Selected Economic Time Series Analysis Using the Fuzzy Linear Regression

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ABSTRACT

The adequacy of mathematical models of economic systems is reduced by the complexity of their structures, the number of parameters and influencing factors. The mathematical regression model assumes that the structure and functional dependence of the input and output variables of the modeled system is precisely defined. However, real systems are complex and indeterminate, and their adequate models must formalize their vague phenomenon. Artificial intelligence methods use fuzzy set mathematics and fuzzy logic approaches to synthesize models of indeterminate systems. We provided our research of defined fuzzy linear regression models using data series of economic variables, namely the evolution of the discount rate, inflation rate and the rate of unemployment between 2019 and 2021. These data series were chosen with regard to the selected economic cycle before, during and after the Covid-19 pandemy. It is precisely due to the cyclical development of the economy that some level of uncertainty and vagueness of data of monitored variables is manifested. Results of the work reflect outputs of the proposed fuzzy regression model of indeterminate variables during the selected time series. These confirmed the assumptions of the authors that there is a mutual interdependence between the selected economic variables, in particular the amount of the discount rate in relation to the inflation rate, the amount of the inflation rate in relation to the rate of unemployment and thus the amount of discount rate in relation to the rate. The existence of time lags in deciding on economic policy measures and their subsequent implementation was also confirmed in all cases, even during the analyzed time series of three years. Only variable unemployment behaved less standardly, as its essence in many respects lies outside of purely pure market mechanism and is under the influence of market inelasticity, legal measures, free movement of labor in the EU, etc.

Keywords: *fuzzy set, fuzzy linear regression, genetic algorithms, time series, discount rate, inflation, unemployment*

JEL Classification: *C22, C51, C65, B22, B23*

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1. INTRODUCTION

Linear regression is a basic and widely used type of predictive analysis of the interdependence of two or more variables. Today, the regression analysis is used in practically every area of economics and applied science.

Linear regression is a mathematical method used to define a set of points on a graph with a straight line. The points representing the measured data are assumed to have their x-coordinates exact, while the ypsilon-coordinates may be subject to random error, while we assume that the dependence of the variables on the individual axes can be graphically expressed by a straight line. If we interpolate the resulting measured points with a straight line, then when subtracting m the graph, there will be a divergency between ypsilon value of the measured point and ypsilon value lying on the straight line. The essence of linear regression is to find such a straight line that the sum of the squares of these deviations is as small as possible [1].

There is not a strong need for a lot of statistics and mathematics for a well and normally functioning business, as well as the system of the national economy. Most relationships are straightforward. Interdependence looks like a straight line in the graph, from the mathematical point of view it is a linear function. If two variables are linked by such a dependency, the value of one of them - dependent variable - could be calculated using the other - independent variable. In essence, it is a „simple" variant of complicated structural models, where we use a series of regression coefficients to assess the links between a large number of variables, which often appear simultaneously at the level of explained and explanatory.

Although linear regression has many practical uses in economics and applied sciences, in practice its use encounters a number of problems, some of which are not very certain and some of which cannot be removed from the analysis [2]. The most common problem is a small data set, whether it is a small number of values of individual variables or their short observation over time [3].

In classically conceived linear regression, the point is that relationships among dependent and independent variables are clearly determined and are just as clearly interpreted and discussed afterwards. Another problem is therefore the vagueness of the data and not always completely certain and clearly defined relationships between them. The interpretation of individual variables, their different content according to static processing, etc. can also be conceived differently [4].

The time lags of individual variables, their measurement over time and the delay in their acquisition and interpretation are a special and distinctive range of issues in linear programming. This is related to the aforementioned issue of small data sets and working with these data. The human point of view, the creation of one‘s own linear regression model and interpretation of results are indispensable, which burdens the accuracy of the outputs.

Since at least 2019, we have also witnessed sudden reversals in the economy and economic policy, which makes the regression outputs imprecise, indeterminate, and sometimes misleading. The so-called black swan enters the observation system, which indicates an event or fact that cannot be predicted, cannot be counted on, and cannot be prevented in advance [5]. When observing economic events in recent decades, we come to the clear conclusion that such phenomena - referred to as black swans - are increasing.

