mathematics

Article

A Comparative Study of Fuzzy Domination and Fuzzy Coloring in an Optimal Approach

Annamalai Meenakshi [1], Adhimoolam Kannan [1,2], Miroslav Mahdal [3], Krishnasamy Karthik [4,*] and Radek Guras [3]

[1] Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai 600062, India; drameenakshi@veltech.edu.in (A.M.); akannan@veltechmultitech.org (A.K.)
[2] Department of Mathematics, Vel Tech Multi Tech Dr. Rangarajan Dr. Sakunthala Engineering College, Chennai 600062, India
[3] Department of Control Systems and Instrumentation, Faculty of Mechanical Engineering, VSB-Technical University of Ostrava, 17. Listopadu 2172/15, 70800 Ostrava, Czech Republic; miroslav.mahdal@vsb.cz (M.M.); radek.guras@vsb.cz (R.G.)
[4] Department of Mechanical Engineering, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai 600062, India
* Correspondence: karthikk@veltech.edu.in

Abstract: An optimal network refers to a computer or communication network designed, configured, and managed to maximize efficiency, performance, and effectiveness while minimizing cost and resource utilization. In a network design and management context, optimal typically implies achieving the best possible outcomes between various factors. This research investigated the use of fuzzy graph edge coloring for various fuzzy graph operations, and it focused on the efficacy and efficiency of the fuzzy network product using the minimal spanning tree and the chromatic index of the fuzzy network product. As a network made of nodes and vertices, measurement with vertices is a parameter for domination, and edge measurement is a parameter for edge coloring, so we used these two parameters in the algorithm. This paper aims to identify an optimal network that can be established using product outcomes. This study shows a way to find an optimal fuzzy network based on comparative optimal parameter domination and edge coloring, which can be elaborated with applications. An algorithm was generated using an optimal approach, which was subsequently implemented in the form of applications.

Keywords: fuzzy coloring; minimum spanning tree; domination number; optimal network

MSC: 05C15; 05C76

check for updates

Citation: Meenakshi, A.; Kannan, A.; Mahdal, M.; Karthik, K.; Guras, R. A Comparative Study of Fuzzy Domination and Fuzzy Coloring in an Optimal Approach. *Mathematics* **2023**, *11*, 4019. https://doi.org/10.3390/math11184019

Academic Editor: Konstantin Kozlov

Received: 24 July 2023
Revised: 10 September 2023
Accepted: 11 September 2023
Published: 21 September 2023

1. Introduction

A mathematical tool known as graph theory plays a vital role in numerous branches of research and technology. A graph typically depicts a real and relevant problem graphically. A graph is a collection of sets (K, L), where K is a collection of non-empty vertices, and L is an edge set. Kaufman (1973) presented the concept of fuzzy graphs, and further, Rosenfeld (1975) interpreted it. Samanta and Pal (2015, 2013) defined various forms of fuzzy graphs. In the literature, there are many ways to color graphs. Fuzzy set theory and fuzzy graph theory have made it possible to model most real-world situations more precisely and adaptably than their classical counterparts [1–7]. More study is being conducted on fuzzy graphs [8–10]. The usual graph model of a network has a collection of nodes joined by edges or connections. The network provides a flexible framework for locating and observing complex systems [11]. The idea of studying complex networks is essential and crosses many academic fields. Real-world problems contain a variety of data that can be represented using a variety of graph types, including fuzzy graphs, intuitionistic fuzzy graphs, and neutrosophic graphs [12–17]. Arif introduced the concepts of the soft overset and the soft over graph [18]. Samanta and others provided an explanation for several recently created ideas about intuitionistic fuzzy graphs (IFGs), as well as a few key concepts that had already been established [19]. Using the concepts of intuitionistic fuzzy sets, intuitionistic fuzzy relations, and index matrices as a foundation, a new generalization of IFGs has been presented [20]. Many authors have studied various types of dominations [21–28] and developed this field of study. The crisp-graph coloring technique has been used to color the α-cuts of these fuzzy graphs. As a result, many crisp graphs are colored for different values of α, and for identical fuzzy graphs, the chromatic index changes depending on the value of α [29]. Additionally, Bershtein and Bozhenuk suggested using the minimax criterion to define the ideal center allocation in fuzzy transportation networks [30]. One study defined the concept of a fuzzy graph and determined the minimum number of colors based on the value of the separation degree [31–38]. A new coloring technique was employed to color a political map and to address a brand-new traffic light coloring issue [39]. A colored vertex on fuzzy graphs was used to color maps. To overcome radio frequency issues, Mahapatra et al. extended the coloring approach to radio fuzzy graphs [40–42]. The relationship (edges) can be more meaningful than the individual (nodes) at times. For instance, links rather than nodes are crucial in fuzzy social networks. An associated concept known as edge coloring is crucial for issues based on uncertainty. Regarding the application of graph coloring to communication systems based on utilizing ring-splits, in addition to comparing the peak throughput of NoCs for circulant and mesh topologies using deadlock-free routing algorithms, the results of high-level modeling were presented in [43]. The suggested method used fewer hardware resources while still achieving minimal transmission delay and enhanced thermal efficiency [44].

The motivation of this research work is to create a new platform to identify an optimal network in order to develop an effective optical network by utilizing various fuzzy graph operations based on edge color and domination parameters, including the operations of residue products, symmetric differences, max products, and lexicographic products. Using comparative studies on domination and edge coloring, our research and analysis aimed to assess the network's strength. We have provided an algorithm to examine the effectiveness and efficiency of the constructed networks, which is the main framework of this research finding. Furthermore, we have developed applications for the product operation of these fuzzy networks.

2. Preliminaries

Definition 1 ([12]). *A Fuzzy graph ($H^*{}_{FG} = (V^*_{FG}, E^*_{FG})$) is a pair of functions ($\sigma_{V^*_{FG}} : V^*_{FG} \to [0,1]$ and $\mu_{V^*_{FG}} : V^*_{FG} \times V^*_{FG} \to [0,1]$) where $\mu_{V^*_{FG}}(b_1, b_2) \leq \min\{\sigma_{V^*_{FG}}(b_1{}^*), \sigma_{V^*_{FG}}(b_2{}^*)\}$ for $b_1{}^*, b_2{}^* \in V*_{FG}$.*

Definition 2 ([12]). *The underlying graph of a fuzzy graph is in the form ($H^*{}_{FG} = (V^*_{FG}, E^*_{FG})$), where $V^*_{FG} = \left\{a_1{}^* \in V^*_{FG} : \sigma_{V^*_{FG}}(a_1{}^*) > 0\right\}$ and $E^*_{FG} = \{(a_1{}^*, a_2{}^*) \in V^*_{FG} \times V^*_{FG} : \mu_{V^*_{FG}}$ $(a_1{}^*, a_2*) > 0\}$.*

Definition 3 ([12]). *A subset (T^*_F) of V^*_{FG} is said to be a dominating set of a fuzzy graph if every vertex in $V^*_{FG} - T^*_{FG}$ is dominated by at least one vertex of V^*_{FG}. The dominating set (T^*_{FG}) is said to be minimal if no proper subset of T^*_{FG} is a dominating set.*

Definition 4 ([12]). *An arc ($a_1 * -a_2*$) is said to be strong if the value of degree of an edge membership of an arc ($a_1 * -a_2*$) is equal to strength of connectedness between $a_1{}^*$ and $a_2{}^*$.*

Definition 5 ([12]). *A vertex $a_1{}^*$ dominates $a_2{}^*$ if there is a strong arc between them.*

Definition 6 ([34]). *A fuzzy graph* $\eta_{ec} = (V_{ec}, \sigma_{ec}, \mu_{ec})$ *is a set that is not empty, together with a pair of functions* $\sigma_{ec} : V_{ec} \to [0, 1]$ *and* $\mu_{ec} : V_{ec} \times V_{ec} \to [0, 1]$, *such that* $x, y \epsilon V_{ec}$, $\mu_{ec}(c, d) \leq \sigma_{ec}(c) \Lambda \sigma_{ec}(d)$, *where* $\sigma_{ec}(c)$ *and* $\mu_{ec}(c, d)$ *represent the vetex membership values and the edge membership values, respectively.*

Definition 7 [35]). *A fuzzy graph* $\eta_{ec} = (V_{ec}, \sigma_{ec}, \mu_{ec})$ *is complete if* $\mu_{ec}(p, q) = min\{\sigma_{ec}(p), \sigma_{ec}(q)\}$ *for all p, q* $\in$ *Vec, where (p, q) represents the edges between the vertices p and q.*

