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The impact of consumer price forecasting behaviour on the bullwhip effect

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As prices fluctuate over time, a strategic consumer may buy more in advance to reduce his or her future needs in anticipation of higher prices in the future, or may choose to postpone a purchase in anticipation of lower prices in the future. We investigate the bullwhip effect from a consumer price forecasting behavioural perspective in the context of a simple two-level supply chain composed of a supplier and a retailer. We consider two different forms for the demand function – linear and iso-elastic demand functions, both depending on the prices in multiple periods. Assuming that the retailer employs an order-up-to inventory policy with exponential smoothing forecasting technology, we derive analytical expressions for the bullwhip effect under the two demand functions, and extend the results to the multiple-retailer case. We find that consumer forecasting behaviour can reduce the bullwhip effect, most significantly when the consumer sensitivity to price changes is medium (approximately 0.5) for both the demand forms. In addition, for iso-elastic demand, the mitigation of the bullwhip effect induced by consumer price forecasting behaviour becomes more significant as the product price sensitivity coefficient and standard deviation of the price decrease. These findings are applicable to the development of managerial strategies by supply chain members that are conducive to bullwhip effect reduction through customer behaviour.

Keywords: bullwhip effect; oscillation; consumer behaviour; price forecasting; linear demand; iso-elastic demand

1. Introduction

The tendency of orders to increase in variability as one moves up a supply chain is known as the bullwhip effect (Lee, Padmanabhan, and Whang 1997a, 1997b; Simchi-Levi, Kaminsky, and Simchi-Levi 2002). This effect can lead to tremendous inefficiencies in a supply chain. Extensive research has been performed on the *operational* causes and remedies of the bullwhip effect. The causes of the effect include demand signal processing, shortage games, order batching and price adjustment (Lee, Padmanabhan, and Whang 1997a). Strategies to alleviate these operational problems include adopting an 'optimal' demand forecasting technique (Alwan, Liu, and Yao 2003), supply chain coordination (Cachon 2003), scheduled ordering policies (Cachon 1999) and price stabilisation (Lee, Padmanabhan, and Whang 1997b).

Forrester (1958) was the first to note the bullwhip effect and identify its possible causes. In the ensuing years, there were many studies investigating different facets of the bullwhip effect. Sterman (1989) discussed the bullwhip effect in the context of the 'Beer Distribution Game' and attributed the phenomenon to the irrational behaviour of the player. Lee, Padmanabhan, and Whang (1997a, 1997b) provided a formal definition of the bullwhip effect and systematically analysed four contributing factors as well as their countermeasures. Assuming that the retailer employed an order-up-to inventory policy with the exponential smoothing (ES) forecasting technique, Chen, Ryan, and Simchi-Levi (2000) quantified the bullwhip effect for two types of demand processes, a first-order autoregressive demand (AR (1)) process and a demand process with a linear trend, and found that the bullwhip effect in the linear trend was significantly higher than that in the AR (1) process. Subsequent papers extended this work by allowing for more general autoregressive demand processes (Graves 1999; Gaur, Giloni, and Seshadri 2005; Gilbert 2005; Disney et al. 2006; Hsiao and Shieh 2006; Dhahri and Chabchoub 2007; Duc, Luong, and Kim 2008; Zhang and Zhao 2010; Zhang and Burke 2011), different inventory policies (Dejonckheere et al. 2003, 2004; Disney and Towill 2003; Disney et al. 2006; Wadhwa, Bibhushan, and Chan 2009; Wang et al. 2010) and different forecasting techniques (Chen et al. 2000; Chatfield et al. 2004; Zhang 2004; Hosoda and Disney 2006, 2012; Agrawal, Sengupta, and Shanker 2009; Wang et al. 2010; Sodhi and Tang 2011; Ma et al. 2013a, 2013b).

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In addition to the theoretical efforts to quantify the bullwhip effect, there have been numerous attempts to validate its existence in empirical studies. Fransoo and Wouters (2000) discussed the practical measurement of the bullwhip effect and provided four ways to aggregate demand data for calculating the bullwhip effect in an industrial project. Niranjan, Wagner, and Aggarwal (2011) proposed a framework to comprehensively describe the information distortion underlying the bullwhip effect through a case study of a real-life automotive supply chain. Cachon, Randall, and Schmidt (2007) sought the bullwhip effect in macroeconomic industry-level data, and found that wholesale industries exhibit a bullwhip effect, but retail and manufacturing industries generally do not exhibit this effect. Bray and Mendelson (2012) investigated the bullwhip effect in a sample of 4689 public US companies operating between 1974 and 2008 and found that about two-thirds of the firms exhibited this effect. Chen and Lee (2012) provided scientific explanation for these different bullwhip observations made in previous studies and discussed how different measures of the bullwhip effect are related to supply chain cost performance. One of the measures proposed to counter the bullwhip effect is price stability, through everyday low price (Lee, Padmanabhan, and Whang 1997a, 1997b); however, Hamister and Suresh (2008) used data from a supermarket scanner to show that utilising fixed rather than dynamic pricing may result in lower profitability and higher bullwhip effect, and they explained this phenomenon through the optimal price and stocking level policy.

Extensive research has also been conducted on consumer behaviour in the presence of variable prices. Ho, Tang, and Bell (1998) developed a normative model describing how rational consumers should shop when the price of the product was random. Aviv and Pazgal (2008) took the consumers' response to price variations into account. They argued that if strategic consumers have information about future price discounts, they may choose to postpone a purchase in anticipation of lower prices in the future. Etzioni et al. (2003) developed the 'Hamlet' programme to study past trends in the variation of airline fares and used this programme to advise consumers on when to buy and when to postpone ticket purchases. Cho, Fan, and Zhou (2009) studied the strategic response of consumers to dynamic prices of perishable products and investigated the impact of strategic consumer behaviour on pricing policies and expected revenue performance.

In many cases, consumers may consider making a purchase for several periods, so that the demand in a given period is a function of both the price in that period and the prices in other periods. Several studies have addressed this issue. Ahn, Gumus, and Kaminsky (2007) demonstrated that in realistic market scenarios, strategic consumers made purchase decisions based not only on the price in the current period but also on that in past and future periods. In the spirit of Ahn, Gumus, and Kaminsky (2007), Rong (2008) assumed that consumers would adjust their orders based on the future trend in price changes inferred from the current trend, and they focused on studying how pricing strategies affect the variability of consumers' orders. Kopalle, Rao, and Assuncao (1996) constructed a demand model that was a function of both the actual prices and the reference prices and developed optimal dynamic pricing policies for the cases where the group of brands is managed by a retailer and where the brands compete in an oligopoly. Similarly, Popescu and Wu (2007) considered the dynamic pricing problem of a monopolist firm in a market where demand is sensitive to the firm's pricing history. These studies are based on the revenue management problem and focus on the analysis and results of dynamic pricing policies and their relation to revenue management. We refer readers to the review of Shen and Su (2007) for a more detailed discussion of consumer behaviour modelling in revenue management.

Unlike the studies mentioned above, this paper investigates supply chain variability, i.e. the bullwhip effect, from a *consumer behaviour* perspective. In particular, we develop insights into the influence of *consumer price forecasting behaviour* on the bullwhip effect. If consumers are not strategic, then their purchasing decisions are based solely on the price in the current period. However, for strategic consumers, they will make purchase decisions based not only on the price in the current period but also on the price trend inferred from the current and previous period. In other words, if strategic consumers anticipate higher prices in the future, they will buy more in advance to reduce their future needs; otherwise, strategic consumers will choose to postpone a purchase if they anticipate lower prices in the future. In this paper, we establish a two-level supply chain composed of a supplier and a retailer and extend the basic model to multi-retailer conditions. We consider linear and iso-elastic demand functions, both depending on the prices in multiple periods. Assuming that the retailer employs an order-up-to inventory policy with ES forecasting technology which uses a weighted moving average of past data as the basis for a forecast, we derive analytical expressions for the bullwhip effect that account for consumer price forecasting behaviour. The results demonstrate that for both demand functions, consumer price forecasting behaviour can reduce the bullwhip effect, especially when the consumer sensitivity to price changes is medium (approximately 0.5). In other words, the bullwhip effect for strategic consumers is lower than that for non-strategic consumers.

This observation has important implications for partners along a supply chain looking to reduce the bullwhip effect. From a behavioural point of view, if the supply chain manager ignores strategic consumers' price forecasting behaviour and thinks that all consumers are non-strategic, they will overstate the bullwhip effect in a supply chain. However,

because the order oscillation is lower for the purchasing decision of a strategic consumer than for that of a non-strategic consumer, our results provide a new way for managers to restrain the bullwhip effect, by inducing non-strategic consumers to become strategic. For example, when the market clearing prices fluctuate, the price in the current period may be higher than that in the previous period; however, if the supply chain manager informs the consumers on their limited inventory or the likelihood of stockouts,[1] strategic consumers still buy more because they think the prices in the future will be ever higher. Conversely, if the price is decreased compared to the previous period, the supply chain manager can inform the consumers that they have sufficient inventory and the likelihood of the discount about the excess inventory. By doing so, the supply chain manager could induce non-strategic consumers to become strategic to buy less in the current period in anticipation of lower prices in the future.

Our research differs from previous studies in two ways. First, this study examines the bullwhip effect from the perspective of consumer price forecasting behaviour, while previous studies focused on the operational causes and remedies of the bullwhip effect. Second, in previous research, the demand was assumed to follow an autoregressive process, and the demand correlation parameter on the bullwhip effect was investigated; see, e.g. Lee, Padmanabhan, and Whang (1997a), Chen et al. (2000), Chen, Ryan, and Simchi-Levi (2000). However, it is difficult to gain managerial insights from this parameter in practice. In this paper, we consider two price-sensitive demand functions, linear and iso-elastic. This approach will allow us to explain the bullwhip effect from a different perspective, based on the impact of demand process characteristics such as the market demand scale and price sensitivity coefficient.

The paper is organised as follows. Section 2 establishes a basic two-level supply chain model composed of a supplier and a retailer, and introduces the improved linear and iso-elastic demand functions as well as the ordering process. In Section 3, we derive analytical expressions for the bullwhip effect for linear and iso-elastic demands considering consumer price forecasting behaviour and draw a comparison between the bullwhip effects for the two demand forms. Section 4 extends the basic model to the multiple-retailer case and analyses the variance of the total order quantity compared to that of the total market demand. Section 5 presents numerical experiments to verify the theoretical findings in Sections 3 and 4. In Section 6, we conclude and suggest several directions for follow-up research.

2. Basic models

Consider a simple two-level supply chain with a supplier and a retailer. The external demand for a single product occurs at the retailer and is price-sensitive. Linear and iso-elastic demand functions are the two forms that are most frequently used when price-sensitive demand must be considered in the inventory model (Petruzzi and Dada 1999; Khouja 2000; Lau, Lau, and Zhou 2006). For both demand forms, these researchers assumed that the demand depends solely on the price in the current period. However, because strategic consumers made purchase decisions based not only on the price in the current period but also on the future price trend inferred from the current and past periods, it is more common in realistic scenarios for the demand in a given period to be a function of both the price in that period and the price in other periods. We first define the notation used in this paper. Our improved linear and iso-elastic demand functions, depending on multi-period prices, are introduced and outlined further below.

p_t	Market clearing price in period t
a	Market demand scale
B	Price sensitivity coefficient
d_t	Market demand in period t
$\hat{d}_t$	Demand prediction in period t
y_t	Order-up-to point in period t
q_t	Order quantity in period t
L	Lead time
D_t^L	Lead-time demand in period t
$\hat{D}_t^L$	Estimate of the mean lead-time demand in period t
$\hat{\sigma}_{e,t}^L$	Estimate of the standard deviation of the L period forecasting error in period t
z	Safety factor
σ	Standard deviation of the market price
r	Consumer sensitivity to price changes
α	Smoothing parameter
BE^{LINEAR}	Bullwhip effect for linear demand
BE^{ISO}	Bullwhip effect for iso-elastic demand

2.1 *Linear demand model*

We consider a market setting in which the retailer sells in a perfectly competitive market and exerts no control over the market clearing price. If consumers are not strategic, then their purchasing decisions are based solely on the price in the current period. The basic linear demand model is therefore given by:

$$d_t = a - bp_t, \tag{1}$$

where $a \geq 0$ and $b \geq 0$.

