Citation for published version:
Li, J, Xiang, Y, Gu, C, Sun, W, Wei, X, Cheng, S & Liu, J 2023, 'Formulation of Locational Marginal Electricity-Carbon Price in Power Systems', *CSEE Journal of Power and Energy Systems*, vol. 9, no. 5, pp. 1968-1972. https://doi.org/10.17775/CSEEJPES.2021.06400

DOI:
10.17775/CSEEJPES.2021.06400

Publication date:
2023

Document Version
Publisher's PDF, also known as Version of record

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
DOI: 10.17775/CSEEJPES.2021.06400

Formulation of Locational Marginal Electricity-Carbon Price in Power Systems

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Abstract—**Decarbonisation of power systems is essential for realising carbon neutrality, in which the economic cost caused by carbon is needed to be qualified. Based on the formulation of locational marginal price (LMP), this paper proposes a locational marginal electricity-carbon price (EC-LMP) model to reveal carbon-related costs caused by power consumers. A carbon-price-integrated optimal power flow (C-OPF) is then developed to maximise the economic efficiency of the power system considering the costs of electricity and carbon. Case studies are presented to demonstrate the new formulation and the results demonstrate the efficacy of the EC-LMP-based C-OPF on decarbonisation and economy.**

Index Terms—**Carbon abatement cost, carbon tariff, locational marginal price, locational marginal electricity-carbon price, optimal power flow.**

I. INTRODUCTION

DECARBONISATION in the power system is essential for realising carbon neutrality. Various technologies and policies are and will be applied to the power system to reduce carbon emissions, which brings additional costs to the power system. Carbon capture and storage technologies (CCS) in coal power plants are estimated to cost 30-45 €/tCO_2 in 2030 [1]. Carbon tariff has been imposed in Minnesota, USA, at 4-34 \$/tCO_2 on coal power from North Dakota [2]. In China, the carbon abatement cost of the power sector with a 5% emission permit is estimated as 216.91 ¥/ tCO_2 in 2016 [3]. Thus, rationally allocating these carbon-related costs to the demand side can efficiently stimulate participants to take their decarbonisation obligation.

Footprint carbon intensity (FCI) [4] and marginal carbon intensity (MCI) [5] are developed to evaluate the carbon intensity in power systems. But these methods can not reveal the carbon-related costs caused by the power consumers, especially when generators have different carbon-related costs. Paper [6] proposed a marginal carbon price based on the nodal carbon intensity, but also ignores the difference of carbon-related costs from generators. The carbon-related costs on the generation side are usually different because of various generation modes, decarbonisation technologies and local policies. The locational marginal pricing (LMP) has been a dominant approach in the power market to define the nodal price, which can reveal the differences of the generation costs from different generators [7]. However, the carbon-related costs and constraints, such as limited carbon-emission permits, are not considered in the conventional LMP.

In a real LMP-based market, the carbon-related costs could be internalised into the bidding price by power plants. Decoupling the carbon-related price from the internalised price with a specific formulation can provide valuable information for decarbonisation in power systems, such as an additional motivation to encourage electricity customers to choose low-carbon power plants.

To address the above problems, this paper proposes a formulation of the locational marginal electricity-carbon price (EC-LMP) to reveal the nodal marginal carbon-related costs as well as the electricity price. The EC-LMP can be obtained by a modified carbon-price-integrated OPF (C-OPF). The costs of carbon abatement, carbon tariff, and carbon-emission permit trading are modelled and added into the C-OPF model. Considering the carbon tax charged by governments from generators is utilised for decarbonisation, the carbon abatement cost in this letter combined the carbon tax and the cost of decarbonisation technologies applied in generators, such as CCS. This also provides sufficient revenue adequacy for the government and power suppliers to reduce or capture carbon emissions. Different from the carbon tax, the carbon tariff in this letter particularly refer to the cross-region tariff, also known as the carbon border tax, such as the carbon tariff applied in America [3]. The cost of carbon-emission permit trading is caused by the cap-and-trade policies, which limits the total carbon emission capacity of power suppliers in certain periods. Besides, since the carbon-emission permits further limit the generation output, the carbon-emission permit is added as a new constraint in the C-OPF.

