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DETERMINATION OF ROCK PARAMETERS USED FOR CALIBRATION OF CONSTITUTIVE MODEL

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ABSTRACT

Mathematical modeling is an important part of designing underground and other structures. However, the accuracy of the obtained results is strongly dependent on the input parameters quality of the selected material (constitutive) model. An insight into this issue is outlined in this article. Special emphasis was placed on various possibilities of laboratory determination of the necessary parameters for calibration of the Mohr-Coulomb and Hoek-Brown constitutive models. Described procedure was also applied to a selected rock type – Čeřínek-type granite of the Moldanubian pluton, a candidate rock in which a deep geological repository for used nuclear fuel and high-level radioactive nuclear waste might be built in a future.

Keywords: Constitutive model; Granite; Hoek-Brown; Mathematical modelling; Mohr-Coulomb.

1 INTRODUCTION

The selection of a suitable material model strongly depends on the specific problem being solved and the availability of quality rock samples. For preliminary analyses, possibly with a lower number of rock samples, rather simpler models are used, for example linear elastic – perfectly plastic, such as the Mohr-Coulomb model. In the case of a more detailed analysis, the simpler model does not sufficiently describe the real behaviour of the rock, and therefore it is necessary to use more sophisticated models, such as the Hoek-Brown model.

The Mohr-Coulomb model (MC) belongs to the basic constitutive models. It describes rock behaviour by using a linear elastic–perfectly plastic work diagram. Five parameters form this model basis: Young's modulus of elasticity E and Poisson’s ratio ν, defining elastic deformations, then cohesion c and angle of internal friction φ, defining the strength of the rock, and also the angle of dilatancy ψ, defining plasticity. Alternatively, the model can also be restricted by tensile strength (Tension cut-off). [1] For all material models, also bulk density ρ is an important parameter for determining the initial stress of the rock due to its own weight.

Compared to the Mohr-Coulomb model, the Hoek-Brown (HB) model defines the rock shear strength as a non-linear function of the stress in the rock, which also directly includes the tensile strength. However, the necessary input parameters of this model are more complex. Young’s modulus of elasticity E_{rm}, Poisson’s ratio ν and angle of dilatancy ψ are the same as for the MC model. The difference is in determination of the rock strength, which is directly given only by the uniaxial compressive strength of the intact rock $|\sigma_{ci}|$ (at GSI = 100 and D = 0 – see further in the text). In addition we need to specify three parameters used for definition of strength curve from $|\sigma_{ci}|$. The first is the empirical parameter m_i used for tensile strength calculation, which can be found either in the literature or determined experimentally. The second one is the Geological Strength Index – GSI, which characterizes the quality and structure of the rock mass. The last one is the disruption factor D, describing the influence of used rock mining technology on its quality. [2]

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2 LABORATORY DETERMINATION OF PARAMETERS FOR MATERIAL MODELS

First, suitable rock samples must be taken, either in the form of rock blocks or cores. Subsequently, test samples are made from them. These are most often cylindrical, but they can also be blocks or cubes. Specimens are defined by the diameter of the body *D* (or sides *A* and *B* for square and rectangular bodies) and by the length of the body *L*. We can also calculate length-to-diameter ratio *L/D*, which defined suitability of specimen for different laboratory tests. After weighing the body, we can simply calculate the bulk density ρ, which is also a basic input for material models.

The basic equipment of the rock mechanics laboratory is a hydraulic press equipped with accurate strain measurement devises and the possibility of applying uniaxial or triaxial stress to the sample. Three destructive tests are important for comprehensive rock behaviour description: compression test, tensile test and triaxial test.

The uniaxial compressive strength σ_c (*UCS*) is the main output of the **uniaxial compressive test** (UCT). The specimen, with a length-to-diameter ratio between 1.0 and 3.0 (depending on the methodology used – see [3], [4] or [5]), can also be fitted with resistance strain gauges to determine the static modulus of elasticity *E* and Poisson's ratio ν.