However, in more realistic market scenarios, it is common for strategic consumers to weigh the benefits of purchasing today against the benefits of waiting to purchase in anticipation of lower prices in the future (or hoarding and purchasing in anticipation of higher prices in the future); in other words, consumers make purchase decisions based not only on the price in the current period but also on the price trend inferred from the current and previous period. We assume in this paper that strategic consumers have memory of the previous period price and exhibit future price forecasting behaviour. A linear demand model that captures this behaviour can be written as follows:

$$d_t = D(p_t, p_{t-1}) = (a - bp_t) + rb(p_t - p_{t-1}), \quad r \in [0, 1), \tag{2}$$

where the market clearing price p_t is independent and identically distributed (i.i.d.) and r denotes the consumer sensitivity to price changes. Note that price sensitivity coefficient b gives the percentage change in quantity demanded in response to price changes. Thus, price sensitivity coefficient is a measure to show the quantity demanded of a product to a change in its price. However, consumer sensitivity factor focuses on consumer reaction to price changes. For a given observed price change, a more sensitive consumer has a higher likelihood of buying or not buying the product. Although the actual market clearing price is i.i.d and its future price evolves independently from the prices of the current period and the previous period, the consumers will make certain responses to the price changes through their own judgment. Therefore, the future price trend made by the consumers will depend on the price of the current period and the previous period. Note that the assumption of an i.i.d. pricing process ensures that the demand process is stationary. Note also that the i.i.d. price can be drawn from any continuous distribution, such as a normal, log-normal or gamma distribution. In the case of linear demand, we assume only that the pricing process is i.i.d.; in the case of iso-elastic demand, we make the stronger assumption that the pricing process is i.i.d. from a log-normal distribution to facilitate the calculation. We will discuss these assumptions further in the next section.

Equation (2) defines an improved linear demand function, accounting for consumer price forecasting behaviour. The first term on the right-hand side of the equation represents the underlying demand and is a decreasing function of p_t, and the second term represents the impact of consumer price forecasting behaviour on the demand. Because future price p_{t+1} has still not been realised in period t, we cannot picture the future price in the second term of Equation (2). However, strategic consumers could predict the future price trend based on the prices in the current and previous periods and make certain response to the price changes. If p_t is higher than p_{t-1}, then strategic consumers believe that the price will be even more higher in the next period. They may, therefore, buy more in advance to reduce their future needs. If p_t is lower than p_{t-1}, then consumers will respond in the opposite manner. A more general setting is to consider the influence of historical multi-period prices. That is, if the current market clearing price is higher than the average historical multi-period prices, then the strategic consumers will think that the price will be even higher in the future, and as a result, the strategic consumers increase their orders in the current period to reduce their future needs. Likewise, the strategic consumers will respond in the opposite manner if the current price is lower than the average historical multi-period prices. However, since the analysis would become more complex, we shall restrict our attention to consider only the influence of the prices in the current period and previous period on the demand. We require the condition $r \geq 0$; if $r = 0$, then the consumers are not strategic in making purchasing decisions. Because Equation (2) can be rewritten as $d_t = a - (1 - r) bp_t - rbp_{t-1}$, $r < 1$ indicates that the demand is negatively correlated with the price in period t.

2.2 *Iso-elastic demand model*

This section introduces the iso-elastic demand model, given by $d_t = ap_t^{-b}$ if consumers are not strategic. However, strategic consumers do not make their purchase decisions based solely on the current period price, p_t; they will adjust their consumptions based on the future price trend inferred from the current and previous period prices. An improved iso-elastic demand function accounting for consumer price forecasting behaviour can be written as follows:

$$d_t = D(p_t, p_{t-1}) = ap_t^{-b} \cdot \left(\frac{p_t}{p_{t-1}} \right)^{rb}, \quad r \in [0, 1), \tag{3}$$

where the variables in Equation (3) have the same definitions as in Equation (2). However, we use $\frac{p_t}{p_{t-1}}$ as price changes and $\left(\frac{p_t}{p_{t-1}}\right)^{rb}$ as consumers' response to the price change for the iso-elastic demand model. If $p_t > p_{t-1}$, then $\frac{p_t}{p_{t-1}} > 1$ and the strategic consumers will make a prediction that the future price p_{t+1} will be even higher; and as a result, they buy more in advance in the current period to reduce their future needs. If $p_t < p_{t-1}$, then the consumers will respond in the opposite manner. If we take the logarithm of both sides of Equation (3), we obtain $\ln d_t = \ln(ap_t^{-b} \cdot (p_t/p_{t-1})^{rb}) = (\ln a - b \ln p_t) + rb(\ln p_t - \ln p_{t-1})$, which has the same linear form as Equation (2).

Recall that we have already assumed that the prices, p_t, are i.i.d. in the linear demand function in Section 2.1. To facilitate the calculation for the iso-elastic demand function, we will make further assumption that the prices are i.i.d. from a log-normal distribution with mean μ and variance σ^2, $\ln p_t \sim N(\mu, \sigma^2)$. Although normal distributions are widely used because of their symmetric properties, a log-normal distribution is more suitable when describing original data (Limpert, Stahel, and Abbt 2001). Limpert, Stahel, and Abbt (2001) note that log-normal distributions are the most commonly used forms when the values cannot be negative and the variances are large. Moreover, log-normal distributions are usually used to describe the price distribution in practice; see, e.g. Laughton and Jacoby (1993), Shrestha et al. (2008), Coad (2009) and Dai (2009).

In most of the previous studies, an autoregressive model was adopted to describe the demand process (Lee, Padmanabhan, and Whang 1997a; Chen et al. 2000; Chen, Ryan, and Simchi-Levi 2000), and the bullwhip effect was investigated as a function of the demand correlation parameter. However, it is difficult to derive managerial insights from this parameter in practice. In this work, we construct two price-sensitive demand forms – linear and iso-elastic demand that incorporate multiple-period prices and thereby account for consumer price forecasting behaviour. We therefore adopt a different perspective and describe the impact of demand process characteristics such as the market demand scale, a; price sensitivity coefficient, b; and consumer sensitivity to price changes, r, on the bullwhip effect. These impact factors have not been analysed in previous studies.

2.3 Ordering process

We assume the following timing of events during the replenishment period. The retailer observes a consumer demand d_{t-1} at the end of period $t-1$, estimates the order-up-to point, y_t, of period t, and places an order of quantity q_t to the supplier at the beginning of period t. After the lead time L, the retailer receives the products from the supplier at the beginning of period $t + L$.

The order-up-to policy is one of the most studied policies in the supply chain model literature (Chen et al. 2000; Chen, Ryan, and Simchi-Levi 2000). The order-up-to level consists of an anticipation stock that is retained to meet the expected lead-time demand and a safety stock for hedging against unexpected demand. This paper assumes that the retailer follows a simple order-up-to inventory policy, in which the order-up-to point, y_t, is given by:

$$y_t = \hat{D}_t^L + z\hat{\sigma}_{e,t}^L, \tag{4}$$

where $\hat{\sigma}_{e,t}^L = \sqrt{\mathrm{Var}(D_t^L - \hat{D}_t^L)}$ and z is the safety factor set to meet a desired service level.

The retailer counts his or her inventory periodically and places an order for q_t to the supplier at the beginning of period t to maintain the inventory at level y_t; therefore:

$$q_t = y_t - y_{t-1} + d_{t-1} = (\hat{D}_t^L - \hat{D}_{t-1}^L) + z(\hat{\sigma}_{e,t}^L - \hat{\sigma}_{e,t-1}^L) + d_{t-1}. \tag{5}$$

Note that the order value can be negative, indicating that excess inventory can be free returned to the supplier; that is, return policy has been considered in our model. However, it should be noted that the return policy assumption is not appropriate in all retail settings. If no return policy has been considered, the retailer's order quantity shown by Equation (5) would become $q_t = \max\{y_t - y_{t-1} + d_{t-1}, 0\}$. Since analytical results are not available using this form, Chen, Ryan, and Simchi-Levi (2000) used simulation to study the impact of the assumption on the bullwhip effect and found that 'the way one treats excess demand and excess inventory has little impact on the increase in variability' (277). Therefore, the variance of the order quantities is not significantly affected by our assumption regarding the return policy of the retailer. Lee, Padmanabhan, and Whang (1997a), Chen et al. (2000), Chen, Ryan, and Simchi-Levi (2000), Zhang (2004), Zhang and Burke (2011) and Ma et al. (2013a, 2013b) also adopted the free return policy assumption to study the bullwhip effect.

At the beginning of period t, the consumer demand d_{t-1} has been realised; however, the lead-time demand, $d_t, d_{t+1}, \cdots, d_{t+L+1}$, has not been realised and must be estimated. The retailer uses the estimate of the mean lead-time demand, $\hat{D}_t^L$, as the basis for his or her order-up-to inventory level. $\hat{D}_t^L$ can be given by $\hat{D}_t^L = \hat{d}_t + \hat{d}_{t+1} + \cdots + \hat{d}_{t+L-1} = \sum_{i=0}^{L-1} \hat{d}_{t+i}$. Note that the retailer should employ certain forecasting techniques to estimate the mean lead-time demand, $\hat{D}_t^L$. ES is one

of the most commonly used forecasting techniques in practice, owing to its ease of use, flexibility and robustness. Many authors have employed the ES technique to analyse the bullwhip effect in the supply chain, such as Chen, Ryan, and Simchi-Levi (2000). In this paper, we assume that the retailer uses an ES technique to estimate the mean demand. ES is a technique that uses a weighted moving average of past data as the basis for forecasting. It should be noted that the demand forecasts for periods $t + i$ $(i = 0, 1, 2, \cdots)$ made at time $t - 1$ are equal using the ES technique;[2] therefore,

$$\hat{d}_t = \hat{d}_{t+1} = \hat{d}_{t+2} = \cdots = \alpha d_{t-1} + (1 - \alpha)\hat{d}_{t-1}, \tag{6}$$

where $\alpha \in (0, 1)$.

Taking the variance of both sides of the Equation (6), we obtain an expression for $\mathrm{Var}(\hat{d}_t)$, $\mathrm{Var}(\hat{d}_t) = (\alpha/(2-\alpha))$ $\mathrm{Var}(d_t) + (2(1-\alpha)/(2 - \alpha))\, \mathrm{Cov}(d_{t-1}, \hat{d}_{t-1})$, and an estimate of the mean lead-time demand:

$$\hat{D}_t^L = \sum_{i=0}^{L-1} \hat{d}_{t+i} = L\hat{d}_t = L(\alpha d_{t-1} + (1 - \alpha)\hat{d}_{t-1}). \tag{7}$$

Substituting Equation (7) into Equation (5), the order quantity given by Equation (5) becomes:

$$\begin{aligned} q_t &= L(\hat{d}_t - \hat{d}_{t-1}) + z(\hat{\sigma}_{e,t}^L - \hat{\sigma}_{e,t-1}^L) + d_{t-1} = L[(\alpha d_{t-1} + (1 - \alpha)\hat{d}_{t-1}) - \hat{d}_{t-1}] + z(\hat{\sigma}_{e,t}^L - \hat{\sigma}_{e,t-1}^L) + d_{t-1} \\ &= (1 + \alpha L)d_{t-1} - \alpha L\hat{d}_{t-1} + z(\hat{\sigma}_{e,t}^L - \hat{\sigma}_{e,t-1}^L). \end{aligned} \tag{8}$$

It should be noted that it is not necessary for the supplier to produce the same q_t ordered by the retailer. The proof has been shown in detail in Appendix 1.

