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On Equilibrium in Monopolistic Competition with Endogenous Labor*

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Abstract

We consider a model of monopolistic competition with several heterogeneous sectors and endogenous labor supply. For low (high) values of the labor supply elasticity, we show that there is always a unique equilibrium. For medium values of the labor supply elasticity, there are either zero or two equilibria.

JEL classification: D43, L13

Keywords: labor elasticity, multiple sectors, multiple equilibria.

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1 Introduction

The monopolistic competition framework introduced by Dixit and Stiglitz (1977) is widely employed in many fields of economics.¹ This framework has been extended in various directions including heterogeneous firms and multiple sectors (see, e.g., Melitz, 2003; Behrens et al., 2020). At the same time, small attention has been paid to the assumption of inelastic labor supply. This paper fills this gap by considering a simple model of monopolistic competition with endogenous labor supply in order to study the link between labor supply and the existence and uniqueness of the equilibrium.

We consider a model of monopolistic competition with multiple sectors, a constant elasticity of substitution within each sector, homogeneous firms, and consumers/workers who can choose how many units of labor to supply. As a result, there is an interplay of the price index in the economy (which is determined by the number of available varieties and their prices) and labor supply. A lower price index induces higher labor supply, which results in more entry into the market and, therefore, a lower price index. In the paper, we argue that this interplay is important for characterizing equilibrium. In particular, we show that the relative size of the Frisch labor elasticity with respect to the elasticities of substitution within sectors plays a crucial role in determining the existence and uniqueness of equilibrium.

We find that there is a unique equilibrium for relatively low or high values of the labor supply elasticity. For medium values of the labor supply elasticity, multiple equilibria are possible (at most two). Our key observation is that the possibility of multiple equilibria arises because of the difference in the elasticities of substitution across sectors and the presence of endogenous labor supply. In particular, different sectors can play a major role in determining the outcome in an equilibrium depending on the values of the price index and the amount of endogenous labor supply. This observation is important for qualitative and quantitative policy analysis in the presence of multiple sectors, which was shown by Behrens et al. (2020) to be crucial for understanding welfare distortions (see also Mankiw and Winston, 1986; Dhingra and Morrow, 2019).

The monopolistic competition framework with elastic labor supply can be found in macroeconomic models in the traditions of New Keynesianism (Blanchard and Kiyotaki, 1987) and business cycles (Bilbiie et al., 2012, 2019). Other applications of this framework include, for instance, the analysis of optimal labor and dividend income taxation in general equilibrium (Colciago, 2016) or the characterization of efficient market structures and optimal tax rules (Etro, 2018). All these papers focus however on a framework with one sector, while the present paper explores the implications of elastic labor supply in a framework with multiple heterogeneous sectors. One exception is a multisectoral monopolistic competition model by Etro and Colciago (2010), who analyze mark-up dynamics in business cycles. Another exception is

¹See Thisse and Ushchev (2018) for the literature review.

Behrens et al. (2020), who considers, however, a model with inelastic labor supply.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we analyze the existence and uniqueness of the equilibrium. Section 4 concludes.

2 The Model

We consider a simple multi-sector model of monopolistic competition with homogeneous consumers/workers who endogenously decide how many units of labor to supply.

2.1 Consumption

We assume that consumers have the following utility function:

$$U = \left(\sum_{j=1}^J \beta_j \left(\int_0^{N_j^E} q_j^{\rho_j}(i) di \right)^{\frac{\sigma-1}{\rho_j \sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \frac{\ell^{1+\gamma}}{1+\gamma},$$

where $J > 1$ is the number of sectors, $q_j(i)$ is the consumption of a variety i produced in sector j , N_j^E is the number of available varieties in sector j , ℓ is the number of labor units supplied, $\rho_j < 1$, (with $\rho_j \neq \rho_{j'}$, $j \neq j'$) represents the constant elasticity of substitution between varieties within sector j , $\sigma > 1$ is the intersectoral elasticity of substitution such that $1 - \sigma(1 - \rho_j) > 0$ for all j (see Behrens et al., 2020), and $\sum_{j=1}^J \beta_j$ is normalized to unity with $\beta_j > 0$. Finally, $1/\gamma > 0$ represents the labor supply elasticity.²

The budget constraint is then given by (labor wage is normalized to unity)

$$\sum_{j=1}^J \int_0^{N_j^E} p_j(i) q_j(i) di = n\ell,$$

where $p_j(i)$ is the price of a variety produced by a firm in sector j and n is labor productivity of consumers.

