Proceedings

OF THE

Sixth Hydraulics Conference

June 13-15, 1955

Arranged by the

IOWA INSTITUTE OF HYDRAULIC RESEARCH

STATE UNIVERSITY OF IOWA

Edited by

LOUIS LANDWEBER

and

PHILIP G. HUBBARD

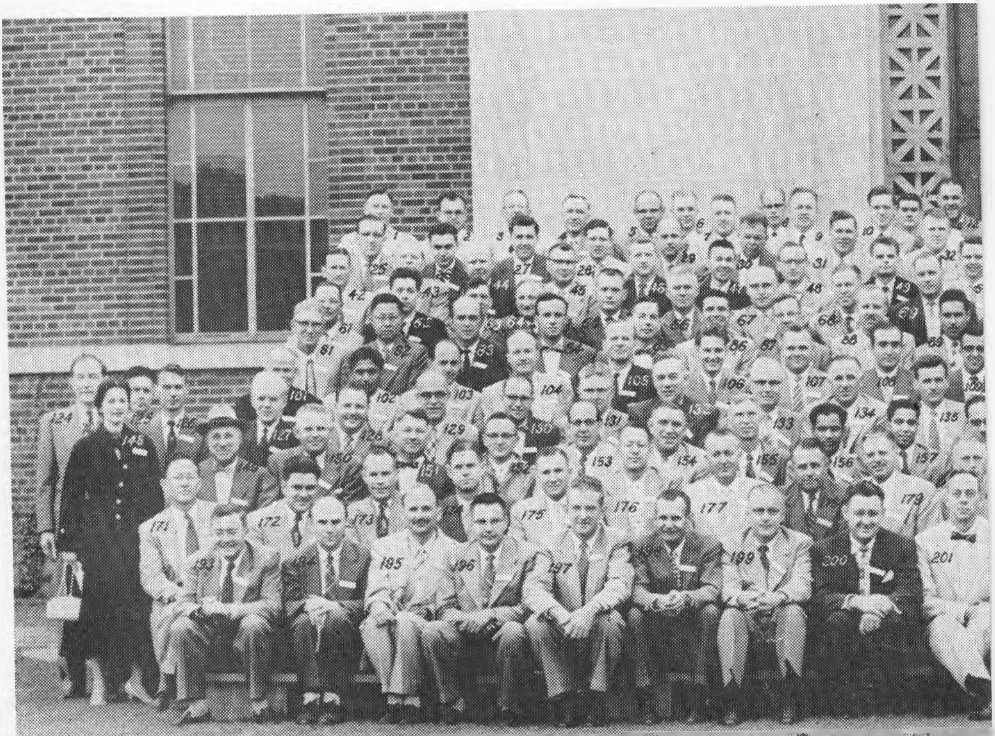
Research Engineers

IOWA INSTITUTE OF HYDRAULIC RESEARCH

Published by the University

Iowa City, Iowa

1956

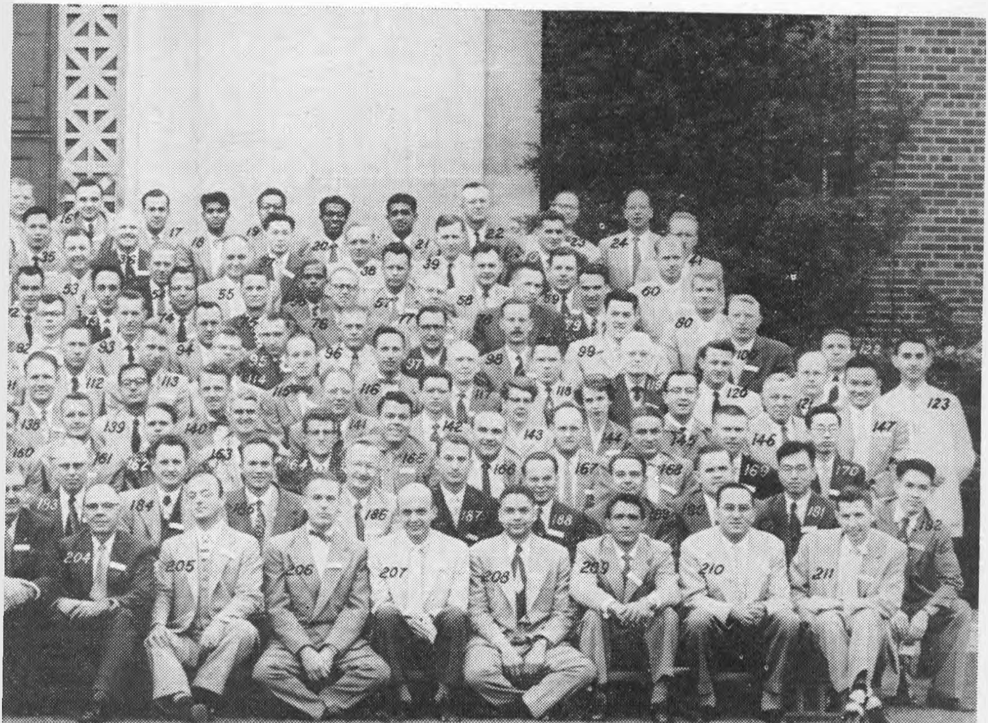


SIXTH HYDR

IOWA INSTITUTE

JUN

1 E.T. Schaefer
2 H.J. Koloseus
3 H.H. Schwob
4 J.B. Saylor
5 L.E. Dickinson
6 C.J. Posey
7 T.F. Maher
8 M.L. Albertson
9 H.S. Kidd
10 E.B. Pickett
11 R.E. Beckett
12 R.D. Schmickle
13 S.W. Mao
14 T.J. Albrecht, Jr.
15 J.A. Roberson
16 D.B. Huber
17 E. Bacci
18 C.R. Guha
19 B. Chanda
20 A.K. Chaudhuri
21 N.P. Rijhwani
22 I.D. Yost
23 E.J. Kennedy
24 E.H. Walker
25 E.R. Leeson
26 J. Davidian
27 W.J. Bauer
28 R.D. Anderson
29 A.J. Moore
30 R. Morgan
31 W.O. Ree
32 G.L. Calehuff
33 J.M. Robertson
34 C.F. Izzard
35 C.S. Song
36 E.C. Clark
37 Y.S. Yu
38 H.O. Wires
39 D.H. Jeffers
40 G. Baltis, Jr.
41 E.C. Balke
42 E.L. Hendricks
43 R.E. Oltman
44 R.W. Carter
45 E.V. Richardson
46 R.J. Longfield
47 G.A. Rusnak
48 R.E. Kaylor
49 R.J. Mulvihill
50 R.L. Player, Jr.
51 N. Shoumatoff
52 C.W. Sullivan
53 R.S. Dart
54 V. Alexander
55 R.N. Weldy
56 K. Seetharamiah
57 R.W. Larson
58 L.R. Sawyer
59 H.M. Erskine
60 F.B. Sessums
61 R. Woodman
62 M. Hug
63 J.M. Killen
64 O. Jurgens
65 L.S. Horihan
66 J.K. Vennard
67 C.R. Forsberg
68 H.W. Feldt
69 M.F. Gade
70 C.A. Lamb
71 J.H. Morgan
72 G.D. Booz
73 O.G. Lara
74 O.J. Larimer
75 J.E. McCall
76 S. Mummey
77 L.W. Furness
78 G.W. Caughran
79 G.L. Bata
80 H.J. Tracy
81 F.D. Paulson
82 Y.T. Siao
83 R.C. Deviney
84 A. Spengos
85 W.L. Moore
86 J.R. Barton
87 D.D. Curtis
88 C.B. Thompson
89 J.E. Martinez
90 K.W. Kirkpatrick
91 W.D. Mitchell
92 R.G. Godfrey
93 W.S. Daniels
94 D.E. Winget
95 C.H. Benson
96 H.L. Hoffman
97 J.C. Dooge
98 K.W. Gent



...NFERENCE
... RESEARCH

114 W.E.HOWLAND
115 J.W.DELLEUR
116 G.E.GIBSON
117 J.C.SCHOVEE
118 W.B.MORGAN
119 D.E.PHILLIPS
120 J.E.FOSTER
121 E.M.LAURSEN
122 M.MARTIN
123 I.M.MOSTAFA
124 L.LANDWEBER
125 D.E.METZLER
126 R.I.MURRAY
127 M.F.ROSE
128 H.A.RAY
129 E.H.SANDHAUS
130 M.R.CARSTENS
131 C.L.BARKER
132 R.G.FOLSOM
133 M.E.NELSON
134 S.FIDELMAN
135 R.J.KENNEDY
136 N.N.GUNAJI
137 D.W.APPEL
138 C.G.DEHAVEN
139 S.S.INCE
140 W.D.BAINES
141 H.P.BROOKS
142 A.TOCH
143 MISS I.E.MILLER
144 MISS M.PETERSEN
145 N.L.BARBAROSSA
146 A.L.ALIN
147 C.S.YIH
148 MRS C.TRUESDELL
149 C.J.MCLEAN
150 E.A.ROEMER
151 W.B.CRAIG
152 G.E.HYDE
153 M.J.WEBSTER
154 D.D.LEWIS
155 H.C.BECKMAN
156 M.A.REDDY
157 A.F.EL KOUH
158 M.G.MOSTAFA
159 S.KOLUPAILA
160 M.V.FEODOROFF
161 J.R.STEVEN
162 B.W.LETOURNEAU
163 J.N.BRADLEY
164 J.C.SCHUSTER
165 W.M.SANGSTER
166 R.L.SMITH
167 M.S.PETERSEN
168 A.HOMYK
169 H.P.JOHNSON
170 M.ARIE
171 J.E.SCHROCK
172 D.E.SCHNEIBLE
173 G.W.EDELEN
174 H.P.THEUS
175 B.GUYTON
176 L.F.INGRAM
177 J.E.GLOVER
178 R.E.MYERS
179 E.G.BARRON
180 R.F.KREISS
181 F.S.WITZIGMAN
182 H.ROUSE
183 J.W.HOWE
184 E.R.TINNEY
185 C.D.SMITH
186 H.A.BABCOCK
187 A.H.BARNES
188 J.ARENDT
189 V.VASQUEZ
190 A.G.ANDERSON
191 H.K.LIU
192 S.C.LING
193 G.E.HARBECK, JR.
194 L.E.NELSON
195 W.J.PRENTICE
196 N.A.KALLIO
197 J.S.CRAGWALL, JR.
198 R.C.SWENGEL
199 W.B.HESS
200 H.C.MCGEE
201 L.BATCHELDER
202 V.R.BENNION
203 M.C.BOYER
204 M.MORRIS
205 C.TRUESDELL
206 J.F.RIPKIN
207 J.S.MCNOWN
208 P.G.HUBBARD
209 J.A.LUISTRO
210 P.EISENBERG
211 J.P.CRAVEN

TABLE OF CONTENTS

INTRODUCTORY REMARKS

by

Hunter Rouse

Iowa Institute of Hydraulic Research

Since the first in its series of triennial conferences, the Iowa Institute has selected for each successive meeting a specific technical theme. The second, in 1942, stressed the similarities in the problems of different professions dealing with fluid motion; the third—postponed till 1946 because of the war—dealt with peacetime applications of wartime research; the fourth, held in 1949, involved the compilation of a reference book on engineering hydraulics; and the fifth, three years thereafter, was devoted to the transportation of sediment. In keeping with this policy, the theme of the Sixth Hydraulics Conference was chosen to be the measurement of fluid flow.

This theme seemed appropriate for two reasons. On the one hand, the previous one and one-half decades had witnessed vast changes in the various techniques of measurement, nearly all of which had incorporated one form or another of electronic gadgetry. New means were thus available for the measurement of depth, velocity, rate of flow, pressure, and shear—whether as mean or as fluctuating quantities. In addition, electrical methods of recording the signals, and of analyzing the records statistically, had been brought to a state of considerable refinement. On the other hand, the same period had also realized great advances in the theoretical analysis of fluid motion, and it seemed timely to assess the role of science in the art of measurement, and as well the relationship of measurement to analysis itself.

In the early days of hydraulics all accomplishment was necessarily empirical, and one heritage of this period in history that persisted well into the present century was the belief that any phenomenon of flow could be understood if only sufficient experimental data were obtained. While this may be true in the limit, it is hardly conducive to efficiency in research, in particular since emphasis is placed upon the quantity at the expense of the quality of the measurements. Fortunately, as the art and the science have progressed, they have had a most salutary reciprocal effect. Improvement in the mechanical aspects of measuring techniques has been paralleled by their more intelligent application in accordance with the dictates of theory. Analysis, in turn, has become increasingly subject not only to experimental verification and subsequent amplification, but as well to experimental guidance at frequent intermediate stages. No longer, in a word, is either considered sufficient unto itself.

A program of fourteen invited papers on the theme of flow measurement was arranged for the Sixth Hydraulics Conference, each of the speakers selected being not only a specialist in his field but also new to the cumulative program roster. Twelve of the papers dealt with particular aspects of instrumentation, and most of these pointed up the complementary influence of the science and the art. Two, however, were purposely directed toward the science rather than the art, in the effort to remind the audience at the outset that measurement is a means and not an end. All fourteen of the papers are presented in full in the following pages. In addition, from a tape recording of the discussion following the twelve papers on instrumentation, the editors have prepared summaries of the essential comments, and these are appended to each paper.

In spite of the conflicts resulting from the steadily growing number of technical meetings, the attendance of the conference reached a comfortable total of 219 engineers and scientists from 35 different states. Twenty-two of them remained at the Institute an additional two and one-half weeks to participate in a special course on laboratory techniques, the accomplishments of which are being published separately.

To avoid any basis for future protests that the date of the next meeting has not been either well chosen or announced sufficiently far ahead of time, let it be noted at this point that each must coincide with the momentary lull between the spring semester and the summer session of the University, and that the Seventh Hydraulics Conference is hence presently scheduled for June 16-18, 1958.

EXPERIENCE, THEORY, AND EXPERIMENT

by

C. Truesdell

Graduate Institute for Mathematics and Mechanics
Indiana University

Science today is much like government or big business: scientists are specialists not only in a single science but even in a single problem, each a senior bureaucrat jealous of interference from the multitude of others whose actions are in turn as isolated as his own, and the multiplied sciences themselves are self-fecundating compartments which reproduce, if at all, by division. Everyone is an expert in something. We are accustomed to speaking of every scientist as a leader in a particular field, but often it is difficult to discern any following. That so many experts turn out so much research that no single person can know it all even in a single field is often brought forward as proving the progress of science. Rate of working, however, is the product of force by velocity and is not necessarily increased if velocity approaches infinity while force approaches zero. There are costly efforts to gather and review the totality of the literature in various fields, but it might be more appropriate to find out who, if anyone, reads the typical paper of today.

Fluid mechanics in its various aspects is divided among several types of engineers, a few physicists, and some mathematicians. Since first I began to study fluids I have had to lose time listening to wrangles among members of these cults, each defending his own while condemning the others. The engineer has the right and duty of knowing fluids as they are met in life for man's direct harm or use; to him, the physicist sets up situations whose only relevance is their ease of study for physicists, while the mathematician is lost in abstraction and arid brain games. The physicist drills out the true principles of fluids, above both the mere detail and empiricism of the engineer and the purism of the mathematician, who for rigor is ever ready to gloss over essential physical aspects. The mathematician has the assurance of correctness and finality; for him the results of physicist and engineer are alike suspect, mere conjecture and ever subject to possible revision. These views are not without their truth in the cellular science of our day. Permit me to reset them in words less apt to reassure the engineers, physicists, and mathematicians in their respective complacencies. The engineer, blinded by the daily need for design or test of this or that device, will not pause to learn enough of the concepts of modern physics or the methods of modern mathematics to find out whether they can be applied to his problems. The physicist, blinded by the oversimplification and the raw guessing now in vogue, despises alike the phenomena which occur in natural, day-to-day situations and the logical standards of precise reasoning. The mathematician, blinded by a century of ever more abstract pure mathematics, has lost the skill and wish to read nature's book. Whether put as praise or put as blame, the foregoing argument, which each of us must endure at every meeting, is barren. I should like to lay before the community of those who study fluids a motion that we hear no more of it. For my part, I promise to try to speak to you not of one of the professions within fluid mechanics today, but of fluid mechanics itself.

Knowledge of fluids is gained through experience, theory, and experiment. Of these, the first and last are often confused. Experience is sometimes dismissed as the uncomprehending rules of thumb of mere artisans, while experiment is exalted as the foundation of science. The empiricism of some physicists of the last century has been embraced by many philosophers of science and educators of our day, particularly those associated with psychology and the biological and social sciences. Students in some of these doctrines are often given long instruction in the "scientific method," which is said to consist in controlled experiments and their statistical evaluation, while theory, if mentioned at all, is subsequent curve-fitting. Experience, the straight impress of nature that observant and rational man gains through his unaided senses as he daily encounters the world and which, by his nowadays all too slighted faculty of reason, he puts in verbal generality, is dismissed as primitive and below science. Many prefer to mine nature's darkest and deepest entrails, closed except to the dearest experimental apparatus or voluminous statistics, while leaving the smiling face of earth unheeded. That modern science is experimental science would follow also from any poll of the scientists themselves: in the biological sciences, little else than experiment exists, while in the physical sciences experiments are often so elaborate that large numbers of persons must be employed upon them. Such force of numbers need not be compelling. A poll of professional musicians would reveal that music today is neither hot jazz nor symphony but sugar stirred in soup. While scientists are more apt than musicians to idolize the means by which they must earn their daily leisure, nevertheless many experimentalists will admit by their actions if not by frank confession that theory is the objective of science. The frustrating and so far vain struggles of biologists and social scientists to organize their subjects upon a basis of mathematical theory is apparent in every conversation among them, while it is the successful theories of the physical sciences that distinguish them from other human endeavors and sometimes cause their enthusiasts to put forward thinly disguised claims to the sole possession of knowledge.

The hydraulic engineer is favored in being alike in daily encounter with experience, theory, and experiment. His tasks and problems arise in common experience, which he dares not desert for more voluptuous realms deep hidden from human eye and touch. The flow of water he describes, understands, and controls in terms of the concepts of theory: velocity, pressure, and density, themselves mathematical ideas expressed in symbols and employed in equations. Either to check and correct the results of theory or to find answers to specific and detailed questions, he has recourse to experimental measurement. For this audience, therefore, I can refrain from further generalities on morals and philosophy. Instead I wish to present you two stories from the history of fluid mechanics. These concern the development of two fundamental concepts. The first concept is one you all use every day, the *static pressure* in a fluid in motion. Its origin is a part of the story of Bernoulli's Theorem, now more than 200 years old. The second concept, *cross-viscosity*, is one with which few of you are yet likely to be familiar; its story begins little more than ten years ago. I will tell you these stories, not in the fashion of those text-book writers who manufacture historical notices so as to bear out their own views of how science ought to have developed, but instead as they really did occur. Since one of these stories is from the earliest period of modern hydraulics and the other is still continuing today, and since despite the lapse of two hundred years between them the general outline is much the same, there will be no need for me to add comments or to draw a moral.

For the early development of hydraulics, I refer you to the excellent history by Rouse and Ince, the first installments of which have now appeared. From it we learn that hydraulic machines are of great antiquity and hence that necessarily man's observation of water flow begins with history. From many centuries of experience we have records of keen observation and reasoning on matters of principle. For example, Theophrastos realized that water waves transport motion, not mass, which suggested to him that sound is a similar undulation of the invisible air. Leonardo da

Vinci made a brilliant comparison between the waves on water and the waves which the wind sends travelling across a wheat field. He went far beyond these isolated remarks in asserting that in general the motions of water and of air are of the same kind. This assertion was based on experience. Leonardo traced streamlines in water by watching small objects cast into the flow, and from his pen we have sketches of waterfalls, river surfaces, and vortices as accurate as a photograph and more beautiful. To follow the motions of air he watched the leaves blown by the wind and even for gentler motions injected smoke into the current. It is now often claimed that Leonardo founded the experimental sciences, but I believe this statement is entirely misleading. Leonardo projected many experiments, some of them reasonable and some of them confused, but he has left us no record of ever having obtained any numerical value by measurement. Rather, he was an observer of undisturbed nature. Like none else he seized upon experience, but experiment lay many decades past his time. In fluid mechanics, as I mentioned, he devised methods for making an existing motion visible, but we have from him no numerical values of discharges or pressures. Leonardo recorded two quantitative statements in hydraulics. One of these is the principle of continuity in its simplest form. The other is the distinction between the rotation of a wheel and the rotation of a potential vortex. Leonardo's words, as always, are vague; if interpreted as strict proportions, his statements are correct, but it is possible that he intended only qualities and inequalities rather than equations. He gives no indication of how he obtained these basic principles, nor does he apply them in any way.

Whether Leonardo's notebooks through private and unacknowledged use influenced the numerous writers on hydraulics who came after him, or whether these discovered anew the facts described earlier by him, will always remain a question in debate. I have described Leonardo's work so as to make clear the keen and abundant experience available before Daniel Bernoulli's time. In addition to these fundamental observations, we must notice the increasing popularity of hydraulic machines from 1500 onward. Both for gainful work and for show or pleasure, pumps, presses, screws, fountains, wheels, conduits, and reservoirs were produced in greater and greater number.

After the principle of continuity, the next theoretical statement made by hydraulic writers was Torricelli's law of efflux, of little use for the design of a pump or wheel. Descartes was the first philosopher to assert that all nature is one great machine, governed by common laws. While nearly all of Descartes' physics is wrong in detail, his grand attempt is the beginning of theory in the modern sense, and as a corollary it began in particular a search for a theory of hydraulics based on mechanical principles. This insistence on generality caused the physicists and geometers to reject all empirical rules and resulted in that separation of hydraulic theory from hydraulic practice, still apparent today, to which Professor Rouse referred in his opening remarks. In nature, the effects of friction, roughness, and turbulence can rarely be neglected and may altogether predominate, just as in ballistics neglect of air resistance would lead to wretched marksmanship. However, just as Galilei's mental abstraction of the medium in which all earthly bodies exist was a necessary forestep to the rational mechanics of solids, a similar abstraction of much of the daily circumstances of water was a necessary preliminary to the rational mechanics of fluids. This abstraction was made by Newton in his celebrated attempt to prove Torricelli's law. While Newton's fiction of the "cataract" is no more than a brilliant course of imagination and hypothesis, it is the first example of hydraulic theory, and as such, even though entirely faulty, it showed the possibility of the field and induced many other savants to attempt the problem. All these later trials were likewise failures. The forces exerted by fluids at rest were by now well known, but to consider the effect of motion on these forces seemed hopelessly difficult.

Such was the scene when Daniel Bernoulli, a young mathematician of twenty-five and already world famous, took up the study of fluids. Throughout all his life Daniel Bernoulli performed both experiments and calculations; while when old he became almost entirely an experimenter, at the period we are discussing he was in the main a mathematician, working not only in mechanics but also in analysis and the theory of numbers. For about five years he gave occasional attention to fluids, and during this time he wrote two important papers on hydraulics before attacking the simultaneous determination of pressure and velocity. The Bernoulli Theorem itself he discovered shortly before 17 July 1730, on which date he wrote to Goldbach as follows: "For my part, I am entirely plunged in water, which furnishes my sole occupation, the some time past I have renounced all that is not hydrostatics or hydraulics.... In these past days I have made a new discovery which can be of great use for the design of conduits for water, but which above all will bring in a new day in physiology: it is to have found the statics of running water, which no one before me has considered, so far as I know.... The problem is to find the effort of water which is pushed with an arbitrary force in an arbitrary tube." He goes on

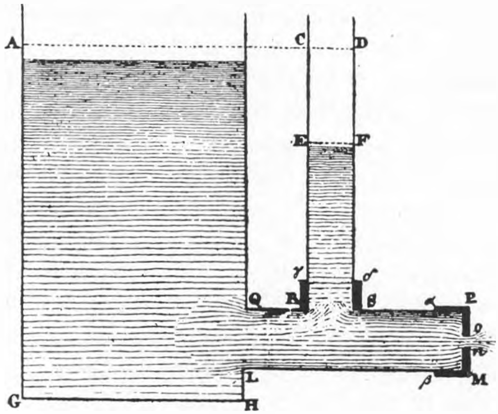


FIG. 1. THIS DRAWING IS NOT BERNOULLI'S ORIGINAL, BUT ONE MADE AFTER IT WHEN HIS LETTER OF 17 JULY 1730 WAS PUBLISHED IN 1843

to explain "one of the simplest cases," the example indicated by Figure 1. For the height SF of the stagnant water over the hole in the tube of running water he obtains

$$SF = \left(1 - \frac{1}{n^2}\right) a \, ,$$

where a is the height of the reservoir and $1/n$ is the ratio of the area of the little hole to the area of the tube. This may not look familiar, but it is in fact the Bernoulli Theorem for this case, with the pressure replaced by the height SF . The advantage of this form is that to test it no velocities need be measured, all quantities being geometric. Bernoulli wrote that an antagonistic senior colleague who belittled his work could not believe the result ". . . until I performed the experiment for him in the presence of other academicians. I made the experiments by means of a very polished iron cylinder which I had caused to be furnished with different covers which had holes of different sizes such as $_a$PMβ ; in the middle of the cylinder was welded a little end of tube γRSδ suitable for supporting a glass tube CRSD . All experiments succeeded perfectly."

Bernoulli's letter makes it clear that his theorem was discovered by theory alone, or, as he put it, *a priori*. Experimental test came afterward. You will note from the diagram that the experiment is devised so as to favor as much as possible the hypotheses under which we now derive Bernoulli's theorem.

Some explanation is required before we can recognize the equation written by Bernoulli as the modern theorem bearing his name.

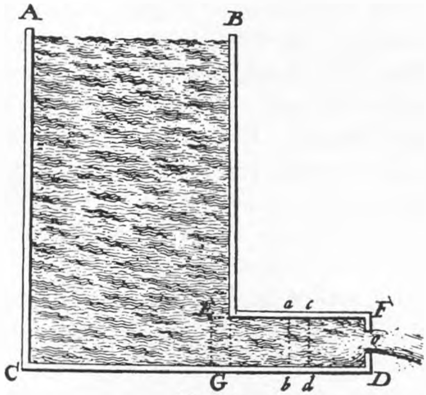


Fig. 2

When we turn to the derivation, even in the improved form published in his often cited but never read *Hydrodynamica,* still more explanation is needed. To repeat Bernoulli's words here would not be helpful. His method is to regard the element EG in Figure 2 as moving down the tube to acdb , where he wishes to find the pressure on the wall. The velocity there is related to the velocity at the small hole o by the principle of continuity. Bernoulli now imagines the tube downstream from ab suddenly to break off or dissolve. The element acdb , thus instantly released, suffers an impulsive acceleration. By using the principle of conservation of energy, Bernoulli calculates this impulse, which in turn he regards as proportional to the pressure on the wall when the tube is not broken off. The argument is intricate; the hypotheses on which it rests are questionable; and the details are confusing.

I have mentioned that Daniel Bernoulli set about at once to

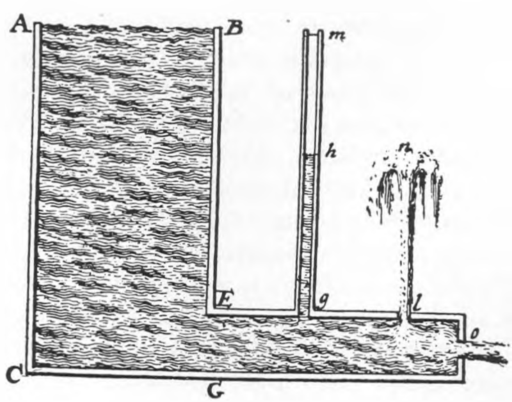


Fig. 3

verify his theorem by experiment. Figure 3 shows the experimental possibilities as presented in the *Hydrodynamica*. It is typical of Daniel Bernoulli not only that theory came before experiment but also that when the result was once confirmed by experiment, he regarded his work as finished. It is easy for us today to see that in fact the equation as written by Bernoulli is not very convenient; Bernoulli's contemporaries saw at once that his derivation was obscure and lacking conviction. What is missing from the equation itself and from the proof is the *internal pressure*, not yet invented. It was this same lack which had prevented Newton from giving an adequate proof of Torricelli's law. In his argument, Bernoulli used four different words, none of them defined, to express the forces exerted by the water upon itself and upon the walls of the tube.

The jealousy of the great Bernoullis among each other is well known. When the publication of Daniel Bernoulli's book in 1738 brought him redoubled fame, his father, the formidable John Bernoulli, who was then seventy-one years old, ill, and with but a decade still to live, grew furious with envy. The problem of proving Torricelli's law he himself had attacked many years before, in vain. All his remaining effort he put out upon the flow of water, and in 1743 he published his own treatise on hydraulics, dating it 1732 and embellishing it with florid and ugly boasting in an effort to steal priority from his son's masterpiece. The details of the controversy I cannot pause to repeat here; let it suffice that while the old man's outrageous procedure would suggest he was a mere thief, in fact his treatise created hydraulics anew. John Bernoulli had as great a talent for mathematics as any man who ever lived. While less successful in discovering the physical principles of a new field of experience, to derive by suitable new concepts and irreproachable reasoning a result already conjectured was something entirely suited to his genius. His innovations were profound. First, he separated the kinematic from the dynamic part of the problem. The principle of continuity and the principle of momentum he used consciously as separate basic postulates, as had none before him. Second, he created the idea of hydraulic pressure. In imagination, he isolated a thin slice of water in a tube and introduced a symbol for the force exerted upon it by the fluid on one side. In this way he achieved a differential equation and integrated it to obtain the Bernoulli Equation for a tube of arbitrary cross-section and position and for not necessarily steady flows. By using the internal pressure he was able to give also a correct derivation from the principle of energy.

The foregoing achievements of old John Bernoulli were not recognized as due to him until last year. Perhaps the reason is that he never explained them with any clarity except in a series of letters to Euler. John Bernoulli lived in a world of challenges, enmities, secret methods, and anagrams. As he wrote to Euler, he derived everything from a certain "principle of the eddy," but even in his letters this is very vague, and in the printed treatise he abstained from expressing it from fear lest the English accuse him of borrowing the "cataract" of Newton. The progress of equations is clear, despite unnecessarily elaborate notations and a mathematical style which was by then obsolete, but mechanical principle is replaced by bombast and boasting.

For Euler, clarity was the hallmark of truth. He saw at once the core of old John Bernoulli's ideas and disrobed them of all vagueness. To him we owe the Bernoulli theorem in the form and terms today in use. To him we owe also the brilliant imagination of the internal pressure in full generality, the pressure field as equipollent to the action of the fluid outside any imaginary closed diaphragm upon that within. This concept, which has been the foundation of all further theory, he achieved ten years after his study of John Bernoulli's hydraulics. To discuss its formation would carry me afield from hydraulics, but I remark upon it in emphasis of the role of imagination and the importance of quantities which can only be thought of and cannot in themselves be measured. Neither is there time to discuss Euler's papers on hydraulic machines, where his grasp of the concept of internal pressure led him not only to detailed analysis of pumps and turbines but also to criteria for avoiding cavitation. These papers were neglected entirely by the hydraulic engineers of the day, and when Euler died in 1783 even a famous physicist of a younger generation characterized his work on fluids as useless in practice and merely exercises in pure mathematics. Euler did not perform experiments except before he was twenty, and thus he was unable to demonstrate the truth of his discoveries to practical men, who in that day despised calculus as being useless higher mathematics. However, Euler was intensely interested in machines, and in 1754 he not only invented the guide wheel for a turbine but even calculated a detailed design and gave a complete hydraulic analysis for the pressure in the rotating machine. Euler wished his turbine to be built, but the engineers at Frederick II's court only reflected the ways of the king himself in scoffing at all of higher mathematics. Euler published two papers, one in French and one in Latin, making his invention free to anyone who paused to read, but in fact it was 190 years before his design was tested. In 1944, long after Euler's guide wheel had been rediscovered and adopted in turbine practice, Ackeret found that a model following Euler's plan reached an efficiency of 71%, as compared with 78-82% for the best modern turbines of similar capacity and head.

It is easy to praise or blame the actions of long ago, since we are free of responsibility in them. When we find parallel events occurring today, with no lesson learned, we are more ready to find excuses. I turn now to the recent discovery of cross-viscosity.

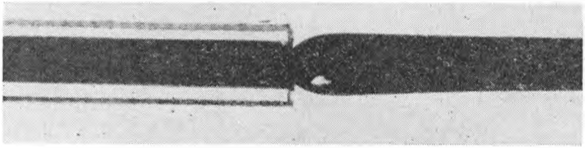


FIG. 4. AFTER MERRINGTON

There are various ways of telling the story, but I prefer to begin with a fact of experience which was a by-product of an experiment. In 1943 Merrington reported that in the course of some measurements of the discharge of rubber solutions or of oils containing metallic soaps he noticed that the fluid column swelled on emerging from the tube (Figure 4). Such an effect obviously cannot follow from any of the classical principles of fluid mechanics. Merrington himself asserted that the fluids in question are visco-elastic and that the swelling is due to their residual elasticity, being in fact recovery from the compression they suffered when forced into the tube. He identified the phenomenon with that observed by Barus in 1893. Barus had cut off perfect cylinders of marine glue extruded from a tube and had found that when left free of external load these cylinders continued to deform and in the end converted themselves into cups. Now Merrington's phenomenon occurs in steady flow. The spring of a portion of visco-elastic substance becomes poorer as time passes. Thus if we compare the swelling of steady flows in longer and longer tubes at the same rate of efflux, for a visco-elastic substance this swelling should diminish, since the portion emerging will then have suffered its compression at more and more remote times in the past. If, however, the swelling is independent of the length of the tube, the phenomenon is not visco-elastic. An experiment should not be difficult, but so far as I know it has never been proposed until today. To return to the story, apparently Merrington did not see any other possible explanation and did not pursue either theory or experiment concerning it.

About the same time several English experimenters noticed a group of new phenomena occurring in rotating fluids. The simplest of these, and the one which really explains all the rest, is that if a rod is rotated in a cup of certain high-polymer solutions, or oils, the fluid climbs up the rod. The results of these experiments were collected and represented schematically by Weissenberg in a diagram published in 1947 (Figure 5). They were not connected with Merrington's phenomenon. Weissenberg proposed an elastic theory which appears to neglect the usual properties of fluids entirely, and some of the other investigators gave semi-quantitative explanations of a chemical nature.

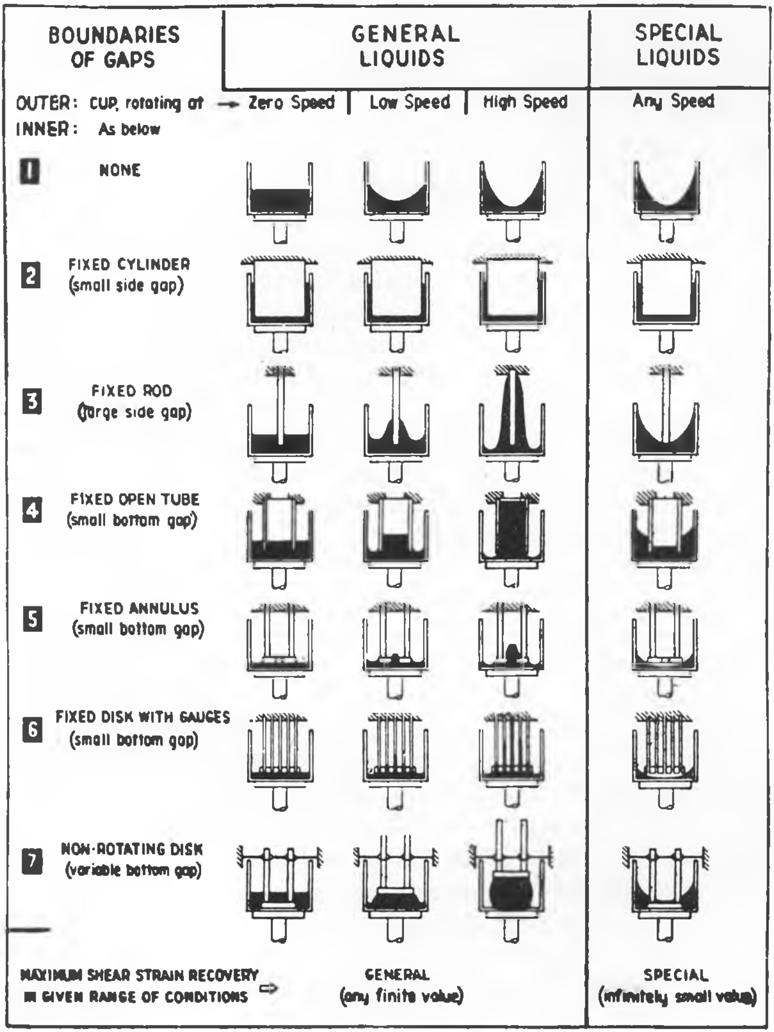


FIG. 5. AFTER WEISSENBERG

Between these two publications, three theorists began to study a subject which turned out to be related to these phenomena of experience. The subject is non-linear viscosity, and the theorists were Reiner, Rivlin, and I. Non-linear viscosity had been studied before, in fact long before, from two different points of view. Some theorists of the last century proposed some general equations as being reasonable, but in fact they did not investigate them sufficiently to get any definite conclusions. More recently a professional group called rheologists had measured departures from linearity in viscometric measurements. The rheologists were accustomed to one dimensional theories in which a single stress component is taken as a non-linear function of a single rate of deformation, but none of this literature could reveal a new phenomenon of a kind not included in the classical theory of viscosity at all.

Reiner is one of this rheological group and has published much work, both theoretical and experimental, of the type described. In 1945 appeared a paper of his of an entirely different character. In it he attempted to apply to fluids the mathematical methods and mechanical concepts used in the general theory of three-dimensional finite elastic strain, an old though little understood branch of mechanics which had been simplified in the previous decade by the introduction of tensor analysis. Reiner had in mind the phenomenon of dilatancy, which Reynolds long ago observed in granular materials: "a definite change of bulk, consequent on a definite change of shape." For example, when one shears wet sand in walking upon it, the footprints are dry, since the volume of the sand mass has increased and thus opened greater voids for the water to sink into. Reiner was able to show that this phenomenon is predicted by the general theory of non-linear viscosity. For his elegant and perspicuous formulation of the general theory itself he acknowledged the assistance of the theoretical physicist Racah.

Rivlin was employed by a laboratory investigating the properties of rubber solutions, and it is possible that his work was motivated by the phenomena published by Merrington and Weissenberg. In any case, he successfully and simply explained them by a theory of incompressible fluids with non-linear viscosity which he published in 1947. This theory is a special case of Reiner's, but Rivlin's work is distinguished not only by greater definiteness and clarity but also by explicit and general solutions for the flow in torsion, tube, and parallel plate viscometers. Particularly surprising is the fact that Merrington's swelling, which Merrington himself regarded as recovery from compression, follows from Rivlin's theory of incompressible fluids. To get the idea behind all these effects it is easiest to consider simple shearing flow. To produce such a flow, according to the general theory, shearing stress is not enough: normal pressures on the shear planes must be supplied as well. This phenomenon is called *cross-viscosity* and is a property altogether independent of shear and bulk viscosity.

Once one has the idea of the general theory of viscosity and a little experience in tensor analysis or matrix algebra, the explanations are not difficult. Each individual phenomenon can also be explained by physical reasoning, but since these physical arguments came only after the really rather simple mathematics was all worked out, I doubt if they are in fact very enlightening, and I will not try to present them.

My own first crude memorandum on non-linear viscosity was issued in 1947, when Reiner's basic paper was already two years in print and Rivlin was far ahead of me. I knew neither their work nor the phenomena of Merrington and Weissenberg. Explanation, though no excuse, for my ignorance may be found in the apathy if not hostility of the world of fluid mechanics toward the subject of non-linear viscosity: all my mentors and colleagues, at that time as now intent upon practical problems and calculations, took no more notice of the work of Reiner and Rivlin than of my own attempts. My formal publication was delayed by the rejection of my paper by a journal of applied mathematics, on the grounds that no one was interested in the subject, the paper would be costly to print, and in any case my work was physics rather than applied mathematics. The terms used by the anonymous referee were so harsh that the only logical alternative to suicide was to give up science forever. While my happening to discover the identity of the referee prevented me from resort to either of these extremes, before arranging for publication abroad I spent eighteen months reviewing the fundamentals of mechanics and trying to learn the processes by which the classical theories had been derived by their discoverers. Priority for non-linear viscosity belongs unquestionably to Reiner and Rivlin, but I speak of my work as well because I have a better knowledge of my own motives and circumstances than of theirs.

For my part, my trouble was that I was employed to study fluids but I could not accept the so-called derivations of the Navier-Stokes equations given in textbooks. It seemed as unreasonable to suppose viscous stress a linear function of rate of deformation as to replace every curve by a straight line. I was aware of claims of departures from the Navier-Stokes equations in certain extreme conditions, but I was more surprised that the Navier-Stokes equations hold at all, and I set out to find the reason. Being naturally both slow and obstinate, I resisted the pressure to calculate or guess useful approximations within accepted theories and to the great annoyance of my superiors and the disgust of my senior colleagues insisted on stopping to think. Textbooks hurry the reader on to accept their conclusions as quickly as possible, often replacing a logical gap by asserting that the result is established by experiment, without a reference. Such evasion is not found in original memoirs dealing with matters of principle. The discoverer or first proponent not only has the task of convincing a skeptical public but also often is close to his struggles to convince himself. Moreover, usually there are no relevant experiments at the time when the theory is first formulated as a plausible model of experience. This statement always surprises believers in the "experimental method". In reply to it they often suggest that if in the history of mechanics theory has usually come before experiment, there must have been many wrong theories proposed. In fact, there were few. Without experience, no explanation definite enough to be considered a mathematical theory is likely to be given; with experience, to expect theorists to propose wholly wrong models suggests a rather limited appreciation for the brains of theorists. Analysis of the dust pile of mechanics reveals few wrong theories but a host of "approximate" or numerical solutions and experimental measurements concerning details and special cases which have lost their interest, as well as many mathematically erroneous "solutions" within correct theories. Today we are piling up this scrap heap so fast that it is difficult to keep the rare cases of fundamental work out from under.

Going back to the great memoir of Stokes on viscosity, I found none of the dogmatism of the modern texts, but instead an honest hesitancy and search for principle. Taking up Stoke's definition of a fluid, I sought by the aid of tensor analysis to put into mathematical form precisely what Stokes had said in words rather than the mere approximations which were all that the mathematics available in his day could easily handle. My work was influenced also by a study of modern general elasticity theory and its mathematics, but while Reiner had attempted to maintain as close a similarity as possible, to me it was the basic conceptual differences which seemed more important. Some earlier writers had spoken loosely of the Navier-Stokes equations as being valid approximately for "small" rates of deformation, just as classical linear elasticity is valid approximately for small strain. This is plain nonsense. Strain is dimensionless and hence can indeed be small, so that the position of linear elasticity with respect to finite elasticity is clear in this formal sense. Not so for fluids, for rate of deformation is of the dimension T^{-1} and hence cannot be absolutely small. It can be small with respect to another rate, but what this standard of comparison should be for a fluid is not obvious. My work on general fluids began at this dilemma, faced it, and resolved it by proposing a theory in which no material parameter of the dimension T can occur. When I learned of the work of Reiner and Rivlin, I found that they had not considered the role of the time, and I was able in some cases to show differences between fluids having a natural time and fluids which do not.

I mentioned the phenomena published by Merrington and Weissenberg, which were soon taken up as evidence for the existence of cross-viscosity. After awhile it was realized that cross-viscosity was not really new. Everyone knows that you cannot stir paint with a rotary motion, for the paint climbs up the rotor. In the paint industry, other methods of stirring were devised and the phenomenon itself apparently regarded as chemical. Here the experience lay before us all, but we were blind to its meaning.

Finally came the time for experiment. All old measurements on non-linear viscosity were made obsolete by the theory, since they measured only small corrections to classical effects and offered no means of detecting even the existence of the new phenomena. The old viscometers have walls supplying lateral pressures of any desired amount with no means of measuring their magnitude. Moreover, it follows very generally from the theory that while departures from the classical first order linear relation is an effect of third order in the rate of deformation, the new phenomena are effects of second order. Thus it is quite possible that fluids previously believed linear in the range tested are in fact non-linear. Precise tests in new instruments designed to show the new effects are necessary, first to measure the modulus of cross-viscosity as a function of the rate of deformation, and second to test the consistency of experiment and theory. Such measurements are now coming into print. Some of these appear to confirm the theory of non-linear viscosity and others do not. In any case we must remember that the classical theory of linear viscosity is a first approximation to several different more general theories, so that not all fluids which obey the classical laws for slow motion can be expected to obey any one particular theory for rapid ones.

The phenomenon of cross-viscosity is altogether typical of non-linear continuum mechanics. Every more or less plausible theory predicts something of this kind. While a few years ago this phenomenon and others like it seemed* outside the domain of mechanics, with the recent development of many new theories we are faced with the opposite difficulty of being unable to use these phenomena as confirming any one theory rather than another. In fact, as I say, these cross effects are typical of mechanics. Everyone knows that if you push a gyroscope, it refuses to move in the direction you push it. This illustrates the general case in mechanics: only when a material is barely stirred out of its sleep will it answer to your wishes and move approximately as you impel it. A century of unquestioning acceptance of linear theories in mechanics has lulled us into expecting response which is not typical. It appears likely that non-linear effects will be discovered in increasing number and may eventually have a greater practical importance than is now foreseen.

My two stories are finished, and I have promised to draw no moral. I hope you will not consider my promise broken if I let Daniel Bernoulli draw a moral. In the quotation which follows, the word "mathematician" occurs, but it comes from late in Bernoulli's life when he was absorbed in experiment, and it obviously refers not to a professional group but to a habit of mind. Here is the quotation: "...there is no philosophy which is not founded upon a knowledge of the phenomena, but to get any profit from this knowledge it is absolutely necessary to be a mathematician."

*Except to the very few persons who knew of certain special results of Poynting (1909-1913) and later writers concerning shear and torsion of elastic materials.

BIBLIOGRAPHY

Ackeret, J., "Untersuchung einer nach den Euler'schen Vorschlagen (1754) gebauten Wasserturbine," *Schweiz. Bauzeitung 123*, 9-15, 1944.

Rouse, H., and Ince, S., "*History of Hydraulics*," separately paginated supplements to *La Houille Blanche*, 1954-5.

Truesdell, C., "A new definition of a fluid, I and II," *J. math. pures appl.* (9) *29*, 215-244, 1950; 30, 111-158, 1951.

Truesdell, C., "The Mechanical Foundations of Elasticity and Fluid Dynamics," *J. Rational Mech. Anal. 1*, 125-300 (1952); *2*, 593-616, 1953. See Chapter V.

Truesdell, C., "A Program of Physical Research in Classical Mechanics," *Z. angew. Math. Phys. 3*, 79-95, 1952.

Truesdell, C., "Rational Fluid Mechanics, 1687-1765," *L. Euleri opera omnia* (II) *12*, IX-CXXV, 1954.

Truesdell, C., "I. Euler's Treatise on Fluid Mechanics (1766); II. The Theory of Aerial Sound, 1687-1788; III. Rational Fluid Mechanics, 1765-1788," *L. Euleri opera omnia* (II) *13*, forthcoming.

Truesdell, C., "Zur Geschichte der inneren Druck," *Phys. Blätter*, forthcoming.

THE LOGIC OF MEASUREMENT

by

GUSTAV BERGMANN

Departments of Philosophy and Psychology,
State University of Iowa

The logic of measurement is a chapter in the philosophy of science. One cannot, therefore, discuss it without possessing a few notions that belong to the philosophy of science in general. My first task is thus, inevitably, to acquaint you with some such notions. In doing this I shall for the most part talk about words, sentences, and language in general. Lest that mode of presentation leave you unnecessarily bewildered, I had better, first of all, say a few words about the so-called linguistic turn, which is probably the most important single thing that has happened in analytical philosophy in this century. I shall limit myself to science and its philosophy. The idea is that while the scientist watches the world, the analytical philosopher watches the scientist watching the world. Let me unpack this formula so that we may see exactly where language comes in. Using the word thing very broadly, one may say that the scientist attends to things, observing them, manipulating them, and so on. Having done that for a while he states what he has seen in words. His language is thus about things, or, more generally, about the world. The analytical philosopher watches and eventually speaks about the scientist's watching and eventually speaking about the world. What, then, one may ask, distinguishes the analytical philosopher from either a psychological student of linguistic behavior or a grammarian? The answer is revealing. There is in our tradition a group of questions that have never ceased to challenge intellectual curiosity such as, say, the nature of causation or the peculiar certainty of deductive inference. These are, of course, the classical philosophical questions. The linguistic philosopher believes that they are all linguistic questions, not in the sense that they are mere questions of grammar or of linguistic behavior, but rather in the sense that the only commonsensical and therefore safe way of answering them begins with an investigation of how language reflects what it is about. These, to be sure, are but poor hints. But I fear that I must let it go at that.

I begin with a dichotomy among words, that is, with a division of all words into two kinds such that each word belongs to one and only one of the two. One of the two classes is exemplified by, say, 'cat', 'color', 'specific heat'; the other, by 'or', 'some', 'is'. The words of the first kind, that exemplified by 'cat' and so on, are called *descriptive*. Those of the second kind, exemplified by 'or' and so on, are called *logical*. The two names, descriptive and logical, allude to the difference between the two kinds. After it has once been pointed out, the difference is, I think, clearly felt. Cats, colors, and specific heats are, to be sure, things very different from each other. Yet they have something in common. They are all "things" or, to put it linguistically, the three words 'cat', 'color', and 'specific heat' each refer to something or name something in a sense in which logical words such as 'or' or 'some' could not by any stretch of the imagination be said to name anything. What the distinction is, is thus clear. To convince one's self that it is worth making one merely has to reflect that in spite of this peculiar feature of logical words, namely, their not referring to anything, no langauge could conceivably get along without them. But let me here add a word of caution that applies equally to all further distinctions. The illustrations I have chosen are clear-cut. Our natural languages, though, English, French, German, and so on, are built for expediency, not for the purpose of exhibiting logical structure. About some words of a natural language there may therefore be doubt whether they belong to the one kind or to the other or, perhaps, depending on how they are used, to both. At this point the analytical philosopher appeals to the schematic languages constructed by the mathematical logician. In such schemata the distinctions are clear-cut. We had better realize, though, that this method raises the question whether one may for the purposes of philosophical analysis without loss or violence replace our natural languages by those schemata. The question must be argued. The answer is affirmative. I shall of course not present the very involved argument on this occasion.

The dichotomy we must consider next is one among sentences. Take the two sentences 'Either it is raining or it is not raining' and 'If everything is green then this is green'. The members of the class they exemplify are called *analytic*. Take next 'Peter is tall' and 'Water if heated boils'. These two sentences belong to the second class. Its members are called *synthetic*. The difference which makes the difference is that while a synthetic sentence says something, an analytical one is tautological or empty. Accordingly, whether a synthetic sentence is true or false depends on what is the case. An analytic sentence, since it says nothing about the world, could not possibly be false. It fits well with this that whether or not a sentence is analytic does not depend on the meaning of the descriptive words which occur in it. Or, to say the same thing positively, whether or not a sentence is analytic depends only on its form, that is, on the logical words in it and on the order and arrangement of the descriptive words it contains. But then, one may wonder why anyone should bother with analytic sentences at all. At first sight it would appear that, saying nothing, they are merely the pathological limiting case of sentences. To understand why the appearance is deceptive, one only needs to remember the crucial role of deductive inference. Take Euclidean geometry. Every Euclidean theorem, those already known as well as those still to be discovered, is a deductive consequence of the Euclidean axioms. What holds for geometry holds equally for every scientific theory. Deductive inference is thus crucial; and there is only one way to explicate what we mean by deductive inference. The conclusion C can be deduced from the premiss P if and only if the compound sentence 'If P then C' is analytic. Presently I shall return to the point; but it will pay if we familiarize ourselves with one more dichotomy among words.

Assume that a foreigner who does not know English very well comes for the first time across the word 'mare'. If he asks us what it means we can do either of two things. If a female horse is at hand we can point at it and say 'This is a mare'. In this case our foreigner learns the meaning of the word directly. Or we could say, without pointing at anything, 'A mare is a female horse'. In this case we have provided the questioner with a *definition*. If he knows what 'horse' and 'female' mean, he will then also know what 'mare' means. Now it is immediately evident that we could not possibly learn the meaning of all words by definition. The meaning of at least some descriptive words, for instance, can be acquired only by becoming acquainted with their referents. On the other hand, it is one of the most important philosophical ideas that we could, in principle, get along with an amazingly small number of undefined words, introducing all others by definitions. Words are thus in principle of two kinds, either *defined* or *undefined* (basic). In a schematic or artificial language the distinction is again clear-cut. To grasp its philosophical significance one merely has to consider that, with one exception, to which I shall attend at the end, even the most abstract descriptive terms of science, with whose referents we are not directly acquainted, can be defined by means of a basic descriptive vocabulary with whose referents we are so acquainted. At the moment, though, I am more interested in the defined logical words. A defined word, by the way, is descriptive if and only if at least one basic descriptive word occurs essentially in its definition; it is logical if only logical words occur in its definition.

Let me take stock. I have introduced you to three dichotomies. Sentences are either analytic or synthetic. Words are either logical or descriptive and either defined or undefined, which yields, in the familiar fashion, four possibilities. Modest as this apparatus is, it permits one to state intelligently and, I hope, intelligibly, one of the most fundamental results of recent philosophical analysis. All arithmetical words, from the integers 1, 2, 3 ..... and the humble '+' to the most complicated notions of higher analysis, are logical words. All arithmetical truths, from the simple '$1 + 1 = 2$' to the most esoteric theorem about Hilbert spaces are analytic. The decisive idea is that, rather surprisingly, the integers themselves and the elementary operations among them can be defined in terms of indubitably logical words such as 'and', 'or,' 'all' and 'some'. The great names connected with this discovery are Peano, Frege, and Russell. The classical document, though of course not the last word, is *Principia Mathematica* by Russell and Whitehead.

Measurement is the assignment of numbers to objects or events according to certain rules. That is why it was necessary to begin as I did. Consider the two statements '$5 + 3 = 8$' and '5 feet *and* 3 feet are 8 feet'. One who understands fully the differences and similarities between these two statements has the key to the logic of measurement; but even to state these differences and similarities one needs our little apparatus. At the moment I shall only mention the differences. '$5 + 3 = 8$' is analytic. The '+' in it, not naming anything, is a logical word of the kind called an operator, that is, it is a word that makes out of two or several words, in this case out of '3' and '5', an expression, in this case the phrase '$5 + 3$ ', which functions itself in many respects like a word. The second statement, '5 feet and 3 feet are 8 feet', is a synthetic sentence of the kind called an empirical law. The 'and' in it is a descriptive operator. What it names, very elliptically, is what we do when we put two straight sticks end to end so that they are in a straight line and then, perhaps, either nail or glue them together. But I notice that I just used several words that need explication.

I shall not undertake to explain what a law is beyond mentioning that, as I use them, the word 'law' and the phrases 'empirical law', 'law of nature', and 'synthetic generality' are synonymous. But it will be necessary for us to distinguish laws from what analytical philosophers call *relations*. Take Boyle's law, $pv = c$. Scientists often say that this law is or establishes a relation between the volume and the pressure of a quantity of gas at a constant temperature. Thus they use 'law' and 'relation' more or less synonymously. This usage blurs a distinction. Strictly speaking, a relation is a character. Being taller, being contiguous, being later are three simple instances of relations. A relation, in other words, is like a property, the only difference being that while a property is exemplified by one thing, a two-term relation is exemplified by two things, a three-term relation, such as betweenness, by three things, and so on. Accordingly, relations are referred to either by words or by phrases which function like words. Laws are expressed by statements. Let 'Peter' be the name of an adult elephant, 'Paul' that of an adult chihuahua. The sentence 'Peter is taller than Paul' says that a certain relation obtains between the individuals Peter and Paul. Accordingly, it contains the relational expression 'is taller than'. But, not being a generality, it is not a law. Take next the sentence 'All crows are black'. It is a generality or law. Yet it does not mention a relation. Accordingly, none of the four words in it is a relation term. 'Crow' and 'black' are the names of descriptive properties; 'all' and 'are' are nonrelational logical words. Take finally 'An adult elephant is taller than an adult chihuahua'. This statement says that every individual of a certain kind stands in a certain relation to every individual of a certain other kind. It is therefore a generality or law. Also, this particular law does mention a relation, as the law 'All crows are black' does not. These examples should go a long way toward convincing anybody that we had better be careful about the way we use 'law' and 'relation'[1].

Relations are either descriptive or logical. Take two straight sticks, put them along side of each other so that one end of the one coincides with one end of the other. If in this position stick a protrudes beyond stick b, we say that a is longer than b. If the two ends we have not put together coincide, so that neither stick protrudes beyond the other, then we say that a and b are equally long. "Longer" and "equally long" are two descriptive relations. As it happens, they are also defined relations. I have, in fact, just defined them in terms of two other relations, namely, coinciding and protruding; nor would it be difficult to define protruding in terms of coinciding and thus our two relations in terms of coinciding alone. Notice also the following empirical law about straight sticks. Of any two sticks, either the first is longer than the second or the

[1] It is worth noticing that every empirical law can be construed as a statement to the effect that the *descriptive* characters (up to and including type n) mentioned in it satisfy a *logical* relation (of type $n + 1$). In this sense every empirical law has a "logical structure." See also footnotes 2 and 3.

second is longer than the first or they are equally long. Let us next provide ourselves with some instances of logical relations. Those obtaining among numbers are a very important kind of such relations. To be "divisible," for instance, is a relation that may or may not obtain between integers. 6 and 3 exemplify this particular relation, 6 and 5 do not. "Sum" is a three-term relation exemplified by 5, 3, and 8 in this order and in the order 3, 5, 8 but not in any other order nor, say, by 5, 3, and 9. Again, to be "larger" and to be "identical" are logical two-term relations among real numbers and numerical expressions. Notice finally the following analytical truth. Of any two real numbers or numerical expressions either the first is larger than the second or the second is larger than the first or they are identical.

I just called attention to an empirical law for straight sticks, "Of any two (straight) sticks, either the first is longer than the second, and so on," and to an analytic truth, "Of any two (real) numbers, either the first is larger than the second, and so on." Clearly, there is some connection between these two generalities; the one, synthetic, about things; the other, analytic, about numbers. Clearly, this connection is essential for measurement, that is, as I put it, for the rules by which numbers are assigned to things or events. Equally clearly, I think, our task is therefore to state this connection both as precisely and as generally as possible. As to precision, I can of course not offer much on this occasion. As to generality, a few preliminary remarks are necessary.

So far I have spoken as if there were only two kinds of things in the world, individuals, whatever that may mean, and characters, either properties or relations, such that the characters are exemplified by individuals. Now we must rid ourselves of this simplification. Properties and relations do in turn have properties and stand in relations to each other. Or, to say the same thing in Russell's words, there are characters of different types. Consider the sentence 'Green is a color'. If, as on this level of abstractness one must, we take our cue from grammar, the very fact that this sentence makes sense (it is even true) indicates that 'color' names a descriptive character of characters. For our purposes, though, the logical characters of characters are of particular interest. Transitivity, for instance, is such a character, as may be seen from our saying, truly, that the descriptive relation of being longer, which obtains among straight sticks, and the logical relation of being larger, which obtains among real numbers, are both transitive. As everybody knows, a two-term relation is called transitive if and only if, for any three things, it obtains between the first and the third provided it obtains between the first and the second as well as between the second and the third. Since I just defined it, transitivity obviously is a defined relation[2]. As to its being logical, one merely has to convince one's self that the definiens, that is, the clause following 'if and only if' in the definition I just wrote down, contains only logical words. This is indeed so. The only two words about which one could have any doubts are 'thing' and 'relation'. In a schematism these two words would be represented by what are called variables of unlimited range. That such variables are logical signs is indeed plausible. Even so, the case must be argued; it can of course be argued; but I am sure you will not expect me to expound so subtle a point in the philosophy of logic proper on this occasion. I instead call your attention to the fact that when we say of a descriptive character that it has a certain logical character, e.g., that being longer is transitive, then we state an empirical law, while when we say the same thing of a logical character, then we state an analytic truth.

We notice, then, that the descriptive relation of being longer, whose field is the class of (straight) sticks, shares a logical property, namely, transitivity, with the logical relation of being larger, whose field is the class of (real) numbers. It is easily shown that the same holds for the descriptive relation of being equally long in the field of sticks and the logical relation of identity in the field of numbers. They, too, share some logical properties, e.g., they are both symmetrical, transitive, and reflexive. Similarly, there are certain logical relations exemplified by longer and equally long in the field of sticks as well as by larger and identical in the field of numbers[3]. I shall express this state of affairs by saying that the two fields, that of sticks and that of numbers, share with respect to these two pairs of relations a certain *structure*, or a certain logical structure. This, by the way, is one specific meaning of that desperately vague word, structure. I hurry to add that the word field, in the sense in which I use it here, also can and must be defined precisely. But I trust no harm will be done if, for brevity's sake, I shall continue to use it without further explication.

---

[2] There are no undefined logical relations.

[3] Let R be the logical relation between two relations r and s that is defined by *Df.*

$$R(r, s) = (x, y, z)\ [r(x, y) \cdot s(x, z) \supset s(y, z)].$$

Substitution of 'equally long' and 'longer' for 'r' and 's' respectively yields, for the case of length, one of the axioms of rank order.

I am now ready to generalize from our example. Consider two classes of objects (I use 'object' very broadly, really only to fill the need for a grammatical "object"), a, b, c, ... and α, β, γ, ..., such that the first is the field of n relations, r_1, r_2, ..., r_n, the second the field of n other relations, ρ_1, ρ_2, ρ_3, ..., ρ_n. Assume that these two groups of relations share a certain logical structure in which r_1 corresponds to ρ_1, r_2 to ρ_2, and so on, up to and including n, as in our example being longer and equally long correspond to being larger and being identical, respectively. If this is so, then it will often be possible to coordinate to each object a of the first field one and only one object α of the second field so that a certain relation r obtains among objects of the first field if and only if the corresponding relation ρ obtains among those objects of the second field which are coordinated to them. Again I must warn you that all this is not as precise as it can and must be made. But again, it will serve our very limited purpose. In the case of measurement, the α and the ρ are of course the numbers and the arithmetical relations among them; the objects a and the relations r are the things we measure and the descriptive relations among these things. What makes measurement possible is that the two fields share a certain logical structure. But then, as we just saw, to say that a descriptive relation or a group of such has a certain logical structure is the same thing as to say that these relations satisfy certain empirical laws. It follows that measurement can be introduced into a field if and only if the relations which obtain among its things fulfill certain empirical laws.

I am virtually certain that some of you are ready to question the value of all this strained and studied generality. Let me anticipate this sort of criticism. With respect to our example I have, roughly speaking, said no more than this. Since any two straight sticks are either equally long or one is longer than the other and since being longer is transitive, one can to each stick so coordinate a number that (1) two sticks are equally long if and only if the numbers coordinated to them are identical and that (2) one is longer than the other if and only if the number coordinated to the former is larger than that coordinated to the latter. (This, by the way, is merely a rank order and therefore not yet as desirable a measurement as can be established in view of the so-called additivity of length.) I grant cheerfully that you have known this before. Yet I insist that there was some point to our labors. The example is obvious. Naturally; examples ought to be obvious. The point is that the formulation I based on it is so general that it comprehends everything and anything any scientist, either physical or biological or behavioral, ever called and, I venture to predict, ever will call measurement. If, for instance, some among you wonder whether the formulation also comprehends what physicists call vectors (behavior scientists speak of multiple scores or profiles), let me remind them that what goes for numbers also goes for ordered pairs, triples, and n-tuples of numbers, if for no other reason than that these latter entities are themselves defined in terms of numbers. The generality we achieved is thus complete; and to have achieved such generality is the same thing as to have analyzed or explicated the logical nature of measurement. In what follows I shall support this contention by showing that some further comments, which are I think of some interest, all flow from our general formulation.

Measurement, I suggested, is the assignment of numbers to things according to certain rules. The word rule in this formula is ambiguous. Yet I used it deliberately, because I was then not ready to dissolve the ambiguity. Now I am. One meaning of 'rule' is that of convention. Rules in this sense are arbitrary or matters of social agreement. Another meaning of 'rule' is that of law of nature; and there is nothing arbitrary or conventional about laws of nature. As to our formula, I now amend it to read that measurement is the assignment of numbers to things according to both laws and conventions. The laws on which measurement is based are of two kinds. To distinguish among them, it will be convenient to introduce a new term. Instead of always speaking laboriously of a field and the descriptive relations within it, I shall, as one usually does, speak of a *dimension*. Measurement may then be based on no other laws than those within the dimension, that is, on no other laws than those connecting its descriptive relations. Such measurement is called *fundamental*. This is the only case we have so far considered. Or measurement may also utilize laws that connect the characters of one dimension with those of others. Such measurement is called *derived*. I shall attend to it presently. As to the conventions entering measurement, we often find that even if we utilize as many empirical laws as we possibly can, the coordination of numbers to things is not yet uniquely determined. Consider two obvious examples. If the empirical laws utilized are those of a rank order and of nothing else as, for instance, in the Mohs hardness scale, then any assignment of numbers that preserves the order is as good as any other. But even in the case of a measurement as desirable as the ordinary measurement of length the choice of the so-called unit is still a matter of convention. The result of the assignment of numbers to the things of a dimension according to certain laws and conventions is called a *scale.* Thus we are led to make another distinction. We had better not confuse a dimension with the several scales that may be constructed to measure it.

Straight sticks can be ranked. In such a rank order a one-inch stick may receive rank 1, a seven-inch stick rank 2, a 19.5-inch stick rank 3, and so on, quite wildly, provided only that the longer stick always receives the higher rank. But sticks can also be measured in the ordinary way, say, in inches. Everyone agrees that the second measurement is more desirable or better than the first. This agreement sets us the task of stating exactly what it is that we mean when we express such preferences. The answer is not difficult. The essence of measurement is that *some* arithmetical relations among the numbers assigned correspond, by virtue of a shared logical structure, to descriptive relations among the things to which they are assigned. The measurement we prefer to others is so constructed that a *maximum* number of arithmetical relations has such descriptive correlates or, as one also says, empirical meaning. In a mere rank order, for instance, the so-called equality of differences has no empirical meaning. Specifically, it makes no sense to say that the difference in hardness between two minerals of Mohs ranks 2 and 4 is equal to that between two minerals of ranks 7 and 9. What holds for a rank-order hardness also holds for a rank-order length. In ordinary length measurement, on the other hand, the equality of differences has a familiar meaning to which I shall presently attend. First, though, I should like to make another point. The first one who saw it was, as far as I know, the great Helmholtz, in his essay "*Ueber Zaehlen und Messen.*"

It follows from our explication that counting is not a species of measuring, or perhaps better, mere counting is not yet measuring. The reason is that numerosity, that is, being of a certain number, is a logical property of classes, not of things, and that the grouping of things in classes, in the sense of 'class' which is here relevant, is arbitrary in a sense in which the grouping of things on the basis of the descriptive characters they exemplify is not. This, however, is not to deny that counting may be an ingredient of measuring. It very often is. If, for instance, we assert that a certain ledge is three inches long, we have counted the layings-off of a unit. But then, these layings-off are the descriptive relational ingredient that is not to be found in mere counting.

The peculiar excellence of ordinary length measurement rests on that feature of the dimension which is known as *additivity.*

Again, I am sure that you know what is involved. To remind yourself, remember the two sentences with which I started, '$5 + 3 = 8$' and '5 feet and 3 feet are 8 feet'. I shall again state the matter as generally as possible. A dimension is called additive if and only if it permits of an operation within it that fulfills two conditions. (1) The operation, called "physical adding," coordinates uniquely to any two objects of the dimension a third, called their "physical sum." (2) The descriptive operator has the logical structure of arithmetical addition. In the case of length the physical operation consists, schematically speaking, in laying two straight sticks end to end in a straight line and then either nailing or gluing them together. Now for two comments. *First:* Notice that I spoke of operations *within the dimension.* What that excludes is best shown by examples. Take temperature, which is a linear but not an additive dimension and assume that a rank-order scale for it has somehow been constructed. Suppose that someone proposes to make this rank order additive by defining as the "sum" of ranks T_1 and T_2 the rank $T_1 + T_2$. We shall point out to him that he has done nothing of the sort, since in his definition no physical operation is mentioned. All he has said, in a rather misleading manner, is that the number $T_1 + T_2$ is the arithmetical sum of the numbers T_1 and T_2. Assume next that someone else proposes as the physical sum of temperatures T_1 and T_2 the temperature T' that prevails in two objects, originally of temperatures T_1 and T_2, respectively, after they have been brought into contact and thermic equilibrium has established itself. This time there is a physical operation. Unfortunately, it does not fulfill our first condition. T', as we all know, is not uniquely determined by T_1 and T_2. This defect, however, can be remedied. If we specify that the two bodies are to be of the same weight and of the same chemical composition, then T' is uniquely determined by T_1 and T_2. Thus, our first condition being fulfilled, it would seem that we have at least a candidate for a physical sum. Again, we all know that however the rank order may have been scaled, this operation does not fulfill our second condition, that is, it does not have the logical structure of addition. This, though, is not the point I want to make. The point is, rather, that even if the second condition were fulfilled, the operation would still not be one within the dimension since, in order to secure the definiteness of T' we had to draw upon extraneous factors, namely, the dimension of weight and chemical composition[4]. *Second:* Notice that I spoke throughout of the additivity, not of a scale, but of a dimension. To grasp this point, consider again length, this time scaled logarithmically, that is, the way we actually measure it when we use a slide rule. With this scale the number assigned to the physical sum is not, as with the ordinary scale, the arithmetical sum but, rather, the arithmetical product of the numbers assigned to its physical constituents. All one can say, therefore, is this. If a dimension is additive, which is an empirical matter and not one of scales, then it *can* always be so scaled that, as in the case of ordinary length, the number assigned to the physical sum is the arithmetical sum of the numbers assigned to its physical constituents.

