the month, then the month will become hollow} \\ \text{late in the month, then the month will become full} \end{cases}$$

In the *Reports* and *Letters* to the Assyrian kings Essarhaddon and Assurbanipal, only the day near middle month was recorded at which moon and sun were seen together. But only a little later, texts record also the time measured between the risings and the settings of sun and moon. (See, for example, Diary -567 I: "On the 14th one god was seen with the other: NA = 4 *uš*").

Intuitively we understand that when NA occurs early (at day number 12 or 13), indicating that opposition of sun and moon also occurred early, then the Babylonian month will also end early. And if full moon takes place late, then that Babylonian month will also tend to end late. But this rule is very crude: a study of 223 months shows that in 48 of these months, NA was measured on day 12 or 13, but only 38 of these months were hollow. If NA was measured on day 15 or 16, then the month was

---

11 HUNGER (1992).

full in 53 out of 73 cases. This is therefore a rather poor rule! Traces of this rule are also found in Section 15 of TU 11.

I shall repeat these first and rough empirical rules in a schematic way:

$$\text{Primitive Rules}: \begin{cases} NA \text{ occurs} \begin{cases} \text{early}: \text{month short} \\ \text{late}: \text{month long} \end{cases} \\ \text{Crescent} \begin{cases} \text{high to the sun}: \text{month short} \\ \text{low to the sun}: \text{month long} \end{cases} \\ NA_N \begin{cases} \text{large}: \text{month short} \\ \text{small}: \text{month long} \end{cases} \end{cases}$$

More Advanced Rules

The new crescent announces the end of a Babylonian month. But above we have seen that the time of its visibility, the quantity NA_N, is also an indicator for the length of the new month. The Babylonian astronomers also noticed this. The known textual material bears witnesses to seven different methods for determining NA_N, and thereby predicting full or hollow months. Below is a survey of the methods using the quantity NA_N for predicting the month length:

- NA_N is found by means of the Goal-Year method (using lunar six data from lunations 18 years earlier) and a little detail within this calculation will determine the length of the new month: Is an addition needed or not.
- NA_N is found from *KUR* through extrapolation. The same value for the daily retardation of the moon is used at the eastern and western horizon, namely a fifteenth of the day length. The size of NA_N decides the day on which the crescent is expected to become visible.
- NA_N is found from its values recorded 1 Saros earlier. The difference between the two values of NA_N, situated 1 Saros apart, is derived from the schematic length of the night.
- NA_N is found (by different methods) from its value one month earlier. In the Atypical text K[12] in the first approximation $NA_N(i+1)$ is found by adding some value t to $NA_N(i)$, where t = t(λ) is a function of the lunar longitude. In section 17 of TU 11 NA_N(II) is found from NA_N(I) and the values of *NA*(VII) and *NA*(VIII), measured 5 1/2 months before the months I and II of interest. The sign of *NA*(VII)–*NA*(VIII) determines the month length. Section 20 seems to find NA_N(V) from NA_N(IV) by calculations which must be erroneous[13] and which I cannot understand.
- NA_N(I) of the new year seems to be inferred from NA_N(I) of some old year in combination with the two values of *KUR*(XII) which occurred a few days

[12] NEUGEBAUER and SACHS (1969), pp. 96–108.

[13] The text tells to calculate some quantity and compare it to 1/2 NA_N in order to decide between full or hollow month. The crucial criteria for full or hollow months is however worthless, since for the calculations reproduced in the text, only one of the inequalities can be true – the other will never occur. Therefore the text must be erroneous.

before the beginning of month I(old) and month I(new). The relative size of *KUR*(old) and *KUR*(new) determines the length of the month. The text does not however specify which year is meant by the “old” year.