Definition 8 ([35]). *A fuzzy graph* $\eta_{ec} = (V_{ec}, \sigma_{ec}, \mu_{ec})$ *is said to be bipartite if vertex set* V_{ec} *is divided into two nonempty sets* V_{ec_1} *and* V_{ec_2}, *such that* $\mu_{ec}(V_{ec_1}, V_{ec_2}) = 0$ *if* $v_{ec_1}, v_{ec_2} \epsilon V_{ec_1}$ *or* $v_{ec_1}, v_{ec_2} \epsilon V_{ec_2}$. *Further, if* $\mu_{ec}(V_{ec_1}, V_{ec_2}) = min\{\sigma_{ec}(v_{ec_1}), \sigma_{ec}(v_{ec_2})\}$ *for all* $v_{ec_1} \epsilon V_{ec_1}$ *and* $v_{ec_2} \epsilon V_{ec_2}$, *then* η_{ec} *is called a fuzzy complete bipartite graph.*

Definition 9 ([29]). *Let H = {*$h_1, h_2, \ldots, h_\lambda$*},* $\lambda \geq 1$ *be the collection of neutral hues. Then, fuzzy set (H, k), where k: H* $\to$ *(0, 1), is known as a collection of fuzzy colors, and 0 <* $k(h_i) \leq 1$*; the color's membership value is the quantity of each element of the combination of* h_i *with the color white. Hence, the color (*h_i*,* $k(h_i)$*) is referred to as the fuzzy color that matches the fundamental color* h_i*. Thus, the* $k(h_i)$ *[*$\leq$*1] amount of* h_i *is mixed with* $1 - k(h_i)$ *to determine how much white color is needed to create the fuzzy color (*h_i*,* $k(h_i)$*). As stated in the definition above, the basic color is the building block from which all other colors are created. For example, green is a fundamental hue. A "fuzzy green" color can be blended to create other colors with 0.8 units of green and 0.2 units of white. This "fuzzy green" is denoted by a green value of 0.8. Similarly, another fuzzy red color (red, 0.6) may be formed by mixing 0.6 units of red with 0.4 units of white, and so on.*

Definition 10 ([29]). *Let* $\eta_{ec} = (V_{ec}, \sigma_{ec}, \mu_{ec})$ *be a connected fuzzy graph and* $c_{ec} = (c_{ec_1}, c_{ec_2}, \ldots, c_{ec_k})$ *be a set of basic colors. Now, two edges are only given two fuzzy colors whose basic colors differ if they are adjacent to one another; otherwise, they may be given fuzzy colors whose basic colors are the same. If the color of any edge is* $(c_{ec_i}, f_{ec_j}(c_{ec_i}))$*, then* C_{ec_i} *is the basic color of edge* e_{ec_j} *= (p, q) and* $f_{ec_j}(c_{ec_i})$ *is its membership value, which is calculated as* $k_{ec_j}(c_{ec_i}) = \frac{\mu_{ec}(p.q)}{\sigma_{ec}(p) \wedge \sigma_{ec(q)}}$, *where* $\sigma_{ec}(p)$ *and* $\sigma_{ec}(q)$ *are the membership values of vertices p and q, respectively. Finally,* $\mu_{ec}(p, q)$ *is the membership value of the edge* e_{ec_j} *, i.e., (p, q) in the fuzzy graph* η_{ec}.

Definition 11 ([29]). *The fuzzy chromatic index of a fuzzy graph is the minimal set of fundamental colors required to color a fuzzy graph. Suppose there are M basic hues at the minimal level, the strengths of edges cannot be described by this chromatic index. For example, when two fuzzy graphs have identical chromatic indices, these graphs cannot be compared using this chromatic index. Hence, there is some weight assigned to the chromatic index. The weight is denoted by Sec, which is defined by*

$$S_{ec} = \sum_{i=1}^{M} \left\{ Max f_{ec_i}(c_{ec_i}) \right\}$$

where the basic color c_{ec_i} *is used to color edge* e_{ec_j} *for some j and the depth of color is* $k_{ec_i}(c_{ec_i})$*. Thus, S is the total of each basic color's maximum membership values. Now, the chromatic index of a fuzzy graph is denoted by (M, S), where M is the minimum number of basic colors to color a graph and S is its weight. We generally follow the operations on fuzzy graph definitions from [13].*

3. Operations on Fuzzy Graphs Using Edge Coloring

Mahabathra et al. [29] introduced Definition 11 to determine the weight of colors in a fuzzy graph. By focusing on the optimality of the fuzzy network, we can compare the formula given in Definition 11 with the sum of the minimal membership value of each color used in the fuzzy network. Hence, this research defines and modifies the formula in terms of the sum of the minimum value of the edge membership values of each color according to the fuzzy network, which yields the optimum value of the created network and helps us determine how effective it is. The chromatic number of a fuzzy graph represents the minimum number of colors required to color the graph. Let us assume that M_{min} is the minimum number of colors used to color the graph. The degree of membership of such crisp graphs is not sufficient to determine the strength of edges, and hence, some weight is associated with the chromatic number. These weights of the edges can influence the coloring process by indicating the strength of association of an edge with a particular color. The weighted minimum of basic colors used is denoted as W_{min} and is defined as

$$W_{\min} = \sum_{p=1}^{M} \left\{ \min g_{e_q}(c_p) \right\}$$

3.1. Residue Product of Two Fuzzy Graphs

Let $RF_1 = (\sigma_{ec_1}, \mu_{ec_1})$ and $RF_2 = (\sigma_{ec_2}, \mu_{ec_2})$ be two fuzzy graph networks of crisp graphs $G_{RF_1} = (V_{ec_1}, E_{ec_1})$ and $G_{RF_2} = (V_{ec_2}, E_{ec_2})$, respectively. Then, its residue product $RF_1 \cdot RF_2 = (\sigma_1 \cdot \sigma_2, \mu_1 \cdot \mu_2)$ is defined as

(i) $\forall\, (a, b) \in V_1 \times V_2, (\sigma_1 \cdot \sigma_2)(a, b) = \sigma_1(a) \wedge \sigma_2(b)$.
(ii) $\forall\, (a, b) \in E_1$ and $c \neq w \in V_2$, $(\mu_1 \cdot \mu_2)((a, c), (b, w)) = \mu_1(a, b)$.

3.1.1. Example

$G_1 = RF_1$ and $G_2 = RF_2$ are the two fuzzy graphs of $G_{RF_1} = (V_{ec_1}, E_{ec_1})$ and $G_{RF_2} = (V_{ec_2}, E_{ec_2})$, depicted in Figures 1 and 2, respectively. Then, the residue product of the fuzzy network is denoted by $RF_1 \cdot RF_2$, as shown in Figure 3.

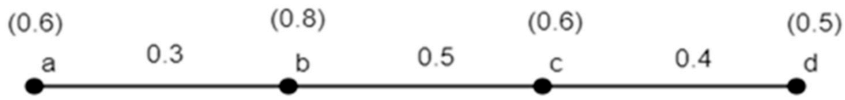

Figure 1. Fuzzy Graph G_1.

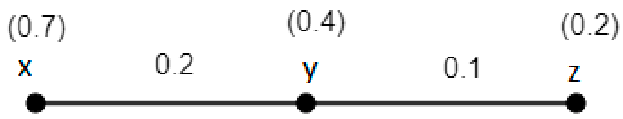

Figure 2. Fuzzy Graph G_2.

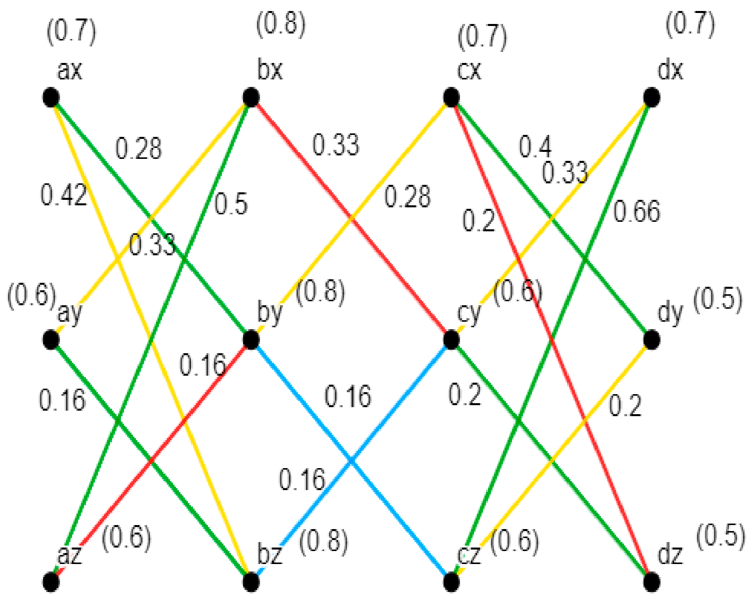

Figure 3. Edge membership value of $RF_1 \cdot RF_2$.

Using Definition 10, the edge membership values of the above constructed $RF_1 \cdot RF_2$ are calculated and shown in Figure 3.

From Figure 3, the minimum number of basic colors used in $RF_1 \cdot RF_2$ is 4. The weight minimum number of basic colors used in constructed residue product $RF_1 \cdot RF_2$ = 0.16 + 0.2 + 0.16 + 0.16 = 0.68. Thus, the W_{min} of $RF_1 \cdot RF_2$ is 0.68.