The utility function implies that, given the prices and income $n\ell$, demand for variety i in sector j is

²Adding parameter η representing the magnitude of the income effect (the effect of a rise in nonlabor income on labor income, see Keane, 2011):

$$U = \frac{\left(\sum_{j=1}^J \beta_j \left(\int_0^{N_j^E} q_j^{\rho_j}(i) di \right)^{\frac{\sigma-1}{\rho_j \sigma}} \right)^{\frac{\sigma(1-\eta)}{\sigma-1}}}{1-\eta} - \frac{\ell^{1+\gamma}}{1+\gamma},$$

does not change the qualitative implications of the model.

$$q_j(i) = n\ell \frac{\beta_j^\sigma P_j^{\frac{1}{1-\rho_j}-\sigma} p_j(i)^{1/(\rho_j-1)}}{\sum_{j=1}^J \beta_j^\sigma P_j^{1-\sigma}}$$

where P_j is the CES price index in sector j given by

$$P_j^{\rho_j/(\rho_j-1)} = \int_0^{N_j^E} p_j(i)^{\rho_j/(\rho_j-1)} di.$$

Taking into account the above expressions, we obtain that

$$U(\ell) = \frac{n\ell}{P} - \frac{\ell^{1+\gamma}}{1+\gamma},$$

where P is the CES price index in the economy given by

$$P = \left(\sum_{j=1}^J \beta_j^\sigma P_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (1)$$

Hence, the optimal labor supply is given by $\ell = (n/P)^{1/\gamma}$.

2.2 Market Equilibrium

The total demand for a variety produced by a firm i in sector j is given by

$$Q_j(i) = \frac{n\ell L \beta_j^\sigma P_j^{\frac{1}{1-\rho_j}-\sigma}}{\sum_{j=1}^J \beta_j^\sigma P_j^{1-\sigma}} p_j(i)^{1/(\rho_j-1)},$$

where L is the total number of consumers in the economy. Given the isoelastic demand, the optimal price is equal to m_j/ρ_j , where m_j is the marginal cost of production in sector j . The free entry into each sector implies that

$$(p_j(i) - m_j)Q_j(i) - f_j = 0,$$

where f_j is the fixed cost of production in sector j .

Hence, the equilibrium in the model is described by (1), the optimal labor supply condition, and the free entry condition given by

$$C_j \beta_j^\sigma \frac{P_j^{\frac{1}{1-\rho_j}-\sigma}}{P^{1-\sigma}} = \frac{f_j}{n\ell L}, \quad (2)$$

where

$$C_j = m_j^{\rho_j/(\rho_j-1)} \frac{1-\rho_j}{\rho_j^{\rho_j/(\rho_j-1)}}.$$

3 Existence and Uniqueness of Equilibrium

The equilibrium conditions (1) and (2) imply that

$$1 = \sum_{j=1}^J A_j(L) (P^{\rho_j} \ell^{1-\rho_j})^{\frac{(\sigma-1)}{1-\sigma(1-\rho_j)}}, \quad (3)$$

where

$$A_j(L) = \beta_j^{\frac{\sigma\rho_j}{1-\sigma(1-\rho_j)}} \left(\frac{f_j}{C_j} \right)^{\frac{1-\sigma}{1-\rho_j-\sigma}} (nL)^{\frac{(\sigma-1)(1-\rho_j)}{1-\sigma(1-\rho_j)}}.$$

Thus, the equilibrium in the model is determined by the intersection of two curves: the PP curve follows from (3) with a negative relationship between P and ℓ (as $1-\sigma(1-\rho_j) > 0$ for all j); the $\ell\ell$ curve is labor supply curve $\ell = (n/P)^{1/\gamma}$, which also implies a negative relationship between P and ℓ . Notice that if we know P and ℓ , we can find P_j from (2).