The rules from TU 11 for finding NA_N are presented below in schematic form:

$$\textbf{Section 14:}\quad \text{Goal-Year Method} \;\to\; NA_N \begin{cases} \text{addition: month hollow} \\ \text{subtraction: month full} \end{cases}$$

[Formulated and valid in case of the old month being full, $NA_N(i-223) \to NA_N(i)$]

$$\textbf{Section 17:}\quad NA_N\,(I) \;\to\; NA_N\,(II) \begin{cases} NA_N\,(II) < NA_N\,(I)\text{: month hollow} \\ NA_N\,(II) > NA_N\,(II)\text{: month full} \end{cases}$$

$$\textbf{Section 18:}\quad NA_N\,(II)[\text{-18 years}] \;\to\; NA_N\,(II) \begin{cases} \text{if larger than 12: hollow} \\ \text{if smaller than 12: full} \end{cases}$$

Section 19: *KUR*(i) extrapolated $\to$ day of new crescent

$$\textbf{Section 22:}\quad NA_N\,(new)\text{ found through} \begin{cases} \text{addition: month hollow} \\ \text{subtraction: month full} \end{cases}$$

We can now ask “who invented these rules”? Was it a scribe from the Seleucid period who knew the Goal-Year method and tried to play around with similar older schemes and methods, or do we have here a genuine collection of newer and older methods? This question is very important since some of the methods seem to apply to astronomical schemes from MUL.APIN, so we might learn here how such schemes were used.

Hermann Hunger gave the same answer to this question as I do, however by arguments which I never thought about. The fact that something is written on a clay tablet gives it a certain value and importance. A scribe who tried out new ideas would never use such a valuable material. Hence, what occurs in cuneiform on a nicely formed clay tablet is important and accepted knowledge.

The reasons why I am convinced that TU 11 contains a collection of methods and rules which were actually developed and used by different astronomers over time are the following: 1) Quite a lot of parallel texts in the British Museum have been found, some of which are considerably older. Most of these texts came from Babylon, while TU 11 originates from Uruk. Also TU 11 itself is a copy, so someone thought that what it contained was worth copying. And 2) when we analyse the methods, we see that they all (as far as we have been able to understand them) reflect the same basic ideas which seem to have been refined over time. The procedures are based on connections between the Lunar Six, on different methods for determining the daily retardation of the moon, and on “similar situations”.

Parallel Texts and Texts with the Same Methods

The tablets BM 42282+42294 were tentatively dated by Irving Finkel to the fifth century B.C. They have passages which correspond to section 14, 16, and 22 of TU 11. Apart from this, the text gives the Goal-Year method in a much clearer and more detailed formulation than what we have found on TU 11. At one point the text disagrees with section 22 (of TU 11): at the place where TU 11 mentions some "new year", the parallel text has "old year".

BM 36782 has passages parallel to section 17, 18, and 19, and BM 36747 which joins to BM 37018 has parts of sections 19 and 20.[14] Except for section 15 and 21, to all the other sections on TU 11 dealing with lunar six and month lengths parallel passages have been found on other tablets.

Structure of the Methods

The different methods (reproduced in condensed form above) have many common features. In all cases the size of (the established) NA_N is an indicator for the month length. And NA_N for the month in question is found from the value of NA_N 1 Saros earlier, 1(?) year earlier, or 1 month earlier; or it is found from *KUR* measured a few days earlier:

(Section 14) $NA_N(I-223) \rightarrow NA_N(i)$

(Section 17) $NA_N(\text{I}) \rightarrow NA_N(\text{II})$

(Section 18) $NA_N(\text{II})_{\text{18 years back}} \rightarrow NA_N(\text{II})$

(Section 19) $KUR(i) \rightarrow NA_N(i+1)$

(Section 22) $NA_N(\text{old}) \rightarrow NA_N(\text{new})$

(Section 20 and 21 also seem to use a NA_N to determine some later NA_N however by arithmetic manipulations which we cannot understand.)

Four of the methods listed above connect values of NA_N measured at special intervals utilizing what we would call the daily change of NA_N, the monthly change of NA_N, the yearly (?) change of NA_N, and the sarosly change of NA_N. The values of these changes are either determined empirically or found by theoretical considerations, or by a combination of both. We use the term ΔNA_N for the daily change of NA_N, and ΔKUR for the daily change of *KUR*, and remind the reader, that these quantities measure the daily retardation of the moon (measured in the west by setting, or in the east by rising, respectively). I want to stress the fact that already on the earliest astronomical texts we find these daily retardations modelled arithmetically: *Enūma Anu Enlil* Tablet XIV[15] and MUL.APIN have tables in which ΔNA_N is approximated by $1/15 \times$ length of the night. A summary of the different ways of finding the changes in NA_N or *KUR* is given below:

The daily change of *KUR*: $\Delta KUR = 1/15 \times$ length of daylight (Section 19)

The daily change of NA_N: $\Delta NA_N = 1/15 \times$ length of night (MUL.APIN)

The daily change of $NA_N(i)$: $\Delta NA_N = (\check{S}U + NA)(i-6)$ (Section 16 and 14)

---

[14] I would like to express my thanks to Clemency Williams for this information.