3.1.2. Find the Weight of the Minimal Spanning Tree Using Kruskal's Algorithm

To find the minimum spanning tree (MST) using the given set of edges, we follow the following steps:

Table 2 shows the weight of the graph, and Table 1 sorts the edges in ascending order based on their weight. We begin by adding the edge ay-bz with a specific weight to the MST. Next, we add the edge by-az to the MST with a weight of 0.16. This edge does not create a cycle within the MST. Moving on, we include the edge by-cz with a weight of 0.16 in the MST. This edge also does not create any cycles. We continue by adding the edge cy-bz, with a weight of 0.16, to the MST. Once again, this edge maintains the property of not creating a cycle. Another edge, cy-dz, with a weight of 0.2, is added to the MST without causing any cycles. The edge dy-cz, with a weight of 0.2, is added to the MST, ensuring that no cycles are formed. We proceed by adding the edge ax-bzy, weight 0.28, to the MST. The inclusion of this edge does not result in any cycles. The edge cx-by, having a weight of 0.28, is included in the MST without creating cycles. Moving forward, we add the edge bx-cy with a weight of 0.33 to the MST; no cycles are introduced by this inclusion. Similarly, the edge dx-cy with weight 0.33 is integrated into the MST without creating any cycles. Lastly, we come across the edge ax-bz with weight 0.42. However, adding this edge would create a cycle within the MST. Thus, we discard it. Having visited all the nodes and ensuring that the number of edges is fewer than the number of nodes, we can conclude that the algorithm can now be stopped.

Table 1. The edges, sorted by weight in ascending order.

Edge	**ay-bz**	**by-az**	**by-cz**	**cy-bz**	**cy-dz**	**dy-cz**	**ax-by**	**cx-by**	**bx-cy**	**dx-cy**	**ax-bz**
Weight	0.16	0.16	0.16	0.16	0.2	0.2	0.28	0.28	0.33	0.33	0.42

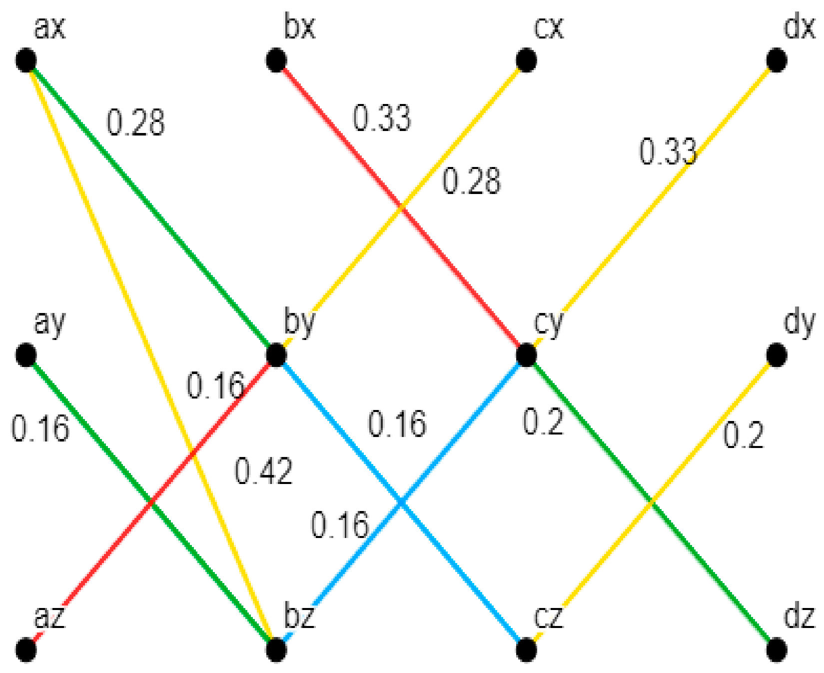

Figure 4. Minimum Spanning Tree of $RF_1 \cdot RF_2$.

Table 2. The weight of a given graph (Figure 4).

Edge	ax-by	ax-bz	bx-cy	cx-by	dx-cy	ay-bz	by-az	by-cz	cy-bz	cy-dz	dy-cz
Weight	0.28	0.42	0.33	0.28	0.33	0.16	0.16	0.16	0.16	0.2	0.2

Using Kruskal's algorithm, the weight of the minimal spanning tree of $RF_1 \cdot\cdot RF_2$ is shown to be 2.68.

3.1.3. Lower Domination Number of $RF_1.RF_2$

It is possible for there to be more than one minimal dominating set in an established network. The set with the lowest domination number among all minimal dominating sets is considered to be the created network's lowest domination number, which allows us to test the network's optimality. Let $G_{DN} = (\sigma_{DN},\ \mu_{DN})$ be the fuzzy graph of the crisp graph $G_{DN} = (V_{DN}, E_{DN})$, and let $S_{DN_1}, S_{DN_2}, \ldots, S_{DNr}$ be the minimal dominating set of G_{DN}. Finally, let the corresponding dominating number be denoted by $\gamma_{DN_1}, \gamma_{DN_2}, \gamma_{DN_3}, \ldots, \gamma_{DN_r}$. Among this, the lowest cardinality of the domination number is called lower domination number and is denoted by $\gamma_{LDN.}$

Let $S_{(1)RF_1\,.\,RF_2}, S_{(2)RF_1.RF_2}, S_{(3)RF_1\,.\,RF_2}$ *and* $S_{(4)RF_1\,.\,RF_2}$ be the sum of the minimal dominating set of $RF_{1.}RF_2$ (refer to Figure 3)

$$S_{(1)RF_1.RF_2} = \{ay, by, cy, dy\};\ S_{(2)RF_1.RF_2} = \{ax, bx, cx, dx\};$$

$$S_{(3)RF_1.RF_2} = \{az, bz, cz, dz\}$$

$$\gamma_{(1)RF_1.RF_2} \text{ of } S_{(1)RF_1.RF_2} = 0.3 + 0.6 + 0.6 + 0.3 = 1.8$$

$$\gamma_{(2)RF_1.RF_2} \text{ of } S_{(2)RF_1.RF_2} = 2.6.$$

$$\gamma_{(2)RF_1.RF_2} \text{ of } S_{(3)RF_1.RF_2} = 2.1.$$

$$\gamma_{LDN-RF_1.RF_2} \text{ of } RF_1\ RF_2 \text{ is } 1.8.$$

3.2. Symmetric Difference of Two Fuzzy Graphs

Let $SDF_1 = (\sigma_{ec_1}, \mu_{ec_1})$ and $SDF_2 = (\sigma_{ec_2}, \mu_{ec_2})$ be two fuzzy graphs of crisp graphs $G_{SDF_1} = (V_{ec_1}, E_{ec_1})\ and G_{SDF_2} = (V_{ec_2}, E_{ec_2})$, respectively. Then, the symmetric difference between SDF$_1$ and SDF$_2$ is denoted by $SDF_1 \oplus SDF_2 = (\sigma_{ec_1} \oplus \sigma_{ec_2}, \mu_{ec_1} \oplus \mu_{ec_2})$ and is defined as follows

1. $\forall (a,b) \in V_1 \times V_{2,}\ \sigma_{SDF_1} \oplus \sigma_{SDF_2}(a,b) = \sigma_{SDF_1}(a) \wedge \sigma_{SDF_2}(b)$.
2. $\forall a \in V_1\ and\ (b,c) \in E_2\ \left(\mu_{SDF_1} \oplus \mu_{SDF_2}\right)((a,b),(a,c) = \sigma_{SDF_1}(a) \wedge \mu_{SDF_2}(b,c)$.
3. $\forall a \in V_2\ and\ (b,c) \in E_1\ \left(\mu_{SDF_1} \oplus \mu_{SDF_2}\right)((b,a),(c,a)) = \mu_{SDF_1}(b,c) \wedge \sigma_{SDF_2}(a)$.
4. $\forall (a,b) \notin E_1\ and\ (c,w) \in E_2,\ \left(\mu_{SDF_1} \oplus \mu_{SDF_2}\right)((a,c),(b,w)) = \min\{\sigma_{SDF_1}(a), \sigma_{SDF_1}(b), \mu_{SDF_2}(c,w)\}$.
5. $\forall (a,b) \in E_1\ and\ (c,w) \notin E_2,\ \left(\mu_{SDF_1} \oplus \mu_{SDF_2}\right)((a,c),(b,w)) = \min\{\mu_{SDF_1}(a,b), \sigma_{SDF_2}(c), \sigma_{SDF_2}(w)\}$.

3.2.1. Example

Let $G_1 = SDF_1$ and $G_2 = SDF_2$ be the two fuzzy graphs of crisp graphs $G_{SDF_1} = (V_{ec_1}, E_{ec_1})\ and\ G_{SDF_2} = (V_{ec_2}, E_{ec_2})$, depicted in Figures 1 and 2, respectively.