We have come upon a new question, namely, whether it is merely a matter of habit that we prefer, as we actually do, except for some very special purposes, the ordinary foot-rule scale to the slide-rule scale of length. The answer, which is again quite general, is this. Other circumstances being equal, we prefer that scale which gives to a maximum number of laws, or, perhaps, to a certain group of laws in which we are specially interested, the simplest mathematical form. In a sense there are thus rational grounds for this kind of preference, too. If I say "in a sense," it is because I, for one, believe that the notion of simplicity itself is by no means simple, or, perhaps better, that it is essentially a psychological notion. This, however, is a long story and a rather controversial one at that. So I shall assume that we know what we mean by mathematical simplicity and show next how this notion operates in the case of derived measurement.

With a few idealizing assumptions it is possible to introduce through fundamental measurement a rank order in the dimension known as density. In the case of nonmixing liquids, for instance, we discover that when we pour any two of them together either one always goes to the top or the one poured last, whichever of the two it may be, stays at the top. On these two empirical relations a rank order can be based. Such a rank order may be quite wild; say, olive oil 1, water 2, concentrated sulfuric acid 17, and so on. Assume

[4] Some might object that by analogy with this example even mass and weight would not be additive and, perhaps, not even rankable within the dimension since, if we use a balance based on the lever principle, we must specify that its arms are of equal length. I would answer that in this case the extraneous dimension, length, is not, as in my example, a relevant property of the objects to be scaled. Or, to speak metaphorically, the balance plays the role of a parameter, not that of a variable.

next that we have also discovered the law that connects this dimension with two other ones, namely, weight and volume, each of which is additive and already provided with a scale very rich in empirical meaning. The law is, of course, that the rank order we established by fundamental measurement is identical with that produced by the quotient of weight over volume. Under the circumstances we may choose these quotients themselves as the numbers of our original rank order. What we achieve by this choice is that the law connecting the three dimensions takes the simple form $d = \frac{W}{V}$. I am sure you see how this example hits two birds with one stone. It shows how arithmetical simplicity enters and it also shows what we mean by derived measurement. For derived measurement, you remember, is measurement that utilizes the laws which connect the characters within a dimension with extraneous ones. Ordinary density is thus a derived scale with weight and volume as the extraneous dimensions. In the case of density, we saw, fundamental measurement within the dimension is, at least in principle, possible. But there is also the possibility of defined magnitudes such that no scale can be based on any relations among them, so that we can in their case no longer strictly speak of a dimension. Even in these cases very satisfactory measurement may be possible. All that is needed is that the defining magnitudes be satisfactorily scaled. The definition then yields automatically a meaningful scaling of the magnitude defined. That is, as it were, the limiting case of derived measurement. The centimeter-gram-second units of theoretical physics are probably the most important application of this idea.

I am ready to sum up. Details and some unavoidable preparations apart, I have asked and answered two questions. The first question was: How is it possible for arithmetical relations among numbers assigned to things to mirror descriptive relations among these things themselves? The answer is: By virtue of a shared logical structure. The second problem with which I dealt is in both question and answer a corollary to the first. The question was: On what rational grounds do we prefer one measurement to another? The answer is: We call that measurement best which manages to endow a maximum of arithmetical relations with empirical meaning. A little reflection will show you that these are the only two problems with which, however sketchily, I have dealt. They are indeed the logical heart of the matter, although they are not the whole of it. In conclusion I shall therefore at least mention four further issues that arise in the logical analysis of measurement.

1. There is an important difference between, say, assigning a number to the momentary strength of an electric current in a wire and assigning numbers to the momentary position or velocity of an electron. The difference is not that in the case of the current we read the number directly from a dial while in the other case, that of the electron, we obtain it by computation from the number or numbers read from one or several dials. The difference is, rather, that the strength of a current can be defined in terms of what we are directly acquainted with while such entities as electrons belong to what is called a model or, more generally, a partially interpreted calculus. It follows that in the case of the electron the numbers assigned depend essentially on the features of the model itself as well as on the way it is fitted to what we are acquainted with. To appreciate the importance of the distinction consider the possibility that by the very way the model is constructed and fitted it yields, even upon the most accurate measurement, not a definite value of, but merely a range for, say, the position of an electron. Clearly, such indeterminacy in assignment must be distinguished from what could reasonably be meant by a lack of accuracy in measurement. As you know, situations whose analysis requires this distinction actually occur in modern physics.

2. No measurement yields or ever will yield a real number. In the ordinary course of events we are satisfied with two or three digits; to ascertain reliably five or six digits is, as every scientist knows, a major effort. All we obtain operationally are thus fractions. In our computations we nevertheless consider those fractions as real numbers or as approximations to such. The advantage of this procedure is essentially computational, that is, logical. That it is advantageous, there is no doubt. The question that arises concerns the legitimacy of the jump from the fractions which we actually obtain through physical operations to the real numbers with which we "operate" verbally. The answer is that whenever we do mention real numbers in synthetic statements we have in effect introduced a partially interpreted calculus.

3. I just spoke several times of accuracy and once of reliability. These words, too, stand in need of elucidation. *Reliability* is a statistical notion, defined by some such terms as, say, the inverse standard deviation of a series of successive measurements. What I mean by *precision* is, simply, the number of digits of a given unit ascertained in a single measurement. The word accuracy is, I believe, often used very inaccurately. The two clear notions are reliability and precision. When we speak of accuracy, we mean

either the one or the other or, perhaps, some ill-defined compound index of both.

4. Imagine a temperature measurement so precise that the heat exchange between the measuring instrument and the system measured cannot be neglected. In this case we do not expect repeated measurements to yield the same result. Quite to the contrary, we would be baffled if they did. This shows that the explication of reliability I just suggested is not yet as general as it could be and, therefore, ought to be. The general issue involved is that of the interaction, or possible interaction, between the object measured and the yardstick with which we measure it. This issue, too, arose in a rather radical form in modern physics. Its analysis leads far beyond the limits of this very elementary discussion.*

REFERENCE

*See G. Bergmann, "The Logic of Quanta," *Am. J. of Physics*, 15, 1947 (reprinted in H. Feigl and M. Brodbeck, eds., *Readings in the Philosophy of Science*, Appleton-Century-Crofts, 1953).

INDIRECT METHODS OF RIVER DISCHARGE MEASUREMENT

by

J. S. Cragwall, Jr.
U. S. Geological Survey, Washington, D. C.

Introduction

The United States Geological Survey is one of the federal agencies designated to collect data pertaining to the water resources of this country. Among its responsibilities is measurement of stream flow. The determination of peak discharges of floods, no small part of the stream-gaging program, becomes an important and challenging task.

Stream flow is usually measured by the familiar current-meter method. Oftentimes during floods, and for many good reasons, however, current-meter measurements cannot be obtained. For definition of floods under such conditions, peak discharges can frequently be measured by so-called indirect methods. It is the purpose of this paper to describe these indirect methods of river-discharge measurement.

Basis of Indirect Methods

Discharge in a reach of channel is related to the water-surface profile and the hydraulic characteristics of the channel. Definition of water-surface profiles from high-water marks, and channel characteristics from a survey of channel size, geometry, and roughness constitute the basis of indirect methods of river-discharge measurement.

The water-surface profile is an important element in an indirect measurement. Not only must its elevation be known, but even more important, its changes within the reach of channel under consideration. These changes in profile are primarily the result of (1) energy losses due to bed roughness, eddies, etc., and (2) acceleration. For a relatively uniform reach of stream channel, the change in water-surface profile results largely from bed roughness. At sudden contractions, such as bridges, culverts, and dams, the surface profile reacts to the influence of acceleration; that is, the change in profile reflects primarily a change in energy from potential to kinetic.

NECESSITY FOR INDIRECT METHODS

The necessity for indirect methods of measurement becomes apparent following any widespread flood of unusual magnitude. The great flood of July 1951 in Kansas and Missouri is a striking example. That flood reached such extremes of stage and discharge that it was found impossible to carry on the normal stream-gaging program in the flooded area. Travel during and immediately following the flood period was at a standstill. Even had it been possible to reach gaging sites, the structures from which measurements would normally have been made were, for the most part, destroyed, overflowed, or bypassed by wide and swift overbank flows, making impossible the measurement of discharge by current meter. Because of these and other difficulties, few direct measurements of peak, or near-peak, flows were made. For a flood of such extreme and widespread proportions, however, it was imperative that discharges be adequately defined.

Soon after the recession of the flood, operations were begun to determine peak discharges by indirect methods at gaging stations and other critical points in the flood area. Engineers from Geological Survey offices all over the country—men experienced in indirect methods of measurement—were quickly dispatched to the flooded area. This group, with the assistance of some personnel from other agencies, State and Federal, manned numerous field parties for securing the necessary surveys, computed the flood records, and prepared reports thereof.

The magnitude of the task and the coverage obtained are illustrated on Fig. 1, a map of the Kansas River basin. Shown thereon are 88 sites where discharge was determined, most of them at established gaging stations. At 50 of those sites, peak flows were defined by indirect methods. Over the entire area encompassed by the flood, discharge was defined at 182 sites, at which, for 68, indirect methods of measurement were applied in defining flood peaks.

This concerted effort was rewarding. In later basin-wide flow comparisons using flood-routing techniques, flood discharges (peaks and volumes) were found consistent to an encouraging degree. Where possible, comparisons with stage-discharge ratings defined by current meter indicated that ratings defined by indirect methods were reliable.

The Kansas-Missouri floods of July 1951 are reported in *Water-Supply Paper* 1139 [1]. That the flood data contained therein were, to a great extent, based on indirect methods of flow measurement, indicates the practical necessity for and the value of indirect

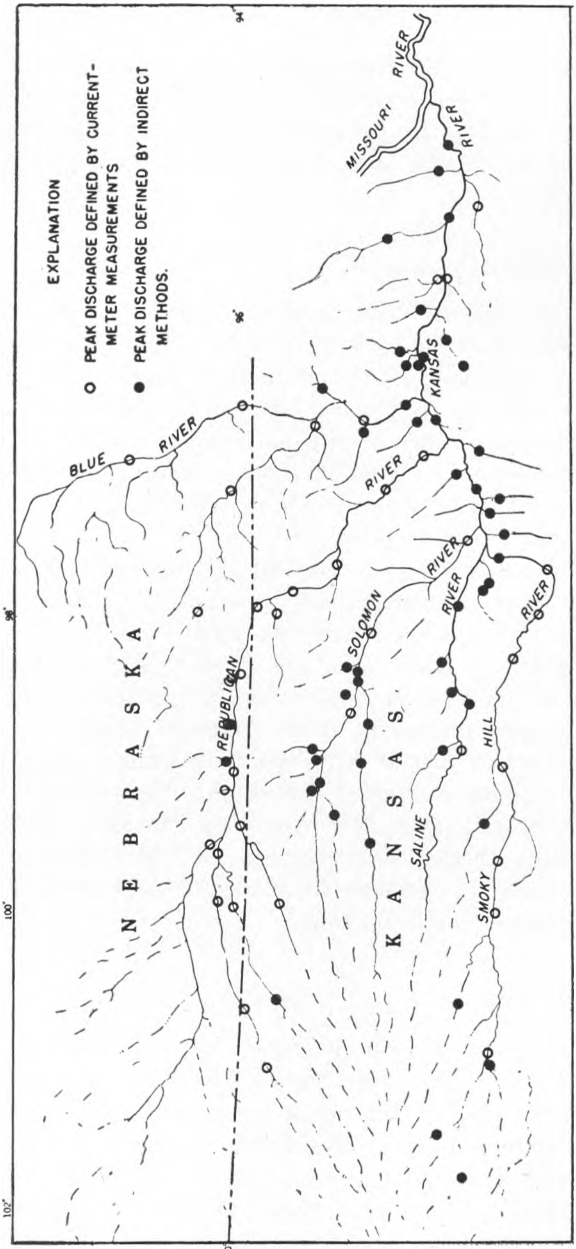


FIG. 1. MAP OF KANSAS RIVER BASIN SHOWING LOCATION OF FLOOD-DETERMINATION POINTS, FLOOD OF JULY 1951

methods. Unquestionably, the quantitative description of the Kansas-Missouri floods of 1951 would not be so complete nor the data nearly so reliable had methods of indirect flow measurement not been applied.

The foregoing is an outstanding single example of the value and necessity for using indirect methods. Their worth can further be attested by the fact that annually, on the average, the Geological Survey makes about 600 determinations of peak discharge by indirect methods.

INDIRECT METHODS CLASSIFIED

Indirect methods of river-discharge measurement are grouped into four major categories for ease of application and reference. These are (1) slope area, (2) contracted opening, (3) flow through culverts, and (4) flow over dams. The four classifications although somewhat arbitrary, have been found convenient in setting up field and office procedures. Occasionally an indirect measurement will involve a combination of methods or another method of solution outside of these general classes.

The slope-area method is the most frequently used, especially on the large rivers, primarily because a natural reach of channel acceptable as a slope-area reach can usually be found. Contracted-opening, culvert, or dam sites are used whenever conditions are favorable. The change in water-surface profile through a slope-area reach results primarily from channel roughness and, hence, the ability to select proper roughness coefficients is a measure of the accuracy of the computed discharge. The contracted-opening, flow-through-culvert, and flow-over-dam methods involve abrupt contractions and, hence, the changes in water-surface profile reflect mainly changes in energy form, and the value of the roughness coefficient becomes less important.

SLOPE-AREA METHOD

In the slope-area method, discharge is computed on the basis of a uniform flow equation involving channel characteristics, water-surface profiles, and a roughness or retardation coefficient. The change in water-surface profile for a uniform reach of channel represents losses caused by bed roughness.

In application of the slope-area method, any one of the well-known variations of the Chézy equation might well be used. The Geological Survey uses the Manning formula. This formula was originally adopted, as it has been by many engineers, because of its simplicity of application. Many years of experience in its use have now been accumulated. That it has become a medium through which reliable results can be obtained, justifies its use.

The Manning formula, written in terms of discharge, is

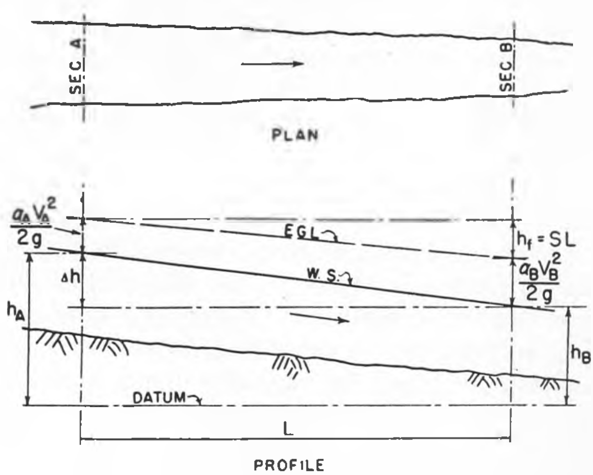
$$Q = \frac{1.486}{n} A R^{2/3} S^{1/2} \qquad (1)$$

where

Q = discharge, in cfs
A = cross-sectional area, in sq ft
R = hydraulic radius in ft
S = energy gradient, or friction slope
n = a roughness or retardation coefficient

Manning's formula, as originally developed, was intended only for uniform flow where the water-surface profile is parallel to the stream bed and the area and hydraulic radius remain constant throughout the reach. In spite of these limitations, since a better solution is lacking, the formula is used for the nonuniform reaches that are invariably encountered in natural channels. The only justification for such use is that of necessity; however, several factors in the formula are modified in an attempt to correct for nonuniformity.

A description of the method used in computing river discharge by the slope-area method is discussed in conjunction with Fig. 2,



$$Q = \frac{1.486}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}} = K S^{\frac{1}{2}}$$

$$K = \sqrt{K_A \cdot K_B} \qquad S = \frac{h_f}{L} = \frac{\Delta h + \left(\frac{a_A V_A^2}{2g} - \frac{a_B V_B^2}{2g}\right)}{L}$$

FIG. 2. DEFINITION SKETCH OF A TWO-SECTION SLOPE-AREA REACH (NOT TO SCALE)

a definition sketch of a two-section reach which is gradually contracting in direction of flow. Cross sections A and B are selected as being representative of the channel between A and B. The term $[(1.486/n)AR^{2/3}]$ contains the several factors descriptive of channel characteristics and is labeled "conveyance" K. For the reach A-B, because it is not truly uniform, conveyance is expressed as the geometric mean of the conveyances of the two sections, in this way: $K = \sqrt{K_A K_B}$

For the contracting reach, the drop in water-surface profile $\triangle h$ is not entirely a result of friction loss h_f, but also reflects the acceleration between sections A and B. It the S term in Manning's formula is taken to represent friction slope only, then the drop in water level Δh must be adjusted by the change in velocity head. Friction slope may be expressed as

$$S = \frac{h_f}{L} = \frac{\Delta h + \left[\frac{\alpha_A V_A{}^2}{2g} - \frac{\alpha_B V_B{}^2}{2g}\right]}{L}$$

The discharge formula, therefore, may be condensed to

$$Q = KS^{1/2} \tag{2}$$

where K represents the average conveyance of the reach and S the energy gradient or friction slope.

In this manner Manning's formula is applied to gradually-varied flow in natural channels. Admittedly, assumptions have been made that greatly oversimplify the complex functions inherent in non-uniform flow. This treatment, however, provides a logical method to which experience can be related, and when a given problem is within the accumulated experience range, it has been found that reliable results can generally be obtained.

Note that a moderately contracting reach is illustrated in Fig. 2. The practice of using such reaches—reaches that are contracting gradually and to moderate degree in the direction of flow—has become standard Survey procedure. Contracting reaches have been found to yield a consistent scale of values in the Survey's roughness-investigation program. On the other hand, expanding reaches have not, probably because of unknown energy losses and velocity distributions associated with the diverging flow patterns. Uniform reaches are not intentionally avoided, but in reality can rarely be found. By searching for slightly contracting reaches, expanding reaches can at least be avoided. Contracting reaches also are favored for other reasons. For example, any errors involved in estimating n affect the discharge result to a lesser degree than in expanding reaches, owing to the effect of the velocity-head correction involved in computing the energy gradient S.

Coefficients

Selection of the roughness coefficient n is a critical step in a slope-area measurement, so much so because personal judgment cannot be entirely eliminated. Moreover, for a natural channel the coefficient is descriptive of losses other than bed roughness. Bank irregularities, channel meanders and curvature, overhanging trees, and many other retarding influences that defy quantitative description are present. Tables of n values as found in many hydraulic texts provide little assistance to the inexperienced, unless very large errors are permissible.

The Geological Survey has attempted to devise a useful and reliable reference index of n values. The project has been underway for several years. For streams representing a wide variety of conditions, and where peak discharges are known, slope-area measurements are obtained for the purpose of computing n values. Comprehensive photographic coverage, in three-dimensional color, is secured for each definition reach. These pictures are duplicated in sufficient quantity to place a reference slide file in each district office of the Surface Water Branch. Such a file enables the less experienced engineer to select an n value for a channel under consideration by a near-realistic and visual comparison of that channel with similar channels having defined coefficients. Values of n ranging from 0.028 to 0.075 are presently included in this reference file; additions to it are continuing.

The roughness-definition program has been limited to simple and single channels of approximate trapezoidal shape, sometimes referred to as unit channels. As would be expected a multiplicity of channel shapes are encountered in the field. Some of these, the so-called compound channels, frequently require subdivision to account for variations in shape or roughness. The conveyance K of a subdivided section is taken as the sum of the conveyances K_1, K_2, ... of each subsection. This procedure focuses attention on the treatment given to the velocity head at each cross section. As noted on Fig. 2 and in the previous discussion on computing friction slope, the velocity-head factor includes the coefficient α. This coefficient is arbitrarily taken as unity for channels of unit shape. For compound, subdivided channels, α for a cross section is estimated by the formula

$$\alpha = \frac{(K_1{}^3/A_1{}^2) + (K_2{}^3/A_2{}^2) + \ldots}{K^3/A^2}$$

The foregoing expression, a means of separating the effect of velocity distribution, allows the coefficient n to be more nearly described as a roughness factor for all channel shapes.

Research

The nationwide program of evaluation of roughness coefficients is a continuing one. While the nucleus of a reference slide file is presently available and more examples are being processed, a broadening of the range of conditions is continually sought. For example, definitive studies on extremely rocky and steep channels and on sand channels are active field projects. Definition is needed for high values of n encountered in heavily brush-covered channels such as those encountered in the southeastern sections of this country.

The work of other investigators concerning resistance to flow in rough channels such as Powell [2, 3], and Robinson and Albertson [4], has been studied with interest. Resistance to flow in pipes has undergone intensive investigation with considerable success. It is hoped that similar treatment of open-channel flow will lead eventually to equations of maximum practical usefulness in natural-channel work.

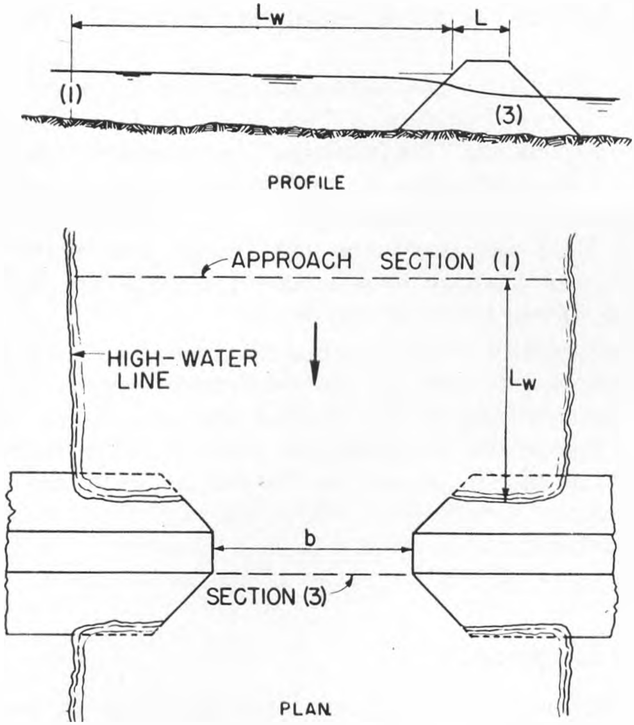
CONTRACTED-OPENING METHOD

A highway or railroad crossing of a river channel is generally so constructed as to impose an abrupt width constriction upon flood flow. At such constrictions the contracted-opening method of measuring peak discharge can be applied.

At an abrupt width constriction in a reach of channel, as shown in the definition sketch of Fig. 3, the change in water-surface profile between an approach section (1) and contracted section (3) results largely from the acceleration within the reach. Because the reach is so short, friction loss is of little importance; thus, the effect of possible errors in selecting roughness coefficients n is greatly minimized.

The drop in water surface between sections 1 and 3 is related primarily to the corresponding change in velocity. By writing the energy and continuity equations between sections 1 and 3, we obtain the discharge formula

$$Q = C\, A_3 \sqrt{2g\, (\Delta h + \frac{a_1\, V_1^2}{2g} - h_f)} \qquad (3)$$



$$Q = CA_3\sqrt{2g\left(\Delta h + \frac{a_1 V_1^2}{2g} - h_f\right)}$$

FIG. 3. DEFINITION SKETCH OF AN OPEN-CHANNEL CONSTRICTION

In this expression,

Q = discharge, in cfs
A_3 = area of section 3, in sq ft
Δh = difference in water-surface elevation, in ft, between sections 1 and 3
$\frac{a_1 V_1^2}{2g}$ = weighted average-velocity head, in ft, at section 1
h_f = friction loss, in ft, between sections 1 and 3
C = a discharge coefficient

For many years the contracted-opening method was patterned after the methods used by the Miami Conservancy District as reported by Ivan E. Houk [5]. The method employed was essentially the same as that embodied in Eq. (3)—a form of the combined energy and continuity equations. Only limited information was available, however, to aid in estimating values of a discharge coefficient.

In February 1951 the Geological Survey initiated a research project at Georgia Institute of Technology on flow through single-opening constrictions. This investigation resulted in a better understanding of the mechanics of flow through width constrictions in open channels and led to development of an improved method for computing peak discharge through bridge waterways. The investigation was reported by Kindsvater and Carter [6, 7], and in *Geological Survey Circular* 284 [8].

The applicability of the method in general, as used previously and as expressed by Eq. (3) was confirmed. Factors largely dealing with the influence of the channel and constriction shapes not previously recognized as significant, however, were found to have an important bearing upon the discharge coefficient C. Thus, emphasis on the experimental evaluation of C for a wide range of variables generally encountered in field problems became the primary objective of the latter phase of the investigation.

Discharge Coefficient

In the laboratory investigation, the discharge of coefficient C in Eq. (3) was taken to represent the combination of a coefficient of contraction, a coefficient accounting for eddy losses due to the contraction, and the velocity-head coefficient α_3 for the contracted section.

The discharge coefficient was found to be a function of certain governing geometric and fluid parameters; i.e.

$$C = \mathrm{f}(\textit{degree of channel contraction, geometry of constriction, Froude number})$$

Of all the factors influencing the coefficient of discharge, the degree of channel-contraction m and the length-width ratio of the constriction L/b were found the most significant.

Perhaps the most important new concept resulting from the laboratory investigation was that pertaining to definition of channel-contraction for channels of irregular shape. It was found that channel contraction could be described as a measure of that part of the total flow which is required to enter the contracted stream from the sides—from the lateral regions upstream from the embankments. Thus m, the channel-contraction ratio, is defined by the equation

$$m = \frac{Q - q}{Q} = 1 - \frac{q}{Q}$$

where Q is the total discharge and q the discharge that could pass through the opening without undergoing contraction.

This concept has particular advantage in connection with bridge-waterway problems in that the degree of channel-contraction for irregular, natural channels can be computed as a ratio of hydraulic conveyances. Even for considerable variations of depth, alignment, and roughness in the approach channel, a procedure for evaluating m is given by

$$m = 1 - \frac{K_q}{K_Q} = \frac{K_a + K_b}{K_Q} \tag{4}$$

The above notation is explained in the definition sketch of Fig. 4.

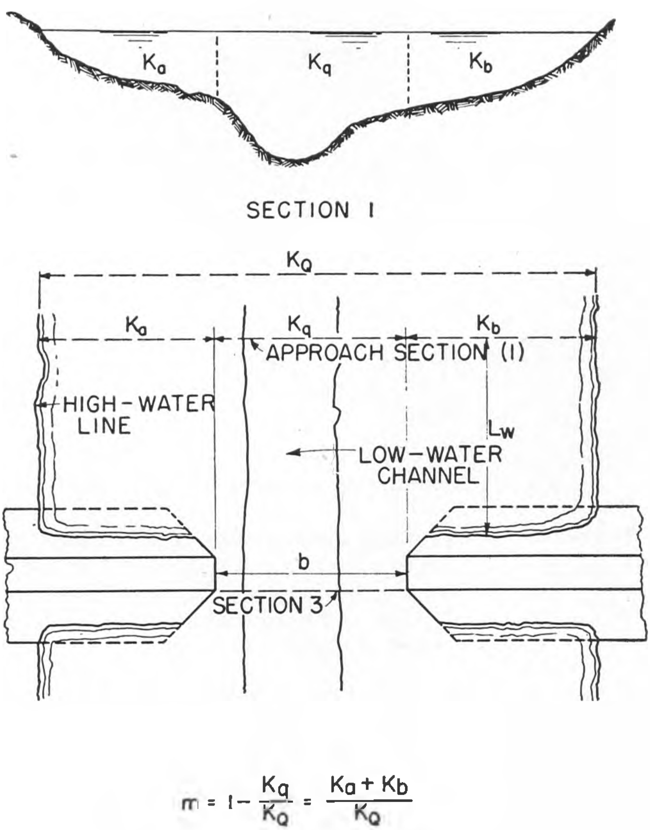


FIG. 4. DEFINITION SKETCH OF CHANNEL-CONTRACTION RATIO

The Froude number F was found to have a relatively minor effect upon C for most geometry types over the range of F tested. The investigation was confined to flow within the tranquil range.

The coefficient C was defined for four constriction types simulating the most frequently encountered forms of highway bridge-opening shapes. These shapes, illustrated in Fig. 5, are classified as:

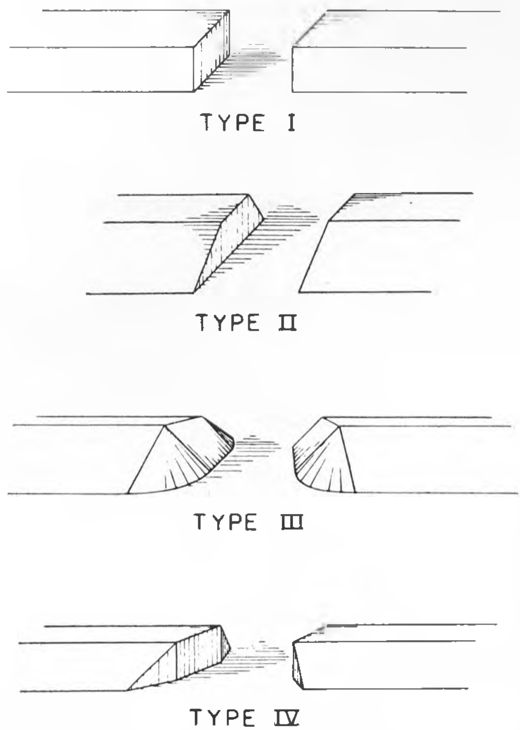
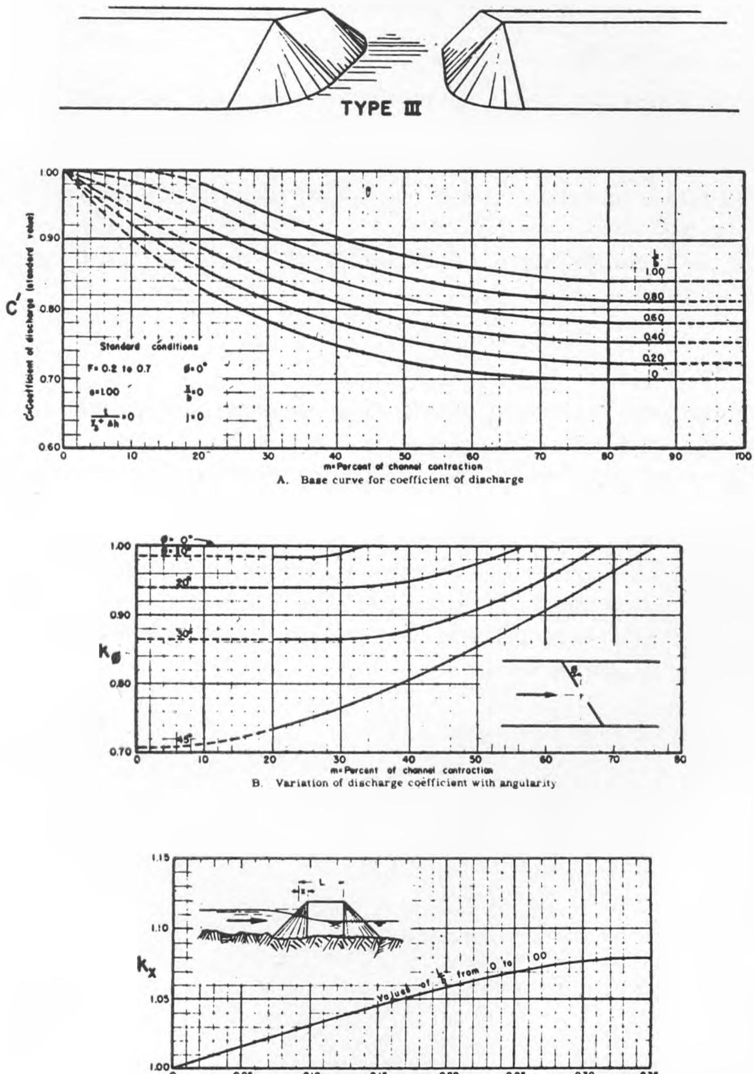


FIG. 5. CONSTRICTION GEOMETRICS CLASSIFIED

Type I — Vertical embankments and abutments
Type II — Sloping embankments, vertical abutments
Type III — Sloping embankments, and abutments
Type IV — Sloping embankments, vertical abutments with wing walls

Curves relating C to pertinent variables for each of these four types of openings may be found in *Geological Survey Circular* 284 (p. 26-34) [8]. Figure 6, for a type III opening only, is included herein as an example. Inasmuch as C was found to depend primarily upon m and L/b, a standard value, C', was related to these variables (see Fig. 6-A) for fixed values of the secondary variables.

DISCHARGE-COEFFICIENT CURVES



A. Base curve for coefficient of discharge

B. Variation of discharge coefficient with angularity

C. Variation of discharge coefficient with $\frac{x}{b}$ ratio

FIG. 6. TYPE III OPENING, EMBANKMENT AND ABUTMENT SLOPE 2 TO 1

If, in a particular problem, the secondary variables deviate from the fixed values shown in the box of Fig 6-A, the value of C' is successively adjusted for each deviation from the standard. For example, the final discharge coefficient is computed as

$$C = C' k_{\phi} k_{x}$$

where a given problem departs from the fixed secondary values only with respect to angularity (Fig. 6-B) and the x/b ratio (Fig. 6-C). Adjustment-coefficient curves for other secondary variables such as effect of area of piles and piers, eccentricity, submergence, etc., are not shown on Fig. 6.

For any combination of variables the limiting value of C is taken as 1.00.

Research

Laboratory investigation of flow through single-opening constrictions, with minor exceptions, was concluded in 1953. Since then a program of field verification of the method has been carried on, with favorable results. Twenty-two contracted-opening surveys at sites where discharges are known have been obtained to date. These cover the wide variety of conditions normally encountered in practice. The following table indicates the comparison of results by indirect-method computation, based upon discharge defined by current-meter measurement.

Difference by indirect method (percentage range)	*Number of verifications*
+15 to +20	1
+10 to +15	1
+ 5 to +10	7
0 to + 5	3
0 to — 5	7
— 5 to —10	1
—10 to —15	1
—15 to —20	1
	22 total

It is encouraging to note from the above tabulation that about 80 percent of the field verifications made to date give results within 10 percent of the respective known discharges. Considering the possibilities of error inherent in the field data and in the discharges measured by current meter, this is believed to be remarkably good.

A laboratory investigation of flow through multiple-opening constrictions is now underway. It is anticipated that the problems and complexities inherent in this more complicated geometrical form will be many. Several years, no doubt, will have elapsed before definitive results are forthcoming.

Flow-Through-Culvert Method

A culvert often can be used as a convenient device for measurement of peak discharge by indirect methods. This is indeed fortunate because of the many difficulties involved in making current-meter measurements of flood flow on the smaller streams.

The flow-through-culvert method is similar to the contracted-opening method in that the change in water-surface profile in the reach between the approach and constricted sections reflects largely the effect of acceleration. Again, friction losses are generally of minor importance. A culvert constriction, however, is such that it may act as a control section; that is, flow may pass through critical depth at the culvert entrance or outlet. The method, therefore, covers conditions of rapid as well as tranquil flow.

Flow classification

The discharge characteristics of a culvert depend upon an evaluation of energy changes between an approach section upstream from the culvert and the control section. Depending upon the location of the control section and the relative height of headwater and tailwater, most culvert flow patterns may be classified in six types. These are described below and defined also in the sketches of Figs. 7-9.

Type I, critical depth at inlet: As indicated on Fig. 7, flow passes through critical depth d_c near the culvert entrance. Culvert barrel flows part full. The headwater-diameter ratio $\frac{h_1 - z}{D}$ is limited to a maximum of 1.5. The slope of the culvert barrel S_o must be greater than the critical slope S and the tailwater elevation h_4 must be less than the elevation of water surface at the control section h_2.

The discharge equation is

$$Q = C\, A_c \sqrt{2g\left(h_1 - z + \frac{V_1^2}{2g} - d_c - h_{f_{1\cdot 2}}\right)} \qquad (5)$$

where A_c is the flow area, in sq ft, at the control section. Other notation is evident in Fig. 7, or has been previously explained.

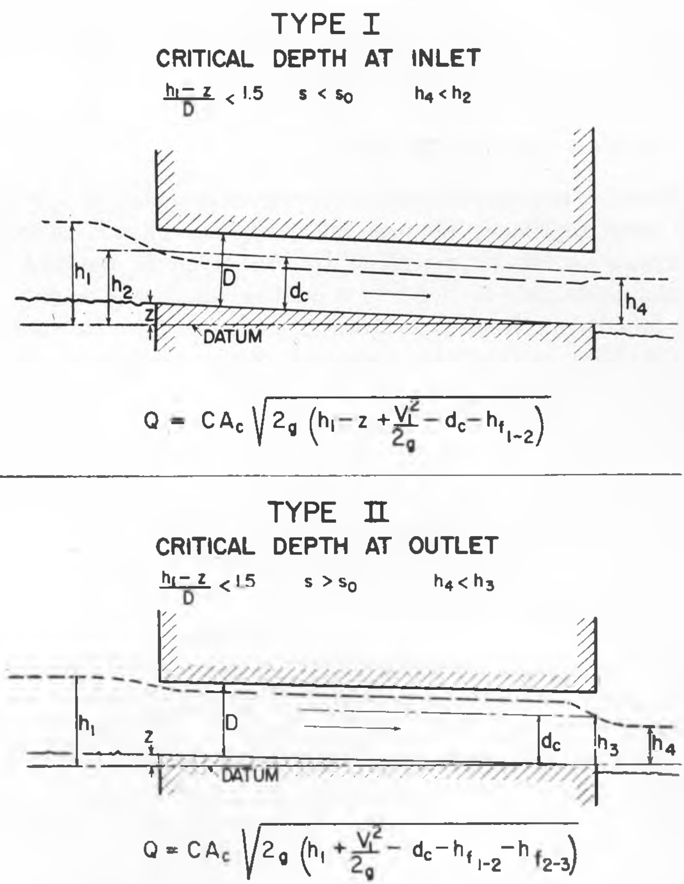


FIG. 7. CLASSIFICATION OF CULVERT FLOW, TYPES I AND II

Type II, critical depth at outlet: Flow passes through critical depth at culvert outlet (see Fig. 7.) Culvert barrel flows part full. The headwater-diameter ratio does not exceed 1.5. Slope of the culvert is less than critical. Tailwater elevation does not exceed the elevation of water surface at the control section h_3.

The discharge equation is

$$Q = C A_c \sqrt{2g \left(h_1 + \frac{V_1^2}{2g} - d_c - h_{f_{1\text{-}2}} - h_{f_{2\text{-}3}}\right)} \qquad (6)$$

Type III, tranquil flow throughout: Culvert barrel flows part full, with the headwater-diameter ratio less than 1.5 (see Fig. 8). The tailwater elevation does not submerge culvert outlet, but does

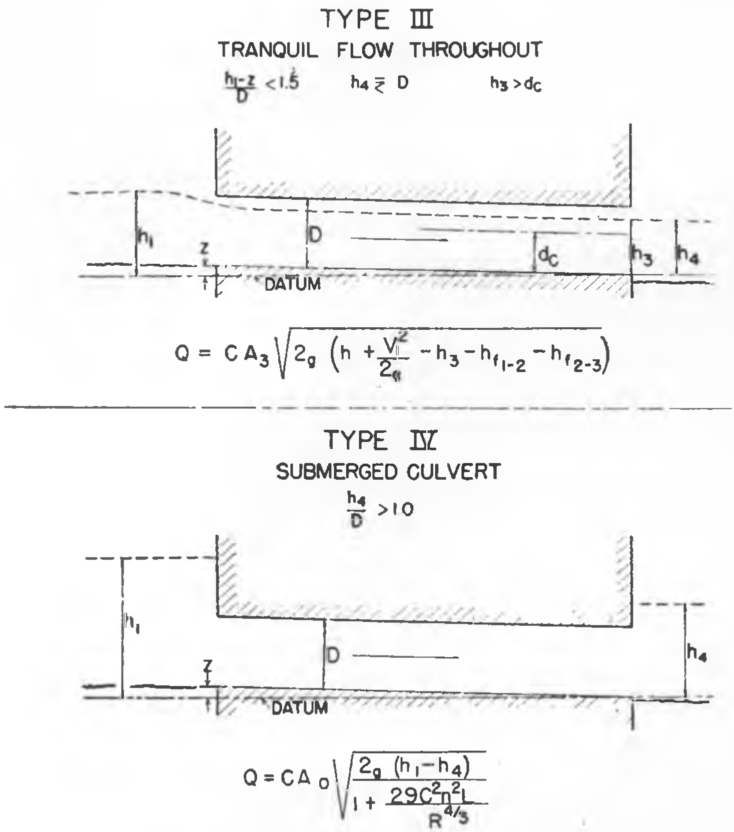


FIG. 8. CLASSIFICATION OF CULVERT FLOW, TYPES III AND IV

exceed the elevation of critical depth at the control section. The discharge equation for this condition is

$$Q = C A_3 \sqrt{2g \left(h_1 + \frac{V_1^2}{2g} - h_3 - h_{f_{1\text{-}2}} - h_{f_{2\text{-}3}}\right)} \tag{7}$$

Type IV, submerged culvert—The tailwater elevation is high enough to submerge culvert outlet; hence the culvert is submerged and flows full (see Fig. 8). The discharge equation is

$$Q = C A_o \sqrt{\frac{2g\,(h_1 - h_4)}{1 + \frac{29C^2n^2L}{R^{3/4}}}} \tag{8}$$

where

A_o = full area, in sq ft, of culvert barrel
L = length, in ft, of culvert barrel

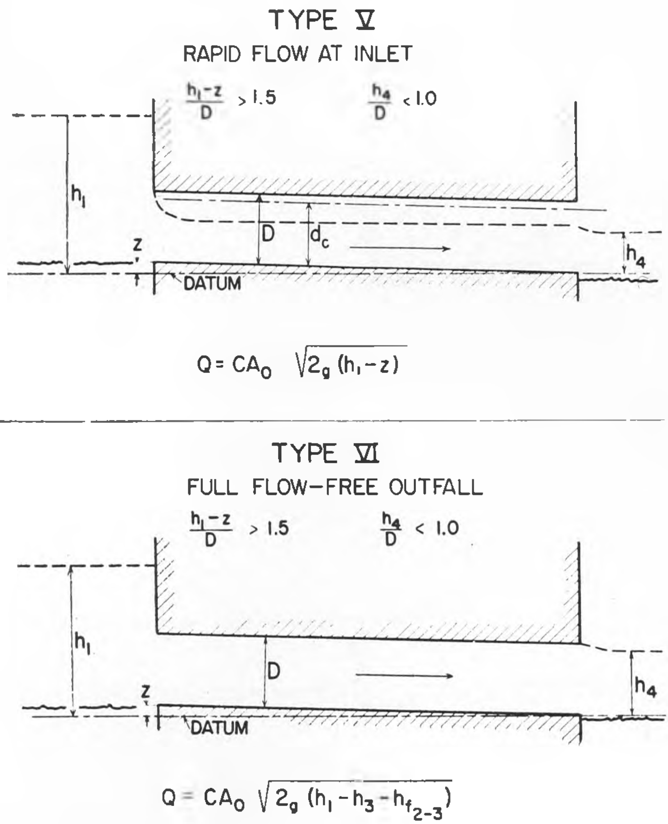


FIG. 9. CLASSIFICATION OF CULVERT FLOW, TYPES V AND VI

Type V, rapid flow at inlet: The headwater-diameter ratio exceeds 1.5 (see Fig. 9). The culvert entrance is such that flow is contracted in a manner similar to sluice- or orifice-type flow. Culvert barrel flows part full and at depth less than critical depth. The tailwater elevation does not submerge culvert outlet. The discharge equation is

$$Q = C\,A_o\sqrt{2g\,(h_1 - z)} \tag{9}$$

Type VI, full flow-free outfall: The headwater-diameter ratio exceeds 1.5 (see Fig. 9). The culvert barrel flows full under pressure. The tailwater elevation does not submerge culvert outlet. The discharge equation is

$$Q = C\,A_o\sqrt{2g\,(h_1 - h_3 - h_{f_{2\text{-}3}})} \tag{10}$$

For flow-types IV-VI a ponded condition in the headwater pool was assumed in writing Eqs: (8) to (10).

During recent years the program of gaging the flow of small streams has been expanded greatly. The use of culverts as a means of indirect measurement of peak discharge focused attention on the need for further research. In problems sometimes encountered the difficulty of distinguishing between flow-types V and VI on basis of field data ordinarily available was apparent. In addition, while discharge coefficients applicable to flow-types IV and VI had been previously defined by Yarnell [9] and Straub and Morris [10, 11], coefficients for flow types I, II, III, and V were somewhat uncertain. In order to fill in certain gaps in published data, during the past year an investigation of flow through culverts has been conducted at the Georgia Institute of Technology by the Geological Survey. The laboratory phase of the investigation has now been completed. The results of the investigation will soon be published in a form suitable for general distribution. The problem of distinguishing between types V and VI flow was not completely solved. The occurrence of flow-types V and VI, however, can be predicted, within range of geometries tested, from a knowledge of entrance geometry of the culvert, the length, slope and roughness of the culvert barrel, and the headwater-diameter ratio.

Discharge coefficients

The discharge-coefficient phase of the laboratory investigation, as previously stated, was confined primarily to flow-types I, II, III, and V. It was found that discharge coefficients vary in magnitude from 0.40 to 0.98, and are a function of the culvert geometry and the degree of channel contraction for any flow type.

Laboratory observation indicated that the discharge coefficients could be grouped for certain combinations of flow types. Coefficients for flow-types I, II, and III form one group, types IV and VI another, and type V, the third. Thus the same form of discharge equation is applicable to each group.

Entrance geometries tested were limited to those types most commonly used in highway-engineering practice. Tests included (1) flush entrances in vertical headwall with varying degrees of entrance rounding or beveling, (2) standard wing-wall entrance, (3) projecting entrance, and (4) mitered entrance set flush with sloping embankments. Results of these tests will be included in the forthcoming report.

Research

The Geological Survey has no immediate plans for further extensive laboratory research on flow through culverts. Other investigators, however, are conducting research on culvert hydraulics. Many of the continuing investigations are pointed to culvert design, the objective being to achieve an entrance form which will insure full-barrel flow (type VI) for low degrees of entrance submergence. Much of this work, undoubtedly, will enable wider usage of culvert structures for indirect methods of flow measurement.

FLOW-OVER-DAM METHOD

The hydraulics of channel controls and spillways has long been a basic tool of the hydraulician and designer. These principles may be adapted to indirect computation of discharge over such structures.

A dam forms a control section at which the discharge is related to the water-surface elevation upstream. Friction loss between an upstream approach section and the control section is generally of minor importance.

For computing peak discharges, the method consists simply of determining by field survey the head on the spillway from high-water profiles, approach-section characteristics, and spillway geometry. Discharge is computed by the well known weir formula,

$$Q = C\, b\, \left(h + \frac{V_1^2}{2g}\right)^{3/2} \qquad (11)$$

where

Q = discharge, in cfs
C = coefficient of discharge
b = spillway width, in ft
h = head on spillway, in ft, adjusted when necessary for friction loss between the approach and spillway sections
$\frac{V_1^2}{2g}$ = velocity head, in ft, in approach section

It is readily apparent that reliability of discharge computed by this method depends in large measure upon selection of the proper coefficient C. For most determinations this coefficient must be estimated by comparison with calibrated spillways of similar shape.

Spillways encountered in the field in indirect-measurement work may be sharp-crested, but more generally are broad or round-crested, or of ogee or irregular shape. For most problems, therefore, the best data available in existing literature must be used in estimating C. Among many possible references, the Survey has found most frequent use for the published works of R. E. Horton [12], the Bureau of Reclamation, [13] and J. N. Bradley [14].

There is frequent need for computing flow over highway embankments. The basic broad-crested weir formula is used, with C selected on basis of results reported by Yarnell and Nagler [15] for flow over railway and highway embankments. The original data of Yarnell and Nagler recently have been reanalyzed by the Survey (for its own use) to define two dimensionless curves; one a curve relating C to the head-breadth ratio for free-fall conditions, and a second, relating correction for submergence to degree of submergence. This dimensionless treatment is more generally applicable than the original method of presentation, insofar as indirect measurement of flow over highway fills is concerned. The weakness of using these relations based on the slightly different geometry representative of railway embankments remains.

Research

The U. S. Geological Survey has an interest in the discharge characteristics of all forms of weirs, whether they be small V-notch plates, masonry mill dams, highway embankments, or major dam spillways. Knowledge of the head-discharge relationships for weirs of all types is essential to the work of the organization, and may occasionally provide the only means of determining an important flood-flow magnitude.

In some cases where peak discharges should be known accurately, the scarcity of information on the discharge characteristics of many forms of weirs and spillways limits the flow-over-dam method to an approximation. In general the discharge coefficient must be determined by experiment, and fortunately, through the application of the principles of dynamic similarity, tests on scale models in the laboratory can be applied to the prototype with confidence.

In 1907 the Geological Survey published *Water Supply Paper* 200 [12] by Robert E. Horton. The importance of Horton's work is demonstrated by the fact that the publication has been reprinted several times and copies have been widely distributed. Horton's work was a complete summary of theoretical and experimental knowledge on the subject as of 1907. In the 48 years since, our theoretical knowledge of fluid behavior and, certainly, our store of empirical knowledge of the discharge of weirs of all forms have increased extensively. But until now, no one has attempted to collect, analyze, correlate, or publish this material in a form which will make it available to the engineering profession.

This task is again being undertaken by the Geological Survey. The project has been designed by and is being carried on under the direction of Professor Carl E. Kindsvater, of the Georgia Institute of Technology. As in the first instance, only a limited amount of original research will be involved. The greater part of the project will consist of collecting, examining, and evaluating all of the data which can be located in this country, and, to some extent, abroad. One of the significant objectives of the proposed study is to correlate these empirical data by methods based on the techniques of modern fluid mechanics. It is envisaged that several years will be required to complete the compilation and research project on coefficients for dams.

An investigation of coefficients applicable to flow over highway embankments is now an active laboratory project at the Georgia Institute of Technology. Various phases of the project will be subjects for masters' theses. The current phase is supported by the J. Waldo Smith fellowship of the American Society of Civil Engineers.

SUMMARY

Indirect methods of measurement offer a practical solution to the determination of peak discharges of natural streams where current-meter measurements cannot be made. While admittedly there are weaknesses and shortcomings in the methods, the necessity for their use and the overall reliability of results fully justify their being called measurements.

Of the four methods described herein, the slope-area method is applied most frequently, with the contracted-opening, flow-through-culvert and flow-over-dam methods following in about that order. Increased reliability of results by the contracted-opening and flow-through-culvert methods is certainly to be expected as a consequence of the recent research. All methods give good results when applied to channel reaches of acceptable standards. Without any doubt it can be said that the overall accuracy of stream flow data published today is greater than ever before. A major contributing factor has been improvement in and greater use of indirect methods of river discharge measurement.

Discussion

Mr. Boyer initiated the discussion by presenting some of his own work on the slope-area method for correlating the Manning coefficient n with the bed roughness, abstracted from a paper published in the Transactions of the AGU in December 1954. Two figures from his paper, plots of y_0/k against $n/y_0^{1/6}$ and $V_{0.2}/V_{0.8}$ against $n/y_0^{1/6}$, were presented as slides. Here k is the average height of bed roughness in the stream and $V_{0.2}$ and $V_{0.8}$ are the velocities at 0.2 and 0.8 depth. He stated that Mr. Stevens of the Soil Conservation Control Council of New Zealand has verified the former plot very well and the latter fairly well, and found that the latter may be a satisfactory method of getting n from lower discharge measurements than those made at the peak of the flood.

Mr. Albertson asked about methods suitable for application to alluvial beds which can continue to scour to equilibrium. Mr. Cragwall replied that the application of indirect methods for computing discharge in alluvial streams has not been encouraging. The Geological Survey has undertaken some studies of scour, which is a difficult problem itself from the instrumentation standpoint, but, so far, it appears that the slope-area method must be used with caution in erodible channels of any type.

Mr. Laursen pointed out two difficulties in the determination of discharge after a flood; first, that the section of an alluvial stream has been changed by the flood, and secondly, where a vegetable screen is present along the low-water bank, as is often the case, it is possible that the water in the overbank is not at the same elevation as the water in the stream. In reply Mr. Cragwall stated that the interpretation of the signiflance of the high-water marks is included in the estimate of the value of n; that in their roughness verification program the high-water mark is measured, so that the effect of the nature of the bank is taken into account. No unexplainable variations in n due to this cause have arisen.

Mr. Kolupaila observed that correction factors for the slope of a channel have been mentioned, and inquired as to what is being done about estimating the velocity-head factor α, which is a difficult problem for open-channels and rivers. Mr. Cragwall pointed out that the procedure used in estimating α had been mentioned in the paper. Such estimates were made only for compound channels, i.e. channels with considerable overbank flow, where roughness varies considerably across the channel. In normal channels, α appears to vary little between upstream and downstream sections. In compound channels, conveyances for 5 or 6 subsections, the number depending upon the shape of the section and the variable roughness across it, are used to calculate α. Thus an attempt is made to incorporate these effects into the estimate of n.

Mr. Barbarossa reported that the Corps of Engineers had measured n values for high flows in alluvial streams as low as 0.013, which is of the order of magnitude of that for concrete. This was attributed to the disappearance of form roughness in such flows. This was mentioned because it appeared to contradict a statement by Mr. Cragwall that he informed his young engineers that n values varied between 0.03 and 0.06. Mr. Cragwall agreed that it is frequently necessary to choose values outside of the range of n from 0.03 to 0.075, and that, in some cases, values as low as 0.012 have been selected by experienced men.

Mr. Banks remarked that there is a recent publication which tends to confirm the Einstein sidewall-elimination method which was published about 14 years ago. He asked how the U.S.G.S. takes sidewall into account in arriving at a final n value. Mr. Cragwall replied that it is hard to describe how n is chosen. He indicated that he has a mental picture of good natural channel of $n = 0.03$, and if other channels differ he increases or decreases this value according to his experience. The method is qualitative, but with the help of 3-dimensional slides it is becoming more consistent.

Mr. Izzard asked Mr. Barbarossa whether actual measurements of the cross-section were taken when the low values of n had been observed, implying that the variation in n might be due to the fact that the bed is lower during higher stages. Mr. Barbarossa replied that that possibility had already been considered, but that the amount of sediment that could be transported could not degrade the bed sufficiently to account for the total change.

REFERENCES

1. "Kansas-Missouri Floods of July 1951," U. S. Geological Survey, *Water-Supply Paper 1139*, 1952.

2. Powell, R. W., "Flow in a Channel of Definite Roughness," *Trans. A.S.C.E.*, v. 111, p. 531-566, 1946.

3. "Resistance to Flow in Rough Channels," *Trans. A.G.U.*, v. 31, p. 575-582, 1950.

4. Robinson, A. R., and Albertson, M. L., "Artificial Roughness Standard for Open Channels," *Trans. A.G.U.*, v. 33, p. 881-888, 1952.

5. Houk, I. E., "Calculation of Flow in Open Channels," State of Ohio, The Miami Conservancy District, Technical Reports, Part IV, 1918.

6. Kindsvater, C. E., and Carter, R. W., "Tranquil Flow through Open-Channel Constrictions," A.S.C.E. convention preprint No. 21, 1952.

7. Kindsvater, C. E., "Tranquil Flow through Open-Channel Constrictions," *A.S.C.E. Separate* 467, 1954.

8. Kindsvater, C. E., Carter, R. W., and Tracy, H. J., "Computation of Peak Discharge at Contractions," *U. S. G. S. Circ.* 284, 1953.

9. Yarnell, D. L., "The Flow of Water through Culverts," Univ. Iowa, Bull. 1, 1926.

10. Straub, L. G., and Morris, H. M., "Hydraulic Tests on Concrete Culvert Pipes," Univ. Minn., Tech. Paper 3, Ser. B, 1950.

11. Straub, L. G., "Hydraulic Tests on Corrugated Metal Pipes," Univ. Minn. Tech. Paper 5, Ser. B, 1950.

12. Horton, R. E., "Weir Experiments Coefficients, and Formulas," *U. S. G. S. Water-Supply Paper* 200, 1907.

13. "Studies of Crests for Overfall Dams," U. S. Bureau of Reclamation, Boulder Canyon Reports, Part VI, Hydraulic Investigations, Bull. 3, 1948.

14. Bradley, J. N., "Discharge Coefficients for Irregular Overfall Spillways," U. S. Bur. Reclamation, Engineering Monographs No. 9, 1952.

15. Yarnell, D. L., and Nagler, F. A., "Flow of Flood Water over Railway and Highway Embankments: U. S. Dept. Agriculture, Bur. Public Roads, Public Roads, v. 11, p. 30-34, 1930.

DEVELOPMENT OF THE ULTRASONIC METHOD FOR MEASUREMENT OF FLUID FLOW

by

R. C. Swengel[1], W. B. Hess[2]

Introduction

Although there are a number of basically different methods available for measuring the flow of fluids, there are situations in which, for one reason or another, the choice may be limited to a very few. One field of application where only a few methods are available is the measurement of large quantities of water, and the required precision, costs and the physical dimensions of the flow passages often preclude the use of these few.

A specific problem, that of measuring hydraulic turbine discharge, provided the incentive for the development of the ultrasonic method of flow measurement. Low-head developments in particular are difficult to measure because the water passages are usually short in length and the geometries of the intakes are unsuitable for existing methods. The means for obtaining performance data on these developments, particularly in the United States, is to step up laboratory tests on homologous models by utilizing formulas derived from basic dimensional relationships plus experience and empirical coefficients to compensate for other less tangible factors that vary with the size ratio. Index tests using combinations of piezometers in the turbine intake have been very helpful in extending performance data over a wide range of head conditions after the index has been proven reliable and its calibration determined from a direct method of field testing. Experience with the use of the scale formulas and index methods have convinced the authors that these methods alone do not provide sufficiently accurate information to realize maximum economies in operations.

When precise performance data are available, economies are obtained from preferential operation of the more efficient units and loading of machines in accordance with their incremental efficiencies. These methods of operation may be applied to a single multiple unit station, an inter-connection of several hydro stations, or a system of combined hydro and steam generating stations.

[1] Consultant, 1133 E. Philadelphia Street, York, Pa.
[2] Senior Engineer, Safe Harbor Water Power Corp., Conestoga 2, Pa.

With units of identical design, variations in performance between individual turbines may seem insignificant; however, field testing of a station with seven 42,500 hp. machines has shown differences of as much as 2 percent in efficiency. The tests in this case were made by the two-type current meter method using propeller type current meters with different pitch of vanes. The test results were very consistent and duplication tests on the same turbine showed a deviation band of less than 0.5 percent.

The scale formula method has been found considerably in error at the higher load points where cavitation begins to influence the performance. Model efficiency tests are usually made in the laboratory at sigma values higher than those existing in prototype operation; therefore step up of the model data do not reflect the effects of cavitation. Incremental operating efficiencies based upon performance from the model therefore can be very misleading.

The plant for which the ultrasonic method was originally intended is not suitable for current meters or any existing method of large flow measurement. The turbines have short and steep intakes and unguided approaches upstream from the head gate section. Volumes of flow exceed 3,000 cubic feet per second and it was not deemed practical to introduce any mechanical structure in these intakes because of the difficulties that would be encountered due to the obstruction of flow and because of the high cost of such an installation.

PRACTICAL CONSIDERATIONS

After establishing the need for a simple and accurate means for measuring large volumes of flow a theoretical study was made of all the known physical laws which might conceivably be used toward this end. In order for any approach to be worth the undertaking, it had to pass the test of two rigorous criteria:

1. Be absolute in method. That is, be capable of use without calibration by any other means or method, yet subject to easy comparison with known standards of the highest order of accuracy.
2. Be capable of measuring by volume, weight or velocity averaged over a prescribed area.

Several methods seemed to offer practical solutions to the above conditions but the use of ultrasound transmission seemed most adaptable to the measurement of large volumes of flow. None of the available literature on ultrasonics made any mention of the measurement of fluid velocity although interferometers for the measurement of absolute velocities of propagation were discussed in detail. Unfortunately, none of these devices could be used to measure flow: Both mechanical and optical interferometers, however, contained the basic requirements of high sensitivity so inherently a part of the new method.

The prerequisite of a high degree of sensitivity arises from the small change in velocity being measured. In water the absolute propagation velocity is approximately 4800 feet per second. In order to measure flow at 1 foot per second to within 1 percent it thus becomes necessary to indicate changes in the absolute propagation velocity on the order of 1 part in 480,000. An interferometer is capable of doing this but only under strict laboratory conditions and with special regard to the maintenance of constant temperatures of both the fluid and the measuring apparatus.

Resolved in practical terms this meant that the mechanical parts of the interferometer must remain constant in spacing to within 2 micro-inches per foot of acoustic path.

It was later discovered that maintenance of constant temperature on the part of both fluid and mechanical apparatus had been considered by workers who had previously been granted patents [1-5] but in no other wise published their findings. In spite of their excellent efforts, the problem of constant transducer (converters of electrical to mechanical or mechanical to electrical energy) spacing remained as great as ever although they achieved a measure of success in reducing the effects of changing propagation velocities with variations in temperature of the fluid. Most of these researchers used three or more transducers to obtain these results.

By utilizing a reversal of transmission between two transducers it was found that errors introduced into the measurement by changes in spacing due to any cause could be kept proportional; i.e. a change in the acoustic path length of 1 percent would produce only a 1 percent error in the flow determinations.

The method as presently used thus measures the differences in the velocities of propagation of compression waves with and against the direction of flow between two reversible transducers as a measure of the velocity of flow [6]. Using these means it is necessary to know the absolute propagation velocity of sound in the fluid. A more basic method utilizing absolute transit times has been derived [7] whereby the propagation velocity in the liquid need not be known.

$$v = \frac{d/t_d - d/t_u}{2} = \frac{d}{2}\left(\frac{1}{t_d} - \frac{1}{t_u}\right)$$

Where v = velocity of flow
d = length of measuring section
t_d = absolute transit time for a compression wave to travel in the downstream direction the length of the measuring section
t_u = absolute transit time for a compression wave to travel in the upstream direction the length of the measuring section

It is believed that further development of the electronic and physical equipment will make the use of the above formula practical. When this is accomplished, the method should find a much wider field of application since it would provide a true measure of velocity regardless of the nature of the fluid.

The second criteria of area measurement has been overcome to a practical degree in the application of the method to rectangular water passages.

TESTS IN ONE-FOOT-DIAMETER PIPE

Many tests were conducted while the equipment went through various stages of development and have shown that the effects of turbulence, sedimentation, variations in temperature and changing contours of flow have but small effects on the accuracy of the method [8].

In the first application of the method an attempt was made to measure flow in a round or cylindrical section. The transducers

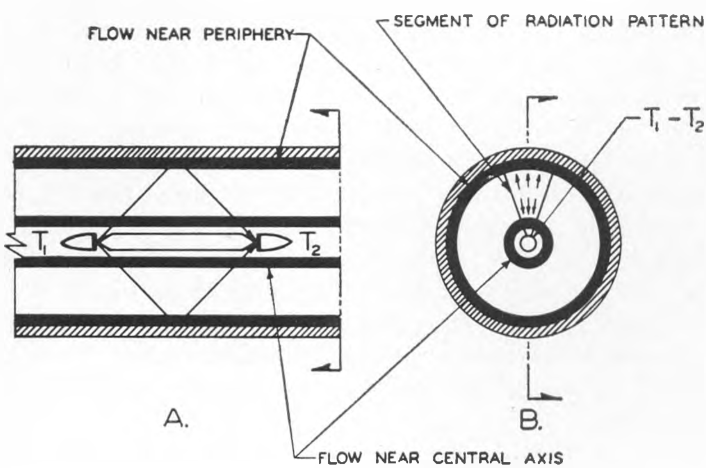


FIG. 1. MEASUREMENTS IN ROUND PIPE

a — Axial Location Transducers T_1 - T_2
b — Rod Pattern and Asymmetrical Flow

were mounted on a streamline form on the central axis of the pipe. Ultrasonic energy radiated from the one transducer out to the wall in the form of a cone and reflected to converge on the other transducer. A line diagram of this installation in shown in Fig. 1a.

While this system proved the basic concept, the precision was poor when compared to a carefully calibrated venturi. Figure 1b is an axial view of the round pipe in which the two darker bands represent water flowing at the same velocity; a small volume of flow near the center but a larger volume of flow at the periphery. The lines and arrows depict the radiation area of a small segment of the transducer as it radiates waves to the wall that converge again on the other transducer. It can be seen that the effect on the radiated energy would be alike for both the inner and outer bands of flow since the segment of radiated energy is narrow when passing through the inner flow and broad through the outer flow ring. Without knowing the true flow contour an accurate measure of the average velocity over the area of the pipe becomes impossible with this radiation pattern.

A photograph of this, the first attempt to use the new method, is shown in Fig. 2. This photograph is also interesting because it shows, in the foreground, an electronic variable frequency meter. The instrument was used to measure the differences in frequency when transmitting with and against the flow while maintaining a constant total number of wavelengths between transducers. Imagine, for example, that the transmission downstream is made at a frequency which at any instant produced 100 waves or wavelengths between the transducer transmitting and that receiving. Since the velocity of propagation is higher with than against the flow a reversal of the direction of transmission in an upstream direction makes necessary a change in the frequency of the power source to a lower frequency to maintain the required 100 wavelengths between transducers. The difference in these two frequencies of transmission can be inserted in a variation of the basic formula to provide an answer in terms of the velocity of flow. The frequency meter reads in cycles per second and serves to establish a time standard of high accuracy which can be further extended by calibration against the National Bureau of Standards Station WWV. Because flow velocities are always related to time the connection is obvious.

Many other techniques for utilizing reverse transmission between two transducers as a measure of flow are possible, but regardless of the means employed, one must use a direct or indirect measure of the absolute transit times or difference in transit times as the indication of velocity.

APPLICATION TO RECTANGULAR WATER-PASSAGES

Because the most immediate problem for practical solution was the measurement of large volume flows in the rectangular intakes of low-head hydroelectric turbines a study was made to determine the method's ability to cope with this situation. A two-dimensional treatment of a rectangular water passage is shown in Fig. 3. The points T_1 and T_2 represent transducers and the circles drawn about T_1 the ultrasonic waves being radiated in the duct. By locating transducers on opposite walls displaced a known projected distance on the principal axis the connecting line of radiated energy becomes that shown by the dashed line. A mathematical treatment [7] assuming uniform flow proves that the effect on the transit time is

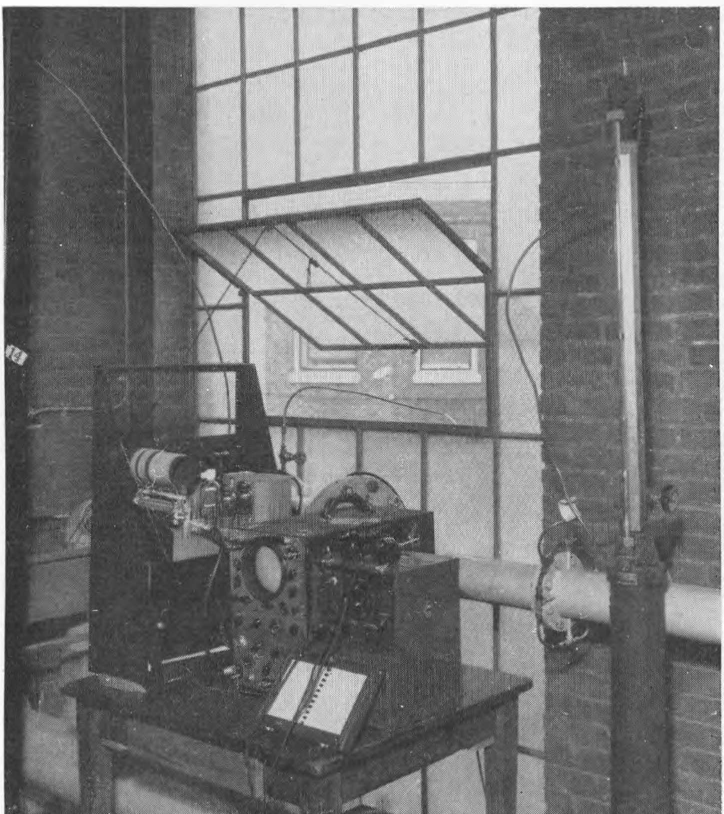


FIG. 2. FIRST TEST ASSEMBLY IN 12-INCH DIAMETER PIPE

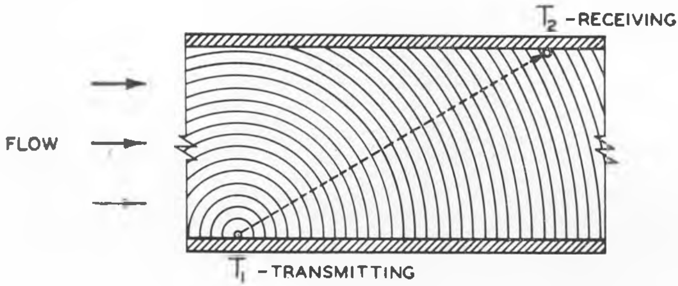


Fig. 3. Measurement at Angle to Flow Axis

the same for this condition as that when the transducers are displaced the same amount directly on the flow axis.

In crossing the duct at an angle further analysis indicates that for flow contours commonly encountered the total transit time will be affected by the average of the velocities along the path. Slight refraction errors are possible for highly distorted flows. These have been investigated and are reported on in detail in reference [7]. The maximum distortion where velocities ranged from 7 to 11 feet per second produced errors of about 2 percent. The negative or positive quantity of these errors were dependent on the physical orientation of the distortion relative to the plane of intersection of the area measurement.

Measurement Over an Area

Figure 4 is a three-dimensional sketch of a rectangular duct and a block diagram of the electronic equipment used for the laboratory and full-scale tests. It will be noted that the transducers are tilted and displaced on the flow axis. The tilt was utilized as a means of overcoming undesired radiation paths which coupled the two transducers by reflecting from the top and bottom walls of the duct. The reflections would be objectionable in the average concrete intake because of dimensional errors in the geometry of the section and because the wavelength employed can be short enough for these walls to appear as very rough mirrors for reflecting ultrasound.

It will be further noted in Fig. 4 that the recorder compares the phase angle of the exciting voltage at the transmitting transducer with that from the transducer receiving the ultrasound via the water path. The direction of transmission was reversed at 5-second intervals by means of the interchanging switch. The difference in the two recorded phase angles was used as the measure of flow velocity.