II. MODELLING OF CARBON-RELATED COSTS

To formulate the EC-LMP, the relation between the generator output and carbon-related costs needs to be first established. The costs of carbon abatement, carbon tariff, and carbon-emission permit widely exist worldwide [1-3] and are considered as the carbon-related costs in this paper. The modelling of each cost is presented as follows:

This work was supported by the National Natural Science Foundation of China (U2166211)

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A. *Carbon Abatement Cost*

Based on the fitted equation in paper [8], the total carbon abatement cost C_t^{CA} can be obtained by a quadratic function (1)-(2).

$$C_t^{CA} = \sum_{n=1}^{N} (\alpha_{l,t} {Ce_{n,t}}^2 + \beta_{l,t} Ce_{n,t}),\ n = 1,2,\dots,N \tag{1}$$

$$Ce_{n,t}(G_{n,t}) = \frac{3.6\vartheta_{cef,\tau_n}}{10^6 \eta_n} G_{n,t} \tag{2}$$

where, $Ce_{n,t}$ is the carbon emission amount of generator n at t^{th} time slot, $\alpha_{l,t}$ and $\beta_{l,t}$ are the coefficients related to the location and time, $G_{n,t}$ is the output of generator n, ϑ_{cef,τ_n}, $\tau_n$and η_n are the emission factors, type and efficiency of generator n.

B. *Carbon-Emission Permit Trading Cost*

The total carbon-emission permit trading cost C_t^{CEP} is determined by the difference of expected carbon emission amount and the carbon-emission permit capacity $\overline{Ce_{n,T}}$ as presented in (3). The carbon-emission permit capacity in the single time slot $\overline{Ce_{n,t}}$ and its alternation $\Delta\overline{Ce_{n,t}}$ caused by the permit trading can be obtained by (4)-(5). If the generator n is selling carbon emission capacity at time t, $\Delta\overline{Ce_{n,t}}$ is negative.

$$C_t^{CEP} = \sum_{n=1}^{N} \frac{\sum_{t=1}^{T} Ce_{n,t}^{pre} - \overline{Ce_{n,T}}}{\sum_{t=1}^{T} Ce_{n,t}^{pre}} Pri_n Ce_{n,t}^{pre} \tag{3}$$

$$\overline{Ce_{n,t}} = Ce_{n,t}^{pre} \overline{Ce_{n,T}} \Big(\sum_{t=1}^{T} Ce_{n,t}^{pre} \Big)^{-1} \tag{4}$$

$$\Delta\overline{Ce_{n,t}} = (Ce_{n,t}^{pre} - \overline{Ce_{n,t}}) + \epsilon_{n,t}^{real} \tag{5}$$

where, T is the interval between carbon emission trading events, Pri_n is the unit carbon emission trading price, $Ce_{n,t}^{pre}$ and $G_{n,t}^{pre}$ are the predicted carbon emission amount and power output without emission constraints, $\epsilon_{n,t}^{real}$ is the rolling variable to revise the carbon emission capacity allocation scheme according to the real situation.