The symmetric difference of fuzzy network $SDF_1 \oplus SDF_2$ is shown in Figure 5.

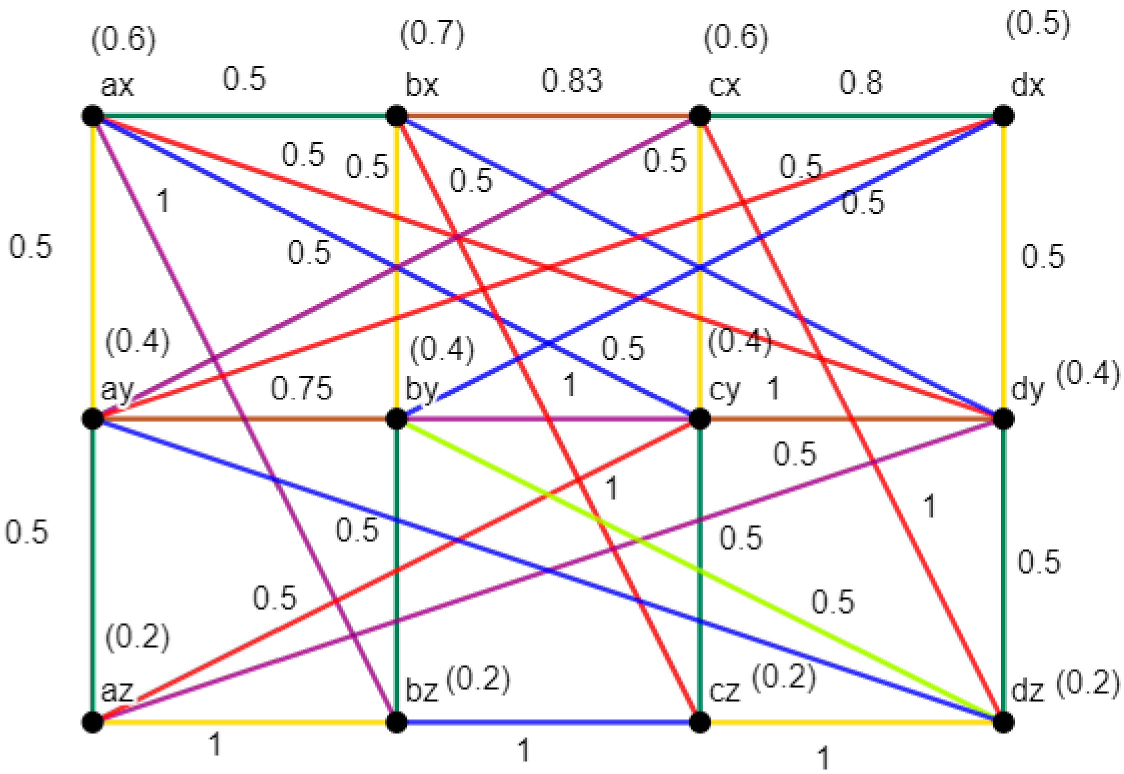

Figure 5. Edge Membership value of $SDF_1 \oplus SDF_2$.

Using Definition 10, the edge membership values of the above constructed $SDF_1 \oplus SDF_2$ are calculated, as shown in Figure 5.

As shown in Figure 5, the minimum number of basic colors used in $SDF_1 \oplus SDF_2$ is 7. The weight minimum number of basic colors used in the constructed symmetric difference network $SDF_1 \oplus SDF_2$ = 0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.75 + 0.5 = 3.75. Thus, the W_{min} of $SDF_1 \oplus SDF_2$ is 3.75.

Using Kruskal's algorithm, the weight of the minimal spanning tree of $SDF_1 \oplus SDF_2$ was found to be 7.

3.2.2. Lower Domination Number of $SDF_1 \oplus SDF_2$

Let $S_{(1)SDF_1 \oplus SDF_2}$, $S_{(2)SDF_1 \oplus SDF_2}$, $S_{(3)SDF_1 \oplus SDF_2}$ *and* $S_{(4)SDF_1 \oplus SDF_2}$ be some of the minimal dominating set of $SDF_1 \oplus SDF_2$ (see Figure 5)

$$S_{(1)SDF_1 \oplus SDF_2} = \{ax, ay\};\ S_{(2)SDF_1 \oplus SDF_2} = \{az, ay\};$$

$$S_{(3)SDF_1 \oplus SDF_2} = \{dx, dy\};\ S_{(4)SDF_1 \oplus SDF_2} = \{dy, dz\}.$$

$$\gamma_{(1)SDF_1 . \oplus SDF_2} \text{ of } S_{(1)SDF_1 \oplus SDF_2} = 2.2$$

$$\gamma_{(2)SDF_1 . \oplus SDF_2} \text{ of } S_{(2)SDF_1 \oplus SDF_2} = 1.6.$$

$$\gamma_{(3)SDF_1 . \oplus SDF_2} \text{ of } S_{(3)SDF_1 \oplus SDF_2} = 2.2.$$

$$\gamma_{(4)SDF_1 . \oplus SDF_2} \text{ of } S_{(4)SDF_1 \oplus SDF_2} = 1.9.$$

$$\gamma_{LDN-SDF_1 \oplus . SDF_2} \text{ of } SDF_1 \oplus SDF_2 \text{ is } 1.6.$$

3.3. Max Product of Two Fuzzy Graphs

Let $MF_1 = (\sigma_{mf_1}, \mu_{mf_1})$ and $MF_2 = (\sigma_{mf_2}, \mu_{mf_2})$ be two fuzzy networks of crisp graphs $G_{MF_1} = (V_{mf_1}, E_{mf_1})$ and $G_{MF_2} = (V_{mf_2}, E_{mf_2})$, respectively. The maximal product of fuzzy graphs MF_1 and MF_2 is represented by $MF_1 * MF_2 = (\sigma_{mf_1} * \sigma_{mf_2}, \mu_{mf_1}*, \mu_{mf_2})$ and is defined as:

(i) $\forall (a, b) \in V_{mf_1} \times V_{mf_2}, \left(\sigma_{mf_1} * \sigma_{mf_2}\right)(a, b) = \sigma_{mf_1}(a) \vee \sigma_{mf_2}(b)$.

(ii) $\forall a \in V_{mf_1}$ and $(b, c) \in E_{mf_2}$, $\left(\mu_{mf_1} * \mu_{mf_2}\right)((a, b), (a, c)) = \sigma_{mf_1}(a) \vee \mu_{mf_2}(b, c)$.

(iii) $\forall a \in V_{mf_2}$ and $(b, c) \in E_{mf_1}$, $\left(\mu_{mf_1} * \mu_{mf_2}\right)((b, a), (c, a)) = \mu_{mf_1}(b, c) \vee \sigma_{mf_2}(a)$.

3.3.1. Example

$G_1 = MF_1$ and $G_2 = MF_2$ defines the two fuzzy graphs of crisp graphs $G_{MF_1} = (V_{ec_1}, E_{ec_1})$ and $G_{MF_2} = (V_{ec_2}, E_{ec_2})$, depicted in Figures 1 and 2, respectively. The max product of the fuzzy network $MF_1 * MF_2$ is shown in Figure 6.

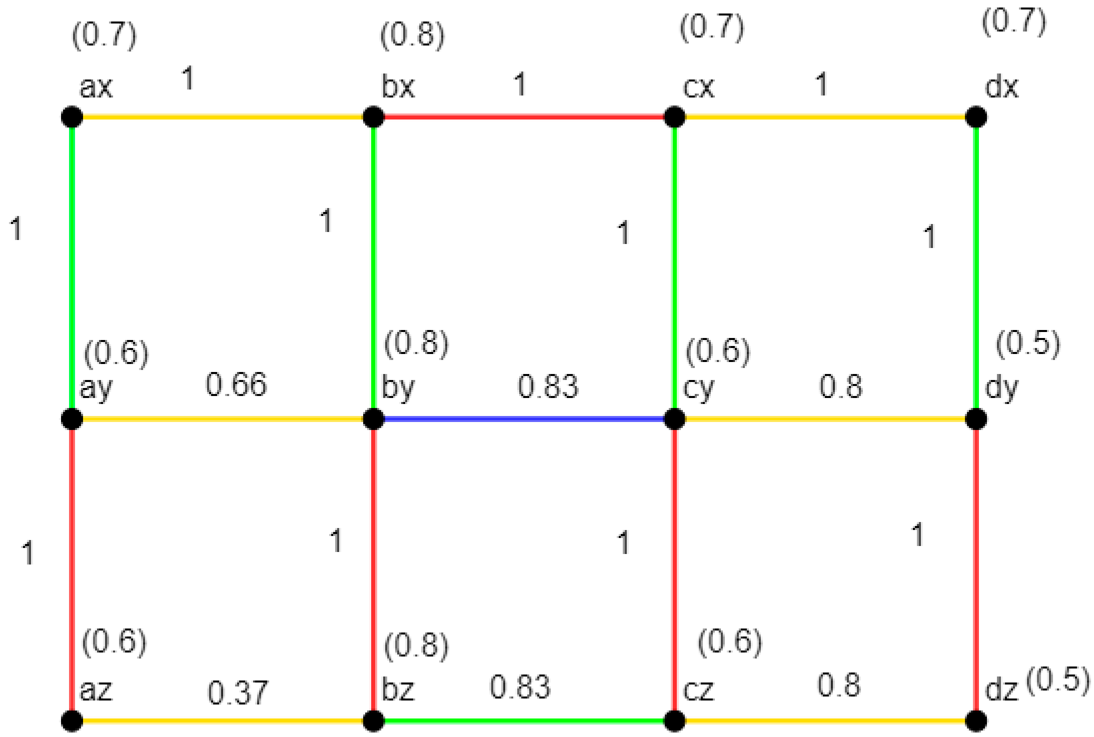

Figure 6. Edge Membership value of $MF_1 * MF_2$.