3. The bullwhip effect for linear and iso-elastic demands

In the previous section, we introduced the improved linear and iso-elastic demand forms and the ordering process. We now develop expressions for the bullwhip effect under the two demand functions.

3.1 *The bullwhip effect for linear demand*

The following lemma will be useful in the derivation of the expression for the bullwhip effect under linear demand.

Lemma 1. The variance of the forecasting error in the lead-time demand remains constant over time under linear demand and is given by:

$$(\hat{\sigma}_{e,t}^{L,\mathrm{LINEAR}})^2 = \left(L + \frac{\alpha}{2 - \alpha}L^2\right)(\sigma_d^{\mathrm{LINEAR}})^2 + 2\left((1 - \alpha)L + \frac{\alpha(1 - \alpha)}{2 - \alpha}L^2 - 1\right)b^2 r(1 - r)(\sigma_p^{\mathrm{LINEAR}})^2, \tag{9}$$

where $(\sigma_d^{\mathrm{LINEAR}})^2 = \mathrm{Var}(d_t)$ and $(\sigma_p^{\mathrm{LINEAR}})^2 = \mathrm{Var}(p_t)$.

Proof: See Appendix 2.

From Lemma 1, we know that $\hat{\sigma}_{e,t}^{L,\mathrm{LINEAR}} = \hat{\sigma}_{e,t-1}^{L,\mathrm{LINEAR}}$, and the order quantity given by Equation (8) becomes, under linear demand,

$$q_t = (1 + \alpha L)d_{t-1} - \alpha L\hat{d}_{t-1}. \tag{10}$$

The tendency of orders to increase in variability as one moves up a supply chain is known as the bullwhip effect (Lee, Padmanabhan, and Whang 1997a, 1997b; Simchi-Levi, Kaminsky, and Simchi-Levi 2002). The demand in period t, d_t, can also be seen as the consumer's order quantity to the retailer. If the variance of the retailer's order quantity q_t is larger than that of the consumer's order quantity d_t, then the bullwhip effect exists. Based on Equation (10), we quantify the bullwhip effect using the measurement $\frac{\mathrm{Var}(q_t)}{\mathrm{Var}(d_t)}$ as shown in Theorem 1.

Theorem 1. If we consider consumer price forecasting behaviour, then the bullwhip effect measurement, BE^{LINEAR}, under linear demand can be expressed as follows:

$$BE^{\text{LINEAR}} = \frac{\text{Var}(q_t)}{\text{Var}(d_t)} = 1 + 2\alpha L + \frac{2\alpha^2 L^2}{2-\alpha} - 2\alpha^2 L\left(1 + \frac{\alpha L}{2-\alpha}\right)\frac{r(1-r)}{2r^2 - 2r + 1}. \tag{11}$$

Proof: See Appendix 3.

From Theorem 1, we know that BE^{LINEAR} is not affected by the market demand scale, a, the price sensitivity coefficient, b, or the price variance, σ^2. However, the effect does depend on the following three parameters: (1) the lead-time parameter, L; (2) the smoothing coefficient parameter, α; and (3) the consumer sensitivity to price changes, r. It is straightforward to show that BE^{LINEAR} is an increasing function of L and α, which means that a smaller lead time or smoother demand forecast can help to reduce the bullwhip effect. A similar conclusion has also been drawn by Chen, Ryan, and Simchi-Levi (2000) for an AR (1) demand process. Because our aim is to analyse the impact of consumer price forecasting behaviour on the bullwhip effect, we shall restrict our attention to the parameter r. The following proposition describes the influence of r on BE^{LINEAR}.

Proposition 1. When consumer price forecasting behaviour is accounted for under linear demand, BE^{LINEAR} is a decreasing function of r when the consumer sensitivity to price changes, $r \in [0, 0.5)$; BE^{LINEAR} is an increasing function of r when $r \in [0.5, 1)$; and BE^{LINEAR} has the minimum value when $r = 0.5$.

Proof: See Appendix 4.

We can explain Proposition 1 as follows. The correlation coefficient of the linear demand process can be given as $\rho_{d_t,d_{t-1}}^{\text{LINEAR}} = \frac{\text{Cov}(d_t,d_{t-1})}{\text{Var}(d_t)} = \frac{b^2 r(1-r)\text{Var}(p_t)}{b^2(2r^2-2r+1)\text{Var}(p_t)} = \frac{-r^2+r}{2r^2-2r+1}$ (see Appendix 3). Because $\frac{\partial \rho_{d_t,d_{t-1}}^{\text{LINEAR}}}{\partial r} = \frac{-2r+1}{(2r^2-2r+1)^2}$, we know that $\rho_{d_t,d_{t-1}}^{\text{LINEAR}}$ is an increasing function of r when $r \in [0, 0.5)$ and is a decreasing function of r when $r \in [0.5, 1)$. From the theoretical perspective, it has been pointed out that the increase in variability (i.e. the bullwhip effect) is a decreasing function of the demand correlation coefficient when demand is forecasted with the ES technique (Chen, Ryan, and Simchi-Levi 2000, 275). Therefore, the larger the correlation coefficient of the demand process, the smaller the bullwhip effect will be. Consequently, BE^{LINEAR} decreases with an increase in r when $r \in [0, 0.5)$ and increases with an increase in r when $r \in [0.5, 1)$. From the practical perspective, notice that Ma et al. (2013a, 2013b) pointed out that the demand process is reflected by a wandering or meandering sequence of observations when the demand correlation coefficient is small; additionally, when the correlation coefficient has a large positive value, neighbouring values in the demand process are similar and the process exhibits less variability. It is easy to deduce that $\rho_{d_t,d_{t-1}}^{\text{LINEAR}} \geq 0$ when $0 \leq r < 1$. In addition, we have also shown that the demand correlation coefficient $\rho_{d_t,d_{t-1}}^{\text{LINEAR}}$ reaches its maximum value when $r = 0.5$. Therefore, the increase in variability will be less when the demand process is smoother and exhibits less variability when $r = 0.5$; in other words, the bullwhip effect is minimal when the consumer sensitivity to price changes, $r = 0.5$.

In addition, we see from the expression for the bullwhip effect, BE^{LINEAR}, that if consumers are not strategic (i.e. $r = 0$), then we have $BE_{r=0}^{\text{LINEAR}} = 1 + 2\alpha L + 2\alpha^2 L^2/(2-\alpha)$, which is the same as the lower bound on the bullwhip effect expressions described in Chen, Ryan, and Simchi-Levi (2000) for an AR(1) process when the demand correlation parameter is zero. However, when $r \in (0, 1)$, it is observed that $2\alpha^2 L(1 + \alpha L/(2-\alpha))r(1-r)/(2r^2 - 2r + 1) > 0$ and therefore $BE_{r\in(0,1)}^{\text{LINEAR}} < BE_{r=0}^{\text{LINEAR}}$. Consequently, unlike non-strategic consumers, strategic consumers benefit the supply chain through bullwhip effect reduction. We can explain this phenomenon as follows. If consumers are not strategic (i.e. $r = 0$), we know from Equation (2) that the market demand d_t is i.i.d. over time. However, for strategic consumers, there is consumer forecasting behaviour and the consumer sensitivity factor $r \neq 0$. Since the demand correlation coefficient $\rho_{d_t,d_{t-1}}^{\text{LINEAR}} = \frac{-r^2+r}{2r^2-2r+1} > 0$ when $0 < r < 1$, the market demands when considering consumer forecasting behaviour are positively correlated, which indicates that neighbouring values in the demand process are similar and the process exhibits market trends. Compared to the i.i.d. demand process for the non-strategic consumers, the demand process when considering consumer forecasting behaviour is smoother and exhibits less variability; thus, the bullwhip effect for strategic consumers when considering consumer forecasting behaviour is less than that for non-strategic consumers. Chen, Ryan, and Simchi-Levi (2000) also pointed out that 'for positively correlated demands, the increase in variability will be less than for i.i.d. demands' (275). Therefore, the consumer forecasting behaviour brings smoother consumer demands and consequently the supply chain exhibits less variability. However, it should be noted that the bullwhip effect is always lower for 'strategic' consumers, but for 'very' strategic customers (price sensitivity close to 1), the benefit is limited. In addition, we know from Proposition 1 that $\min(BE^{\text{LINEAR}}) = BE_{r=0.5}^{\text{LINEAR}} = 1 + 2\alpha L + \alpha^2 L(L-1) > 1$. Therefore, although consumer price forecasting behaviour can reduce the bullwhip effect, it cannot completely eliminate the effect.

3.2 *The bullwhip effect for iso-elastic demand*

Our approach to developing the expression for the bullwhip effect under iso-elastic demand is the same as that used for linear demand. For the assumed order-up-to inventory policy and ES forecasting technique, we derive the retailer's order quantity given by Equation (8). The following lemma will be useful in our derivation of the bullwhip effect for iso-elastic demand.

Lemma 2. The variance of the forecasting error in the lead-time demand remains constant over time under iso-elastic demand and is given by:

$$\left(\hat{\sigma}_{e,t}^{L,\text{ISO}}\right)^2 = \left(L + \frac{\alpha}{2-\alpha}L^2\right)\left(\sigma_d^{\text{ISO}}\right)^2 + 2\left((1-\alpha)L + \frac{\alpha(1-\alpha)}{2-\alpha}L^2 - 1\right)a^2 e^{-2b\mu+(r^2-r+1)b^2\sigma^2}\left(1 - e^{(r^2-r)b^2\sigma^2}\right), \tag{12}$$

where $\left(\sigma_d^{\text{ISO}}\right)^2 = \text{Var}(d_t)$.

Proof: See Appendix 5.

Using Lemma 2, the order quantity given in Equation (8) becomes, under iso-elastic demand,

$$q_t = (1 + \alpha L)d_{t-1} - \alpha L \hat{d}_{t-1}. \tag{13}$$

Theorem 2. If we consider consumer price forecasting behaviour, then the bullwhip effect measurement, BE^{ISO}, under iso-elastic demand can be expressed as follows:

$$BE^{\text{ISO}} = \frac{\text{Var}(q_t)}{\text{Var}(d_t)} = 1 + 2\alpha L + \frac{2\alpha^2 L^2}{2-\alpha} - 2\alpha^2 L\left(1 + \frac{\alpha L}{2-\alpha}\right)\frac{e^{(-r^2+r)b^2\sigma^2} - 1}{e^{(2r^2-2r+1)b^2\sigma^2} - 1}. \tag{14}$$

Proof: See Appendix 6.

From Theorem 2, we know that BE^{ISO} depends on the following five parameters: the price sensitivity coefficient, b; the lead time, L; the smoothing coefficient, α; the consumer sensitivity to price changes, r; and the variance, σ^2. The market demand scale, a, has no effect on BE^{ISO}. It can be shown that BE^{ISO} is an increasing function of L and α. However, we will restrict our attention to the influence of the parameters b, r and σ on BE^{ISO}. For $L = 2$ and $\alpha = 0.5$, Figure 1 shows the relations between BE^{ISO} and r for different values of b and σ.[3]

From Figure 1(a) (or Figure 1(b)), we can see that BE^{ISO} decreases by increasing the value of r from zero to 0.5 and increases by increasing the value of r from 0.5 to one for various values of b (or σ). In addition, it can be shown that BE^{ISO} increases with increasing b (or σ), which means that consumer price forecasting behaviour will have a greater impact on reducing the bullwhip effect when the product price sensitivity coefficient is small or when the price has a distribution of small standard deviations.