Through its methods, statistics tries to describe things as accurately as possible and draw accurate conclusions. However, economic and social science variables in general can never be measured exactly. If we were to get down to very small scales, we wouldn‘t be able to cope with a 100% accurate description anyway, because on the one hand, the description would be very complex and difficult to understand, and its reliability would be very low.

It is possible to describe the environment, where uncertainty is omnipresent, with vague linguistic terms, even if we want to manage a system or simply predict its behavior. In the 1970s, L. A. Zadeh came up with fuzzy set theory, which affects precisely these vague - fuzzy – concepts [6].

The application of fuzzy set theory has been a great contribution in the field of modeling complex uncertain systems [7]. The development of the indeterminate regression model is the development of the model of vagueness, using the formalization of uncertainty rather than numerical intervals [8]. Regression models reflecting the vagueness of the modelled systems using approaches of fuzzy set theory are called fuzzy regression models [9], [10], [11], [12]. The indeterminate nature of the fuzzy regression model is represented by the fuzzy output values and the fuzzy regression coefficients in the form of specialized fuzzy sets - fuzzy numbers [8]. Modern and powerful artificial intelligence methods are used to identify the structures and parameters of fuzzy models [13].

The fuzzy regression analysis presented in this paper is built on analysis of time-specific data series of economic variables. The independent variable is the amount of the interest discount rate (DIS) and the dependent variables are the amount of inflation rate (INF) and amount of the rate of unemployment (UNE) in the Czech Republic in the years 2019 to 2021. These data were selected with regard to their topicality and with regard to the global economic downturn due to the Covid-19 pandemic. The data with which the study works were obtained from the official database of the Czech National Bank (DIS) and the Czech Statistical Office (INF and UNE).

The selection of the selected macroeconomic variables was made with regard to real practice in the area of the state's economic policy and also with regard to their seasonality. Discount rate is interest rate announced by central banks. For this rate, the central bank provides discount loans to commercial banks, these are mostly short-term loans with a maturity of up to 3 months. Through this rate, the central bank regulates and moderates the amount of short-term loans on the interbank market, i.e. the price of money.

The increase in the discount rate is an anti-inflationary measure, as it increases the cost of servicing the loan, increases repayments, and thus dampens the demand for loans. And even if commercial banks can obtain credit resources elsewhere, primarily from their clients in the form of deposits, this is a very fundamental intervention within the framework of the discount policy. A decrease in the discount rate works in the opposite direction – it reduces the amount of installments and makes loans more affordable, thus supporting the demand for them.

The price of money and its quantity in circulation is directly related to the level of the inflation rate. Inflation is always and everywhere a monetary phenomenon, in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output [14]. Even if inflation rate reacts to adjustment of the central bank's interest rates with a certain time delay, the mutual interrelationship is quite obvious and scientifically proven. When adjusting interest rates, central banks consider other circumstances and connections in addition to the rate of inflation, such as the cost of mortgages, the effect on the rate of economic growth, etc., however, inflation is the primary and priority goal of monetary policy.

The situation regarding unemployment and its rate is also quite complex. If the central bank raises interest rates, it generally makes investment more expensive, which can lead to a decline in economic growth and an increase in unemployment. However, the labor market can behave non-standardly here as well. The cause may be low elasticity of the labor market, low wages elasticity, strong effect of labour unions, the setting of the social system, unemployment benefits, the existence of the gray economy, etc. These connections will be carefully discussed in the results of the work.

The following section of text is organized as follows. Chapter 2 concisely presents the principal concept and concept of fuzzy regression linear analysis, chapter 3 is devoted to the presentation of fuzzy regression model used in the work and a proper fuzzy regression analysis series of the selected macroeconomic variables. The article traditionally ends with the Conclusions section.

2. FUZZY REGRESSION ANALYSIS OF ECONOMIC TIME SERIES

The development of the indeterminate regression model is the development of the model of vagueness [8]. Regression models reflecting the vagueness of the modelled systems using approaches of fuzzy set theory are called fuzzy regression models [9], [11], [12]. The indeterminate nature of the fuzzy regression model is represented by the fuzzy output values and the fuzzy regression coefficients in the form of specialized fuzzy sets - fuzzy numbers [8]. The fuzzy linear regression model has the opportunity to express not only the analytical linear approximation of multivariate functions, but also the size of its uncertainty (vagueness, fuzziness) in the form of an indeterminate possibility area.