Using Definition 10, the edge membership values of the above constructed $MF_1 * MF_2$ were calculated, as shown in Figure 6.

As shown in Figure 6, the minimum number of basic colors used in $MF_1 * MF_2$ is 4. The weight minimum number of basic colors used in the constructed maximal product network $MF_1 * MF_2 = 0.37 + 1 + 0.83 + 0.83 = 3.03$. Thus, the W_{min} of $MF_1 * MF_2$ is 3.03.

Using Kruskal's algorithm, the weight of the minimal spanning tree of $MF_1 * MF_2$ was found to be 9.29.

3.3.2. Lower Domination Number of $MF_1 * MF_2$

Let $S_{(1)MF_1*MF_2}, S_{(2)MF_1*MF_2}, S_{(3)MF_1*MF_2}$ *and* $S_{(4)MF_1*MF_2}$ be some of the minimal dominating set of $MF_1 * MF_2$ (see Figure 6)

$$S_{(1)MF_1*MF_2} = \{ay, cx, cz, dy\} S_{(2)MF_1*MF_2} = \{ay, bx, by, dy\};$$

$$S_{(3)MF_1*MF_2} = \{bx, cx, ay, dy\};\ S_{(4)MF_1*MF_2} = \{ax, bz, cx, dz\}.$$

$$\gamma_{(1)MF_1.*MF_2}\,\text{of}S_{(1)MF_1*MF_2} = 6.5; \gamma_{(2)MF_1.*MF_2}\ \text{of}\ S_{(2)MF_1*MF_2} = 6.7;$$

$$\gamma_{(3)SMF_1.*MF_2}\,\text{of}S_{(3)MF_1*MF_2} = 7.2; \gamma_{(4)MF_1.*MF_2}\ \text{of}\ S_{(4)MF_1*MF_2} = 5.7;$$

$$\gamma_{LDN-MF_1.*MF_2}\ \text{of}\ MF_1 * MF_2\ \text{is}\ 5.7.$$

3.4. Lexicographic Product of Two Fuzzy Graphs

Let $LF_1 = (M_1, P_1)$ and $LF_2 = (M_2, P_2)$ be two fuzzy graphs of the crisp graphs $G_{LF_1} = (V_{ec_1}, E_{ec_1})$ and $G_{LF_2} = (V_{ec_2}, E_{ec_2})$ respectively.

The lexicographic product of the two graphs is denoted as $LF_1 \cdot LF_2$ in fuzzy graph pair (M, P), such that

(i) $M(a_1, b_2) = \min(M_1(a_1), M_2(b_2)), \forall (a_1, b_2) \in M_1 \times M_2$.
(ii) $P((x, b_2)(x, d_2) = \min(M_1(x), M_2(b_2 d_2)\ , \forall x \in M_1, b_2 d_2 \in P_2$.
(iii) $P((a_1, b_2)(c_1, d_2)) = \min(P_1(a_1 c_1), P_2(b_2 d_2)), \forall a_1 b_1 \in P_1$ and $b_2 d_2 \in P_2$.

3.4.1. Example

Let $G_1 = LF_1$ and $G_2 = LF_2$ be two fuzzy graphs of crisp graphs $G_{LF_1} = (V_{ec_1}, E_{ec_1})$ and $G_{LF_2} = (V_{ec_2}, E_{ec_2})$, depicted in Figures 1 and 2, respectively. The lexicographic product of the fuzzy network is denoted by $LF_1 \cdot LF_2$, as shown in Figure 7.

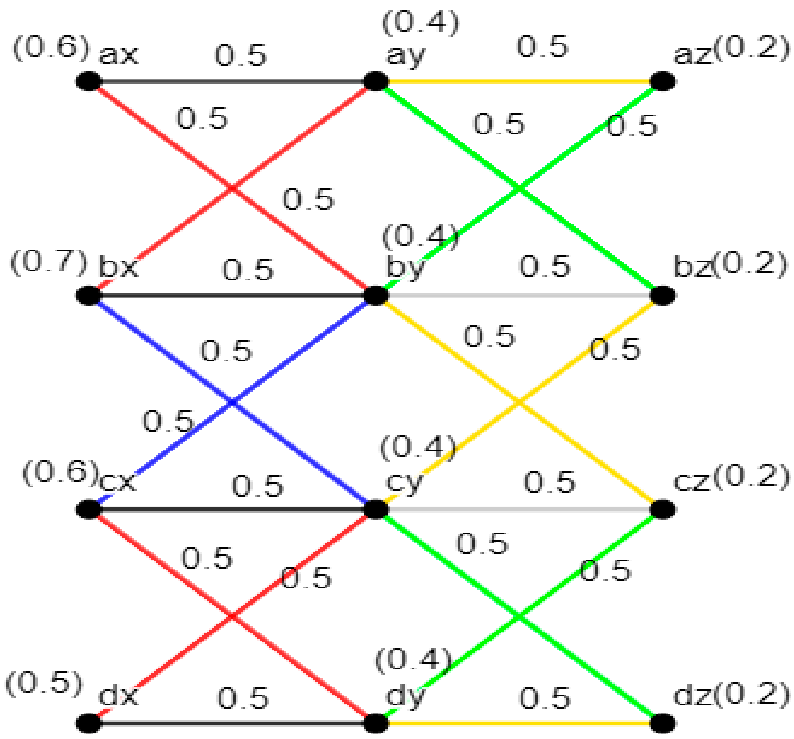

Figure 7. Edge Membership value of $LF_1 \cdot LF_2$.

Using Definition 10, the edge membership values of the above constructed $LF_1 \cdot LF_2$ were calculated, as shown in Figure 7.

As shown in Figure 7, the minimum number of basic colors used in $LF_1 \cdot LF_2$ is 6. The weight minimum number of basic colors used in the constructed lexicographic product of network $LF_1 \cdot LF_2$ = 0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.5 = 3. Thus, the W_{min} of $LF_1 \cdot LF_2$ is 3.

Using Kruskal's algorithm, the weight of the minimal spanning tree of $LF_1 \cdot LF_2$ was found to be 5.5.

3.4.2. Lower Domination Number of $LF_1 \cdot LF_2$

Let $S_{LF_1 * LF_2}$ be the minimal dominating set (only one dominating set) of $LF_1 \cdot LF_2$ (see Figure 7)

$$S_{LF_1 * LF_2} = \{ay, by, cy, dy\}.\gamma_{LDN-LF_1.*LF_2} \text{ of } S_{LF_1 * LF_2} \text{ is } 3.$$

Algorithm 1 provides comparative studies of fuzzy domination and fuzzy coloring in operations of fuzzy networks.

4. Algorithm to Find an Optimal Network

4.1. Algorithm

This algorithm explores how to determine an optimal network using the product operations of a fuzzy network.