The tensile strength of rock σ_t can be determined in different ways. The easiest way is to use correlations for determination of the tensile strength based on the uniaxial compressive strength of the rock. Direct measurement of tensile strength is relatively demanding on laboratory equipment, and therefore it is more often replaced by the **Brazilian indirect tensile test (**BITT, Figure 1). Compared to a direct tensile test, it is very easy to perform. In its simplest form, a sample with a length-to-diameter ratio of *L/D* = 0.5 is place in the press on the body shell. There are also additional devices used for better control of the application of stress to the sample. Although the strength obtained this way does not perfectly match to the direct tensile strength, it is sufficient for most applications. As a last option for the tensile strength determination, a **modified tensile test** (MTT) can be used, which is based on the use of a larger diameter cylindrical body specifically modified (Figure 2). The body is then inserted into the press on the supports and loaded, while tensile failure occurs on the inner ring.

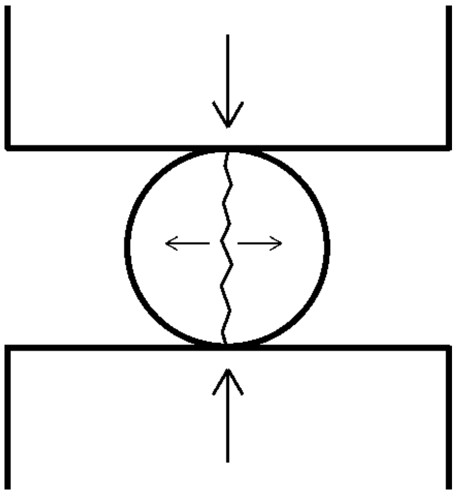

Figure 1. *Sample failure scheme in the simplest form of the Brazilian test [6]*

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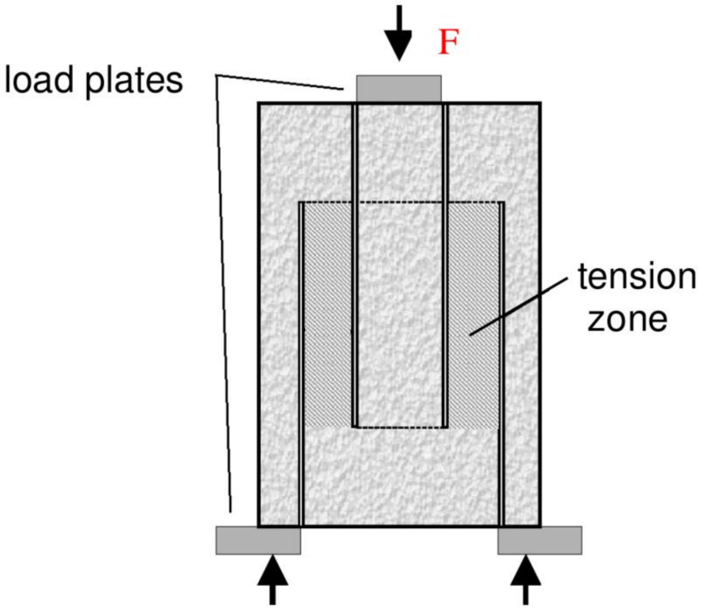

Figure 2. *Modified tensile test scheme (MTT), adapted from [7]*

The compression triaxial test (CTT) also takes into account the rock behaviour under triaxial loading compared to the uniaxial compression test. There are two options for this test, the so-called “true triaxial”, where the stresses in all three directions are different ($\sigma_1 \neq \sigma_2 \neq \sigma_3$) and a "false triaxial", where the horizontal stresses are the same [$\sigma_1 \neq (\sigma_2 = \sigma_3)$]. Performing a true triaxial test requires the use of cubes or prisms and complex testing equipment. For practical uses, a false triaxial test is more often carried out, usually in a massive steel chamber referred to as a *Hoek cell* (Figure 3). To determine the cohesion c and the angle of internal friction φ, needed in the material models, at least three CTT tests for different chamber pressures must be carried out.