Fuzzy regression modelling

The shape of fuzzy linear regression model is given by

$$\widetilde{Y} = \widetilde{A}_0 x_0 + \widetilde{A}_1 x_1 + \ldots + \widetilde{A}_n x_n = \sum_{i=0}^{n} \widetilde{A}_i x_i \qquad i = 0,1,\ldots,n \tag{1}$$

where $(\widetilde{A}_0, \widetilde{A}_1, \ldots, \widetilde{A}_n)$ are fuzzy regression coefficients (fuzzy numbers). The fuzzy number $\widetilde{A}$ is defined using its triangular shape membership function $\mu_{\widetilde{A}}(x)$ - Figure 1a

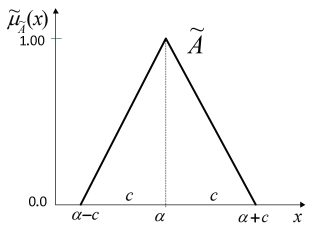


Figure 1a. Fuzzy regression coefficient $\widetilde{A}$

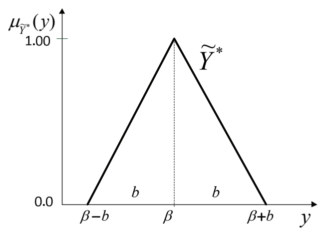


Figure 1b. Estimated fuzzy value $\widetilde{Y}^*$

where α is the mean value (core) of fuzzy number $\widetilde{A}$ and c is a half of the width of the carrier bearing $\widetilde{A}\{\alpha, c\}$.

The output variable $\widetilde{Y}$ of fuzzy regression model (1) is fuzzy number defined using the triangular membership function – Figure 1b. The estimated value $\widetilde{Y}^*$ is defined in the form $\widetilde{Y}^*\{\beta, b\}$, respectively.

The observed value $\widetilde{Y}^0$ is defined in the form $\widetilde{Y}^0\{y^0, d\}$. Estimated value β is the mean value (core) of estimated output fuzzy number $\widetilde{Y}^*$ and *b* is a half of the width of the carrier bearing $\widetilde{Y}^*\{\beta, b\}$.

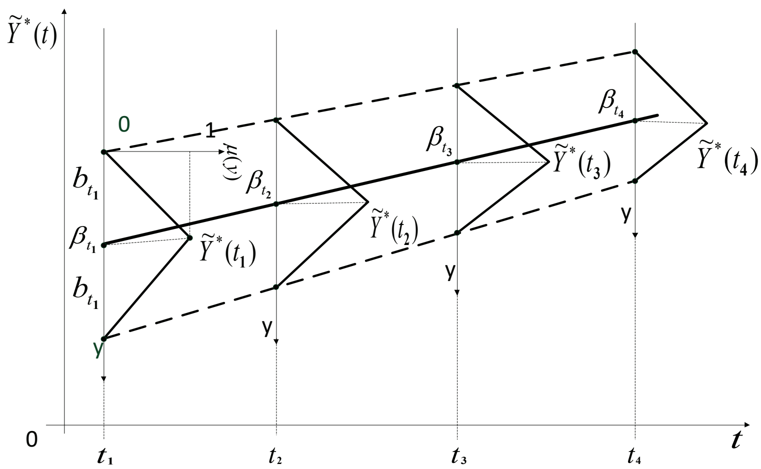
Finding values α_i and c_i as searched parameters of fuzzy regression coefficients $\widetilde{A}_i$ is defined as an optimization issue (see next).

Time series fuzzy regession modelling

The graph of a one-dimensional fuzzy regression function we can see in Figure 2 together with the appropriate linear approximation and the possibility area of the estimated fuzzy output $\widetilde{Y}^*$

Fuzzy Linear Regression Function

Figure 2.