Proposition 2. When consumer price forecasting behaviour is accounted for under iso-elastic demand, BE^{ISO} is a decreasing function of r when the consumer sensitivity to price changes, $r \in [0, 0.5)$; BE^{ISO} is an increasing function of r when $r \in [0.5, 1)$; and BE^{ISO} has the minimum value when $r = 0.5$.

Proof: As in the proof of Proposition 1, we can prove Proposition 2 by taking the 1st and the 2nd partial derivatives with respect to r on both sides of Equation (14). This completes the proof. $\qquad\square$

Similarly, the iso-elastic demand correlation coefficient can be given as:

$$\rho_{d_t,d_{t-1}}^{\text{ISO}} = \frac{\text{Cov}(d_t, d_{t-1})}{\text{Var}(d_t)} = \frac{a^2 e^{-2b\mu+(r^2-r+1)b^2\sigma^2}\left(1 - e^{(r^2-r)b^2\sigma^2}\right)}{a^2 e^{-2b\mu+(2r^2-2r+1)b^2\sigma^2}\left(e^{(2r^2-2r+1)b^2\sigma^2} - 1\right)} = \frac{e^{(-r^2+r)b^2\sigma^2} - 1}{e^{(2r^2-2r+1)b^2\sigma^2} - 1} \qquad \text{(see Appendix 6).}$$

It can be seen that the numerator of $\rho_{d_t,d_{t-1}}^{\text{ISO}}$, $e^{(-r^2+r)b^2\sigma^2} - 1$, is an increasing function of r when $r \in [0, 0.5)$ and a decreasing function of r when $r \in [0.5, 1)$; however, the denominator of $\rho_{d_t,d_{t-1}}^{\text{ISO}}$, $e^{(2r^2-2r+1)b^2\sigma^2} - 1$, is a decreasing function of r when $r \in [0, 0.5)$ and an increasing function of r when $r \in [0.5, 1)$. Therefore, it can be deduced that $\rho_{d_t,d_{t-1}}^{\text{ISO}}$ increases with an increase in r when $r \in [0, 0.5)$ and decreases with an increase in r when $r \in [0.5, 1)$. Following the same approach to explain Proposition 1, we can explain Proposition 2, both from a theoretical perspective and a practical perspective. However, the analysis is not reported here.
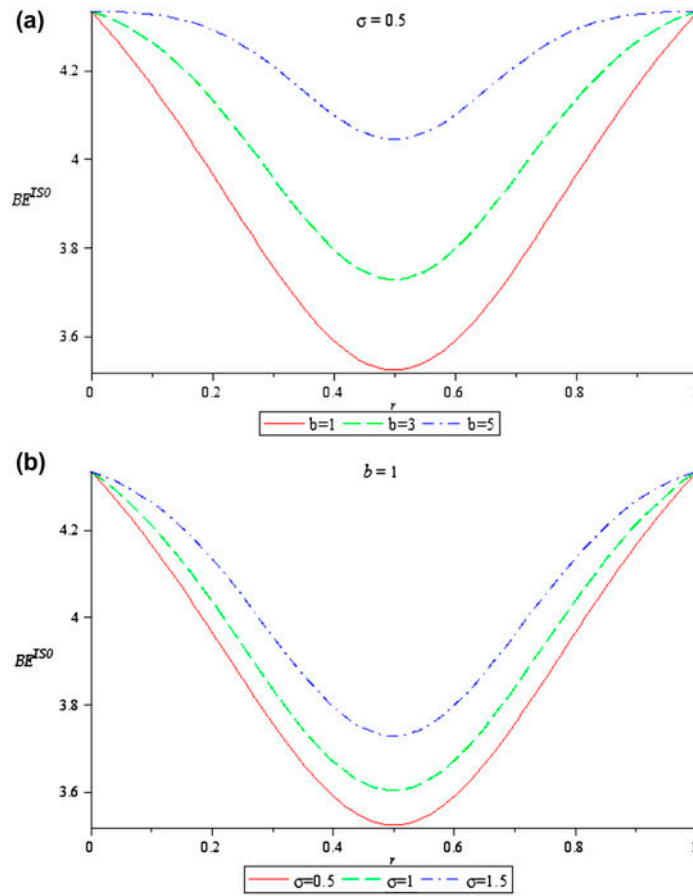
Figure 1.   The influence of r on BE^{ISO} for different values of b and σ when $L = 2$ and $\alpha = 0.5$. (a) The influence of r on BE^{ISO} for $b = 1$, 3 and 5 when $L = 2$, $\alpha = 0.5$ and $\sigma = 0.5$. (b) The influence of r on BE^{ISO} for $\sigma = 0.5$, 1 and 1.5 when $L = 2$, $\alpha = 0.5$ and $b = 1$.

When $r = 0$, we obtain the bullwhip effect expression $BE_{r=0}^{ISO}$ for non-strategic consumers. In this case, BE^{ISO} reduces to $BE_{r=0}^{ISO} = 1 + 2\alpha L + 2\alpha^2 L^2/(2 - \alpha)$, which is equal to the bullwhip effect for linear demand, $BE_{r=0}^{ISO} = BE_{r=0}^{LINEAR}$. In addition, by comparing $BE_{r \in (0,1)}^{ISO}$ to $BE_{r=0}^{ISO}$, it can be shown that consumer price forecasting behaviour can help to reduce the bullwhip effect but not completely eliminate it.

Note that the bullwhip effect is the same for both forms of the demand when consumers are not strategic. A natural question is which type of demand causes more variability when consumers are strategic? To address this point, we compare the bullwhip effect under the two demand forms when $r \neq 0$.

3.3 *Comparison of the bullwhip effects under different demand forms*

To compare the bullwhip effect under the two demand forms, we assume that there are two two-level supply chains, as described in Section 2, where each supply chain distributes the same single product. The first retailer, i.e. the retailer in the first supply chain, faces a linear demand form, while the second retailer faces an iso-elastic demand form. Assuming that the retail prices are i.i.d. from a log-normal distribution in the market for both retailers, we deduce the following theorem:

Theorem 3. Let BE^{LINEAR} (BE^{ISO}) be the bullwhip effect under linear demand (iso-elastic demand), assuming that the prices of both retailers are i.i.d. from a log-normal distribution. Then:
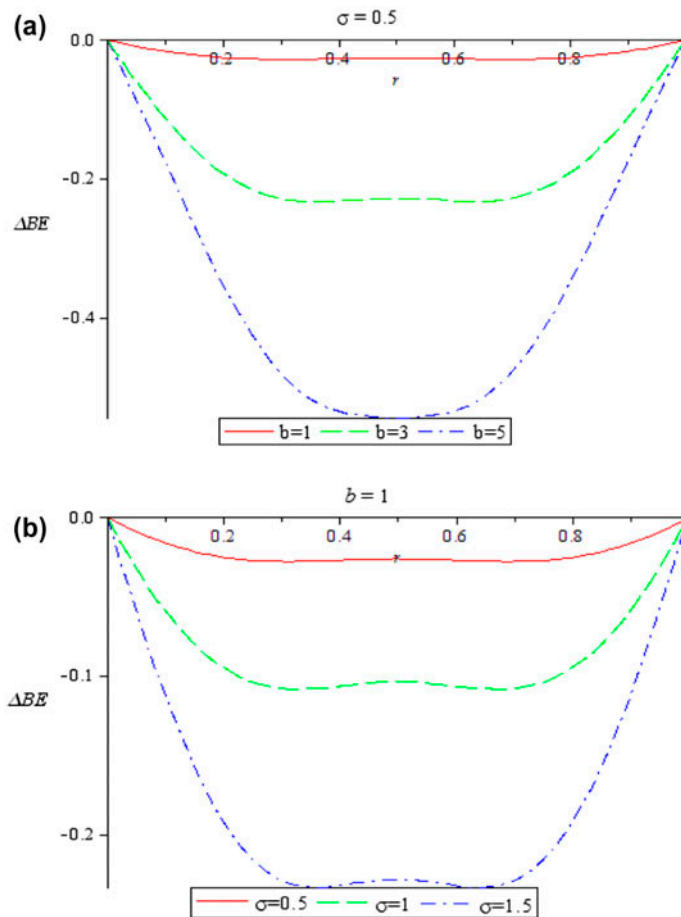
$$BE^{LINEAR} \leq BE^{ISO}, \quad r \in [0, 1).$$

Figure 2. The influence of r on ΔBE for different values of b and σ when $L = 2$ and $\alpha = 0.5$. (a) The influence of r on ΔBE for $b = 1$, 3 and 5 when $L = 2$, $\alpha = 0.5$ and $\sigma = 0.5$. (b) The influence of r on ΔBE for $\sigma = 0.5$, 1 and 1.5 when $L = 2$, $\alpha = 0.5$ and $b = 1$.

Proof: See Appendix 7.

Theorem 3 indicates that the orders placed by the first retailer are no more variable than those placed by second retailer, although both retailers use the same ordering process and forecasting technique. Let $\Delta BE = BE^{\text{LINEAR}} - BE^{\text{ISO}}$. Figure 2 illustrates how ΔBE changes with r for different values of b and σ when $L = 2$ and $\alpha = 0.5$.

From Figure 2, it can be observed that the higher the price sensitivity coefficient (or the standard deviation of the price), the greater the enhancement of the bullwhip effect for the second retailer above that for the first retailer. In addition, when the consumer sensitivity to price changes, r, has a value near $r = 0.5$, the gap in the bullwhip effect between the two retailers is more appreciable.

4. Multi-retailer model

Most of the previous studies, such as Lee, Padmanabhan, and Whang (1997a), Chen, Ryan, and Simchi-Levi (2000) and Zhang (2004), used a simple one-supplier and one-retailer model to investigate the bullwhip effect. We also considered a supply chain with a supplier and a retailer in Sections 2 and 3 in this paper. However, this simple model fails to capture many of the complexities involved in real-world supply chains. By extending our results to the multiple-retailer case in which each retailer distributes a single product, we can explore the influence of an increase in the number of retailers on the bullwhip effect when considering consumer price forecasting behaviour. We analyse this problem through the variance of the total order quantity compared to that of the total market demand.

Assume that there are n retailers and one supplier, that all retailers with the same lead time use ES technology, and that the retail prices of different retailers are independent. To facilitate the analysis, we assume that the market demand

scale and the price sensitivity coefficient are identical for different products. If strategic consumers display price forecasting behaviour, then the retailer i ($1 \le i \le n$) faces the following market demand:

$$d_t^i = D(p_t^i, p_{t-1}^i), \tag{15}$$

where d_t^i is the market demand of retailer i in period t, p_t^i is the retail price of retailer i in period t and p_t^i is i.i.d. over time for any retailer i.

Note that if the retail prices of different retailers are independent, it is straightforward to obtain that $\mathrm{Cov}(d_t^i, d_t^j) = \mathrm{Cov}(D(p_t^i, p_{t-1}^i), D(p_t^j, p_{t-1}^j)) = 0$, $\forall i \ne j$. That is, the demands for different retailers are not linearly dependent, but this does not mean that there are no relations among them. For instance, if retailer i sells more products, then the sales quantities of other retailers will be restrained because the total market demand scale is bounded. However, because our aim is to determine whether the number of retailers influences the bullwhip effect, we shall restrict our discussion to facilitate the analysis.

We can write the variance of the total market demand, $\mathrm{Var}\left(\sum_{i=1}^n d_t^i\right)$, as:

$$\mathrm{Var}\left(\sum_{i=1}^n d_t^i\right) = \sum_{i=1}^n \mathrm{Var}(d_t^i) + 2\sum_{i=1}^{n-1}\sum_{j=1}^{n-i} \mathrm{Cov}(d_t^i, d_t^{i+j}) = \sum_{i=1}^n \mathrm{Var}(d_t^i). \tag{16}$$

The order quantity of retailer i is given by:

$$q_t^i = (1 + \alpha L)d_{t-1}^i - \alpha L \hat{d}_{t-1}^i. \tag{17}$$

We can therefore express the total order quantity as:

$$Q = \sum_{i=1}^n q_t^i = (1 + \alpha L)\sum_{i=1}^n d_{t-1}^i - \alpha L \sum_{i=1}^n \hat{d}_{t-1}^i. \tag{18}$$

Using Equation (18), the variance amplifications of the total order quantity under linear and iso-elastic demands are given in Theorem 4 and Theorem 5, respectively.