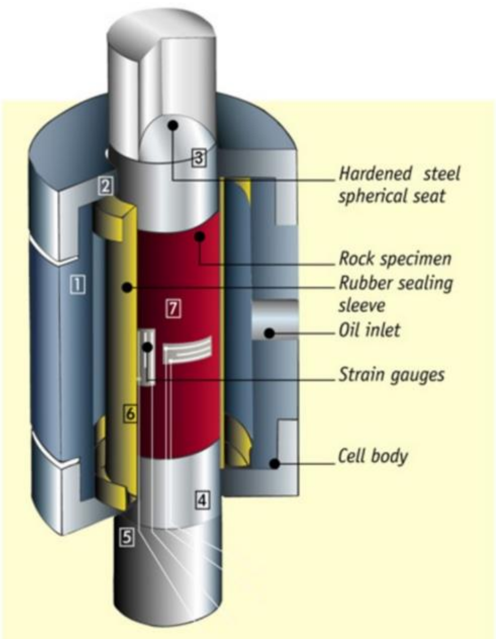

Figure 3. *Hoek´s cell scheme, adapted from [8]*

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3 IMPLEMENTATION ON A SPECIFIC TYPE OF ROCK

Medium to coarse-grained two-mica Čeřínek-type granite of the Moldanubian pluton was chosen for our study. Granite was taken at Čertův Hrádek locality near Jihlava [6]. This site is located in the central part of defined polygon for a potential site for a deep geological repository for used nuclear fuel and high-level radioactive nuclear waste [9]. Granite of the Čerínek type also has naturally increased radioactivity. The examined rock microstructure was evaluated by studying thin sections using a polarizing microscope (Figure 4).