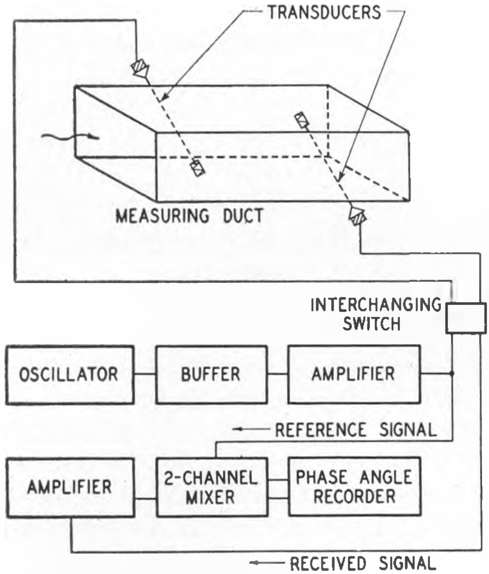


FIG. 4. BLOCK DIAGRAM OF EQUIPMENT USED

In order to radiate in even amounts over an area the transducers were designed as physical counterparts of electromagnetic antennae [9]. When they are mounted on opposing walls of the duct as shown in Fig. 4 the area between them becomes the thin invisible curtain affected by the velocities of flow. The transducer rod transmitting can be considered as an infinite number of point source radiators with an infinite number of receiving counterparts on the other transducer rod. A line connecting any one of the point sources with its receiving counterpart is lengthened or shortened an amount which is an average of the velocities encountered. If no flow exists, all transmission paths reach the true transducing element, a crystal in this case, simultaneously.

The crystal mentioned is only one of several transducing means which can be employed as driving and sensing elements. Compression waves in the long rods are created or sensed by these transducers in reciprocal fashion.

A sketch showing the solution to another problem is shown in Fig. 5. This problem resulted from the fact that no material of which the rods might be made is lossless to the passage of ultrasound. This results in a falling off of the intensity of the ultrasonic waves as they proceed down the rod and away from the crystal transducers. To align two such transducer rods in a parallel fashion and with the crystal drivers on the same ends would thus result in a very non-uniform distribution of ultrasonic energy over the area between the rods. Figure 5 therefore shows the crystal drivers and rods aligned in an opposed fashion with the energy paths a, b and c all traversing an equal length water path and an equal total length of rod. This opposed mounting provides an even distribution of ultrasonic energy over the complete area between the rods and no errors can possibly occur from this source.

Figure 5 likewise serves to illustrate another point which is a test of the measure of parallelism of the transducer rods. This measurement is accomplished by observing the power output of the receiving transducer as the water in the passage is slowly raised or lowered. If the area is uniformly and adequately covered, the incremental changes in power output with the height of the water will be uniform. An uneven change in received signal would be an indication of a faulty installation. The amount of non-parallelism that can be tolerated depends upon the frequency used to obtain a minimum required degree of sensitivity and accuracy. This somewhat limiting condition can be reduced to a negligible degree by correlating the area and maximum flow velocities with the design of the electronic equipment.

With respect to area measurements another interesting phenomenon has been observed which it should be possible to use to advantage. This is the change in the power level of the received signal with distorted or turbulent flow. Very little was known on this subject when the testing was first started, and even to date no quantitative data have been recorded. The remarkable fact is that

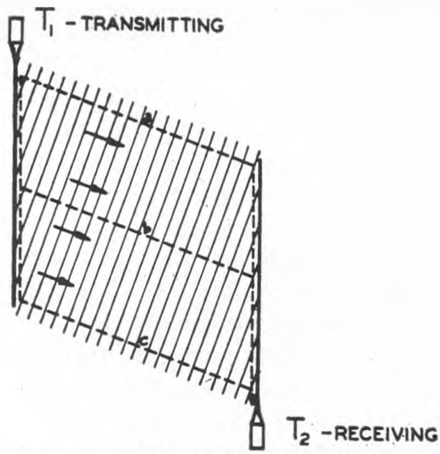


FIG. 5. EQUAL ENERGY DISTRIBUTION OVER AREA
a, b, c — EQUAL ENERGY PATHS

extreme turbulence seemed to have less of an effect on the power level of the transmitted signal than highly distorted flow although even the latter reduced the output by only a very few percent. With precisely controlled conditions it should be possible to establish some sort of a distortion index which would be indicative of the degree of distortion encountered by the ultrasonic waves. It would not, however, be possible to establish the shape of the contour using such data but rather its relation in degree to known flow contours.

ELECTRONIC EQUIPMENT

The electronic equipment has very little in its design that is unusual. The main consideration is the complete isolation of the reference and receiving transducer channels [10]. Any cross coupling between these links can produce erroneous results. These errors would not be apparent even with zero flow velocity, therefore each channel should be appraised separately for pick-up from the other.

It is possible to substitute lightly coupled dummy loads for the transducers and thus establish the correct functioning of the equipment in a zero flow condition. The frequency of operation can be easily checked against a frequency standard or the previously mentioned radio station WWV.

POTENTIAL USES

A photograph of the first large scale application is included as Fig. 6. The intake shown is the middle bay of three which comprise the intakes to a 42,500 horsepower adjustable blade-type turbine at the Safe Harbor Water Power Corp. plant, Conestoga, R. D. #2, Pa. The transducer can be seen as a thin line on the wall facing one of the authors. It was operated at 25 kilocycles, and flows in this single bay reached values of 3,000 cubic feet per second.

At one time experimental transducers were installed in a flume for testing the method's ability to measure in an open channel. A drought at the most inappropriate time prevented any save static or no-flow tests from being made, but all indications justified the belief that the results would have been the equal of those in the closed systems. Several techniques are available and have been used to evaluate accurately the correctness of the closed system installations. These same techniques were used to check the open flume installation mentioned above.

Dr. Julia Herrick of the Mayo Clinic is working with the method

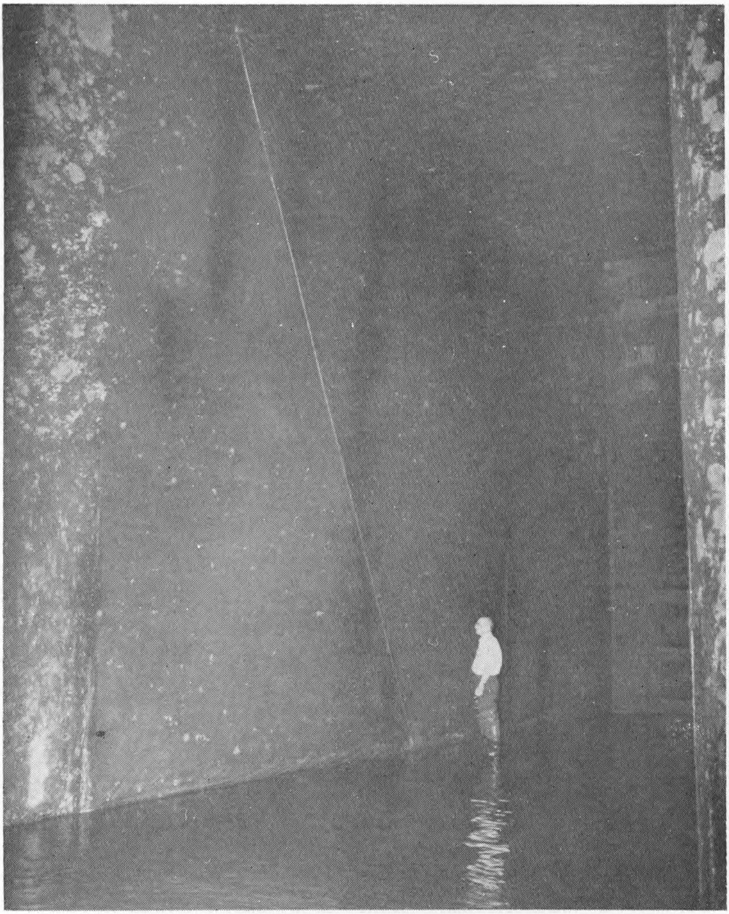


FIG. 6. 30 FOOT TRANSDUCER INSTALLED IN 42,500-HORSEPOWER TURBINE INTAKE

in its adaptation to the measurement of blood flow where total flows of fractions of a cubic centimeter per second are being investigated.

Dr. F. H. Middleton and Wen-Hsiung Li of Johns Hopkins University's Institute for Cooperative Research have completed the development of an estuarine current meter using the method. This meter was designed to measure from 0.08 fps to 5.1 fps, a velocity ratio of 1 to 64. This instrument is remarkable in that only 9 electronic tubes are used and the flow readings over a long period of time are automatically recorded on film for later processing.

It is also understood that one large company is investigating the method's adaptation to the measurement of highly viscous flow where no other method has ever been completely successful.

In applying the method to gaseous flow there will probably be many new problems to overcome, the most significant anticipated being that of changes in the speed of propagation with changes in the intensity level of the ultrasound.

A demonstration flowmeter of the utmost simplicity has been shown [11] which measures air flow using two quite ordinary loudspeakers as transducers.

ACKNOWLEDGMENTS

The authors would like to acknowledge the very important assistance in the development and testing of this new method so freely given by Dr. S. K. Waldorf, Engineer of Research for Pennsylvania Water and Power Company, Lancaster, Pennsylvania.

DISCUSSION

Mr. Barron of the U. S. Geological Survey emphasized their urgent need for instruments to measure discharges in rivers where the usual stage-discharge relationships become grossly inaccurate, such as flat-sloped streams and those where backwater affects the surface slopes independently of the discharge. As an example he cited their problems on the Ohio, where the fall is sometimes only a few tenths of a foot between stations 20 miles apart, and errors in its determination may easily amount to as much as 20%. Discharges below about 20,000 cfs must be estimated under these conditions at most Ohio River Gaging Stations. The problem is complicated still more by unsteady conditions, in which case a calibration of the slope reach will be very complicated, costly, and possibly unattainable by the usual methods.

Because most attempts to record stream velocities continuously have proved impractical to date, they are intensely interested in the ultrasonic method. From their discussions with Mr. Swengel and others, they are quite hopeful that this measuring technique can be successfully applied in natural stream channels. Their present plans are to measure the mean velocity on a line between transducers placed on opposite banks of the river, rather than integrate across the entire cross-section. Their successful use of observations at a single point in determining total discharge gives them confidence in this possibility, since a line-velocity measurement across the main portion of the flowing water would be an even better parameter of the flow.

The immediate goal of the Geological Survey is to write a practical and workable set of specifications leading to a contract for the development of ultrasonic velocity-measuring equipment. To do this, they need more definite information concerning the following characteristics of the ultrasonic method: Maximum range of transmission, maximum width-depth ratio attainable, range of velocities, practical accuracy over entire range of velocity, long-term stability, and amount of maintenance required.

When an instrument becomes available, the first installation may be made in the lower Sacramento River because of the urgent need for information to be used in their program to prevent salt-water intrusion. If this test installation is successful, then other applications would be numerous.

Mr. Tinney inquired about the cost of a unit, because this would be an important factor for a small laboratory. Mr. Swengel's reply was that theirs cost several thousand dollars, but was purely an experimental unit. The work at Johns Hopkins indicates that many simplifications could probably be made, but he would rather not guess at the eventual cost.

Mr. Mitchell asked about applications using hydraulic fluids or very small-scale installations. Mr. Swengel knew of one company which was now working with highly viscous fluids, and of Dr. Herrick's work at the Mayo Clinic measuring blood flow at velocities as low as a fraction of a centimeter per second. From the range of these studies, it appears that there is not much of a limit to either the scale of the systems which can be studied or the velocity range.

Mr. Strasberg wanted to know whether the flow must be uniform in the direction parallel to the rods. The speaker replied that the maximum errors occurred when this flow was non-uniform, but that in rivers the various errors cancelled each other within one-half of one percent.

In response to Mr. Shoumatoff's question concerning the effect of turbulence, the speaker stated that no real error was ever ascribed to turbulence as such, and that its only effect was to lower the signal slightly.

PATENTS ON ULTRASONIC APPARATUS

1. Fiorenzi, Antonio, "MetedoAcustico per la Misura della Velocita dei Fluidi in Movimento entro Condotti Chiusi, con Particolaro Riguardo All'Acqua Fluente Nelle Tubazioni Forzate Degli Impianti Idroelettrici," Italian Patent, Brevetto Industriale N. 287653 Rilasciato il 31 luglio 1931.

2. Hartig, H. E., "Fluid Meter," U. S. Patent 2,015,933, Oct. 1, 1935, U. S. Patent Office, Washington, D. C.

3. Hartig, H. E., "Fluid Meter and Method of Measuring the Rate of Flow of

Fluids," U. S. Patent 2,151,203, March 21, 1939, U. S. Patent Office, Washington, D. C.

4. Wolff, I., "Air Speed Indicator," U. S. Patent 2,274,262, February 24, 1942, U. S. Patent Office, Washington, D. C.

5. Cafarelli, N. J., Jr., "Fluid Velocity Indicator," U. S. Patent 2,328,546, September 7, 1943, U. S. Patent Office, Washington, D. C.

6. Swengel, R. C., "Fluid Velocity Measuring Apparatus," U. S. Patent Applied for.

REFERENCES

7. Swengel, R. C., Hess, W. B., and Waldorf, S. K., "Principles and Application of the Ultrasonic Flowmeter," Winter General Meeting American Institute of Electrical Engineers, February 1955. *AIEE Transactions Paper* 55-28, *Power Apparatus and Systems*, April 1955, No. 17, pages 112-118.

8. Swengel, R. C., Hess W. B., and Waldorf, S. K., "The Ultrasonic Measurement of Hydraulic Turbine Discharge," *American Society of Mechanical Engineers, Transactions Paper* 54A-54, Annual Meeting of the American Society of Mechanical Engineers, December, 1954.

9. Swengel, R. C., "Antenna-Type Transducers for Ultrasonic Flowmetering," 1955 Institute of Radio Engineers National Convention, March 1955. *IRE Convention Record*, June 1955.

10. Hess, W. B., Swengel, R. C., and Waldorf, S. K., "Measuring Water Velocity by an Ultrasonic Method," *American Institute of Electrical Engineers, Miscel. Paper 50-214*, October 1950. Abstract in *Electrical Engineering, Vol. 69, No. 11*, November 1950, page 983.

11. Swengel, R. C., Hess, W. B., and Waldorf, S. K., "Demonstration of the Principles of the Ultrasonic Flowmeter," *American Institute of Electrical Engineers, Conference Paper C54-470*, AIEE Middle Eastern District Meeting, October 1954. *Electrical Engineering*, December 1954, Vol. 73, No. 12, pages 1082-1084.

AUTOMATIC INSTRUMENTATION OF THE MISSISSIPPI BASIN MODEL

by

H. C. McGee

Waterways Experiment Station Suboffice, Jackson, Mississippi

Introduction

The automatic instrumentation of the Mississippi Basin Model is within itself an engineering feat. It is an outstanding achievement of the combined efforts of engineers of the Waterways Experiment Station and instrument manufacturers. From the earliest stages of the development of the model, automatic instrumentation was considered desirable and necessary. There were skeptics in the beginning, and some even who said it could not be done; but today, the completed sections of the model (and ultimately the whole model) can be operated by automatic instruments synchronized by a single timing device. However, to understand thoroughly the magnitude of this undertaking, it is first essential to review the purpose and development of the Mississippi Basin Model.

The need for some means of developing plans for coordinating reservoir operation and integrating other flood-control problems throughout the entire Mississippi basin became evident to Major General Eugene Reybold while he was District Engineer at Memphis during the 1937 flood. Later while Chief of the Corps of Engineers, General Reybold initiated the plan which he had earlier envisioned for a comprehensive model of the Mississippi River and its tributaries; thus, the Corps of Engineers began the design and construction of the Mississippi Basin Model in August 1943 at the Waterways Experiment Station Suboffice, Jackson, Mississippi.

The model will be, when completed, a reproduction to scale of the Mississippi River from Hannibal, Missouri, to the Gulf of Mexico and its major tributaries (Fig. 1). The streams that have already been constructed and are in operable condition are the Missouri River from Sioux City, Iowa, to the mouth; the Arkansas River from Tulsa, Oklahoma, to Pine Bluff, Arkansas; and the Mississippi River from Hannibal, Missouri, to Tiptonville, Tennessee, including the Ohio River to the mouth of the Wabash, the Cumberland River upstream to Old Hickory Dam, and the Tennessee River upstream to Pickwick Dam. The completed sections constitute about 25 percent of the streams to be constructed.

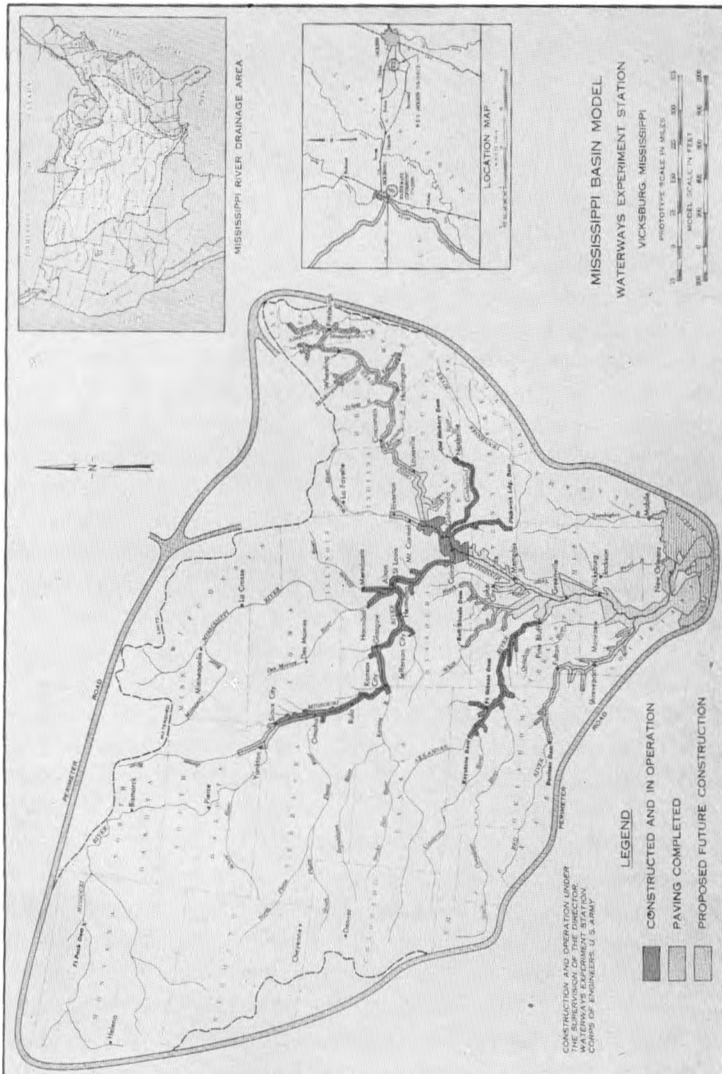

MISSISSIPPI RIVER DRAINAGE AREA
LOCATION MAP
MISSISSIPPI BASIN MODEL
WATERWAYS EXPERIMENT STATION
VICKSBURG, MISSISSIPPI
PERIMETER ROAD
LEGEND
CONSTRUCTED AND IN OPERATION
PAVING COMPLETED
PROPOSED FUTURE CONSTRUCTION
CONSTRUCTION AND OPERATION UNDER THE SUPERVISION OF THE DIRECTOR WATERWAYS EXPERIMENT STATION CORPS OF ENGINEERS, U.S. ARMY

FIG. 1

Operating as a unit, the model will be used primarily as originally conceived to assist in developing plans for coordinated operation of all flood-control works in the basin. The studies will include tests to determine how the operation of the existing or proposed systems of reservoirs, floodways, and other works can be coordinated, and to indicate what alterations or changes are needed in flood-control works to accomplish the greatest overall flood reduction and protection. In addition to the basin-wide studies projected for the future, studies of local and regional flood-control problems are currently being made on the sections of the model that are in operable condition and will continue to be made when the model is completed. The testing programs, both basin-wide and local, will consist principally of tests conducted with flood waves.

In the Mississippi Basin the streams vary from alluvial streams with broad valleys and slow flood-moving characteristics of the plains and delta to the steep mountain streams subject to flash floods in narrow valleys of the Upper Ohio basin. Flood waves in the streams of the plains and delta, which sometime extend over a period of months, could be reproduced accurately in the model with manually operated instruments; but it was considered extremely difficult, if not impossible, to introduce flash floods accurately on the mountain streams and chart their movements with manual instruments. For this reason automatic instrumentation appeared to be a requisite for conducting successful tests of flood waves on these streams.

The enormous amount of man power required to operate the model manually as projected was another factor which made automatic instrumentation desirable. A staff of approximately 600 would have been necessary for total manual operation. Training and maintaining a force of this size with the usual rate of turnover in personnel would have been a difficult and most expensive operation; hence, manual operation was considered undesirable from the standpoint of labor and cost.

Approximately four years, 1943 to 1947, were devoted to the study of the problems of automatic instrumentation and to the testing of commercial and pilot instruments. Instrument manufacturers throughout the United States (about 125) were consulted. Instruments, both commercial and pilot models, embodying various design principles were considered and tested. Weirs, fixed orifices, variable orifices, and rotameters were some of the types tested for inflow instruments. The principal unit considered for programming flows was a device which follows a chart of the inflow hydrograph plotted in electrically-conductive ink. Various types of stage measuring devices based on the operating principles of float mechanisms, pressure cells or diaphragms, point gages, and displacer or bellows units were considered. Most of the stage recording devices considered consisted of strip-chart, printed-record, and photograph types. The rotameter was considered for use as an outflow instrument as well as for inflow.

A thorough investigation of the commercial instrument market disclosed that instruments available did not have the accuracy of measurement or the range required for use on the model. The scales of the model imposed rigorous accuracy requirements on the instruments. Model scales that directly affected the accuracy requirements were the vertical, time, and discharge scales which have ratios, model to prototype, as follows: vertical 1 to 100, time 1 to 267, and discharge 1 to 1,500,000. For instance, the range of the rotameter for measuring discharge is a ratio of 10 to 1, whereas the range of discharge on streams reproduced in the model have a ratio of maximum to minimum of 50-1 to 1,000-1. With a tenth of a foot in prototype stage represented by 1/1,000 of a foot in the model, an hour by 13½ seconds, and discharge of a thousand cfs by 7/10,000 of a cfs, it can readily be seen that instruments must possess great precision capabilities to measure accurately such minute quantities. In general, the specifications for the model instruments presented extraordinary problems for the instrument companies, and it finally became evident that the instruments would have to be specially designed to accomplish the required results.

For the sake of economy, instruments that could be interchanged were considered necessary. Interchangeability of instruments would permit flexibility in operation as instruments could be moved from one station to another as needed. This stipulation also imposed rigid requirements in the manufacture of the instruments as they would have to be designed to operate at any gaging station. Complete interchangeability was not accomplished in all of the different types of instruments, but it was accomplished to a beneficial degree.

Near the end of the fourth year of research enough knowledge and information had been acquired to write the specifications for the automatic instruments. The manufacturing companies were invited to bid on these instruments and to submit designs for instruments other than those described in the specifications that could accomplish the desired results. Consequently, some of the instruments acquired and now in use were not described in the specifications.

On 20 July 1948, contracts were awarded to Infilco, Inc., Chicago, Illinois (now of Tucson, Arizona) for 76 inflow instruments which included the inflow controllers and programmers, and to Leupold and Stevens Instruments, Portland, Oregon, for 160 stage instruments which included the stage transmitters and recorders.

Description of Instruments

General

The model is designed and constructed so that when it is completed it can be operated as a unit or in sections. A recirculating water supply system provides water for the operation. This system consists of a large sump, two 2500 gpm pumps, a 50,000 gallon water tank, and approximately 22,000 feet of water supply and return lines varying in size from 4 to 14 inches. To operate the model as a unit will require a discharge of about 2.2 cfs or 1,000 gpm which represents the peak flow required to reproduce the project flood in the Lower Mississippi River. To operate separate sections simultaneously and independently, the discharge requirements are considerably greater and depend on the number of sections in operation at one time and the type of tests being conducted.

Centrally located instrument houses on the major streams provide control centers for the automatic instrumentation (Fig. 2). Four of these control houses have been constructed—one at St. Louis for the Mississippi River, one at Kansas City for the Missouri River, one at Fort Smith for the Arkansas River and one at Cincinnati for the Ohio River. Programmers located in the control

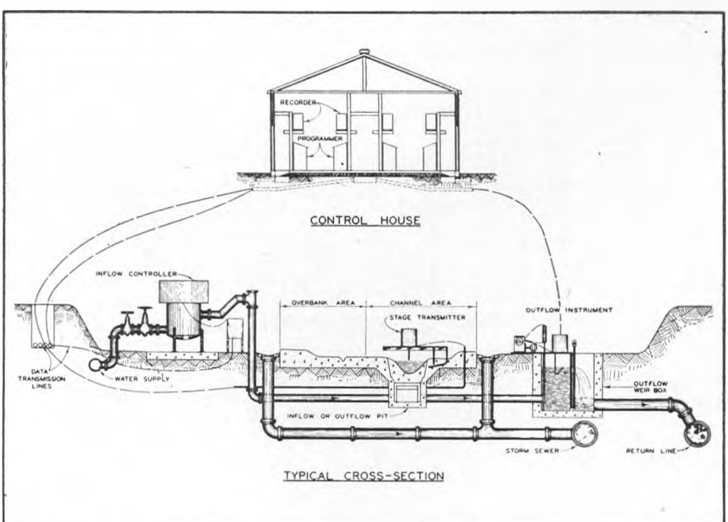


Fig. 2. Instrumentation for Mississippi Basin Model

houses regulate over an underground system of electrical cable the introduction of flood waves on the model through the inflow controllers; recorders in the control houses make permanent records of instantaneous stages received electrically from stage transmitters at the gaging stations on the model streams as the movement of the flood waves progresses; and the action of motorized valves programmed from the control house maintains stages at the control gages immediately upstream from the outflows—the points of diversion of the water from the model. Stage transmitters measure the head over calibrated weirs at the outflow points and transmit the measurements to recorders in the control houses from which the discharges are determined. The operation of the instruments in the control houses and on the model are synchronized by a master timer, and the month, day, and hour of the flood period are accurately registered by calendars also controlled by the master timer.

Inflow Instruments

The inflow instrument unit consists of a controller at the inflow point on the model and a programmer in the control house. The inflow controller is a multiple orifice device for measuring flows that vary in accordance with a predetermined program. The principal features of the inflow controller are presented schematically in Fig. 3, and a picture of the controller with the cover and a section of the tank removed is shown in Fig. 4. The orifice plate divides the

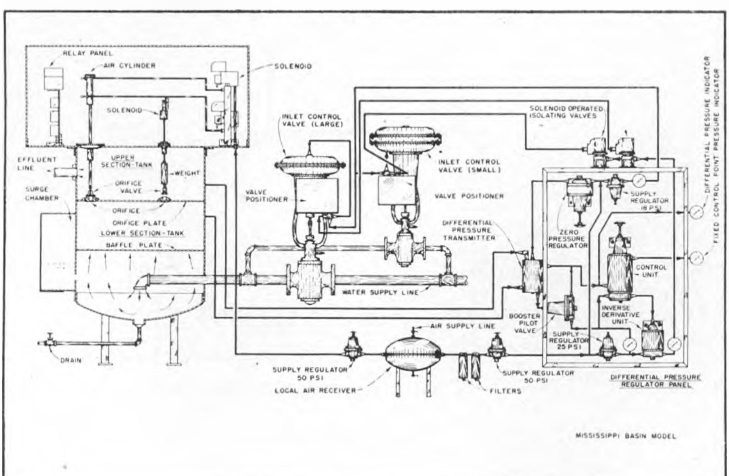


FIG. 3. SCHEMATIC DIAGRAM OF INFLOW CONTROLLER

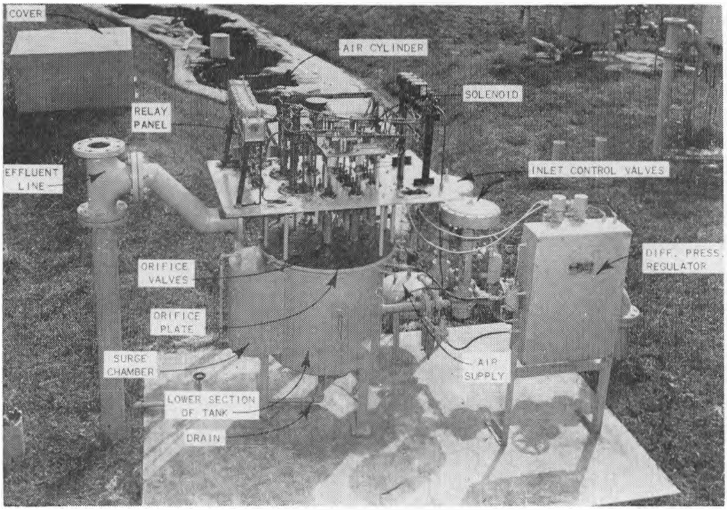


FIG. 4. INFLOW CONTROLLER WITH COVER AND UPPER SECTION OF TANK REMOVED

tank into upper and lower sections. An effluent pipe connects the upper section to the model inflow point and the lower section is connected to the water supply.

The orifice plate contains 13, and in some cases 14, orifices. Each orifice is opened and closed by raising and lowering the metal disc attached to the valve stem. The discs for the smaller orifices are raised by solenoids and lowered by gravity—weights fastened to the valve stem seat the discs against the water supply pressure. For the larger orifices the discs are raised and lowered by pneumatic cylinders.

The capacities of the orifices in gallons per minute for the entire range in all instruments are as follows:

600	60	6	0.6	0.06	0.006
300	30	3	0.3	0.03	0.003
200	20	2	0.2	0.02	
100	10	1	0.1	0.01	

From the 22 orifice capacities listed above, the 13 or 14 orifices in each controller are selected consecutively from the lowest to the highest required for the range of flows at a particular inflow point. The series of integers 1, 2, 3, 6 was selected because a full range of integers from 1 to 9 could be obtained by selection of these integers or a combination of them. This permits a uniform progression or degression in the selection of orifice capacities.

The two factors, area and pressure, that determine the flow through the inflow controller are selected to give a workable combination. The area of each orifice is fixed and the sizes of the orifices vary from 0.0077″ to 4.0769″ in diameter. With these orifice sizes a differential pressure across the orifice plate (difference in pressure in the upper and the lower sections of the tank) of 4 psi, or an equivalent head of 111 inches of water, is required to obtain and maintain the orifice flows listed in the above paragraph. Since the orifice areas are fixed and are accurately machined, the attainment of the desired orifice flows with a minimum of error is dependent on the maintenance of the required differential pressure across the orifice plate.

The components of the inflow controller (shown in Fig. 3) that maintain the required differential pressure are the surge chamber, the inlet control valves and valve positioners, piezometers connected to the upper and lower sections of the tank and the differential pressure regulator. The surge chamber dampens the momentary fluctuations in the supply pressure and prevents harmonic vibration in the differential pressure control system.

Normally, two inlet control valves are required for each controller—one small and one large valve. The number of control valves necessary is determined by the range in discharge for the particular inflow point. Each valve provides control through a portion of the range. The control valves equipped with valve positioners are pneumatically operated diaphragm valves manufactured commercially by Fisher Governor Company. The signal pressure for the operation of the diaphragm is supplied by the differential pressure regulator and varies in intensity from 3 to 18 psi. Usually the small valve operates in the signal pressure range of 3 to 5 psi and the large valve in the range from 5 to 18 psi. The range through which the valve operates is controlled by the force of the opposing spring on the diaphragm; the range can be changed by changing the tension of this spring.

The valve positioner, which is a bellows-type instrument manufactured by Moore Products Company, eliminates the non-linear characteristics in the operation of the valve components caused by friction. Hence, the valve opening is proportional to the intensity of signal pressure.

The differential pressure regulator that supplies the signal pressure is composed of commercial units also manufactured by Moore Products Company and consists of a differential pressure transmitter and booster pilot valve, a zero-pressure regulator, a control unit, and an inverse derivative unit. Each unit is a bellows-type instrument that has linear characteristics and that operates on the principle of pressure balance.

The transmitter, with the aid of the booster-pilot valve and zero-pressure regulator, measures, amplifies, and transmits to the control unit in the form of a measured variable pressure the variation in differential pressure from the 111 inches of water. In response, the control unit compares the measured variable pressure to a fixed control-point pressure and provides changes in the signal pressure proportional, but in the opposite direction, to the variation in differential pressure. The inverse derivative unit filters transitory pulsations and stabilizes the signal pressure.

The inflow programmer is an electrical switch device consisting of switches that are operated automatically or manually. These switches energize or de-energize circuits that actuate the orifice valve solenoids and air cylinders in the inflow controller.

A schematic diagram of the programmer is presented in Fig. 5 and a picture of it with the covers removed in Fig. 6. Leaf switches in the switch box which are opened and closed pneumatically as a perforated program roll passes over the tracker bar control automatic operation; toggle switches on the control panel permit manual operation. The automatic feature of the programmer is based on the vacuum principle originally used in player pianos thirty or forty years ago and in recent years to operate automatic typewriters for typing form letters.

The tracker bar has a series of identified slots, one for each switch, connected by rubber hose to the switch pouches in the

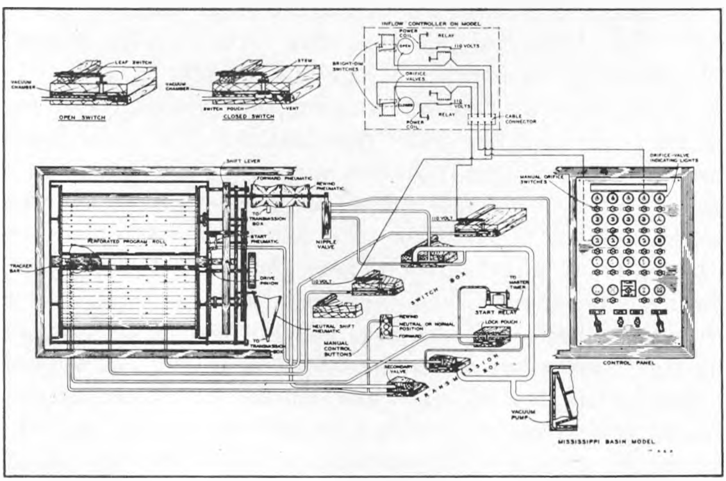


FIG. 5. SCHEMATIC DIAGRAM OF THE INFLOW PROGRAMMER

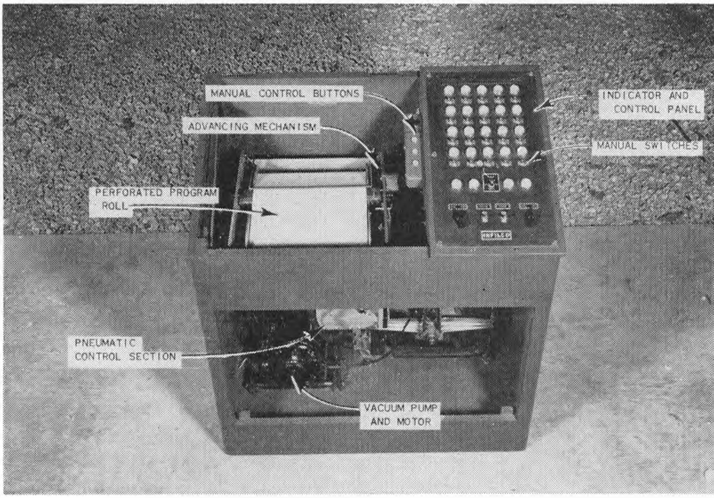


FIG. 6. INFLOW PROGRAMMER WITH COVERS REMOVED

switch box. The switch pouches are encased in a vacuum chamber in the switch box connected to the leaf switch by a stem; the vacuum is provided by an electrically driven vacuum pump consisting of four bellows. The bellows work in succession providing continuous action. When a perforation appears over the tracker bar, an impulse of air enters the switch pouch; the combination of atmospheric pressure on the under side of the pouch and pump suction on the upper side causes the pouch to rise, lifting the stem and closing the leaf switch. As soon as the perforations cease to appear and the opening in the tracker bar is closed, the remaining air in the pouch is withdrawn through a small vent to the vacuum chamber. The pouch and the stem then return to the normal down position breaking the contact of the leaf switch.

The gear transmission for moving the program roll over the tracker bar is governed by four "pneumatics." The word pneumatics are used in this sense and for the purpose of this paper is a term borrowed from the manufacturer of the programmer, and each pneumatic includes an air tube, the pouch, the valve, and bellows which operate the gear shifts. These pneumatics place the transmission in high speed forward, high speed reverse, neutral or normal forward speed, and stop position, as appropriate perforations appear on the tracker bar or as the forward, rewind, or neutral keys on the left of the control panel are depressed. By the operation of these keys the program roll can be shifted to any set of perforations in the program for starting, or with perforations for the high speed forward pneumatic, the program roll can be advanced to the beginning set of perforations in the program and rewound with appropriate perforations for the high speed reverse pneumatic at the end of the program.

In automatic operation of the model, the program roll is advanced from one set of perforations to the next at the end of each 13.5 seconds. This is done by an impulse from the master timer to the start relay in the programmer, which operates the start pneumatic that engages the drive pinion for normal forward speed and starts the program roll. Then as the stop perforation in the next set of perforations appears on the tracker bar, the stop pneumatic through its action disengages the drive pinion, stopping the program roll; thus, the program roll is advanced to succeeding sets of perforations at model intervals corresponding to an hour.

The program roll is a perforated record of an inflow hydrograph for a particular inflow point. Each set of perforations on the program roll represents a special grouping of the orifice openings to obtain the model equivalents (gallons per minute) of the prototype discharge for a specific hour in a flood hydrograph. There is a possible series of perforations for each orifice in the controller, the two inlet valves, and the three pneumatic gears—forward, rewind, and stop. The lamps on the control panel indicate the leaf switches that are closed and the brightness of the light denotes whether the orifice opened in response.

The perforations are made by a machine resembling a typewriter, shown in Fig. 7. On the right of the perforator, as the machine is termed, there is an indicator panel with a lamp similar to

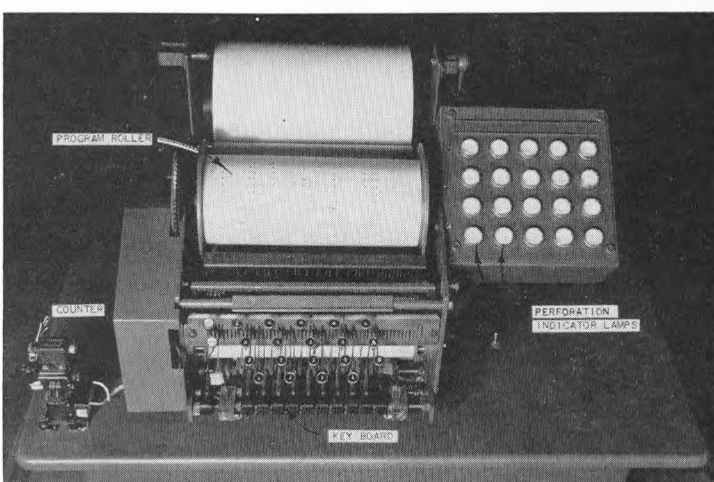


FIG. 7. PROGRAM PERFORATOR

the lamps on the control panel of the programmer for each key. When the perforator keys are depressed they operate electrical switches, which complete circuits to the indicator-panel lighting lamps that correspond to the orifices in the inflow controller. Thus, the perforations in the program roll can be quickly checked against the gallons per minute required for that particular model hour. (Correction tabs are provided also for errors made in perforations.) The switches remain closed until the perforator feed key is depressed and the roll is advanced to the next set of perforations.

Stage Instruments

The stage instrument unit is composed of a transmitter and recorder with a telemetering circuit consisting of two selsyn motors—one in the transmitter and one in the recorder—connected by an electrical cable. The stage transmitter is located over the model channel at the gaging station corresponding to the gaging station on the prototype stream and the recorder is in the control house. The transmitter "feels" the water surface with an electronic sensing probe and transmits the elevation of the water surface to the recorder by means of the telemetering circuit. A schematic diagram of the transmitter is presented in Fig. 8 and a picture with the covers removed is shown in Fig. 9.

The principal features of the transmitter (shown in Fig. 9) are a two-phase reversible motor geared to a precision screw and the transmitting selsyns, and two electronic circuits that control the

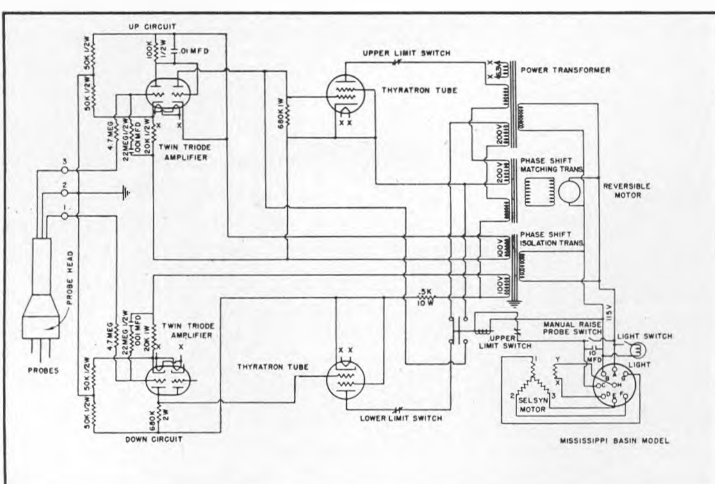


FIG. 8. SCHEMATIC DIAGRAM OF STAGE TRANSMITTER

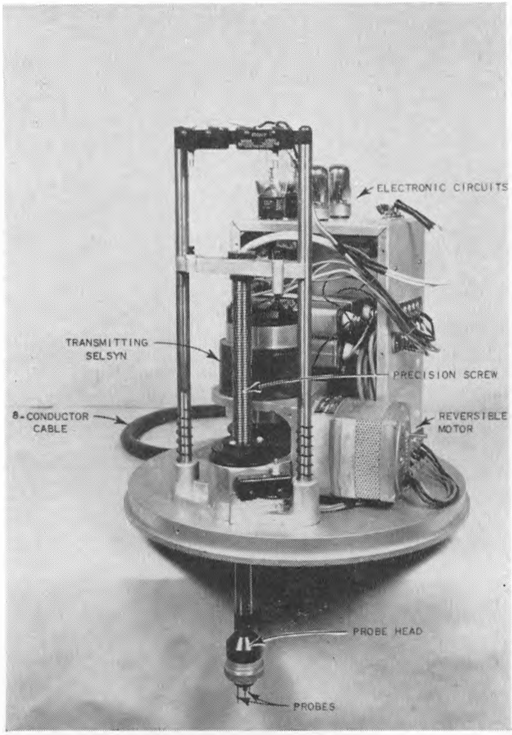


Fig. 9. Stage Transmitter with Cover Removed

operation of the motor through the action of the water on the sensing probe. The probe head on the lower end of the precision screw contains three probes—a rod-like probe that is the common ground for the electronic circuits and two needle probes that regulate the electronic circuits; one of the needle probes is 0.004 inch longer than the other. Each of the electronic circuits contains a twin-triode amplifier tube and a thyratron tube. If the needle probes are not touching the water when the instrument is put into operation, the triode in the downward circuit passes current that fires the thyratron and starts the motor to run the precision screw down. As the long needle probe touches the water, a bias is applied to the grid of the triode that cuts the voltage applied to the control grid of the thyratron, stopping the motor. When the water rises and touches the short needle probe the flow of current in the upward circuit (the circuit that raises the precision screw) begins firing the thyratron in this circuit and starts the motor in the opposite direction.

The travel of the precision screw is 0.700 ft; this represents a prototype range of 70 ft. The maximum rate of travel is 52 seconds

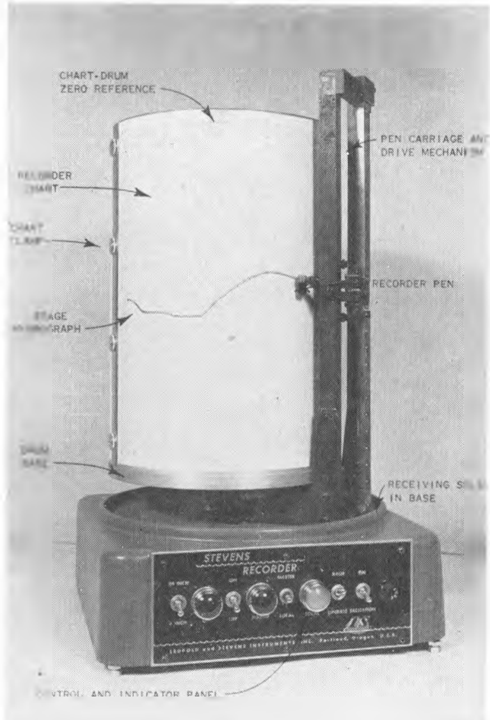


FIG. 10. STAGE RECORDER

for the full vertical range of 0.700 ft in the upward direction and 57 seconds in the downward direction. The surface of the base plate is machined for establishing the zero; bolt heads that can be moved up or down for obtaining the desired zero support the base plate of the transmitter on the gage mount.

As the precision screw travels up and down with needle probes sensing the water surface, the rotor of the transmitting selsyn is rotated through certain angles; the rotor of the receiving selsyn in the recorder is also rotated through the same angles. A pen on the recorder is made to travel vertically with the precision screw in the transmitter by a gear train connected to the rotor of the receiving selsyn (Fig. 10). The movement of the pen is recorded in ink on a chart fastened to a drum that is rotated horizontally and clockwise. Thus the stage hydrograph is drawn on the chart for the particular gaging station. The travel of the pen is 14 inches for the 0.700 ft travel of the precision screw. The base on which the drum rests is driven by electrical clock works rotating the drum. The rate of rotation is selective—1 and 6 inches per hour for some of the recorders and 6 and 24 inches per hour for the others. Normally, the rotation speed of 6 inches per hour is used and at this rate slightly over 6 hours can be recorded on a chart or a prototype period of 70 days. The effective length of the chart is about 31 inches.

Outflow Instruments

An outflow is provided on the model mainly to measure the discharge at a selected point on a stream or to make releases from a reservoir. In addition to providing means for measuring the discharge, means must also be provided to simulate the control that would be supplied by the stream below the outflow. This control is usually provided by programming the stages at or near the outflow in accordance with a predetermined rating curve (stage versus discharge), or in accordance with a predetermined stage hydrograph. Both methods of control are used from time to time in the testing programs on the Mississippi Basin Model. Releases from reservoirs are usually made in accordance with a predetermined stage or discharge program.

Originally, and to some extent now, the control at outflow points was maintained by manual manipulation of a tail-gate weir. The head over the tail-gate weir was read by means of a hook gage or recorded automatically by a stage transmitter and recorder. With a previously developed rating curve, the discharges for the outflow point were obtained. Recently an automatic control for programming either stage or discharge hydrographs has been developed. This control is built around a hydraulic regulator, a commercial device manufactured by Askania Regulator Company. A schematic diagram of the installation for programming both stage and discharge is shown in Fig. 11 and a picture of a model installation in Fig. 12.

The nucleus of the hydraulic regulator is an oil jet that discharges under a pressure of about 100 psi into two closely adjacent orifices that are connected by tubes to opposite ends of a double acting hydraulic cylinder. The jet pipe from which the oil is discharged is pivoted and free to move through a small horizontal angle. When the jet is centered on the orifices the pressures on the piston in the hydraulic cylinder are balanced and the piston is stationary regardless of its position.

The movement of the jet through the horizontal angle is controlled by the deflection of a diaphragm. On one side of the diaphragm is a signal force that is opposed by a counteracting force

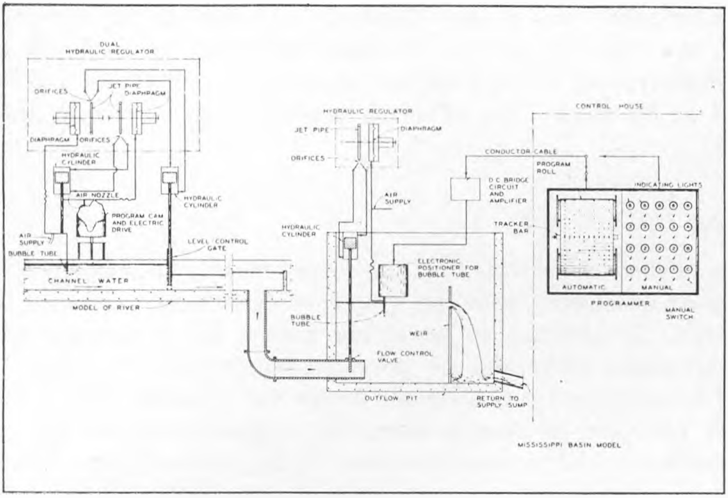


FIG. 11. SCHEMATIC DIAGRAM OF THE INSTALLATION OF THE OUTFLOW INSTRUMENTS

developed by a spring. The diaphragm is attached by a shaft to the jet pipe which moves with the deflection of the diaphragm.

The signal force is developed by the head over the end of a bubble tube set in the channel which discharges an accurately measured stream of air; this head at the balance point as used on the model is exactly 2 inches. (However, it can be made any reasonable amount by adjusting the spring to balance the signal with the

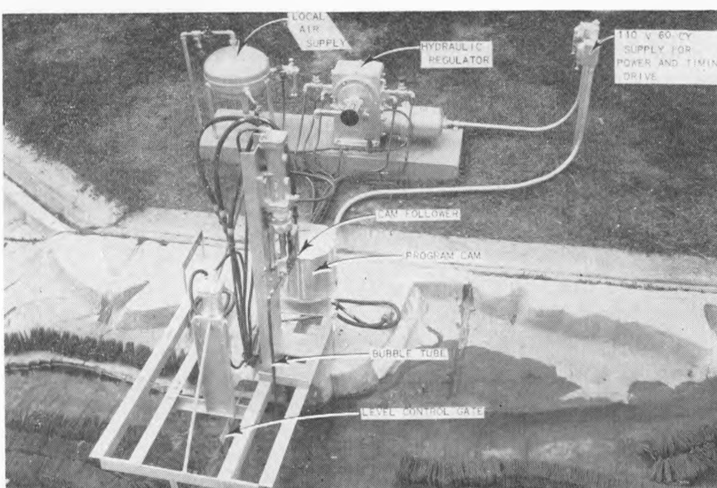


FIG. 12. MODEL INSTALLATION OF THE HYDRAULIC REGULATOR AND CAM PROGRAMMING DEVICE

pressures balanced on the piston in the hydraulic cylinder.) Any change in the head over the bubble tube from the balance point will deflect the diaphragm and move the jet pipe in the direction of the greater force; this unbalances the force on the piston in the hydraulic cylinder and moves it in the direction of the greater force. The piston is attached to the valve stem of a gate valve in the outflow pipe or to a rod connected to a gate in the model channel (Fig. 11). The gate is moved in a direction to return the head on the end of the bubble tube to the balance position. When the head over the end of the bubble tube has been returned to the balance position of exactly 2 inches, the jet emanating from the jet pipe is centered on the two orifices and the system is in balance again.

The stage or water surface in the model channel, or the head over a weir, is 2 inches above the end of the bubble tube. By programming the elevation of the bubble tube, a program of stage or head can be maintained at the bubble-tube point. Two methods have been used on the model to program the elevation of the bubble tube.

One method used is an oil jet set up with a double acting hydraulic cylinder and a diaphragm similar to the one described for operating the valve gate (portion of diagram on left of Fig. 11). The bubble tube is fastened to the piston rod of the hydraulic cylinder. A cam follower that is positioned by the top edge of a cam cut to the stage hydrograph and that furnishes the signal pressure to the diaphragm is also fastened to the piston rod. The signal pressure on the diaphragm is developed by an air jet impinging on an orifice connected by a tube to the diaphragm chamber. At the balance point for the two forces, signal pressure and spring, the cam is partially cutting the air jet. The cam is circular and is driven by electrical clockworks. As the cam rotates, the top edge is moved away from the balance point in the air jet. This unbalances the forces acting on the diaphragm moving the oil jet away from one orifice and toward the other and in turn unbalancing the forces acting on the piston in the hydraulic cylinder, moving it in the direction to bring the air jet on the cam back to the balance point. In this manner the air jet is made to follow the top edge of the cam carrying with it the bubble tube, since they are both rigidly connected to the piston rod of the hydraulic cylinder. The movement of the bubble tube causes the system controlling the valve gate or channel gate to change the water-surface elevation to agree with the new elevation of the bubble tube. Thus a head program over a weir or a stage program in the channel can be maintained.

Another method of programming stage and discharge is shown schematically on the right in Fig. 11. This method makes use of a programmer described under inflow instruments, a wheatstone bridge, and a bubble tube positioner. On one side of the wheatstone bridge are twelve fixed precision resistors that can be programmed into and out of the circuit by the programmer; on the other side is a variable precision resistor, the movable arm of which is connected by a gear train to a reversible motor. The output of the bridge is amplified and applied to the motor to move the arm of the variable resistor to balance the selected fixed resistance in the bridge circuit. The motor is geared also to a precision screw to which is attached the bubble tube.

The various fixed resistors are selected such that, as they are programmed in the circuit, they will cause the motor to move the precision screw, in balancing the variable resistance, the following vertical distances:

0.6 ft	0.06 ft	0.006 ft
0.3 ft	0.03 ft	0.003 ft
0.2 ft	0.02 ft	0.002 ft
0.1 ft	0.01 ft	0.001 ft

By programming the fixed resistors in the wheatstone-bridge circuit, which is done by a perforated program roll in the programmer, the elevation of the end of the bubble tube is programmed. In turn, through the action of the hydraulic regulator apparatus, the stage in a channel or the head over a weir is programmed.

Master Timer and Calendar

The master timer provides the timing functions in accordance with the time scale for the entire model and synchronizes the operation of all instruments (Fig. 13). It is an electromechanical device that consists of a series of cams driven by a synchronous motor with suitable reduction gears to produce preselected speeds determined by the model-time scale. The teeth on these cams actuate microswitches which, with their associated relay circuits, furnish three timing signals: a 0.25-second signal every 13½ seconds for actuating the program roll in the inflow programmer; a 2-second signal, also, at the end of each 13½ seconds for programming the date and hour on the calendar; and a 2-second audible signal at the end of each model day.

The calendar is a date-hour indicator designed so that the model time can be continuously and visually indicated in the control house during tests (Fig. 14). It consists of a series of relay circuits that operate lamps indicating the month, day, and hour on the annunci-

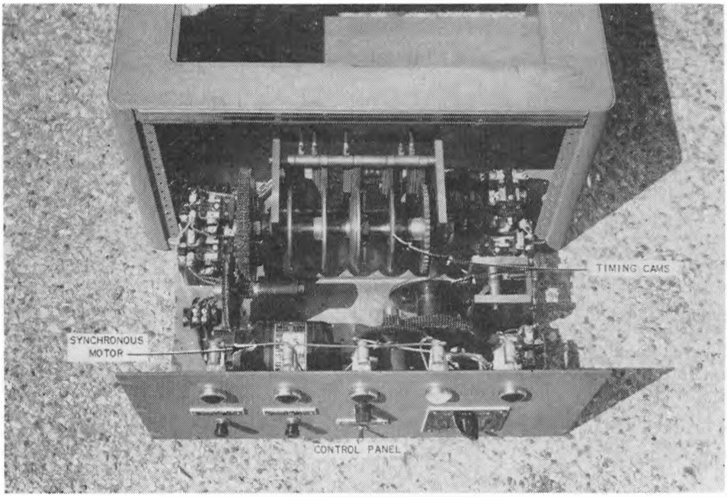


Fig. 13. Master Timer

ator panel. The calendar also relays audible hourly signals to the model which are selective for 2, 4, 6, or 12 hour periods.

The operation of all instruments is governed from the control house. In preparing for a test the timer, the calendar, the program rolls in the inflow programmers, and the recorder charts on the stage recorders are set for the month, day, and hour of starting time; then, by means of a single switch, all instruments in the control house and on the model can be started simultaneously.

Fig. 14. Calendar - Date - Hour Indicator

ACCURACY

It is difficult, if not impossible, to evaluate the absolute accuracy of the combined system of instrumentation operating as a unit on the model because of the many variables that affect the data obtained with the instruments. These data are the results of the aggregate effects of the instrumentation and of the model. The separate effects of the instruments and the model are difficult to isolate and it is possible only in rare instances. However, relative accuracy of the instrumentation as a unit and the accuracy of the instruments operated individually under controlled conditions are more readily obtainable.

Prior to beginning formal testing programs the model streams are subjected to a series of verification tests in which the model roughness is adjusted to reproduce the relationship between stages

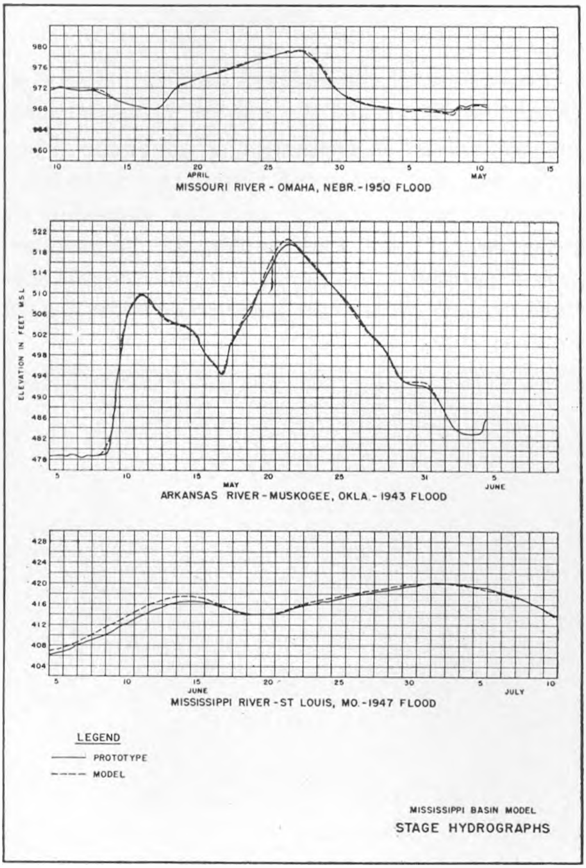


FIG. 15. STAGE HYDROGRAPHS

and discharges at the various gaging stations for selected floods of record. The accuracy with which these floods are reproduced is a verification of the composite accuracy of the model, the instrumentation, and the prototype flood data. Typical model stage and discharge hydrographs compared to prototype hydrographs for selected verification floods are shown on Figs. 15 and 16. The maximum difference in the model and prototype hydrographs is only about five percent. When this difference is apportioned to the instrumentation, model, and prototype data as errors, the errors become comparatively small for each of these factors. The accuracy of the majority of the prototype discharge measurements and ratings as indicated in the records is within 10 percent. The model reproduction of these floods is well within the accuracy of the prototype data.

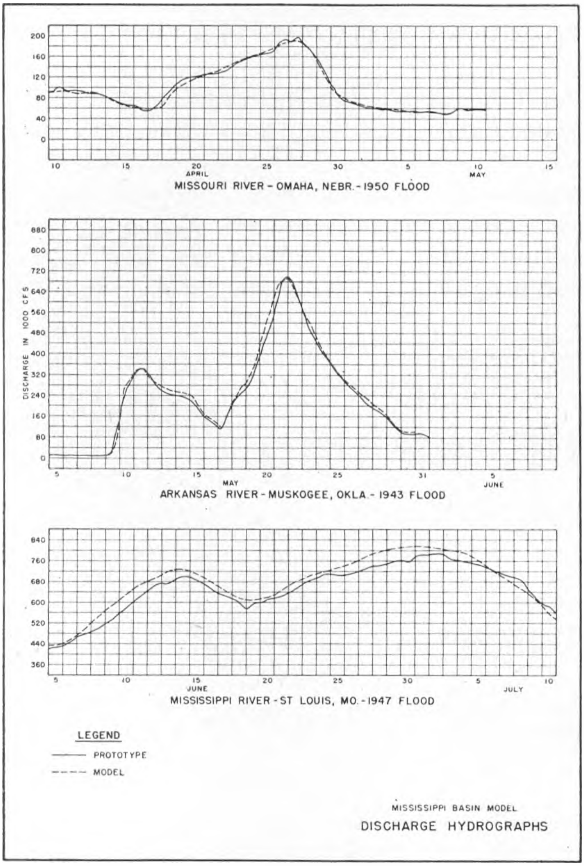


FIG. 16. DISCHARGE HYDROGRAPHS

The accuracy with which identical tests can be repeated on the model is another indication of the accuracy of the instrumentation as a unit. Generally, the differences in these tests do not exceed one percent and in numbers of instances the repeated test appears to be an identical reproduction of the original test.

Flood-control models of the type of the Mississippi Basin Model have been in use at the Waterways Experiment Station for the past 20 years. The original models of this type were operated manually. The relative accuracy of manually operated instruments and automatic instruments has been obtained from a comparison of the results of tests on the original models of this type and on the Mississippi Basin Model. The reproduction of verification floods by both methods of instrumentation was accomplished with approximately the same degree of accuracy. However, the accuracy of repeating identical tests with automatic instruments is greater than with the manually operated instruments. Automatic instruments eliminate the human equation which affects the data obtained with the manual instruments.

Extensive tests were made with individual instrument units under controlled conditions for the purpose of determining the accuracy. Sixty-three hydrograph tests were conducted with twelve inflow controllers and programmers. Three series of tests were conducted: one series consisting of the rising leg of inflow hydrographs; a second series of the falling leg; and the third series of complete hydrographs, both rising and falling legs. The percent of

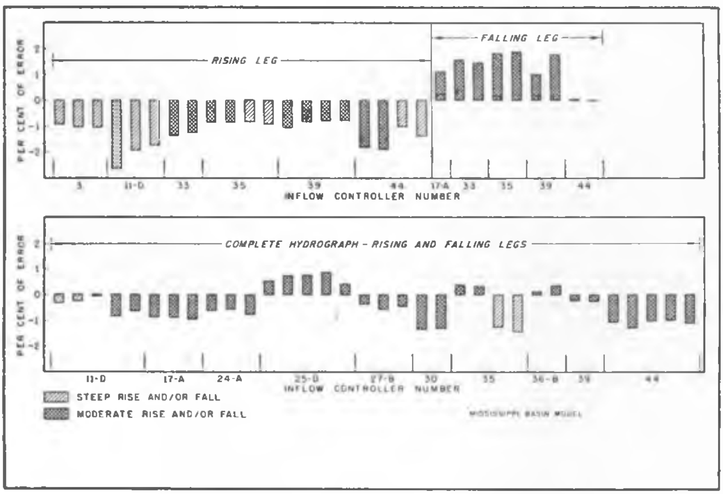


FIG. 17. PERCENT OF ERROR IN TOTAL VOLUME OF INFLOW HYDROGRAPHS PROGRAMMED AND MEASURED BY INFLOW INSTRUMENT UNITS

error in total volume measured by the instrument unit in each of these tests is shown as a bar in Fig. 17. The rate of rise or fall, whether steep or moderate, is shown by the shading of the bars. In the tests with hydrographs of the rising leg, the pressure dropped momentarily at the beginning of each 13.5-second period as the orifice area was increased resulting in a smaller measured volume than the programmed volume. The reverse occurred in the tests with hydrographs of the falling leg.

Tests, which numbered several hundred, were conducted with stage instrument units under laboratory conditions to determine the accuracy of these instruments in following a rising and falling water surface. These tests indicated that the stage instrument units would follow and record hydrographs with maximum errors of less than 0.001 ft under these conditions. The accuracy on the model was somewhat less.

In the tests conducted with the outflow instruments, an accuracy of maintaining stages in a channel or head over a weir within 0.001 to 0.002 ft was indicated.

The cost of automatic operation of the Mississippi Basin Model compared to the cost of manual operation has been reduced about 50 percent. It is estimated that the cost of the instruments in use on the model has been amortized by the saving in salaries that would have been required for manual operation.

Additional instruments consisting of 120 inflow controllers and programmers, about 300 stage transmitters and recorders, and 102 hydraulic regulators with the programming device will be required for completion of the model. These instruments will provide for operating the main stem of the major tributaries and the Mississippi River.

Discussion

The discussion was initiated by Mr. Murray who asked why a vacuum-type of control rather than an electrical one was used. Mr. McGee replied that specifications were drawn up and submitted to the manufacturers, and the method used was the least costly that would satisfy the requirements.

Mr. Wires then inquired whether the 0.004″-probe gap implied an accuracy of that magnitude or if this determined the limits of a continuous hunting process. Mr. McGee indicated that it is the latter and that the maximum error is less than 0.001 foot under laboratory conditions, although the accuracy is poorer on the model.

Mr. Rouse requested comment on difficulties encountered due to the settlement of the model and means that were used to combat this. Mr. McGee admitted that there had been difficulty with settling of the model due to the fact that the Jackson clay formation in that area expands and contracts considerably with changes in moisture. It was found that, after the Missouri section of the model had been constructed, instead of settling, it rose by about 0.03 to 0.07 foot. It has been necessary to insert piles in the sub-grade to a depth of about 10 feet, set the slabs on the piles with a void of about 0.4″ under the slabs and to install screw-jacks on top of the slabs that could be adjusted as required. This procedure is being used for the entire model.

Mr. Landweber inquired about the scaling laws which were used in setting up the model. Mr. McGee stated that a modified Froude law is used. Because they were not able to scale the roughness, however, it was necessary to increase the discharge from a ratio of 1:2,000,000 as required by the modified Froude law, to a ratio of 1:1,500,000. Before testing begins, though, the model is made to reproduce a flood of record under the same conditions that that flood occurred. Only bulk movements of flood waters are studied with the model, not velocity distributions or other local phenomena.

Mr. Truesdell then asked what is the accuracy of predictions. Mr. McGee replied that it was about five thousandths of a foot, which represents five tenths of a foot in the prototype. In some cases they have duplicated floods within 2 or 3 tenths of the prototype elevations.

Mr. Bauer wished to know what kind of adjustments were made on the model in order to reproduce floods of record. In reply Mr. McGee said that first the model was molded according to the available hydrographic and topographic surveys. Then roughness is installed in the channel and on the overbank. Screen wire, which represents timber, is installed on the basis of aerial photographs, and its density adjusted in order to obtain the stage-height versus discharge relationships measured at the various gaging stations.

Mr. Mostafa pointed out the change in roughness in some rivers, such as the Missouri, when the flood goes out and asked how this was taken into account in the model which has a constant roughness. Mr. McGee was aware of this phenomenon. He stated that roughness is put on the bottom of the channel and quite often it is observed that roughness is not needed on the sides. As the flood rises, the roughness of the bottom is sufficient to take care of the roughness requirements. In most channels, however, roughness is needed all around the wetted perimeter of the cross-section. This is adjusted to give the correct stage-discharge relationship at various stations. He further went on to say that the procedure had been verified on several streams for more than one flood; on the upper portion of the Mississippi for '43, '44, and '47 floods, and on several floods on the Cumberland River, all at the same time scale.

Mr. Albertson continued the same line of inquiry as Mr. Mostafa. It was his experience also that in alluvial rivers the roughness changes with the stage because of the change in the pattern of the sand dunes, as a result of which, at higher stages, the river channel may appear to be smooth. He asked how this was taken into account in the model? Mr. McGee then essentially repeated his reply to Mr. Mostafa's question, that, by putting fairly large pegs on the bottom of the channel and little or none on the sides, the condition of diminishing effective roughness with increasing stage is simulated. This procedure has been justified by the reasonable degree of accuracy with which various floods of record have been reproduced.

Mr. McGee concluded the discussion by showing two hydrographs of floods at Muskogee on the Arkansas River with which the model was in very close agreement. He admitted however that, with considerable movement of the bed load and the formation of bars, which might change the bed level by two or three feet between floods, the model, which has a fixed bed, would be able to reproduce only one flood. This was the case in the St. Louis area of the Mississippi River.

RECENT DEVELOPMENTS IN ELECTRONIC INSTRUMENTATION

by

PHILIP G. HUBBARD

At the Third Hydraulics Conference in 1946, Dr. S. W. Grinnell presented a paper [1] in which he discussed the role of electronics in hydraulic research and the work of some of the pioneers in the field up to that time. He analyzed the particular virtues and deficiencies of electronic instruments and reviewed current techniques in the field as a guide to those who found it necessary to supplement traditional methods of measurement. Nearly a decade has passed since that time, and a review of developments during this period is considered in order, both as a guide to those who wish to take advantage of current knowledge and equipment and as a clue to those who intend to participate in future developments along this line. For the sake of brevity, the title of this paper has been made much broader than the actual subject matter, and limitations can now be fully stated: Recent developments are those which have arisen or matured within roughly the last decade; only instrumentation which is closely related to hydraulic engineering will be included; and finally, the topics to be considered are meant to be only representative, certainly not exhaustive. In particular, instruments and techniques which are being presented in other papers of the present conference will be omitted or only mentioned briefly, for obvious reasons.

GENERAL OBSERVATIONS

For the research worker confronted with the problem of devising methods to measure various physical quantities under laboratory conditions, the required understanding of the details of electronic circuitry has been greatly decreased during the last decade. Because of the demands of industrial research, particularly in the aircraft and flow-process industries, a multiplicity of sensing elements which can be applied directly to hydraulic measurements is now commercially available. For processing the signals from these elements, entire assemblies of electronic components which formerly had to be designed and refined in the laboratory can now be purchased directly from a choice of manufacturers in a highly reliable form. It is therefore possible for an engineer with imagination and skill in mechanical design to assemble satisfactory electronic systems with little more than a "block-diagram" understanding of the electronic components. There are, of course, many problems whose solution demands a much deeper understanding of electronic principles, and obviously many more which to date have defied solution.

A cursory look at available statistics shows that the business of developing better sensing elements and simpler-to-operate electronic components is a highly-competitive field which has assumed giant proportions. From an annual volume of about one and one-half billion dollars in 1947, the instrumentation industry has grown until it now boasts a volume of more than four billion dollars [2] and has produced vigorous branches in computers and automation. Because of the increased use of automatic measuring and controlling devices, some even refer to present developments as the second industrial revolution. In 1946, an efficient instrumentalist had to keep abreast of the rapid release of a rich but unrefined storehouse of information which had developed in the confined atmosphere of wartime research. In 1955, an efficient instrumentalist must keep abreast of the rapidly increasing array of commercially available and highly refined instruments which are the product of the intervening years.

The author of a recent technical article surveys the range of available instruments and concludes that "the most important frontier of modern instrumentation is in application" [3]. This remark is highly significant and applies to a great many of the problems of hydraulics, especially in what may be considered the extremities of the field: the operation of model structures and the solution of hydrodynamic and other integro-differential equations.