[15] EAE is a great canonical omen series. Tablet XIV with astronomical schemes has been published in AL-RAWI and GEORGE (1991).

The monthly change of NA_N: function of lunar longitude (Atypical Text K).

The monthly change: $NA_N(\mathrm{II}) - NA_N(\mathrm{I}) = (NA(\mathrm{VIII}) - NA(\mathrm{VII}))$ (Section 17)

The yearly(?) change of NA_N: $(KUR(\mathrm{old}) - KUR(\mathrm{new}))$ (Section 22)

The sarosly change: $NA_N(i+223) - NA_N(\mathrm{i}) = 1/3(\check{S}U + NA)(i-6)$ (Section 14)

The sarosly change of $NA_N = 1/30 \times \mathrm{night}$ (Section 18)

Of course, the accuracy of the predicted value of NA_N depends on how good the applied method approximates the changes of NA_N. The best approximation to ΔNA_N is found in the Goal-Year method, which uses $\check{S}U + NA$, the daily retardation of the moon measured at full moon 5 1/2 months earlier. Test calculations have shown that $\Delta NA_N(\mathrm{i})$ is optimally approximated by $(\check{S}U + NA)(i-6)$.[16] To me this fine method seems to be the end result of refinements and a combination of older practices. In any case sections 9–12, 15, 17, and 18 have elements reflecting some empirical know-how, which in a skilled manipulation shows up in the Goal-Year method. Section 9–12 and 18 connects lunar events which are situated 1 Saros apart in order to make predictions.

The Goal-Year method and the method in Section 17 connect NA_N with quantities which occurred half a year earlier.[17] It is wise to do so, because the conditions under which the new crescent is seen in month (i) are very similar to the conditions of the full moon when it sets for the first time after sunrise in month $(i-6)$.[18] This can be illustrated by comparing Figure 1 with Figure 4 (in the Appendix). Figure 1 shows the situation at the western horizon at the very beginning of month I, while Figure 4 shows the situation around full moon 5 1/2 months earlier, in the middle of month VII. The antisun $\overline{\mathrm{j}}$ indicates the place where the opposition took place. The inclination of the ecliptic is the same at the two events, and so is the movement of the moon relative to the sun and antisun, respectively: Arc(●,2) ≈ Arc($2_{\check{S}\acute{U}}$, 2_{NA}). The same will, for example, hold for the situation of NA_N in month VII compared to NA in month I. In these cases, the ecliptic would be low, inclined by some 34° to the horizon. We have ignored here the lunar latitude; but remark that it will be about the same in the two situations, since 5 1/2 mean synodic months ≈ 6 mean draconitic months.

Section 15 (which also gave the simple rule: new crescent high, month short; new crescent low, month long), has traces of this knowledge about "similar situations" in its last passage. It says: *From month I onwards, the first days [the moon is] high, the fourteenth days [the moon is] low; from month VII on, the first days [the moon is] low, the fourteenth days [the moon is] high*. What is expressed here reflects some knowledge about the inclination of the path of moon and sun: That it stands steep to the horizon when the new crescent of month I is observed, and also when the full moon sets in the middle of month VII. And that it is flat to the horizon when the setting full moon is observed in month I, but also when the new

---

16 See BRACK-BERNSEN (1999), Figure 7.

17 Section 17 tells us to calculate $NA_N(\mathrm{I}) - (NA(\mathrm{VIII}) - NA(\mathrm{VII}))$ [$= NA_N(\mathrm{II})$].

18 The longitude and latitude of the moon as well as the lunar velocity will be about the same in the two situations. For further details see BRACK-BERNSEN (1999).

crescent becomes visible near the western horizon announcing the beginning of month VII.