Theorem 4. Assume that the retail prices of different retailers are i.i.d. If all the retailers face the same lead time, then under linear demand, the variance of the total order quantity compared to that of the total demand can be expressed as follows (when considering consumer price forecasting behaviour):

$$BE^{\mathrm{LINEAR}} = \frac{\mathrm{Var}(Q)}{\mathrm{Var}\left(\sum_{i=1}^n d_t^i\right)} = \frac{\mathrm{Var}(q_t^i)}{\mathrm{Var}(d_t^i)} = 1 + 2\alpha L + \frac{2\alpha^2 L^2}{2 - \alpha} - 2\alpha^2 L\left(1 + \frac{\alpha L}{2 - \alpha}\right)\frac{r(1 - r)}{2r^2 - 2r + 1}. \tag{19}$$

Proof: See Appendix 8.

As prices fluctuate over time, a strategic consumer may buy more in advance to reduce his or her future needs in anticipation of higher prices in the future, or may choose to postpone a purchase in anticipation of lower prices in the future. This research used the consumer sensitivity factor r to investigate the bullwhip effect when price changes. Note that by Theorem 4 and Theorem 1, the bullwhip effect for the total order quantity is the same as that for the order quantity of a single retailer for the linear demand form, which means that an increase in the number of retailers does not affect the demand variability amplification from the retailers to the supplier as one moves up the supply chain. Consequently, the number of retailers has no influence on the relationship between consumer price forecasting behaviour and the bullwhip effect.

Theorem 5. Assume that the retail prices of different retailers are i.i.d. from a log-normal distribution with mean μ_i and variance σ_i^2. If all the retailers face the same lead time, then under iso-elastic demand, the variance of the total order quantity compared to that of the total demand can be expressed as follows (when considering consumer price forecasting behaviour):

$$BE^{\mathrm{ISO}} = \frac{\mathrm{Var}(Q)}{\mathrm{Var}\left(\sum_{i=1}^n d_t^i\right)} = 1 + 2\alpha L + \frac{2\alpha^2 L^2}{2 - \alpha} - 2\alpha^2 L\left(1 + \frac{\alpha L}{2 - \alpha}\right) \times \frac{\sum_{i=1}^n e^{-2b\mu_i + (r^2 - r + 1)b^2\sigma_i^2}\left(1 - e^{(r^2 - r)b^2\sigma_i^2}\right)}{\sum_{i=1}^n e^{-2b\mu_i + (2r^2 - 2r + 1)b^2\sigma_i^2}\left(e^{(2r^2 - 2r + 1)b^2\sigma_i^2} - 1\right)}. \tag{20}$$

Proof: See Appendix 9.

In particular, when $\mu_i = \mu$ and $\sigma_i^2 = \sigma^2$, Equation (20) reduces to:

$$BE^{\text{ISO}} = \frac{\text{Var}(q_t^i)}{\text{Var}(d_t^i)} = 1 + 2\alpha L + \frac{2\alpha^2 L^2}{2 - \alpha} - 2\alpha^2 L \left(1 + \frac{\alpha L}{2 - \alpha}\right) \frac{e^{(-r^2 + r)b^2\sigma^2} - 1}{e^{(2r^2 - 2r + 1)b^2\sigma^2} - 1}. \tag{21}$$

Note that the right-hand side of Equation (21) is identical to that of Equation (14) in Theorem 2. Therefore, if the retail prices of different retailers are independent and have the same distribution, then an increase in the number of retailers also does not affect the bullwhip effect for the iso-elastic demand form. From the above analysis, we conclude that for both demand forms, consumer price forecasting behaviour will have the same inhibiting effect on the supply chain variability even if the number of retailers is increased. Fransoo and Wouters (2000, 82) used an industrial project to show practical ways for calculating the bullwhip effect in a supply chain that has P products and M outlets. The authors found that the bullwhip effect for aggregating over the products is smaller than that for aggregating over the outlets and products. However, the difference is very small and the deviation (1.6%) can be neglected. On the other hand, it should be noted that Chen and Lee (2012, 779) considered a typical one-retailer and one-supplier supply chain model that distributes N products. The authors aggregated over the products to measure the bullwhip effect and found that the product aggregation can mask the severity of individual bullwhip. However, we considered in our research that there are one supplier and n retailers in a more general supply chain in which each retailer distributes a single product. We aggregated over the retailers to measure the bullwhip effect and found that the number of retailers has no influence on the relationship between consumer price forecasting behaviour and the bullwhip effect. Our future research may investigate the impact of product aggregation and retailer aggregation on the bullwhip effect.

5. Numerical experiments

In the preceding sections, we have performed a theoretical analysis of the influence of consumer price forecasting behaviour on the bullwhip effect in the context of the basic (one-retailer) and multi-retailer models. In this section, we provide numerical experiments to verify our analysis and illustrate the impact of the consumer sensitivity to price changes, r, on the bullwhip effects in linear and iso-elastic demand models for different values of b and σ.

We first generated the market prices using Matlab 7.0 for the basic model, which are i.i.d. from a log-normal distribution with $\mu = 0$, for 2000 consecutive time periods with $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 1.5$, respectively. The price data are shown in Figure 3. In all our experiments, we fixed $a = 100$, $L = 2$ and $\alpha = 0.5$. We varied the parameters b and r over the values $b \in \{1, 2, 3, 4, 5\}$ and $r \in \{0, 0.1, 0.3, 0.5, 0.7, 0.9\}$. Given the above parameters, we computed the corresponding demands using Equations (2) and (3), the order quantities using Equations (10) and (13), and finally the bullwhip effects for the two demand forms. In addition, we also considered a two-retailer model and generated another stream of random prices following the same approach as described for the basic model. The bullwhip effects were computed as the variance of the total order quantities compared to those of the total demands. Figure 4 shows the influence of r on BE^{LINEAR} for specified values of σ in the basic and two-retailer models,[4] and Figure 5 shows the influence of r on BE^{ISO} for various values of b and σ in both models.

We can make the following observations from Figures 4 and 5: (1) For both demand forms and in both the basic and two-retailer models, the simulated bullwhip effect values when $r = 0$ are greater than or equal to those when $r \neq 0$. (2) When $r \neq 0$, the values of the bullwhip effects BE^{LINEAR} and BE^{ISO} reach their minimum when $r = 0.5$ in our experiments. (3) The values of the bullwhip effect under linear demand, BE^{LINEAR}, are always less than or equal to those under iso-elastic demand, BE^{ISO}.[5] (4) The price sensitivity coefficient, b, and the standard deviation of the price, σ, generally have no influence on BE^{LINEAR}.[6] However, BE^{ISO} always reaches its smallest value when b (or σ) is small. (5) Comparing the values in Figure 4(a) (or Figure 5(a)) with those in Figure 4(b) (or Figure 5(b)), we find that an increase in the number of retailers (from 1 to 2) has no obvious influence on BE^{LINEAR} (or BE^{ISO}). These observations are consistent with the theoretical findings presented in Sections 3 and 4.

Because strategic consumers have memory of the price in the previous period and exhibit future price forecasting behaviour, our experiment results indicate that market demand process is smoother when considering consumer forecasting behaviour and strategic consumers lead to a reduced bullwhip effect. The benefit is most significant when the consumer sensitivity to price changes is medium (approximately 0.5). In addition, as for iso-elastic demand, if the product price sensitivity coefficient or the standard deviation of the price is small, then consumer price forecasting behaviour causes a greater reduction in the bullwhip effect.
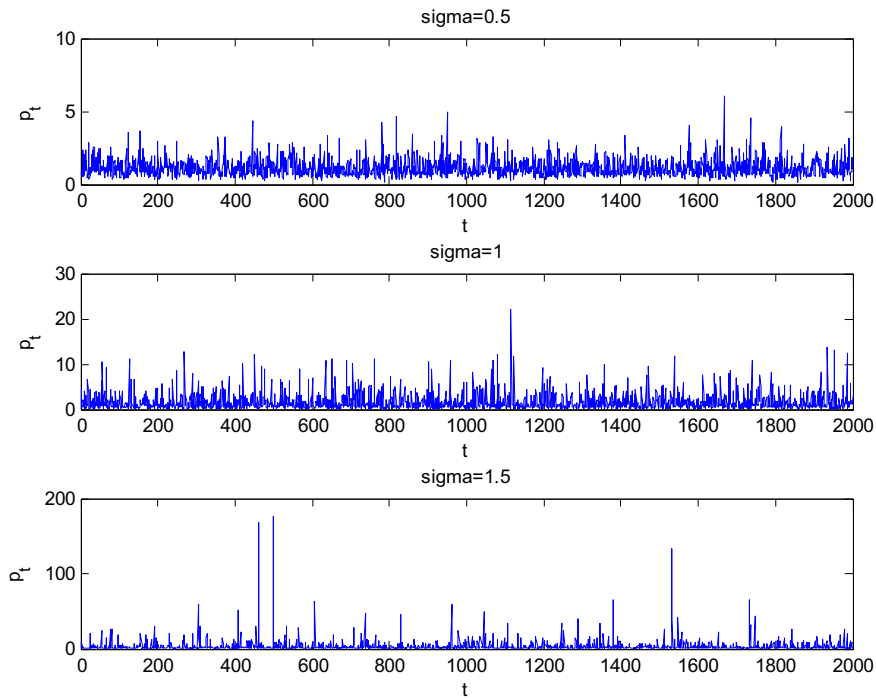
Figure 3. The market prices when p_t are i.i.d. from a log-normal distribution when $\mu = 0$ and $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 1.5$ for 2000 consecutive time periods.
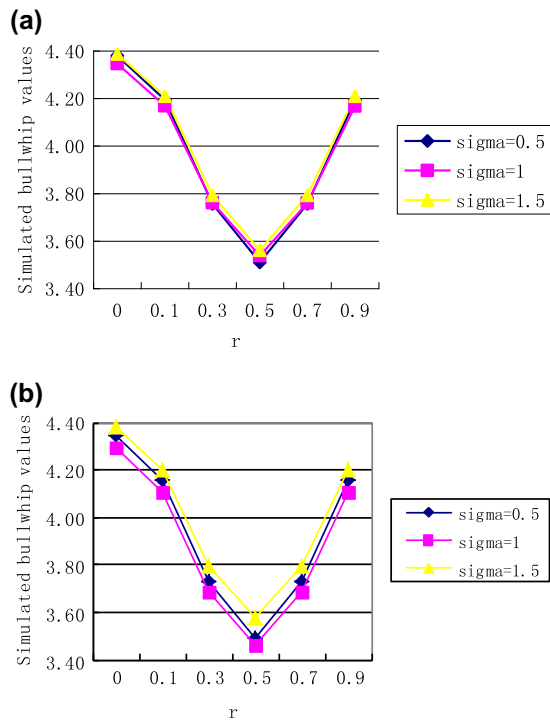