As typical examples of techniques which are being applied to model operation, tidal fluctuations are now being produced automatically from a rectangular plot on paper of the desired stage relationships versus time, while different parts of a model are scheduled to reproduce desired conditions by means of feed-back loops in which the stage at one point, for example, controls the discharge at another [4]. At the measuring end, operations such as averaging, integrating or applying correction factors can be handled electronically with considerable savings in time and money. Final results can be recorded on charts, on magnetic tape, on oscilloscope tubes or, in keeping with latest trends, as digital data on punched cards, decade counters, or even automatically typed sheets.

For the analyst who has suffered the frustrating experience of managing to express the relationships between parameters in equation form only to find that useful solutions will require years of work, analog and digital computers are now available which will quickly provide solutions. The ability of these computers to check their own operation and indicate faults automatically has improved so that they exhibit a freedom from error hundreds of times better than skilled workers. Since large general-purpose computers may cost more than a million dollars and produce answers at a fantastic rate, they are generally located at convenient centers of activity where they can serve many analysts on a rental basis. Special-purpose or "small" computers (in the trade, those which cost less than $50,000) are often supplied complete with an intensive short course on their operation and presumably can be kept, like a slide rule, at the analyst's elbow.

SPECIFIC APPLICATIONS

In order to discuss some recent developments in more detail, it will be necessary to narrow the range of problems considerably. Because of the continually recurring need to measure velocity, pressure, and vertical displacement of a liquid surface or channel bed, these have been chosen as examples. The range will be further restricted by considering measurements made under research laboratory conditions as opposed to routine industrial measurements or field work under adverse conditions.

Electronic instruments are used in measuring fluid pressures in three categories: very low, rapidly varying, and where a record or remote indication is necessary. Several manufacturers supply instruments which will indicate the instantaneous values of pressures above about 1 psi with very fast time response and high accuracy. Extreme care has been taken to reduce the size of the sensitive portion and insure freedom from errors due to extraneous variables such as vibration and temperature variations. For measurements in this range, therefore, it is recommended that commercially available devices be considered before starting to design a sensing element. In Fig. 1, a typical pressure cell of this type is illustrated.

If measurements in the range below 1 psi are to be attempted, two alternatives should be considered: to measure only the fluctuating component if possible, in which case commercially available hydrophones with sensitivities down to background noise can be obtained; or, if small changes in the mean level must also be known, to arrange components such as sensitive bellows or diaphragms so that they convert pressure differences to linear motion which can be detected with displacement detectors which, again, are available commercially in several forms. As an example only, a linear variable

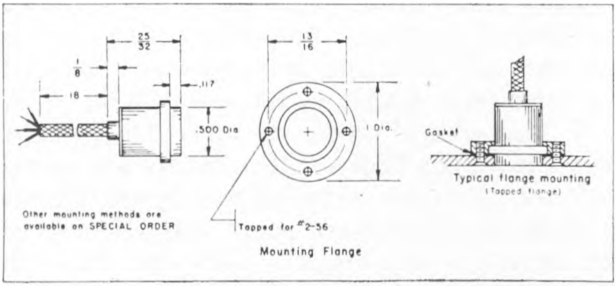


FIG. 1

differential transformer which reliably and accurately detects displacements of less than 10^{-4} inch is shown in Fig. 2. The problem of recording the results is simply a matter of selecting from a wealth of material. A recorder which happens to be available at the Institute is shown in Fig. 3.

Lest the conclusion be reached that all pressure measurements are now simply routine, a few words of caution concerning possible pitfalls should be noted. If the dimensions of the system under study are quite small and if fluctuations in pressure are rapid, considerable study and design are often necessary to assure freedom from inertial and viscous effects which can cause very large errors. A complete analysis of the factors which must be considered in this regard could well be the subject of a paper many times the length of this one, however, and will therefore not be even attempted now.

In the field of measuring liquid surface elevations in gage wells, manometers, or open channels (including waves), one recent de-

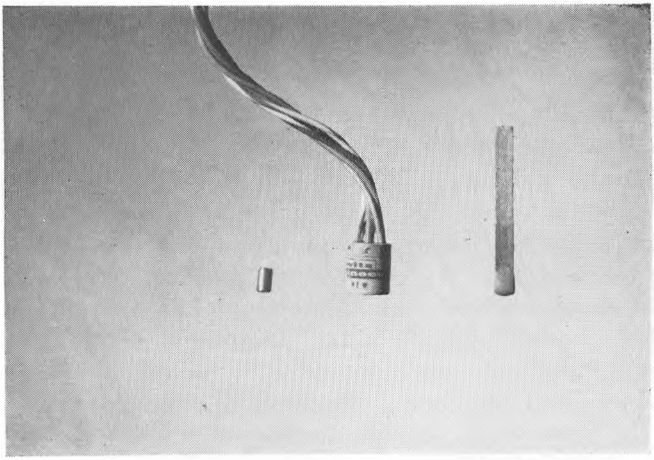


FIG. 2

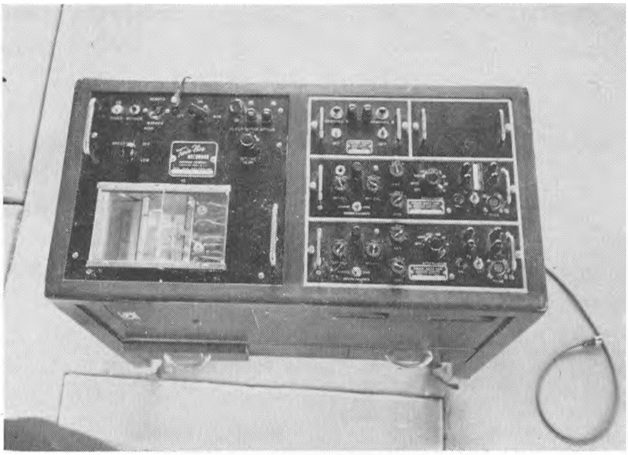


FIG. 3

velopment is the perfection of capacitive-type detectors consisting of very fine wires coated with a thin dielectric. When these wires are stretched between two points, one beneath and one above the surface, the capacitance from the wire to the liquid varies linearly with the immersed length. These fine wires usually do not create a noticeable disturbance in the system, respond as quickly as necessary, and the capacitance can be made nearly independent of the ambient temperature or conductivity of the fluid. Their chief virtue, of course, is simplicity.

Another liquid-level detector which has been highly developed recently is the vibrating-needle device which enjoys considerable popularity in Europe [5]. A related European development which to the writer's knowledge has not been employed in this country is the "wave-height analyser" of the Hydraulics Research Station in England [6]. The sensing element of this analyser is a series of vertical wires in graduated lengths which contact the moving water surface. When wires are immersed or exposed by the changing water surface, resistors connecting the wires are progressively shunted or vice versa. If a steady current is passed through the system thus formed, the voltage varies in a step pattern which is converted to pulses and finally to indications on a counter. It is expected that this novel device will enjoy considerable popularity because of its basic simplicity and high probable accuracy.

When the surface elevation to be measured is not the liquid-air interface but the interface between water-saturated earth and the water, the problems of accurate and dependable measurement are much farther from satisfactory solution. First, because of the heterogeneous nature of the bed material and the possibility of partial suspension in flowing water, simply specifying the location of the bed becomes a matter of statistical definition before any measurements are attempted. Secondly, practically none of the physical characteristics of the surrounding media go through a sharply defined change as the point of measurement crosses the interface. In the face of these facts, it is not surprising that the best available measurements of bed level are only 1/10 to 1/100 as precise as corresponding measurements of the liquid surface; nevertheless, there is recent progress to report in this field. At the Institute, for example, the bed level at a fixed object such as a bridge pier or abutment is being measured by means of a series of small electrodes embedded in the surface on a vertical line [7]. If the electrical impedance from each of these electrodes to a large electrode fixed in the water is measured, a definite difference is evident as the interface is crossed. In keeping with earlier remarks, this change is not abrupt, but becomes increasingly sharp as the particle size in the bed and the vertical dimension of the electrode decrease. When the water is moving, however, the gradually varying concentration of suspended material complicates the problem of defining the bed level.

For another recent development in electronic bed-level detection, attention can again be directed to work being done abroad. In both England and France, investigators have independently developed instruments operating on the change in electrical impedance between two immersed electrodes as they approach the bed [8]. An associated pair of electrodes compensates for changes in the conductivity of the water and, under favorable conditions, a reproducibility of indication less than 1 mm is reported. No mention was made of adapting the device to automatic recording, but this should be a relatively simple step using modern servo devices.

Of all problems of hydraulic measurement probably none has received more attention during the past decade than the measurement of velocity, and probably none is farther from satisfactory solution throughout the range of normally-encountered conditions. With one or two possible exceptions, none of the techniques being used is based on an entirely new principle, but several variations and improvements on old methods are worthy of notice.

One of the oldest and most direct methods of velocity determination is to inject a neutrally bouyant but optically distinct material into the stream and to observe its subsequent motion. At least two new methods of introducing the tracer electrically have come to the writer's attention, and are especially useful in observations of the boundary layer. Wortmann, in Germany reports on the use of a very fine tellurium wire stretched across the stream at a point where it can be observed optically [9]. With this wire as the cathode and any convenient metal as the anode, a pulse of current lasting a few milliseconds is passed through the water. This pulse dislodges ionized particles of tellurium which are transformed into elementary tellurium by a secondary reaction and proceed with the flow in what appears to be a dense black line. The distortion of this line with passage of time is related to the velocity, of course, and can be observed photographically for quantitative results. By using due care, the particles produced have fall velocities of less than 0.1 mm/second and diffusion velocities of less than 10^{-3} mm/second.

A superficially similar method utilizing an entirely different principle to produce similar results has been reported in a thesis by Geller at Mississippi State College [10]. As in the Wortmann method, a fine wire is stretched across the flow and used as the cathode to pass a pulse of current through the water. The wire was made of platinum, however, and operated by producing a fine line of very small bubbles of gas through electrolysis, instead of ionized particles. The actuating pulse was provided by discharging a condenser through the water, and better results were obtained by adding salt or acid to the water to increase its conductivity; errors due to bubble bouyancy were eliminated by using vertical flow past the measuring point. Besides the novel method of introducing the line of bubbles, the thesis also described an improved method of establishing a time base by repeating the pulses at controlled short intervals of time so that a single film exposure included several lines of bubbles. Furthermore, by combining the repetitive-pulse technique with motion pictures, unsteady flow patterns were successfully measured.

The method which is intrinsically most nearly perfect for measuring velocities is that of electromagnetic induction. It is capable of indicating over an extremely wide range, has zero lag, is practically independent of conductivity, temperature, pressure, and gaseous or solid impurities, and indicates the true vector component of velocity. During the many decades that the principle has been understood, however, much work by many different investigators has failed to produce a practical instrument for any but very limited conditions. Work on the method has been continued, however, and several papers describing the results are to be found in the literature of the last decade. Two of these are of particular interest to hydraulic engineers: One describing a probe-type instrument for use in pipes at the University of California [11], and one describing several forms of instruments used by Electricite de France [12]. Their work was significant and produced useful results, but many problems still must be solved before the method can find widespread application in hydraulic measurements. The principle has been mentioned in the present paper primarily because of its tremendous potential.

Proceeding to a method which is less direct than either of the preceding ones, the well-known stagnation tube has come in for its share of development through electronic auxiliaries, and has actually been used to measure turbulence in several laboratories. At the Massachusetts Institute of Technology, for example, a small stagnation tube has been connected to a variable-capacitance sensing element and used to measure turbulence in high-velocity flow at shallow depths [13]. At the Iowa Institute, several forms of modified pitot-static tubes have been developed, some of which indicate fluctuations in the direction as well as the magnitude of the velocity [14]. They have limited application in the laboratory primarily because of the relatively low sensitivity of the fast-response types and the difficulty of producing them in sufficiently small sizes.

No discussion of electronic instruments for measuring velocity would be complete, of course, without including the hot-wire anemometer and its close relative, the hot-film anemometer, which is particularly well adapted to measurements in water [15], [16]. Although these instruments are less direct indicators of velocity than any of those already discussed, they have produced excellent results and are considered most likely to be used soon in general laboratory investigations of turbulent flow. Operating techniques which minimize the influence of secondary variables have been developed, and long-time stability has been achieved. The size and response time are adequate for practically all conditions which will be encountered in laboratory work, although their use in field studies is limited by the frangibility of the sensing element.

PROSPECTS FOR THE FUTURE

A discussion of recent developments naturally leads to the consideration of future developments either expected or hoped for, a task which is pleasant but somewhat hazardous. Firstly, what are the major unsolved problems of measurement confronting hydraulic engineers? The following list is almost certainly incomplete, but represents the writer's assessment of the problem, with no particular signifiance applied to the order of presentation.

1. An in-place sediment-concentration analyzer.
2. A size-frequency sediment analyzer for use in large-scale studies.
3. Improved velocity meters for field use, especially for very low velocities and very rapid or very intense fluctuations.
4. A better stage recorder for the field.
5. Instruments to measure pressure and pressure-velocity correlation in turbulent flow.

Finally, what current developments in instrumentation are likely to have the most influence on future developments in hydraulic measurements? The comments made above apply equally well to the following list.

1. Increased use of digital-type indicators with a resultant decrease in operator-judgment factors.
2. Increased use of automatic computing, compensating, and recording techniques.
3. Transistorized circuitry for light-weight, long-life field instruments.
4. Speed-changing recording techniques to make better use of standard analyzing equipment.
5. Greatly expanded use of servo-type instruments for control and measurement in hydraulic models.

DISCUSSION

Mr. Baines remarked that all of the examples cited rely primarily on electrical techniques, and that some electrical modifications of standard techniques have also been produced. For example, much work has been done on the midget current meter in an effort to reduce friction and record the revolutions remotely. Two organizations have developed propeller-type meters, one metal, one plastic, and have independently perfected nearly identical electrical circuits to detect the speed. A carrier current is passed through a wire near the blades, and passage of each blade produces a blip in the carrier. This is converted to a pulse which actuates a standard counter. The counters operate for periods up to 100 seconds and display the total number of pulses during the period.

Mr. Baines then asked about the present status of the hot-film anemometer, its frequency response, and temperature-velocity correlations. The speaker replied that any desired frequency response could be obtained up to at least 50 kilocycles, but that 1 kilocycle was usually adequate for the conditions encountered in water. He then referred the questioner to the recent Ph.D. dissertation by S. C. Ling for additional information on frequency response and the various correlations which could be measured.

Mr. Harbeck asked if the speaker cared to evaluate the vibrating-string type of instrument for measuring pressures, and the speaker emphasized the advantages of using commercially available devices wherever possible. These are available in many forms, but he knew of none using the vibrating-string principle. Although this principle and many others are well understood and probably reliable, most development work involves a great deal of time eliminating the effects of extraneous variables.

Mr. Folsom corrected the speaker's error in displacing Messrs. Grossman and Charwat from their true home at the University of California in Berkeley. He was also gratified by the favorable reference to the electromagnetic induction method and agreed that it should be exploited more thoroughly. With regard to the speaker's prediction concerning a shift to digital recording, he has already put all of his laboratory manometers on Veeder-Root counters. Random errors in the experimental results disappeared as if by magic.

Mr. Gent asked if the cloud of bubbles produced by the platinum wire could be produced by discharging current directly through the water. The speaker said that this was an unlikely possibility, but that production of cavitation bubbles by producing intense standing waves in the water was a possible means of eliminating the troublesome wire in the fluid. When intense standing waves are produced by a crystal or magnetostriction transducer, the fluid actually vaporizes in the pressure loops.

Mr. Shoumatoff was interested in the probable behavior of a hot-wire or hot-film anemometer in a fluid containing cellulose fibers, and in the precision of the vibrating-needle technique for measuring viscosity. The speaker recommended a hot-film anemometer with a conical tip to minimize the tendency to collect fibrous material, but was not familiar with the vibrating-needle method of measuring viscosity. Mr. Shoumatoff said that their attempts to obtain an instantaneous measure of viscosity with the vibrating needle had been unsuccessful.

Mr. DeHaven reported that their experience with the Fiscotron at Phillips Petroleum had also been unsatisfactory.

REFERENCES

1. Grinnell, S. W., "The Application of Electronics to Hydraulic Research," *Proc. Third Hydr. Conf.*, 1946.

2. Editorial, *Instrument and Apparatus News*, Vol. 2, No. 1, 1954.

3. Editorial, *Instrument and Apparatus News*, Vol. 1, No. 5, 1953.

4. Wilkie, M. J., "Some Automatic Devices for Use on Tidal Models," *Jour. Sci. Instr.*, Vol. 30, Aug., 1953.

5. Gridel, H., "The Accurate Measuring and Recording of Stable or Fluctuating Water Levels by Means of Continually Vibrating Limnimetric Points," La Houille Blanche, Special B/1950, p. 709.

6. Wilkie, M. J., and King, R.F.J., "A Wave-Height Analyzer," *Jour. Sci. Instr.*, Vol. 30, 1953.

7. Hubbard, P. G., "Field Measurement of Bridge-Pier Scour," *Proc. Hwy. Res. Board*, Vol. 34, 1955.

8. Wilkie, M. J., and King R.F.J., "A Bed Level Indicator for Detecting the Boundary of a Body of Water," *Jour. Sci. Instr.*, Vol. 31, 1954.

9. Wortmann, F. X., "Eine Methode zur Beobachtung und Messungen mit Tellur," *Zeit. f. Ang. Phy.* 5 Band, 6 Heft, 1953.

10. Geller, E. W., "An Electrochemical Method of Visualizing the Boundary Layer," Mississippi State College M.S. Thesis, August, 1954.

11. Grossman, L. M., and Charwat, A. F., "The Measurement of Turbulent Velocity Fluctuations by the Method of Electromagnetic Induction," *Rev. Sci. Instr.*, Vol. 23, 1952.

12. Remenieras, G., and Hermant, C., "Mesure Electromagnetique des Vitesses dans les Liquides," *La Houille Blanche*, No. Special B/1954.

13. Ippen, A. T., Tankin, R. S., and Raichlen, S., "Turbulence Measurements in Free Surface Flow with an Impact Tube-Pressure Transducer Combination," *M.I.T. Tech.* Rpt. No. 20, July, 1955.

14. Hubbard, P. G., Appel, D. W., and Ling, S. C., "Instruments for Measuring Large-Scale Turbulence in Water," *Iowa Inst. of Hydr. Res. Rpt.* Oct. 1952.

15. Hubbard, P. G., "Constant-Temperature Hot-Wire Anemometry with Application to Measurements in Water," S.U.I. Ph. D. Dissertation, 1954.

16. Ling, S. C., "Measurement of Flow Characteristics by the Hot-Film Technique," S.U.I. Ph. D. Dissertation, 1955.

PHOTOGRAPHIC ANALYSIS OF FLUID FLOW

by

John H. Waddell

Fairchild Camera and Instrument Corp., Syosset, L. I., N. Y.

The flow of gases, fluids and solids is a transient phenomenon which is extremely difficult to observe with the eye. If the eye is replaced with a lens and the rods and cones are replaced by a photosensitive material, such as photographic film or paper, a permanent record can be made. With the understanding of a few fundamental rules, a definite and excellent record can be obtained which then can be interpreted.

Before starting a study of flow patterns one must determine whether the phenomenon is in a steady or a transitory state. If the event occurring is in a steady state, still photography will suffice for recording it. If conditions are changing, sequence or motion pictures will fill the requirement. If the mechanical action (which, of course, includes the flow of liquids, gases and solids) is aperiodic, it becomes even more apparent that a photographic record is necessary. Obviously, the stroboscope is an ideal instrument for the visual observation of periodic events, but for erratic or discontinuous phenomena, the stroboscope alone is not sufficient for their examination.

Mechanical actions are defined as those which can be seen or recorded directly. Electrical actions are those phenomena which might be invisible or intangible, such as, pressures, sound, etc. The latter can be converted, through a transducer, into electrical energy which then can be seen as voltage changes on a cathode-ray oscilloscope or similar recording mediums. These definitions will become more important as the discussion proceeds to simultaneous recording of the flow patterns and pressures.

The time interval for an event to take place is the next factor to be considered.

A classical story exists concerning the study of the aerodynamic characteristics of a guided missile. Schlieren pictures were required of a scale model in a wind tunnel. The missile was to go from zero velocity to Mach 2.5 in a very short time. One engineer stated that one picture should be taken as the velocity became measurable and a second picture at Mach 2.5 when steady state was reached; that the passage of the missile through Mach 1 was so fast that there would be no effect on the missile's behavior. A second engineer felt that a continuous record from 0 to 2.5 would be better for a complete analysis. A high-speed motion picture was made from Mach 0 to 2.5. As the missile passed through Mach 1, the violence of its passage was so great, it was apparent that the flight of the missile would be affected. The film record was such that a good analysis was possible and subsequent redesign produced a missile with better performance. But what it is important to note is that there is practically no event occurring which cannot be photographed.

The time of exposure is related to the field size and velocity. Especially is it important in the case of a single still picture that it be "crystal-clear sharp". For example, in photographing a moving object through two field widths, the first a 1-foot field and the second a 10-foot field, the apparent velocity will be ten times greater for the first than for the second. Therefore, the exposure time would have to be ten times as fast for the first in order to get a picture as sharp as the second one. It is true that the image size of the second will be but one-tenth the size of the first and that if it were enlarged to the same size as the first, the movement during equal exposures would be the same. The second picture actually seems sharper because of its reduced size. In observing Fig. 1, it is obvious that $BB_1 = 10\ AA_1$, and in any unit time of exposure the object will move an equal distance on AA_1 and BB_1.

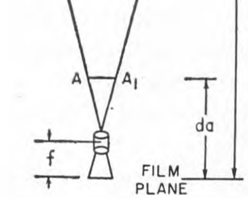


FIG. 1

The reduction factor is

$$R = \frac{d - f}{f}$$

where R is the number of times reduction
d is the distance from subject to film plane
f is the focal length of the lens in the same units as used in "d".

If the subject was moving at 10 feet per second, the following table can be computed for various exposure times:

Exposure time	*Object movement during exposure*	*Object movement during exposure at 10 times reduction*
1 second	10 feet	1 foot
.1	1 foot	1.2 inches
.01	1.2 inches	.12 "
.001	.12 "	.012 "
.0001	.012 "	.0012 "
.00001	.0012 "	.00012 "
.000001	.00012 "	.000012 "

A sharp picture will be obtained if there is image movement of less than .001 inch. Therefore, for all practical purposes, an exposure time of .00001 second or 10 microseconds will make a sharp picture at one-to-one reduction while at ten-to-one reduction, 100 microseconds would be satisfactory. This same calculation is made for still, sequential and motion pictures.

There are times, however, in which pictures are obtained with a smeared image along the axis of movement. When measurements are to be made by comparing a sequence of two or more pictures, the leading edge of the smear should be used.

(In projecting motion pictures as such, the little bit of smear adds to the illusion of motion;—those motion pictures having very sharp pictures have a tendency to appear stroboscopic.)

The time magnification factor is important in determining picture-taking rates for sequential and motion pictures. In silent motion pictures, 160 frames or pictures are required for a 10-second sequence and 240 pictures are necessary for sound pictures. For frame-by-frame analysis, a smaller number of pictures may be required. Therefore, for projection,—we define

$$\frac{P_t}{P_p} = M$$

where P_t is the picture taking rate
P_p is the picture projection rate
M is the time magnification.

At 4800 pictures per second, the time magnification factor will be 300 times for silent projectors and 200 times for sound projection. For time-lapse pictures where 1 picture per second may be taken, the time-magnification factor would be .0625 for silent pictures and .0417 for sound pictures.

As to field size, the width of the field will be inversely proportional to the focal length oi tne lens. Where explosions might take place, the longer focal length lenses offer greater protection for the equipment. For a rapid approximation for field width or distance from the lens to the subject for a given field width, the following formula, based on similar triangles, will prove useful:

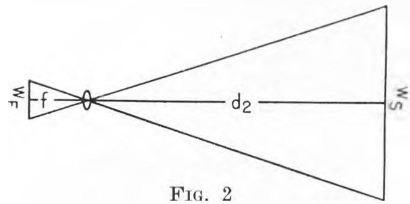


FIG. 2

$$\frac{W_f}{W_s} = \frac{f}{d'}$$

where W_f is the width of the film
W_s is the width of the field
f is the focal length of the lens
d' is the distance from the lens to the subject

Illumination of the subject plays an extremely important part in the study of flow patterns. In the case of liquids and gases, transmitted light is the basic method; while for the flow of solids, reflected light is generally used. To secure negatives of equal background density, the amount of transmitted light required will be about 1/40 that required for reflected-light pictures when using the camera lens at the same stop and the camera at the same picture-taking rate.

Practically there are but few basic lighting schemes for taking flow pictures;—though each engineer often modifies the over-all system to meet his own needs.

Specular light will give the best results from the standpoint of the flow patterns in clear liquids and gases. The specular light is light travelling in straight lines—preferably originating from a point source.

Diffuse light is scattered light and is best illustrated by light coming from a frosted lamp or from opal glass. The light can, or may, enter the clear glass in a straight line, but as it strikes the diffusing surface, it is broken up and no longer travels in the same direction as it entered, as is shown in Fig. 3.

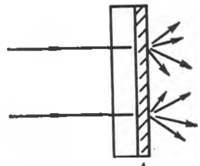


FIG. 3

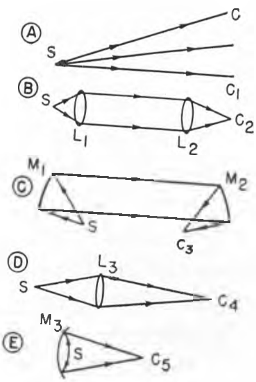


FIG. 4

There are various methods used for producing optical systems furnishing specular or quasi-specular light. The more specular the light, the sharper the flow pattern. Good observation of the flow pattern depends on the variation of the index of refraction in the medium due to pressure and/or temperature changes. Since these lines of differences may be very small and sharply delineated, the straighter the path of the light, the better.

In Fig. 4, the basic systems are illustrated. In A, a point source S is placed at some distance from the camera lens which is at C_I. If the subject is placed between the points S and C_1 but nearer C_1 and the lens focused on the plane of subject, a shadowgraph picture will be obtained. Such a shadowgraph will show the flow patterns in gases and transparent liquids, as well as dispersion of solids in liquids and the phenomena of solution and mixing.

Light sources which are generally used for this purpose are:

1. Ribbon-filament incandescent lamps
2. High pressure mercury-vapor lamps*
3. Zirconiun arc lamps
4. Spark
5. Electronic flash

In B and C lenses or mirrors are placed so that L_1 and M_1 will form a collimated beam of light and then L_2 and M_2 will re-image the source at C_2 and C_3. The subject is placed in the collimated beam, and the camera lens focused upon the subject. (A reference for this basic optical system will be indicated in the discussion of schlieren photography.)

Simplified systems are shown in D and E. D employs a single condensing lens. A large diameter lens is usually used so that the

* For any picture taking rate above 16 per second, d-c should be used.

background density is produced uniformly over the negative. The subject is usually placed between the condensing lens and the camera lens. (Two typical condensing lenses could be the 12″ f.l. 6″ diameter and the 21″ f.l. 14″ diameter.) When the lens is replaced by a mirror, as in E, the subject is placed between the lamp and the camera lens. Field size will determine the diameter of the mirror to be used.

From these basic systems, there will be slight deviations for shadowgraph and schlieren photography. In both shadowgraph and schlieren, the differences in the index of refraction within the medium are recorded. The shadowgraph pictures will show gross changes in index which would be produced and could be photographed by one of the five basic specular optical systems. If a stop is placed in the focal plane of the subject or at the principal focus of the lens, a schlieren picture is produced which shows even minute differences in the index. The stop can be either a knife edge or an iris diaphragm at the principal focus of the lens.

The schlieren system can be modified to produce color schlieren by introducing a continuous spectrum at the plane of the filament or slit. A second slit can be added at the plane of the spectrum to produce a narrow band of the color spectrum.

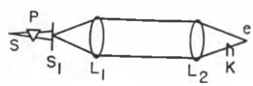


FIG. 5

Figure 5 shows the schematic employing the "B" system of Fig. 4, but with the 60° prism (P) interposed between S and S_1; S_1 is the adjustable slit and K the knife edge at the principle focus of the lens.

The diffuse light source or optical system is used both for transmitted and reflected light subjects. These systems are easier to set up than the specular systems. Typical ones are shown in Fig. 6.

A diffuse lighting system was used for the photography of the effect of missiles entering water with both still cameras and high-speed motion-picture cameras. The tank used for this purpose is about 4 feet high. The back of the tank toward the lights is opal glass so that an evenly diffused surface is obtained. Photo-flood lamps are used to back-light the tank. The camera is placed in such a position that the impact point, the splash and the trajectory of the missile through the water can be photographed. The sche-

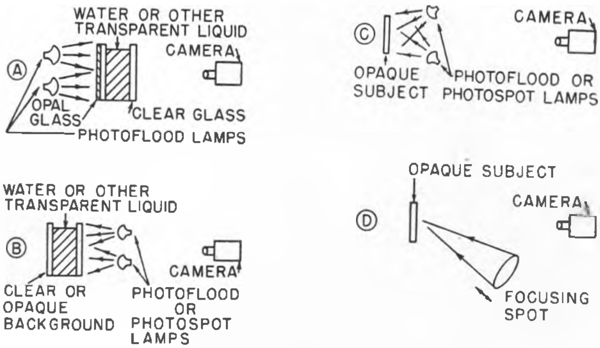


FIG. 6.

matic setup is shown in Fig. 6(a). The missile was photographed in silhouette. See Fig. 9.

Front lighting is generally used for the photography of liquid and gaseous flow and for the study of cavitation and bubble formation,—though occasionally front- and back-lighting will produce better pictures. The front-lighting setup is shown in Fig. 6(b), and some examples in Fig. 10.

Where opaque subjects are being photographed, for example, the flow of solids, Fig. 6(c) and (d) illustrate the basic methods. Figure 6(d) is especially applicable to the photography of small areas. In the case that mirror surfaces are encountered, the light must be brought in on the optical axis of the lens, but most surfaces are rough enough to permit the lights to be used at an angle. No matter how fine the scratches are, the light will be broken up so that the subject can be identified. For the flow of solids, time-lapse photography may be resorted to, where the picture-taking frequency may be 1 per hour, per day or per week. A typical example of this would be silver migration through phenol plastic.

In some cases, light sources themselves are photographed. Neon and argon glow lamps can be used effectively to study intake velocities and patterns into chambers. In the succeeding part of this paper describing how specific liquid studies are made, the effect of flow may or may not be present. Furthermore, many of the techniques described are applicable to gas studies.

The pattern of the flow of a liquid into a partitioned chamber could not be seen because of the all metal construction of the housing. A board (black) was made up with a number of neon lamps mounted on it. One side of the lamp terminals was wired in parallel and the other side of the individual lamps connected to selected positions in the chamber. With the chamber as shown in

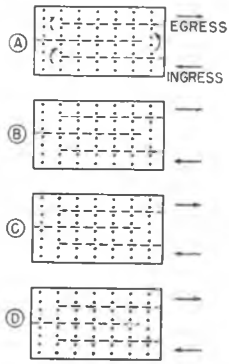


FIG. 7

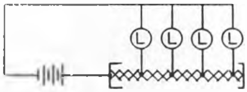


FIG. 8

Fig. 7(a) and the lamps mounted so that the partitioned chamber was clearly outlined, the water would close the circuit on the affected lamps as it rushed into the chamber. The schematic of the circuit used is shown in Fig. 8. A motion-picture camera taking 16 pictures per second photographed the progressive action. In Fig. 7(b) the water flow has started and two air pockets are noted. In 7(c), other pockets have developed and some eliminated, while in 7(d) the water has started to flow from the egress. These films were analyzed in a film reader, picture-by-picture. Master sheets were prepared prior to the reading for easy transcription. From the data obtained, the water position could be plotted against elapsed time.

Cavitation studies have always presented many problems from the standpoint of ship's hull and propeller design. In working on this project, a number of different techniques have been employed. In cavitation, high-speed motion pictures, high-speed still pictures and high-speed sequence have been used extensively.

Cavitation and the presence of gas bubbles in liquids or vapor in air require similar treatment. Schlieren photography has been used as well as shadowgraphy the best to portray these phenomena. The ebullition of air in a propeller wake and the formation of vapor trails by aircraft are typical of these reactions.

One of the finest examples of this type of work has been done by Werner Kraus of the Bayer Chemical Works in Germany. He employed color schlieren to study mixing and stirring processes. He used an a-c arc lamp as a source with a prism for creating the spectrum. The schlieren slit was placed at the principal focus of the objective lens. The mirror scheme shown in Fig. 3(c) was applied. This method is known as the Loepler process and was pushed to its present valuable state by Dr. Hubert Schardin. He used both still cameras and motion-picture cameras to get his pictures. In a note from him, he points out that they were interested in qualitative results only, but that J. G. Van de Vesse, T. H. Delft, Netherlands, has been using this method for quantitative measurements.

The Naval Ordnance Laboratory, at Silver Spring, Maryland, has been using the method shown in Fig. 6(a) for studying the cavitation formed by missiles falling into water and other liquids. The missiles are of different shapes and enter the water at varying velocities. The rotating prism high speed cameras were used to photograph these events.

The David Taylor Model Basin has been studying propeller and hull design in towing tanks and water tunnels. High-speed micro-flash (electronic flash) is used to get the tip action where the cavitation action begins. High-speed motion pictures are made of comparatively narrow fields (up to five feet) and normal-speed motion pictures are made of the gross effects on the full run through the towing tank. High-speed motion pictures and micro-flash pictures are made by looking through windows into the water tunnel.

One word of caution with respect to looking into windows;—plastic windows will cause more trouble than glass. A barium crown or "Aquaplate" glass can best be used for windows. This glass is clear and not tinted green as is ordinary plate glass. The external surface (air side) should be coated by the fluoride, if possible. Condensation can also be very troublesome and a double window with a dehydrating agent such as silica gel helps in overcoming this problem.

As one is using a camera looking into water, the focusing scale is modified because of the difference in index of refraction between air and water. The scale distance is multiplied by a factor of .75. If the fixed focus on a camera is normally 24 feet in air, the fixed focus becomes 18 feet in water.

One window should be kept clear for the camera and other ports used for the lights. The ports for the lights should be kept as near as possible to the optical axis of the lens. Highly concentrated spots are desirable for penetrating the water. Among these lamps are:

(a) PH750R
(b) 300 watt
(c) #4560 Airplane landing lamp
(d) General Radio "Microflash" unit

The California Institute of Technology has used the Edgerton electronic-flash unit up to 20,000 flashes per second for cavitation studies. In order to secure the high rate, several power supplies were used in tandem. A continuously moving film was used and hence with a synchronizer on the camera, ribbon-like pictures were obtained.

Other typical problems which have been extensively studied are:

The Waterways Experiment Station at Vicksburg, Tenn., has been making motion studies of water flow by normal- and high-speed photography. The flow of water over dams and levees, through tunnels and through water courses, such as the Mississippi River Basin model, are recorded.

Dr. Paul Fye, now of the Naval Ordnance Laboratory and formerly of Woods Hole Oceanographic Institute, has been photographing under-water explosions. In the earlier pictures, high-

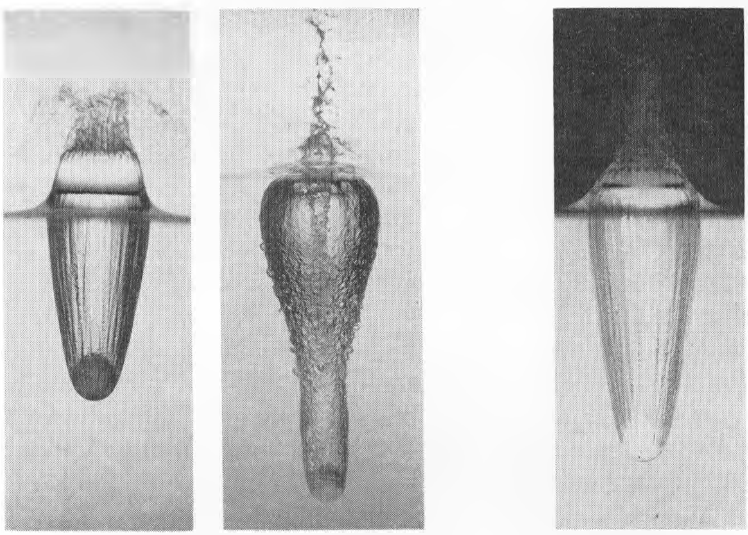


FIG. 9. AIR-WATER ENTRY OF A SPHERE, COURTESY OF NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

speed rotating-prism cameras were lowered in water-tight bells to where the test discharge was to take place. Jigged onto the bell was the explosive and also photoflash lamps. The sequence of operation was as follows:

1. The camera was started.
2. When the camera was at full speed, the photoflash lamp (G.E. #50 or equivalent) was ignited.

3. As the photoflash lamp was approaching its peak, the charge was fired.
4. The camera was stopped with a limiting switch when "the fireworks were over."

A weighted white sheet was generally suspended beyond the charge to get better contrast.

(Note: By means of a high-speed oscillographic camera placed on board the ship and connected to a transducer, simultaneous recordings of pressure and the mechanical effects of the explosion could be made. In order to validate the readings, zero time should be placed on both camera films. The zero time could be initiated from another pair of contacts on the firing switch. A 1,000-cycle oscillator with sufficient power to drive two neon or argon lamps (¼ watt) would furnish "the pips" for timing. When a camera is kept in the dark, the timing lights may not fire. In order to overcome this, a 28-volt d-c bias which is on all of the time will permit the lamps to fire when the 1,000-cycle "pips" come through the circuit.)

Fye later submerged a modified Bowen-Knapp camera to 12,000 feet to secure a sequential series of underwater explosions.

With a continuous camera, the film will be running in a vertical plane or along the "Y" axis. Therefore, the input to the cathode-ray oscilloscope will be on the "X" axis only. (On older oscilloscopes, the tube was rotated 90° and the high gain "Y" amplifier was used. Newer scopes have equal gain "X" and "Y" amplifiers.) A P11 coating on the scope tube with good accelerating voltage will give good records.

The effects of water currents on the ocean bed have been studied by Dr. Ewing and later Dr. Harold Edgerton. Ewing used an incandescent source and Edgerton has been using both incandescent and electronic flash. Rubicoff has been working off southern France with incandescent lights and electronic flash. Edgerton has been working at depths up to 16,000 feet and there are plans to go even deeper.

In 1943, the question arose as to what happened to a torpedo when it became entangled in a net. As an exploratory test, a series of high-speed motion pictures was made at Silver Springs, Florida, in daylight. At that time only Eastman Super XX film was available. At depths down to 10 feet, utilizing the white sandy bottom as a reflector, full exposures were obtained at 1,000 pictures per second with a rotating-prism camera. A fair exposure was obtained at 4,000 pictures per second. With the fast films available today, 4,000 pictures per second can be obtained in sunlight.

Later Chesterman of the Royal Naval Scientific Service made

FIG. 10. CAVITATION PHOTOGRAPHY BY ELECTRONIC FLASH, COURTESY OF THE ORDNANCE RESEARCH LABORATORY, STATE COLLEGE, PENNSYLVANIA

beautiful pictures of propeller cavitation and torpedo ejection from the submarine's torpedo tube. These same types of pictures have been made by the United States Navy.

At the Morris Dam installation, the Navy has been studying the effect of impact of torpedoes as they strike the water. The velocities of the torpedo in air would correspond to that of a destroyer- or aircraft-launched torpedo. Motion pictures, both normal- and high-speed, document the event.

The photographic-instrumentation field is expanding rapidly. Cameras are being designed to occupy minimum space and yet be extremely rugged. Auxiliary equipment is being designed to work with the cameras effectively and efficiently. New light sources are being developed. Films are being made that have greater speed and finer grain. Color films are coming onto the market which may equal the speed of black-and-white film. New film bases are being developed which will allow operation of equipment at 65°F.

There is one criticism to be offered, however. The present equipment that is available for the actual analysis of the films is the weakest link of the chain in all these studies.

Motion pictures can be observed qualitatively by projecting them in a standard projector. One manufacturer has placed a hand crank and a frame counter on the projector which permits the observer to advance the pictures one-by-one. They, however, did not calibrate the focal length of the lens. With the tolerances of manufacture of the lenses, $\pm$ 4% error can be introduced unless there is a scale in the original picture. This lens should be replaced by a calibrated photographic objective—not a projection lens.

For frame-by-frame analysis, film readers are available but these are sometimes awkward to use. The magnification factor may be some odd number such as 17 or 23 times. These readers were designed primarily for microfilm transcription.

One instrument company has designed a 10-times magnification reader for 16mm film. Movable cross-hairs for vertical and horizontal measurements are read on direct reading dials to the nearest .001 inch. The film is observed on a translucent screen.

There is a gap from these to the very elaborate laboratory units which cost from $15,000 upward.

In conclusion, photography is playing a very important role in the recording of the transient events that occur in fluid flow. As is indicated by the papers that are presented before the hydraulics conferences, many phases of photography are used to illustrate practically every paper.

It will be to everyone's advantage to contact various manufacturers with his problems. This allows the manufacturer to design equipment which is built for the task at hand and not "jury-rigged". The future needs are not modification of existing equipment, but engineering requirements for design.

USE OF RADIOISOTOPES IN HYDRAULIC STUDIES

by

Oscar M. Bizzell

Chief, Technical Developments Branch
Isotopes Division, USAEC

Radioactive tracers provide tools long needed by hydraulic engineers. Their powers are unaffected by changing chemical and physical conditions and they can be detected in very low concentrations, much lower than is possible with dye or salt. Radiotracers give a more accurate picture of actual flow-through curves of basins and conduits by bringing out the longer tails of the build-up and fall-off curves which go undetected by other means [1]. Also, density effects from radioisotopes are vanishingly small as only micro-amounts of the material are required. In most cases radiotracers may be followed by taking readings through the pipe or tank wall, thus leaving the flow pattern undisturbed. The following discussion alludes to hydraulics in its broadest sense, that is, the general behavior of all liquids. Although the examples cited deal with many different liquids, the basic principles can usually be applied to problems in water measurement.

Principles of Use

It would be difficult to conceive of a more sensitive or versatile tool than the radioactive atom. The most familiar use of radioisotopes is as sources of radiation. Here the type of emitted radiation, its energy and the half-life are the principal considerations, and generally the right combination of the three can be found to meet most needs.

The second principal use of radioisotopes is as tracer atoms. The radiation from radioisotopes gives investigators an extremely sensitive means of detecting their presence and hence their movement through physical or physical-chemical transfer, or in a chemical reaction. Here the chemical form of the radiomaterial as well as its radiation and half-life determine its usefulness. Since radioisotopes of most of the elements are available, a suitable tracer can be found for most purposes.

The fundamental principles involved may be reduced to three major types or modes of use as shown in Fig. 1.

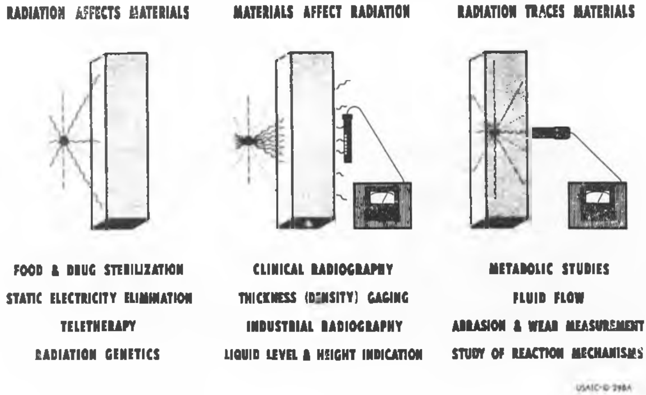


FIG. 1. BASIC PRINCIPLES OF RADIOISOTOPE UTILIZATION

Effect of Radiation on Materials

In the first type of use the radioisotope is used simply as a fixed source of radiation much as radium and X-ray machines are used. The ability of radiation to alter a material is indeed important in many ways. This type of use, however, does not lend itself to a testing procedure and will not be discussed further.

Effects of Materials on Radiation

In the second type of use (Fig. 1) the effect of the target material on the radiation furnishes information about the material. Here the application is based on detecting or measuring the radiation which penetrates or is reflected from the material. This presents an ideal setup for a testing procedure, especially since the amount of radiomaterial used is so small that the radiation does not alter the material under test. This type of use is the basis for most of the testing procedures now utilizing radioisotopes.

Tracing Materials with Radiation

In the third type of use (Fig. 1) the radioisotope serves as a tracer to follow the complicated course of material in bulk or the individual batches of atoms in chemical or biological reactions. The thing labeled and traced may be water running through a pipe, sugar being utilized in a human being, a raw product for milk production in a cow's body, or an atom transferring from one kind of molecule to another in a chemical reaction.

SIMPLICITY OF USE

These modes of use illustrate the simplicity of the principles involved. The basic requirements are a radiation emitter and a radiation detector. Lest this be an over-simplification, however, it should be emphasized that "emitter" and "detector" represent concepts which are far from simple. Even so, the materials and equipment needed for most applications, such as specially prepared sources, radiolabeled organic and inorganic compounds, shielding, and electronic circuitry, are readily available from commercial suppliers in as convenient forms as the manufacturer can make them. Further, the equipment and attendent services are continually being improved.

SENSITIVITY AND SPECIFICITY OF RADIOISOTOPES

Radioisotopes permit materials to be traced in minute quantities —a millionth to a hundred-millionth of the amount detectable by other means. It is easy to detect radiation from isotopes diluted with a billion or ten-billion times as much non-radioactive material. Some isotopes are detectable after dilutions of more than a trillion.

Even more important than sensitivity is the specificity of radioactive tracer atoms. They can label a specific batch of atoms and enable it to be traced through a series of chemical or physical processes. This permits the sorting out or untangling of complicated processes which can be followed in no other way.

The sensitivity of radioisotope detection, the specificity of the tracer method, and the unique radiation characteristics of individual radioactive species permit radioisotopes to be used as powerful analytical tools in at least three major ways. These may be referred to as "tracer analysis", "isotope dilution analysis", and "activation analysis". The first two analytical techniques, as applied to hydraulic problems, are comparable in many ways to the salt-velocity method and the salt-dilution method, respectively.

EXAMPLES OF APPLICATIONS

Radioisotopes have been used to determine flow rate, volume, flow pattern, efficiency of separation, thoroughness of mixing, leakage, and other associated hydraulic problems. These determinations can often be made quite easily where other methods fail.

In many hydraulic applications of radioisotopes, a sharply defined peak in the counting rate must be obtained. It is essential, therefore, that the radioisotopes be injected quickly, in as small a volume as practicable and without introducing air into the system. The radioisotope injector, shown in Fig. 2, illustrates one type of commercially available apparatus that meets these requirements [2]. The injector uses small CO_2 cartridges to inject the tracer and flushing liquid into the pipe in less than one second. In one particular use, 18 cc of tracer solutions and 36 cc of flushing solution are placed in a capsule in a radio-chemical laboratory. At the pipe-line site, the operator places the loaded capsule and a CO_2 cartridge into the injector housing on the pipe. By operating a trigger handle to release the CO_2 gas, the tracer may then be injected when needed.

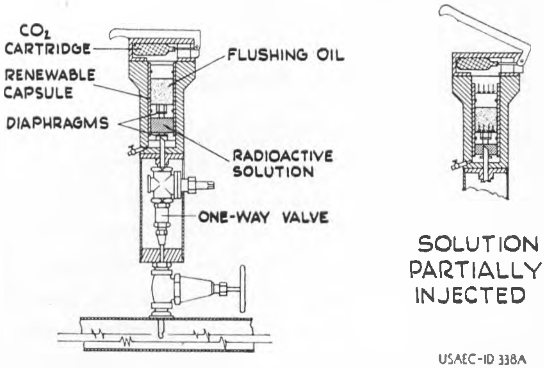


FIG. 2. RADIOISOTOPE INJECTOR FOR PIPELINE FLOW PROBLEMS

Measuring Flow Rates

Two point method: The general method, illustrated in Fig. 3, for measuring liquid- or gas-flow rate is timing a surge of tracer between two points separated by a determinable volume [3]. This is most easily done on a straight section of pipe of known dimension, free from branch connections. The radioisotope is injected quickly, close to the point where it will be timed, in order to obtain a sharply defined peak in the counting rate as the tracer passes the counter. The counters at the two points are connected to a single amplifier so that they both record on the same chart. The flow rate is equal to the volume between the two counters divided by the time between the peaks.

Integrated count method: Radioisotopes can also be used to measure flow rate by recording integral counts [4]. This method, illustrated in Fig. 4, uses only one detector and eliminates the need for determining pipe volume. It is based on the principle that the total number of gamma rays registered by a radiation detector on

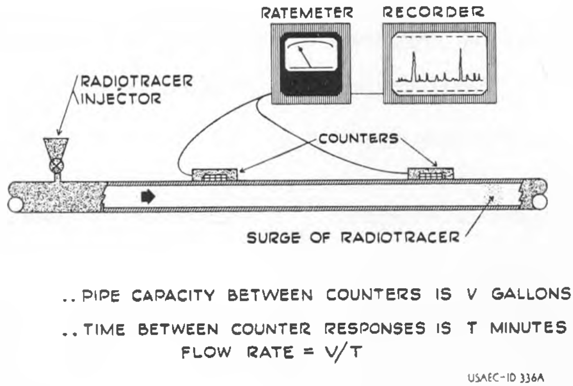


FIG. 3. DETERMINING FLOW RATE BY PEAK TIMING USING RADIOACTIVE TRACER

a pipe passing a definite quantity of radioisotopes is inversely proportional to the flow velocity. Fewer counts are recorded when the isotope passes rapidly. The integrated count is independent of the variation in isotope concentration along the stream, as long as the flow rate is constant. The validity of the method is due to the sensitivity of the radioisotope method which permits measuring the longer tails of the build-up and fade-out curves that are undetectable with other tracers. To translate the total counts to an absolute determination of flow rate, it is necessary to calibrate the counting setup on a particular type and size of pipe involved. This is done by filling a cut section of pipe with a radiotracer solution at known concentration and measuring the counting rate under the same conditions as in field measurements. The counting rate depends on the concentration and may be expressed as:

$$\text{Counts/second} = \text{calibration constant} \times \text{millicuries/gallon}$$

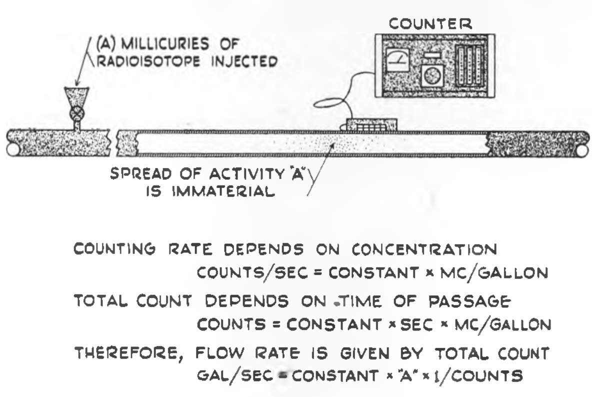


FIG. 4. MEASURING FLOW RATE BY INTEGRATED COUNT USING RADIOISOTOPE TRACER

The total count from the passing liquid depends on the time of passage and may be expressed by rearranging this equation as:

$$\text{Counts} = \text{constant} \times \text{sec} \times \text{millicuries/gallon}$$

Therefore, the flow rate is given by the total count and may be expressed as:

$$\text{gallons/sec} = \text{constant} \times \text{millicuries} \times 1/\text{counts}$$

Measuring Liquid Volume

Measurement of volume in a closed system under dynamic conditions may present difficult problems. However, radioisotopes readily lend themselves to these determinations in several different ways. They can be used both as external sources of radiation for liquid-level gaging and as ratiotracers for calculating individual volumes in multiphase systems. In the latter case a mixing action either by external circulation or by internal stirring is necessary.

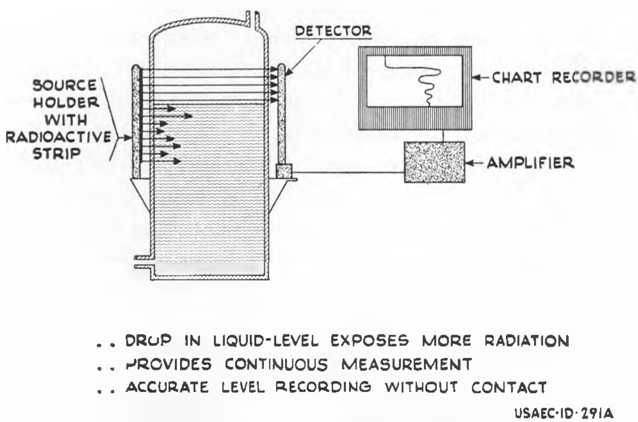


FIG. 5. LIQUID LEVEL GAGE, AN INDUSTRIAL USE OF RADIOISOTOPES

Liquid level gages, illustrated in Fig. 5, are perhaps the most common use of radioactive sources in hydraulic problems. As an example, the gamma radiation from radiocobalt has been used to determine the height or volume of molten metal in a cupola. The source is mounted on one side of the cupola and the radiation detector on the other side. By noting the intensity of the measured radiation, it can be easily determined whether the height of the material inside the cupola is above or below the height at which the source and the detector are set. The same technique has also been used to determine, under dynamic conditions, the height of materials in high-pressure, high-temperature autoclaves and various other fluid systems. The primary advantage of this type of measurement is that it eliminates the effects of high temperature, high pressure, and corrosion on the gage.

Radiotracer dilution method: Another radioisotope method for determining volumes within closed systems is similar to the salt-dilution method well known to hydraulic engineers. Volume may be found by adding a known quantity of radioisotope to an unknown quantity of liquid. Samples are taken from the unknown volume after a steady state has been reached. By comparing these with standards prepared by dilution, total volume can be readily determined. This method, known as the isotope-dilution technique, is widely used by radiochemists and physicists. Due to the sensitivity of the radioisotope method a very small quantity of radioactivity can be used to determine accurately and economically the volume of large amounts of liquid.

Exponential method: Another radiotracer method is used to determine volumes within tanks or systems through which there is a known constant flow [4]. A radiotracer is put into the incoming line and complete mixing of the incoming stream with the vessel contents is assumed. Mathematically the tracer concentration in the tank falls off exponentially with a rate determined by throughput R and volume V as follows:

$$\frac{d \ln C}{dt} = \frac{R}{V}$$

where C is the counting rate. The left-hand member is simply the slope of the straight-line plot of counting rate vs. time on semilog paper. The volume is therefore found from

$$V = R/\text{slope}$$

Circulating-loop method: Volume of a circulating-loop system can be determined by a variation of the method for determining flow rate by peak timing, described above [4]. Here, however, the flow rates must be known in order to determine the total volume in the system. Also, it is essential that mixing be slow compared to circulation rate. A concentrated slug of radioactivity, quickly injected, is observed in repeated cycles. Time between successive tracer-peak appearances, multiplied by circulation rate, gives the total volume of the circulating liquid.

Tracing Flow Patterns

Control of liquid levels: In oil-well operations any means of obtaining information regarding conditions in the bore-hole has considerable economical significance. As an example, the rate of oil production depends to a large extent on the porosity of the formation. This porosity is reduced with age as particles of limestone are deposited in the interstices. When production drops to an uneconomical level, the well can be treated with hydrochloric acid under pressure to dissolve some of the carbonate and reestablish the flow.

If a radiotracer is added to the acid, as illustrated in Fig. 6, the operator may determine the depth of the acid level without

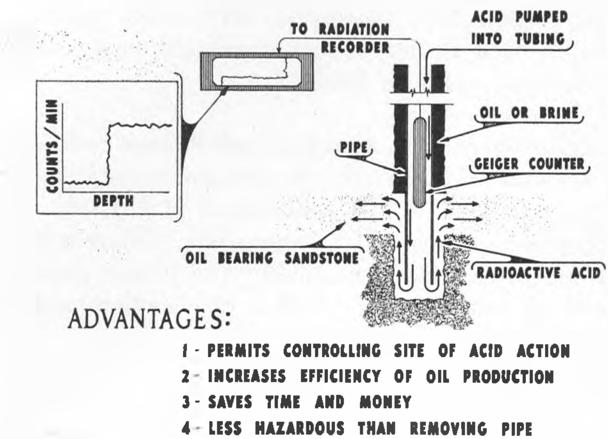


FIG. 6. RADIOACTIVE ISOTOPES FOR CONTROL OF OIL-WELL ACIDIZING

removing the sections of 2-inch pipe through which the acid has been pumped into the well. Once the acid has reached the level at which the radiation detector is suspended (at the formation to be treated), a signal indicates to the operator that pressure should be applied to force the acid through the oil-bearing strata.

Water flooding operations: In another oil-field use involving water flooding, radiotracers are injected into water-input wells and, after underground migration, are measured at surrounding oil-production wells [6]. These measurements have led to successful determination of relative rates and patterns of flow of injected water between water-input and oil-production wells and detection of zones of excessive water entry into oil-production wells.

Measuring Efficiency of Separation

Radioisotopes may be used to measure the efficiency of separation in complex systems [4]. For example, an admixture of products A plus B may be separated in an evaporator, as illustrated in Fig. 7. Radioisotope tracers for either A or B are injected into the evaporator feed line. To check for efficiency of separation a counter is attached to the pipe carrying the untagged separated product and the presence of any radioactivity is measured. Another counter attached to the pipe carrying the tagged products shows the radioactivity in that stream. By integrating the counts on both streams and making corrections for the different flow rates, a quantitative measure of the amount of cross-contamination of the products may be obtained.

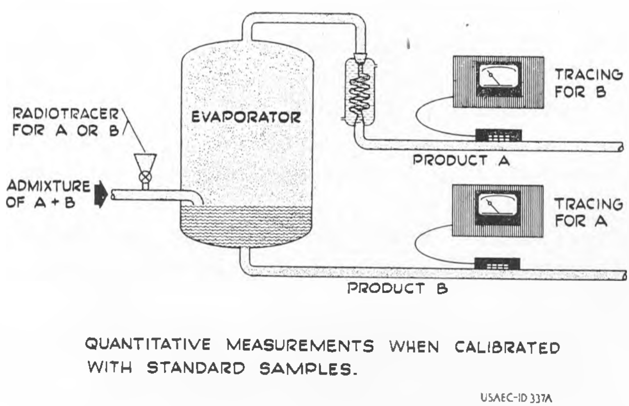
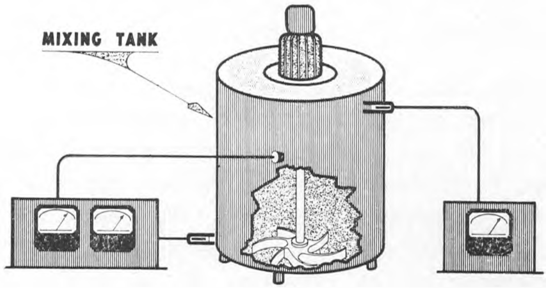


FIG. 7. MEASURING EFFICIENCY OF SEPARATION USING RADIOACTIVE TRACER

Determining Uniformity of Mixing

Another possible application of the radioactive tracer technique is measuring uniformity of mixing. When large volumes of liquids are involved, sampling from all parts of the mixer may present many difficulties. By fitting radiation detectors in strategic places around the mixing tank and incorporating a short-lived gamma-ray-emitting radioisotope into one of the constituents, the concentration of that constituent at the various places can be compared. With this information, time of mixing necessary for a desired degree of uniformity can be determined. A simplified diagram of this possible method is illustrated in Fig. 8. The method could be readily used in either a continuous or a batch process.



ADVANTAGES:

1 - UNIFORMITY OF MIXING EASILY ASSURED

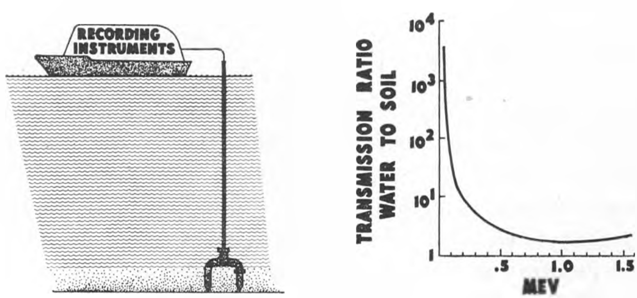
USAEC-ID 34A 2 - EXCESSIVE MIXING TIME ELIMINATED

FIG. 8. RADIOACTIVE ISOTOPES FOR DETERMINING THOROUGHNESS OF MIXING

Measuring Silt Density in Water

Measuring silt density in large bodies of water often involves major difficulties in access and sampling. Such measurements have been considerably simplified, however, by an interesting application of the radioisotope technique. A device for this particular application was developed jointly by the AEC Isotopes Division and the TVA [6].

Even the "softest" rays from available gamma-ray emitters penetrated water and water-saturated silt with nearly equal ease and thus did not distinguish between them. However, the easily absorbed X-rays, or bremsstrahlung, produced in a secondary process by beta-emitting isotopes, were found to give a satisfactory attenuation ratio between water and silt.



ADVANTAGES: 1—MORE ACCURATE

2—MEASUREMENTS EASILY AND QUICKLY MADE

USAEC-ID 227A 3—ELIMINATES COLLECTING SAMPLES

FIG. 9. BREMSSTRAHLUNG GAUGE FOR MEASURING SILT DENSITY

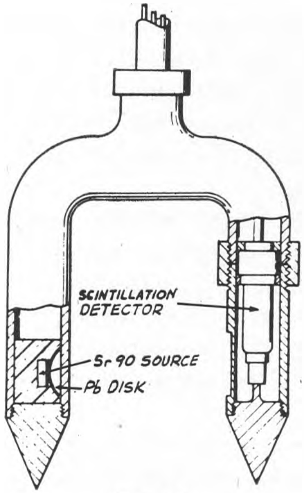


FIG. 10

The device which evolved, shown in use in Fig. 9 and detailed in Fig. 10, is a 2-pronged probe with a 20-millicurie Strontium 90 source in one prong and a scintillation counter 12 inches away in the other. Beta particles from the source produce the desired X-rays as they penetrate a lead disk. The counter is connected, through a long cable, to a counting-rate meter in the operator's boat or shore station.

The general principle and approach revealed in this application may easily be applied to many other hydraulic problems.

Leak Testing

Leaks of any liquid from one channel to another can be readily detected with radiotracers and, under the proper conditions, can be measured quantitatively. This technique has been widely used in detecting and measuring leaks in heat exchangers and other similar closed-liquid systems.

Perhaps the simplest application is the use of radioiodine or radiosodium in detecting leaks in buried pipes. In this case, a small quantity of radioisotope is introduced into the pipeline and followed by means of its radiation until it ceases to travel on leaving the otherwise closed system. In a number of instances this test has made it possible to find and repair leaks in a house or other building with a minimum disruption of the structure. Furthermore, the test gives quick and reliable results where other techniques often fail. A personal experience with a leak location problem illustrates the simplicity of the technique. A puzzling leak developed in a water pipe in an Oak Ridge church just prior to Christmas. The leaking pipe, beneath five inches of concrete, was spilling three gallons of water per minute into the foundation of the building. Cancellation of several Church activities in the building appeared imminent.

Before resorting to more drastic leak-location procedures, it was decided to try the radiotracer technique. A ten-gallon bucket of water, as illustrated in Fig. 11, was mounted on a ladder, and connected to an outside faucet through a garden hose. After closing the system at the water main, 2½ millicuries of Iodine 131 solution were injected into the hose from a lead shielded syringe.

The only flow was thus from the elevated bucket to the leak and the radioiodine was, of course, carried with it. The radiation was easily detected and followed above the concrete floor until it came to a stop in the men's lavatory. After the water pipe was flushed to dispel any remaining radioactivity, a hole was cut into the floor at the indicated spot. The leak was found directly beneath it. The entire job cost less than $10, whereas an estimated $1000 to $1500 would have been required by conventional methods.

POSSIBLE FUTURE USES OF RADIOISOTOPES

Recent developments in low-level radiation counting (counting just above the natural or background radiation level) hold considerable promise for a large number of additional applications. Using these very sensitive methods, tracer tests can be carried out during actual processing in a plant, and so little radiomaterial need be used that the products reaching the general public would be completely safe.

Special electronic circuits that cancel out the natural radiation background permit an easy measurement of harmless levels of radioactivity. Samples for measurement are usually placed directly inside the counter or the scintillation fluid.

Nature has herself shown how useful radioisotopes can be. For example, it is possible to distinguish recently living wood from wood long dead by virtue of radiocarbon produced through the action of cosmic rays on nitrogen in the air. Growing things absorb and use the radioisotope along with normal carbon. At death, absorption stops. Using the decay of these naturally occurring radioisotopes as a timing device, the age of archeological relics can be quite accurately determined. In a similar way, rain water

FIG. 11a. TO LOCATE A PUZZLING LEAK BENEATH THE CONCRETE FLOOR IN KERN METHODIST CHURCH, OSCAR BIZZELL INJECTS 2½ MILLICURIES OF IODINE 131 FROM A LEAD SHIELDED SYRINGE INTO A GARDEN HOSE SERVING AS A PRESSURE HEAD. JAMES HITCH MONITORS THE OPERATION WITH A "CUTIE PIE" WHILE REV. G. WILSON ELLIOTT WATCHES.

FIG. 11b. HITCH WITH A SCINTILLATION COUNTER AND BIZZELL WITH A GEIGER COUNTER TRACK THE RADIOACTIVE SIGNAL ACROSS THE CONCRETE FLOOR. THE MORE SENSITIVE SCINTILLATION COUNTER WAS FOUND TO BE BEST SUITED FOR THIS TYPE OF MEASUREMENT.

FIG. 11c. AFTER TRACKING THE RADIATION THROUGH SEVERAL ROOMS AND AROUND BENDS IN THE PIPE, THE SIGNAL CAME TO A STOP IN A LAVATORY. A BULLS EYE ON THE FLOOR INDICATES THE SPOT ABOVE THE LEAK.

can be distinguished from ground water by the radioactivity of naturally occurring radiohydrogen, or tritium, also formed by cosmic radiation.

Many progressive engineers are already using radioisotopes to solve a multitude of difficult problems. Recent developments in low-level counting techniques now make it possible to extend the use of radioactive tracers to many new problems in which they were previously considered too hazardous because of the amounts required. There is no doubt but that radioisotopes hold the key to many present-day hydraulic problems.

DISCUSSION

The discussion was initiated by Mr. Bauer who was interested in knowing whether the dilution technique, in which a known quantity of radioactive fluid is used, could be applied to measure discharge in rivers. Mr. Bizzell thought that the dilution would be so great for such a tremendously large volume of water that the method would be inaccurate, or that so great a quantity would be required that the cost would be prohibitive. The method should be restricted to pipe lines, basins, etc., or to determine dilution in a river within a fairly short distance downstream. For example, he continued, at Oak Ridge, the natural waste from the Oak Ridge National Laboratory reacter is held for a "cooling" period in a settling basin and then dumped into the Clinch River. It is extremely difficult to find any trace of radioactivity in the waste even a short distance downstream.

Mr. McLean then remarked that a few years ago he had considered using radioactive isotopes in connection with some pump tests, but that at that time these isotopes were not available. He inquired whether such isotopes are available now and whether they could be used in an 84-inch pipe in which water for cooling condensers is being recirculated. In reply, Mr. Bizzell first called attention to the fact that one must consider whether or not the water is potable. The National Committee on Radiation Protection has rather stringent requirements about how much added radioactivity can be present. This is detailed in National Bureau of Standards Book 52, which lists the amount of various isotopes. Some radioactive isotopes are relatively non-hazardous, such as

Fig. 11d. After the leak is repaired, water samples are analyzed to assure that the radioiodine is below the maximum permissible concentration for drinking water. Maximum permissible concentrations of radioisotopes in water are published in National Bureau of Standards Handbook No. 52, available from the U. S. Government Printing Office, Washington, 25, D.C., price 20 cents.

sodium 24 which is widely dispersed in the body and eliminated quite rapidly, whereas iodine 131 which goes straight to the thyroid gland and tends to stay there, is relatively hazardous. One could tolerate about 1000 times as much sodium 24 as iodine 131 in potable water. For Mr. McLean's application he suggested the use of sodium 24. Although radioactive isotopes are now available from the Brookhaven and Argonne laboratories, they are primarily available as processed materials from the Oak Ridge National Laboratory which can furnish over 100 different radioactive isotopes. He stated, further, that Oak Ridge has the function to license radioactive-isotopes for non-agency uses. The primary criteria for licensing are the experience of the prospective user in handling the materials and the safety factors involved. The materials can be readily obtained if assurance is given that the operator is sufficiently well-trained so that he will avoid accidental exposure, and that the precautions are such that none would be harmed by the radioactive isotopes. The materials are quite inexpensive. The material used to trace the leak was $2.25 worth of radioactive iodine.

Mr. McLean then asked whether the trail-out part of the curve is obtained as the radioactive concentration is increased. Mr. Bizzell's reply was in the affirmative, that the dilution technique with radioactive-isotopes is more sensitive than with salt.

Mr. Bizzell then thought it would be of interest to mention that there have been recent developments in techniques of low-level counting, on which Dr. Libby, a member of the Atomic Energy Commission, has done considerable work. This opens up a tremendous field of application where a safe level of radioactive isotope is still measurable.

In conclusion, Mr. Bizzell mentioned some additional applications, such as the demarcation of the interfaces of oils in pipelines. It is possible, without actually injecting a radioactive tracer in a pipeline, to determine the location of an interface with great accuracy by using a radioactive source outside of the pipe and measuring the radiation reflected from the interface.

BIBLIOGRAPHY

1. Archibald, Ralph S., "Radioactive Tracers in Flow Tests," *Journal of the Boston Society of Civil Engineers*, Vol. 37, 49-116, 1950.

2. Kohl, Jerome and Newacheck, R. L., "Isotopes and Interfaces," *Petroleum Engineer*, Vol. 26, March 1954.

3. Metcalf, G. H., U. S. Patent 2,631,242 (1953).

4. Hull, D. E., "Using Tracers in Refinery Control," *Nucleonics*, Vol. 13, No. 4, April 1955.

5. Watkins, J. W., and Mardock, E. S., "Use of Radioactive Iodine as a Tracer in Water Flooding Operations," *Journal of Petroleum Technology*, Vol. 6, 117-124, September 1954.