Algorithm 1: Find an optimal network using the operations of a fuzzy network

---

Input: Two fuzzy networks $FG_1 = (\sigma_{c_1}, \mu_{c_1})$ and $FG_2 = (\sigma_{c_2}, \mu_{c_2})$ of crisp graphs $G_{F_1} = (V_{c_1}, E_{c_1})$ *and* $G_{F_2} = (V_{c_2}, E_{c_2})$, respectively.
Output: Optimal fuzzy network
Begin
Step 1: Construct a collection of finite networks, e.g., $N_1, N_2, \ldots, N_r$, by performing separate operations on a fuzzy network with vertex sets $V = V_{c_1} \times V_{c_2}$.
Step 2: Calculate $k_{ec_j}(c_{ec_i}) = \frac{\mu_{ec}(p.q)}{\sigma_{ec}(p) \wedge \sigma_{ec(q)}}$ of all edges. Vertices are to be labeled as 1, 2, 3,. . ., n
Step 3: Find the membership function value for each node and edge of $N_1, N_2, \ldots, N_r$ using the operations applied to the constructed network.
Steps 4: In the constructed network, e.g., N_1, identify vertex '1′ of the maximum degree and color all its incident edges such that no two incident edges receive the same color.
Step 5: Next, focus the direct neighbors of vertex 1, color the incident edges of the neighboring vertex, and label them as 12, 13, . . ., 1m, if there are m neighbors. If the number of neighbors is less than m, then use the minimum number of the same color used for vertex '1′.
Step 6: Proceed with Step 5 again until all the edges receive the colors. From the above step, we get the minimum number of colors used to color the given network. Steps 4, 5, and 6 will continue for the other constructed networks $N_2, \ldots, N_r$ to find the minimum number of colors used to color the edges of the networks.
Step 7: Find the minimal edge coloring set with the minimum number of colors used to color the established network and the corresponding optimal weight of the colors.
Step 8: Find the lower domination number of established network $N_1, N_2, \ldots, N_r$ and let it be $\gamma_{LDN_1}, \gamma_{LDN_2}, \gamma_{LDN_3}, \ldots, \gamma_{LDN_r}$, respectively.
Step 9: Find the minimum number of colors used to color the established network, e.g., $S_{ec_1}, S_{ec_2} \ldots S_{ec_r}$, and its corresponding weight, e.g., $WS_{ec_1}, WS_{ec_2}, \ldots, WS_{ec_r}$ of $N_1, N_2, \ldots, N_r$, respectively.
Step 10: Determine the minimal spanning tree of the constructed networks, e.g., $ST_1, ST_2, \ldots, ST_r$ of $N_1, N_2, \ldots, N_r$, respectively, and let its corresponding minimum weight of $ST_1, ST_2, \ldots, ST_r$ be $M_{ST_1}, M_{ST_2}, \ldots, M_{ST_r}$
Step 11: Let O_{DN_i} denote the sum of the lower domination number of established network N_i and the minimum weight of spanning tree N_i, that is, $O_{DN_i} = \gamma_{LDN_i} + M_{ST_i}$. O_{N_i} is the sum of the weight of the basic colors used in establised network N_i and the minimum weight of spanning tree N_i, that is

$$O_{N_i} = WS_{ec_i} + M_{ST_i}$$

Step 12: Optimal value of the established network using domination, $O_{opt-DN} = \min\{O_{DN_i}\}\ i = 1, 2, \ldots, r$, and optimal value of the established network using chromatic index, $O_{opt-CI} = \min\{O_{N_i}\}\ i = 1, 2, \ldots, r$.
Step-13: Effective optimal value of the established network, $O_{eff-opt-val} = \min\left\{O_{opt-DN}, O_{opt-CI}\right\}\ i = 1, 2, \ldots, r$.
End

4.2. Flow Chart of the Algorithm

Figure 8, shows the flow chart of algorithm. We derived the following from Section 3.

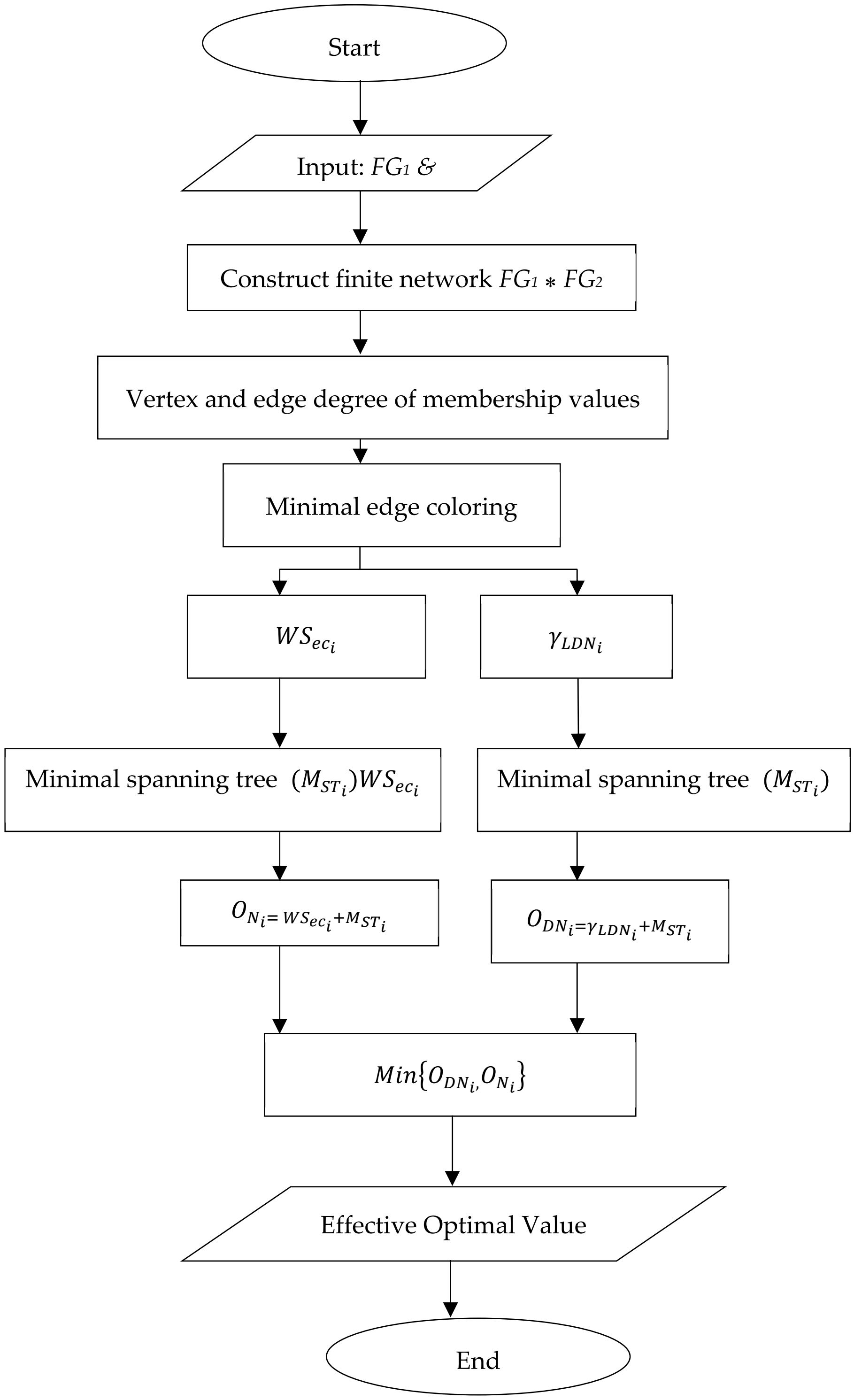

Figure 8. Flow Chart of the Algorithm.

Let WS_{ec_1}, WS_{ec_2}, WS_{ec_3}, and WS_{ec_4} be the weight of the minimum number of basic colors used in the residue product, symmetric difference, max product, and lexicographic, respectively, and let its corresponding minimum weight of the spanning tree be M_{ST_1}, M_{ST_2}, M_{ST_3} and M_{ST_4}. Let O_{N_1}, O_{N_2}, O_{N_3}, and O_{N_4} be the sum of the weight of the minimum number of basic colors used in the established network and the minimum weight of the spanning tree of the residue product, symmetric difference, max product, and lexicographic, respectively.

$$WS_{ec_1} \text{ of } RF_1 \cdot RF_2 \text{ is } 4.68, \text{ and } M_{ST_1} \text{ is } 2.68. \text{ Hence } O_{N_1} = 4.68 + 2.68 = 7.36.$$

$$WS_{ec_2} \text{ of } SDF_1 \oplus SDF_2 \text{ is } 10.75 \text{ and } M_{ST_2} \text{ is } 7. \text{ Hence } O_{N_2} = 10.75 + 7 = 17.75.$$

$$WS_{ec_3} \text{of } MF_1 * MF_2 \text{ is } 7.03 \text{ and } M_{ST_3} \text{ is } 9.29. \text{ Hence } O_{N_3} = 7.03 + 9.29 = 16.32.$$

$$WS_{ec_3} \text{ of } LF_1 \cdot LF_2 \text{ is } 9, \text{ and } M_{ST_3} \text{ is } 5.5. \text{ Hence } O_{N_4} = 9 + 5.5 = 14.5.$$

Let $O_{DN_1}, O_{DN_2}, O_{DN_3}, \textit{and}\ O_{DN_4}$ be the sum of the lower domination number and minimum weight of the spanning tree of residue product, symmetric difference, max product, and lexicographic, respectively

$$\gamma_{LDN-RF_1.RF_2} \text{ of } RF_1 \cdot RF_2 \text{ is } 1.8,\ \gamma_{LDN-SDF_1 \oplus .SDF_2} \text{ of } SDF_1 \oplus SDF_2 \text{ is } 1.6.$$

$$\gamma_{LDN-MF_1.*MF_2} \text{ of } MF_1 * MF_2 \text{ is } 5.7 \text{ and } \gamma_{LDN-LF_1.*LF_2} \text{ of}$$

$$S_{LF_1 * LF_2} is\ 3.$$

$$\text{Hence, } O_{DN_1} = 1.8{+}2.68 = 4.48,\ O_{DN_2} = 1.6{+}\ 7 = 8.6,\ O_{DN_3} = 5.7 + 9.29 = 14.99 \text{ and}$$

$$O_{DN_4} = 3 + 5.5 = 8.5$$

The optimal value of the established network : $O_{opt-CI} = min_i\{O_{N_i}\}, i = 1,2,3,4 = 3.36$.