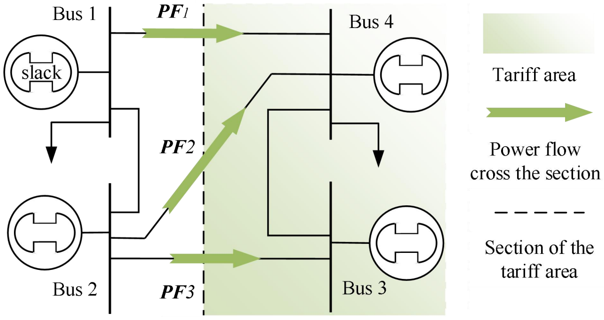

Fig. 1. Tariff area and the power flow across the section

C. *Carbon Tariff Cost*

The carbon tariff is added on the demand side to restrict the competitiveness of the high-emission generators. Thus, to 1) reveal the effect of carbon tariff on generators in a competitive market, and 2) establish the relation between power generation and carbon tariff cost, the carbon tariff paid by demand side is converted to the generation side. The active power generated outside and consumed inside the tariff area is defined as tariff-charged power in this letter. The tariff charged power can be presented by the vector sum of power flows across the tariff area, given as $(\boldsymbol{PF}1 + \boldsymbol{PF}2 + \boldsymbol{PF}3)$ and shown in Fig. 1. It is the total tariff-charged power from all outside generators. For a certain outside generator n, the tariff-charged power can be obtained by using $(\boldsymbol{PF}1_n + \boldsymbol{PF}2_n + \boldsymbol{PF}3_n)$, where $\boldsymbol{PF}1_n$ is a portion of $\boldsymbol{PF}1$ and generated by generator n. $\boldsymbol{PF}1_n$ can be obtained by the product of the power transmission distribution factor and the output of generator n. Thus, the total carbon tariff cost $C_{N'}^{CT}$ can be converted into the equivalent penalty cost of the generators by (6).

$$C_{N',t}^{CT} = \sum_{n'=1}^{N_{out}} Pri_{ct} Ce_{n',t} \left(\sum_{l=1}^{L_s} \xi_{n',l}^{PTDF} G_{n',t} \right), n' \in N' \tag{6}$$

where, $\xi_{n',l}^{PTDF}$ is the power transfer distribution factor, Pri_{ct} is the unit carbon tariff price, L_s is the set of power lines cross the tariff area section, N' is the set of the generators outside the tariff area.

III. FORMULATION OF EC-LMP

The conventional OPF is to minimise the total generation costs C_t^{PG} as shown in (7) with the related constraints (8)-(10). Since the transmission line's resistance is much less than reactance, the transmission loss is ignored in this paper. Thus, the LMP of bus m, π_m^{LMP}, can be obtained by the Lagrangian multipliers as (11). As the power transfer distribution factors on the reference bus are zero, π_{Re}^{LMP} is equal to λ_0. Then, the π_m^{LMP} can be obtained by the summation of LMP on reference bus π_{Re}^{LMP} and the line congestion costs μ_m^{LMP} [9], as presented in (12)-(13). Since the resistance is much less than the reactance on power lines in transmission networks, the marginal power loss is ignored to simplify the formulation.

$$f_{opf} = \min C_t^{PG} = \min \sum_{n=1}^{N} (a_n {G_{n,t}}^2 + b_n G_{n,t} + G_{n,t}) \tag{7}$$

$$\text{s.t.} \sum_{m=1}^{M} (\omega_{mn} G_{n,t} - D_{m,t}) = 0\!:\ \lambda_0 \tag{8}$$

$$\underline{PF_l} \le \sum_{m=1}^{M} \xi_{m,l}^{PTDF} (\omega_{mn} G_{n,t} - D_{m,t}) \le \overline{PF_l}\!: \underline{\mu_{1,l}}, \overline{\mu_{1,l}} \tag{9}$$

$$\underline{G_{lim,n}} \le G_{n,t} \le \overline{G_{lim,n}}\!: \underline{\mu_{2,n}}, \overline{\mu_{2,n}} \tag{10}$$

$$\pi_m^{LMP} = \frac{\partial C_t^{PG^*}}{\partial D_{m,t}} = \lambda_0 + \sum_{l=1}^{L} \xi_{m,l}^{PTDF} (\underline{\mu_{1,l}} - \overline{\mu_{1,l}}) \tag{11}$$

$$\pi_m^{LMP} = \pi_{Re}^{LMP} + \mu_m^{LMP} \tag{12}$$

$$\pi_{Re}^{LMP} = \frac{\partial C_t^{PG^*}}{\partial D_{Re}}, \mu_m^{LMP} = \sum_{l=1}^{L} \xi_{m,l}^{PTDF} \frac{\partial C_t^{PG^*}}{\partial \overline{PF_l}} \tag{13}$$

where, M and L are the numbers of buses and lines in the system, a_n, b_n, and c_n are the constant coefficients, D_m and D_{Re} are the load at bus m and the reference bus, if generator n is located at bus m, $\omega_{mn} = 1$, otherwise, $\omega_{mn} = 0$, $\overline{PF_l}$ is the maximum power flow on line l, $\overline{G_{lim,n}}$ and $\underline{G_{lim,n}}$ are the maximum and minimum output of generator n, λ_0, $\underline{\mu_{1,l}}$, $\overline{\mu_{1,l}}$, $\underline{\mu_{2,n}}$ and $\overline{\mu_{2,n}}$ are the Lagrangian multipliers, the superscript asterisk of variables is to indicate the optimal value of them.