6. Morgan, G. W., Hitch, J. W., Bizzell, O. M. and Manov, G. G., U. S. patent application filed.

MEASUREMENT OF MICROPRESSURES IN LIQUIDS

by

PHILLIP EISENBERG,* JOHN P. CRAVEN,** AND JAMES A. LUISTRO**

INTRODUCTION

The measurement of very small static pressures (of the order of 0.001 inch of water) can be accomplished with instruments that are essentially uncomplicated. The measurement of pressures of such magnitude at *low* frequencies, i.e., from a fraction of a cycle per second to the order of 10 to 20 cycles per second, is, by comparison, a formidable problem. In this frequency range, it is generally not possible to take advantage of piezoelectric or magnetostrictive effects, for example, and the problem reduces to one of detecting the (very small) motion of a mechanical system deforming in response to the applied pressure.

For measurements in the interior of liquids, diaphragms of some type must usually be employed in a leak-proof container. The design of the instrument then resolves itself into consideration of essentially three elements: the diaphragm and its behavior as affected by the transducer system; the response of the fluid system in the container or housing and the feedback to the diaphragm; and, the transducer and recording system.

Early in World War II, the David Taylor Model Basin was called upon to make such measurements in support of the development of countermeasures against pressure mines. The pressure mine is actuated by the change in ambient pressure associated with the flow field about a ship as it passes in close proximity to the mine. Combined with magnetic and acoustic influences, the pressure signal required can be coded into the mine mechanism to make a particularly vicious and selective weapon which can single out for attack a ship of certain size. Some of the history of this type of mine warfare and examples of ship and mine characteristics are given in reference [1]. A typical example of the pressure "signature" recorded on the bottom of a channel for a ship model moving in shallow water is shown in Fig. 1.

Model studies were initiated in 1940 with two objectives: the determination of the characteristic pressure signatures of Naval and merchant vessels in order to establish safe speeds for transit

* Office of Naval Research.
** David Taylor Model Basin.

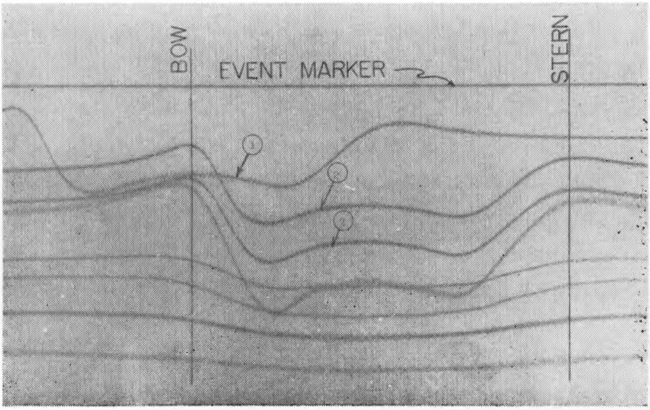


FIG. 1. TYPICAL PRESSURE SIGNATURE OF A SHIP MODEL IN SHALLOW WATER
1. CENTERLINE SIGNATURE OF A MODEL APPROACHING THE RANGE
2. CENTERLINE SIGNATURE OF A MODEL OVER THE RANGE
3. SIGNATURES ABEAM OF THE MODEL OVER THE RANGE

of minefields sown with mines of given firing rules; and, concurrently, the development of specific devices designed to produce pressure fields suitable for sweeping.

In addition to the need for measurements for the above application, data of this type were also required in the investigation of the forces associated with the movement of ships in shallow water and restricted channels. Thus, while the early work during World War II was carried out with temporary equipment, there existed a need of a sufficiently long range nature to lead to the installation early in 1944 of an essentially permanent pressure-measuring range.

It is the purpose of this paper to describe the equipment which comprises the present DTMB micropressure range and in this context discuss the treatment of the various problems mentioned above as encountered in developing this installation. Hence, not all possible methods of accomplishing the purpose will be discussed here but only those considered as practicable for the DTMB problem.

THE REQUIRED INSTRUMENT CHARACTERISTICS

In the installation described here, measurements of pressure changes were desired over an operating range of 2 inches of water with a resolution of 0.001 inch of water under ambient water heads of ½ to 10 feet. It was required that the response of the system be insensitive to the absolute magnitude of this ambient head, be essentially linear throughout the operating range of pressures, and be reasonably flat over a range of 0 to 20 cycles per second.

In addition to the requirements specific to the DTMB installation, other general requirements peculiar to systems placed under water for appreciable lengths of time are the following. The system must be insensitive to temperature changes; in the case of the DTMB installation, in which a closed gas system is used, particular care must be taken to eliminate such changes. Provision must be made for electrical stability of the transducer and recording system, and the gages must be designed to eliminate difficulties due to the presence of moisture. Watertight integrity must be maintained and, for systems exposed for long periods, noncorrodible materials are essential. Finally, for permanent installation of a range containing a large number of gages in a basin in which the water level is often changed for various experimental purposes, the system should be self-monitoring and self-adjusting to balance out the ambient head. This is required in order to protect diaphragms from damage resulting from excessive pressures and to provide an easy means of establishing the reference pressure level.* Consequently, such an installation requires a remote means of calibration.

Aside from the instrument and recording problems outlined in the foregoing, the measurement of such small pressures requires that special attention be given to the experimental environment, i.e., the facility in which the experiment is conducted and the method of gage installation. This will be discussed briefly before proceeding to the description of the DTMB micropressure range, itself.

Possible Errors Associated with the Experimental Environment

The facilities employed at DTMB for micropressure measurements are two rectangular towing basins. The larger one, in which the permanent range is installed, the "Shallow Basin," is 260 feet long by 52 feet wide with a water depth which may be varied from 0 to 10 feet; the smaller of the two, which is used with portable equipment, is 140 feet long by 10 feet wide with a water depth variable between 0 and 6 feet.

The possible sources of error associated with the characteristics of these facilities are: the modifications to the pressure field associated with flow about the gage housings, the limited distance available for the establishment of steady-state conditions, and modification of the flow field associated with wall proximity.

The difficulties resulting from flow about the housing may be

*As will be shown, thin diaphragms are required for large sensitivity, so that sufficient strength to withstand the maximum possible head may be incompatible with sensitivity requirements. Furthermore, the linear range of the system may be asymmetrical with respect to the zero position depending upon the operating head.

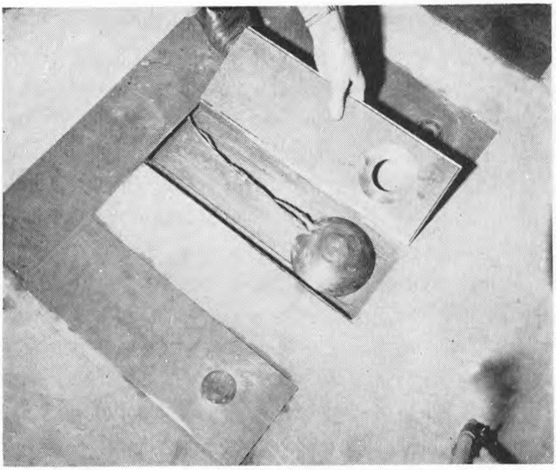


FIG. 2. INSTALLATION OF TRANSDUCERS IN THE DAVID TAYLOR MODEL BASIN SHALLOW BASIN

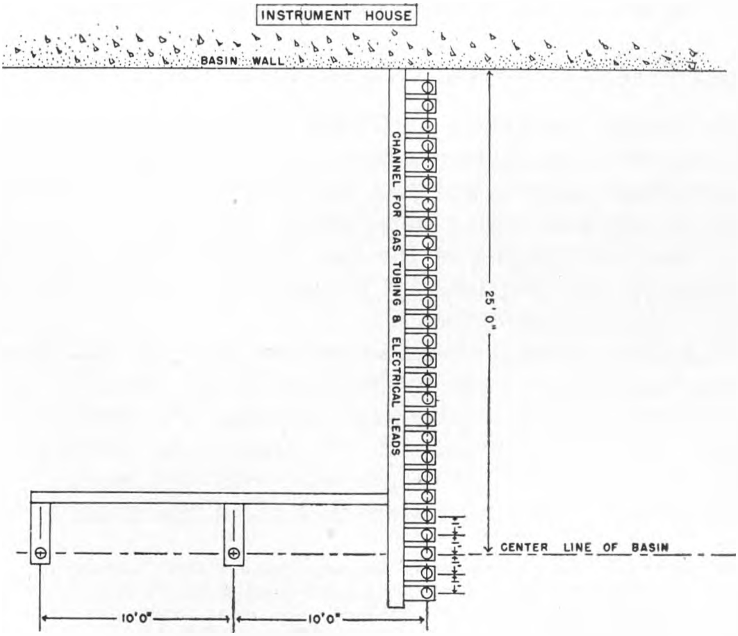


FIG. 3. LAYOUT OF THE MICROPRESSURE RANGE IN THE DTMB SHALLOW BASIN

eliminated by flush mounting where possible. In the DTMB installation, this is accomplished by placing the gages in channels cut into the basin floor. These gages are covered and held in place with heavy brass cover plates as shown in Fig. 2. The cover plates are of sufficient weight to prevent their displacement under the forces acting during the passage of a model. For use in channels in which it is not possible to arrange flush mounting, a number of gages have been designed with faired contours to minimize errors due to the local flow conditions.

While the model basins at the David Taylor Model Basin have hard (concrete) bottoms, and the water level is varied to obtain different depths, it may sometimes be necessary to use a false bottom to obtain depth variation, as was sometimes done in the earliest experiments relating to the mine problem. In this case, extreme care must be exercised in assuring rigidity of the bottom and in sealing the gaps between the false bottom and the basin walls. Under the loading produced by the pressure field of a model, deflections large enough to significantly alter the desired pressures may be produced if adequate stiffening is not provided. The presence of even very small gaps allows flow which again may alter the pressure field significantly.

The transducers are arranged on the basin floor in the manner shown in Fig. 3. Since three or more transducers are located on the basin centerline along the path of the models, comparison of the records at successive stations discloses whether steady-state conditions have been attained during a given run.

It is, of course, well known that wall corrections must often be applied to data taken in facilities of the type considered here, and it is not the intention to discuss such corrections in detail. However, it is well to point out that in measurements of the magnitudes considered here such corrections assume a much greater importance than in the types of measurements ordinarily made in model basins. For methods used in correcting pressure field data, reference may be made to a report by Borden [2] and references therein.

THE TRANSDUCER PROBLEM; TYPES SELECTED FOR THE DTMB MICROPRESSURE RANGE

When applied to an ideal pressure transducer, the pressure signal created by the model and the influences of the facility will be converted to an electrical signal having an output proportional to the applied pressure. Since appreciable distortion, attenuation and phase lag may result in liquid-filled instrument lines, it is desirable that the sensing element be located at the point of measurement. Hence, a diaphragm or bellows whose deflections are detected by electrical means was determined to be the most practical method for satisfying this requirement and that of flush mounting without excessive deflections. The design of the diaphragm then becomes a problem of finding a suitable compromise between the requirements for deflections that are easily detectable and high frequency response. While the latter is compatible with the need for small diameters (to obtain measurements representative of conditions at a point, i.e., diameters very small compared with the distance over which significant changes in pressure may occur), the requirement for large deflections is not. Although the actual design is influenced by the method used for detecting the deflections, these points may be illustrated by examining the diaphragm alone.

The deflection w_c at the center of a thin circular diaphragm or plate clamped rigidly around the circumference and loaded uniformly by an excess pressure Δ_p is given by [3]

$$w_c = \frac{3}{16}\frac{\Delta p}{E}(1 - \nu^2)\frac{a^4}{h^3}$$

where E is the modulus of elasticity and ν is Poisson's ratio for the material, a is the radius and h the thickness of the diaphragm.

The fundamental frequency ω_o of such a diaphragm is, approximately [4],

$$\omega_o = 10.21\,\frac{h}{a^2}\sqrt{\frac{E}{12\,\rho_m\,(1 - \nu^2)}}$$

where ρ_m is the mass density of the material plus the added mass associated with the motion of the ambient fluid. Thus, where high sensitivity (large deflection per unit of pressure change) is obtained by making a large and h small, just the opposite relation is required for high frequencies, i.e., h large and a small.

The first transducer system employed at the Taylor Model Basin was based on the change in the natural frequency associated with a variation of the inductance in an inductance-capacitance circuit. In particular, this system was designed to detect the motion of a circular copper diaphragm placed near an inductive coil [5]. The motion of the diaphragm under changing pressures induced a change in an oscillating signal which was measured as the deviation from the tuned frequency. Development of a suitable diaphragm and mounting method was a very difficult process. The diaphragm finally developed was of 0.004 inch rolled copper, one inch in diameter, soldered to a steel ring. However, only one diaphragm in four was satisfactory in spite of careful preparation and stress relieving; buckling due to thermal stresses and hardspots due to the rolling process or non-uniform stress relieving accounted for most of the failures. Although this system was used essentially as described in reference [5] throughout the war years of the program, the problems of diaphragm fabrication and limitations in linearity and stability of the circuits led in 1946 to examination of other systems.

Condenser type transducers were considered as an alternate solution; such a system had already been used successfully by the Admiralty Experiment Works in England [1, 6]*. Variation in capacitance due to displacement of the diaphragm acting as one plate of a condenser may be used for frequency or amplitude modulation of a sharply tuned signal, and very high sensitivity may be obtained. However, it was feared that difficulties would arise as a result of moisture accumulation in the gap between plates over long periods of immersion, non-linearity of output signal over the required pressure range, temperature drift, and the problem of the capacitance of long cables leading from the gage housing to the recording equipment. Since much experience had been gained at the Taylor Model Basin with resistance-wire strain gages by this time and because of the anticipated difficulties with condenser-type gages, the latter were abandoned for the present application. Subsequently, however, condenser-type gages were developed for other applications [7].

Attention was then turned to the application of resistance-wire strain gages for measurement of micropressures. This was motivated by the previous successful use of such gages in dynamometer and balance equipment and by the availability of a highly sensitive and stable carrier-type strain indicator [8]. The first attempts centered on the use of gages of the SR-4 type bonded to metal strips in various configurations to sense the movement of a diaphragm. It was soon apparent, however, that the restraints introduced by such methods were too large for attainment of the sensitivities required. Development was then undertaken of methods for mounting unbonded strain wires in suitable configurations. Shortly after the latter program began, the Statham Laboratories of Los Angeles, California, announced the development of a suc-

*Reference 6 is of additional interest since it describes what is evidently one of the first successful applications of the hot-wire for practical measurements of velocities in water.

cessful system employing unbonded wires, and early in 1948 it was decided to use Statham gages fabricated to TMB specifications.

In addition to the specifications for the gage housing required to satisfy requirements for long periods of immersion and for the diaphragm to attain the needed frequency response, the following specifications were laid down to insure the attainment of required sensitivity when used with the DTMB strain indicators:

"1. Each micropressure gage should have two active strain sensitive resistance elements arranged so that they form two adjacent arms of a Wheatstone bridge. The remaining two arms of the bridge are in the recording system and are inactive. If more than two active resistance elements are used, they should be wired in parallel so as to form effectively the two adjacent arms of a bridge. This will permit compensation for temperature variations.

"2. The resistance of one arm of the Wheatstone bridge consisting of one element or set of elements should be as near 120 ohms as is practicable.

"3. The strain sensitive resistance elements should be made from an alloy having a low temperature coefficient such as Ad-

FIG. 4. TOP VIEW OF STATHAM LABORATORIES P-18 PRESSURE TRANSDUCER FABRICATED FOR THE DTMB MICROPRESSURE RANGE

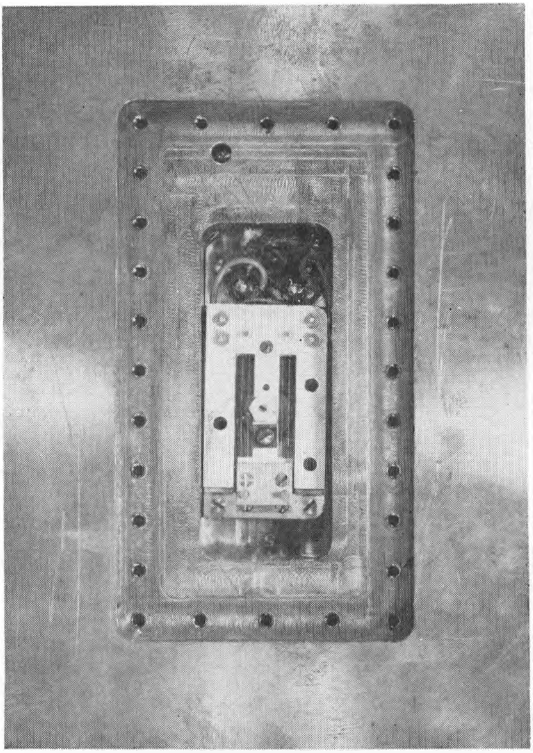


FIG. 5. BOTTOM VIEW OF PRESSURE TRANSDUCER WITH PROTECTIVE PLATES REMOVED TO EXPOSE THE UNBONDED STRAIN WIRES

vance or Constantan and should have a very stable strain-resistance coefficient.

"4. A change of resistance of not less than 50 micro-ohms per ohm for a pressure change of plus or minus 0.009 pounds per square inch is required for each of the two active strain-resistive resistance elements.

"5. The variations of the mechanical system due to the hysteresis, vibration, and thermal variations must not cause a variation in output voltage greater than 2% of the output voltage at a pressure of 0.009 pounds per square inch."

THE RESISTANCE-WIRE STRAIN-SENSITIVE TRANSDUCERS AND THE AMPLIFYING AND RECORDING EQUIPMENT

The transducers fabricated to the specifications outlined in the foregoing are illustrated in Figures 2 and 4. The corrugated diaphragm has an active diameter of 2 inches and is formed of tinned brass having a thickness of 0.0015 inch. The unbonded strain wires may be seen in Fig. 5 which is a photograph of the bottom of the transducer with the watertight cover plate removed. These strain wires are 0.003 inch in diameter with an electrical resistance of approximately 120 ohms per arm wired in a two-arm bridge circuit and activated through a mechanical linkage by movement of the diaphragm. The natural frequency of the diaphragm-linkage combination in water is approximately 23* cycles per second. However, the true resonant frequency of the entire system depends not only on the mechanical characteristics of the transducer itself but also on the response characteristics of the internal gas system. This problem and the measures which must be taken to insure the necessary behavior of the gas system are discussed in the appendix.

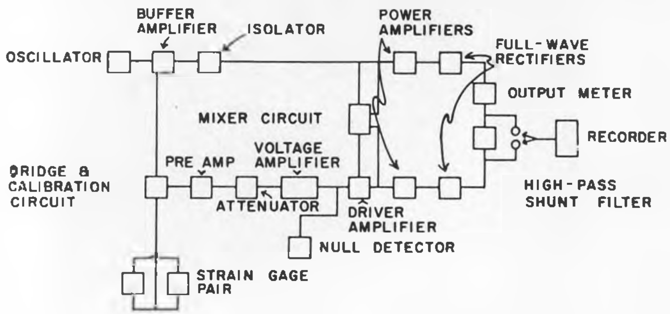


FIG. 6. BLOCK DIAGRAM OF THE DTMB TYPE 1-A STRAIN INDICATOR USED WITH THE MICROPRESSURE TRANSDUCERS

Since the characteristics and complete circuits of the DTMB strain indicator used with these transducers have been described elsewhere [8], only a brief description of the components will be given here. A block diagram of the amplifying and recording system is shown in Fig. 6. The carrier frequency is generated by a sinusoidal oscillator which drives the buffer amplifier; the latter furnishes power to excite the bridge and also delivers a voltage through the isolator, to the mixer circuit. The output or signal voltage from the bridge is passed by the preamplifier to the attenuator. The output voltage of the attenuator is further amplified by a fixed-gain amplifier. The amplified signal voltage is impressed on the null detector and also passes through the driver to the mixer circuit, which is a polarity discriminating demodulator. The composite output voltage from the mixer circuit drives two independent power amplifiers, which in turn operate into full-wave rectifiers and thence

* Employed more recently is Statham Model P18-A which has a diaphragm with an active diameter of about 2½″. The natural frequency of this model is 35 c.p.s.—the difference between models being due primarily to differences in the linkage systems. The characteristics of Model P18-A are used in the computations in the appendix.

through a high-pass filter and milliammeter to a multi-channel oscillograph.

A typical calibration of the pressure transducers at a single gain is shown in Fig. 7. Although the nominal operating range is ± 2 inches, the output is actually useable over a much wider range —being linear up to ± 8 inches of water. This is also the operating limit—stops being provided to prevent further movement of the diaphragm under greater pressures and, thus, to prevent damage to the diaphragm should there be malfunctioning of the internal gas system.

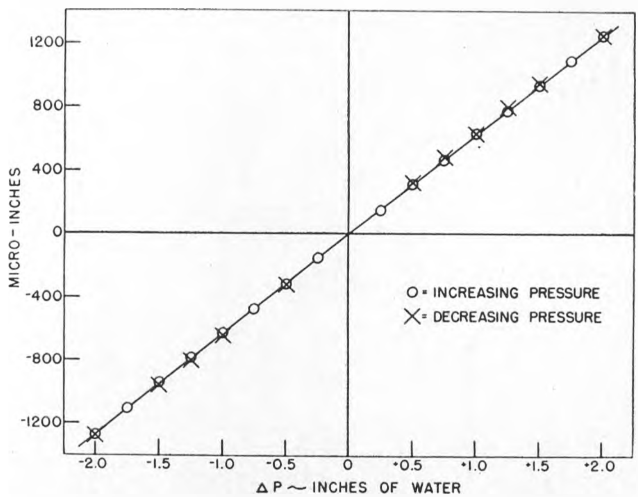


Fig. 7. Typical Calibration of a Pressure Transducer

The Internal Gas Reference, Monitoring, and Calibration System

All of the transducers in the installation described are pressurized initially to a pressure equivalent to the ambient, undisturbed water head so that only the deviations (pressure changes) are detected. The internal gas system must accomplish a three-fold purpose: provide the static reference level to which the pressure changes are referred; be capable of monitoring the internal pressure to insure that the ambient head is always balanced; and, be useable for in-place calibration. Before the design of the multichannel micropressure range described here, the early inductance-type gages were pressurized with air which was simultaneously fed to a submerged bell. The appearance of air bubbles escaping from under this bell indicated when the ambient head was balanced.

PART NO.	DESCRIPTION	PART NO.	DESCRIPTION
1-10	TRANSDUCER CUTOFF	21	SOLENOID BYPASS VALVE
11	MASTER CUTOFF FOR TRANSDUCERS	22	INLET SOLENOID VALVE
12	TOTAL HEAD MANOMETER CUTOFF	23	EXHAUST SOLENOID VALVE
13	SOLENOID VALVE MASTER CUTOFF	24	WATER TRAP DRAIN VALVE
14	EXHAUST VALVE	25	INLET THROTTLE VALVE
15	CALIBRATING BELLOWS CUTOFF	26	EXHAUST THROTTLE VALVE
16	EXHAUST VALVE	27	MOISTURE TRAP BYPASS VALVE
17	CALIBRATING BELLOWS CUTOFF	28	AUXILIARY BALANCING CHAMBER CUTOFF
18	AUTOMATIC BALANCING CHAMBER (3 WAY VALVE)	29	AUXILIARY BALANCING CHAMBER VENT VALVE
19	MICRO-MANOMETER STATIC REFERENCE TANK CUTOFF	30	BELLOWS CUTOFF VALVE
20	MICRO-MANOMETER CALIBRATION SCALE CUTOFF		

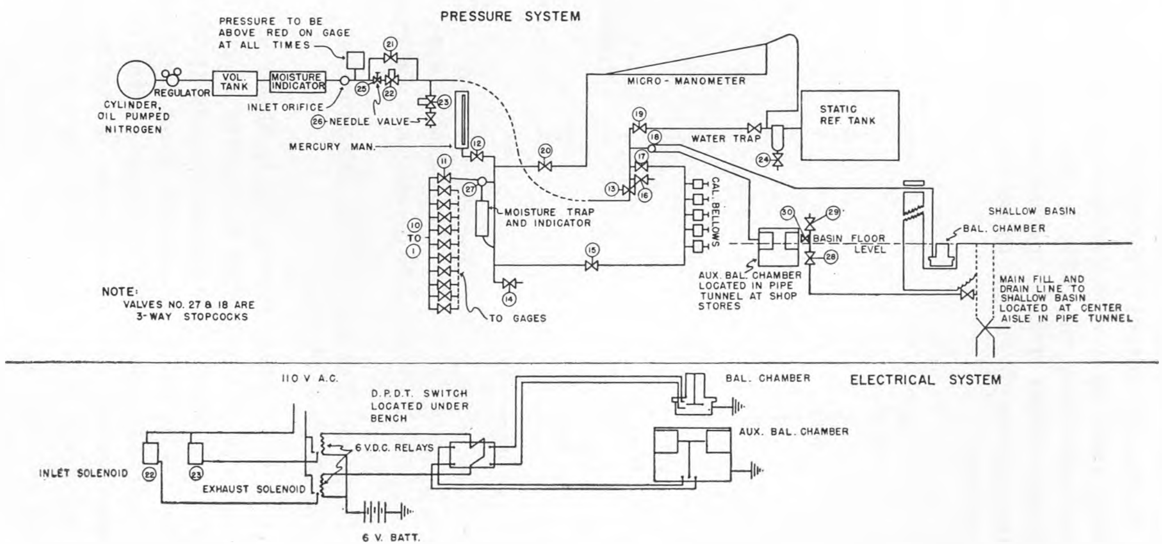


FIG. 8. SCHEMATIC DIAGRAM OF THE GAS PRESSURE REFERENCE, MONITORING, AND CALIBRATION SYSTEM

Since the bell was open at the bottom, it was able also to compensate for small changes in head. Calibration was accomplished by changing pressure internally by changing the volume of the system with a small bellows. It was necessary, of course, to remove this system from the basin whenever it was not in use.

For a permanent multi-gage installation to be operated under large changes of ambient head and to be left unattended over long periods of time, a closed, gas-tight system was required. A schematic diagram of the gas reference, monitoring, and calibrating system designed for simultaneous operation of ten transducers is shown in Fig. 8. The control console from which this system is operated is shown in Fig. 9. The control loops for each of the required functions may be identified from the following description.

The control console embodies three major sections: the supply and exhaust, the control and calibration, and the transducer feed. Each of these sections can be isolated from the others and vented to the atmosphere separately, if necessary (thus, making it possible to repair or remove any single component without disturbing the system). Due to the high humidity prevailing in the basin building and the long immersion of the transducers, oil-pumped nitrogen is used as the gas in the system rather than air compressed at the test site. As added protection for the transducers, the gas is passed through a series of silica gel dehydrators. The latter are

FIG. 9. CONTROL CONSOLE FOR DTMB MICROPRESSURE RANGE

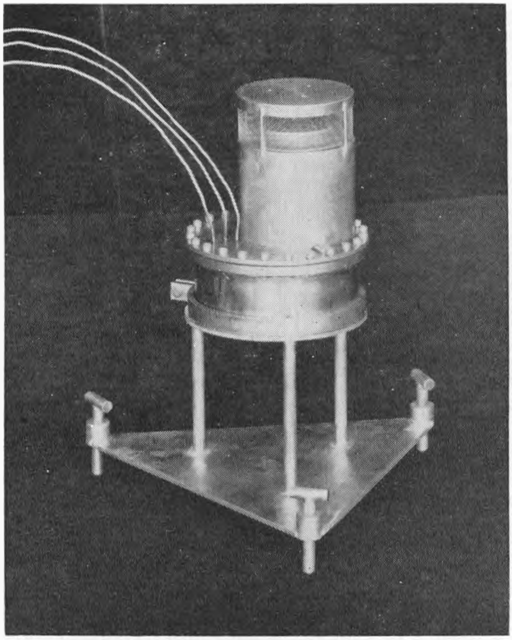


FIG. 10. THE SUBMERGED AUTOMATIC BALANCING CHAMBER MOUNTED IN LEVELING STAND

required because of the presence of a water surface in the calibrating manometer.

The hydrostatic head on the basin is compensated by means of the automatic balancing supply and exhaust system. Control is maintained by a balancing chamber, Fig. 10, submerged in the basin. Through a mechanical linkage, Fig. 11, motion of the bellows actuates the relays (seen on the backboard of the control console in Fig. 9) which in turn activate the appropriate inlet or exhaust solenoid valves. Thus, when there is an increase in water level, the bellows of the balancing chamber is compressed, the inlet solenoid is actuated, and gas is fed into the system until equilibrium is restored. Conversely, a falling water level allows the bellows to expand as a result of the excess internal pressure, the exhaust solenoid is actuated, and the excess nitrogen is exhausted to the atmosphere. The nitrogen reservoir pressure is maintained at a pressure equivalent to a few feet greater than the maximum water depth in the Shallow Basin.

The electrical contacts in the balancing chamber are set so that the circuits are made by changes of ± 0.07 inch of water head, and the pressure lag is just sufficient to return the floating contact to a central position between the contacts for the two circuits. Although smaller limits of control may be obtained, the value of 0.07 inch was chosen as compatible with the average roughness of the concrete basin floor. Stability of the gas circuit for the balancing chamber was attained by the use of needle valves near the inlet and exhaust valves and by proper choice of gas lead from the console to the chamber as selected by test. The balancing chamber is protected against overpressure by internal and external stops incorporated into the watertight base and into the protective covering for the bellows. During calibration and operation of the range, this circuit is isolated from the internal pressure system so that activation (caused by waves, for example) does not occur during a test. Since this balancing chamber is inaccessible for quick, emergency repair, a less sensitive but easily accessible standby balancing chamber has been added. The latter utilizes opposed bellows, one carrying internally the full head on the basin by direct communication with the water in the basin, the other operated in the same manner as that of the system described above.

The control console is equipped for in-place static calibration of the transducers. Incremental changes in pressure are obtained by changing the volume of a bank of small Hydron bellows (shown to the right in Fig. 9). The change in pressure is monitored on the slant-tube manometer which is connected to the calibrating bellows and transducer manifold on one side and to a gas reservoir or reference volume on the other. Thus, the response of the transducers as recorded on an oscillograph record is obtained directly in terms of a pressure change.

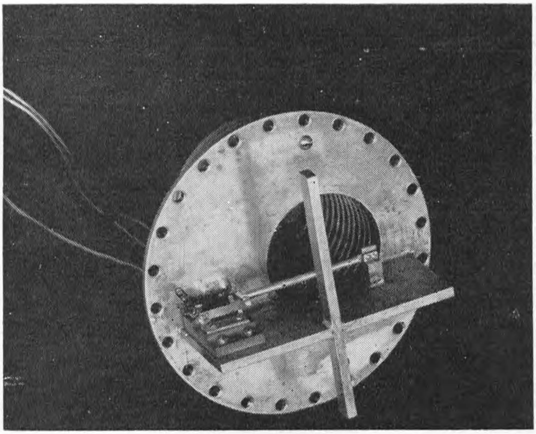


FIG. 11. INTERIOR OF BALANCING CHAMBER SHOWING MECHANICAL LINKAGES AND ELECTRICAL CONTACTS

Errors Associated with the Internal Gas System

Since the gas system is completely closed and limited in overall size, errors will be introduced if there are temperature fluctuations and are inherent as a result of internal volume changes not associated with temperature changes. Since the resolution of the system is to be of the order of 0.001 inch of water, the maximum temperature fluctuation, within the range of temperatures and total pressures encountered at DTMB, must be less than 0.001°F during the course of a test run. To accomplish this, the entire gas supply and control console are encased in an enclosure of ¾-inch plywood and ½-inch Cellotex (not shown in Fig 9). The console, together with the amplifiers and recording instruments, are further housed in an insulated room in which the temperature fluctuation is limited to plus or minus 1°F.

Changes in internal volume not associated with temperature fluctuations are associated with the displacement of liquid in the slant-tube manometer during calibration. The gas-reference volume tends to minimize the error caused by the volumetric change in this manometer. In this installation, the reference volume is 3 cubic feet so that a change of pressure equivalent to 0.1 inch of water produces a change in volume of 0.049 cubic inches when the manometer is set for a slope of 1/10. This will cause a change in the reference tank of approximately 5 percent of the *differential pressure* at the maximum head of 10 feet in the basin, and all calibrations must be corrected for this effect. For smaller total heads and smaller manometer magnifications, the error is correspondingly less, being negligible for a total head of about 4 feet and a magnification of ¼, for example.

Errors in frequency response associated with the gas system are discussed in detail in the appendix.

Concluding Remarks

The DTMB micropressure range essentially as described here has been in operation for the past twelve years with only one major overhaul and such inactivation as needed to incorporate improvements. In spite of the adverse conditions under which many of the components operate, the reliability has been most satisfactory.

A portable micropressure range based on this system has also been constructed for use primarily in the small basin mentioned previously, but suitable for use in any of the DTMB facilities.

Acknowledgments

The early inductance gages were developed by E. Plesset and C. Starr and the first gas system by A. Kalinske and J. M. Robertson. The present multi-transducer micropressure range was designed by P. Eisenberg; improvements were added by J. P. Craven and J. A. Luistro who also designed the portable range.

Discussion

Mr. Robertson, who had been mentioned by Mr. Eisenberg for his early work on the micro-pressure system reminisced that when he had last heard about the project, in 1942, the system had consisted of one unit instead of ten. He re-emphasized the point that in a system where small, transient pressures are being measured it is necessary to consider the frequency responses of all parts of the system, electrical, electronic, and mechanical. If the frequency response is poor, the measurements may be deceptive. When he had worked on the problem he had made a brief study of other systems that had been used to study transient pressures. One of the earliest ones, he found, was made by Capt. Geeler and reported in his book on *Why Wave Action*, in which he described pressure measurements taken with a diaphragm type pickup. Because of the possibility that his transients could have been in tune with the natural frequencies of the system, Mr. Robertson questioned the reliability of Capt. Geeler's work. A system where a large volume at the diaphragm is not available and a limited volume with a long tube is used for calibration may lead to difficulty.

Mr. Yih wondered whether fish could actuate a pressure sensitive mine, but Mr. Eisenberg had never heard of a fish being blown up by one.

Mr. Calehuff then raised the question of hysteresis in the diaphragm and asked how it had been avoided. Mr. Eisenberg agreed that there had been a great deal of difficulty in the early days of the project because the gages were made by soldering two steel rings. A man at the Taylor Model Basin had developed the skill and technique of applying a process of rolling, soldering, and annealing for stress relief which on the average yielded a successful diaphragm in one of three attempts. With the Statham gage, however, this difficulty does not seem to exist, probably because a corrugated diaphragm, screwed onto the gage, is used.

Dr. Craven remarked, at this point, that a corrugated diaphragm is difficult to machine.

Mr. Ingram then raised the question as to whether the operation of the gages with a common back pressure induced any errors; whether there was interaction between the diaphragms. Mr. Eisenberg was of the opinion that in a reach as long as in this system the interaction is negligible.

Finally, at the request of Mr. Robertson, Mr. Eisenberg sketched and described a corrugated diaphragm.

REFERENCES

1. Gawn, R. W. L., "Some Experiments in Connection with Mine Warfare," *Trans. of the Institution of Naval Architects*, Vol. 88, p. 148, 1946.

2. Borden, A., "Wall Corrections for Flow About Two- and Three-Dimensional Symmetrical Bodies in Rectangular Channels of Infinite and Finite Lengths," *David Taylor Model Basin Report 864*, December 1954.

3. Timoshenko, S., *Theory of Plates and Shells*, McGraw-Hill Book Company, New York, 1940, p. 58 et. seq.

4. Timoshenko, S., *Vibration Problems in Engineering*, D. Van Nostrand Co., New York, 1937, p. 426 et seq.

5. Plesset, Ernst, "The Application of Frequency Deviation Methods to Measurements of Mechanical Changes," *David Taylor Model Basin Report R-79*, February 1943.

6. Richardson, E. G., "Measurements of Water Flow and Pressure Set Up by Ships in Motion," *Trans. North East Coast Institution of Engineers and Shipbuilders*, Vol. 64, 1948, pp. 273-288.

7. Perls, Thomas A., "A New Condenser-Type Pressure Gage," *David Taylor Model Basin Report No. 625*, June 1949.

8. Cook, George W., "A Carrier-Type Strain Indicator," *David Taylor Model Basin Report No. 565*, November 1946.

9. Iberall, Arthur S., "Attenuation of Oscillatory Pressure in Instrument Lines," *Journal of Research of the National Bureau of Standards*, Vol. 45, July 1950.

APPENDIX

The frequency response of the transducer is a function of the mechanical impedance of the gage diaphragm and strain-wire system and of the "acoustic impedance" of the instrument lines and reference volume. The gage system shown schematically in Fig. 12 may be divided into four components, the mechanical elements of the transducer, the instrument volume, the instrument gas line, and the reference volume.

The diaphragm, its attached strain wires and the mounts for the wire form a complex mechanical-spring system such that an

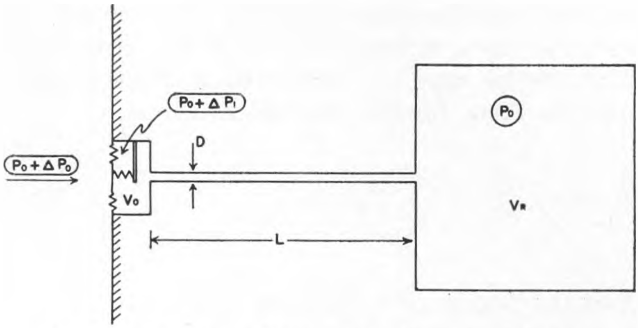


FIG. 12. SCHEMATIC DIAGRAM OF PRESSURE SYSTEM

approximation based on diaphragm properties alone is not applicable. Practically, the problem may be greatly simplified by considering the system as a simple spring whose constants are determined empirically. The equation for such a simple spring system is

$$(\Delta p_0 - \Delta p_1) A_0 = k x_{max} \sqrt{\left[\frac{c\omega}{k}\right]^2 + \left[1 - \frac{m}{k}\omega^2\right]^2}\; e^{i(\omega t - \phi_1)} \tag{1}$$

Here Δp_0 is the applied external pressure increment,
Δp_1 is the resulting internal pressure increment,
A_0 is the effective area of the diaphragm,
k is the spring constant,
c is the damping coefficient,
m is the effective mass of the spring,
ϕ_1 is the phase angle equal to $\tan^{-1}\left[\frac{c\omega/k}{1 - m\omega^2/k}\right]$, and
x is the deflection of the center of the diaphragm.

It is not necessary to determine all of the constants in this expression since the apparent or recorded pressure Δp_r may be determined by a static calibration:

$$\Delta p_r = \frac{k}{A_0} x = \frac{k}{A_0} x_{max}\, e^{i\omega t} \tag{2}$$

Therefore

$$\frac{\Delta p_0 - \Delta p_1}{\Delta p_r} = \sqrt{\left[\frac{c\omega}{k}\right]^2 + \left[1 - \frac{m}{k}\omega^2\right]^2}\; e^{-i\phi_1} \tag{3}$$

The magnitude of the internal pressure increment Δp_1 differs from zero as a result of the finite geometry of the instrument and instrument line and the compressibility and viscosity of the gas. Since the surface area to volume ratio of the instrument and line is large the process may be considered isothermal and the ideal gas law applied. Thus, for the instrument volume

$$\frac{\Delta p_1}{p_0} = -\frac{\Delta V}{V_o} + \int_{-\infty}^{t} \frac{Q\,dt}{V_o} \tag{4}$$

where ΔV is the change in instrument volume
V_o is the instrument volume, and
Q is the volume rate of gas flow from the instrument volume into the instrument line.

As the recorded pressure is simply a measure of diaphragm deflection, we may write

$$\frac{\Delta V}{V_o} = k_2 \frac{\Delta p_r}{p_o} \tag{5}$$

Here $k_2 V_o/p_o$ is an experimentally determined constant. If we consider the pressure increments to vary in a simple harmonic manner we may differentiate equation (4) with respect to time and with equation (5) determine

$$\frac{\Delta p_1 + \dfrac{i p_o Q}{\omega V_o}}{\Delta p_r} = -k_2 \tag{6}$$

The flow Q through the instrument lines has been determined by Iberall (9). He assumed that the flow follows Poiseuille's law of viscous resistance,

$$\frac{\partial p}{\partial x} = -\frac{128}{\pi}\frac{\mu}{D^4} Q \tag{7}$$

where $\frac{\partial p}{\partial x}$ is the pressure gradient in the instrument line,
D is the diameter of the line, and
μ is the dynamic viscosity of the gas.

If it is further assumed that the reference volume V_R is very large, then it can be determined from Iberall's analysis that

$$\frac{\Delta p_1}{\frac{128\,\mu\,Q}{\pi\,D^4}} = \frac{L}{A'}\,\frac{(tanh^2 A' + tan^2 A')^{1/2}}{1 + tanh\,A'\,tan\,A'}\quad e^{i\phi_2} \tag{8}$$

where

$$\phi_2 = tan^{-1}\left[\frac{tan\,A' - tanh\,A'}{tan\,A' + tanh\,A'}\right]$$

$$A' = 4\,\frac{L}{D}\left(\frac{\omega\mu}{p_o}\right)^{1/2}$$

and L is the length of the instrument line.

Equations (3), (6) and (8) may be solved simultaneously to determine the response characteristics of the system.

$$\frac{\Delta p_0}{\Delta p_r} = z_1\,e^{i\phi_1} - k_2\left[1 + \frac{p_0}{V_{o\omega}}\,z_2 e^{i(\pi/2-\phi_2)}\right]^{-1} \tag{9}$$

where

$$z_1 = \sqrt{\left(\frac{C\omega}{k}\right)^2 + \left(1 - \frac{m}{k}\,\omega^2\right)^2}$$

and

$$z_2 = \frac{\pi D^3}{32}\left(\frac{\omega}{p_o\mu}\right)^{1/2}\frac{1 + tanh\,A'\,tan\,A'}{(\,tanh^2 A' + tan^2 A')^{1/2}}$$

The response curves for the TMB range are shown in Fig. 13. Three conditions are shown

(a) the frequency response of the gage without instrument lines,
(b) the frequency response of the system as originally installed, and
(c) the frequency response of the system if modified by reducing the length of piping.

The spring constants were determined experimentally by striking the gage in water and measuring the resulting natural frequency and exponential decay time. The volume coefficient was determined by a static calibration with the reference volume sealed. The system characteristics employed are listed in Table I. At the present time the instrument range has been modified by the addition of a volume reservoir at the instrument. This has the effect of reducing the length of the instrument line to zero.

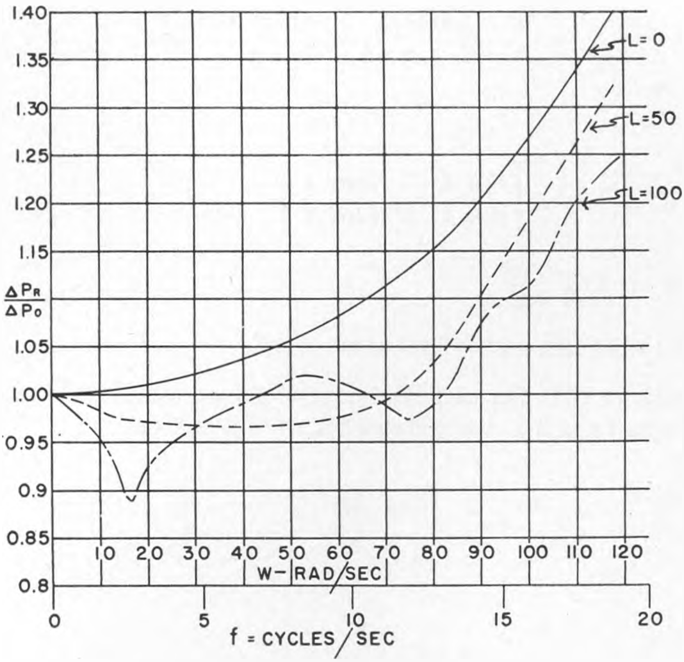


FIG. 13. RESPONSE CHARACTERISTICS OF DTMB MICROPRESSURE RANGE

TABLE I

RESPONSE CHARACTERISTICS OF DTMB MICROPRESSURE RANGE FOR $p_o = 19$ p.s.i.a.

$k/m = 4.8 \times 10^4$
$c/m = 3.22$
$L = 100$ ft.
$V_R = 3$ ft.^{3}
V_o (before modification) $= 1.37$ in^3
k_2 (before modification) $= 0.121$
V_o (after modification) $= 29.6$ in^3
k_2 (after modification) $= 0.0056$

POTENTIAL-FLOW ANALOGS AND COMPUTERS

by

S. C. Ling

Introduction

An analogy is often used to explain an unfamiliar phenomenon in terms of a better known counterpart. In particular, if two physical systems are described by the same mathematical relationships, quantitative results for one system can be obtained by studying the other system as an analog. Consequently, the physical system which performs the numerical calculations using analogous variables of the prototype system is known as an analog computer.

For inviscid irrotational fluid motion [1], in which the tangential stress on a fluid element due to viscous shear is absent, the velocity component in any direction can be expressed as the corresponding space derivative of a velocity potential ϕ. This type of flow, generally known as potential flow, is described mathematically by the equation

$$\nabla^2 \phi = o \tag{1}$$

where ∇^2 represents the Laplacian operator

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

More generally the potential satisfies

$$div\ (k\ grad\ \phi) = o \tag{2}$$

where k is the permeability coefficient of the flow field, and is taken as unity for fluid motion in free space.

Although water, the fluid which is of primary concern to hydraulic engineers, cannot be considered as inviscid, for many flow conditions where the thickness of the laminar or turbulent boundary layer due to viscous shear between the fluid and the flow boundary is thin as compared to the geometrical size of the flow boundary under investigation, it is generally safe to assume that the flow velocity at the outer edge of this boundary layer is essentially that due to potential flow, and the pressure on the flow boundary is the same as that outside the boundary layer [2]. This therefore forms the basis upon which potential flow theory can be applied to some problems which involve the study of pressure distribution along solid boundaries. Examples of fundamental importance in hydraulic engineering are: designing boundary profiles that will avoid or minimize the destructive phenomena of cavitation, such as the investigation of boundary transitions for flow inlets and contractions; and testing profiles for head forms and hydrofoils. A second type of problem that can be treated as potential motion is the seepage type of flow in which the fluid acceleration is negligible so that the velocity potential ϕ can be considered as equivalent to the force potential; that is, ϕ is equal to pressure head plus elevation head

$$\phi = \left[\frac{P}{\gamma} + \mathrm{h} \right]$$

The gradient of this potential times the permeability coefficient k of the soil bed is the flow velocity in the corresponding direction

$$U_n = -\mathrm{k} \frac{\partial \phi}{\partial n} \tag{3}$$

For a number of potential flow problems which involve simple boundary values the exact mathematical solutions can be determined, but for most problems that are of practical interest the boundary values are too complicated for obtaining solutions in the orthodox manner. Consequently, the integration of Eq. (2) is generally carried out by the analog method because of the ease and simplicity with which useful solutions can be obtained.

PRINCIPLE OF ANALOG

The mathematical expression of Eq. (2) not only describes the potential field of fluid flow but also a number of physical fields such as the electric potential field, the magnetic potential field, and the thermal potential field in various transmitting media. Any of these fields may be used as an analog to study another through proper scaling of the physical constants. For example, the differential equation for an electric potential field is

$$div\ (g\ grad\ E) = o \tag{4}$$

where E is the electric potential and g is the conductivity of the electrical field. If one lets

$$E = A\phi \text{ and } g = Bk$$

and substitutes them into Eq. (4), the resultant expression is identical to Eq. (2) for the fluid counterpart with A and B as the scaling constants. In most flow problems, the solutions sought can be expressed in dimensionless ratios so that an exact knowledge of these scaling factors is unnecessary. For instance, the solution can be stated in the form of velocity ratio U/U_0, where U is the fluid velocity at any point of the field and U_0 is the reference velocity, usually taken as the uniform flow velocity that exists in the flow field. If n is the distance normal to the equipotential surface, then

$$\frac{U}{U_o} = \frac{\dfrac{\partial \phi}{\partial n}}{\left.\dfrac{\partial \phi}{\partial n}\right|_o} = \frac{\dfrac{\partial E}{\partial n}}{\left.\dfrac{\partial E}{\partial n}\right|_o} \tag{5}$$

Hence the velocity ratio U/U_0 of the prototype system is simply the ratio of the electric potential gradients at the respective points in the model system.

The corresponding pressure p of the flow field may be obtained from the Bernoulli relationship

$$\frac{p - p_o}{\rho U^2{}_o/2} = 1 - \left(\frac{U}{U_o}\right)^2 = 1 - \left[\frac{\dfrac{\partial E}{\partial n}}{\left.\dfrac{\partial E}{\partial n}\right|_o}\right]^2 \tag{6}$$

where p_o is the pressure at the point where the reference velocity is U_0.

Unlike mathematical solutions which operate upon numerical values, the analog method works on physical quantities. Consequently, the accuracy of the solutions obtained depends on the precision with which these physical quantities can be set up and measured. In consideration of this fact, the electric potential analog is used almost exclusively because quantities such as the electrical potential, the electrical current and the electrical resistance can all be measured with great precision by simple instruments that are either available commercially or found in most laboratories. This paper will therefore be confined to the discussion of various electrical analogs which are designed to solve steady-state potential flow problems.

The difference in various electrical analogs lies primarily in the use of different media for the conducting field. Those most commonly used are the conducting paper, the electrolytic tank, and the resistance network. The application of each of these techniques will be discussed.

CONDUCTING PAPER AND ELECTROLYTIC TANK METHODS

Perhaps the oldest known potential analog is the work of Kirchhoff [3] who used thin copper sheet as the electrically conducting medium. His work was published in 1845, but only in recent years has his technique received increasing attention [4] partly due to intensive interest in the application of potential theory in many fields of engineering and partly due to the availability of more uniform conducting materials. Examples of the latter are the paper which is used in making tape resistors and the paper which is made for use in teleprinters. In fact laboratory units complete with accessories for solving simple two-dimensional flow fields are available commercially from the General Electric Company.

Since a paper conductor is essentially a two-dimensional medium, it can solve only flow problems of a similar nature; i.e., two-dimensional flows, which satisfy the equations

$$\nabla^2 \phi = o \text{ and } \nabla^2 \psi = o \tag{7}$$

where ψ is the stream function and ∇^2 is the Laplacian operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Since the differential equations for ϕ and ψ are similar they can be solved separately by the electrical potential analog. As an illustration, consider the solution of a two-dimensional flow-inlet problem shown in Fig. 1.

The dark line A-B-C-D-E-F-G represents the outline of the conducting paper; the sides A-B and F-G represent the two dimensional conduit; B-C the inlet transition; C-D is the wall of reservoir and E-F is the floor of reservoir or the centerline of a pipe heading from a semi-infinite reservoir. A-G represents an equipotential line which should be straight and normal to the velocity vector of the uniform flow in the conduit, while D-E represents an equipotential line at a distance R from the inlet. The latter line is an arc of a circle with radius R centering at F, if R is large compared with the inlet opening C-F. A conducting paint applied to these equipotential boundaries may serve as electrodes between which an ac voltage is impressed to set up the flow field. The four-dial decade potentiometer which is calibrated to read down to 1/10000 part of the total applied potential, the sensitive null detector such as the Ballantine Voltmeter with a sensitivity of .0001 volts and the probe P, as indicated in Fig. 1, are typical accessories required to trace the equipotential lines ϕ_0, ϕ_1, --- ϕ_m or to measure the potential at any position in the field. From these readings, velocity

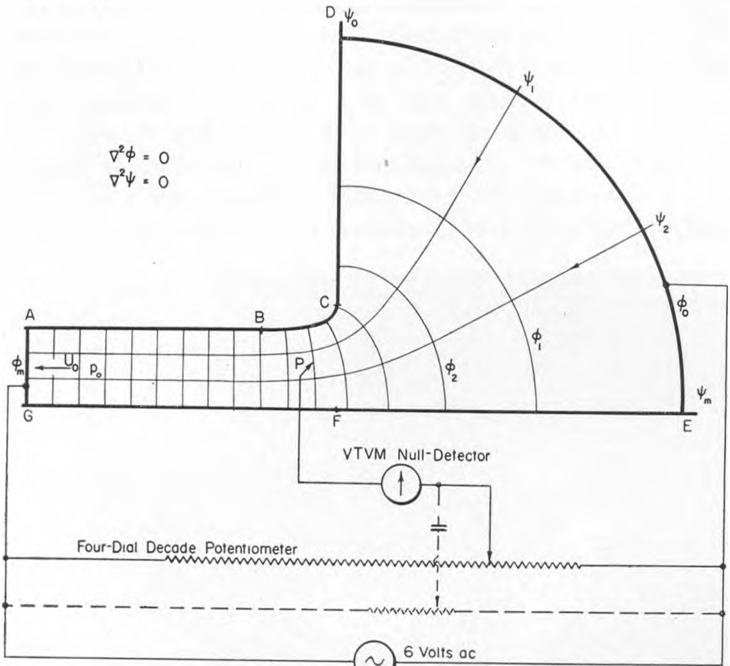


FIG. 1

ratios and pressure ratios of the corresponding fluid systems can be obtained through the use of Eqs. (5) and (6). From Eq. (7) it is further possible to trace the stream lines ψ_1, ψ_2, --- ψ_m by impressing the ac potential along ψ_0 and ψ_m instead of along ϕ_0 and ϕ_m.

The main advantage of using the conducting paper technique is its simplicity and low cost. However, due to the fact that perfectly uniform conducting paper is still not readily available, and the accuracy of cutting out or setting the boundary profiles is somewhat dependent on human factors, the paper technique is not considered ideal for precision work.

The other type of electrical analog commonly used [5] is the so-called electrolytic tank method, in which an electrolytic solution serves as the conducting field. The solution is usually held in a glass or plastic tank whose shape and form are dictated by the boundary values of the flow problem. Figure 2 shows the picture of one such tank built for the study of inlet transitions from an infinite reservoir wall to a square or to a circular conduit. The tank for holding the electrolyte is made of 5/16″ lucite plates glued together with chloroform. The transition boundary between the reservoir wall and the conduit is made detachable so that different transition forms can be tested. This plastic tank is placed on a ¾" plate glass which in turn is set on a rigid wooden frame work in a manner that permits the tank to lie in a precise horizontal plane. When the tank is filled with electrolyte it will represent the flow field of a two-dimensional inlet transition, or when tilted at a 15° angle from the horizontal position, the electrolyte will represent the flow field of a symmetrical sector of a circular-con-

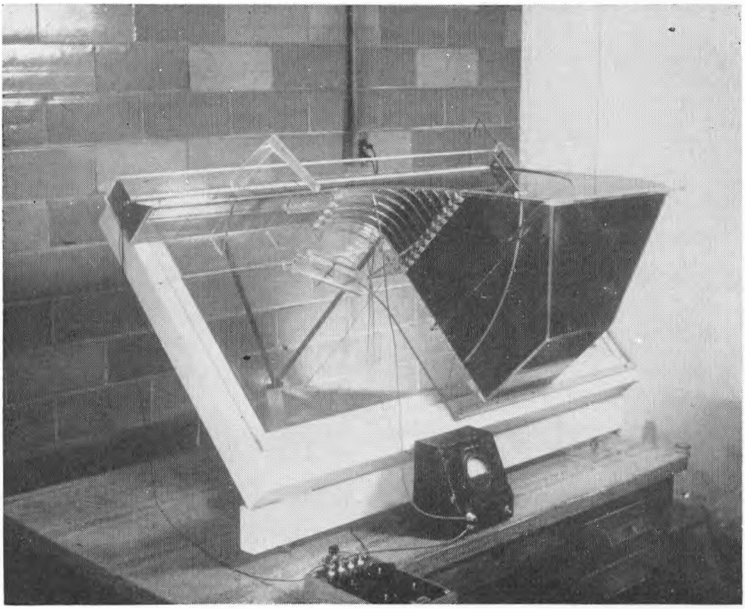


FIG. 2

duit and its inlet transition. With the tank tilted at 45° from the horizontal position, as shown in Fig. 2, the electrolyte represents the flow field which is a segment between planes of symmetry of a square conduit and its inlet transition. Also, varying the liquid level in the tank permits various contraction ratios (i.e. the size of inlet to the size of conduit) to be studied. Six volts ac at 60 cycles was impressed between one electrode at the left end of the tank, representing an equipotential plane of uniform flow in the conduit, and the other electrode made of wire mesh at the right end of the tank, representing part of an equipotential sphere some distance from the inlet opening. The resultant electrical potential distribution along the flow boundary is detected by electrodes made of No. 36 gage copper wires embedded normal to and flush with the boundary surfaces. These electrodes are placed at precisely

½-inch center-to-center spacing in an orthogonal pattern, thus permitting the total velocity vector U_n at any point along the flow boundary to be evaluated; that is

$$U_n = \frac{\partial \phi}{\partial n} = \sqrt{\left(\frac{\triangle \phi}{\triangle x}\right)^2 + \left(\frac{\triangle \phi}{\triangle y}\right)^2} \tag{8}$$

where $\triangle\phi/\triangle x$ and $\triangle\phi/\triangle y$ are the potential gradients measured between the corresponding pairs of orthogonal electrodes.

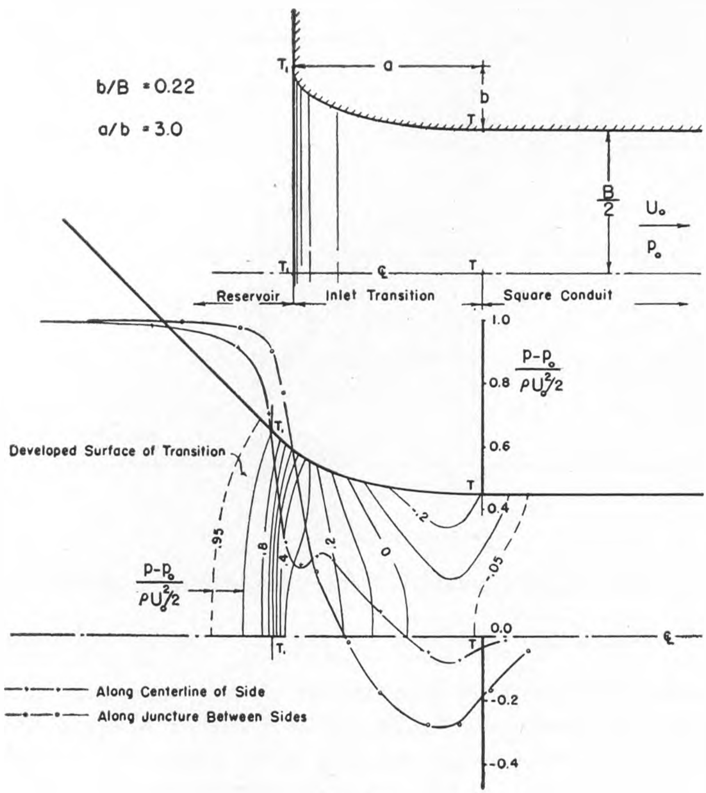


FIG. 3

A typical experimental result is shown in Fig. 3 in which the dimensionless pressure distribution along the boundary of a square conduit with a 3:1 elliptic transition curve is plotted in terms of dimensionless pressure parameter $\frac{p - p_o}{\rho U_o^2/2}$. It is interesting to note that the lowest pressure always occurs at the corners of the conduit; it is no coincidence that the cavitational erosion found in prototype inlet structures of high dams was also located at the position predicted by the electric analog.

From the viewpoint of structural economy, the most efficient transition curve is of course one that gives the least negative pressure with a shortest length of transition, provided that a complicated curve does not lead to excessive form costs. The elliptic transition curves were chosen for the study because of their form and functional simplicity. The results obtained by electrical analog are shown in Figs. 4, 5 and 6 for two-dimensional-,

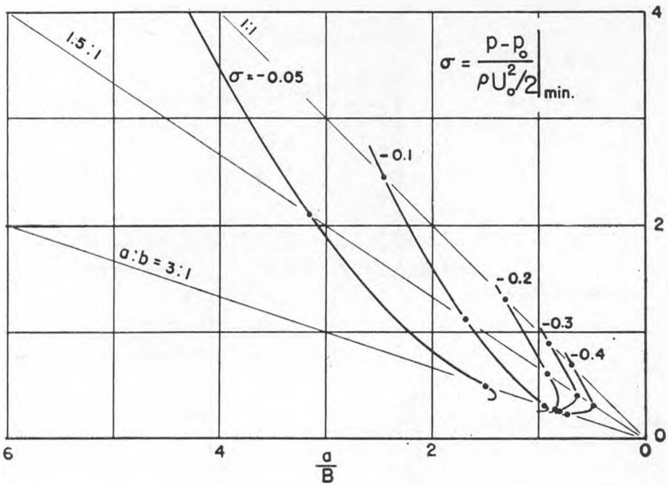


FIG. 4

circular-, and square-conduit inlet transitions respectively. The loci of cavitation parameters $\sigma = \left.\frac{p - p_o}{\rho U_o^2/2}\right|_{min.}$, or the minimum pressure ratios that exist in a particular elliptic transition geometry, are plotted as functions of a/B and b/B ratios, where a and b are lengths of the semi-major and the semi-minor axes respectively of the ellipses, and B is the width or the diameter of the conduit. From these figures one can obtain the required elliptic form and size for a given cavitation parameter σ. Also one notes that a 3:1 to 4:1 elliptic transition is the most efficient form to use. Elliptic forms with slenderness ratio greater than 3:1 have not been investigated, but from the trend of the above figures one can see that for higher slenderness ratios longer transition curves would be required to provide for the same cavitation number σ.

The difficulties that arise from the use of an electrolytic solution as a conducting medium are many. Among the major problems

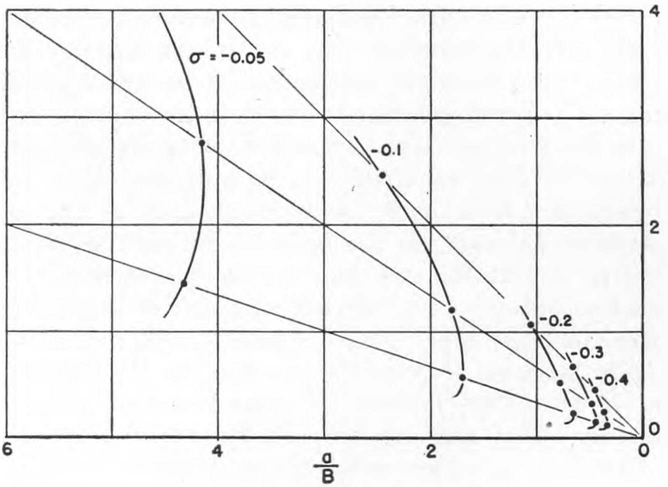


Fig. 5

encountered are 1) the necessity of a water-tight tank to hold the liquid, 2) the requirement of precision leveling of the tank in case the free-liquid surface is used as one of the flow boundaries, and 3) the presence of errors due to the polarization layer formed on the electrodes. However, with due care in designing the equipment, inaccuracies due to the first two problems can be effectively eliminated. The third problem can be taken care of by the use of concentrated copper-sulfate solution, in the order of 100 gr. of $CuSO_4$-

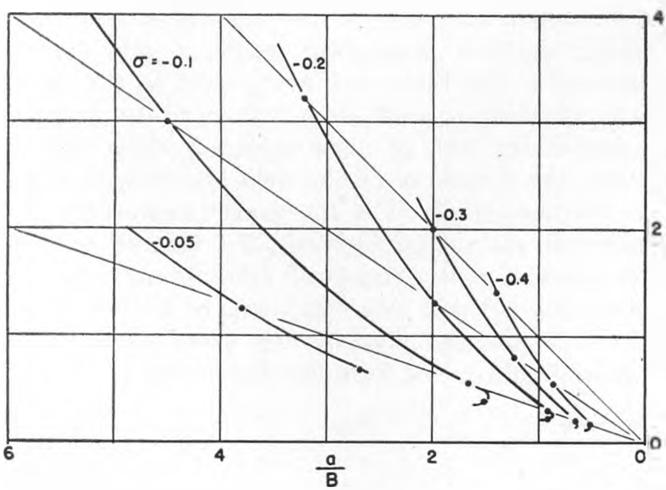


Fig. 6

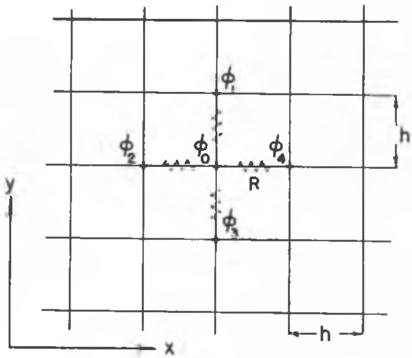
$\cdot 5H_2O$ per liter of water with few drops of concentrated sulfuric acid added to stabilize the solution. An electrolyte made in this proportion will not only keep the copper electrodes clean and increase the conductivity of the solution with resultant better impedance matching (or more sensitivity) to the measuring circuits, but, above all, it reduces the time required for the polarized film to reach a stabilized condition from hours or even days, as in the case of a weak electrolytic solution, to the order of a fraction of a second. The equivalent circuit for this film can be represented by an unwanted resistor plus an extra capacitor combined in parallel. This extra resistor not only takes time to reach an equilibrium value but causes a false increase of potential gradient in the vicinity of the electrodes. If ac voltage is used to excite the analog, the electrolytic condenser effect will cause a quadrature current to flow in the field. However, by using an impedance-bridge system as shown by the dotted line in Fig. 1, one can balance out the quadrature voltages due to this quadrature current and thus permit the desired potential distribution in the analog to be measured.

With proper care and selection of model scale the electrolytic tank method can be considered as a precision analog for the study of 3-dimensional flow problems. Accuracy better than $\pm$ 1.0 percent in the measurement of velocity ratios and $\pm$ 2.0 percent in the measurement of pressure ratios can be readily achieved.

RESISTANCE NETWORK COMPUTER

A still more versatile potential-flow analog, which may be considered as an analog computer, is the resistance network [6]. Instead of using uniform conducting media, a grid system of resistors connected in the form of a net is used as the field. This is equivalent to dividing a continuum into a finite grid system of flow paths, and if the sizes of these grids are made infinitely small compared with the geometry of the field one obtains essentially a continuum. Mathematically it is the exact counterpart of the well known numerical relaxation method [7] (better known as the Hardy Cross Method or the Southwell relaxation method) in which the field is usually divided into small square lattices, Fig. 7. The basic Eq. (7) of potential flow for the two-dimensional field can be written in finite difference form for this lattice as

$$\nabla^2 \phi \Big|_o = \frac{\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o}{h^2} + \frac{h^2}{12}\left[\frac{d^4\phi}{dx^4} + \frac{d^4\phi}{dy^4}\right] = o \tag{9}$$



$$\nabla^2\phi\Big|_0 = \frac{\phi_1+\phi_2+\phi_3+\phi_4-4\phi_0}{h^2} + \frac{h^2}{12}\left(\frac{\delta^4\phi}{\delta x^4}+\frac{\delta^4\phi}{\delta y^4}\right)_0 + \cdots = 0$$

$$\nabla^2\phi\Big|_0 \doteq \frac{\phi_1+\phi_2+\phi_3+\phi_4-4\phi_0}{h^2} = 0$$

$$\frac{\phi_1-\phi_0}{R}+\frac{\phi_2-\phi_0}{R}+\frac{\phi_3-\phi_0}{R}+\frac{\phi_4-\phi_0}{R} = \frac{\phi_1+\phi_2+\phi_3+\phi_4-4\phi_0}{R} = 0$$

FIG. 7

neglecting the higher order terms one has

$$\nabla^2\phi\Big|_o = \frac{\phi_1+\phi_2+\phi_3+\phi_4-4\phi_o}{h^2} = o \tag{10}$$

where h is the lattice size, and ϕ_0, ϕ_1, ϕ_2, ϕ_3, ϕ_4 are the potentials at the node points 0, 1, 2, 3, 4 respectively. Through a tedious mathematical accounting system the ϕ function at each node point is relaxed step by step until it satisfies both the boundary values as well as Eq. (10), throughout the field. Suppose that the grids of Fig. 7 were to be replaced by resistors whose value R is made proportional to h. Then from the law of continuity the total electric currents flowing into the node 0 must be equal to zero or

$$\frac{\phi_1-\phi_o}{R}+\frac{\phi_2-\phi_o}{R}+\frac{\phi_3-\phi_o}{R}+\frac{\phi_4-\phi_o}{R}$$

$$= \frac{\phi_1+\phi_2+\phi_3+\phi_4-4\phi_o}{R} = o \tag{11}$$

One notes the identity of Eq. (11) to Eq. (10). Furthermore the resistance network represented by Eq. (11) is self-consistent; that is once the values of ϕ_1, ϕ_2, ϕ_3, ϕ_4 are given ϕ_0 will adjust itself to satisfy the equation. In the case of a complete network, the potential ϕ at each node point will adjust itself to the correct value once the boundary values are fixed. Consequently the solution obtained by the resistance network is equivalent to that obtained by the numerical integration, but without the tedious relaxation processes.

Neither the relaxation method nor the network analog is absolutely correct, because both Eqs. (10) and (11) omit the term $\frac{h^2}{12}\left[\frac{\partial^4 \phi}{\partial x^4} + \frac{\partial^4 \phi}{\partial y^4}\right]$ and other higher order terms of Eq. (9). However, if due care is taken to provide for sufficiently small nets such that the potential distribution is nearly linear within each net, then the error due to neglecting these higher order terms will be insignificant.

Against this approximation of finite difference operation, the advantages gained by using the resistance network are many. First of all, the boundary geometry can be represented by simple resistance values, and consequently the human elements involved in the construction of models, as in the case of the paper analog and the electrolytic tank analog can be eliminated. This ease of setting up the flow boundaries permits the solution of many flow problems involving previously unknown flow boundaries that will satisfy certain preset flow conditions. The free flow surface of an overfall under gravitational action, profiles of constant velocity-inlet transitions, and the water-table surfaces of seepage flow are some of the potential-flow problems that can be ideally handled. The process involves successive cut and try steps which quickly converge to the correct solution [8, 9]. Second, the structure of the network permits any flow fields whose permeability k is either a constant or a function of space to be accurately set up; that is, the conductivity $g(x, y)$ of the resistance network can be distributed in accordance with the permeability $k(x, y)$. For the special case of 3-dimensional flow where the flow contains an axis of symmetry, a sector of such a field can be represented by a two-dimensional network whose conductivity $g(r)$ is made proportional to the radial distance r from the axis. Third, the accuracy of the resistance network analog is inherently high. By using precision resistors with a consistency of $\pm$ 0.5 percent, one can easily estimate the corresponding space distance of each unit net with the same order of accuracy. With the same precision provided in the measurement of potentials between each node of the network, one can obtain an accuracy of $\pm$ 1.0 percent in the calculation of potential gradients. Consequently the solutions obtained by the resistance network are highly reproducible and free from human factors.