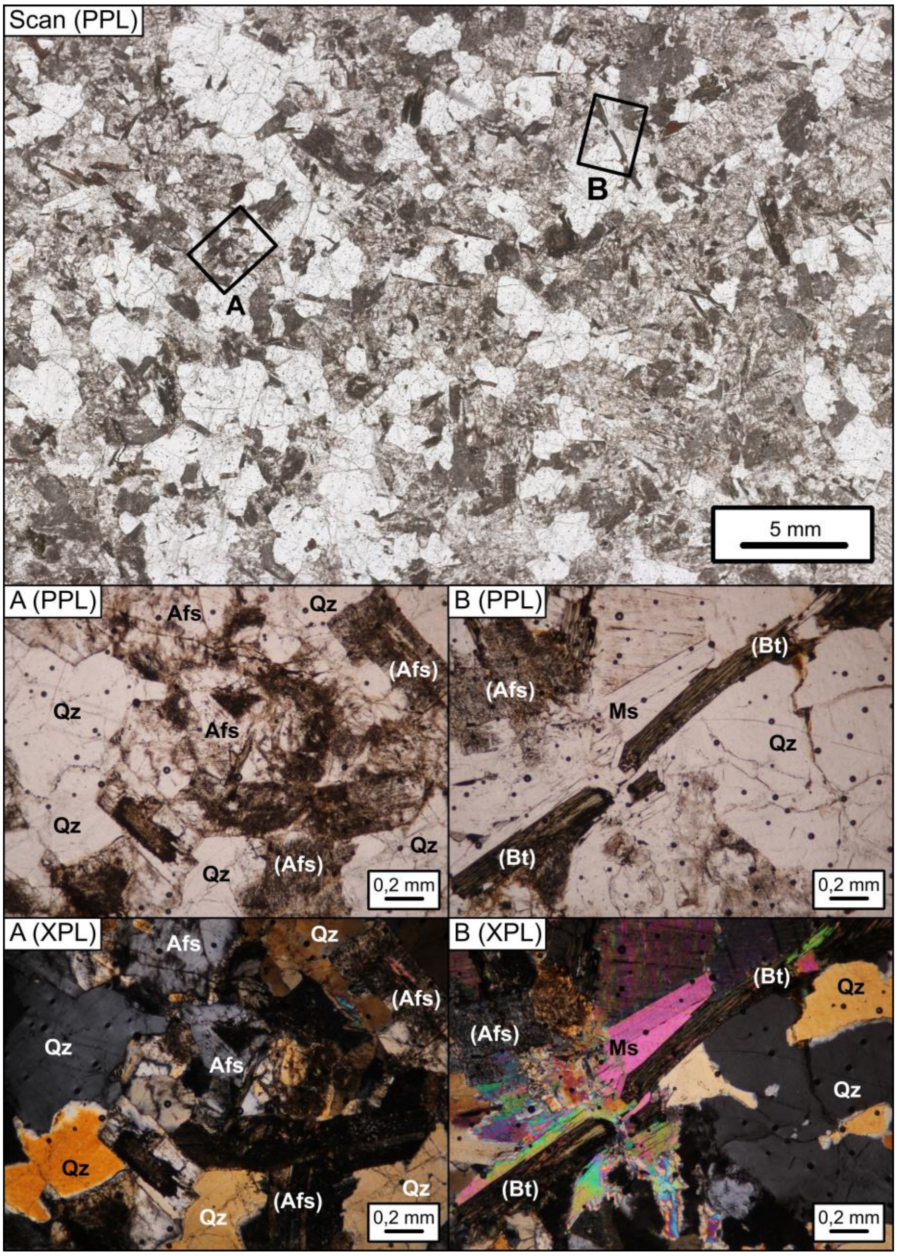

Figure 4: *Thin section images of Čeřínek-type granite. Part A shows different stages of sericitization of alkali feldspar, part B documents two-mica granite characteristic with conspicuous chloritization of biotite along cracks. Abbreviations: PPL – plane polarized light, XPL – crossed polarized light, Qz – quartz, Afs – alkali feldspar, (Afs) – altered alkali feldspar, (Bt) – altered biotite, Ms – muscovite*

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The individual minerals of Čeřínek-type granite are affected by variable degrees of alteration, which can have a negative effect on the strength characteristics. [6]

Cores samples were obtained from the granite blocks using a wet core drilling. Drill cores were then cut to the required length on a circular saw with a diamond cutting tool. A total of nine cylindrical specimens were produced (Figure 5). The sample length-to-diameter ratio of 2.0 was used for pressure and triaxial testing and 0.5 for Brazilian testing, respectively.

Figure 5. *A set of tested samples [6]*

A total of three tests of each type were carried out. During simple compression tests, the bodies were fitted with two vertical and one horizontal resistance strain gauges and loaded according to ČSN EN 14580 [10] to determine the deformation characteristics. A loading rate of 200 N/s was used for uniaxial and triaxial tests, a rate of 50 N/s for the Brazilian tests.

Triaxial tests were carried out in a Hoek's cell in the load-controlled mode, with confining pressures of 5, 10 and 15 MPa.

Average values of particular characteristics were determined from the measured values (Table 1). Cohesion *c* and internal friction angle *φ* were obtained by determining the Mohr-Coulomb failure criterion by displaying the obtained strength characteristics in the form of Mohr's circles.

Table 1. *Summary of laboratory test results*

characteristic	symbol	unit	diameter	deviation	measurement
bulk density	ρ	[kg/m^3]	2585	15	9
uniaxial compressive strength	σ_c *(UCS)*	[MPa]	92.7	10.3	3
static modulus of elasticity	E	[GPa]	33.93	0.47	3
Poisson's ratio	v	[-]	0.19	0.00	2
Brazilian indirect tensile strength	σ_t *(BITS)*	[MPa]	6.2	2.6	3
cohesion	c	[MPa]	14.5	-	-
angle of internal friction	φ	[o]	53.5	-	-

4 DETERMINED PARAMETERS OF CONSTITUTIVE MODELS

The parameters of the Mohr-Coulomb model were determined directly by evaluating laboratory tests. For the Hoek-Brown model, two variants are presented. The first variant of Hoek-Brown model was calibrated by altering the model strength parameters (mainly $|\sigma_{ci}|$ and m_i, *GSI* was determined according to the rock character in its natural deposit and *D* is equal to zero, which corresponds to minimal rock disturbance), until the best correspondence between the HB failure envelope and Mohr's circles of laboratory test results was established. For the second variant, $|\sigma_{ci}|$ equals the laboratory determined *UCS* value and m_i was obtained by dividing BITS and UCS. GSI value of 100 was used to gain failure envelope independent of the rock mass discontinuity system, more suited for the intact laboratory samples. Table 2 summarizes the set parameters.

Table 2. *Determined parameters of the constitutive models*

Mohr-Coulomb model		**Hoek-Brown model – calibrated on Mohr's circles (left) and directly calculated from laboratory test results (right)**		
$\gamma_{sat} = \gamma_{unsat}$ [kN/m^3]	25,85	$\gamma_{sat} = \gamma_{unsat}$ [kN/m^3]	25.85	25.85
E [GPa]	33,93	E_{rm} [GPa]	33.93	33.93
v [-]	0,19	v [-]	0.19	0.19
c_{ref} [MPa]	14,5	$\lvert\sigma_{ci}\rvert$ [MPa]	480	92.7
φ [°]	53,5	m_i [-]	20	14.95
ψ [°]	0	GSI [-]	70	100
σ_t [MPa]	6,2	D [-]	0	0
-	-	ψ_{max} [°]	0	0