Figure 4. (a) The influence of r on BE^{LINEAR} for different values of σ when $\mu = 0$, $a = 1000$, $L = 2$ and $\alpha = 0.5$ in the basic model. (b) The influence of r on BE^{LINEAR} for different values of σ when $\mu = 0$, $a = 1000$, $L = 2$ and $\alpha = 0.5$ in the two-retailer model.
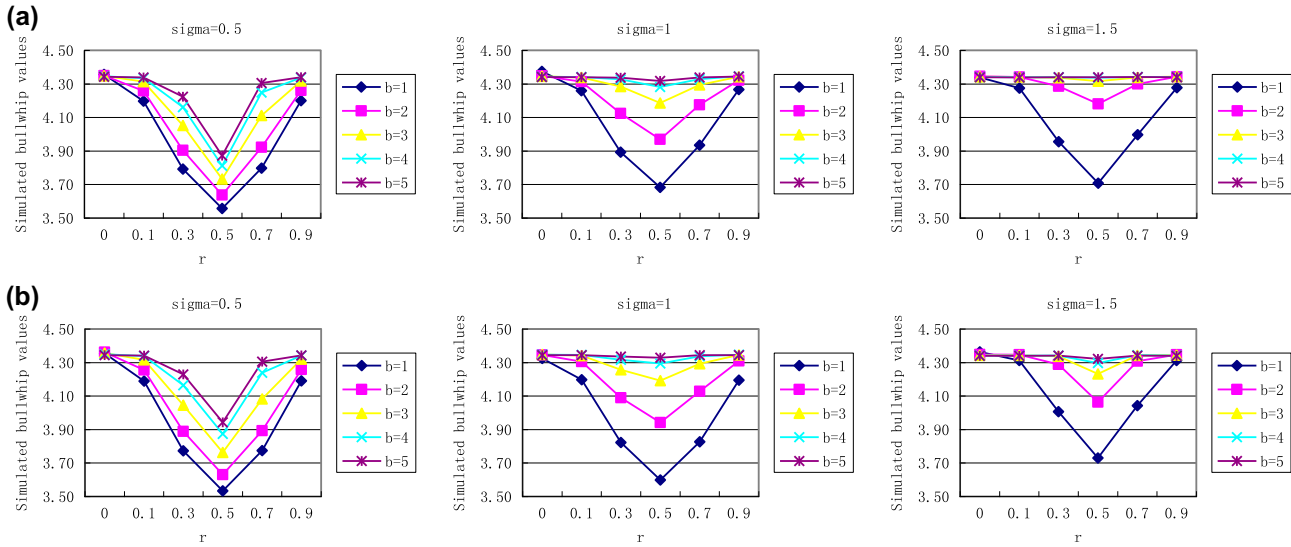
(a)

(b)

Figure 5. (a) The influence of r on BE^{ISO} for different values of b and σ when $\mu = 0$, $a = 1000$, $L = 2$ and $\alpha = 0.5$ in the basic model. (b) The influence of r on BE^{ISO} for different values of b and σ when $\mu = 0$, $a = 1000$, $L = 2$ and $\alpha = 0.5$ in the two-retailer model.

6. Extension

In this section, we examine the impact of consumer price forecasting behaviour on supply chain cost, which would provide a comprehensive argument in favour of the proposed forecasting behaviour.

We have investigated the impact of consumer price forecasting on the bullwhip effect in a two-level supply chain composed of a supplier and a retailer. Because the bullwhip effect can lead to misguided capacity plans, missed production schedules and inactive transportations from upstream businesses, this effect will primarily influence the supplier's cost (Ma et al. 2013b). In the previous sections, we found that the bullwhip effect is lower compared to the situation without consumer price forecasting behaviour for both linear and iso-elastic demand models. Therefore, we believe that consumer forecasting behaviour could reduce the supplier's cost.

We now discuss how consumer forecasting behaviour influences the retailer's cost. Zipkin (1995) and Lee, So, and Tang (2000) have pointed out that the retailer's expected inventory holding and shortage cost is a function of $\hat{\sigma}_{e,t}^{L}$ (the standard deviation of forecasting error) and some other terms. The latter ones depend exclusively on the cost factors C_h and C_b, where C_h denotes the inventory holding-cost rate and C_b denotes back order penalty cost rate. Therefore, we can develop the expressions for retailer's expected inventory holding and shortage cost, $C_{\text{Retailer}}^{\text{LINEAR}}$ and $C_{\text{Retailer}}^{\text{ISO}}$, for linear and iso-elastic demand models:

$$C_{\text{Retailer}}^{\text{LINEAR}} = \hat{\sigma}_{e,t}^{L,\text{LINEAR}} \cdot f_1(C_h, C_b), \tag{22}$$

and

$$C_{\text{Retailer}}^{\text{ISO}} = \hat{\sigma}_{e,t}^{L,\text{ISO}} \cdot f_2(C_h, C_b), \tag{23}$$

where $\hat{\sigma}_{e,t}^{L,\text{LINEAR}}$ (resp. $\hat{\sigma}_{e,t}^{L,\text{ISO}}$) is the standard deviation of forecasting error for linear (resp. iso-elastic) demand and can be given by Equation (9) (resp. Equation (12)), $f_1(C_h, C_b)$ and $f_2(C_h, C_b)$ are the terms that depend exclusively on C_h and C_b.

Because $\dfrac{\partial C_{\text{Retailer}}^{\text{LINEAR}}}{\partial r} = \dfrac{2b^2}{2-\alpha}[2(L(L-1)\alpha^2 + (2L-1)\alpha + 2)r - (L(L-1)\alpha^2 + (2L-1)\alpha + 2)] \cdot f_1(C_h, C_b)$ from Equation (22), we know that $\dfrac{\partial C_{\text{Retailer}}^{\text{LINEAR}}}{\partial r} < 0$ when $r \in [0, 0.5)$ and $\dfrac{\partial C_{\text{Retailer}}^{\text{LINEAR}}}{\partial r} \geq 0$ when $r \in [0.5, 1)$. Thus, $C_{\text{Retailer}}^{\text{LINEAR}}$ is a decreasing function of r when $r \in [0, 0.5)$ and is an increasing function of r when $r \in [0.5, 1)$. Similarly, we can derive the same relationship between $C_{\text{Retailer}}^{\text{ISO}}$ and r. Consequently, consumer price forecasting behaviour could reduce the

retailer's cost for both demand forms, particularly when the consumer sensitivity to price changes is 0.5 (which corresponds to the situation where the correlation between demands is highest and the demand process is smoothest).

7. Conclusions

This paper establishes two forms of demand models (linear and iso-elastic) that incorporate multiple-period prices based on the purchasing decisions of a strategic consumer. We derive analytical expressions for the bullwhip effect under the two demand forms, accounting for consumer price forecasting behaviour, and extend the results to the multiple-retailer case. The results demonstrate that:

(1) For linear demand, neither the market demand scale nor the price sensitivity influences the bullwhip effect. However, consumer price forecasting behaviour helps to reduce the bullwhip effect, most significantly when the consumer sensitivity to price changes is medium (approximately 0.5).

(2) For iso-elastic demand, the market demand scale again has no effect on the bullwhip effect. However, the reduction of the bullwhip effect via consumer price forecasting behaviour is significant, especially when the price sensitivity coefficient is small, the consumer sensitivity to price changes is medium (approximately 0.5) or the standard deviation of the prices is small.

(3) For both demand forms, an increase in the number of retailers does not affect the amplification in the demand variability from the retailers to the supplier. Consequently, consumer price forecasting behaviour will have the same inhibiting effect on the supply chain variability for any number of retailers.

(4) We expand our study of consumer price forecasting behaviour on supply chain cost. We provide evidence that the strategic consumer's response to price changes results in lower supply chain cost.

Our findings regarding bullwhip effect reduction are important as they encourage companies to develop systems that will induce consumers to be strategic. For example, when the market clearing prices fluctuate and the price in the current period is higher than that in the previous periods, the supply chain manager can inform the consumers about their limited inventory or the likelihood of stock-outs through commercials or emails. By doing so, the consumers may think that the product prices in the future will be ever higher, and as a result, the supply chain manager can induce non-strategic consumers to become strategic to make more purchases in the current period. Conversely, the supply chain manager can do the opposite way. These marketing behaviours adopted by the manager can benefit the supply chain through bullwhip effect reduction and inventory-related cost reduction.

The research presented in this paper suggests several future directions to enhance our understanding of the influence of consumer price forecasting behaviour on the bullwhip effect. First, our model considers only the order-up-to inventory policy and the ES forecasting technique; other inventory policies and forecasting technologies require further study. Second, this paper considers only the influence of the prices in the current and previous periods on the demand. The additional influence of historic multi-period prices is a direction worth pursuing. Finally, extending the two-level supply chain to multi-level chains would be another fruitful direction for follow-up to this study.

Notes

1. Note that this is only the marketing behaviour. The supply chain may still have sufficient inventory.
2. Note that the demand forecasts for periods $t+i$ ($i=0, 1, \ldots$) may be not equal when different forecasting technique is adopted, such as the Minimum Mean Squared Error (MMSE) technique. We refer readers to Ma et al. (2013b) to get a more comprehensive understanding when demand is forecast with MMSE, Moving Average and ES techniques, respectively.
3. For different values of L and α, we can derive similar relations between BE^{ISO} and r; this additional numerical analysis is not reported for brevity.
4. For a given value of σ, there is no difference in BE^{LINEAR} when b is varied between $b=1$, 2, 3, 4 and 5 in either the basic or two-retailer models in our experiments.
5. This observation follows from a comparison of the values in Figure 4(a) (or (b)) with those in Figure 5(a) (or (b)).

6. Note that when we investigate the impact of r on BE^{LINEAR} for the different values of σ shown in Figure 4(a), we find that the differences range from −0.76% (0.16%) to 0.82% (1.46%) if we compare the $\sigma = 1$ ($\sigma = 1.5$) case with the $\sigma = 0.5$ case in the basic model. These differences are caused by the random character of the input price data and the limited experiment time periods. Similar observations also apply to Figure 4(b).
7. Previous research, such as Ma et al. (2013a), adopted the same approach to simplify the analysis.

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Appendix: Proofs of lemmas, theorems and propositions

Appendix 1: Derivation of supplier's production quantity

We first consider the following time of events. At the beginning of period t, the supplier receives and ships the required order quantity q_t to the retailer. The supplier calculates its order-up-to level y_t^s for period t and produces q_t^s at the beginning of period t according to its current inventory level. After the production lead time L^s, the supplier fills its inventory with the production quantity q_t^s at the beginning of period $t + L^s$. If we assume that the supplier also adopts the order-up-to inventory policy, the production quantity for the supplier in an order-up-to system is as follows:

$$q_t^s = y_t^s - y_{t-1}^s + q_t, \tag{A.1}$$

where $y_t^s = \hat{D}_t^{L^s} + z^s \hat{\sigma}_{e,t}^{L^s}$, $\hat{D}_t^{L^s}$ is an estimate of the mean lead-time demand and can be given as $\sum_{i=1}^{L^s} \hat{q}_{t+i}$, $\hat{\sigma}_{e,t}^{L^s}$ is an estimate of the standard deviation of the L^s period forecasting error in period t and can be given as $\sqrt{\text{Var}(D_t^{L^s} - \hat{D}_t^{L^s})}$, and z^s is the safety factor. To simplify the analysis, we set z^s to zero.[7] Then from Equation (A.1), we can achieve the following equation:

$$q_t^s = \hat{D}_t^{L^s} - \hat{D}_{t-1}^{L^s} + q_t. \tag{A.2}$$

We assume that the supplier also employs the ES forecasting technology to estimate the mean lead-time demand $\hat{D}_t^{L^s}$. Thus,

$$\hat{D}_t^{L^s} = \sum_{i=1}^{L^s} \hat{q}_{t+i} = L^s(\alpha q_t + (1 - \alpha)\hat{q}_t), \tag{A.3}$$

where $\hat{q}_{t+i} = \alpha q_t + (1 - \alpha)\hat{q}_t$, $i = 1, 2 \cdots$.