In order to set up a network with proper resistance values for various boundary forms and permeabilities of flow media, some rules or equations are required. The most complete and general equations known to the writer are due to Tschiassny [10] who derived a basic equation by replacing a triangular element of continuum with an equivalent resistance net. This elementary triangle is then the basis upon which a flow field with complicated boundary geometry can be constructed.

Figure 8 shows an elementary two-dimensional triangular continuum $A_1A_2A_3$ which can, in the sense of finite differences, be replaced by a triangular network of resistors with conductances g_1, g_2 and g_3.

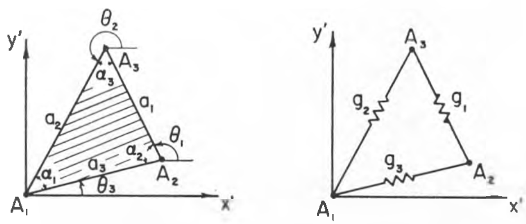


FIG. 8

Let $k_{x'x'}$ and $k_{y'y'}$ be the principal conductivities of the continuum, where x' and y' are the arbitrary Cartesian coordinates which coincide with the principal axes of the conductivity tensor. Assuming only that the continuum considered is so small that the potential distribution is linear within that region, a general expression for the conductance of the resistor element is

$$g_3 = \tfrac{1}{2}\left[\frac{k_{x'x'} + k_{y'y'}}{2}\ cot\ a_3 + \frac{k_{x'x'} - k_{y'y'}}{2}\ \frac{cos\ (\theta_1 + \theta_2)}{sin\ a_3}\right] \tag{12}$$

The same equation applies to $g_1$and g_2 after proper rotation of the indices.

One notes three important characteristics of Eq. (12): First, if the field is isotropic, that is $k_{x'x'} = k_{y'y} = k$, then the equation reduces to

$$g_3 = \frac{k}{2}\ cot\ a_3 \tag{13}$$

Second, if angle a is acute, right, and obtuse then the conductance of the resistor on the side opposite the angle is positive, zero and negative respectively. In order to prevent negative conductances, obtuse angles should be avoided. Third, the conductance is independent of the absolute size of the triangle.

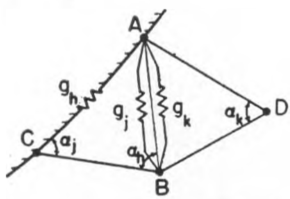
To combine these triangular elements into a complete flow field, one notes that there are always two conductances to be added up between two nodes unless both nodes are on the boundary.

$$k_{x'x'} \quad , \quad k_{y'y'}$$

$$g_s = \frac{1}{2}\left[\frac{k_{x'x'}+k_{y'y'}}{2}\cot\alpha_s + \frac{k_{x'x'}-k_{y'y'}}{2}\,\frac{\cos(\theta_1+\theta_2)}{\sin\alpha_s}\right]$$

$$k_{x'x'} = k_{y'y'} = k$$

$$g = \frac{k}{2}\cot\alpha_s$$



$$g = g_j + g_k$$

$$R = \frac{1}{g}$$

FIG. 9

Figure 9 shows two adjacent elements with common nodes A and B from which it may be seen that the total conductance between A and B is $g = g_i + g_k$, or the total resistance value is $R = 1/g$.

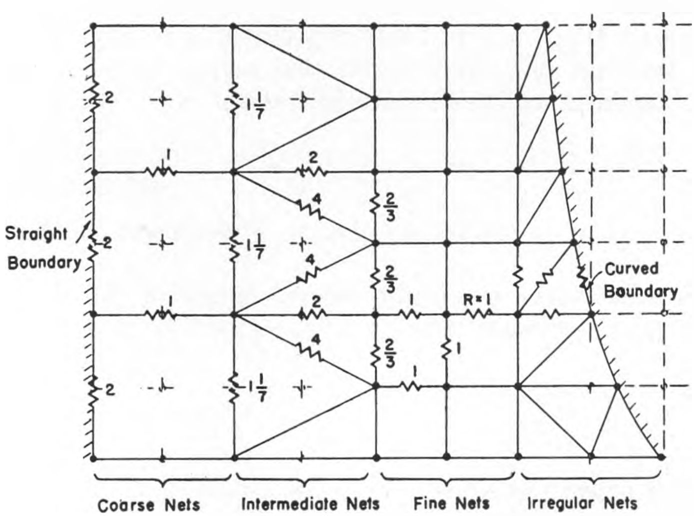


FIG. 10

It may be further shown that if the angles a_i and a_k are right angles, the conductance for the diagonal A-B of a square element will be zero. Figure 10 depicts a typical square-net system representing a flow field. A coarse network may be joined to a finer one by an intermediate network, and the elements in the vicinity of a curved boundary may be represented by irregular nets. The fineness of network required, of course depends on the linearity of potential distribution within each element of net considered. As a rule, finer nets are required where the potential distribution is found to be nonlinear.

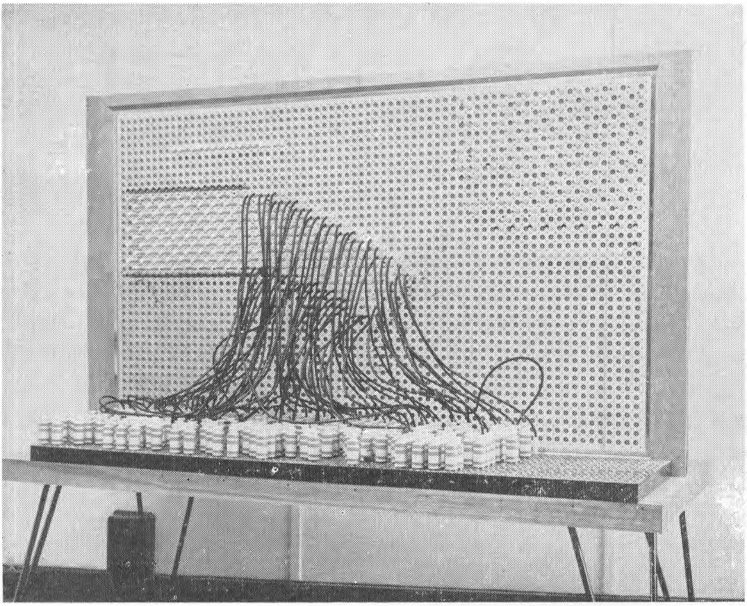


Fig. 11

A computer based on this network principle is shown in Fig. 11. The vertical panel contains a grid of sockets into which resistor elements may be plugged to form the flow field. The board is divided into 70 squares wide and 40 squares high which gives a total of 2,800 square nets. A four-pin socket is provided at the center of each net, and they are interconnected as shown in Fig. 12. A one-pin socket is also provided at each node point for the measurement of potentials or for the feeding in of boundary values. The panel is made of ¼ inch clear Lucite plate, so a to-scale graph can be hung on the opposite side of the panel to provide the operator with a visual reference to the flow field. For regular square elements,

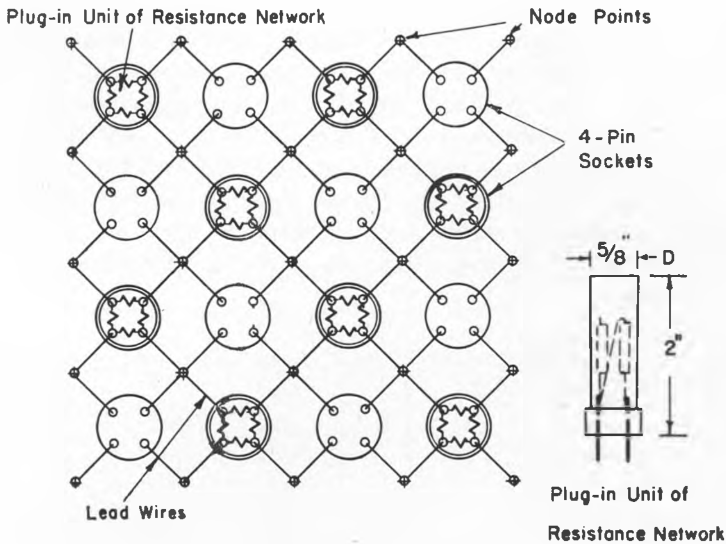


FIG. 12

four 1000-ohm precision resistors are mounted in a ring on a four-pin plug which forms the basic plug-in unit. These units are then plugged into alternate sockets on the panel, as shown in Fig. 12, to form a network of square nets. For irregular-net elements a special board is provided on the table of the computer (see Fig. 11). On this board 78 units of irregular nets can be set up. Each irregular net may consist of from one to five units of adjustable resistors which are special 10,000-ohms 3-dial decades made by the Telex Company. Each decade unit is only 1½ inches diameter by 3 inches high, and is provided with a convenient plug-in base. The necessary number of these decade units is plugged into the board on the table to form a particular irregular net element, and the precise resistance values are then simply dialed out on the decades. A plug-in cable connects this irregular unit to the main network on the vertical panel.

This liberal use of plug-in units permits a maximum of flexibility in the setting up of a problem, as well as a maximum utilization of the costly precision resistors and decade elements. Future modifications can also be performed with a minimum of alterations because all major parts are separated into special units.

Conclusions

Three of the most commonly used analogs for the solution of potential-flow problems have been presented. Emphasis is placed upon the precision and on the methods of setting up the fields of these analogs. Much of the distrust of all forms of analogs in the past has been based on inability of providing an absolute check on the accuracy of results obtained. It is therefore a primary object of this paper to show that if proper care and attention are used in setting up the analog, and its tolerance and characteristics are understood, there is no reason why the results obtained cannot be reproducible and accepted with confidence.

The main characteristics of the three analogs are summarized in the following table:

Advantages	*Disadvantages*
Conducting Paper	
Simple and cheap; applicable to complicated boundary geometries, especially to cut and try solution for correct boundary forms.	Limited to two-dimensional problems; accuracy is limited by the uniformity of conducting material and the human factors.
Electrolytic Tank	
Capable of handling three-dimensional flow problems in a simple manner; reasonably accurate if handled properly.	Needs complicated water-tight tanks; construction of precision boundary forms is time consuming; accuracy involves human factors.
Resistance Network	
High accuracy, and free from human factors; extremely flexible in the setting up of boundary values and fields.	More expensive; finite difference manner of representing a continuum needs special attention.

A literature survey has revealed an intensive progress in the art of electrical analogs, and the published papers on this subject are numerous; most of these papers deal with very specialized problems in other fields of engineering. Also there are many other forms of analogs and applications which are considered beyond the scope of this paper and consequently have had to be omitted.

ACKNOWLEDGMENTS

The writer wishes to express his gratitude to Mr. Michel Hug for his many brilliant ideas and his work on the resistance network analog, and to the staff members of the Iowa Institute of Hydraulic Research, who have contributed to the improvements of these analogs. This work was supported by the Office of Naval Research under Contract N8onr-500, T. O. III., and the David Taylor Model Basin, U.S. Navy.

DISCUSSION

Mr. McNown initiated the discussion by emphasizing the need for the solutions of a great many potential-flow problems. He stated that the analogs probably offered considerably more power in the attack on these problems than theoretical methods which, in part at least, have been exhausted. The relaxation approach is numerical by nature but it offers serious limitations in the time and patience required. However, he warned anyone who wished to use the electrolytic tank first to take steps to have a man with the dexterity and skill of Mr. Ling to set up the models.

Mr. Murray then inquired if circulation can be superposed in the analog. Mr. Ling replied in the affirmative. For the case in point the flow would be two-dimensional for which the paper-conducting technique would be suitable. Circulation is superposed upon the field by modifying the values of the input potentials on the boundaries in accordance with a desired or trial circulation.

Baines pointed out that free-streamline problems are the ones for which potential flow solutions are most commonly needed in practical work, and that they are more difficult than problems with fixed boundaries to handle by relaxation, and asked which of the three techniques is best for this purpose. Mr. Ling thought that the paper technique is easier for approximate solutions, but that the network analog is superior because it also does not require the construction of special models and it is more precise.

Mr. Baines inquired further about application to problems involving gravitational effects. Mr. Ling asserted that this could also be done very simply by measuring the current flowing through the net, instead of the potential gradient, and determining the velocities along a trial boundary from the current readings.

An unidentified discusser commented that the network is not necessarily limited to the analysis of potential flows and inquired whether analogs have been used to study other problems. He also called attention to the fact that M. Germain at the University of Brussels had made an electrolytic tank for two-dimensional problems, equipped with an automatic sensing probe and a pantograph system, which automatically plotted the field on a sheet of paper. Mr. Ling replied that there are many differential equations which can be solved by networks, as has been shown by Mr. Kron of G.E. who has contributed most to this art. By using network principles, time-dependent and other resistance elements can be used to solve various types of differential equations, such as for the fluttering of a wing. The network described here is a very simple type because it is designed to solve only one equation. In response to the second remark he stated that there are many papers describing automatic analog systems, but that these are of little interest in connection with the solution of hydraulics problems where interest is focussed upon the velocity and pressure distributions on solid boundaries rather than upon the flow characteristics in the entire field.

Mr. Bauer wished to know the order of magnitude of the time required to set up and solve a typical flow problem, such as for a two-dimensional inlet, or an analog computer. Mr. Ling indicated that problems with a definite boundary form could be solved in less than a day, but that flow problems for which a trial and error procedure is used would take more time; although he estimated the analog method to be 20 times faster than the relaxation process.

Mr. DeHaven inquired about the adaptability of the analog to 3-dimensional problems. Mr. Ling assured him that it was applicable, but that the set-up became much more complicated since many more boards are required. Nevertheless such systems have been built.

Mr. McPherson called attention to the fact that there is a paper analog manufactured by G.E. available on the market for about a hundred dollars, which includes a rectifier, a voltmeter, a pantograph, a mounting board, and about a five years' supply of paper. Mr. Calehuff contributed the additional information that the G.E. computer is now available from Sunshine Electric in Philadelphia.

Finally, Mr. McPherson asked whether, in order to duplicate seepage flow studies, it would be necessary to have resistances with a squared characteristic. Mr. Ling stated that that was not necessary, that in Darcy's law, the equation for the pressure drop, one could operate on the force potential instead of on the velocity potential. Mr. McPherson also mentioned that there are many electric-power network analyzers throughout the country in all metropolitan areas which might be available for the solution of potential flow problems.

REFERENCES

1. Rouse, H., *Fluid Mechanics for Hydraulic Engineers*, McGraw-Hill, 1938.

2. Goldstein, S., *Modern Developments in Fluid Dynamics*, Oxford Press, 1950, p. 116-118.

3. Kirchhoff, G., Ann. Phys., Lpz., 64, p. 497, (1845); Collected Papers, Lpz., 1882, p. 56.

4. Liebmann, G., "Electrical Analogues," *Br. J. App. Physics*, V. 4, N. 7, July 1953, p. 193-200.

5. Hubbard, P. G., and Ling, S. C., "Hydrodynamic Problems in Three Dimensions," ASCE Paper No. 2569, Trans., Vol. 118, 1953, p. 1046.

6. Keron, G., "Equivalent Circuits of Compressible and Incompressible Fluid Flow Fields, *J. of Aero. Aci.*, V. 12, 1945, p. 221-231.

7. Grinter, L. E., *Numerical Methods of Analysis in Engineering*, Macmillian Co., 1949.

8. Schreebeli, M., "Nouvelles Methodes de Calcul Pratique des Ecoulements de Filtration non Permanents a Surface Libre," *La Houille Blanche*, No. Special B/1953.

9. McNown, J. S., et alii, "Applications of Relaxation Technique in Fluid Mechanics," ASCE, Pro., Vol. 79, Sep. No. 223, 1953.

10. Tasny-Tschiassny, L., "The Triangulation of a Two-Dimensional Continuum for the Purpose of the Approximate Solution of Second-Order Partial Differential Equations," *J. App. Phys.*, May, 1949, V. 20, p. 419-424.

INSTRUMENTATION FOR STUDIES OF LOW-VELOCITY WINDS

by

JOHN F. RIPKEN
University of Minnesota

INTRODUCTION

Physical measurement of the magnitude and direction of low wind velocities is a problem encountered in a wide variety of technical studies.

This paper will attempt to briefly summarize an investigation conducted at the St. Anthony Falls Hydraulic Laboratory of the University of Minnesota for the purpose of selecting the anemometer most suited to a specific end use. The end use in this case required a field measuring instrument capable of giving reasonable sensitivity, response, and accuracy for the velocity magnitude and direction of winds ranging from 0 to 50 fps. The output signal was to be electrical and capable of remote transmission, and the sensory instrument was to be capable of operating with a non-rigid support or balloon suspension.

The report describes several pertinent existing instruments disclosed by a literature survey of the subject, together with the physical studies conducted at St. Anthony Falls to modify or adapt certain of these instruments to the specific problem.

Although many basic physical phenomena have been employed for indicating the character of a wind, the most practical instrument developments of the past have in general been motivated by either the dynamic pressure or the thermal cooling action of a wind.

This paper is therefore roughly divided into consideration of these two systems of motivating anemometers, plus a third more recent type represented by the ionization anemometer.

DYNAMIC PRESSURE ANEMOMETERS

General Considerations

The pressure or force exerted by a wind has been used as a basis for wind-velocity measurement since the earliest attempts to develop anemometers. The forms of such instruments have been widely varied and certain of these today, as in the past, serve to give us most of the factual wind knowledge that we have. In general, these instruments may be assumed to fall into three basic classes: those which measure, in effect, the unit dynamic pressure of the wind; those which measure the total dynamic force on a directly exposed area; and those which measure the speed or frequency of a pressure pulse. Devices of the first class are rather limited in form and are perhaps best represented by the common Pitot tube. Devices of the second class are numerous in form but may be considered to have two basic subdivisions, notably: those with essentially stationary pressure areas, such as the pressure plate or pendulum devices; and those with moving or rotating pressure areas, such as the cup or propeller anemometers. The third class is a modern type as represented by the vortex trail and the sonic anemometers.

No consideration will be given herein to the first class or unit pressure type of device because of the very small signal values resulting from their use with low-velocity winds and the problems involved in accurate measurement of such pressures under field conditions.

A number of useful forms of the second and third classes are described, including a drag-sphere anemometer development of the St. Anthony Falls Hydraulic Laboratory.

Pendulum Force-Measuring Devices

Although a unit pressure type of measurement proves physically difficult with low wind values, a consideration of the conventional drag-force equation for a wind acting on an exposed body indicates that the drag-force may be made a measurable value by exposure of a sufficient body area.

This principle is recognized in the mechanical pendulum anemometer, consisting of a plate so mounted that it rotates about a vertical axis to face the wind, while remaining free to swing about a horizontal axis above the center of gravity but in the plane of the plate. The plate swings upward to permit its weight to balance the wind force. An attached arc is calibrated in units of wind velocity. The device is undesirable as an accurate velocity instrument because of its unstable flutter in turbulent winds, limited velocity range, non-linear scale, and confusing sensitivity to the vertical wind component. While the instrument is simple and inexpensive, current interest is slight and chiefly academic. Described by R. Hook in 1667, it is notable as the oldest of mechanical anemometers.

A modified portable version of this device is manufactured today by the Illinois Testing Laboratories of Chicago, Illinois. This instrument, known as the "Alnor Velometer", consists of a small box containing a curved duct which is largely obstructed by a pivot-mounted, lightweight flat plate with a restoring force supplied by a hairspring. The swing of the spring-loaded flat plate, which is due to a stream of air entering the box through an orifice, has been calibrated on a graduated arc. With proper selection of orifice size, the instrument is calibrated to read velocities from 20 to 24,000 fpm with an accuracy of 3 percent of full scale. The lack of an inherent directional signal eliminated this unit from development in the program.

The Normal Pressure Plate Anemometer

The normal pressure plate anemometer of Sherlock and Stout [1]* is fundamentally similar to the pendulum types in that the displacement of a spring-loaded pressure plate serves as a calibrated index of the wind velocity. It differs from the pendulum in that the exposed area (and force) is quite large and remains essentially normal to the wind, and the area displacement is small and is directly and sensitively converted to an electrical output signal. A version of this type of instrument is shown in Fig. 1 and consists of the 8- by 9-in. pressure plate A which is kept normal to the horizontal wind by the weather vane C causing rotation about the ball bearing and slip rings at the top of the supporting mast. The pressure plate is attached at its lower edge to the support frame through a flexure hinge. The plate pressure is conveyed through a push rod to a stiff restoring spring D. Armature E which is attached to spring D varies the electrical inpedance of the coil F as the air gap or proximity of E varies with the wind pressure. The electrical value is applied to an A-C bridge circuit so that the resulting out-of-balance current is an index of the velocity.

This instrument was designed for the purpose of measuring storm gusts with velocities between 30 and 75 mph and proved to have a response time of about 1/8 sec. While this excellent instrument was designed for high velocity winds, it is conceivable that appropriate redesign could materially reduce the workable velocity range.

*Numbers in brackets in the text refer to the corresponding numbers in the Reference List, while numbers in brackets following figure titles similarly designate the source if other than the St. Anthony Falls Hydraulic Laboratory.

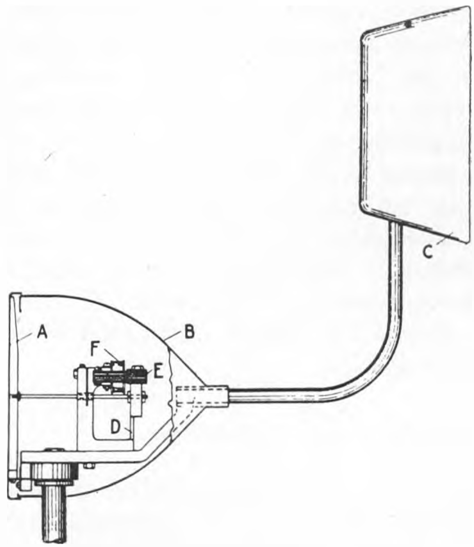


FIG. 1. THE NORMAL PRESSURE PLATE ANEMOMETER OF SHERLOCK AND STOUT [1]

The Drag-Sphere Anemometer

In an attempt to design a drag-area type of anemometer with sensitivity to low winds, quick time response, and with a measure of direction as well as magnitude, a preliminary design study was conducted at the St. Anthony Falls Hydraulic Laboratory. The result is shown in a schematic sketch in Fig. 2 and consisted of a 6-in. diameter lightweight drag sphere A connected via a plate B to flexure rod C which was in turn anchored to D. Since the cantilevered flexure rod was of circular section, the magnitude of its deflection was a measure of the horizontal wind speed with the displacement being determined by the direction of the wind.

The two electrical strain measuring devices employed are shown as F in the figure and were disposed 90 degrees apart in the horizontal plane and mounted on a rigid housing tube E. The strain units consisted of standard linear variable differential transformers and provided an accurate and sensitive way of electrically measuring a mechanical motion through push rod G in such a way that both the magnitude and direction of the applied wind could be interpreted from the output electrical signal.

The 6-in. sphere exposed to velocities from 1 to 50 fps operated in the fairly constant range of the spherical drag coefficient C_D lying between the low-end viscosity effects and the high-end flow separation criticals. The 3/16-in. diameter flexure rod permitted measurable deflections within the range of the linear transformers.

Additional studies of this type of device have disclosed similar developments by Vershinski [2], who employed a very similar unit for measuring turbulence pulsations in flowing water; by Knapp [3], who used a related form for detecting the direction and magnitude of water currents near the bottom of a harbor; and by the Baldwin-Lima-Hamilton Corporation [4] wind-velocity pickup for measurement of air currents to 600 mph. All of these devices produce an electrical output signal.

The drag sphere was not pursued in development because of the difficulties associated with its use on a non-rigid mounting.

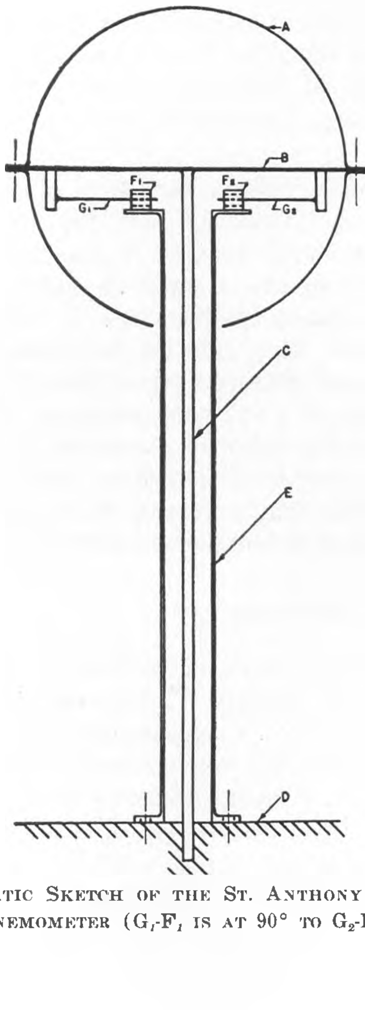


FIG. 2. DIAGRAMMATIC SKETCH OF THE ST. ANTHONY FALLS DRAG-SPHERE ANEMOMETER (G_1-F_1 IS AT 90° TO G_2-F_2)

The Vortex-Trail Anemometer

The natural occurrence of audible sounds as a function of the wind's velocity is a common observation. These so-called aeolian sounds or pressure phenomena are generally the evidence of the pulse period of the flow separation eddies which trail from behind various geometries when exposed to a flow. The eddies, which periodically form and shed from both downstream sides of a cylinder held normal to a wind, are a common form of this. This so-called Karman vortex trail has a shedding frequency which is usually expressed in a dimensionless quantity known as the Strouhal number S. In this quantity, $S = nd/V$, where n is the shedding frequency from one side of the cylinder, d is the cylinder diameter, and V is the stream velocity. The Strouhal number will apparently depend on all of those quantities which influence separation, including body geometry, Reynolds number, stream turbulence, body roughness, etc.

Roshko [5], working with a variety of cylinders in a low-turbulence-wind tunnel found, however, that the value of S was primarily dependent on the Reynolds number of the cylinder in the manner shown in Fig. 3. From this it may be concluded that for a substantial range of selected conditions, S is approximately equal to 0.2 or that $V \approx 5nd$. This concept has been utilized by Roshko [5], and Liepmann and Skinner [6] in the design of sensitive anemometers consisting of a cylinder equipped with a pulse-sensing device in the form of a hot-wire anemometer. The unit was used for accurate measurement of velocities down to 1 ft per sec. It appears quite possible that extended development of this type of instrument might lead to more practical forms.

Sonic Phasemeter Anemometer

If a sound pressure wave of velocity v is impressed upon a moving air stream of velocity V, the wave will travel with the stream at a velocity $(v + V)$ and against the stream at a velocity $(v - V)$. If the transmitted sound wave is compared with the wave received at a detector, a fixed distance s from the transmitter, the difference in phase of the two waves will yield a time difference Δt.

The wind velocity in such a case will be a function of the velocity v of sound in air under the existing conditions, the established base line distance s and the time difference Δt. Fortunately the velocity of sound in air is relatively insensitive to most of the more common atmospheric variations with the exception of temperature,

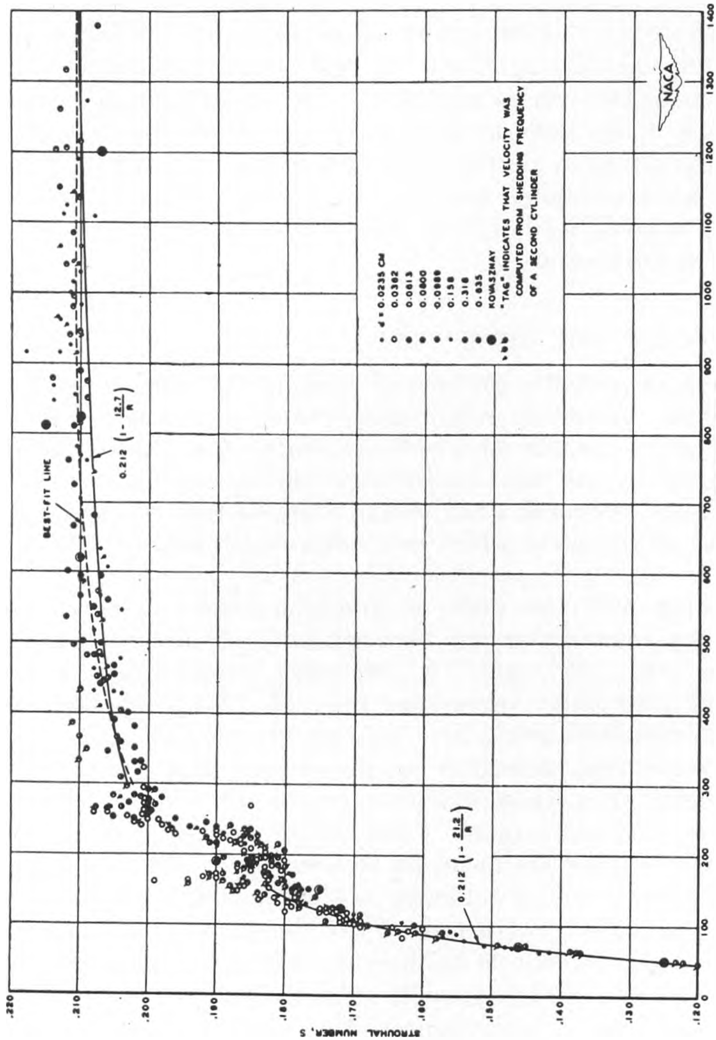


FIG. 3. VORTEX SHEDDING FREQUENCY OF CYLINDERS AS DETERMINED BY ROSHKO [5]

and since s can be quite easily established to a high order of accuracy, only the time difference $\triangle t$ poses a measuring problem.

The value of $\triangle t$ involved in measuring wind velocities of 1 to 50 fps over a base line of a few feet may be roughly calculated as of the order of 10^{-3} to 10^{-5} sec. While this is a minute physical value, it can, in the case of repetitive signals, be measured by modern electronic devices known as phasemeters. Brief accounts of such developments by Corby [7] and Kalmus [8], indicate that very low velocities can be measured with very good time response, and the use of multiple detector stations will permit the measurement of the direction as well as the magnitude of a wind.

The sonic phasemeter anemometer appears to have a number of attractive possibilities but is in need of extensive further development and simplification.

The Rotational Cup Anemometer

The cup anemometer consists of a number of cups affixed to the ends of light horizontal radial arms which are attached in turn to a hub which rotates on a vertical shaft. The differential drag force exerted by the wind on the cups varies with the orientation of the cup and produces a net torque force on the wheel leading to a terminal rotary speed which is a calibrated function of the wind speed.

Beginning with the work of Robinson over a hundred years ago, the cup anemometer has been subjected to a continual study [9], [10], [11], [12], and [13], resulting in the stable, effective instrument used today to procure the bulk of our meteorological data. Anemometers which are required to energize a mechanical counter or electrical generator usually employ large cups of about 5-in. diameter on a wheel diameter (center to center of cups) of about three times this value. Units which are designed for sensitive research studies are about 40 percent of the size of the larger units and activate but do not energize the counting mechanism.

The lightweight construction of the sensitive units limits their speeds to winds of about 90 fps because of centrifugal forces and a continual reversal of stresses with each revolution.

When equipped with precision ball or jewel bearings, the starting speeds of some of these units are as low as ½ fps with running speeds of ¼ fps [11], [14], [15], and [16]. The response time of the instrument is, however, rather slow requiring that a change in wind speed must persist for a travel length of about 20 wheel diameters before the wheel acquires the new velocity. The cup wheel also tends to over-register in gusty winds. The peripheral speed of the wheel at the cup centerline is usually between 30 and 50 percent of the wind speed.

Both the large and small cup anemometers are available as standard production items by a number of makers. Although cup anemometers have been flown from kites with workable results, they were not considered an answer to the needs of the program.

The Vane, Windmill, and Propeller Anemometers

This type of anemometer is essentially a windmill consisting of blade areas arranged at the end of radial arms which are in turn attached at their inner ends to a hub which is free to rotate about a bearing spindle. In most cases the plane of the rotor is positioned perpendicular to the wind, and the spindle is horizontal and in line with the wind. The blades are positioned so the aerodynamic lift force of the wind causes rotation about the spindle at a terminal rate which is a linear function of the wind velocity.

The blade at a mean radius usually moves at a rotary velocity about equal to that of the wind, thus producing relatively high rotary speeds with good sensitivity and readability at low velocities. Since the blades run with the wind with only a light and steady aerodynamic loading, the blading can be made very light with resulting sensitivity to about $\frac{1}{2}$ fps and responsiveness (to $\frac{1}{6}$ sec).

Sensitive research units of the windmill type have been developed by Richard [13] and Fergusson [13].

A small portable unit of this type is made by a number of manufacturers for use in ventilation studies and is sometimes called a vane anemometer. These units must be manually oriented in the air stream and have a protective cylindrical shroud around the blading. They usually have: a horizontal shaft, a shroud diameter of about 4 in., about 8 blades, a speed range of about 1 to 50 fps, and a mechanical counter dial indicating either the wind velocity or the number of feet of wind which has passed through the instrument in a given operating period. A quite complete discussion of this type of instrument has been given by Ower [17].

A larger modern unit, the "Aerovane" by Bendix-Friez, of Baltimore, Maryland is a commercial development of the three-bladed propeller type and is capable of being used at permanent meteorological stations to provide full velocity and directional information. The large molded plastic propeller drives a directly connected electrical generator which transmits a signal for remote reading or recording. The generator unit is contained in a streamlined housing which serves also as the support for the lightweight, aerodynamically stable, responsive, directional vane which keeps the propeller oriented in the wind. The vertical spindle also contains a self-synchronous motor for transmission of the directional signal. The unit has been tested to show a starting speed of about 2 fps and has sustained wind-tunnel speeds of 300 fps. The mass inertia of this rugged generating unit requires that a change of wind speed must persist for a travel length of about 20 ft before the propeller acquires its new speed.

The characteristics of the unit are described by Conover [18]. This is a relatively heavy unit designed for rigid mounting. Its use in the development program was not pursued.

THERMAL ANEMOMETERS

General Considerations

The thermal anemometer is a device which measures wind velocity as a function of the rate at which heat is removed from a heated body placed in the wind. It involves no moving mechanical parts and is based on the extensive developments found in the field of thermal measurements wherein thermal-electrical conversion permits ready use of a wide variety of accurate and responsive indicating or recording electrical components. The thermal anemometer has proved especially adaptable to low-velocity wind measurements because of its inherent sensitivity in this range.

In the following discussion we shall consider three of the many forms of thermal anemometers that have been devised. These three are believed to be the most significant and include the hot wire, the Thermistor and the heated thermocouple. Emphasis will be given to the Thermistor studies conducted at the St. Anthony Falls Hydraulic Laboratory.

The Hot-Wire Anemometer

This instrument consists of a small-diameter, short, cylindrical wire which is electrically heated and exposed to a moving air stream in such a way that the rate of thermal dissipation from the cylinder surface may be electrically indicated through the heating circuit.

Since all electrical systems for detection of thermal change are essentially a measurement of electrical resistance, hot-wire instruments are usually arranged to maintain either a constant temperature (this is equivalent to a constant resistance) or a constant current in the exposed member by use of a bridge circuit. King [19] established that for a wire placed normal to the wind and with a constant temperature operation $i^2 = i_o^2 + K\sqrt{V}$, where i is the electrical heating current, i_o is the current at zero velocity, V is the stream velocity and K is a calibration constant dependent on the properties of the wire, the air, and the circuit.

From the equation it is apparent that the fourth root relation between the current and velocity provides a very high degree of sensitivity at low air speeds. This is graphically demonstrated in the test data of Fig. 4, as taken from Ower [17].

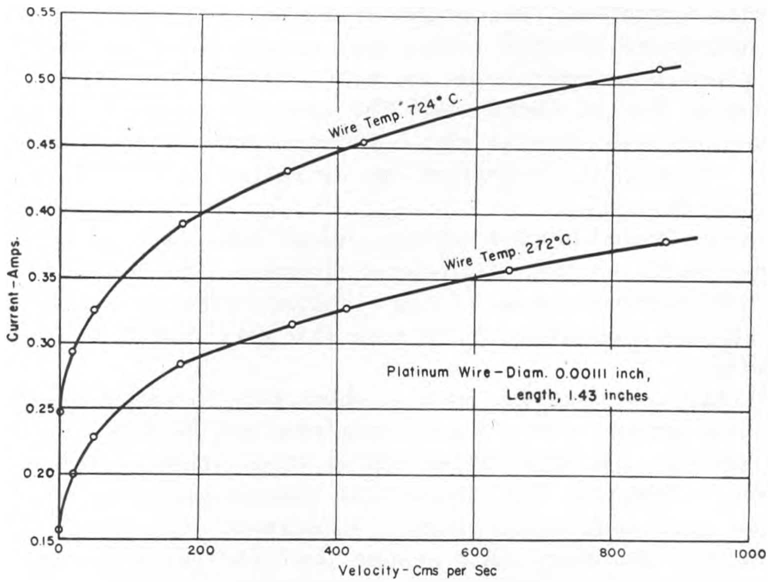


FIG. 4. TYPICAL CHARACTERISTIC CURVES FOR A HOT-WIRE ANEMOMETER POSITIONED NORMAL TO THE AIR STREAM AND OPERATED UNDER CONSTANT TEMPERATURE CONDITIONS [17]

The equation constant K has proved to involve a large number of variables which cannot be analytically determined and reliance must be placed on physical calibrations similar to Fig. 4.

The usual form of the sensory element of the instrument consists of a fine wire ranging from 0.0005 to 0.005 in. in diameter and a length ranging from about ⅛ to ½-inch. Since accurate measurement of resistance with a Wheatstone bridge circuit increases with the magnitude of the resistance, use of a wire of minimum diameter and maximum length is desirable. However, in practice this selection is a compromise, including consideration of structural strength, ruggedness, thermal stability, chemical stability, electrical stability, etc. Platinum is the most frequent answer to these considerations.

The wire is usually heated to a value varying from 400 to 900 F with the more measurable values and sensitivity associated with the higher temperatures. Unfortunately, the higher temperatures also involve a high current demand, a structural weakening, and danger of melting.

The useful velocity of various designs of this instrument has ranged from $\frac{1}{4}$ fps to supersonic values, and because of the small sensory mass the units exhibit an unusual rapidity of response to velocity fluctuations. This characteristic has been exploited to make the instrument the chief tool in the examination of air turbulence, and commercial instruments are now available with a time of response as low as 0.0003 sec. The measuring sensitivity is, unfortunately, accompanied with a physical delicacy and time instability which limits instrument use to refined laboratory environs or research studies.

An additional property of this unique instrument is also found in its inherent sensitivity to wind direction. The convective heat loss and electrical output of the cylindrical wire are a function of the angular orientation of the wire axis with respect to the wind direction.

Weske [20], working with a single wire, established good directional sensing within a given quadrant and Ferrari [21], working with two parallel, closely spaced wires extended this to 180 degrees. Goldstein [22] states that angular values in the latter system could be obtained within $\pm \frac{1}{4}$ degrees.

Ferrari also described a system involving two wires arranged as a "V" with a vertex angle of 10 degrees. These wires when pointed into the wind by manual rotation were very sensitive to asymmetry of position, permitting angular determinations to a fraction of one degree.

Additional data on the hot-wire anemometer may be found in the summaries of Goldstein [22] and Willis [23]. The fragile nature and unstable performance of the hot-wire anemometer did not lend itself for use in the development program.

The Thermistor Anemometer

The Thermistor anemometer measures an electric voltage (or resistance) as the result of the convective cooling action of a wind on a self-heated proprietary resistance element known as a Thermistor. The general nature of the instrument action is quite similar to that of the hot-wire anemometer, but the inherent properties of the Thermistor produce a greater sensitivity, ruggedness, and signal value.

The Thermistor, which is a semiconductor compounded largely of metallic oxides, differs from the conventional thermal-electric conductor metals in that: (a) the specific resistance is relatively very high, (b) the rate of resistance change with temperature is as much as 500 times as high as that of the competitive metal platinum, and (c) unlike metals, the resistance decreases as the temperature increases.

The Thermistor, which has a long history, is only a recent commercial offering because of earlier difficulties with production quality control. However, it is available today in a variety of shapes, sizes, and compoundings.

A tiny spherical bead form of the material has been investigated by Hales [24] and Sanford [25] as an anemometer with the conclusion that it provided a meter which was relatively sensitive, relatively responsive, stable, simple, compact, reasonably rugged, and inexpensive.

Typical fundamental data for a unit of this type is shown in Figs. 5 and 6 as measured at the SAF Laboratory.

In Fig. 5 the static resistance-temperature curve was obtained by heating the unit in an oven with self-heating being avoided. The volt-ampere curve, on the other hand, demonstrates the linear Ohm's law relation in the unheated left end region with a progressive deviation as the current becomes sufficient to cause self-heating. The temperature values on the latter curve were computed from the resistance-temperature curve.

The dynamic data of Fig. 6 was obtained from a whirling boom-test facility with the bridge manually balanced to maintain the bead at roughly 250 F (the resistance being held constant at 2000 ohms). The Thermistor current was measured by a series ammeter and the signal voltage calculated therefrom.

Subsequent operations with this unit eliminated the manual balancing operation by introduction of the self-balancing bridge shown in Fig. 7 and linearization of the output signal was accomplished as per Fig. 8. Ambient temperature compensation was provided by the battery in the circuit with manual adjustment to cancel the zero velocity current flow. All of the above operations employed the constant-temperature type of circuit operation rather than constant-current because of certain associated control problems. A constant current circuit was, however, employed in most of the directional studies to be discussed.

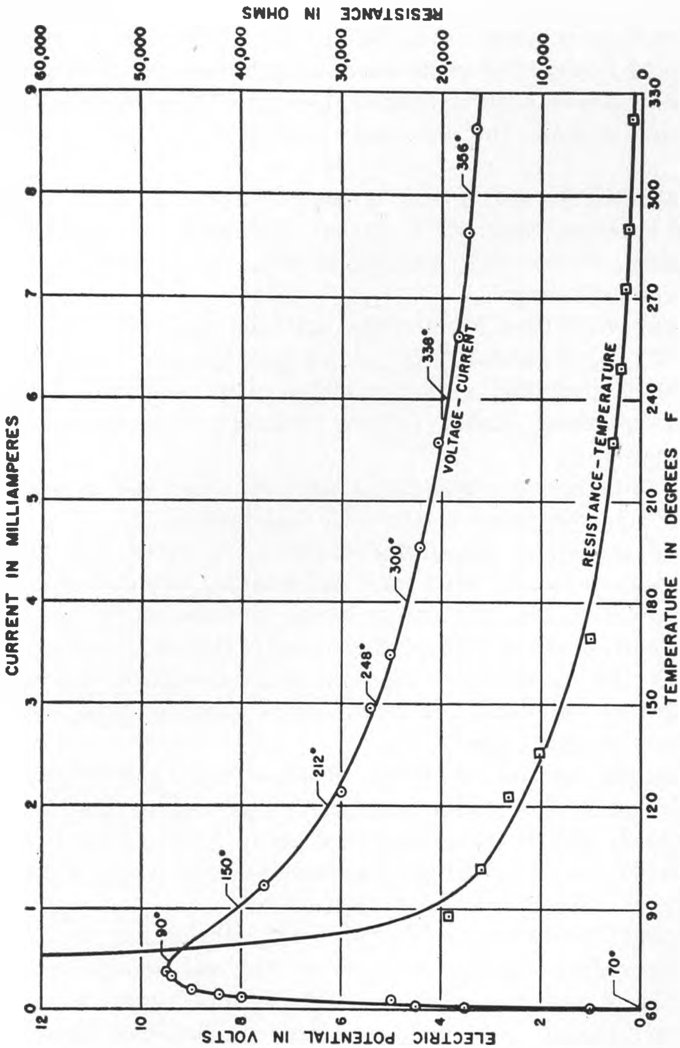


FIG. 5. ELECTRICAL CHARACTERISTICS OF WESTERN ELECTRIC D 176980 BEAD THERMISTOR

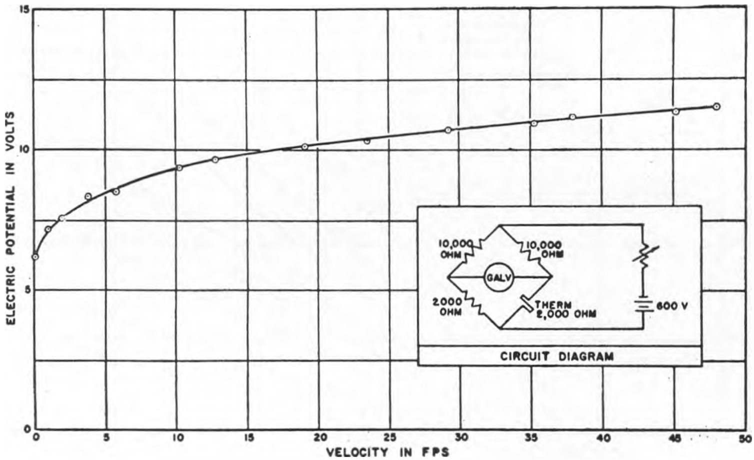


FIG. 6. VELOCITY CALIBRATION AT ROOM TEMPERATURE OF WESTERN ELECTRIC D 176980 BEAD THERMISTOR FOR CONSTANT-TEMPERATURE (OR RESISTANCE) OPERATION

While the bead Thermistor just described proved to be an adequate velocity measure, it possessed no directional properties as required by the Laboratory program, therefore, concurrent studies were conducted on a cylindrical-rod shape. These studies were based on King's [19] finding of directional sensitivity in his hot-wire studies of cylindrical forms.

For this purpose the longest and thinnest form of rod Thermistor commercially available was selected for study. This unit was rod No. 517613-1 as made by Friez Instrument Division, Bendix Aviation Corporation, Baltimore, Maryland, and was 0.018-in. diameter and 1.5-in. long.

The directional characteristics of a single rod rotated in a plane

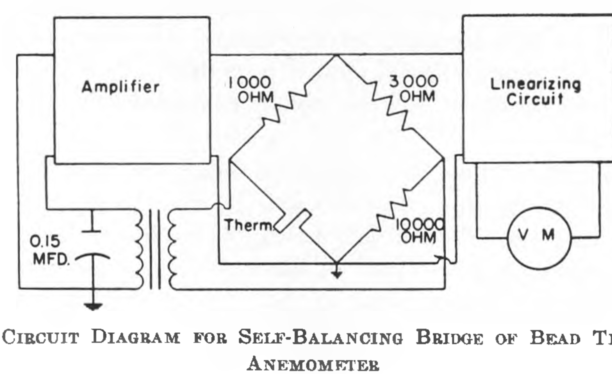


FIG. 7. CIRCUIT DIAGRAM FOR SELF-BALANCING BRIDGE OF BEAD THERMISTOR ANEMOMETER

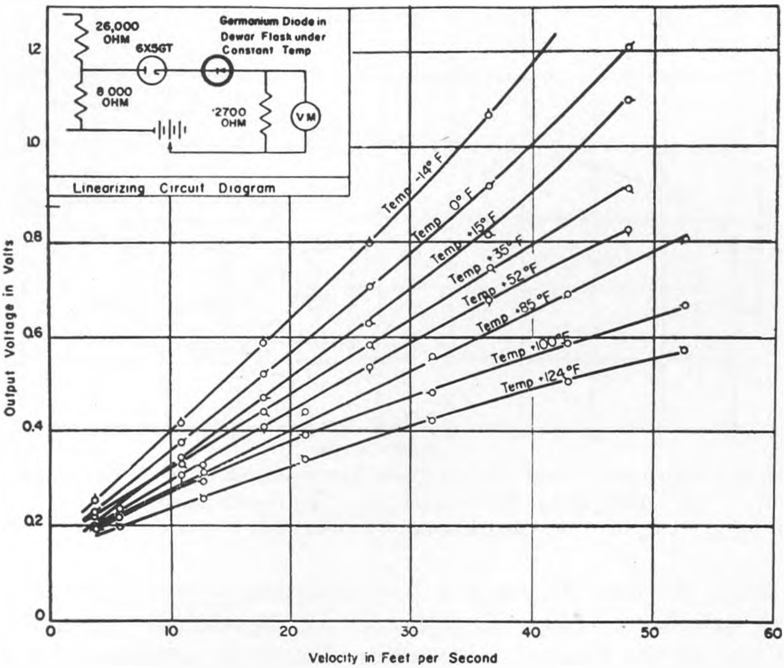


FIG. 8. TYPICAL CALIBRATION OF LINEARIZING CIRCUIT FOR BEAD THERMISTOR ANEMOMETER

parallel to the wind and about an axis perpendicular to the center of the rod axis, are shown in Fig. 9. This demonstrates a definite ability to measure direction as a voltage sine function for any 90-degree quadrant of the horizon but with no discrimination as to which of the four quadrants. The discrimination was therefore extended by utilizing two parallel rod Thermistors rotated about their longitudinal axis of symmetry. The circuit and resulting data are shown in Fig. 10. In this, the longitudinal rod axis was vertical and normal to the direction of horizontal air flow, and the rods were spaced at about 0.06-in. center to center. The introduction of a positive and negative voltage signal extended the discrimination to 180 degrees.

A correction of the lobe shape to nearly a sine function was accomplished by inclining the rod pair to $\Omega = 15$ degrees as shown in Fig. 11, and the discrimination was extended to 360 degrees of horizon by combining two pair of inclined rods as shown in Fig. 12 with the circuit as shown in Fig. 13. Tests on this assembly are currently under way using a Selsyn output with dial indicator.

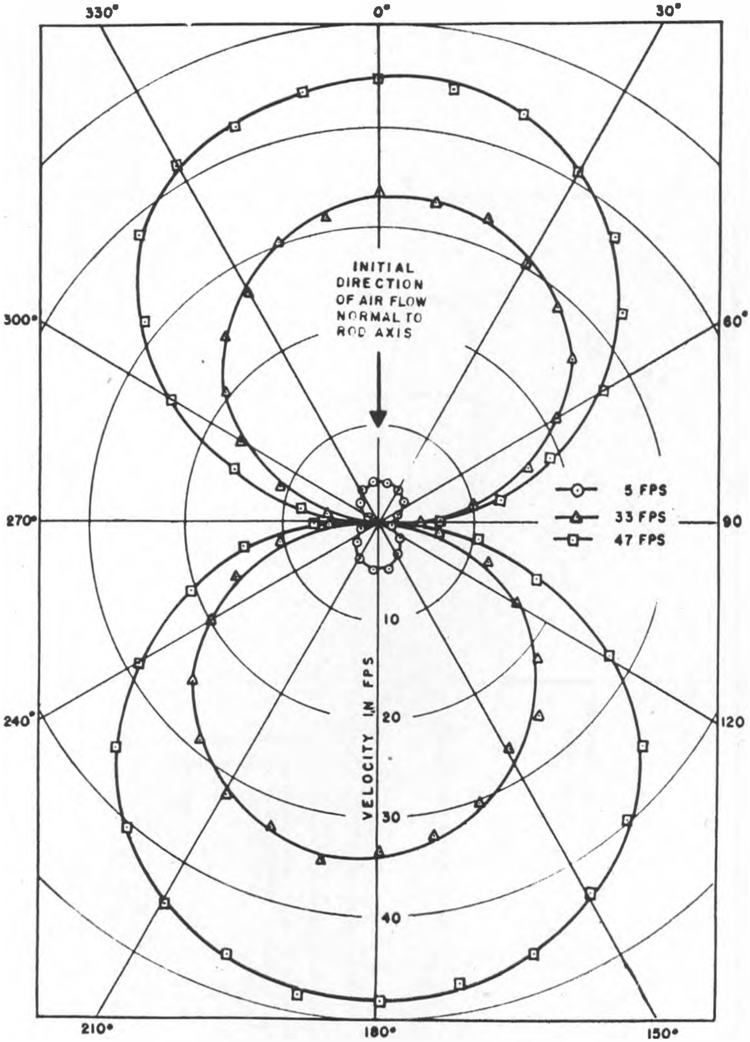


FIG. 9. DIRECTIONAL CHARACTERISTICS OF A SINGLE BENDIX-FRIEZ 517613-1 ROD THERMISTOR ROTATED IN A WIND UNDER CONSTANT-TEMPERATURE CONDITIONS

The tests indicate that a full 360 degrees of directional sensing will be achieved with an angular error of less than 10 degrees.

The foregoing directional studies relate to a field determination of the horizontal components of a wind. However, since natural winds normally possess some third dimension or vertical component, it is apparent from the data of Fig. 11 that such a vertical component will materially obscure the determination of the horizontal direction. As a corrective for this, the two rod pairs shown

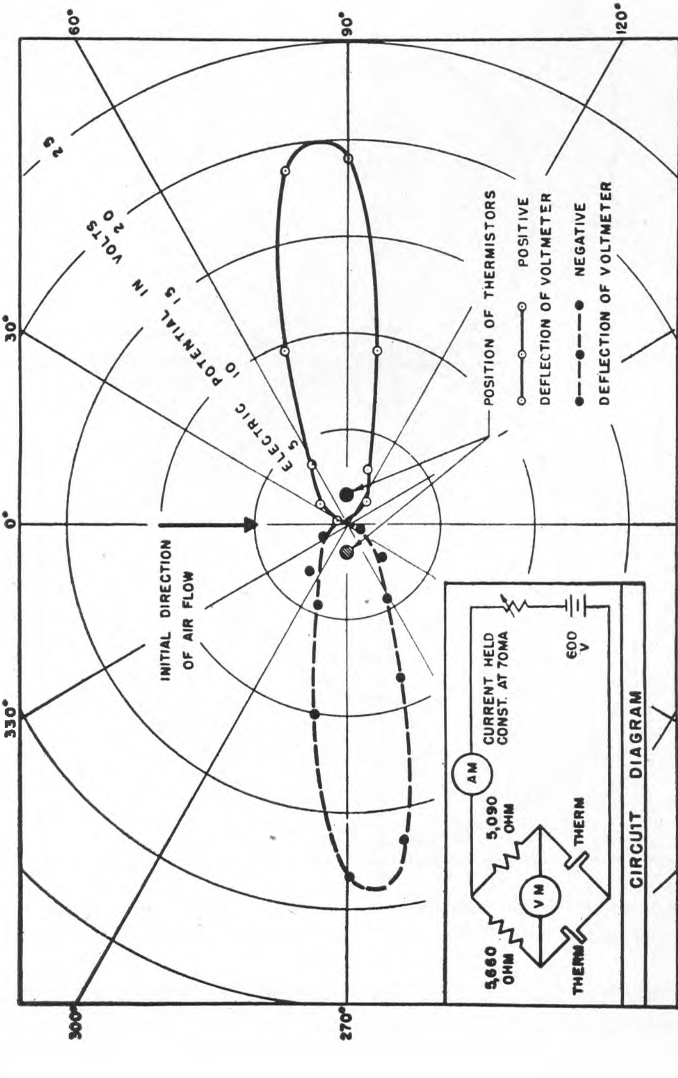


FIG. 10. DIRECTIONAL CHARACTERISTICS OF A PARALLEL PAIR OF BENDIX-FRIEZ 517613-1 ROD THERMISTORS ROTATED IN A WIND UNDER CONSTANT CURRENT CONDITIONS

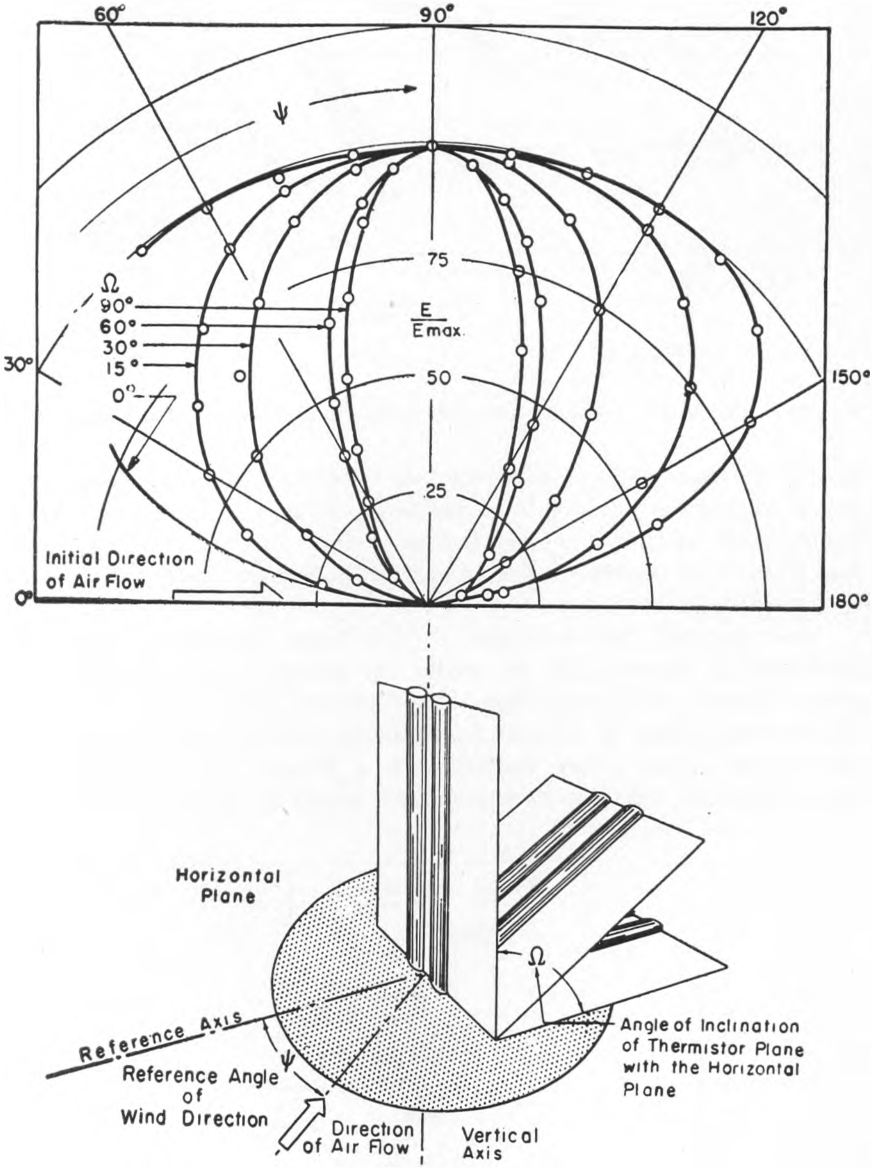


FIG. 11. DIRECTIONAL CHARACTERISTICS AND ARRANGEMENT OF A TILTED PAIR OF ROD THERMISTORS

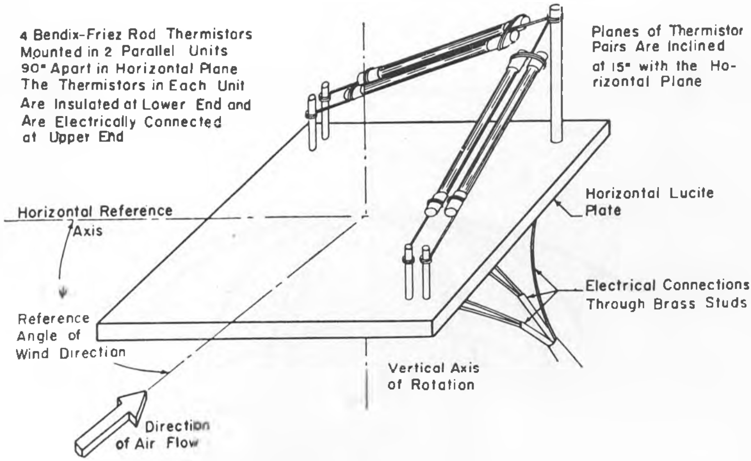


FIG. 12. DIRECTIONAL SENSING ASSEMBLY USING TWO PAIRS OF ROD THERMISTORS

in Fig. 12 were housed between two parallel horizontal disks. The rolled-edge disks were of 5-in. diameter, spaced ½-in. apart. Wind-tunnel tests of this assembly indicated that vertical wind angles of less than ± 30 degrees failed to seriously obscure the determination of horizontal direction.

The thermal time response of the bead Thermistor has been evaluated at about 1/20 sec while the parallel rod assemblies required from 1 to 7 sec. Some experiments were made to reduce the thermal mass of the rod by reducing its size through use of a thin metal support core coated with a Thermistor overlay. Pilot tests indicated that the time-constant might be reduced by 80 per-

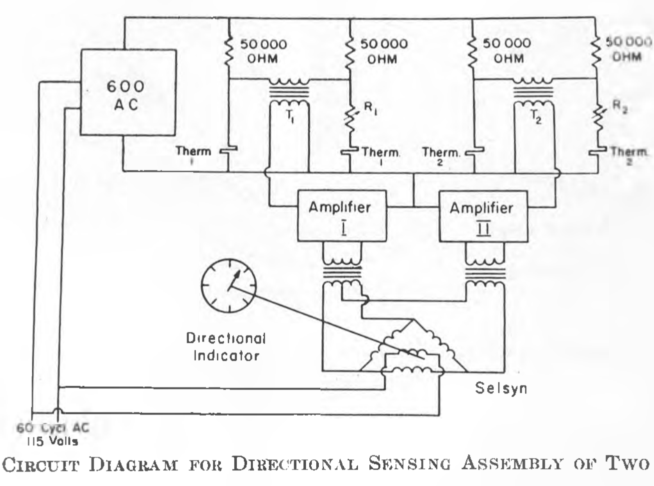


FIG. 13. CIRCUIT DIAGRAM FOR DIRECTIONAL SENSING ASSEMBLY OF TWO ROD PAIRS

cent with such means. The Thermistor coating material is a recent development giving promise of many useful instrument applications.

Because of the demonstrated ability to answer most of the needs of the development program, the Thermistor anemometer has been selected as the final choice for the program.

The Heated-Thermocouple Anemometer

The thermocouple, long used in the field of thermal measurements, depends on the fact that when two selected dissimilar metallic wires are fused at their ends, application of heat to the fused junction will generate an electromotive force to support a flow of current across the junction. Since the total emf generated is only a small absolute value, accurate measurement of the temperature influence requires accurate measurement of a very small voltage. To amplify this voltage for increased accuracy, it is customary to arrange a number of thermocouples in series and, thus multiply the effect. Such a device with a large number of "hot" and "cold" junctions is known as a thermopile.

The special modifications which permit this device to be used for air velocity measurements consist basically of heating the hot junctions of the thermocouple or pile with a measured supply of heat from an external source. Exposure of the heated thermocouple to the air stream whose velocity is to be measured will result in convective cooling of the heated junction and a consequent drop in the emf impressed across the junctions of the pile. This relation results in an emf which is inversely proportional to the air velocity. The emf may be read as velocity on a suitably calibrated voltmeter.

In the earlier forms of this instrument the metered heat supply for the hot junction was in the indirect form of a selected electric current fed into a resistance heating coil which surrounded or contacted the hot junction. In the modern form [26] and [27], heating is accomplished by superimposing a suitable alternating current directly across the thermopile itself while simultaneously maintaining a direct current measurement of the developed thermocouple emf. This simplification and reduction of the sensory probe mass reduces the thermal lag and improves the time response of the instrument to fluctuating velocities.

The general nature of the calibration curve obtained with a heated thermocouple anemometer is demonstrated in Fig. 14, which is taken from [27]. The velocity equations covering this occurrence stem from the relations which King [19] established for the hot-

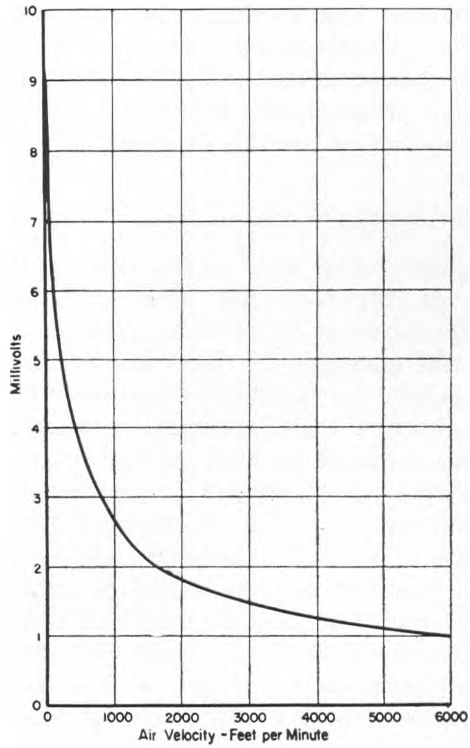


FIG. 14. VELOCITY CHARACTERISTICS OF A TWO ELEMENT THERMOCOUPLE ANEMOMETER [27]

wire anemometer and again give a fourth power equation leading to high sensitivity in the low velocity range. The instruments can be made very versatile in range by simple adjustment of the heating current and various forms have been designed for velocities varying from 0.08 fps to 600 fps.

The time response of the instrument is among other things a function of the wire size and wind velocity and practical field instruments with a wire of 0.003-in. diameter have established a time response of the order of 0.10 sec.

A durable, all-weather, non-directional field instrument is shown in Fig. 15. The small blobs on the diagonal wires are the hot-junction thermocouples. This instrument is commercially produced by the Hastings Instrument Company of Hampton, Virginia and is provided with compensating circuitry making it adaptable to a wide range of conditions.

A smaller portable battery operated probe unit for use in air conditioning studies is commercially produced by the Illinois Testing Laboratories of Chicago, Illinois.

The geometry of the thermocouple affords none of the directional sensitivity inherent to the cylindrical form of the hot wire and the Thermistor rod anemometers and there is no evidence of thermocouple research with this objective. This lack of inherent directional sensitivity, plus a relatively weak basic signal, eliminated the instrument from further development in this program.

IONIZATION ANEMOMETERS

The structure of air which has been exposed to certain electrification processes is said to be ionized if it contains a greater than normal number of free ions.

In an ionization anemometer the ionized air particles are used as an inherent tracer of relative air motion in much the same

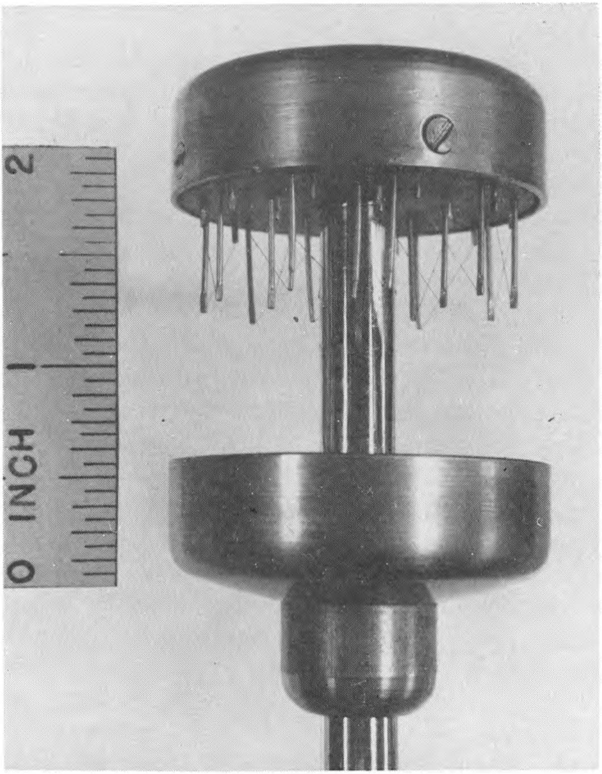


FIG. 15. NONDIRECTIONAL ANEMOMETER OF THE HEATED THERMOPILE TYPE AS MADE BY THE HASTINGS INSTRUMENT COMPANY

manner that dye or confetti are used as visual tracers and salt brine as an electrical tracer in motion studies in water.

In discussing an anemometer of this type, consideration must be given first to the method of ionization or production of the tracer material, and second, to the method of detection or measurement of this tracer.

A review of the first consideration establishes that while numerous methods are known for achieving ionization in air, only the methods employing bombardment by emissions from natural radioactive materials and excitation by an electric field of high potential gradient have proved of practical use with anemometers. A considerable study [28] and [29] of the latter method has established that production of a powerful field by use of spaced electrodes with high voltages (corona, glow and spark devices) can produce certain desired effects but at the expense of hazardous conditions and poor relative stability. The most practical low-velocity developments have, therefore, employed ionization by radioactive materials.

A review of the methods of measuring the tracer movement indicates that this may be done either by determining the manner in which the wind deflects the steady flow of ions between two exposed electrodes, or by marking the progress of a series of drifting ionized clouds in the stream. The latter system which is analogous to the salt-velocity method in water has been given considerable study [30] and [31] with the end conclusion that it was quite successful at high velocities, but the inherent mobility and diffusion of the ions destroyed the measurable character of the ion cloud if the wind velocity was less than about 30 fps. However, evidence [32] was found demonstrating that low-velocity winds could be measured as a function of the manner in which the wind deflected a steady stream of ions passing between two electrodes.

An anemometer of this type for measurement of velocity magnitude in a two dimensional or horizontal wind was evolved by Lovelock and Wasilewska [32] and subjected to an extended study at the St. Anthony Falls Hydraulic Laboratory. This unit, as shown in Fig. 16, consisted of two parallel, circular disks of about 1½-in. diameter separated by a predetermined gap.

The ionization of the passing air stream was achieved by exposure to Alpha-particle emissions from the radioactive coating of polonium electroplated on the upper disk. Polonium was selected because of its relative availability and low cost, and because it emitted a large number of alpha particles with a range of only about 1½ inch.