The optimal value of the established network using domination: $O_{opt-DN} = \min\{O_{DN_i}\}$ $i = 1,2,3,4 = 4.48$

The effective optimal value of the established network: $O_{eff-opt-val} = \min\{O_{opt-DN}, O_{opt-CI}\}$ $i = 1,2,3,4 = 3.36$.

Comparison between the optimal value using the weight of the minimum basic colors used and the domination number of the established network, as shown in Table 3.

Table 3. Comparison of O_{DN_i} and O_{N_i}.

S. No	Established Network	O_{DN_i}	O_{N_i}
1	$RF_1 \cdot RF_2$	4.48	3.36
2	$SDF_1 \oplus SDF_2$	7.6	7.75
3	$MF_1 {*} MF_2$	14.99	12.32
4	$LF_1 \cdot LF_2$	8.5	8.5

Hence the effective optimal value of the established network using edge coloring is more effective than the network using the domination number.

4.3. Applications of a Fuzzy Graph in a Social Network

Social networks have been around for a very long time. They apply the simple process of extending the number of individuals you know by connecting with peers and so on. Indeed, many of us now utilize social media platforms like Facebook and Twitter to promote our present and potential enterprises. People who want to connect with other business-related acquaintances frequently go to sites like LinkedIn. LinkedIn is a social media website built especially for professional networking to assist people in finding a job, locating sales leads, and connect with possible business partners. Let us assume the presence of group of people in a network, where $G_3 \cdot G_4$ is the network of organizations linked between them. When these two networks collaborate, it is via a new fuzzy network created by lexicographic product operation $G_3 \cdot G_4$. Here, the organizations that collaborated with the people, if an applicant's demands meets the organization's needs, are represented as vertices, and the information shared between them is represented as edges. Let us consider a, b, c, and d as people with distinct skills and x, y, and z as organizations; see Table 4.

Table 4. Roles of vertices.

S. No	Vertices	Characteristic of Each Vertex
1	a	Technical skills
2	b	Data-driven skills
3	c	Management skills
4	d	Marketing skills
5	x	Technology Company
6	y	Consulting firms
7	z	Retail and consumer goods company

Every applicant satisfies the minimum demand of the organization in terms of their skills, so they are allowed to see what they are working on. Applying lexicographic product operation, LinkedIn provides more personalized networking recommendations.

Above Figures 9 and 10 shows the fuzzy graph G_3 and G_4. When these networks collaborate, they share information, form business-to-business relationships, provide services to find jobs, and share market products in the network. A network of lexicographic products of two graphs was constructed, as shown in Figure 7. For example, if "ax" is a technology organization with one person with technical skills, they may have expertise in the job of data scientist and they may share information about the needs of the organization with "ay "(Big data developer) and" by" (Private Equity Analyst). With experts in the fields of technical and data-driven skills, they collaborate to meet their own needs and, likewise, share information. Using lexicographic products provides a comprehensive view that can potentially reduce the complexity of analyses. This fully connected network makes employees as well the organization more flexible in terms of sharing information that may be depicted using six distinct colors, where each color represents the most efficiently shared information; also, time management is taken into consideration, while the colors of the edges, such as red, green, yellow and blue, provide information regarding the minimum possible weight of the edges. The amount of information shared through edge coloring is effective in networking; the chromatic index of the edges is the minimum that leads to the optimization of the network. Vertices ay, by, cy, and dy seem to have a significant influence and dominate in the network with minimum time management. The information is used to curate relevant collaborations, thereby increasing knowledge sharing as well as the profits benefits for the organizations and making the network more optimal. Hence, using domination in the connected network, i.e., the colored minimum weighted edges, enhances the allocation of resources by considering the timing and coordination of interactions. The minimum weight of the edges contributes to more harmonious network dynamics, ultimately improving the experience of users and achieving the network goals more effectively.

Figure 9. Fuzzy Graph G_3.

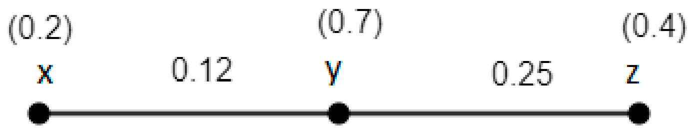

Figure 10. Fuzzy Graph G_4.

4.4. Applications of Fuzzy Graph Coloring in a Communication Network

Fuzzy graph coloring is used to build fault-tolerant routing schemes with acyclic subnetwork in on-chip networks. Fuzzy graph coloring is a mathematical technique that extends traditional graph coloring by allowing vertices to be partially colored with fractional or fuzzy values, rather than being strictly assigned a single color. This approach has been used in various applications, including fault-tolerant routing schemes in on-chip networks with acyclic subnetworks. Fuzzy graph coloring can be applied in this context in the following ways:

4.4.1. Fault-Tolerant Routing in On-Chip Networks

On-chip networks are essential components of modern microprocessors, enabling communication among different cores and memory banks on a single chip. To ensure reliability in these networks, fault-tolerant routing schemes are employed. These schemes aim to find alternative routes in the presence of faults (e.g., faulty links or routers) to maintain communication.

4.4.2. Acyclic Subnetworks

In some on-chip network topologies, acyclic subnetworks are used. Acyclic networks have no cycles, meaning there are no closed paths or loops in the network. This property simplifies routing but can introduce challenges in fault tolerance, since alternative paths must be carefully chosen to avoid loops.

4.4.3. Fuzzy Graph Coloring

Fuzzy graph coloring can be applied to model and optimize the assignment of communication routes within an on-chip network, taking into account the acyclic subnetwork structure and fault tolerance requirements. It works as follows:

Vertex Coloring

In a traditional graph coloring problem, each vertex (representing a router or node in the network) is assigned a single color, and adjacent vertices cannot have the same color. In fuzzy graph coloring, vertices are assigned fractional colors, allowing multiple vertices to share the same color value to a certain degree.

Edge Weights

In the context of fault-tolerant routing, edge weights can represent the reliability or quality of network links. Faulty or less reliable links are assigned lower weights.

4.4.4. Optimization Objective

The goal is to minimize the total conflict among adjacent vertices while maximizing the reliability of communication paths. This can be formulated as an optimization problem where the fractional coloring of vertices and the selection of routes are jointly optimized.

4.4.5. Fault Tolerance

By allowing vertices to share colors to some extent and optimizing the assignment of routes based on edge weights (reliability), the fuzzy graph coloring approach can help in finding fault-tolerant routing schemes. The algorithm can find alternative routes that avoid faulty components while maintaining the acyclic nature of the subnetwork.

4.4.6. Implementation

Implementing fuzzy graph coloring for fault-tolerant routing schemes in on-chip networks may require specialized algorithms and software tools. This implementation may consider factors like the network topology, fault models, and communication patterns specific to the on-chip network architecture.

Fuzzy graph coloring can be a useful technique for designing fault-tolerant routing schemes in on-chip networks with acyclic subnetworks. It enables the optimization of communication paths while considering fault tolerance and network reliability, ultimately improving the robustness and performance of on-chip communication systems.

5. Conclusions

The implications of this study extend beyond theoretical frameworks and hold practical significance. The insights gained from our research have the potential to inform decision-making processes, optimize calculations, and offer valuable guidance in various real-world applications. This paper has described operations such as residue product, symmetric difference, max product, and lexicographic on fuzzy graphs. Additionally, we have tried to discover some of their characteristics to determine how effective they are, and we have discussed how the lexicographic operation may be used in the real world while using a minimal spanning tree approach, which was developed to perform activities with minimal strength in a social network. The study will eventually cover additional processes and include methods for maximizing the created network's efficiency. In the future, we will extend our studies to include intuitionistic fuzzy networks and neutrosophic networks.

Author Contributions: Conceptualization, A.M. and A.K.; Data curation, A.M. and A.K. Formal analysis, A.M., K.K., M.M. and A.K.; Investigation, A.M., K.K. and R.G.; Methodology, A.M. and A.K. Supervision, A.M.; Visualization, A.M., R.G. and A.K. Writing—original draft, A.M. and A.K. Writing—review and editing, K.K., M.M. and R.G. All authors have read and agreed to the published version of the manuscript.

Funding: This article was supported by the European Regional Development Fund in the Research Centre of Advanced Mechatronic Systems project, CZ.02.1.01/0.0/0.0/16_019/0000867 within the Operational Programme Research, Development, and Education.