It can be seen from Equations (A.2) and (A.3) that $q_t^s \neq q_t$. Thus, the supplier does not produce the same q_t ordered by the retailer. $\square$

Appendix 2: Proof of Lemma 1

$$(\hat{\sigma}_{e,t}^{L,\text{LINEAR}})^2 = \text{Var}(D_t^L - \hat{D}_t^L) = \text{Var}(D_t^L) + \text{Var}(\hat{D}_t^L) - 2\text{Cov}(D_t^L, \hat{D}_t^L), \tag{B.1}$$

where

$$\text{Var}(D_t^L) = \text{Var}\left(\sum_{i=0}^{L-1} d_{t+i}\right) = \sum_{i=0}^{L-1} \text{Var}(d_{t+i}) + 2\sum_{i=0}^{L-2}\sum_{j=1}^{L-1-i} \text{Cov}(d_{t+i}, d_{t+i+j}) = L(\sigma_d^{\text{LINEAR}})^2 + 2(L-1)\text{Cov}(d_t, d_{t+1})$$
$$= L(\sigma_d^{\text{LINEAR}})^2 + 2(L-1)b^2 r(1-r)(\sigma_p^{\text{LINEAR}})^2, \tag{B.2}$$

$$\text{Var}(\hat{D}_t^L) = L^2\text{Var}(\hat{d}_t) = L^2\left(\frac{\alpha}{2-\alpha}\text{Var}(d_t) + \frac{2(1-\alpha)}{2-\alpha}\text{Cov}(d_{t-1}, \hat{d}_{t-1})\right)$$
$$= L^2\left(\frac{\alpha}{2-\alpha}\text{Var}(d_t) + \frac{2\alpha(1-\alpha)}{2-\alpha}\text{Cov}(d_{t-1}, d_{t-2})\right) = L^2\left(\frac{\alpha}{2-\alpha}(\sigma_d^{\text{LINEAR}})^2 + \frac{2\alpha(1-\alpha)}{2-\alpha}b^2 r(1-r)(\sigma_p^{\text{LINEAR}})^2\right), \tag{B.3}$$

$$\text{Cov}(D_t^L, \hat{D}_t^L) = \text{Cov}\left(\sum_{i=0}^{L-1} d_{t+i}, L\hat{d}_t\right) = L\sum_{i=0}^{L-1} \text{Cov}(d_{t+i}, \hat{d}_t) = L\alpha\text{Cov}(d_t, d_{t-1}) = L\alpha b^2 r(1-r)(\sigma_p^{\text{LINEAR}})^2, \tag{B.4}$$

and

$$\text{Cov}(d_t, d_{t+i}) = \text{Cov}((a - bp_t) + rb(p_t - p_{t-1}), (a - bp_{t+i}) + rb(p_{t+i} - p_{t+i-1})) = 0, \quad i \geq 2,$$

$$\text{Cov}(d_t, d_{t+1}) = \text{Cov}(d_{t-1}, d_{t-2}) = \text{Cov}(d_t, d_{t-1}) = \text{Cov}((a - bp_t) + rb(p_t - p_{t-1}), (a - bp_{t-1}) + rb(p_{t-1} - p_{t-2}))$$
$$= \text{Cov}(-b(1-r)p_t - rbp_{t-1}, -b(1-r)p_{t-1} - rbp_{t-2}) = b^2 r(1-r)\text{Var}(p_t),$$

$$\text{Var}(\hat{d}_t) = \frac{\alpha}{2-\alpha}\text{Var}(d_t) + \frac{2(1-\alpha)}{2-\alpha}\text{Cov}(d_{t-1}, \hat{d}_{t-1}).$$

Substituting Equations (B.2–B.4) into Equation (B.1), we see that σ_d^{LINEAR} and σ_p^{LINEAR} are time independent. Consequently, the variance of the forecasting error in the lead-time demand remains constant over time as well, and this completes the proof. $\qquad\square$

Appendix 3: Proof of Theorem 1

The variance of the order quantity can be derived from Equation (10):

$$
\begin{aligned}
\text{Var}(q_t) &= (1 + \alpha L)^2 \text{Var}(d_{t-1}) + \alpha^2 L^2 \text{Var}(\hat{d}_{t-1}) - 2\alpha L(1 + \alpha L)\text{Cov}(d_{t-1}, \hat{d}_{t-1}) \\
&= (1 + \alpha L)^2 \text{Var}(d_t) + \alpha^2 L^2 \text{Var}(\hat{d}_t) - 2\alpha L(1 + \alpha L)\text{Cov}(d_{t-1}, \hat{d}_{t-1}) \\
&= \left(1 + 2\alpha L + \frac{2\alpha^2 L^2}{2 - \alpha}\right)\text{Var}(d_t) - 2\alpha L\left(1 + \frac{\alpha L}{2 - \alpha}\right)\text{Cov}(d_{t-1}, \hat{d}_{t-1}) \\
&= \left(1 + 2\alpha L + \frac{2\alpha^2 L^2}{2 - \alpha}\right)\text{Var}(d_t) - 2\alpha^2 L\left(1 + \frac{\alpha L}{2 - \alpha}\right)\text{Cov}(d_{t-1}, d_{t-2}) \\
&= \left(1 + 2\alpha L + \frac{2\alpha^2 L^2}{2 - \alpha}\right)\text{Var}(d_t) - 2\alpha^2 L\left(1 + \frac{\alpha L}{2 - \alpha}\right)b^2 r(1 - r)\text{Var}(p_t),
\end{aligned} \tag{C.1}
$$

where $\text{Cov}(d_{t-1}, d_{t-2}) = b^2 r(1 - r)\text{Var}(p_t)$.

From Equation (2), we obtain the relations between the market demand variance, $\text{Var}(d_t)$, and the market price variance, $\text{Var}(p_t)$:

$$
\text{Var}(d_t) = \text{Var}((a - bp_t) + rb(p_t - p_{t-1})) = \text{Var}(-b(1 - r)p_t - rbp_{t-1}) = b^2(2r^2 - 2r + 1)\text{Var}(p_t). \tag{C.2}
$$

From Equation (C.1) and (C.2), we can prove the theorem. This completes the proof. $\qquad\square$

Appendix 4: Proof of Proposition 1

We take the 1st and the 2nd partial derivatives with respect to r on both sides of Equation (11):

$$
\frac{\partial(BE^{\text{LINEAR}})}{\partial r} = \frac{2\alpha^2 L(1 + \alpha L/(2 - \alpha))(2r - 1)}{(2r^2 - 2r + 1)^2}, \tag{D.1}
$$

$$
\frac{\partial^2(BE^{\text{LINEAR}})}{\partial r^2} = 4\alpha^2 L\left(1 + \frac{\alpha L}{2 - \alpha}\right)\frac{(-6r^2 + 6r - 1)}{(2r^2 - 2r + 1)^3}. \tag{D.2}
$$

Therefore, when $r \in [0, 0.5)$, $\partial(BE^{\text{LINEAR}})/\partial r < 0$, and BE^{LINEAR} is a decreasing function of r; when $r \in [0.5, 1)$, $\partial(BE^{\text{LINEAR}})/\partial r \geq 0$, and BE^{LINEAR} is an increasing function of r. Because $\frac{\partial(BE^{\text{LINEAR}})}{\partial r}\big|_{r=0.5} = 0$, $\frac{\partial^2(BE^{\text{LINEAR}})}{\partial r^2}\big|_{r=0.5} > 0$, BE^{LINEAR} has a unique extremum point (local minimum) when $r = 0.5$. Since $BE_{r=0.5}^{\text{LINEAR}} < BE_{r=0}^{\text{LINEAR}}$ and $BE_{r=0.5}^{\text{LINEAR}} < \lim_{r \to 1^-} BE^{\text{LINEAR}}$ when $r \in [0, 1)$, BE^{LINEAR} has a unique global minimum when $r = 0.5$. This completes the proof. $\qquad\square$

The following lemma will be useful in the proofs below.

Lemma 3. Let X be a random variable with a log-normal distribution with mean μ and variance σ^2. The *k-th* moment and second central moment of X are then $e^{k\mu + k^2\sigma^2/2}$ and $e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$, respectively.

Proof: $Y = \ln X \sim N(\mu, \sigma^2)$. Therefore, $X = e^Y$, and the *k-th* moment is given by:

$$
E(X^k) = E(e^{kY}) = \int_{-\infty}^{+\infty} e^{ky} \cdot 1/(\sigma\sqrt{2\pi}) \cdot e^{-(y-\mu)^2/(2\sigma^2)} dy
$$

$$
= (1/(\sigma\sqrt{2\pi}))e^{k\mu + k^2\sigma^2/2} \int_{-\infty}^{+\infty} e^{-(y-\mu-k\sigma^2)^2/(2\sigma^2)} dy = e^{k\mu + k^2\sigma^2/2}.
$$

We obtain the second central moment as follows:
$E(X - EX)^2 = EX^2 - (EX)^2 = DX = e^{2\mu + 2\sigma^2} - (e^{\mu + \sigma^2/2})^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$. This completes the proof. $\qquad\square$

Appendix 5: Proof of Lemma 2

$$
(\hat{\sigma}_{e,t}^{L,\text{ISO}})^2 = \text{Var}(D_t^L - \hat{D}_t^L) = \text{Var}(D_t^L) + \text{Var}(\hat{D}_t^L) - 2\text{Cov}(D_t^L, \hat{D}_t^L), \tag{E.1}
$$

where

$$\text{Var}(D_t^L) = L(\sigma_d^{\text{ISO}})^2 + 2(L-1)\text{Cov}(d_t, d_{t+1}) = L(\sigma_d^{\text{ISO}})^2 + 2(L-1)a^2 e^{-2b\mu + (r^2 - r + 1)b^2\sigma^2}(1 - e^{(r^2 - r)b^2\sigma^2}), \quad \text{(E.2)}$$

$$\text{Var}(\hat{D}_t^L) = L^2 \left(\frac{\alpha}{2-\alpha}\text{Var}(d_t) + \frac{2\alpha(1-\alpha)}{2-\alpha}\text{Cov}(d_{t-1}, d_{t-2}) \right)$$
$$= L^2 \left[\frac{\alpha}{2-\alpha}(\sigma_d^{\text{ISO}})^2 + \frac{2\alpha(1-\alpha)}{2-\alpha} \cdot a^2 e^{-2b\mu + (r^2 - r + 1)b^2\sigma^2}(1 - e^{(r^2 - r)b^2\sigma^2}) \right], \quad \text{(E.3)}$$

$$\text{Cov}(D_t^L, \hat{D}_t^L) = L\alpha \text{Cov}(d_t, d_{t-1}) = L\alpha \cdot a^2 e^{-2b\mu + (r^2 - r + 1)b^2\sigma^2}(1 - e^{(r^2 - r)b^2\sigma^2}), \quad \text{(E.4)}$$

and

$$\text{Cov}(d_t, d_{t+i}) = a^2 \text{Cov}(p_t^{-(1-r)b} \cdot p_{t-1}^{-rb}, p_{t+i}^{-(1-r)b} \cdot p_{t+i-1}^{-rb}) = 0, \quad i \geq 2, \quad \text{(E.5)}$$

$$\text{Cov}(d_t, d_{t+1}) = \text{Cov}(d_{t-1}, d_{t-2}) = Cov(d_t, d_{t-1}) = a^2 \text{Cov}(p_t^{-(1-r)b} \cdot p_{t-1}^{-rb}, p_{t-1}^{-(1-r)b} \cdot p_{t-2}^{-rb})$$
$$= a^2 [E(p_t^{-(1-r)b} p_{t-1}^{-b} p_{t-2}^{-rb}) - E(p_t^{-(1-r)b} p_{t-1}^{-rb}) E(p_{t-1}^{-(1-r)b} p_{t-2}^{-rb})]$$
$$= a^2 E(p_t^{-(1-r)b}) E(p_t^{-rb}) [E(p_t^{-b}) - E(p_t^{-(1-r)b}) E(p_t^{-rb})]. \quad \text{(E.6)}$$