In Mesopotamia there was a strong tradition of preserving old wisdom. Ancient tablets were copied over and over again. I am sure that TU 11 is a collection of genuine rules from both older and more recent times, and I hope that my arguments will convince the reader too.

Appendix

The "Lunar Six"

The Babylonians specified a moment of a day by its time difference to sunrise or sunset. They gave special attention to the movement of the moon in the days around opposition or conjunction. The time of these events is not directly observable, so what the Babylonians observed were the differences in time between the rising and setting of the sun and moon in the days around opposition and conjunction. A. Sachs called these time differences the "Lunar Six".[19] The four intervals relating to the full moon, which we call the "Lunar Four", are the following:

ŠÚ = time from moonset to sunrise, measured at last moonset before sunrise.

NA = time from sunrise to moonset, measured at first moonset after sunrise.

ME = time from moonrise to sunset, measured at last moonrise before sunset.

GE_6 = time from sunset to moonrise, measured at first moonrise after sunset.

The setting moon is not visible before conjunction, and its rising is not visible after conjunction. Therefore only two time intervals were observed around new moon.
1) On the evening when the new crescent became visible, indicating the first day of the month:

NA_N = the time between sunset and the setting of the moon, when it has become visible for the first time after conjunction[20] (Figure1).

And 2) at the end of the month:

KUR = the time from moonrise to sunrise, when the rising moon is visible for the last time before conjunction.

---

19 SACHS (1948), p. 281.

20 In the texts with which we are working, this interval is called *NA*, but it occurs always together with an indication that it is the *NA* of the first day or the *NA* at the beginning of the month. We put this identification into the name, calling it *NA* (of the new crescent), or NA_N. We do this in order to be as precise as the Babylonian texts. There the term *NA* is also used for a time interval in the middle of the month, but always identified by calling it the *NA* of day 14 or the *NA* opposite the sun.

The "Goal-Year" Method[21]

These time intervals between the rising and setting of the sun and moon are obvious and easy to observe. From a theoretical point of view, however, they are very complicated quantities. They depend on the time of the conjunction or the opposition: when it takes place in comparison to sunset or sunrise. They also depend on the position of the full or new moon in the ecliptic, and on the lunar velocity and latitude.[22]

It was therefore very surprising and exciting to find out that the Babylonians had developed an easy, elegant and very exact empirical method for the prediction of these time intervals. They had noted that, in comparison to sunset or sunrise, a syzygy would occur 1/3 day later than the one that took place a Saros earlier. And they had realized that ($\check{S}\acute{U} + NA$) measured the daily delay of the moon at setting, and that ($ME + GE_6$) measured its delay at rising. In addition, they must have remarked that the daily delay of the moon would repeat after one Saros. All these connections are implicitly used in what we have called the Goal-Year Method. Below the method is reproduced in the form of mathematical equations:

$$(NA_N)_i = (NA_N)_{i\text{-}223} - 1/3(\check{S}\acute{U} + NA)_{i\text{-}229} \quad (1)$$

$$\check{S}\acute{U}_i = \check{S}\acute{U}_{i\text{-}223} + 1/3(\check{S}\acute{U} + NA)_{i\text{-}223} \quad (2)$$

$$NA_i = NA_{i\text{-}223} - 1/3(\check{S}\acute{U} + NA)_{i\text{-}223} \quad (3)$$

$$ME_i = ME_{i\text{-}223} + 1/3(ME + GE_6)_{i\text{-}223} \quad (4)$$

$$(GE_6)_i = (GE_6)_{i\text{-}223} - 1/3(ME + GE_6)_{i\text{-}223} \quad (5)$$

$$KUR_i = KUR_{i\text{-}223} + 1/3(ME + GE_6)_{i\text{-}229} \quad (6)$$

Corrections: Sometimes the results found by these calculations are preliminary. If $NA_N < 10$ *uš*, then wait a day:

$$\text{corrected } (NA_N)_i = \text{preliminary } (NA_N)_i + (\check{S}\acute{U} + NA)_{i\text{-}229}.$$

(The visibility limit of NA_N is given in both sections 14 and 16 as 10 *uš*.)