Data Availability Statement: The data availability of this research work is based on the published research article of the relevant topics.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Akram, M.; Waseem, N.; Dudek, W.A. Certain types of edge m-polar fuzzy graphs. *Iran J. Fuzzy Syst.* **2017**, *14*, 27–50.
2. Mahapatra, T.; Ghorai, G.; Pal, M. Competition graphs under interval-valued m-polar fuzzy environment and its application. *Comput. Appl. Math.* **2022**, *41*, 285. [CrossRef]
3. Mahapatra, T.; Pal, M. An investigation on m-polar fuzzy threshold graph and its application on resource power controlling system. *J. Ambient Intell. Humaniz. Comput.* **2022**, *13*, 501–514. [CrossRef]
4. Zadeh, L. Fuzzy sets. *Inf. Control.* **1965**, *8*, 338–353. [CrossRef]
5. Buckley, J.J.; Eslami, E. *An Introduction to Fuzzy Logic and Fuzzy Sets*; Springer Science & Business Media: New York, NY, USA, 2002.
6. Talebi, A.A.; Rashmanlou, H. Isomorphism on interval-valued fuzzy graphs. *Ann. Fuzzy Math. Inform.* **2013**, *6*, 47–58.
7. Samanta, S.; Pal, M. Fuzzy planar graphs. *IEEE Trans. Fuzzy Syst.* **2015**, *23*, 1936–1942. [CrossRef]
8. Akram, M.; Jun, Y.B.; Al-Shehrie, N.O.; Sarwar, S. N-structures applied to graphs. *World Appl. Sci. J.* **2013**, *22*, 1–8.
9. Karunambigai, M.G.; Akram, M.; Sivasankar, S.; Palanive, K. Balanced intuitionistic fuzzy graphs. *Appl. Math. Sci.* **2013**, *7*, 2501–2514. [CrossRef]
10. Rangasamy, P.; Akram, M.; Thilagavathi, S. Intuitionistic fuzzy shortest hyperpath in a network. *Inf. Process. Lett.* **2013**, *113*, 599–603. [CrossRef]
11. Islam, S.R.; Maity, S.; Pal, M. Comment on wiener index of a fuzzy graph and application to illegal immigration networks. *Fuzzy Sets Syst.* **2020**, *384*, 148–151. [CrossRef]
12. Kumaran, N.; Meenakshi, A.; Mahdal, M.; Prakash, J.U.; Guras, R. Application of Fuzzy Network Using Efficient Domination. *Mathematics* **2023**, *11*, 2258. [CrossRef]

13. Meenakshi, A.; Mythreyi, O.; Bramila, M.; Kannan, A.; Senbagamalar, J. Application of neutrosophic optimal network using operations. *J. Intell. Fuzzy Syst.* **2023**, *45*, 421–433. [CrossRef]
14. Yang, H.L.; Guo, Z.L.; She, Y.; Liao, X. On single valued neutrosophic relations. *J. Intell. Fuzzy Syst.* **2016**, *30*, 1045–1056. [CrossRef]
15. Atanassov, K.T. *On Intuitionistic Fuzzy Sets Theory*; Springer: Berlin, Germany, 2012.
16. Broumi, S.; Smarandache, F. New Distance and Similarity Measures of Interval Neutrosophic Sets. In Proceedings of the 17th International Conference on Information Fusion, Salamanca, Spain, 7–10 July 2014.
17. Islam, S.R.; Pal, M. Hyper-Wiener index for fuzzy graph and its application in share market. *J. Intell. Fuzzy Syst.* **2021**, *41*, 2073–2083. [CrossRef]
18. Arif, N.E. Domination (set and Number) in Neutrosophic Soft over Graphs. *Wasit J. Pure Sci.* **2022**, *1*, 26–43.
19. Rashmanlou, H.; Samanta, S.; Pal, M.; Borzooei, R.A. Intuitionistic fuzzy graphs with categorical properties. *Fuzzy Inf. Eng.* **2015**, *7*, 317–334. [CrossRef]
20. Shannon, A.; Atanassov, K. On a generalization of intuitionistic fuzzy graphs. *NIFS* **2006**, *12*, 24–29.
21. Bondy, J.A.; Murty, U.S.R. *Graph Theory with Applications*; Elsevier: New York, NY, USA, 1982.
22. Ore, O. *Theory of Graphs*; American Mathematical Society Colloquium Publications: Providence, RI, USA, 1962.
23. Haynes, T.W.; Hedetniemi, S.; Slater, P. *Fundamentals of in Domination in Graphs*; Springer International Publishing: New York, NY, USA, 1998.
24. Swaminathan, V.; Dharmalingam, K.M. Degree equitable domination on graphs. *Kragujev. J. Math.* **2011**, *35*, 191–197.
25. Meenakshi, A. Equitable domination of complement of inflated graph. *AIP Conf. Proc.* **2019**, *2112*, 1–6.
26. Meenakshi, A.; Babujee, J.B. Paired Equitable domination in graphs. *Int. J. Pure Appl. Math.* **2016**, *109*, 75–81.
27. Meenakshi, A. Paired Equitable domination in inflated graph. *Int. J. Innov. Technol. Explor. Eng.* **2019**, *8*, 9–16.
28. Meenakshi, A.; Mythreyi, O. Applications of Neutrosophic social network using max product networks. *J. Intell. Fuzzy Syst.* **2023**, *45*, 407–420. [CrossRef]
29. Mahapatra, R.; Samanta, S.; Pal, M. Applications of edge colouring of fuzzy graphs. *Informatica* **2020**, *31*, 313–330. [CrossRef]
30. Bershtein, L.; Bozhenyuk, A.; Rozenberg, I. Optimum allocation of centers in transportation networks using fuzzy graph bases. In Proceedings of the 8th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT-13), Milano, Italy, 11–13 September 2013; pp. 270–275.
31. Mahapatra, T.; Ghorai, G.; Pal, M. Fuzzy fractional coloring of fuzzy graph with its application. *J. Ambient. Intell. Humaniz. Comput.* **2020**, *11*, 5771–5784. [CrossRef]
32. Mahapatra, R.; Samanta, S.; Pal, M.; Lee, J.G.; Khan, S.K.; Naseem, U.; Bhadoria, R.S. Colouring of COVID-19 affected region based on fuzzy directed graphs. *Comput. Mater. Contin.* **2021**, *68*, 1219–1233. [CrossRef]
33. Lewis, R. *A Guide to Graph Colouring*; Springer: London, UK, 2016.
34. Malaguti, E.; Toth, P. A survey on vertex coloring problems. *Int. Trans. Oper. Res.* **2010**, *17*, 1–34. [CrossRef]
35. Galinier, P.; Hamiez, J.P.; Hao, J.K.; Porumbel, D. Recent advances in graph vertex coloring. In *Handbook of Optimization: From Classical to Modern Approach*; Springer: Berlin, Germany, 2013; pp. 505–528.
36. Lih, K.W. The equitable coloring of graphs. *Handb. Comb. Optim.* **1998**, *1*, 2015–2038.
37. Leighton, F.T. A graph coloring algorithm for large scheduling problems. *J. Res. Natl. Bur. Stand.* **1979**, *84*, 489. [CrossRef]
38. Furmanczyk, H.; Kubale, M. Equitable coloring of graphs. *Contemp. Math.* **2004**, *352*, 35–54.
39. Samanta, S.; Pramanik, T.; Pal, M. Fuzzy colouring of fuzzy graphs. *Afr. Mat.* **2016**, *27*, 37–50. [CrossRef]
40. Chang, G.J.; Chen, S.H.; Hsu, C.Y.; Hung, C.M.; Lai, H.L. Strong edge-coloring for jellyfish graphs. *Discret. Math.* **2015**, *338*, 2348–2355. [CrossRef]
41. Kishore, A.; Sunitha, M.S. *On Injective Coloring of Graphs and Chromaticity of Fuzzy Graphs*; LAP Lambert Academic Publishing: Saarbrücken, Germany, 2016.
42. Rosyida, I.; Indrati, C.R.; Sugeng, K.A. A new approach for determining fuzzy chromatic number of fuzzy graph. *J. Intell. Fuzzy Syst.* **2015**, *28*, 2331–2341. [CrossRef]
43. Romanov, A.Y.; Myachin, N.M.; Lezhnev, E.V.; Ivannikov, A.D.; El-Mesady, A. Ring-Split: Deadlock-Free Routing Algorithm for Circulant Networks-on-Chip. *Micromachines* **2023**, *14*, 141. [CrossRef] [PubMed]
44. Li, Z.; Shen, R.; Yi, M.; Song, Y.; Wang, X.; Du, G.; Huang, Z. Hotspots Reduction for GALS NoC Using a Low-Latency Multistage Packet Reordering Approach. *Micromachines* **2023**, *14*, 444. [CrossRef]