To represent the carbon-related costs, the objective function of C-OPF is integrated with the cost of carbon abatement, carbon tariff, and carbon-emission permit trading, as presented in (14). ξ_n^t is the carbon-neutral index, which will be 100% in the target year of net-zero carbon emissions, such as 2050 in China and the UK. $\boldsymbol{\mu_t}$ is a logic vector, given by (15). For any pricing method, if it exists in the scenario, the corresponding variable (one of μ_t^{CA}, μ_t^{CEP} and μ_t^{CT}) is set as 1, otherwise, 0. For example, if only carbon abatement cost and carbon tariff are considered, $\mu_t^{CA} = 1$, $\mu_t^{CEP} = 0$ and $\mu_t^{CT} = 1$. Considering the carbon-emission permit also reduces the carbon emissions, ξ_n^t is revised as $\widetilde{\xi_n^t}$ in (16). The carbon-emission permit capacity is added into the constraints of C-OPF optimisation, as shown in (17). Then, the carbon-related LMP (C-LMP) π_m^{CLMP} can be obtained by the summation of the carbon-related costs on reference bus π_{Re}^{CLMP}, the line congestion costs μ_m^{CLMP} and the carbon tariff cost, as shown in (18)-(20).

$$f_{copf} = \min\left(C_t^{PG} + \boldsymbol{\mu_t}\left[C_t^{CA}\left(\widetilde{\xi_n^t} Ce_{n,t}\right), C_t^{CEP}, C_{N',t}^{CT}\right]\right) \tag{14}$$

$$\boldsymbol{\mu_t} = [\mu_t^{CA}, \mu_t^{CEP}, \mu_t^{CT}]^T \tag{15}$$

$$\widetilde{\xi_n^t} = \xi_n^t - \frac{\sum_{t=1}^{T} Ce_{n,t}^{pre} - \overline{Ce_{n,T}}}{10^{-2}\sum_{t=1}^{T} Ce_{n,t}^{pre}} \tag{16}$$

$$\text{s.t. (8)-(10), and } Ce_n \le \overline{Ce_{n,t}} + \Delta\overline{Ce_{n,t}}\colon \overline{\mu_{3,n}} \tag{17}$$

$$\pi_m^{CLMP} = \pi_{Re}^{CLMP} + \mu_m^{CLMP} + \varphi_m \frac{\Delta C_{N',t}^{CT\ *}}{\Delta D_{m,t}} \tag{18}$$

$$\pi_{Re}^{CLMP} = \frac{\partial\left(\boldsymbol{\mu_t}\left[C_t^{CA^*}\left(\widetilde{\xi_n^t} Ce_{n,t}{}^*\right), C_t^{CEP^*}, 0\right]\right)}{\partial D_{Re}} \tag{19}$$

$$\mu_m^{CLMP} = \sum_{l=1}^{L} \xi_{m,l}^{PTDF} \frac{\partial\left(\boldsymbol{\mu_t}\left[C_t^{CA^*}\left(\widetilde{\xi_n^t} Ce_{n,t}{}^*\right), C_t^{CEP^*}, 0\right]\right)}{\partial \overline{PF_l}} \tag{20}$$

where, $\mu_t^{CA}, \mu_t^{CEP}, \mu_t^{CT}$ are logic variables of the existence of carbon abatement cost, carbon-emission permit cost and carbon tariff cost; if bus m is located inside the tariff area, $\varphi_m = 1$, otherwise, $\varphi_m = 0$; $\overline{\mu_{3,n}}$ is the Lagrangian multiplier.