$$(NA_N)_i = (NA_N)_{i\text{-}223} + 2/3(\check{S}\acute{U} + NA)_{i\text{-}229} \quad \text{(1 corrected)}$$

Except for the few cases of eclipses, the time of opposition or conjunction cannot be directly observed. The Babylonians used the Lunar Six to get information on the relative position of the sun and moon around full moon and new moon. They used the observable ($\check{S}\acute{U} + NA$) as the daily retardation of the setting moon. The following considerations will illustrate that it is quite obvious to do so:

In the very few "ideal" cases, in which opposition takes place at the moment of sunrise, the full moon will set at the Western horizon while the sun will rise at the Eastern horizon. At the next morning, the moon will set about an hour after sunrise. And this time interval from sunrise to moonset evidently measures the retardation of the moon on the day of opposition.

---

21 In TU 11 these rules were expressed more or less explicitly (in section 14 and 16) – some of them were reconstructed; on the parallel tablet BM 42282+42294 the rules are formulated more clearly.

22 For more details, see BRACK-BERNSEN and SCHMIDT (1994).

Normally, the opposition does not take place at the moment of sunset (or sunrise), therefore normally the daily retardation of the sun is split up into two intervals: *ŠÚ*, the time from moonset to sunrise measured on the last morning before opposition, and *NA*, the time from sunrise to moonset measured on the next morning, the first after opposition. Obviously, their sum (*ŠÚ* + *NA*) measures, how much later, in comparison to sunrise, the moon sets on the morning of *NA* than on the morning before.

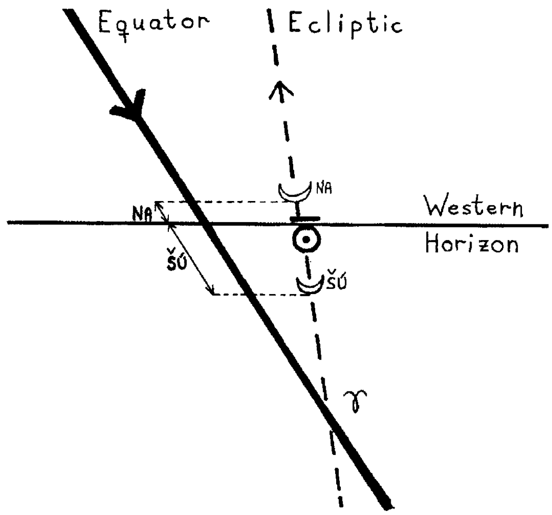

Figure 4: The situation at the Western horizon at sunrise in the days around an opposition taking place in (the middle of) month VII. The dashed line depicts the ecliptic, the path along which sun and moon moves, the direction of motion is indicated by the arrow. The positions of moon and "anti-sun" are shown on two mornings on which *ŠÚ* and *NA* are measured.

Figure 4 illustrates the normal case when opposition does not take place at sunrise. For the sake of simplicity we introduce the symbol $\overline{\mathrm{j}}$ for the "anti-sun", which we define as the point on the ecliptic situated directly opposite the sun. At the exact moment when the sun rises, $\overline{\mathrm{j}}$ sets, and vice versa. It marks the point of the ecliptic at which the opposition takes place. The daily retardation of the setting moon is measured by the arc (*ŠÚ* + *NA*) of the equator.

The retardation of the rising moon at the day of opposition is determined in a similar way: as the sum (ME + GE_6) of *ME*, the time from last moonrise before opposition to sunset, and of GE_6, the time from sunset to first moonrise after opposition.

The Empirical Foundation of the Goal-Year Method

The Goal-Year method uses $(\check{S}\acute{U} + NA)(i - 223)$ as the daily retardation of the setting moon around opposition *i*. It uses $(\check{S}\acute{U} + NA)(i - 229)$ for the daily retardation of the new crescent in month (*i*), and it takes a third of the daily retardation as the change after 1 Saros of *ŠÚ*, *NA*, and NA_N, respectively. This method is based on the following empirical recognition: The daily retardation at full moon will in a good approximation repeat after 1 Saros:

$$(\check{S}\acute{U} + NA)(i) \approx (\check{S}\acute{U} + NA)(i - 223)$$

The daily retardation of the new crescent is approximately equal to the daily retardation of the setting moon in its full phase, measured 5 1/2 months earlier.

$$\Delta NA_N(i) \approx (\check{S}\acute{U} + NA)(i - 6) \approx (\check{S}\acute{U} + NA)(i - 229).$$

The time of lunar eclipses in comparison to sunset is shifted by 1/3 of a day after 1 Saros. Or generally for each lunar month:

223 synodic months = 1 Saros is 1/3 day longer than a whole number of solar days.