If labor supply is inelastic, as usually assumed in monopolistic competition models, there is a unique equilibrium (the equation in (3) has a unique solution with respect to P). Under endogenous labor supply, however, changes in P affect the labor supply, which in turn affects the price index P through entry. This leads to the possibility of multiple equilibria.

To explore this question more formally, we substitute $\ell = (n/P)^{1/\gamma}$ into the PP curve, deriving an equation for P given by

$$0 = -1 + \sum_{j=1}^J A_j(L) n^{\frac{(1-\rho_j)(\sigma-1)}{\gamma(1-\sigma(1-\rho_j))}} (P^{\gamma\rho_j-1+\rho_j})^{\frac{(\sigma-1)}{\gamma(1-\sigma(1-\rho_j))}}. \quad (4)$$

First, note that if there exists j_0 such that $\rho_{j_0}/(1-\rho_{j_0}) = 1/\gamma$ and $A_{j_0}(L) n^{\frac{(1-\rho_{j_0})(\sigma-1)}{\gamma(1-\sigma(1-\rho_{j_0}))}} - 1 \geq 0$, then equation (4) has no positive roots (the right-hand side is strictly positive for any finite positive P), implying that there is no equilibrium in the model. In the further analysis, we do not consider such cases and, therefore, without loss of generality, assume that $\gamma\rho_j - 1 + \rho_j$ is different from zero for all j . Note also that equation (4) is a polynomial-like equation with irrational powers. This allows us to apply a generalized Descartes' rule of signs to study the number of its positive roots (see, e.g., Haukkanen and Tossavainen, 2011). If we denote $\rho_{min} = \min_j \rho_j$ and $\rho_{max} = \max_j \rho_j$, then we obtain the following proposition.

Proposition 1. *1) If $\rho_{min}/(1-\rho_{min}) > 1/\gamma$ or $1/\gamma > \rho_{max}/(1-\rho_{max})$, there exists a unique equilibrium. 2) If $\rho_{max}/(1-\rho_{max}) > 1/\gamma > \rho_{min}/(1-\rho_{min})$, then there exists a threshold \bar{L} such that if $L > \bar{L}$, there is no equilibrium, and if $L < \bar{L}$, there are two equilibria.*

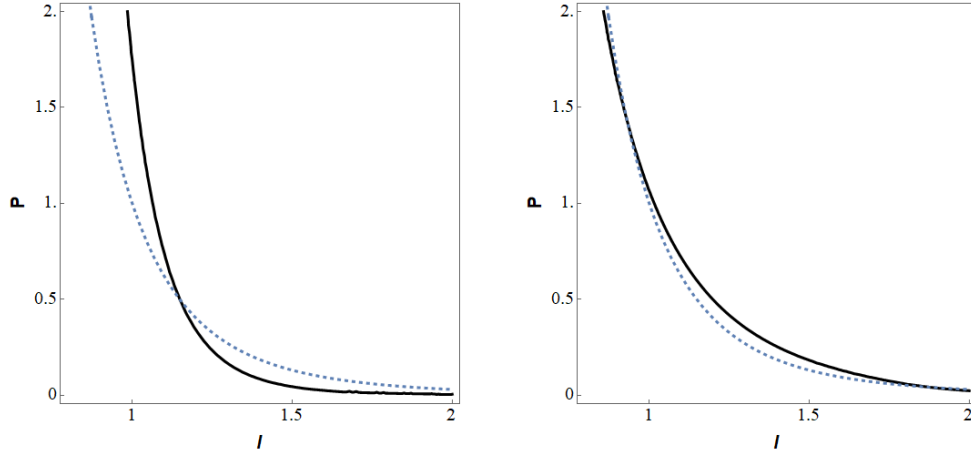


Figure 1: The number of equilibria in two-sector model: $\ell\ell$ curve is dashed and PP curve is solid; $\rho_1 = \rho_2 = 0.9$ in the case of the unique equilibrium (left) and $\rho_1 = 0.8$ and $\rho_2 = 0.9$ in the case of two equilibria (right); $A_1(L) = 0.5, A_2(L) = 0.2, \sigma = 4.5, \gamma = 0.2, n = 1$.