Therefore, combined with the original electricity price in (12)-(13), the EC-LMP on bus m, π_m^{EC-LMP}, can be calculated by the sum of LMP and C-LMP, as (21). The C-LMP can also be obtained by (18)-(20) in the OPF model with the original objective function in (7) subjected to (17). However, the carbon-electricity combined cost is not minimum in this situation, which could bring comparatively high carbon prices for some generators and, consequently, power consumers.

$$\pi_m^{EC-LMP} = \pi_m^{CLMP} + \pi_m^{LMP} \tag{21}$$

IV. CASE STUDY

In this section, the proposed C-OPF and EC-LMP are performed on a modified PJM 5-bus system in Fig. 2 [10]. The details of the generators are presented in Table I (Table I and II have been uploaded to an opensource website as: https://github.com/JunLong-Li/Paper_figures_tables/blob/main/Tables for EC-LMP.docx). Generator A at buses #1 and #4 are set as gas power plants. ξ_n^t is set as 40%. α_t and β_t are 0.603 and 1.951 ¥/tCO₂ according to the carbon abatement costs in China, 2020 [8]. Pri_n, Pri_{ct} are set as 15 ¥/tCO₂. The reference bus is set as bus #5. The results of the EC-LMP with OPF and C-OPF are compared in Fig. 2. The overall results of this case study are listed in Table II.

As shown in Fig. 2, the OPF and C-OPF lead to different results of generators' output, power flow, line congestion and EC-LMP. And the EC-LMP is decoupled and presented into conventional LMP and C-LMP, shown as electricity charging slice and decarbonisation charging slice respectively.

Firstly, for the generators, running with OPF, the power generation cost strongly limits the output of generators A and D though they are the most low-emission generator. However, the output of generators A and D increases 48.1% in total in C-OPF because of lower carbon-related costs, while the generation amount of high-emission generators C and E decreases. Generator B is strictly limited by its comparatively high generation and carbon-related costs in both of the two modes.

Secondly, the power flow is also changed by the altered generation schemes. The absolute value of the power flow on the branch between bus #4 and #5 is reduced in C-OPF, which greatly alleviates the transmission pressure on this branch and eliminates the congestion on it. In OPF, the carbon-related line congestion cost for bus #1 to #4 are all negative as [-0.2832¥, -0.6650¥, -0.8118¥, -1.2153¥]. This indicates that load increment on any bus can reduce the carbon-related costs in the whole system with the line congestion. This is because any load increment would require more output from the generators with comparatively lower carbon costs, such as generator D. Optimised by C-OPF, the low-carbon generators are allowed to generate more electricity, which alleviates the line congestion in this case.

Thirdly, for the EC-LMP, the prices on buses #1 and #4 are the lowest in OPF. This is caused by the power line congestion and the comparatively lower C-LMP. As the power users on these two consume more power from the low-emission generators, they will be allocated with less decarbonisation obligation. Additionally, the load increment at bus #4 can reduce the output of generator E because of the transmission congestion on the line between bus #5 and bus #4. This leads to a negative carbon tariff cost at bus #4 as a reward for its contribution to decarbonisation, with a value of -0.69¥. Besides, the prices on buses #1 and #4 increase in C-OPF because of the higher marginal generation cost of generators A and D, while others are decreased for no congestion costs and much less C-LMP. Though all the LMP in C-OPF charges more, the total costs of generators and customers are both declined, which can be seen as a better Pareto Optimality.

As for Table II, compared with OPF, although the generation cost increases 2.32% in C-OPF, the carbon-related cost decreases 13.96%, which leads to a 2.19% reduction of the total electricity-carbon cost on the generation side. This leads to less costs on power users, which decreases 3.82%. As the