Acknowledgements

At this place I express my thanks to the DFG (Deutsche Forschungsgemeinschaft) for supporting this work. I warmly appreciate a careful reading of the manuscript by Claire O'Reilly and thank Irving Finkel, Christopher Walker, and Hermann Hunger for identifying and translating parallel texts, and Peter Huber for making his computed lunar files available. Finally I thank John Steele and Hermann Hunger for valuable corrections and suggestions to this paper.

Abbreviations

LBAT = PINCHES, STRASSMAIER, and SACHS (1955)

References

AABOE, Asger; BRITTON, John P.; HENDERSON, Janice A.; NEUGEBAUER, Otto, and SACHS, Abraham J. 1991. *Saros Cycle Dates and Related Babylonian Astronomical Texts* (Transactions of the American Philosophical Society 81/6). Philadelphia: American Philosophical Society.

BRACK-BERNSEN, Lis. 1997. *Zur Entstehung der babylonischen Mondtheorie, Beobachtung und theoretische Berechnung von Mondphasen* (Boethius 40). Stuttgart: Franz Steiner Verlag.

— 1999. "Goal-Year Tablets: Lunar Data and Predictions". In: Noel M. SWERDLOW, *Ancient Astronomy and Celestial Divination*: 149–177. Cambridge, MA: MIT Press.

— and SCHMIDT, Olaf. 1994. "On the foundations of the Babylonian column Φ: Astronomical significance of partial sums of the Lunar Four". *Centaurus* 37: 183–209.

HUNGER, Hermann. 1992. *Astrological Reports to Assyrian Kings* (State Archives of Assyria 8). Helsinki: Helsinki University Press.

— and PINGREE, David. 1989. *MUL.APIN, an astronomical compendium in cuneiform* (Archiv für Orientforschung Beiheft 24). Horn: Ferdinand Bergern & Söhne.

NEUGEBAUER, Otto. 1947. “Studies in ancient astronomy VIII. The water clock in Babylonian astronomy”. *Isis* 37: 37–43.

— and SACHS, Abraham. 1969. “Some Atypical Astronomical Cuneiform Texts II”. *Journal of Cuneiform Studies* 22: 92–113.

PINCHES, Theopholus G.; STRASSMAIER, Johann N., and SACHS, Abraham J. 1955. *Late Babylonian Astronomical and Related Texts*. Providence: Brown University Press.

AL-RAWI, Farouk N. H. and GEORGE, Andrew R. 1991. “Enuma Anu Enlil XIV and Other Early Astronomical Tables”. *Archiv für Orientforschung* 38/39: 52–73.

SACHS, Abraham J. 1948. “A classification of the Babylonian astronomical tablets of the Seleucid period”. *Journal of Cuneiform Studies* 2: 271–290.

— and HUNGER, Hermann, 1988, *Astronomical diaries and related text from Babylonia*, Volume I (1988), II (1989) and III (1996). Wien: Österreichische Akademie der Wissenschaften.

STEELE, John M. 2000a. *Observations and Predictions of Eclipse Times by Early Astronomers*. Dordrecht: Kluwer Academic Publishers.

— 2000b. “Eclipse Prediction in Mesopotamia”, *Archive for History of Exact Science* 54: 421–454.

THUREAU-DANGIN, François.1922. *Tablettes d'Uruk* (Textes Cunéiformes 6). Paris: Musée du Louvre.

VAN DER WAERDEN, Bartel L. 1949. “Dauer der Nacht und Zeit des Monduntergangs in den Tafeln des Nabu-zuqup-GI.NA.”. *Zeitschrift für Assyriologie* 49: 307 ff.

— 1951. “Babylonian Astronomy III: The earliest astronomical Computations”. *Journal of Near Eastern Studies* 10: 20–34.