According to Lemma 3, under general conditions, $E(p_t^k) = e^{k\mu + k^2\sigma^2/2}$, and Equation (E.6) can be simplified to

$$\text{Cov}(d_t, d_{t+1}) = \text{Cov}(d_{t-1}, d_{t-2}) = \text{Cov}(d_t, d_{t-1}) = a^2 e^{-2b\mu + (r^2 - r + 1)b^2\sigma^2}(1 - e^{(r^2 - r)b^2\sigma^2}). \quad \text{(E.7)}$$

By substituting Equations (E.2–E.4) into Equation (E.1). This completes the proof. □

Appendix 6: Proof of Theorem 2

Similar to the expression for the variance of the order quantity under linear demand, the variance of the order quantity under iso-elastic demand can be derived from Equation (13):

$$\text{Var}(q_t) = \left(1 + 2\alpha L + \frac{2\alpha^2 L^2}{2-\alpha} \right) \text{Var}(d_t) - 2\alpha^2 L \left(1 + \frac{\alpha L}{2-\alpha} \right) \text{Cov}(d_{t-1}, d_{t-2})$$
$$= \left(1 + 2\alpha L + \frac{2\alpha^2 L^2}{2-\alpha} \right) \text{Var}(d_t) - 2\alpha^2 L \left(1 + \frac{\alpha L}{2-\alpha} \right) \cdot a^2 e^{-2b\mu + (r^2 - r + 1)b^2\sigma^2}(1 - e^{(r^2 - r)b^2\sigma^2}). \quad \text{(F.1)}$$

From the equation $d_t = ap^{-b} \cdot (p_t/p_{t-1})^{rb} = ap_t^{-(1-r)b} p_{t-1}^{-rb}$ and Lemma 3, we obtain:

$$\text{Var}(d_t) = a^2 \text{Var}(p_t^{-(1-r)b} \cdot p_{t-1}^{-rb}) = a^2 \{ E(p_t^{-(1-r)b} p_{t-1}^{-rb})^2 - [E(p_t^{-(1-r)b} p_{t-1}^{-rb})]^2 \}$$
$$= a^2 \{ E(p_t^{-2(1-r)b}) E(p_t^{-2rb}) - [E(p_t^{-(1-r)b})]^2 [E(p_t^{-rb})]^2 \} = a^2 e^{-2b\mu + (2r^2 - 2r + 1)b^2\sigma^2}(e^{(2r^2 - 2r + 1)b^2\sigma^2} - 1). \quad \text{(F.2)}$$

From Equation (F.1) and (F.2), we can prove the theorem. This completes the proof. □

Appendix 7: Proof of Theorem 3

From Equations (11) and (14), we obtain:

$$\Delta BE = BE^{\text{LINEAR}} - BE^{\text{ISO}} = 2\alpha^2 L \left(1 + \frac{\alpha L}{2-\alpha} \right) \left(\frac{e^{(-r^2 + r)b^2\sigma^2} - 1}{e^{(2r^2 - 2r + 1)b^2\sigma^2} - 1} - \frac{r(1-r)}{2r^2 - 2r + 1} \right). \quad \text{(G.1)}$$

To prove Theorem 3, we need to show that $\Delta BE \leq 0$. As $0 < \alpha < 1$ and $L > 0$, we must prove that:

$$\frac{e^{(-r^2 + r)b^2\sigma^2} - 1}{e^{(2r^2 - 2r + 1)b^2\sigma^2} - 1} \leq \frac{-r^2 + r}{2r^2 - 2r + 1}. \quad \text{(G.2)}$$

We set $m = (-r^2 + r)b^2\sigma^2$ and $n = (2r^2 - 2r + 1)b^2\sigma^2$. Because $m_{\max} = b^2\sigma^2/4 \leq n_{\min} = b^2\sigma^2/2$ for $r \in [0, 1)$, $0 \leq m \leq n$. Equation (G.2) can be transformed to:

$$\frac{e^m - 1}{e^n - 1} \leq \frac{(-r^2 + r)b^2\sigma^2}{(2r^2 - 2r + 1)b^2\sigma^2} = \frac{m}{n}, \quad 0 \leq m \leq n. \tag{G.3}$$

As $e^x - 1 = x + x^2/2! + \cdots + x^n/n! + \cdots$, $\forall x$, Equation (G.3) can be transformed to:

$$\frac{e^m - 1}{e^n - 1} = \frac{m + m^2/2! + \cdots}{n + n^2/2! + \cdots} \leq \frac{m}{n}, \quad 0 \leq m \leq n. \tag{G.4}$$

When $0 \leq m \leq n$, it is straightforward to show that $n(m + m^2/2! + \cdots) = mn + m^2n/2! + \cdots \leq mn + mn^2/2! + \cdots = m(n + n^2/2! + \cdots)$, i.e. $n(m + m^2/2! + \cdots) \leq m(n + n^2/2! + \cdots)$. This proves Equation (G.4). □

Appendix 8: Proof of Theorem 4

The market demand faced by retailer i is:

$$d_t^i = (a - bp_t^i) + rb(p_t^i - p_{t-1}^i). \tag{H.1}$$

From Equation (18), we can obtain the total order variance:

$$\mathrm{Var}(Q) = (1 + \alpha L)^2 \mathrm{Var}\left(\sum_{i=1}^{n} d_{t-1}^i\right) + \alpha^2 L^2 \mathrm{Var}\left(\sum_{i=1}^{n} \hat{d}_{t-1}^i\right) - 2\alpha L(1 + \alpha L)\mathrm{Cov}\left(\sum_{i=1}^{n} d_{t-1}^i, \sum_{i=1}^{n} \hat{d}_{t-1}^i\right)$$

$$= (1 + \alpha L)^2 \mathrm{Var}\left(\sum_{i=1}^{n} d_t^i\right) + \alpha^2 L^2 \mathrm{Var}\left(\sum_{i=1}^{n} \hat{d}_t^i\right) - 2\alpha L(1 + \alpha L) \sum_{i=1}^{n} \mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i), \tag{H.2}$$

where $\mathrm{Cov}\left(\sum_{i=1}^{n} d_{t-1}^i, \sum_{i=1}^{n} \hat{d}_{t-1}^i\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{Cov}\left(d_{t-1}^i, \hat{d}_{t-1}^j\right) = \sum_{i=1}^{n} \mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i)$.

Additionally,

$$\mathrm{Var}\left(\sum_{i=1}^{n} \hat{d}_t^i\right) = \sum_{i=1}^{n} \mathrm{Var}(\hat{d}_t^i) + 2\sum_{i=1}^{n-1}\sum_{j=1}^{n-i} \mathrm{Cov}(\hat{d}_t^i, \hat{d}_t^{i+j}) = \sum_{i=1}^{n} \mathrm{Var}(\hat{d}_t^i)$$

$$= \sum_{i=1}^{n} [(\alpha/(2-\alpha))\mathrm{Var}(d_t^i) + (2(1-\alpha)/(2-\alpha))\mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i)]$$

$$= (\alpha/(2-\alpha))\mathrm{Var}\left(\sum_{i=1}^{n} d_t^i\right) + (2(1-\alpha)/(2-\alpha)) \sum_{i=1}^{n} \mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i), \tag{H.3}$$

where $\mathrm{Cov}(\hat{d}_t^i, \hat{d}_t^{i+j}) = 0$, $\mathrm{Var}(\hat{d}_t^i) = (\alpha/(2-\alpha))\mathrm{Var}(d_t^i) + (2(1-\alpha)/(2-\alpha))\mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i)$, $\sum_{i=1}^{n} \mathrm{Var}(d_t^i) = \mathrm{Var}(\sum_{i=1}^{n} d_t^i)$.

Substituting Equation (H.3) into (H.2), we obtain:

$$\mathrm{Var}(Q) = (1 + 2\alpha L + 2\alpha^2 L^2/(2-\alpha))\mathrm{Var}\left(\sum_{i=1}^{n} d_t^i\right) - 2\alpha L(1 + \alpha L/(2-\alpha)) \sum_{i=1}^{n} \mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i). \tag{H.4}$$

Additionally,

$$\sum_{i=1}^{n} \mathrm{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i) = \alpha \sum_{i=1}^{n} \mathrm{Cov}(d_{t-1}^i, d_{t-2}^i) = \alpha b^2 r(1-r) \sum_{i=1}^{n} \mathrm{Var}(p_t^i) = (\alpha r(1-r)/(2r^2 - 2r + 1)) \sum_{i=1}^{n} \mathrm{Var}(d_t^i)$$

$$= (\alpha r(1-r)/(2r^2 - 2r + 1))\mathrm{Var}\left(\sum_{i=1}^{n} d_t^i\right), \tag{H.5}$$

where $\mathrm{Cov}(d_{t-1}^i, d_{t-2}^i) = b^2 r(1-r)\mathrm{Var}(p_t^i)$, $\mathrm{Var}(p_t^i) = \mathrm{Var}(d_t^i)/(b^2(2r^2 - 2r + 1))$ (see Equation (C.2)), $\sum_{i=1}^{n} \mathrm{Var}(d_t^i) = \mathrm{Var}(\sum_{i=1}^{n} d_t^i)$.

Substituting Equation (H.5) into Equation (H.4) and dividing by $\mathrm{Var}(\sum_{i=1}^{n} d_t^i)$ on both sides of Equation (H.4), we obtain the expression in Theorem 4. This completes the proof. □

Appendix 9: Proof of Theorem 5

The market demand faced by retailer i is:

$$d_t^i = a(p_t^i)^{-b} \cdot (p_t^i/p_{t-1}^i)^{rb}. \tag{I.1}$$

Similarly, the variance of the total order quantity, Q, in the iso-elastic case is:

$$\text{Var}(Q) = (1 + 2\alpha L + 2\alpha^2 L^2/(2-\alpha))\text{Var}\left(\sum_{i=1}^{n} d_t^i\right) - 2\alpha L(1 + \alpha L/(2-\alpha))\sum_{i=1}^{n}\text{Cov}(d_{t-1}^i, \hat{d}_{t-1}^i). \tag{I.2}$$

Additionally,

$$\sum_{i=1}^{n}\text{Cov}\left(d_{t-1}^i, \hat{d}_{t-1}^i\right) = \alpha\sum_{i=1}^{n}\text{Cov}\left(d_{t-1}^i, d_{t-2}^i\right) = a^2\alpha\sum_{i=1}^{n}\left[e^{-2b\mu_i + \left(r^2 - r + 1\right)b^2\sigma_i^2}\left(1 - e^{(r^2 - r)b^2\sigma_i^2}\right)\right], \tag{I.3}$$

$$\text{Var}(\sum_{i=1}^{n} d_t^i) = \sum_{i=1}^{n}\text{Var}(d_t^i) = a^2\sum_{i=1}^{n}[e^{-2b\mu_i + (2r^2 - 2r + 1)b^2\sigma_i^2}(e^{(2r^2 - 2r + 1)b^2\sigma_i^2} - 1)], \tag{I.4}$$

where $\text{Cov}(d_{t-1}^i, d_{t-2}^i) = a^2 e^{-2b\mu_i + (r^2 - r + 1)b^2\sigma_i^2}(1 - e^{(r^2 - r)b^2\sigma_i^2})$ (see Equation (E.7)) and $\text{Var}(d_t^i) = a^2 e^{-2b\mu_i + (2r^2 - 2r + 1)b^2\sigma_i^2}(e^{(2r^2 - 2r + 1)b^2\sigma_i^2} - 1)$ (see Equation (F.2)).

Substituting Equations (I.3) and (I.4) into Equation (I.2) and dividing by $\text{Var}(\sum_{i=1}^{n} d_t^i)$ on both sides of Equation (I.2), we can prove the theorem. This completes the proof. □