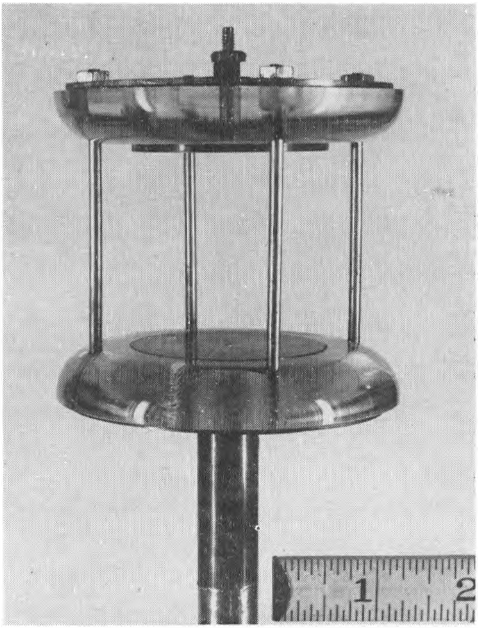


Fig. 16. The Parallel-Plate Ionization Anemometer

The ionized particles created near the disk were attracted to the emitter plate or the opposite disk by the electric field imposed on these electrodes from an external source.

In the Laboratory tests the spacing of the disks and the applied voltage were varied as the instrument was exposed to various velocities in a wind tunnel.

Under these conditions of continuous ion production and migration to the electrodes, a measurable flow of current was sustained between the electrodes. Since the passing wind tended to sweep some of the ions beyond the attractive range of the electrodes and promoted neutralization through increased ion recombination, a particular selection of ion production, disk spacing, applied voltage, and wind speed produced a specific flow of current. Typical data for tests of this type with a $5/8$-in. disk spacing are shown in Figs. 17 and 18. Figure 17 demonstrates that if the voltage is sufficiently high and the wind speed sufficiently low, virtually all of the ions will be attracted to the disks to support a stable, limiting current flow. The anemometer may be operated with either constant current or constant voltage, but the former system exhibits the better linearity of signal shown in Fig. 18.

In parallel with the foregoing velocity measuring studies, an attempt was made to provide an ionization instrument with di-

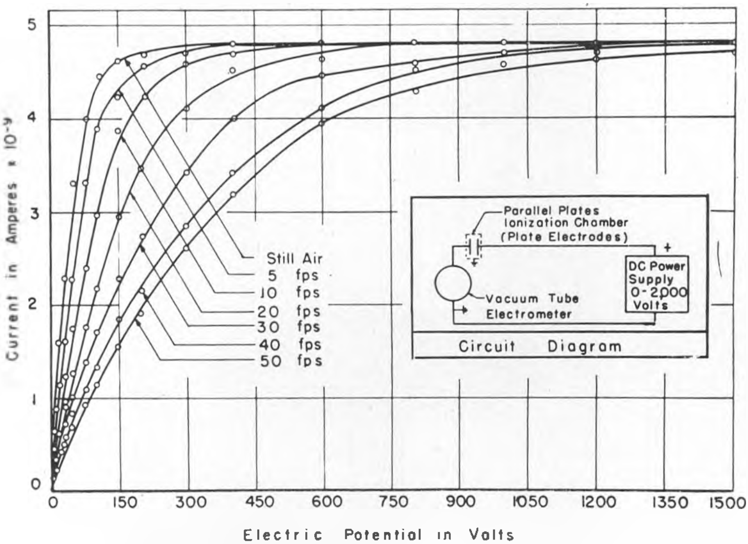


FIG. 17. VOLT-AMPERE CHARACTERISTICS OF THE PARALLEL-PLATE IONIZATION ANEMOMETER

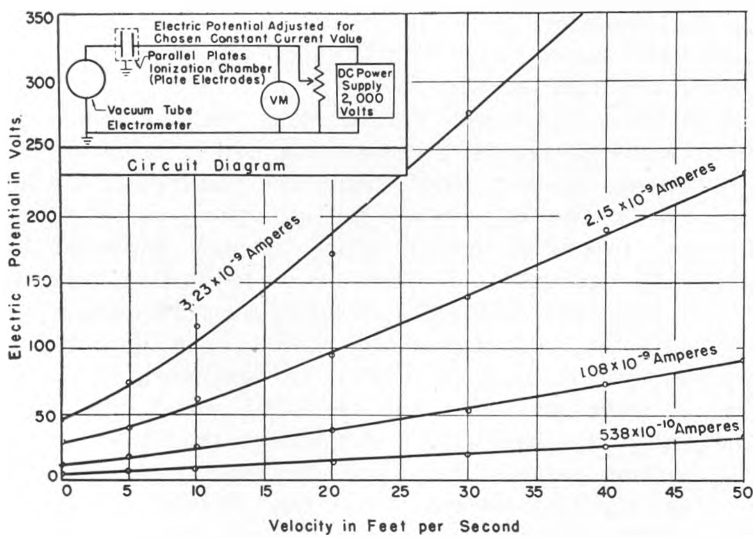


FIG. 18. VELOCITY CHARACTERISTICS OF THE PARALLEL-PLATE IONIZATION ANEMOMETER WITH CONSTANT-CURRENT OPERATION

rectional sensitivity. This development was principally based on the fact that if the charged electrodes consisted of two parallel permeable screens placed normal to the plane of a wind, the measured rate of collection of ionized particles existent between the screens would vary depending on the angular direction of the wind with respect to the planes of the screens. A two-screen arrangement of this kind produced a directional signal which was roughly analogous to that of the single rod Thermistor shown in Fig. 9, and similarly was useful only for directional measurement within a quadrant of the horizon. This limited discrimination of direction was extended to 180 degrees of horizon by addition of a third parallel screen as shown in Fig. 19.

In this unit the ionization was produced by the polonium coated disk mounted on the top plate and the current meter was connected to the center screen, thus receiving ion flow from the two screened ion chambers. The current was either positive or negative depending on the excess number of either positive or negative ions reaching the screen. The circuit and test data for the three-screen arrangement is shown in Fig. 20 which is roughly analogous to the paired Thermistor rod data as shown in Fig. 10. It is believed that a further extension of unique directional sensing to a full 360 de-

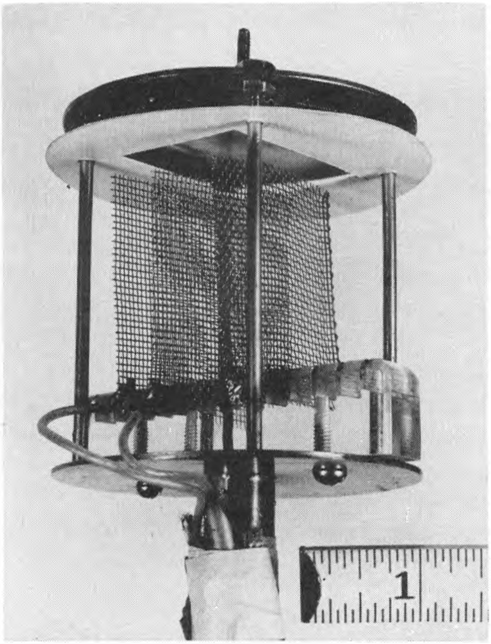


FIG. 19. DIRECTIONAL IONIZATION ANEMOMETER WITH THREE SCREENS

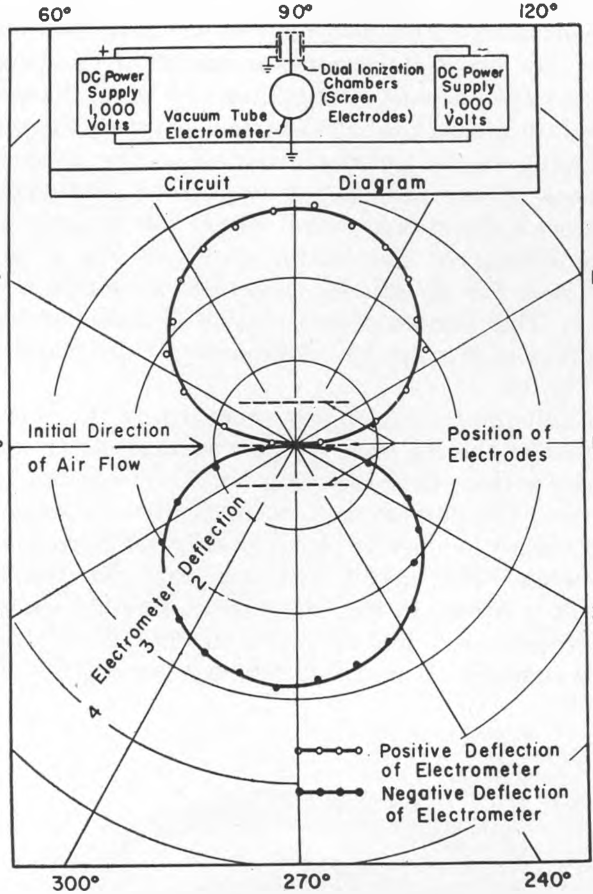


FIG. 20. DIRECTIONAL CHARACTERISTICS OF THE THREE-SCREEN IONIZATION ANEMOMETER

grees of horizon can be accomplished by orienting two such three-screen units 90 degrees apart and providing a suitable circuit in a manner similar to the general treatment of the two pair Thermistor rod assembly shown in Fig. 12.

Because of the field problems associated with the radiation hazard, high operating voltages, and weak inherent signals in this device, neither the velocity nor directional possibilities were fully pursued in the developments at St. Anthony Falls. The inherent sensitivity and responsiveness of the device are, however, very promising.

ACKNOWLEDGMENT

The survey studies discussed herein were initially sponsored by the Ballistic Research Laboratories of the Army Ordnance Corps with the subsequent research and development studies sponsored by the Evans Signal Laboratory of the Army Signal Corps.

Particular credit is due John M. Killen of the St. Anthony Falls Laboratory Staff for his contribution to the clarification of the electrical aspects of the survey discussions and for his prime contributions to all phases of the investigations later conducted at St. Anthony Falls.

DISCUSSION

Mr. Craven was impressed by the way in which this paper illustrated the principles stated in the earlier paper by Mr. Bergmann regarding the difference between direct and indirect measurement. He cited the ionization and oil-droplet techniques as good examples of direct methods in which a particle is identified and its time of passage between two known points is measured. When an indirect approach is used, it is necessary to know the law involved, such as the Bernoulli principle or a combination of the Bernoulli and viscous principles as with a drag sphere. Other examples of the indirect approach are the utilization of known laws for the travel of sound in a fluid or the laws of heat transfer for a thermistor or a hot-wire anemometer. In choosing a velocity-measuring instrument one has to decide whether to use a direct method or to use a law, and if it is to be the latter, to examine the law and ascertain exactly what the instrument indicates. He considered Mr. Ripken's paper an excellent example of the correct approach since he first considered the direct method, then the Bernoulli principle, and finally the law of heat transfer and its application in the final instrument.

Mr. Calehuff contributed information concerning the use of thermistors in boundary-layer studies at ORL after reading of Baines' work. Bead thermistors were used in circuits similar to those of the speaker, and either an oscilloscope or a Ballantine voltmeter was used to indicate the change in level of the AC component of the signals. They noted changes of 30 db in the signal level between the free stream and the boundary layer, and similar signal changes between the free stream and the wake of a model torpedo which was in the 48-inch tunnel at the time. He found that the indications of intermittency using this method were an interesting supplement to observations of intermittency by other methods. They plan still more applications of thermistors to determine where the boundary-layer transition occurs on models.

Mr. Ripken expressed approval of the further application of thermistors and mentioned the great improvements which had been made in their manufacture in recent years. He expects still more improvements and more extensive applications to follow soon.

Mr. Baines discussed the problems they had in using the self-heated thermistors to measure turbulence in water, and mentioned the speaker's success in using self heating. His program using external heating methods had run down because of the difficulties involved in perfecting the proper system, but not before they had found British thermistors superior to those of Western Electric with regard to quality and cost.

The speaker remarked that external heating had been used in a great many thermal anemometers with excellent results, but that the larger thermal mass in the sensitive head reduced high-frequency response to a point below what they needed. They generally use direct heating and try to select an optimum temperature which is high enough to reduce the influence of ambient temperature changes but low enough to avoid erratic operation of the thermistors due to their being overheated. They have selected 250° to 350° F. as the most workable range.

Mr. Moore asked whether a device consisting of a thermistor for magnitude detection and a vane for direction could be used in air on the time scale of the speaker's studies. Mr. Ripken knew of commercial and experimental devices using this technique, but had found them suitable only when a rigid mounting was available. With their non-rigid balloon mounting, the problem of maintaining the orientation of a mechanical device was quite severe; similarly, a cup anemometer with associated vanes can be used with kites at various angles of inclination, but this also poses difficulties.

Mr. Shoumatoff wanted to know how the characteristics of the thermistors compared with those of foil-type instruments. The speaker replied that the response of thermistors is slower, and that he had experimented with a semi-conductor paint. This was less satisfactory than regular thermistors because of insufficient quality control, but should yield higher signal levels than platinum because of the very high temperature coefficient.

Mr. Toch inquired about the manner in which the direction of the non-rigid support was controlled, and the speaker explained it as a system of tethers which form essentially a tripod arranged to obtain high lift and low drag from the balloons. By mounting the instrument at the tether point, its rotation is kept small and stable control can be maintained from the ground. It is not perfect for rotational or lateral stability, but was considered adequate for the studies.

REFERENCES

1. Sherlock, R. H., and Stout, M. B., "An Anemometer for a Study of Wind Gusts," *University of Michigan Research Bulletin* No. 20, 1931.
2. Vershinski, N. V., "Recording Velocity Vector Pulsations in Fluids," (O Registratsiyi Pulsatsiy Vektora Skorosti V Zhidkostykh). *Reports of the Academy of Sciences USSR* (Doklady Akademii Nauk SSSR) Vol. 73, No. 5, pp. 941-43. 1950. Translation No. 15, St. Anthony Falls Hydraulic Laboratory, by Meir Pilch.
3. Knapp, R. T., "Wave Produced Motion of Moored Ships," *Proceedings of the Second Conference on Coastal Engineering*, pp. 57-60. 1952.
4. Anon., "Wind Sensing Unit Measures Air Currents," *Industrial Laboratories*, p. 21. Sept. 1954.
5. Roshko, A., "On the Development of Turbulent Waves from Vortex Streets," *National Advisory Committee for Aeronautics Technical Note* No. 2913. March 1953.
6. Liepmann, H. W. and Skinner, G. T., "Shearing Stress Measurements by Means of a Heated Element," *National Advisory Committee for Aeronautics Technical Note* No. 3268. November 1954.
7. Corby, R. E., "Acoustic Anemometer Anemoscope," *Electronics*, Vol. 23, pp. 88-90. January 1950.
8. Kalmus, H. P., "Electronic Flowmeter System," *Review of Scientific Instruments*, Vol. 25, pp. 201-206. March 1954.
9. Patterson, J., "The Cup Anemometer," *Transactions of the Royal Society of Canada*, Vol. 20, Section 3. January 1926.
10. Marvin, C. F., "Recent Advances in Anemometry," *Monthly Weather Review*, Vol. 62, pp. 115-120. 1934.
11. Sheppard, P. A., "An Improved Design of Cup Anemometer," *Journal of Scientific Instruments*, Vol. 17, pp. 218-221. September 1940.
12. Spilhaus, A. F., "Analysis of the Cup Anemometer," *Massachusetts Institute of Technology Professional Notes* No. 7, 1934.
13. Fergusson, S. P., "Experimental Studies of Anemometers," *Harvard Meteorological Studies* No. 4, Harvard University Press, 1939.
14. Latimer, W. L., "Micrometeorological Instruments," *Handbook on Aerosols*, U. S. Atomic Energy Commission, Washington, 1950.
15. Anon., "Portable Anemometer," *Journal of Scientific Instruments*, Vol. 21, p. 50. 1944.
16. West, G., "Wind Recorder for Microclimatology," *Electronics*, June 1952.
17. Ower, E., *The Measurement of Air Flow*, London: Chapman and Hall. 1949.
18. Conover, J. H., "Tests of the Friez Aerovane in the Natural Wind at Blue Hill Observatory," *Bulletin of the American Meteorological Society*, pp. 523-531. November 1946.

19. King, L. V., "On the Convection of Heat from Small Cylinders in a Stream of Fluid," *Philosophical Transactions of the Royal Society of London*, Series A, Vol. 214, pp. 373-432. 1914.

20. Weske, J. R., "Methods of Measurement of High Air Velocities by the Hot-Wire Method," *National Advisory Committee for Aeronautics Technical Note* No. 880, February 1943.

21. Ferrari, C., "Electrical Equipment for the Experimental Study of the Dynamics of Fluids," (Sociata Italiana Per il Progresso Delle Scienze,) *Roma*, XVI, 1938. Translated as *National Advisory Committee for Aeronautics Technical Memo* No. 1006, March 1942.

22. Goldstein, S., *Modern Developments in Fluid Mechanics.* Oxford University Press, Vol. I. 1938.

23. Willis, J. B., "Review of Hot Wire Anemometry," *Australian Council for Aeronautics Report* No. ACA-19, October 1945.

24. Hales, W. B., "Thermistors as Instruments of Thermometry and Anemometry," *Bulletin of the American Meteorological Society*, Vol. 29, pp. 494-499. December 1948.

25. Sanford, E. R., "A Wind Tunnel Investigation of the Limitations of Thermistor Anemometry," *Journal of Meteorology*, Vol. 8, pp. 182-190. June 1951.

26. Hastings, C. E. and Wcislo, C. R., "A Compensated Thermal Anemometer and Flowmeter," American Institute of Electrical Engineers Miscellaneous Paper 51-149, March 1951. (This unpublished paper appears in a brief published summary of the same title in *Electrical Engineering*, p. 597. July 1951.)

27. Hastings, C. E., "A New Type Instrument for Measuring Air Velocity," American Institute of Electrical Engineers Miscellaneous Paper 49-23, December 1948. (This unpublished paper appears in a brief published summary as "Measuring Air Velocity with a Thermopile", *Electrical Engineering*, p. 582. July 1949.)

28. Fucks, W., "Investigation of the Operating Properties of the Leakage Current Anemometer," *National Advisory Committee for Aeronautics Technical Memo* No. 1178. October 1947.

29. Morgan, A. J. A. and Vrebalovich, T., "Investigation of Direct and Alternating Current Flow Anemometers," Guggenheim Aeronautical Laboratory, California Institute of Technology. August 1950.

30. Mellen, Glenn L., "Gas Flow Speedometer," *Electronics*, pp. 80-81. February 1950.

31. Boyd, B., Dorsch, R. G., and Brodie, G. H., "True Airspeed Measurement by Ionization—Tracer Technique," *National Advisory Committee for Aeronautics Research Memo* No. RME52C31. July 1952.

32. Lovelock, J. E. and Wasilewska, E. M., "An Ionization Anemometer," *Journal of Scientific Instruments*, Vol. 26, pp. 367-370. 1949.

ULTRASONIC MEASUREMENT OF DEPTH

by

LAURENCE BATCHELDER

Raytheon Maunfacturing Company

Echo sounding consists merely in transmitting a short burst of sound under water and measuring the time required for the echo to return. It is one of the simplest of the products of this electronic age.

Soundings used to be taken by hand with a lead line, but this was cumbersome and inaccurate. A line might shrink or stretch according to its wetness, and it would not hang straight unless the ship was stopped. In deep water the drag on a long line made it difficult or impossible to detect the loss of tension when the sounding weight struck bottom. This difficulty led to the invention of many ingenious devices in which the maximum hydrostatic pressure would register in the lead, but none of these was as accurate as a measurement of the time of travel. An expendable sounding bomb was invented in Germany and used to some extent in the 1920's. To take a sounding a small bomb was tossed overboard. It would arm itself after it was wet and would detonate on contact with the bottom. The sinking time of the bomb could be measured by an ordinary stop watch and the depth calculated from the supposedly known sinking rate.

Echo sounding by measuring the travel time of sound waves is so obvious that it is hard to say when it was first invented. In fact the English language seems to have anticipated the invention. We have "sound" meaning to measure depth and "sound" meaning acoustic energy. These homonyms come from the old French, but from different words. Actual echo sounding required instrumentation and the courage to experiment. It was first tried about 1912 in this country by Fessenden and separately in France by Langevin. As a result of the Titanic disaster in 1912, Fessenden was concerned with detecting icebergs by underwater echoes but he obtained far stronger echoes from the sea bottom. Langevin had developed quartz crystals for ultrasonic generators and, since they radiate far more efficiently in water than in air, he applied them to the problem of depth sounding.

Exploitation of these experiments needed to await development of practical means for indicating the echo time which ranges from a millisecond to several seconds, according to the depth of water.

All the mechanical and optical art of oscilloscopes and oscillographs was reviewed and many ingenious depth indicators were tried. But there is a world of difference between a laboratory instrument and a practical sea going apparatus. The two most successful solutions to the time measurement problem were the red light indicator which came in the middle 1920's and the graphic recorder in the early 1930's.

The red light indicator (Fig. 1) contains a neon lamp which travels at constant speed around a circular depth scale. The sound bursts are transmitted as the neon lamp passes zero on the scale, and the lamp flashes at the instant each echo is received. The retina of the eye retains the image of the flash long enough for

FIG. 1. RED LIGHT DEPTH INDICATOR (RAYTHEON TYPE DE-111)

the observer to read the position on the scale where the flash occurred. The recorder (Fig. 2) is essentially the same in principle, but the neon lamp is replaced by a stylus which travels across a calibrated chart scale. The echo causes the stylus to mark the paper electrically, and the mark is permanent. The memory feature of the recorder has considerable advantage. It draws a permanent record which is easier to interpret than a sequence of momentary flashes. However, from the commercial viewpoint the special paper required for the recorder is an appreciable expense, and many customers still prefer the economy of the red light indicator.

With any indicator or recorder it is possible to expand the scale in order to magnify the region of the echo while suppressing the range closer to the surface. This is particularly convenient with a cathode-ray oscilloscope, which is useful for detailed study of the echo structure. However, a cathode-ray tube indicator is seldom used because it does not have enough advantage to justify its size and cost. Yet it may prove to be the only practical indicator for scale model soundings in tanks where the time of travel is very short.

Before going further into the details of design and performance

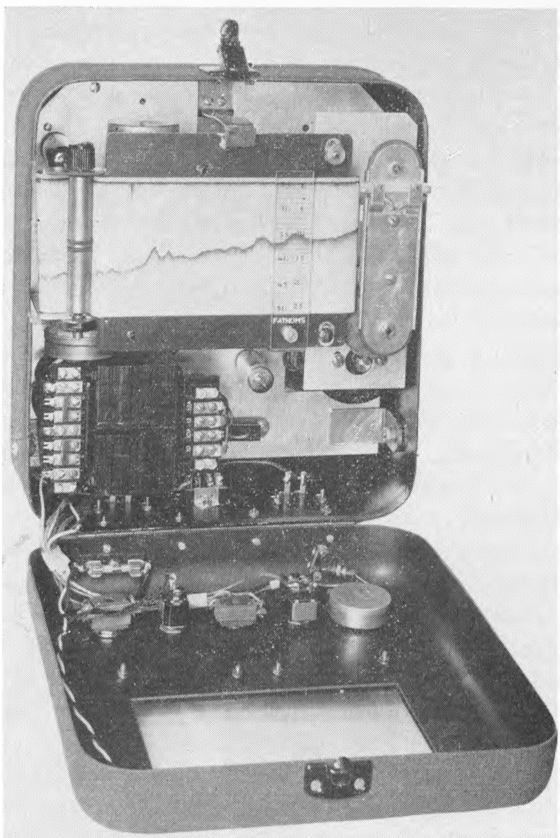


FIG. 2. DEPTH RECORDER (RAYTHEON TYPE DE-112)

let us consider briefly the purposes for which depth sounding has been developed. It is primarily an aid to navigation. It is obviously a protection against running aground, either when a pilot does not know where he is or when he has no chart of the shoals and rocks. It is particularly useful in finding and following a harbor channel in a fog. When out of sight of land, a navigator often fixes his position by comparing his soundings with the chart.

Here a single sounding is of little value, but when the course and speed are known and the soundings are tabulated for a brief period, they can usually be matched with some unique position on the chart.

Soundings are equally essential to the surveys which provide the basis for navigation. The Navy Hydrographic Office and the U. S. Coast and Geodetic Survey use echo sounding extensively in preparing navigator's charts. They use essentially the same equipment as the navigator but take particular care to maintain precise control of the speed of the indicator or recording mechanism. They also observe water temperature and salinity since these variables affect the velocity of sound. In harbors and inland waters the survey function is carried out by the Army Corps of Engineers and the U. S. Geological Survey. Here again the equipment is similar except that it is designed for relatively shallow water. Increasing attention must be paid to the elevation of the water surface which varies with the tide and with the flow of streams. Occasionally depth sounding locates submerged wrecks, either for protective charting or as a preliminary to salvage operation.

Depth sounding is invaluable to the fishing industry, both as a navigation instrument and as a means for finding fish. Most fish caught commercially swim a few feet above the bottom in depths of 50 to 100 fathoms, on the sloping sides of submarine ridges or banks. They are caught by trawling parallel to the ridge, maintaining constant depth, with a net rigged to drag or just clear the bottom. Continuous sounding is almost essential because the fish congregate along a contour of constant depth. Unless the fisherman is lucky enough to find a school of fish he may trawl for hours and catch practically nothing. This wastes valuable time on the fishing grounds and risks unnecessary damage to the net. However, by careful attention to the depth sounder it is often possible to detect echoes from the schools of fish or even from individual fish. Various fish finding equipments are available commercially. Some are essentially depth sounders with a beam of sound directed downwards. Others search with a horizontal beam, much as naval vessels search submarines with sonar. It seems fair to say, however, that all fish finders are only marginal in performance, because fish, even in schools, reflect very little sound. Better equipments for finding fish are being developed but economics pose a major obstacle. It is an important endeavor, however, because the world's population is rapidly approaching limitations in the food supply. Oceanographers believe that there are many more fish in the ocean than we have yet learned to catch.

Echo sounding has application also to geology. For different kinds of bottom, the reflection coefficient differs not only in its value but also in the way it varies as a function of the angle of incidence. The reflection of sound is much like that of light. There is specular reflection, as with a mirror in optics, where sound is reflected at an angle equal to the angle of incidence. There is also diffuse reflection in which the sound is scattered in all directions from a rough surface. Both phenomena occur on the bottom of a waterway, partly because the interface is never truly smooth, and partly because the particles which compose the bottom move and interact in a random manner. In addition, an appreciable fraction of the incident sound is transmitted into the bottom and absorbed. In sonar applications, one is generally concerned with specular reflection at normal incidence, or when incidence is oblique, with the component of scattered sound which returns to the source. Regardless of the angle of incidence, the intensity of this returning sound is proportional to the back scattering coefficient.

The back scattering coefficient is a function of the angle of incidence. This has been measured for a variety of harbor bottoms which, except for rocks, are comparatively smooth [1]. The reflection from a rocky bottom tends to produce rather uniformly diffuse scattering, either because the rocks are spherical, or if jagged, because they are oriented at random. Smooth sand or mud bottoms, on the other hand, are characterized by a relatively large proportion of specular reflection. There are no data for the rippled sandy bottom of a river. Here one would expect an intermediate condition, but the reflection coefficient should depend also on the direction of the ripples. While sand and mud are alike in producing relatively large proportion of specular reflection, mud has a smaller coefficient because it absorbs more of the sound. The presently available data are nearly enough to permit a classification of the bottom by echo sounding providing one takes the trouble to measure the reflection for various angles incidences.

Mud reflects less sound than sand because more of the sound penetrates the softer material. As long as the mud is homogeneous, the sound will propagate through it, but will be rapidly attenuated by the viscosity of the mud. If a harder stratum underlies the mud some of the sound will be reflected upward. When the mud is not too thick this reflected sound gets back into the water with an intensity sufficient to cause a discernable second echo. Often the underlying stratum is not distinct and the density changes gradually from clear water through soft ooze to hard soil. In such cases the echo is stretched out in time. Occasionally the upper ooze is so soft that it is difficult to define where the bottom really begins, and then it is equally difficult to determine with a lead line. The softness of the bottom can often be judged by the quality of the echo, without resource to quantitative measurements, but this requires experience. In a few instances these phenomena have been used to study sedimentation with a view to determining the geological age of a body of water. Observations in Lake Michigan show acoustic penetration through at least 7 fathoms of clay overlying till [2]. In the Gulf of Maine sediment ranging up to 50 fathoms of thickness has been found [3]. Both of these studies were made with essentially standard commercial equipment. If the geologists are interested in pursuing this sort of investigation, more basic research is needed, directed particularly toward the choice of an optimum frequency of sound for penetrating various types of mud and silt.

The application of depth sounding to hydraulic research is the one to which this paper is primarily directed. This new application poses certain problems which have not occurred before. One is to measure the shape of the bottom in detail in order to study scour patterns and ripples. Another is to integrate depth over a cross-section of a stream in order to facilitate computation of volumetric flow. Both of these measurements may require greater accuracy than has been needed in navigation and even survey applications. Finally the hydraulic engineer wants soundings, not only in real streams, but also in small scale laboratory flumes.

In the matter of accuracy, the variations of the velocity of sound come immediately to mind. The velocity is a function of temperature, salinity, and hydrostatic pressure [4]. Hydrostatic pressure increases the velocity slightly more than one part per million for each meter increase in the depth, so this variation may be neglected completely in shallow water. Figure 3 shows how the velocity increases with temperature and salinity. The two salinities shown are not extreme, but represent the range which is usually encountered in ocean waters. The hydraulic engineer works mostly with fresh water, where salinity may be neglected. His major concern is temperature, which causes a total variation of some 7 percent in the working range of natural water.

When there is a vertical temperature gradient in the water, it is quite a complicated thing to correct exactly for its effect on the depth sounding echo time. The reciprocal of the velocity of sound must be calculated as function of depth, and this reciprocal integrated along the sounding path to find the total time of travel. In oceans and lakes appreciable thermal gradients occur because the surface water is warmed by the sun or cooled by winter air. The warmer water is less dense and remains at the surface, and any surface-cooled water sinks to the bottom. In rivers and streams, however, there is probably enough turbulence to stir the water and produce a reasonably uniform temperature from surface to bottom. When this is true it is sufficient to determine the average temperature from a few measurements, and to assume that the average velocity exists along the entire path.

One way of overcoming the uncertainty in the velocity of sound

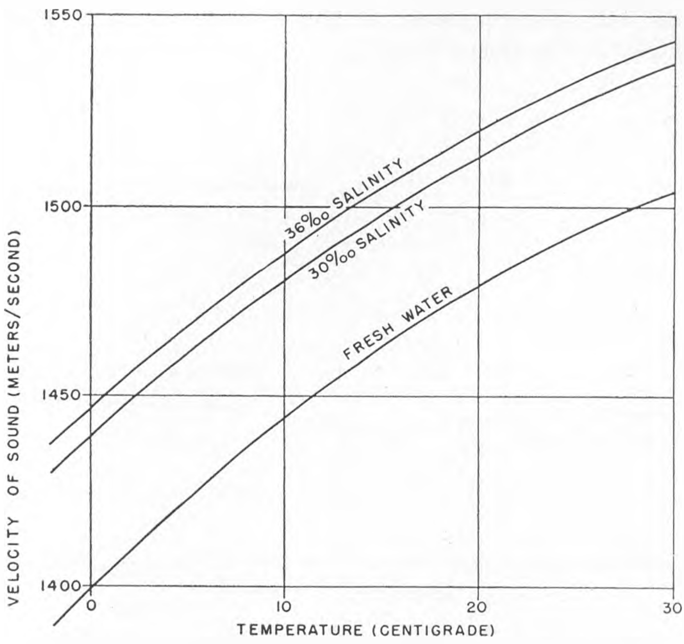


FIG. 3. VELOCITY OF SOUND IN WATER AS A FUNCTION OF SALINITY AND TEMPERATURE

is to calibrate the echo sounder with echoes from a horizontal bar lowered on chains to a known depth. This method is used extensively in the survey work of the Army Corps of Engineers. It corrects the errors in the time of travel, but does not eliminate inaccuracies due to the gradual onset of the echoes.

It is difficult to measure echo time precisely because the echo cannot start and stop suddenly. No one has succeeded in building an efficient sound projector that is free of resonance. Furthermore, the receiving system usually requires electrical tuning, or narrow-banding, to exclude much of the noise that is generated by the wash of water against the hull of a moving vessel. In any resonant system, mechanical or electrical, the response is not instantaneous, but gradual, as an exponential function of time (Fig. 4). Whatever the indicating device, it requires a certain threshold to distinguish an echo from the background noise. The indication of the echo must be delayed until this threshold level is reached. If the delay were constant, it would cause little trouble, but obviously the delay varies with the intensity of the echo. A strong echo, arriving at time t_0 is indicated at t_1, but a weaker echo (10 db weaker in Fig. 4) is not indicated until t_2. Since the echo intensity may vary as much as 20 db with the character of the bottom, there is always some uncertainty in the delay time.

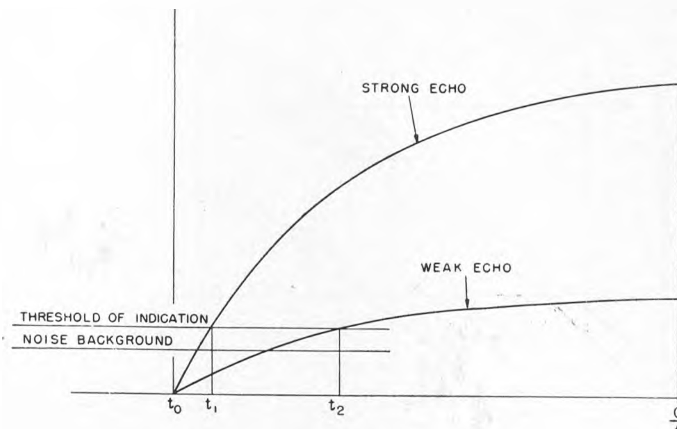


FIG. 4. RESPONSE AS AN EXPONENTIAL FUNCTION OF TIME, AND DELAY INTRODUCED BY A FINITE THRESHOLD

The amount of delay, and its variability, can be reduced to some extent by lowering the threshold, but background noise always imposes a limitation to this approach. The rise time of a response depends on the frequency and on the effective Q value of the system. Since a signal attains 95 percent of its final value in Q periods of the oscillation the delay increases inversely as the frequency. Errors in depth measurement from this cause often amount to 1 percent in ordinary echo sounders. In hydraulic research, particularly in the laboratory, the problems seem quite different. In shallow water the frequency of the sound may be increased, perhaps by an order of magnitude, before excessive attenuation limits the performance. If operating conditions are relatively quiet, the receiver may be designed to accept a much wider band of noise, and consequently contribute little to the Q of the system. These design changes should make the uncertainty in depth quite negligible.

Another cause of a gradually rising echo intensity involves the geometry illustrated in Fig. 5. The sound is tranmitted downward in some sort of a beam with an intensity proportional to the directivity function $\delta(\theta)$. This is a function of the angle θ from the axis. It is assumed for simplicity that the bottom is level, and that the ship is on an even keel, so that the normal incidence occurs. Traveling with a velocity c, the sound goes down a distance d and returns as an echo. At a time

$$t_0 = 2d/c \tag{1}$$

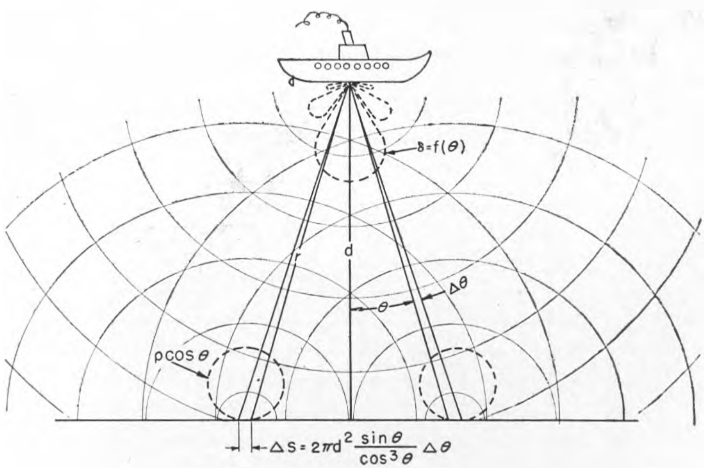


FIG. 5. GEOMETRY OF ECHO SOUNDING OVER A HORIZONTAL BOTTOM

after the transmission began, the echo starts to appear from the nearest point directly beneath the ship. The first sound returns from only an infinitesimal area of the bottom and contains only an infinitesimal amount of energy. As time continues, a progressively larger area of the bottom contributes reflected sound, and the echo increases accordingly. At any time t, the instantaneous echo power is the resultant of contributions from all of the bottom within a cone of angle θ, where

$$\sec\theta = t/t_0 \tag{2}$$

To treat this problem mathematically requires some assumption about how the back scattering coefficient of the bottom varies with the angle of incidence. One might naturally assume Lambert's cosine law. Actually the reflection from a smooth bottom is more specular than that, and from a rocky bottom it is more uniformly diffuse. Thus, if one wants to generalize by assuming an average bottom, Lambert's law appears to be a very fair compromise. With this assumption it can be shown [5] that the echo power arriving at any instant t is

$$w = 2\pi\, w_0\, \rho \frac{A_c^2}{\lambda^2 d^2} \frac{1}{t_0} \int_{t_0}^{t} \delta^2(\theta) \cos^5 \theta \times 10^{-2ad \sec \theta}\, dt \qquad (3)$$

Where w_0 = total power transmitted

ρ = reflection factor of the bottom for normal incidence

A_c = capture area of the transducer

λ = wavelength of sound

d = depth

a = coefficient of attenuation of sound in water

The definite integral is the only factor in equation (3) which varies with time. Figure 6 shows how it behaves for a few arbitrarily

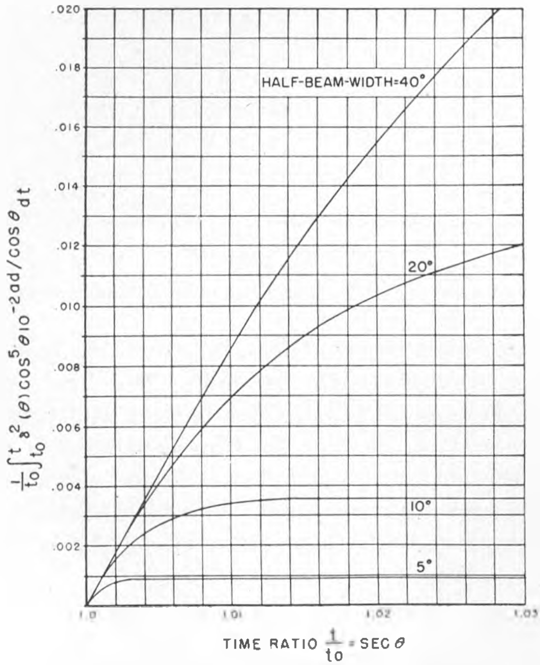


FIG. 6. VALUE OF THE DEFINITE INTEGRAL IN EQUATION (3) AS A FUNCTION OF NORMALIZED TIME, FOR THE CASE ad=0.01 BEL. THE DIRECTIVITY IS THAT OF A CIRCULAR PISTON, WITH THE BEAM WIDTHS AS NOTED AT THE —10 db LEVEL

chosen widths of the sound beam δ (θ), and for one moderate value of attenuation. The origin represents the time t_0 when the echo begins to arrive, and the abscissa is the normalized time ratio t/t_0. The time required for the echo to reach any arbitrary value is thus proportional to the echo time t_0, and the resulting error in depth measurement is proportional to the depth itself. Regardless of the beam width, all these curves have the same initial slope, which is equal to the attenuation factor 10^{-2ad}. In a sounding equipment designed for good accuracy the indicator threshold would presumably occur at a level so low that this initial slope still persists. Figure 6 shows only the time dependent portion of expression (3). For the best accuracy we want the fastest rise of the echo. Therefore, we should maximize

$$t_0 \left[\frac{dw}{dt} \right]_{t=0} = 2\pi\rho\, W_0 \frac{A_c^2}{\lambda^2 d^2} 10^{-2ad} \tag{4}$$

The reflection factor ρ is a property of the sea bottom over which we have no control. We can always gain by increasing the transmitted power W_0, but this is expensive. Also if the size of the projector is fixed, W_0 is limited by cavitation to the order of 1 watt per square centimeter. Regardless of the transmitting power, the other parameters in equation (4) should be chosen wisely. The wavelength λ is inversely proportional to the frequency while the attenuation factor a per unit distance increases roughly in proportion to the square of the frequency. Obviously, there is an optimum frequency for which $10^{-2ad}/\lambda^2$ is maximum. The optimum frequency increases as the depth diminishes. Also, it is considerably higher for fresh water than for salt because sea water has more than 50 times as much attenuation over most of the useful range of frequencies [6].

The characteristic rise due to the geometrical effect, shown in Fig. 6, was calculated without regard to the transient response of resonant systems. Both effects are present, and operate additively on the echo. Since resonance causes a truly exponential rise, and the geometric effect causes a rise of very similar shape, the echo will build up in almost exponential fashion providing the signal continues indefinitely as shown at (a) in Fig. 7. Of course, the signal does not continue, but has a limited duration, as in Fig. 7(b). The same causes which contribute to a gradual rise at the beginning of the echo also produce a gradual decay, or tail, at the end. This may be demonstrated by considering the limited signal of Fig. 7(b) to be composed of positive and negative unit functions,

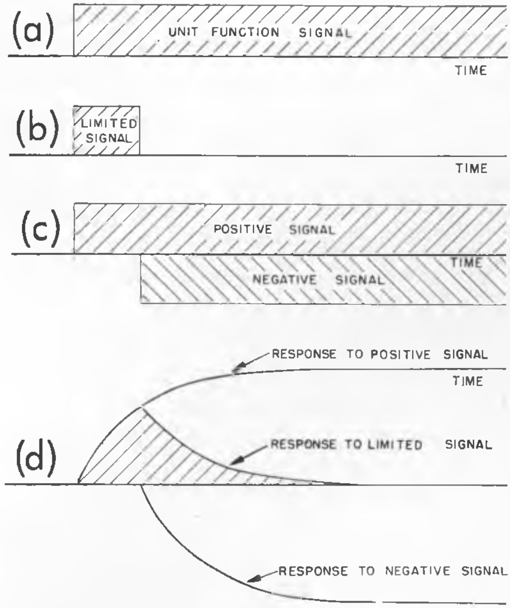


FIG. 7. COMPOSITION OF THE TRANSMITTED SIGNAL AND THE CORRESPONDING ECHO RESPONSE

displaced in time, as in Fig. 7(c). The responses to these two signals are alike, but opposite in sign and displaced in time as in Fig. 7(d). The algebraic sum of the two responses is the response to the limited signal. While the tail of the echo does not affect the accuracy of depth measurement, it may obscure some of the information about the character of the bottom.

The problem of maximizing the rate of rise of the echo is not quite as simple as this discussion would imply. The capture area A_c is related to the wavelength and to the physical area of the transducer face. The relationship is rather complicated, and involves the directivity of the sound beam which we must consider also from other viewpoints. To some extent one can increase A_c merely by paying the dollar cost of a large transducer. However, cost is not the only limitation. The directivity is proportional to the ratio A_c/λ^2. If this ratio were too large, the sound beam would be too narrow for the roll and pitch of a ship at sea. If a stable platform were available, the narrower the beam the better for most research applications. In this case there would be some limit to the sharpness of a sound beam which could be obtained in practice. Since this problem has not arisen in the past there is no experience to indicate what that limit is.

A typical transducer beam pattern is shown in Figure 8. Here the relative response in decibels is plotted as a function of the angle from the transducer axis. The pattern consists of one main beam surrounded by several minor lobes. The designer can easily control the width of the main beam, but in general, he cannot eliminate the minor lobes. The pattern shown in Figure 8 applies either in transmitting or receiving. The relative intensity of an echo would be indicated by doubling the decibel scale of this plot. Thus an echo from the direction of the first minor lobe would be some 40 db weaker than the echo on the axis. If the bottom is smooth and hard, delayed echoes from the minor lobes can occasionally be distinguished, but usually they are masked by the tail of the earlier echo from the central beam.

Soundings which show the bottom contour in accurate detail require a high degree of angular resolution. A depth sounder tends always to display the nearest point of the bottom. Suppose the bottom has a step, as in Fig. 9, and consider what would happen when cruising over it. For a considerable distance, the step is nearest to the surface. With a broad sound beam the depth sounder would pick up the echo from the step, and would indicate the apparent bottom as the hyperbolic curve. The region of true bottom beneath this curve would be obscured. If the water were shallower, the depth sounder would be closer to the step, and the obscured region would be shorter. But in the extreme case the hyperbola would degenerate only to a line of 45 degree slope. Of course the situation is not as hopeless as Fig. 9 implies. The echo from the

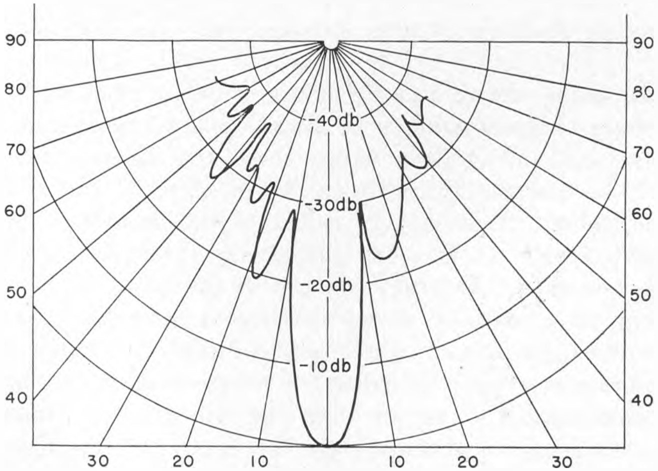


Fig. 8. Directivity Pattern of Raytheon Type 2155 Transducer at 210 kcs

true bottom would also be present, this would be indicated at its proper time, and might be discernible below the hyperbolic trace. However, we must remember that every echo has a tail, and that the tail of the step echo would probably overlap the true bottom echo. A narrow sound beam would help to suppress the undesired echo from the step, but complete suppression is never possible because of the minor lobes.

A step such as drawn in Fig. 9 seldom occurs on a natural bottom, but it illustrates the difficulty in echo sounding whenever the bottom is not level. The fact that the depth sounder measures the distance to the nearest point introduces an error in true depth of a smooth but sloping bottom [7]. The only sure way to sound

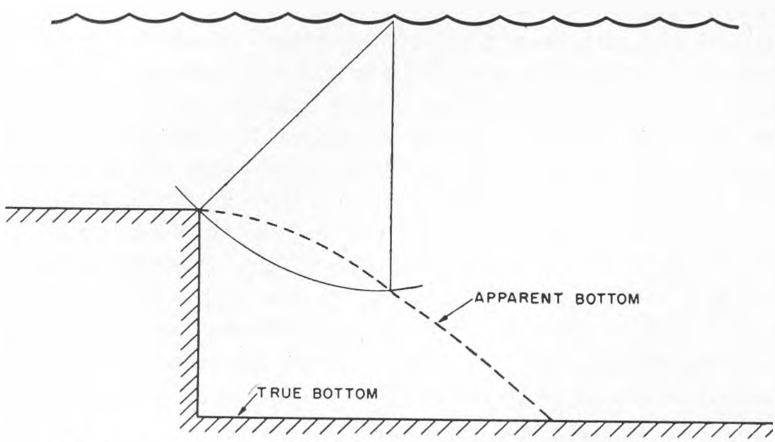


FIG. 9. GEOMETRY OF ECHO SOUNDING OVER A STEP IN THE BOTTOM

the deepest point of any hollow, such as a scour pattern, is to lower the transducer deep enough to be always below the center of curvature of the bottom surface. Even then, the sloping sides of the hollow will be distorted slightly in the display. If the true bottom contour is known, it is easy to calculate the distortion introduced in the echo trace. It is very difficult to interpret a given echo trace and determine the exact contour of the bottom.

The ripples caused by water flowing on a sandy bottom seem much too fine for this sort of geometrical analysis. From this point of view, the ripples might be called the microstructure of the bottom. If their wavelength is shorter than the width of the sound beam, the indicated echo would show only the depth to the crests. While an equal amount of sound would return from the intervening valleys, this would be obscured in the tail of the earlier echo from the crests.

The ripples stretch the echo out in time, but in view of the other factors which also stretch it out, it does not seem practical to measure the average depth of water over a rippled bottom. It is possible, however, that useful information about the amplitude and wavelength of the ripples might be obtained from a study of the back scattering as a function of the angle of incidence.

It is quite another problem to integrate depth on a gross scale and calculate the cross section of a stream. Here the recorded trace can be integrated with a planimeter, providing the depth of water is great enough to make a mechanical recorder practical. A recorder mechanism can drive a stylus reliably with a straight-line motion at speeds up to 24 inches/second. At this speed the scale factor is 1 inch of paper chart for 10 feet of water depth. For a more expanded scale, faster recorders have been made in which the stylus is mounted on the end of a rigid arm and travels across the paper in the arc of a circle. Such instruments produce a valid record but the chart is distorted and the depth graduations are not uniformly spaced. With a chart of this type the trace would be difficult to integrate with a planimeter.

For depths less than 10 feet, as in laboratory flumes, the echo time is so short that a cathode-ray tube seems far more suitable than any mechanical indicator recorder. A cathode-ray tube does not supply a permanent record to be integrated, but this function could be accomplished photographically. It is also possible to integrate a series of soundings electronically in a variety of different circuits. As a matter of fact there are various circuits which measure time electronically. Many of these have been used in cheaper depth sounding equipments to indicate depth by a simple electric meter. They have not been discussed in this paper because, in general, they are quite unsuited to research applications. The chief difficulty is that an electronic circuit gives a false indication whenever it receives a burst of noise which is comparable in intensity to an echo. In laboratory flumes, however, there is every reason to expect that the background noise would be much quieter than on a moving vessel. If so, electronic depth sounders should be completely reliable. The accuracy of their time measuring circuits is controlled primarily by economics. Some electronic counter circuits are available which, although relatively expensive, are capable of greater accuracy than even mechanical instruments. These electronic counting circuits are readily adapted to the integration of a sequence of soundings to yield a direct measure of the area of a cross section.

In connection with echo sounding in a small scale flume, it is worth noting the existence of another kind of equipment which has been developed for quite a different purpose. The performance of a radar on a bombing plane can be simulated by ultrasonic echo ranging in a tank of water [8]. Radio waves travel roughly 200,000 times as fast as sound waves in water, so simulation results when all distances are scaled down by this factor. The ultrasonic transducer is transported through the water according to the simulated motion of the airplane. A relief map of a target area, such as a city, is placed on the bottom of the tank. Ultrasonic echoes are presented on a cathode-ray tube plan position indicator, just as the real radar echoes would appear on a bombing mission. This system provides a realistic picture of the target, but it suffers from the same limitations as radar when sounding into a concave region. The ultrasonic radar simulator is cited here, not to imply that the problem of depth sounding in small tanks has been solved, but merely to point out that there may be techniques in that area which could be useful in the hydraulic laboratory.

CONCLUSION

This review of the current art of ultrasonic depth sounding has been written in a critical vein in order to emphasize the limitations which might concern the hydraulic engineer. Conventional equipment designed for general navigation purposes does not appear to meet all the requirements of hydraulic research. However, these needs could probably be satisfied with appropriate equipment especially designed for the purpose. The most significant departure from conventional design would be to increase the ultrasonic frequency by an order of magnitude in order to improve the resolution of the echo sounding system. Such a change is made possible by the limited depth of the rivers and flumes where the special equipment would be used. Cathode-ray tube indicators and electronic counters are worthy of consideration for shoal soundings with quiet operating conditions.

DISCUSSION

Mr. Bradley was interested in the accuracy obtainable in the range 10 to 40 feet of water, and the cost; the speaker estimated 2 percent for standard commercial equipment and ½ percent with special equipment. Cost figures should be obtained from a manufacturer, but elements cost approximately $250.

Mr. Kolupaila wanted to know if soundings could be made from an airplane or if both transmitter and receiver had to be immersed in water. The speaker answered that practically all of the energy from a transducer in air would be reflected at the water surface because of the tremendous difference in acoustic impedances.

Referring to the speaker's mention of two cases in which a firm bottom was detected below alluvial material, Mr. Schneible asked if equipment was commercially available for detecting rock formations 10 to 50 feet below alluvial stream beds. After ascertaining that the question referred to wet beds, Mr. Batchelder emphasized his earlier statement concerning better transmission in wet soils. He thought that it should be possible to make such measurements with commercial equipment, but felt that the question was one to be answered by a geologist. Mr. Schneible had found, in his experience at sites where the depth of rock had already been determined by other methods, that the fathometer gave such indications, but not consistently. He now wanted to know if special equipment was in existence and if the type used in Lake Michigan was a secret type developed for the navy. The speaker assured him that it was a standard fathometer and was used by the University of Illinois. He also referred to a paper by Firstak at the University of Chicago.

Mr. Bengal spoke of his experience in using the equipment for soundings in San Francisco Bay and along the lower channels of the Sacramento and several other rivers. They found that they could measure the depth of the very light and porous mud (75 to 80 lb/ft^3) found in San Francisco Bay, with the hard blue clay or sand bottom showing up as a very sharp line.

In response to a question by Mr. Liu concerning application of the fathometer to detect the movement of dunes, the speaker stated that continuous observations at one point would yield the desired information.

Referring to slide No. 9, Mr. Shoumatoff recalled a similar instance in which a form asserted to be that of a huge lizard was observed during soundings in Lock Ness. Mr. Batchelder had been assured by a sound expert of the British Admiralty that it could be nothing but "the monster."

REFERENCES

1. Urick, R. J., "The backscattering of sound from a harbor bottom," *J. Acoust. Soc. Am. 26*, 231 (1954).

2. Hough, J. L., "Fathogram indications of bottom materials in Lake Michigan," *J. Sedimentary Petrology, 22*, 162 (1952).

3. Murray, H. W., "Topography of the Gulf of Maine," *Bull. Geol. Soc. Am. 58*, 153 (1947).

4. Del Grosso, V. A., "The velocity of sound in sea water at zero depth," *Naval Research Laboratory Report* 4002 (June 11, 1952).

5. Batchelder, L., "Optimum signal characteristics for distance measurement by echoes," *Proc. National Electronics Conference*, 7, 29 (1951).

6. Beranek, L. L., *Acoustic Measurements* (John Wiley and Sons, Inc., New York, 1949), p. 76.

7. Schärfe, J., "Besonderheiten bei Echolotungen über unebenem Grund," *Fischereiwelt*, 7, 99, (1952).

8. Rosenberg, P., "Supersonic Training Device" U. S. Patent 2,518,938 (August 15, 1950).

ULTRASONIC INSTRUMENTATION IN CAVITATION RESEARCH

BY

M. Strasberg

David Taylor Model Basin, Washington 7, D. C.

Introduction

In the study of cavitation, a problem of fundamental interest is an understanding of the conditions required for its onset. The conditions to which I refer are the local conditions which must exist at any point in the water, such as the local pressure and temperature, for the formation of a cavity at that point in the water. This problem is of interest to both the hydraulics engineer and the hydrodynamicist, in the design of cavitation-free equipment and in determining the conditions and scaling laws for cavitation model testing.

At the Taylor Model Basin we are utilizing electro-acoustic equipment, operating at ultrasonic frequencies, to investigate the influence of undissolved air cavities in the water on the conditions required for cavitation. Before describing our equipment, it may be useful to review briefly some of the contemporary ideas on cavitation inception, to provide the background for an understanding of the purpose of our apparatus.

For many years it was assumed that cavitation occurred in a hydraulic flow when the local pressure dropped to the vapor pressure of the water. However, it is now recognized that the cavitation inception pressure does not have a fixed value. Indeed, under certain circumstances, particularly those found in water tunnels, inception may require a negative pressure or tension [1, 2].

Measurements of the inception pressure made under static conditions, by applying steady suction or tension to undisturbed water, have resulted in an amazing range of values, ranging from vapor pressure to minus 300 atmospheres [3]. On the other hand, theoretical estimates made by the physicists, based on calculations of the inherent strength of water, predict negative pressures as large as —500 to —10,000 atmospheres for inception. However, such large negative values have never been observed experimentally by any method.

To explain why the measured inception pressures are so variable and yet less negative than the theoretical values, it has been postulated that the onset of cavitation requires the existence of microscopic non-homegeneities or nuclei in the water [4]. The nuclei are generally assumed to be undissolved air cavities whose sizes may range from 10^{-3} to 10^{-5} inches. The cavitation process is considered to be the growth of these air-filled nuclei to visible size. The variation of the inception pressure is attributed to a variation in size of the original nuclei.

AIR-FILLED NUCLEI

A nucleus can grow to visible size in two ways: (1) by the relatively slow diffusion of dissolved air out of the water into the nucleus, or (2) by its sudden explosive expansion into a large cavity. The slow growth by diffusion results in a permanent bubble filled primarily with air, and is accordingly called "gaseous cavitation". The sudden explosive expansion, on the other hand, results in a cavity filled primarily with water vapor, since diffusion of air cannot keep up with the expansion, and is accordingly called "vaporous cavitation". In vaporous cavitation, the cavities collapse and disappear, due to the condensation of the water vapor, after leaving the low-pressure cavitating region.

Sometimes a collapsing vapor cavity does leave a permanent air bubble, much smaller than the original cavity, as a result of the diffusion of some air into the vapor cavity during its short life [5].

If it is assumed that these air nuclei are small free bubbles of air, it is possible to estimate theoretically the critical pressure for inception of either gaseous or vaporous cavitation. Theoretical values of the critical pressure are shown in Fig. 1 as a function of the radius of the bubble nucleus. The critical pressure for gaseous cavitation depends not only on the nucleus size but also on the amount of air dissolved in the water; curves are shown for three air contents. The critical pressure for vaporous cavitation depends only on the size of the nucleus. (It is possible, however, that the nucleus size itself may depend on the air content of the water.)

If the nuclei are small, both gaseous and vaporous cavitation require negative pressures of equal magnitude. If the nucleus bubbles are large, gaseous cavitation can occur at pressures above the vapor pressure provided the bubble remains at the required pressure long enough to accommodate the slow diffusion process. Vaporous cavitation, in contrast, always requires pressures below vapor pressure, but the growth is rapid.

Photographs of cavities growing rapidly by vaporous cavitation

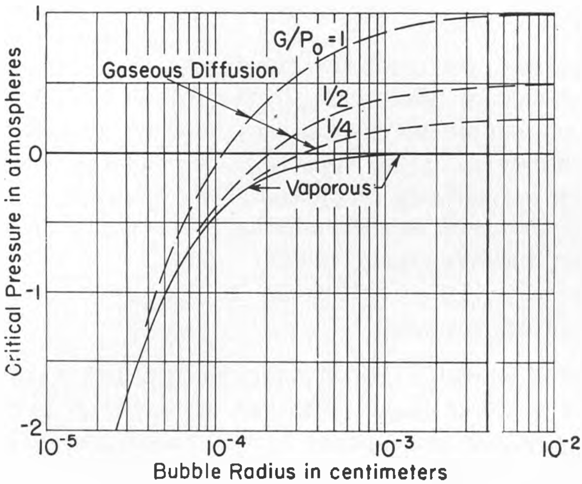


FIG. 1. THE CRITICAL PRESSURE FOR GASEOUS AND VAPOROUS CAVITATION AS A FUNCTION OF THE RADIUS OF THE AIR-FILLED NUCLEUS. THE NUCLEUS SIZE IS THE SIZE AT ATMOSPHERIC EXTERNAL PRESSURE. THE CRITICAL PRESSURE IS GIVEN RELATIVE TO THE VAPOR PRESSURE. THE THREE CURVES FOR GASEOUS CAVITATION ARE FOR THREE VALUES OF AIR CONTENT, THE SATURATION PRESSURE OF THE DISSOLVED AIR BEING INDICATED ON THE CURVES.

are fairly common, those of Knapp and Hollander [6] being typical. Recently, some photographs showing the slow growth typical of gaseous cavitation were published by Parkin and Kermeen [3].

The hypothesis that water contains air-filled nuclei, whose sizes may vary from one batch of water to another, thus explains the observed variations in the measured values of the inception pressure. However, there is no direct evidence that such nuclei actually exist in water. On the contrary, it would ordinarily be assumed that air cavities would spontaneously disappear by dissolving in the water.

According to diffusion theory, an air bubble of 10^{-3} cm radius should dissolve in about 7 seconds, even in saturated water; smaller bubbles should dissolve even faster [7].

Accordingly, it is of interest to find some direct method for detecting air-filled cavities in water, and for determining their effect on the onset of cavitation. The apparatus to be described was developed to perform these functions.

Apparatus

The apparatus performs two functions:

1. To detect air-filled nuclei in the water.
2. To determine the critical pressure for cavitation.

In the present form of the apparatus, both of these measurements are made on a relatively small sample of undisturbed water. The detection of air nuclei and the setting-up of cavitation are both performed with high-frequency sound.

Detection of Air Cavities

Air cavities in water absorb sound energy, and it is this property that is used to detect them. When an air cavity is in a sound field, the cavity pulsates in response to the fluctuating sound pressure. The resulting expansion and contraction of the air converts some of the sound energy into heat. The quantitative theory of sound absorption is well understood [8], and only the results pertinent to the apparatus need be discussed here.

The amount of absorption of a body of water containing air cavities may be determined by putting some sound energy into the water and measuring the rate at which sound energy decays, i.e., by observing the reverberation decay. The principle of the method may be illustrated by a simple experiment. If a glass of water is tapped, it will ring with a clear note of perhaps one-second duration. If bubbles are now formed in the water by inserting an "Alka-Seltzer" tablet, tapping will only result in a dull thud. The difference is due to the absorption of the sound energy by the bubbles. This principle, in refined form, is utilized in our apparatus.

Electro-acoustic apparatus for measuring sound absorption is relatively conventional in acoustics, that described by Moen [9] being typical. Our equipment is shown schematically in Fig. 2. The water sample is contained in a 12-liter spherical Pyrex flask about 12 inches in diameter. Two small piezo-electric ceramic cylinders, ¼-inch diameter and ¼-inch long are cemented to the

The piezo-electric cylinders are cast ceramics consisting principally of barium titanate. They are obtainable commercially from sources such as the Brush Development Company and the Gulton Manufacturing Company.

outer wall of the sphere. One of these cylinders acts as the sound source and is supplied with electrical random noise from a noise generator and power amplifier. The second piezo-electric cylinder acts as sound receiver. Its output is amplified, passed through an attenuator and then rectified into a DC signal by a tuned loga-

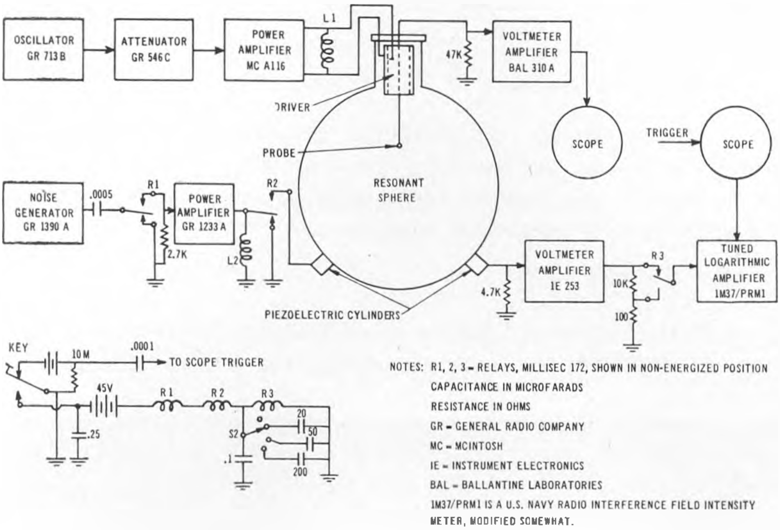


FIG. 2. BLOCK DIAGRAM OF THE ELECTRONIC EQUIPMENT. THE LOWER PORTION SHOWS THE EQUIPMENT FOR MEASURING THE ACOUSTIC ABSORPTION OF THE WATER, WHEREAS THE UPPER LINE SHOWS THE EQUIPMENT FOR CAUSING CAVITATION WITH HIGH-INTENSITY ULTRASONICS. ALL THE COMPONENT INSTRUMENTS ARE COMMERCIALLY AVAILABLE, EXCEPT FOR THE TUNED LOGARITHMIC AMPLIFIER WHICH IS A SOMEWHAT MODIFIED ITEM OF NAVY EQUIPMENT.

rithmic amplifier. The DC output of this amplifier, which is proportional to the logarithm of the AC voltage from the sound receiver, is observed as a trace on a DC cathode-ray oscilloscope. Three relay switches are arranged in the circuit so that depressing a hand key turns on the noise signal to the sound source and inserts attenuation into the receiving circuit. Releasing the key cuts off the noise source, triggers the horizontal time base of the oscilloscope and, after an adjustable time delay, removes the attenuation from the receiving circuit. When the key is released, the resulting display on the 'scope shows the decay of the sound in the sphere. Since the sound decays exponentially with time, the logarithm of the sound amplitude falls linearly with time, so that the logarithmic amplifier provides a decay trace on the 'scope which is approximately a straight line. The decay rate is given by the slope of the trace, in terms of the known sweep rate of the horizontal time base and the known logarithmic sensitivity of the vertical axis.

A photograph of the apparatus appears as Fig. 3.

Absorption of sound at a given signal frequency is associated primarily with bubbles which resonate near that frequency. The resonant frequency f_r of an air bubble in water is given by the numerical relation

$$f_r R = 330 \text{ cm/sec} \tag{1}$$

where R is the radius. Accordingly, by varying the frequency it is possible to detect bubbles of different sizes. The rate of sound decay caused by gas bubbles resonating at a frequency f_r is given by the approximate numerical relation

$$d_r \simeq 10^{-14} N_r/f_r^2 \tag{2}$$

where f_r is the signal frequency in cycles per second and N_r is the number of bubbles per cm^3 resonating in the frequency range

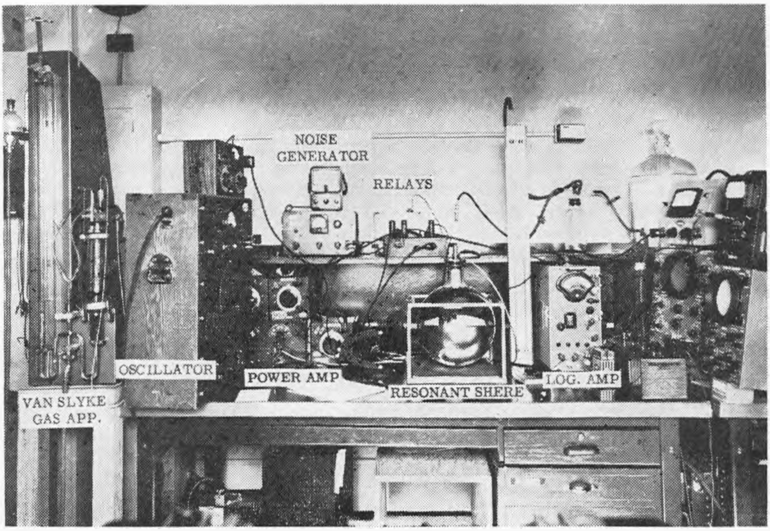


FIG. 3. PHOTOGRAPH OF THE APPARATUS

$\frac{2}{3} f_r$ to $2f_r$. It is apparent that the smaller the bubbles, the more of them are required for a given decay rate. The decay rate d_r is in decibels per second, an increment of 10 decibels corresponding to a factor of 10 in sound energy; e.g., 50 decibels per second means that the energy drops to 10^{-5} of its original value in a second.

Sound decay is caused in the apparatus not only by bubbles but also by sound absorption inherent to the water itself and in the walls of the sphere. Accordingly, the total decay rate d is equal to $(d_r + d_o)$, where d_o is the minimum or background decay rate of the apparatus in the absence of bubbles. This background decay sets a limit to the sensitivity of the apparatus for detection of bubbles. The background decay may vary erratically from day to day by as much as 20%. To detect bubbles with certainty, enough must be present to increase the decay rate by a substantial amount, say by 50% over the average background rate. The number of bubbles which will cause such an increase can be estimated by equating d_r in Eq. (2) to ½ d_o. Accordingly, the minimum detectable number of bubbles, or threshold of detectability, is given by

$$(N_r)\ \text{min} = 5 \times 10^{-15}\, d_o f_r^2 = 5 \times 10^{-10}\, d_o/R^2 \qquad (3)$$

where again N_r is the number of bubbles per cm³ with radii ranging from ½ to 3/2 R, R being in cm, and f_r is the resonant frequency of the bubble in cycles per second. If fewer bubbles are present than the quantity given by Eq. (3), they will not be detected.

Ultrasonic Cavitation

The measurements of cavitation inception are made in the same apparatus, simultaneous with the bubble detection. The cavitation is induced by setting up a high-intensity sound field in water. If the sound pressure has an amplitude p_s, the instantaneous total pressure is $P_o + p_s \sin 2\pi ft$, where P_o is the static pressure. If this oscillating pressure drops to the critical pressure P_c for cavitation, cavitation occurs. Accordingly, the critical pressure is given in terms of the sound pressure at cavitation by the relation $P_c = P_o - p_s$.

The sound field is generated by a large driver consisting of a 1½-inch diameter piezo-electric ceramic cylinder held in the neck of the spherical flask. The open bottom end of the driver cylinder projects about ¼ inch below the water surface. The sphere is filled with water just to the neck, to keep the shape of the water as nearly spherical as possible. The driver receives its electrical power from a 30-watt power amplifier excited at a frequency of about 25 kc/sec by a controllable oscillator. This section of the equipment is similar to that used by Galloway [10] for cavitation studies.

By proper choice of the sound frequency, a sound field is generated with maximum sound pressure at the center. This has the advantage of causing cavitation only at the center of the sphere, away from any solid boundaries. The spherical body of water is highly resonant at these frequencies. The exact frequency is found by experiment, slowly tuning the oscillator to resonance. For our apparatus, a resonance near 25 kc/sec is used. The half-power bandwidth of the resonance is only about 1 c/sec; an extremely stable oscillator is necessary to hold the resonance. The large amplification at resonance results in high sound pressures with relatively small electrical power.

The sound pressure in the water is measured with a sound probe. The probe consists of a tiny piezo-electric cylinder $\frac{1}{16}$ inch in diameter, at the end of a brass tube. The electrical output of the probe is measured with an electronic voltmeter and also observed on an oscilloscope. The sensitivity of the probe is known from a calibration performed by conventional acoustic techniques, so that its voltage output can be converted to sound pressure. The probe is not placed at the center of the sphere, for this would cause the cavitation to form on the probe rather than in the water. The probe is held several inches off center, and the indicated pressure at this point is multiplied by a correction factor to obtain the pressure in the cavitation zone at the center.