Proof. Note that equation (4) can be written as

$$\sum_{k=1}^{J+1} a_k P^{\alpha_k} = 0, \quad (5)$$

where $a_k \in \mathbb{R} \setminus \{0\}$ and $\alpha_k \in \mathbb{R}$. In the above, we rearrange coefficients such that the powers have an increasing order, i.e., $\alpha_k < \alpha_{k+1}$. Note that, for some \hat{k} , $a_{\hat{k}} = -1$ (with $\alpha_{\hat{k}} = 0$), while for all $k \neq \hat{k}$, $a_k > 0$. If $\rho_{min}/(1 - \rho_{min}) > 1/\gamma$, we have $\alpha_k > 0$ for all $k \neq \hat{k}$ and $\hat{k} = 1$. Hence, the coefficients of equation (5) change signs once. According to the generalized Descartes' rule of signs, this implies that equation (5) has at most one root (Theorem 2.2 in Haukkanen and Tossavainen, 2011). The equation indeed has one root as its left hand side is a continuous function, which is negative for $P = 0$ and positive for $P \rightarrow \infty$. The same argument applies if $1/\gamma > \rho_{max}/(1 - \rho_{max})$, implying that $\alpha_k < 0$ for all $k \neq \hat{k}$ with $\hat{k} = J + 1$. In this case, the coefficients of equation also change signs once and there is again one root.

When $\rho_{max}/(1 - \rho_{max}) > 1/\gamma > \rho_{min}/(1 - \rho_{min})$, α_k is negative for $k < \hat{k}$ and positive for $k > \hat{k}$. In other words, the coefficients a_k change signs in the ordered polynomial twice: positive-negative-positive. According to the generalized Descartes' rule of signs, this implies that the number of positive roots in (5) is at most two. To understand when there exist two equilibria, we show that the right-hand side of (4) as a function of P has a minimum. Indeed, note that the value of the right-hand side goes to infinity when $P \rightarrow 0$ or $P \rightarrow +\infty$. As it is a continuous function on $(0, +\infty)$, it has to have a minimum there. Let us define \bar{L} as the value of L such that the value of the right-hand side at the minimum is equal to zero. This is possible, as $A_j(L)$ is strictly increasing in L . Thus, for $L > \bar{L}$, there is no solution of (4), while if $L < \bar{L}$, there are two solutions. \square

The two cases of Proposition 1 are illustrated in Figure 1, where the $\ell\ell$ curve is dashed

and the PP curve is solid. The left subfigure illustrates the case where there is only one equilibrium possible, whereas the right subfigure shows the case with two possible equilibria. Note that such multiplicity of equilibria is not possible in the specification with one sector or homogeneous multiple sectors. It is also interesting to observe that though the intersectoral “strategic interplay” allows bending the PP curve in a flexible way, even with multiple J sectors there exists not more than two equilibria.

4 Conclusion

We study in this paper the existence and uniqueness of the equilibrium in the monopolistic competition framework with several heterogeneous sectors and endogenous labor supply. We show that for low (high) values of labor supply elasticity the equilibrium is unique, while for medium values of the labor elasticity, multiple equilibria are possible.

The estimates of the elasticity of substitution ρ_j and the labor supply elasticity $1/\gamma$ vary a lot across studies. Broda and Weinstein (2006) report $\rho_j/(1 - \rho_j) \in [0.2, 21]$ for U.S. data, Behrens et al. (2020) report $\rho_j/(1 - \rho_j) \in [1.9, 30]$ for French data and $\rho_j/(1 - \rho_j) \in [1.3, 52]$ for UK data. The estimates of the labor supply elasticity typically lie in the range $1/\gamma \in [0.3, 1.8]$ (see Saez et al., 2012; Mertens and Montiel Olea, 2018). These estimates suggest that while empirically more relevant case is likely to be the one leading to a unique equilibrium, the case of multiple equilibria cannot be ruled out. Furthermore, the estimates of ρ_j depend highly on the level of aggregation. Hence, it is important to take into account the possibility of multiple equilibria when modeling the market with multi-sector monopolistic competition and endogenous labor supply.

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