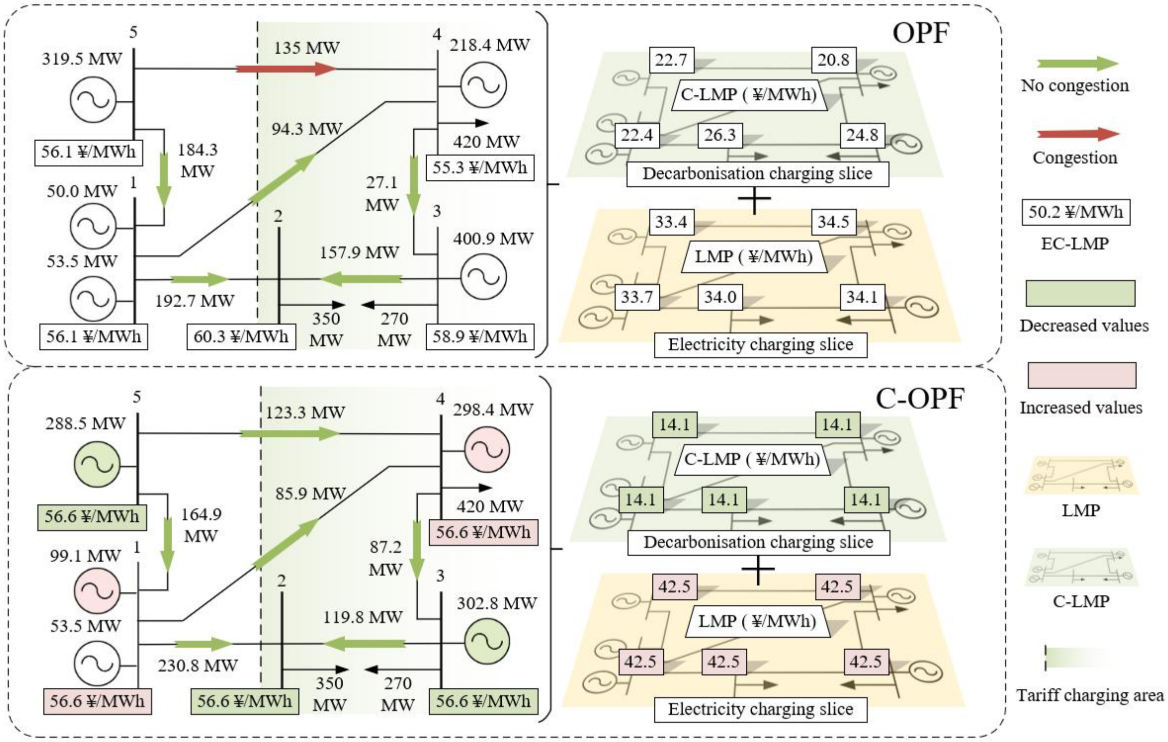

Fig. 2. The results and decoupled slices of the EC-LMP with OPF and C-OPF

carbon-related costs are highly related to the carbon emission amount, the emission amount also falls by 4%.

It is worthy to mention that the carbon-related congestion cost is -1893¥ in OPF for the congestion limits the output of high-emission generator E and requires more generation amount from low-emission generator D. Although this situation is caused by the maximum transmission limit on power lines, this can also be artificially created by policies, such as carbon permit allocated on transmission lines or regional sections. In this way, carbon emissions could be further reduced in power systems.

In conclusion, the EC-LMP provides a more stimulant solution for the participants in the competitive power market to reduce the carbon emissions caused by them. Besides, unlike the OPF oriented by minimum electricity generation cost, the EC-LMP-based C-OPF considers the economic value of the decarbonisation cost. In this way, the carbon emission in the power system, and the total costs on the generation side and consumer side are all reduced.

V. CONCLUSION

This letter presents a formulation of EC-LMP to allocate the carbon-related costs to the demand side. Correspondingly, combined with the carbon-emission permit constraints, a C-OPF is developed to obtain the EC-LMP and minimise the sum of power generation cost and carbon-related costs.

Test results show the efficacy of the EC-LMP on rationally allocating the carbon-related cost to the demand side. The buses that consume power from high-emission generators are allocated with higher C-LMP for more decarbonisation responsibility. And the load which can reduce the carbon-related costs gets rewards for its contribution to decarbonisation. With the EC-LMP, the decarbonisation technologies and policies in power systems could be more efficient for the clear price signals of carbon-related costs in the competitive power markets. Besides, the C-OPF is testified to efficiently reduce the carbon emission amount and the costs on both generation side and consumer side. This could be a viable solution for enhancing the market equilibrium of the future carbon-introduced competitive power market.

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