In Figure 6, the failure envelopes of both selected models are compared with the results of laboratory tests displayed through Mohr's circles. The Mohr-Coulomb model well matches the whole set of circles. Compared to the Mohr-Coulomb model, the first variant of Hoek-Brown model shows a significantly lower tensile strength of the rock, which is a result of the calibration. It corresponds well to Mohr's circles of uniaxial and triaxial compressive tests. The second HB model variant matches the correct tensile strength value, however, it neglects

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the strengthening effect of confining stress simulated by triaxial tests, which is necessary for the correct deep underground rock behaviour simulation).

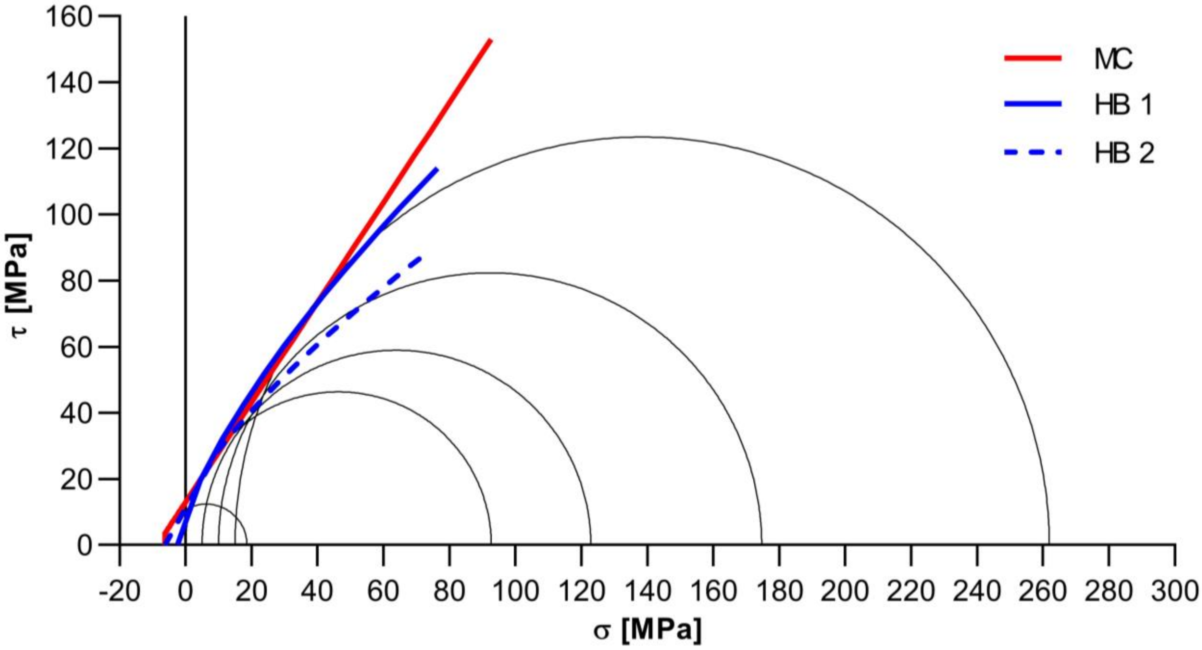

Figure 6. *Failure criterion of both models (MC = Mohr-Coulomb, HB = Hoek-Brown) in σ-τ and their comparison with laboratory test results in the form of Mohr's circles*

5 CONCLUSIONS

Within the article, the procedure required for determining the material model characteristics was summarized in a simplified form. This is a very important step for designing a structure using the finite element method, which can significantly affect the accuracy of the final mathematical model.

When comparing both constitutive models included in this article, certain differences are evident. The Mohr-Coulomb model simplifies the rock non-linear behaviour with a linear failure envelope, which for the tested granite corresponds well with the presented laboratory results. The Hoek-Brown model calibration process is more complex. Correct determination of the model parameters may require a calibration as demonstrated by the first variant of the model. This can result in lower tensile strength value of the constitutive model and unrealistic values of some parameters to match the laboratory test results. However, if the parameters of the model are directly calculated from laboratory test results, the failure envelope may not reflect the rock strengthening well, as demonstrated by the second variant.

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