It may be well to consider how the value of critical pressure for ultrasonic cavitation is related to the critical pressure for hydraulic cavitation. In hydraulic cavitation the water is at a low pressure for a time which may vary from 1 to 100 milliseconds, depending on the velocity and the size of the cavitating region. In ultrasonic cavitation, on the other hand, the water is at low pressure for very short periods which repeat periodically; at 25 kc/sec, for example, the low pressure exists for periods shorter than 0.012 milliseconds.

The critical pressure for vaporous cavitation, plotted in the previously-discussed Fig. 1, was calculated with the assumption that the pressure changes are very slow so that the water is at the low cavitation pressure for a long time. A calculation of the critical pressure for ultrasonic cavitation, taking account of the rapid fluctuations in pressure, [11] leads to the same values of critical pressure as for the slow pressure changes. This is an important result, for it indicates that the critical pressure for vaporous cavitation does not depend on the duration of the low pressure.

This is not true, of course, for extremely short durations or extremely high ultrasonic frequencies, say durations less than 5 microseconds or frequencies over 100 kc/sec.

Accordingly, the critical pressure for ultrasonic vaporous cavitation should be equal to the critical pressure for hydraulic cavitation.

For gaseous cavitation, however, there is no relation between ultrasonic cavitation and hydraulic cavitation. Although gaseous cavitation can be induced by ultrasonics, the process is more complicated than that of hydraulic gaseous cavitation.

PROCEDURE AND TYPICAL RESULTS

For a typical set of measurements, tap water is drawn directly from the city supply into the spherical flask. The decay measurements for bubble detection are made over a period of several days as the water stands undisturbed in the sphere. Simultaneously, measurements are made of the inception pressure for ultrasonic cavitation.

The onset of cavitation is determined by increasing the sound pressure in small increments until cavitation starts. The voltage from the sound probe, just before the onset of cavitation, indicates the inception sound pressure. Both gaseous and vaporous cavitation are observed. For gaseous cavitation, the onset is observed visually by the appearance of a small bubble at the center of the sphere. The onset of vaporous cavitation is apparent by the sudden explosive formation of a cavity, accompanied by a "ping" which can be heard in the air.

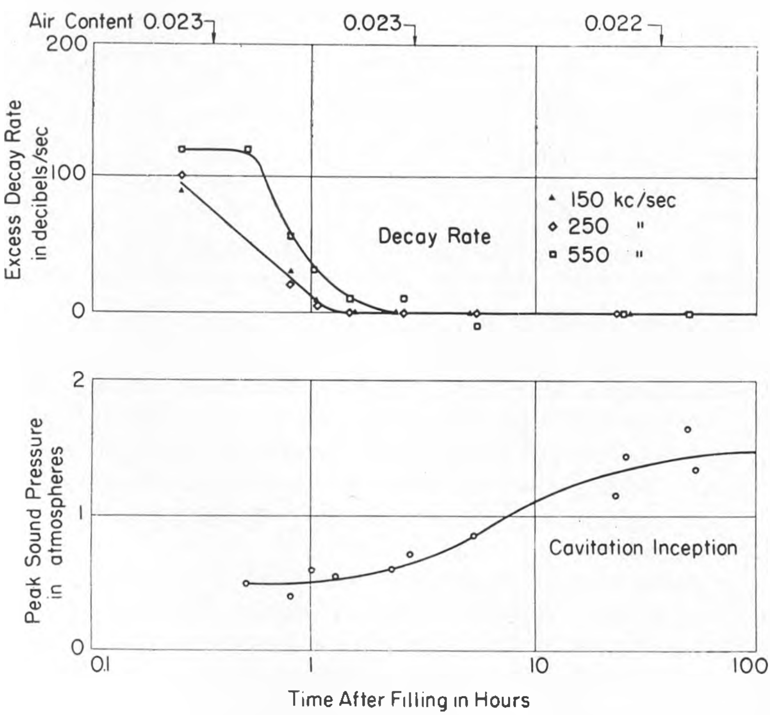


FIG. 4. THE DECAY RATE AND THE INCEPTION PRESSURE FOR ULTRASONIC CAVITATION, AS A FUNCTION OF THE TIME AFTER FILLING THE SPHERE WITH FRESH TAP WATER

Some typical results are shown in Fig. 4. The upper curves

These results were obtained in the course of measurements performed in partial fulfillment of the requirements for the Ph.D. at the Catholic University of America.

show the decay rate measured at three frequencies as a function of the time after filling. The plotted "Excess Decay Rate" is the measured total rate minus the background rate inherent to the apparatus. The excess decay falls to zero in several hours. This means that the content of air bubbles has become too small to be detectable. An upper bound on the number of bubbles which can be present and yet avoid detection can be estimated by substituting into Eq. (3) the measured background decay rates. Upper bounds on the number of bubbles N_r and their volume concentration V_r are indicated in the table below. These values may be considered to be the thresholds of detectability for our apparatus.

TABLE I
Thresholds of Detectability for Bubbles

Frequency kc/sec	Background Decay db/sec	Bubble Radius R cm	Number* N_r per cm³	Volume** V_r cm³/cm³
150	20	2.2×10^{-3}	0.002	1×10^{-10}
250	40	1.3×10^{-3}	0.01	1×10^{-10}
550	200	6×10^{-4}	0.3	2×10^{-10}

*N_r is the number of bubbles per cm³ with radii ranging from 1/2 to 3/2 R.
**V_r is the total volume per cm³ of bubbles whose radii range from 1/2 to 3/2 R.

The lower curve of Fig. 4 shows the sound pressure for cavitation, measured with the same sample of water. The inception pressure changes with time. The measurements during the first few hours resulted in gaseous cavitation, whereas the later measurements at negative critical pressure resulted in vaporous cavitation.

These results, showing how the cavitation characteristics of the water change as it stands, are only illustrative of the type of data obtainable with the apparatus.

For determining the effect of dissolved air, the water can be deaerated without removal from the sphere. The sound intensity is set at a maximum while applying a vacuum to the water in the sphere. This results in violent gaseous cavitation. The appearance of the water during deaeration is shown in Fig. 5. The total air content after partial deaeration is determined by syphoning a 10 cm³ sample into the Van Slyke Gas Apparatus visible at the left side of Fig. 3.

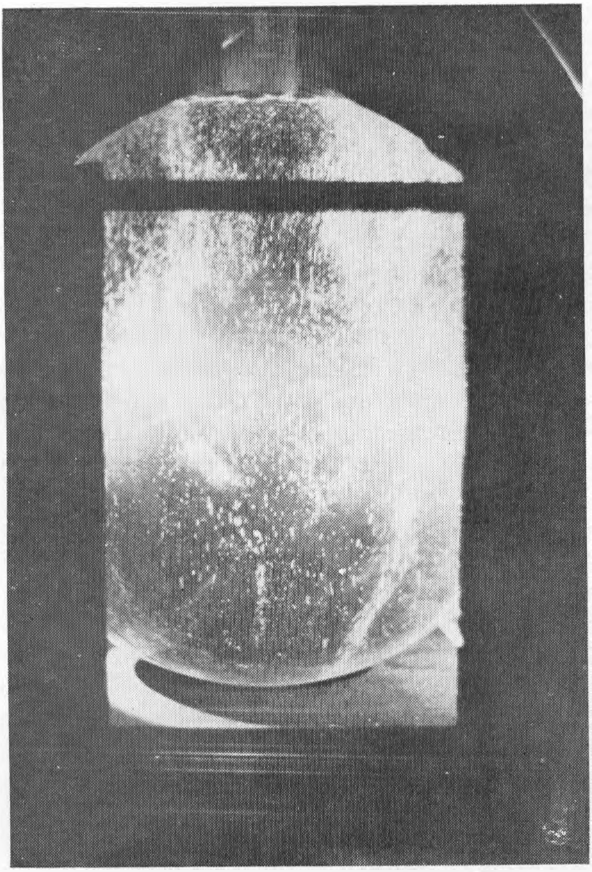


FIG. 5. ULTRASONIC DEAERATION OF THE WATER WITHIN THE SPHERICAL FLASK

FUTURE DEVELOPMENT

The present apparatus is limited to measurements on a batch of water standing undisturbed. For hydraulic applications it would be desirable to detect bubbles and measure the cavitation inception pressure in a stream of moving water.

For measurements in a stream of water, the spherical resonator could be replaced by a cylindrical resonator open at both ends. The water could then stream axially through the cylinder. The bubble detection and the onset of ultrasonic cavitation could be observed in the flowing water. Such a device might be useful for characterizing the water in a water tunnel.

Discussion

Mr. Eisenberg added some comments to clarify the question of bubbles and stability, stating that bubbles will be in equilibrium over a specific range of sizes which are related to the vapor pressure and partial pressure (of air). Bubbles larger than this range of sizes will remain, while the smaller ones will dissolve. Those within the range will be dynamically unstable so that they will grow, and when pressure is applied these bubbles are actually in a state of equilibrium in which the required pressure is somewhat less than vapor pressure. This is demonstrated by the same conditions of stability holding. Though the mechanism by which such nuclei are stabilized is still an open question, long before Harvey, Herzfeld, etc., suggested these mechanisms, people working with problems of surface chemistry had pointed out that certain materials such as detergents may provide stability. He mentioned the cases where air is not needed for stability and stated that infinite energies would be required to dissolve a vapor on that premise even without air.

Regarding the maximum tensions attainable in liquids, he said that the value of 10,000 atmospheres mentioned by Mr. Strasberg was based on the old theory of a ruptured molecule, while according to modern theories—he mentioned the "hole theory"—fluctuations are possible and explained in terms of the crystal structure. In most materials the estimates of 1000 atmospheres which are made seem to be more realistic, although, as Mr. Strasberg pointed out, even this magnitude has never been experimentally observed.

Mr. Eisenberg mentioned, without wishing to discuss, the experiments 3 or 4 years previously by Dr. Glaser at the University of Michigan in which he used a "bubble chamber" to detect sub-atomic particles. This is the liquid analog of a cloud chamber, and in it, paths of the particles are shown by nuclei that open up in superheated water when it is bombarded. His main point was to express the possibility that cosmic radiation in the atmosphere may determine the limit of tension attainable in experiments with the rotating tube and other devices.

Mr. Strasberg emphasized the fact that the critical pressure in his curve was corrected for vapor pressure of the water. With regard to the possibility of the cavities containing only water vapor, he didn't wish to exclude it entirely but stated that his data show a very definite variation in critical pressure with air content which would seem to require that the cavity itself contain air which varies with air content.

Mr. Ripken expressed the opinion that the techniques discussed in the paper furnish a good example of the effective use of an indirect method where the dimensions of the things to be examined are extremely minute and apparently direct methods are impossible. In view of the indirectness, the good correlation of the data is a high tribute to what has been done. It should be noted that the experiments described herein more or less describe the static scale of things and that there is a broad area of obscurity as to what happens when we transfer to dynamic problems, particularly with regard to derivations based on surface-tension phenomena which have been measured only in the steady case. We have no knowledge which would indicate that surface tensions could not be drastically different as functions of time when fluids are in motion. Mr. Strasberg replied that he usually referred to instantaneous variations or to the fact that water might be flowing.

Mr. Ripken concluded by saying that anything which disturbs the rate at which energy can be transferred through the surface is important, since surface tension is an energy condition and an energy exchange must take place with the bulk of the fluid to cause a change at the surface.

Mr. Strasberg stated that the equipment could be redesigned to permit its use with flowing water by replacing the sphere with an open-ended cylinder. By using the resonant frequency of the cylinder, exactly the same kind of measurements could be performed and the results could be checked using flowing water.

REFERENCES

1. Crump, S. F., "Determination of Critical Pressures for the Inception of Cavitation in Fresh and Sea Water as Influenced by Air Content," David Taylor Model Basin Report 575 (October 1949).

2. Parkin, B. R., and Kermeen, R. W., "Incipient Cavitation and Boundary Layer Interaction," Hydrodynamics Laboratory, Cal. Inst. of Tech. Report E 35.2 (December 1953).

3. Briggs, L. J., "Limiting Negative Pressure of Water," *Jour. App. Phys.*, Vol. 21, p. 721 (1950).

4. Harvey, E. N., et al, "Bubble Formation in Animals," *Jour. Cellular and Comparative Physiology*, Vol. 24, pp. 1 and 23 (1944).

5. Pode, L., "The Diffusion of Air into a Pulsating Cavitation Bubble," David Taylor Model Basin Report 804 (March 1955).

6. Knapp, R. T., and Hollander, A., "Laboratory Investigations of the Mechanism of Cavitation," *Trans. A.S.M.E.*, Vol. 70, p. 419 (1948).

7. Epstein P. S., and Plesset, M. S., "On the stability of gas bubbles in liquid-gas solution," *Jour. Chem. Phys.*, Vol. 18, p. 1505 (November 1950).

8. Carstensen, E. L., and Foldy, L. L., "Propagation of Sound Through a Liquid Containing Bubbles," *Jour. Acous. Soc. Amer.*, Vol. 19, p. 481 (1947).

9. Moen, C. J., "Ultrasonic Absorption in Liquids," *Jour. Acous. Soc. Amer.*, Vol. 23, p. 62 (January 1951).

10. Galloway, W. J., "An Experimental Study of Acoustically Induced Cavitation in Liquids," *Jour. Acous. Soc. Amer.*, Vol. 26, p. 849 (September 1954).

11. Noltingk, B. E., and Neppiras, E. A., "Cavitation Produced by Ultrasonics," *Proc. Phys. Soc. B*, London, Vol. 63, p. 674 (1950) and Vol. 64, p. 1032 (1951).

ELECTROLYTIC MEASUREMENT OF LOW VELOCITIES IN WATER

by

M. C. Boyer

Introduction

There has long been a need for a device to measure low velocities in flowing water. Present instruments, including the Pitot tube, hot-wire anemometer, and blade or cup meters, are insensitive, erratic, or unstable in their response to low velocities, below approximately 0.5 foot per second. The writer began the study of electrolytic methods for the measurement of low velocities in 1946. In 1947 he presented a thesis [1] which included the results of a study of change in resistance of an electrolysis cell formed in water, caused by movement of the water. The principle of operation was not understood at that time and the investigation was continued.

By 1953 a somewhat clearer understanding had been acquired, at which time a paper was presented at the Third Midwestern Conference on Fluid Mechanics [2], describing the results of the investigation to that time. It had been discovered that the apparent resistance change of the cell occurred at the cathode and was related to the movement of dissolved oxygen molecules past it. This effect was observed as a voltage drop between the cathode and anode. That voltage drop was related to the velocity of the water by calibration in a towing tank, and an instrument was designed.

Further understanding was gained from the literature on polarography, which is the science of detecting the presence of various elements and compounds in an electrolyte by means of characteristic shapes of current-voltage curves for electrolysis cells containing these materials in solution. On the basis of the knowledge thus gained the factors involved in the design and operation of the instrument were recognized and it was then possible to separate and study the variables. The information thus obtained led to the development of an instrument for the electrolytic measurement of low velocities in water which shows promise.

Principle of Operation

In order to understand the operation of this device it is necessary to have some understanding of an electrolysis cell. When two electrodes are immersed in an electrolyte and a direct current from an external source passed through the cell, there will be a drop in voltage between the electrodes, indicating resistance to the flow of current through the cell. This voltage drop occurs in three steps, a sharp drop at the anode, where the electrons leave the electrolyte, a lesser drop through the electrolyte, and a second sharp drop at the cathode, where the electrons enter the electrolyte. Since in this study the important voltage change is that at the cathode and for reasons of stability, which will be discussed later, the voltage drop at that electrode is measured against a standard calomel reference electrode. The latter will maintain a constant voltage drop between itself and the electrolyte for a wide variation in electrolyte composition. This reference electrode is formed in a glass container, the bottom of which is covered with a pool of mercury. Over the surface of the mercury is a layer of calomel, and the container is filled with saturated NaCl solution. Connection with the electrolyte in which it is immersed is made through a finely perforated glass disk in the side of the container. Electrical connection is made with the mercury pool through an immersed tungsten wire. The voltage drop between the mercury surface and the saturated NaCl solution remains constant and since there is no resistance to current flow between that solution and the electrolyte no further drop occurs. Thus is maintained a constant reference electrode.

Figure 1 shows schematically the voltage drops from anode to reference electrode and from the reference electrode to the cathode across an electrolysis cell formed in tap water, with the electrode materials used in this study. The current through the cell was 200 microamperes, the temperature 70 deg. F., the water contained

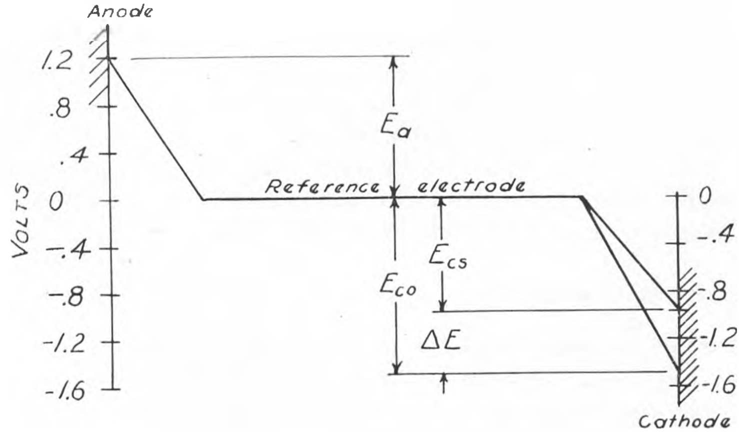


FIG. 1

approximately 300 parts per million of dissolved solids. For quiescent water the voltage difference E_a between the anode and reference electrode was 1.2 volts, the potential of the anode being positive with respect to the reference electrode. The drop E_{co} between the reference electrode and the cathode was 1.5 volts, the cathode being negative with respect to the reference electrode.

If the water is rapidly stirred, the voltage drop between anode and reference electrode remains the same, but that between the reference electrode and the cathode (E_{cs}) became 0.9 volt. This change ΔE is the result of the contact of dissolved oxygen molecules with the cathode. These molecules, while electrically neutral in solution, will, on contact with a negatively charged surface whose voltage is sufficiently below that of the water, take from that surface 4 electrons per molecule, thus facilitating the flow of electric current. If the total current through the cell remains constant, this easier flow will be observed as a smaller voltage drop between the cathode and reference electrode. This voltage drop can be related experimentally to the velocity of water past the cathode, thus permitting the use of the cell as a velocity-measuring device.

The change in voltage at the surface of the cathode with movement of the electrolyte past it is a phenomenon of considerable complexity. It has found practical usefulness in the determination of the presence of various ions in an electrolyte [3]. It will probably be best understood from a study of the relation between the current passing through the cell and the voltage difference between the cathode and the reference electrode. Curves of relation have been developed using a conducting film electrode, to be described in detail later, in tap water and in sea water, both quiescent and stirred. Stirring was accomplished with a magnetic stirring rod 2 inches in length, rotated at a speed of approximately 10 revolutions per second in a container 3 inches in diameter. The data are shown in Fig. 2.

At zero current flowing in the cell, the voltage difference between the cathode and the reference electrode is 0.35 volt. As current flow begins, the water remaining quiescent, the voltage increases with increase in current, following the section of the curve AB. All ions in the water and the oxygen molecules are participating in the current transfer, by the process of ionic migration. The current is sufficiently small, so that no concentration gradient is set up between the cathode surface and the liquid surrounding it for any ion. At a current of 20 microamperes per square centimeter, oxygen, which is the most easily reduced of any element at the cathode, begins to develop a concentration gradient. This is shown by the section BC, which indicates that the voltage drop is increasing rapidly with small increase in current. Because of the formation of a concentration gradient, the oxygen molecules contact the cathode in progressively lesser numbers, until the rate of such contact is a constant and a function of the diffusion rate in the liquid.

Following the establishment of this diffusion gradient of oxygen, the other ions continue to carry the current and the voltage-current curve rises again at a rate essentially parallel to the section AB, until another ion establishes a diffusion gradient, resulting in a

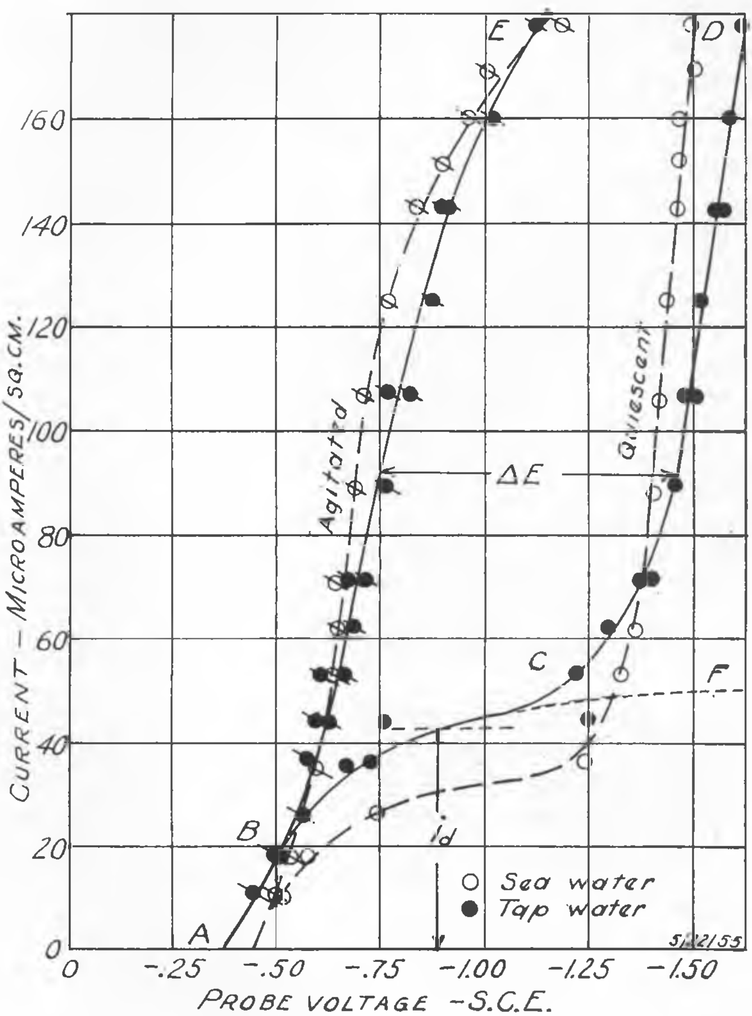


FIG. 2

second plateau. If the oxygen molecule were the only means of transferring electrons from the cathode, the curve would follow CF.

If the electrolyte is rapidly stirred, a different effect is noted. With stirring, the oxygen molecules are carried to the cathode more rapidly than they are reduced, hence no concentration gradient may form and the curve continues at practically the same slope as the section AB. With sufficiently high currents, even rapid stirring will not supply a surplus of oxygen molecules, and the curve begins to flatten, showing the formation of a concentration gradient, at E. This voltage difference between quiescence and rapid stirring is a function of the velocity of water movement past the cathode and can be calibrated experimentally. It is this principle upon which the operation of the device described in this paper is based.

Instrument Development

Electrode Material

Once the principle of operation of the device was recognized and evaluated, the development centered about the determination of an electrode material which would be both responsive and stable. Many difficulties were encountered. In early experiments the voltage across the cell was used, but variations in anode potentials upset the readings. Platinum and gold wire were first tried for the cathode, while carbon, aluminum, zinc, iron, brass, the calomel reference electrode, and the metal of the flume were used as anode. Most of these combinations were responsive, but could not be made stable. Attention was then directed to aluminum, a material nearly at the opposite end of the electromotive series of elements from the noble metals, and having a small voltage difference at the liquid-solid interface. Somewhat better stability was obtained, but neither stability nor response were completely satisfactory [2].

The writer took the problem to Dr. Karl Kammermeyer, head of the Department of Chemical Engineering, State University of Iowa, who has been experimenting with conducting films on non-conducting base materials. These films are highly polished being only 3-5 angstroms thick, resistant to the action of all the common reagents, and have a relatively high voltage drop at the liquid-solid interface. These films are available commercially on ¼-inch rods, which can readily be made into electrodes.

Two probes were used in the tests described in this paper. Earlier experiments were made with a probe formed from a single rod from which the coating was partly removed by grinding, so that opposite quarter surfaces remained. This was used primarily to determine the effects of the different variables. It was mounted in a glass holder, in a rubber stopper, and placed in a 300-cc bottle, in which the water could be magnetically stirred. One surface was made the cathode, the other the anode. It was found that an anode of this material was stable, so that voltage changes were observed directly across the cell for the earlier work. Later, the reference electrode was adopted for these studies as well as for the recording of velocities in water.

The probe used for the velocity determinations consisted of a cathode formed by a semicylindrical conducting film surface, presented to the flow with the cylinder at right angles to it, so that the flow would pass over the film surface. The anode was a cylinder of the same material, with its axis parallel to the direction of flow so that the sweeping action over the surface would be minimized.

Electrical Circuitry

The external electrical circuit may be designed in either of two ways: with a low-voltage direct-current source, holding the voltage drop across the cell a constant; or with a high-voltage direct-current source and a high limiting resistor in series, producing a constant current through the cell. In the former, the changes produced by the movement of water past the probe are shown as changes in the current flow, while in the latter the changes are noted in the voltage across the cell or with respect to a reference electrode apart from the circuit. The method of constant current was adopted, as it was much easier to amplify the voltage changes so that they could be graphically recorded than to produce a voltage signal from the variable current, then to amplify and record.

It was discovered that much of the instability in the earlier instruments was caused by variations in the voltage drop from the anode to the water, apparently caused by anodic reactions with dissolved matter in the water. It was then decided to measure the voltage at the cathode against a standard reference electrode, consisting of a calomel cell, previously described. This system materially improved the stability of the instrument.

The circuitry selected is shown in Fig. 3. The current to the cell is supplied by a 45-volt battery, connected in series with a variable resistor covering a range from 0.1 to 2.5 megohms, which will give a current range from 450 down to 18 microamperes. The current can be held closely constant. A voltmeter and voltage change recorder are connected between the cathode and the standard reference electrode, thus completing the recording circuit.

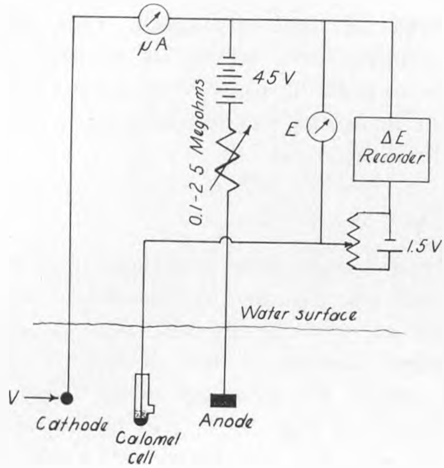


FIG. 3

EFFECTS OF VARIABLES OF THE WATER

There are many variables of the water, in addition to its dissolved oxygen content, on which this instrument's operation depends. They include the water temperature, hydrogen-ion concentration, the presence of dissolved solids, salt, principally, in sea water. Each was investigated in turn, holding the others as nearly constant as possible.

Dissolved Oxygen

Water practically free of oxygen was produced by boiling and cooling under a nitrogen atmosphere, which was maintained thereafter. For each test oxygen-free water was blended with varying ratios of oxygen-saturated water of the same chemical makeup. The water contained approximately 500 ppm of NaCl, to provide ionization. Dissolved-oxygen concentrations from approximately 1 to over 8 parts per million by weight, were produced. Voltages across the cell were observed when the water was quiescent and agitated. The difference is a function of the concentration, as shown in Fig. 4. Two anodes were used in these experiments, the coated rod and the calomel electrode, the solid curve shows the results of the first combination, the dashed curve the second. The dissolved oxygen was determined for each sample by chemical titration, following the Winkler method. The current density was 60 microamperes per square centimeter. It will be noted that above approximately 4 parts per million of dissolved oxygen the effect of oxygen on the voltage change is relatively small. This would indicate that water at room temperature, which, if saturated, would contain about 8 parts per million of dissolved oxygen, could have a considerable variation in actual oxygen content without effect on the instrument's calibration.

Water Temperature

The effect of temperature was investigated under two conditions: First, the water was warmed to 116 degrees F., and allowed to cool while the readings of volt-change between quiescent and stirred water were recorded; second, it was cooled to 32 degrees F. and allowed to warm, while the readings were taken. The results of these tests are shown in Fig. 5. It will be noted that the change decreased by 6 percent over the range, at a rate of 0.075 percent per degree. It is not thought that such a change would be serious in the calibration of the instrument and could be allowed for. The results agree with those of Brdicka [4], who reported that the potential usually shifted toward a more positive value (easier reduction) with increase in temperature. Also, the diffusion rate, which measures the change in voltage along the curve BC in Fig. 3, will be greater, hence the voltage change will be less with increasing temperature.

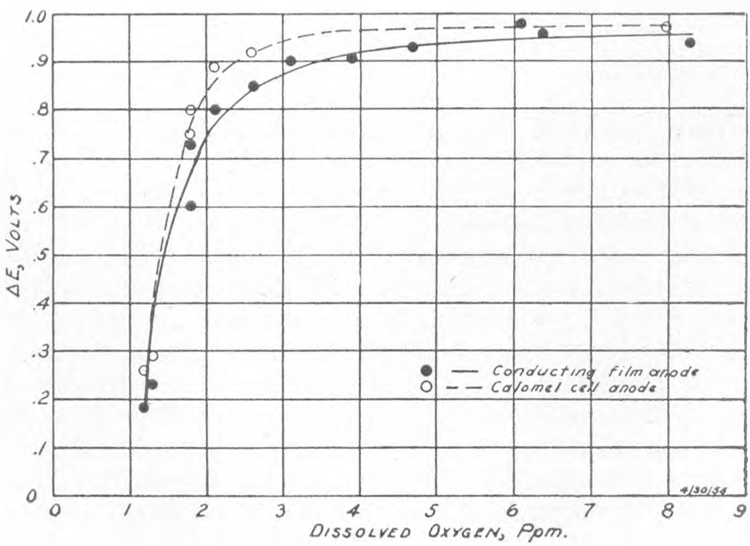


FIG. 4

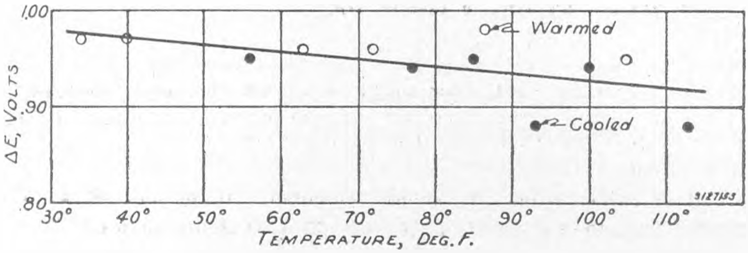


FIG. 5

Hydrogen-Ion Concentration

The effect of hydrogen-ion concentration was observed in water samples made progressively more basic or acid through addition of ammonium hydroxide and acetic acid. Determinations were made over a range of pH from 5 to 11.5. The results are shown in Fig. 6. While the voltage of the cathode relative to the reference electrode became larger with increasing pH, for both the quiescent and stirred water, the voltage change remained constant. Kolthoff and Lingane (Reference [3], p. 405) show polarographic waves of oxygen in current-voltage curves for pH concentrations between 3.0 and 12.0 pH, for quiescent water. These also indicate that the voltage referred to the reference electrode increased with increasing pH.

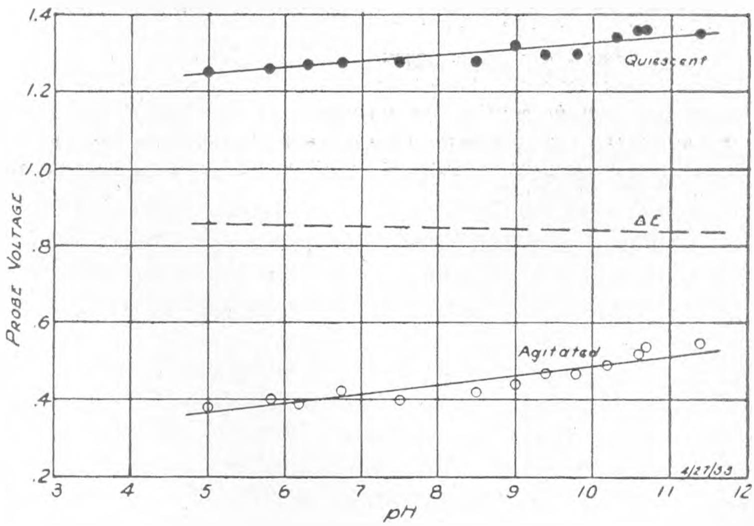


FIG. 6

Effects of Other Dissolved Substances

Other substances in the water besides dissolved oxygen are reduced at the cathode. The voltages at which these reductions begin are generally more negative than that for oxygen and little difficulty with the interference of those other substances in the oxygen reduction is anticipated, with the possible exception of hydrogen. The approximate voltages, referred to the standard calomel electrode, at which reduction of the more common elements begins is shown in Table I.

TABLE I

Approximate voltage, referred to the standard calomel electrode, at which reduction in water of common elements will begin, after Lingane and Kolthoff [3], Vol. 2.

Element	Voltage	Element	Voltage
Oxygen	—0.5	Aluminum	—1.8
Lead	.6	Calcium	2.2
Copper	.6	Magnesium	2.2
Hydrogen	1.0	Sodium	2.2
Manganese	1.5	Potassium	2.4
Iron	1.7		

Of considerably more importance is the fact that as the oxygen atoms are reduced they combine with water molecules to form hydroxyl ions in the diffusion layer, according to the equation

$$O + H_2O + 2e \rightarrow 2OH^-$$

These ions will react with certain metals in the water and produce hydroxides, which, if insoluble, may be deposited on the cathode. Principal among these is $Ca(OH)_2$, which is very insoluble, being only 2 percent that of $Ca(Cl)_2$ at room temperature. Further, if glass is used as the nonconducting support for the electrodes, it will be attacked by the hydroxyl ions, with the removal of sodium and calcium from the surface. Further investigation of this aspect is indicated.

The effect of NaCl was studied. To samples of distilled water were added successively greater amounts of NaCl, producing a range in salinity from approximately 200 to 350,000 parts per million. The results of this are shown in Fig. 7, as voltage across the cell versus the salt concentration. As the salt content was increased, the voltage drop across the cell decreased for both the quiescent and stirred water, while the voltage change increased

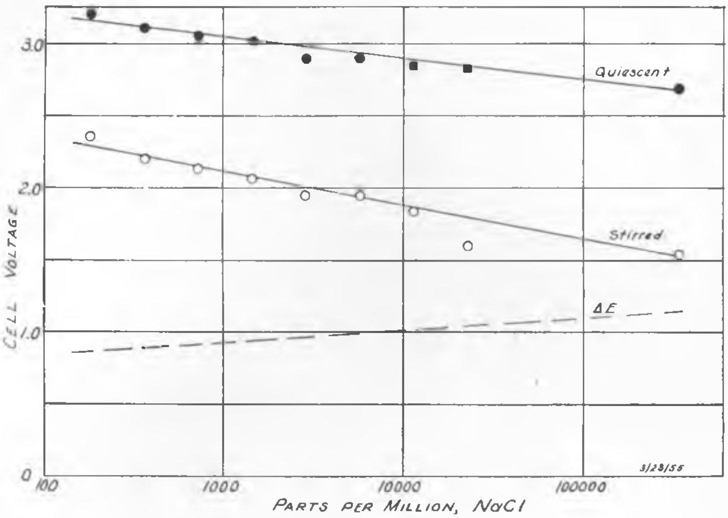


FIG. 7

slightly. However the effect was small and not thought to present a serious problem. If the device is to be used in sea water it should be calibrated in that medium.

INSTRUMENT DESIGN

The design of the sensing device for the instrument described in this paper was limited by the availability of the conducting material, which could be obtained only as a coating on a 1/4-inch rod. The cathode was made by mounting a short section of the rod at right angles to the direction of the water movement, so that the flow was across the surface of the conducting film in intimate contact with it. The anode was of the same material, mounted to the rear of the cathode. The reference electrode consisted of a standard calomel electrode, described earlier. This reference electrode was mounted between the cathode and anode, so that the ohmic drop in the water between the cathode and reference electrode would be small.

Following design of this sensing device it was discovered that a method of putting a highly polished platinum surface on glass was available, thus permitting the conducting platinum film to be placed on a probe of any desired shape. A glass probe was formed with leads to the front and rear ends, which were made hemispherical. One end was coated with the shiny platinum to an area of approxi-

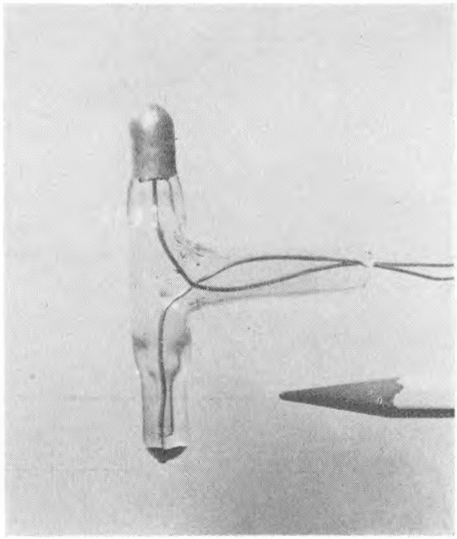


FIG. 8

mately one square centimeter, the other with a porous coating of several square centimeters, Fig. 8. While this probe was used for demonstration, it is not believed that the platinum will prove as stable as the conducting film previously experimented with. Such a film can also be put on a probe of this form.

The electric circuitry has been previously described. The voltmeter can be any standard high-impedance meter. A standard direct-current power amplifier and graphic recorder complete the circuit for recording. For field use a battery-operated amplifier and bridge circuit has been designed. It measures the voltage change between cathode and anode and can with little modification be adapted to measure that between the cathode and reference electrode instead.

CALIBRATION AND STABILITY

The conducting film probe described above was mounted on a towing-tank carriage at the Iowa Institute and moved through still water at speeds varying between 0.02 and 2 feet per second. Calibrations were carried out at three current densities, 133, 200, and 267 microamperes per square centimeter. The results are shown in Fig. 9. For velocities below 0.4 foot per second the calibration for each current density is a straight line on logarithmic plotting, the upper range increasing as the current is increased.

Stability was the problem of greatest concern in this study.

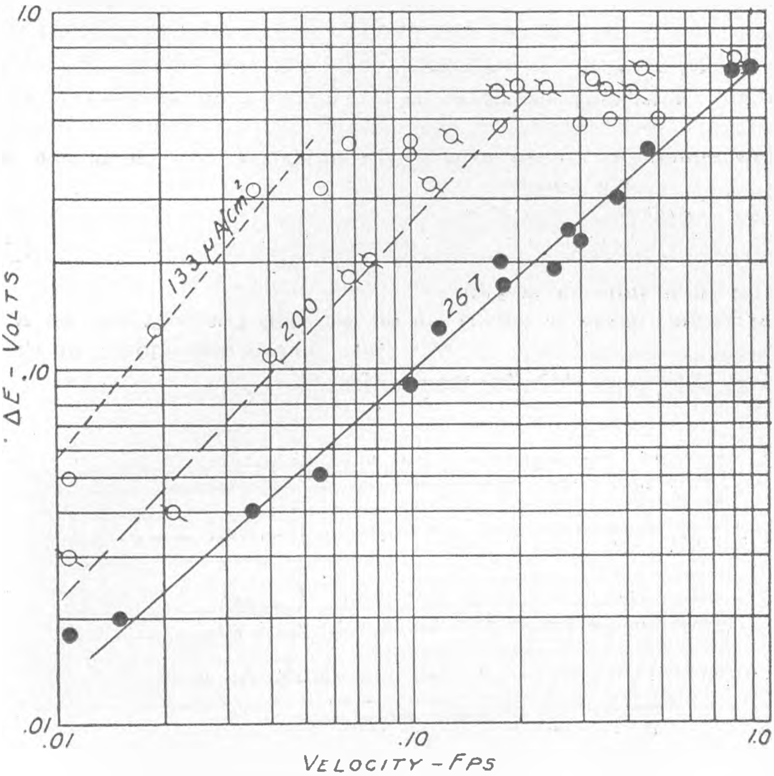


FIG. 9

Various combinations of cathode and anode were tried and their stability determined by making graphic recordings of the voltage behavior under conditions of constant and zero velocity. These tests were made at room temperature, using tap water and a probe current of 200 microamperes per square centimeter. The results are shown in Figs. 10a and 10b. Graph (a) is for the conducting film cathode and calomel anode, the voltage across the cell being recorded. It will be noted that the voltage drop for zero velocity drifted slowly, while that for constant velocity was nearly constant. The voltage after the water motion was stopped returned to a somewhat smaller value, indicating some instability. Graphs (b), (c), and (d) were for the conducting film (BK) cathode and brass anode, aluminum wire cathode and calomel anode, and gold cathode and calomel anode. Not only were these combinations unstable, their response was not as great as that for the BK-calomel electrode combination.

A test for stability of approximately 2 hours duration was then run with the BK cathode and anode. The results are shown in Fig. 11. Good stability is indicated. The voltage during agitation remained essentially constant, as did that during the period of zero velocity. More important, the voltage drop returned to its former value when the liquid was again agitated after a period of 90 minutes of zero velocity.

Stability tests were also run for the BK cathode and calomel reference electrode. Essentially the same results were achieved. Using this combination, graphs were made of the action of waves in a flume, using a period of 30 seconds between waves, and of turbulence downstream from a jet. These are shown in Fig. 12. Reasonably good stability was indicated.

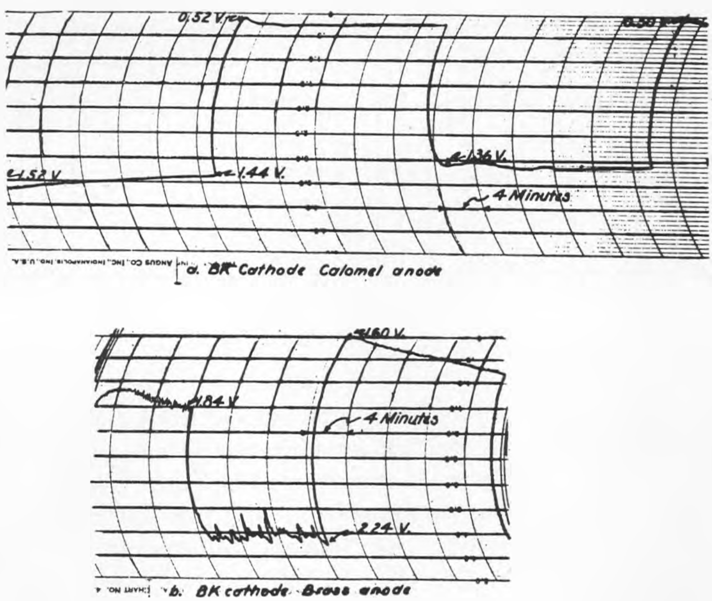
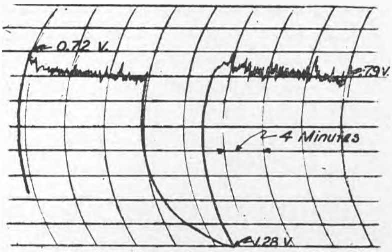


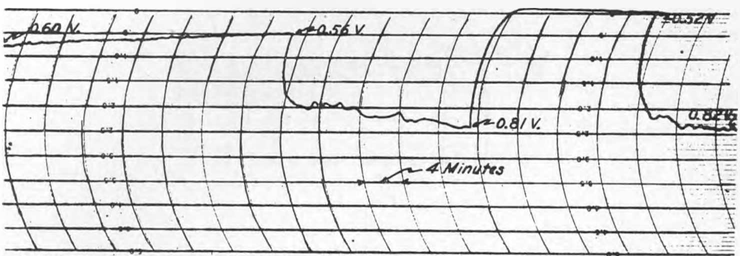
FIG. 10 A

CONCLUSIONS

It is believed that the instrument described will prove useful in the determination of low velocities in flowing water. So far as the writer knows, it is a new application of an old principle, and as such will require much further study and experimentation before it will have reached the stage of development of other velocity measuring instruments—the hot-wire anemometer, for example. Since the

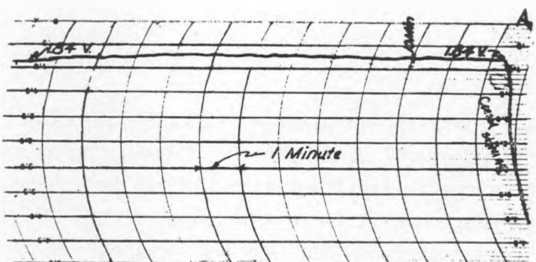


c. Aluminum cathode Calomel anode

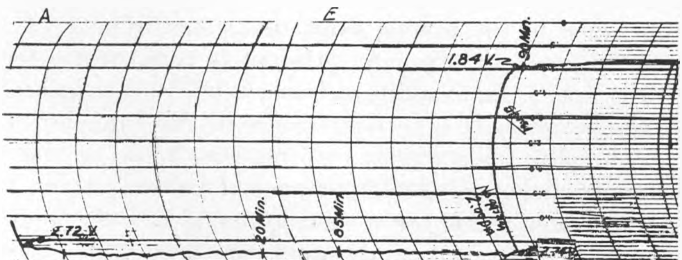


d. Gold cathode Calomel anode

FIG. 10 B



e. BK cathode and anode



BK cathode and anode-Cont.

FIG. 11

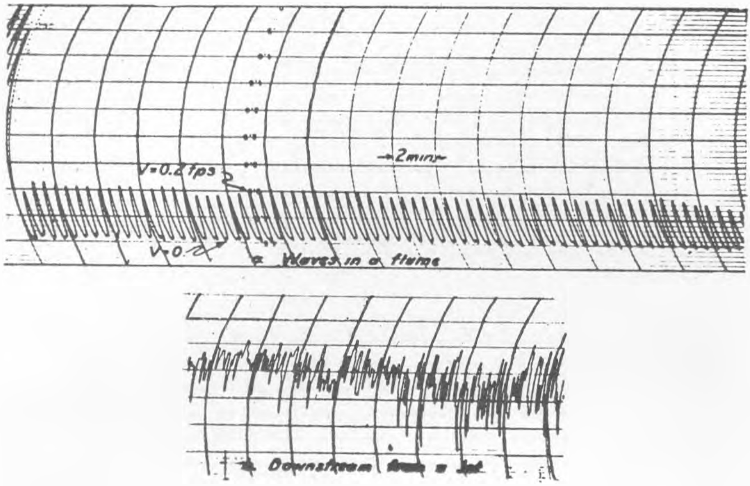


FIG. 12

makeup of the water in a closed circuit in a hydraulics laboratory can be closely controlled, calibration of the device and its use under such conditions should prove satisfactory. So far as its use in the field is concerned, only further experimentation and development will provide the answer.

DISCUSSION

Mr. Ripken emphasized the importance of Mr. Boyer's work because nothing is presently available for measuring velocities around 1 or 2 feet per second. He then asked what accuracy could be expected in measuring velocities in the range from 0.01 to 1 foot per second after compensating for known sources of error and instability. Mr. Boyer replied that he did not yet know the answer to this difficult question, but stated that in his measurements using tap water in the towing tank, occasional indications were off by 15 or 20 percent without any attempt to compensate for various things. He hopes that additional work he is now planning will improve the accuracy to 5 percent. Since no instrument is presently available, one with an accuracy of 5 percent would be a great help.

Mr. Craven discussed the importance of knowing the law governing the indication whenever a measurement is quite indirect. He felt that a more detailed study would be necessary to determine the extent to which dissolved oxygen and other phenomena affect the indications. Mr. Boyer agreed heartily, and emphasized the fact that he had looked into the mathematics of the subject and found that differential equations had been developed for concentration gradients near cylinders and spheres. Also, some of the work done in Germany was confirmed by his own results.

Mr. Hubbard remarked that the frequency response of 20 cycles per second, though low, was probably adequate for the velocity range below 1 foot per second where the electrolytic method is applicable.

Mr. Gent wanted to know whether the electrodes had to be spaced about the instrument or whether they could be built in each side of a model so that there is no obstruction to the water. Mr. Boyer believed that a sweeping action very near the electrodes was very important so that any arrangement which placed them within a thick boundary layer would not be satisfactory.

It was then asked if any thought had been given to measuring the extreme variations in direction which usually accompanied low velocities in the field. Mr. Boyer recognized the need for this type of indication and felt that it was principally a hydraulic problem to make the flow past the sensing element depend upon the angularity of the flow.

Mr. Kolupaila asked whether the instrument could be modified to work at higher velocities by changing the electrode spacing. In reply, Mr. Boyer stated that the concentration gradient at the cathode was the primary factor in the instrument's indication, and that this gradient reached a limiting condition which did not depend on electrode spacing.

Mr. Hubbard commented on the great improvement which has resulted from the use of a reference electrode to eliminate the undependable anode effect.

Mr. Kalinske stated that this type of instrument is more or less a standard for measuring dissolved oxygen in quiescent liquids and is widely used by chemists and biologists for this purpose. However, it is also influenced by oxidizing and reducing substances which are present, such as chlorine, or any biological organisms that may be oxidizing or reducing organic matter, so that there are many secondary effects which will influence the readings besides the velocity of the fluid.

Mr. Boyer concluded by saying that he has had trouble with copper depositing on his cathode and that in polarographic research the oxygen is usually removed because it obscures the presence of many other substances. Similar problems exist at the anode, and chlorine did cause some trouble in the early experimental work.

REFERENCES

1. Boyer, M. C., "The Measurement of Velocity of Flowing Water by Electrical Methods," State University of Iowa M.S. Thesis, 1947.

2. Boyer, M. C., and Lonsdale, E. M., "The Measurement of Low Water Velocities by Electrolytic Means," Third Midwestern Conference on Fluid Mechanics, University of Minnesota, 1953.

3. Kolthoff, I. M., and Lingane, V. V., "Polarography," Interscience Publishers, New York, 1952.

4. Brdicka, R., "Collection of Czechoslovakian Chem. Comm." (see p. 234 of Ref. [3] above).

COLLEGE OF ENGINEERING
STATE UNIVERSITY OF IOWA

Complete undergraduate courses are offered in Chemical, Civil, Commercial, Electrical, and Mechanical Engineering, as well as graduate courses in these fields and in Mechanics and Hydraulics. For detailed information, application may be made to T. H. McCarrel, Registrar, State University of Iowa, Iowa City, Iowa.

IOWA INSTITUTE OF HYDRAULIC RESEARCH

The Institute was organized to coordinate the talents and facilities available at the University of Iowa for the investigation of problems in the fields of fluid mechanics, hydrology, and hydraulic engineering. Through this medium the University has cooperated with government agencies, technical societies, and industrial concerns throughout the country and abroad. Correspondence regarding the services of the Institute should be addressed to Hunter Rouse, Director, Iowa Institute of Hydraulic Research, Iowa City, Iowa.

STUDIES IN ENGINEERING

Bulletin 1. "The Flow of Water Through Culverts," by D. L. Yarnell, F. A. Nagler, and S. M. Woodward, 1926. 128 pages, 26 figures, 23 plates. Out of print.

Bulletin 2. "Laboratory Tests on Hydraulic Models of the Hastings Dam," by Martin E. Nelson, 1932. 72 pages, 40 figures. Out of print.

Bulletin 3. "Tests of Anchorages for Reinforcing Bars," by Chesley J. Posey, 1933. 32 pages, 18 figures, price $0.50.

Bulletin 4. "The Physical and Anti-Knock Properties of Gasoline Blends," by Theodore R. Thoren, 1934. 32 pages, 13 figures, price $0.35.

Bulletin 5. "The Transportation of Detritus by Flowing Water — I," by F. T. Mavis, Chitty Ho, and Yun-Cheng Tu, 1935. 56 pages, 15 figures, price $0.50.

Bulletin 6. "An Investigation of Some Hand Motions Used in Factory Work," by Ralph M. Barnes, 1936. 60 pages, 22 figures. Out of print.

Bulletin 7. "A Study of the Permeability of Sand," by F. T. Mavis and Edward F. Wilsey, 1936. 32 pages, 12 figures, price $0.35.

Bulletin 8. "Radiation Intensities and Heat-Transfer in Boiler Furnaces," by Huber O. Croft and C. F. Schmarje, 1936. 32 pages, 17 figures, price $0.35.

Bulletin 9. "A Summary of Hydrologic Data, Ralston Creek Watershed, 1924-35," by F. T. Mavis and Edward Soucek, 1936. 72 pages, 25 figures, price $0.50.

Bulletin 10. "Report on Hydraulics and Pneumatics of Plumbing Drainage Systems — I," by F. M. Dawson and A. A. Kalinske, 1937. 32 pages, 5 figures, price $0.35.

Bulletin 11. "The Transportation of Detritus by Flowing Water — II," by F. T. Mavis, Te-Yun Liu, and Edward Soucek, 1937. 32 pages, 8 figures, price $0.35.

Bulletin 12. "Studies of Hand Motions and Rhythm Appearing in Factory Work," by Ralph M. Barnes and Marvin E. Mundel, 1938. 64 pages, 24 figures. Out of print.

Bulletin 13. "Hydraulic Tests of Small Diffusers," by F. T. Mavis, Andreas Luksch, and Hsi-Hou Chang, 1938. 32 pages, 16 figures, price $0.25.

Bulletin 14. "A Study in Flood Waves," by Elmer E. Moots, 1938. 32 pages, 7 figures, price $0.25.

Bulletin 15. "The Road Map of Hydraulic Engineering in Iowa," by E. W. Lane and Edward Soucek, 1938. 16 pages, 4 figures, price $0.25.

Bulletin 16. "A Study of Hand Motions Used in Small Assembly Work," by Ralph M. Barnes and Marvin E. Mundel, 1939. 68 pages, 33 figures, price $0.50.

Bulletin 17. "A Study of Simultaneous Symmetrical Hand Motions," by Ralph M. Barnes and Marvin E. Mundel, 1939. 40 pages, 15 figures, price $0.40.

Bulletin 18. "Percolation and Capillary Movements of Water Through Sand Prisms," by F. T. Mavis and Tsung-Pei Tsui, 1939. 32 pages, 13 figures, price $0.25.

Bulletin 19. "Two Decades of Hydraulics at the University of Iowa, Abstracts of Theses, Publications, and Research Reports, 1919-1938," edited by F. T. Mavis, 1939. 84 pages, price $0.50.

Bulletin 20. "Proceedings of Hydraulics Conference," edited by J. W. Howe, 1940. 260 pages, 84 figures. Out of print.

Bulletin 21. "Studies of One and Two-Handed Work," by Ralph M. Barnes, Marvin E. Mundel, and John M. MacKenzie, 1940. 68 pages, 31 figures, price $0.50.

Bulletin 22. "The Study of the Effect of Practice on the Elements of a Factory Operation," by Ralph M. Barnes and James S. Perkins with the assistance and collaboration of J. M. Juran, 1940. 96 pages, 34 figures. Out of print.

Bulletin 23. "An Annotated Bibliography of Fishways," by Paul Nemenyi, 1941. 72 pages, 12 figures. Out of print.

Bulletin 24. "An Investigation of Fishways," by A. M. McLeod and Paul Nemenyi, 1941. 64 pages, 15 figures, 6 plates, price $0.50.

Bulletin 25. "The Electrostatic Effect and the Heat Transmission of a Tube," by Melvin R. Wahlert and Huber O. Croft, 1941. 40 pages, 10 figures, price $0.40.

Bulletin 26. "Investigations of the Iowa Institute of Hydraulic Research," 1939-1940, edited by J. W. Howe, 1941. 64 pages, 15 figures, price $0.40.

Bulletin 27. "Proceedings of the Second Hydraulics Conference," edited by J. W. Howe and Hunter Rouse, 1943. 352 pages, 167 figures. Out of print.

Bulletin 28. "The Preparation of Stoker Coals from Iowa Screenings," by H. L. Olin, 1942. 64 pages, 22 figures, price $0.50.

Bulletin 29. "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, 1945. 40 pages, 18 figures, price $0.40.

Bulletin 30. "The Iowa Institute of Hydraulic Research," 1946. 80 pages, 36 figures, price $0.50.

Bulletin 31. "Proceedings of the Third Hydraulics Conference," edited by J. W. Howe and John S. McNown, 1947. 332 pages, 163 figures, price $2.50.

Bulletin 32. "Cavitation and Pressure Distribution — Head Forms at Zero Angle of Yaw," by Hunter Rouse and John S. McNown, 1948. 70 pages, 53 figures, price $1.00.

Bulletin 33. "Third Decade of Hydraulics at the State University of Iowa," edited by M. C. Boyer, 1949. 84 pages, 8 figures, price $0.50.

Bulletin 34. "Proceedings of the Fifth Hydraulics Conference," edited by John S. McNown and M. C. Boyer, 1953. 302 pages, 139 figures, price $3.50.

Bulletin 35. "Free-Streamline Analyses of Transition Flow and Jet Deflection," edited by John S. McNown and Chia-Shun Yih, 1953. 82 pages, 37 figures, price $1.00.

Bulletin 36. "Proceedings of the Sixth Hydraulics Conference," edited by Louis Landweber and Philip G. Hubbard, 1956. 276 pages, 131 figures, price $3.50.

REPRINTS

Institute articles which have appeared since publication of Bulletin 33 in 1949

79. "Pressure Distributions from Theoretical Approximations of the Flow Pattern," by John S. McNown and En-Yun Hsu; *Heat Transfer and Fluid Mechanics Institute 1949*, pp. 65-76, published by the A.S.M.E. Price $0.25.

81. "Influence of Boundary Proximity on the Drag of Spheres," by J. S. McNown, H. M. Lee, M. B. McPherson, and S. M. Engez; *Proc. of the VII International Congress for Applied Mechanics*, pp. 17-29, London 1948. Price $0.30.

82. "Application of the Electrical Analogy in Fluid Mechanics Research," by Philip G. Hubbard; *The Review of Scientific Instruments*, Vol. 20, No. 11, pp. 802-807, November 1949. Price $0.15.

83. "Characteristics of Irrotational Flow through Axially Symmetric Orifices," by Hunter Rouse and Abdel-Hadi Abul-Fetouh; *Journal of Applied Mechanics*, Vol. 17, No. 4, pp. 421-426, 1950. Price $0.10.

84. "How the Vortex Affects Orifice Discharge," by C. J. Posey and Hsieh-Ching Hsu; *Eng. News-Rec.*, Vol. 144, No. 10, p. 30, March 9, 1950. Price $0.05.

85. "Effects of Particle Shape on Settling Velocity at Low Reynolds Numbers," by John S. McNown and Jamil Malaika; *Trans. A.G.U.*, Vol. 31, pp. 74-82, 1950. Price $0.15.

87. "Effect of a Partial Cutoff on Seepage Rates," by John S. McNown and En-Yun Hsu; *Trans. A.G.U.*, Vol. 31, No. 3, pp. 468-472, June 1950. Price $0.10.

88. "Diffusion of Submerged Jets," by M. L. Albertson, Y. B. Dai, R. A. Jensen, and Hunter Rouse; *Trans. A.S.C.E.*, Vol. 115, p. 639, 1950. Price $0.25.

89. "Model Studies Aid in Design of Brazilian Hydro Project," by E. M. Laursen; *Civil Eng.*, pp. 44-47, January 1951. Price $0.15.

91. "Design of Channel Expansions," by Hunter Rouse, B. V. Bhoota, and En-Yun Hsu, in Symposium on High-Velocity Flow in Open Channels, *Trans. A.S.C.E.*, Vol. 116, p. 265, 1951. Price $0.50.

92. "Progress Report on Model Studies of Scour around Bridge Piers and Abutments," by E. M. Laursen; Thirtieth Annual Meeting of Highway Research Board, Washington, D. C., pp. 1-11, January 1951. Price $0.20.

93. "Critical Regimes of Flows with Density Stratification," by A. Craya; *Tellus*, Vol. 3, No. 1, pp. 28-42, February 1951. Price $0.10.

94. "An Investigation of Flow through Screens," by W. D. Baines and E. G. Peterson; *Trans. A.S.M.E.*, Vol. 73, p. 467, 1951. Price $0.15.

95. "Approximation of Axisymmetric Body Forms for Specified Pressure Distributions," by John S. McNown and En-Yun Hsu; *Journal of Applied Physics*, Vol. 22, No. 7, pp. 864-868, July 1951. Price $0.10.

96. "Application of Conformal Mapping to Divided Flow," by John S. McNown and En-Yun Hsu; *Proc. First Midwestern Conference on Fluid Dynamics*, pp. 143-155, 1950. Price $0.10.

98. "Model Techniques in Meteorological Research," by Hunter Rouse; *Compendium of Meteorology*, American Meteorological Society, pp. 1249-1254, 1952. Price $0.10.

99. "Evaluation of the Critical Regime in Stratified Flow," by A. Craya; *Trans. A.G.U.*, Vol. 32, No. 6, pp. 891-897, December 1951. Price $0.10.

100. "Particles in Slow Motion," by J. S. McNown; *La Houille Blanche*, Vol. 6, No. 5, pp. 701-722, 1951. Price $0.40.

101. "Present-Day Trends in Hydraulics," by Hunter Rouse; *Applied Mechanics* Reviews, pp. 49-50, February 1952. Price $0.05.

102. "Numerical Analysis of Continuous Unsteady Flow in Open Channels," by Pin-Nam Lin; *Trans. A.G.U.*, Vol. 33, No. 2, pp. 226-234, April 1952. Price $.05.

103. "Experimental Investigation of Fire Monitors and Nozzles," by Hunter Rouse, J. W. Howe, and D. E. Metzler; *Trans. A.S.C.E.*, Vol. 117, pp. 1147-1188, 1952. Price $0.25.

104. "Wind Pressure on Elementary Building Forms Evaluated by Model Tests," by J. W. Howe; *Civil Eng.*, pp. 42-46, May 1952. Price $0.15.

105. "Air-Tunnel Studies of Diffusion in Urban Areas," by Hunter Rouse; *Meteorological Monographs*, Vol. 1, No. 4, pp. 39-41, November 1951. Price $0.15.

106. "Seiche in Harbours," by J. S. McNown and P. Danel; *The Dock and Harbour Authority*, pp. 177-180, October 1952. Price $0.15.

107. "Model Studies of Scour Around Bridge Piers and Abutments—Second Progress Report," by Emmett M. Laursen and Arthur Toch; *Highway Research Board Proceedings*, pp. 82-87, December 1951. Price $0.20.

108. "Fluctuation Analysis of Turbulence Measurements," by C. J. Posey; *Proceedings of the Second Midwestern Conference on Fluid Mechanics*, pp. 49-55, 1952. Price $0.25.

109. "Sediment Concentration and Fall Velocity," by J. S. McNown and P. N. Lin; *Proceedings of the Second Midwestern Conference on Fluid Mechanics*, pp. 401-411, 1952. Price $0.30.

110. "Gravitational Convection from a Boundary Source," by H. Rouse, C. S. Yih and H. W. Humphreys; *Tellus*, Vol. 4, No. 3, pp. 201-210, August 1952. Price $0.30.

111. "Waves and Seiche in Idealized Ports," by John S. McNown; U. S. Department of Commerce, National Bureau of Standards, Circular 521, pp. 153-164, November 1952. Price $0.30.

112. "On the Growth and Decay of a Vortex Filament," by Hunter Rouse and H. C. Hsu; *Proceedings of the First U. S. National Congress of Applied Mechanics*, pp. 741-746, June 1951. Price $0.20.

113. "Drag of Spheres Within Cylindrical Boundaries," by J. S. McNown and J. T. Newlin; *Proceedings of the First U. S. National Congress of Applied Mechanics*, pp. 801-806, June 1951. Price $0.20.

114. "Free Convection Due to a Point Source of Heat," by Chia-Shun Yih; *Proceedings of the First U. S. National Congress of Applied Mechanics*, pp. 941-947, June 1951. Price $0.25.

REPRINTS

Institute articles which have appeared since publication of Bulletin 34 in 1953

115. "Free Convection over Parallel Sources of Heat," by Hunter Rouse, W. D. Baines, and H. W. Humphreys; *Proc. of the Physical Society*, B, Vol. LXVI, pp. 393-399, 1953. Price $0.10.

116. "Cavitation in the Mixing Zone of a Submerged Jet," by Hunter Rouse; *La Houille Blanche*, pp. 9-19, Janvier-Fevrier 1953. Price $0.10.

117. "Hydrodynamic Earthquake Forces on Submerged Structures," by John S. McNown; *Proc. of the Third Midwestern Conference on Fluid Mechanics*, pp. 233-242, 1953. Price $0.10.

118. "The Measurement of Low Water Velocities by Electrolytic Means," by M. C. Boyer and E. M. Lonsdale; *Proc. of the Third Midwestern Conference on Fluid Mechanics*, pp. 455-462, 1953. Price $0.10.

119. "The Spectrum Equation of Two-Dimensional Isotropic Turbulence," by Francis R. Hama; *Proc. of the Third Midwestern Conference on Fluid Mechanics*, pp. 427-433, 1953. Price $0.10.

120. "A Generalized Model Study of Scour around Bridge Piers and Abutments," by E. M. Laursen and Arthur Toch; *Proc. Minnesota International Hydraulics Convention*, pp. 123-131, Sept. 1953. Price $0.15.

121. "The Transportation of Sand in Pipes," by John P. Craven and Harry H. Ambrose; *Proc. of the Fifth Hydraulics Conference*, pp. 67-88, 1953. Price $0.25.

122. "An Instrument for Rapid Size-Frequency Analysis of Sediment," by David W. Appel; *Proc. of the Fifth Hydraulics Conference*, pp. 287-302, 1953. Price $0.20.

123. "Observations on the Nature of Scour," by Emmett M. Laursen; *Proc. of the Fifth Hydraulics Conference*, pp. 179-197, 1953. Price $0.20.

124. "On the Velocity Distribution in the Laminar Sublayer and Transition Region in Turbulent Shear Flows," by Francis R. Hama; *Journal of the Aeronautical Sciences*, Vol. 20, No. 9, pp. 648-649, 1953. Price $0.15.

125. "Mechanics of Manifold Flow," by John S. McNown; *Trans. A.S.C.E.*, Vol. 119, pp. 1103-1142, 1954. Price $0.40.

126. "Hydrodynamic Problems in Three Dimensions," by Philip G. Hubbard and Sung-Ching Ling; *Proc. A.S.C.E.*, Vol. 78, pp. 1-9, 1952. Price $0.25.

127. "Applications of the Relaxation Technique in Fluid Mechanics," by John S. McNown, En-Yun Hsu, and Chia-Shun Yih; *Trans. A.S.C.E.*, Vol. 120, pp. 650-686, 1955. Price $0.45.

128. "Stationary Waves in Water Flowing over a Rough Surface," by Chia-Shun Yih; *Trans. A.G.U.*, Vol. 34, No. 6, pp. 889-892, 1953. Price $0.10.

129. "Temperature Distribution in Laminar Stagnation-Point Flow with Axi-symmetry," by Chia-Shun Yih; *Journal of the Aeronautical Sciences*, Vol. 21, No. 1, pp. 37-42, 1954. Price $0.15.

130. "Particle Shape and Settling Velocity," by J. S. McNown, J. Malaika, and H. R. Pramanik; *International Association for Hydraulic Research—Fourth Meeting*, Appendix 25, pp. 511-522, 1951. Price $0.05.

131. "Measurement of Velocity and Pressure Fluctuations in the Turbulent Flow of Air and Water," by Hunter Rouse; *Mémoires sur la Mécanique des Fluides offerts à M. D. Riabouchinsky à l'occasion de son Jubilé Scientifique*, Publications Scientifique et Techniques du Ministère de L'Air, 1954.

132. "Stability of Two-Dimensional Parallel Flows for Three-Dimensional Disturbances," by Chia-Shun Yih; *Quarterly of Applied Mathematics*, Vol. XII, No. 4, pp. 434-435, 1955. Price $0.15.

133. "Seiche in Rectangular Ports," by Julien Kravtchenko and John S. McNown; *Quarterly of Applied Mathematics*, Vol. XIII, No. 1, pp. 19-26, 1955. Price $0.30.

134. "Form Drag of Composite Surfaces," by Hunter Rouse and Tien-To Siao; *Proc. of the Second U. S. National Congress of Applied Mechanics*, pp. 743-749, June 1954. Price $0.20.

135. "Stability of Parallel Laminar Flow with a Free Surface," by Chia-Shun Yih; *Proc. of the Second U. S. National Congress of Applied Mechanics*, pp. 623-628, June 1954. Price $0.20.

136. "Boundary-Layer Characteristics for Smooth and Rough Surfaces," by Francis R. Hama; *Trans. S.N.A.M.E.*, Vol. 62, pp. 333-358, 1954. Price $0.35.

137. "Hydraulic Jump in a Fluid System of Two Layers," by Chia-Shun Yih and C. R. Guha; *Tellus*, Vol. VII, No. 3, pp. 358-366, 1955. Price $0.05.

138. "Field Measurement of Bridge-Pier Scour," by Philip G. Hubbard, and "Model-Prototype Comparison of Bridge-Pier Scour," by Emmett M. Laursen; *Highway Research Board Proceedings*, pp. 184-193, January 1955. Price $0.15.

139. "Turbulent Diffusion across a Density Discontinuity," by Hunter Rouse and J. Dodu; *La Houille Blanche*, No. 4, pp. 522-532, Aug.-Sept. 1955. Price $0.20.

140. "Inlets for Square Conduits," by John S. McNown and Sung-Ching Ling; *La Houille Blanche*, No. 5, pp. 775-781, Oct.-Nov. 1955. Price $0.15.

141. "Free Convection due to Boundary Sources," by Chia-Shun Yih; *Proc. of the First Symposium on the Use of Models in Geophysical Fluid Dynamics*, pp. 117-133, 1953.

142. "On a Generalization of Taylor's Virtual Mass Relation for Rankine Bodies," by L. Landweber; *Quarterly of Applied Mathematics*, Vol. XIV, No. 1, pp. 51-56, 1956. Price $0.15.

143. "A Comparison of the Added Masses of Streamlined Bodies and Prolate Spheroids," by L. Landweber and A. Winzer; *Forschungshefte für Schiffstechnik*, Bd. 3, pp. 180-183, April 1956. Price $0.20.

144. "Seven Exploratory Studies in Hydraulics," edited by Hunter Rouse; *Proc. A.S.C.E.*, Vol. 82, pp. 1-35, 1956. Price $0.30.