The fuzzy time series regression model has the ability to express its trend and seasonal cycles, respectively. The fuzzy linear regression model of a time series trend is given

$$\widetilde{Y} = \widetilde{A}_0 + \widetilde{A}_1 t \qquad t = 1, 2, \ldots \tag{2}$$

The value of a seasonal deviation in every month MSD (as fuzzy number) is calculated for each year $r = 1,2,...,L$ and for each month k = 1, 2, …, 12 as the difference between the trend value and the actual value to be estimated

$$MSD = (\widetilde{Y}^0_{r,k} - \widetilde{Y}^*_{r,k}),\quad r = 1,2,...,L,\ k = 1, 2, \dots , 12 \tag{3}$$

The central value of fuzzy number MSD is calculated as the difference of the central values $\widetilde{Y}^0_{r,k}; \widetilde{Y}^*_{r,k}$, the fuzziness is calculated as the sum of fuzziness of fuzzy numbers $\widetilde{Y}^0_{r,k}; \widetilde{Y}^*_{r,k}$.

The seasonal cycle is then defined as the time series of 12 seasonal deviations for 12 months. A seasonal deviation for a given month k = 1, 2,..., 12 is calculated as the average value of the month of year r = 1 , 2, ... , L of the considered time series.

$$\widetilde{Y}^*_k = \frac{1}{L}\sum_{r=1}^{L} (\widetilde{Y}^0_{r,k} - \widetilde{Y}^*_{r,k}),\quad r = 1,2,...,L,\ k = 1, 2, \dots , 12 \tag{4}$$

The values of monthly deviations are calculated as fuzzy numbers. The core of fuzzy number $\widetilde{Y}^*_k$ is calculated as the mean difference of the cores, the uncertainty is calculated as the mean of the sum of fuzziness. Thus, we calculate 12 fuzzy numbers, which pass into the timeline of 12 months as a curve of cores and their possibility areas.

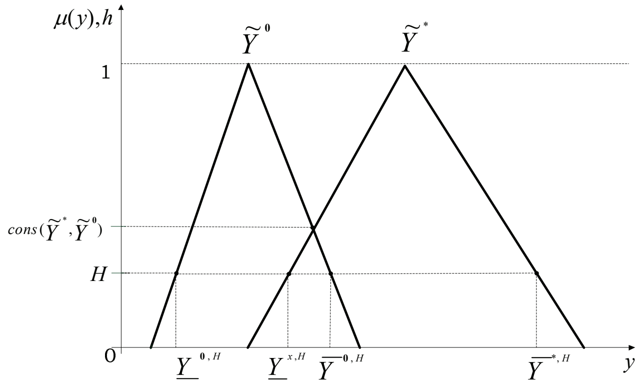
Identification of fuzzy regression model

Fitness of the linear regression fuzzy model to the given data is measured through the Bass-Kwakernaaks´s index H – see Figure 3 [9], [15]. Adequacy of the observed and estimated values is conditioned by the relation (3) – the maximum intersection (consistency) of two fuzzy sets – the estimated $\widetilde{Y}^*$ and the examined $\widetilde{Y}^0$ - must be higher than the aspiration limit H (see Figure 3) [10].

$$\max_y \left\{\mu_{\widetilde{Y}^0}(y) \wedge \mu_{\widetilde{Y}^*}(y)\right\} = Cons(\widetilde{Y}^0, \widetilde{Y}^*) \geq H \tag{5}$$

Adequacy of Linear Regression Model

Figure 3



The requirement on adequacy of the estimated and observed values (5) will be complemented by the requirement on minimum possible total uncertainty of the identified fuzzy regression function

$$\sum_{i=0}^{n}\sum_{j=1}^{m} c_{i,j} \rightarrow \min, \quad i = 0, 1, \dots, n, \;\; j = 1, 2, \dots, m \tag{6}$$

where $i = 1,2,\dots,n$ is the number of input values of the regression function and $j = 1, 2, \dots, m$ is the number of observations. Then we can set the optimization problem - minimization of fuzzy model vagueness (6) under the condition (5). For the identification of the mean value (core) α_i of fuzzy number $\widetilde{A}_i$ the minimization of the fitness function J_1 is defined in the form

$$\min J_1 = \min \frac{1}{m}\sum_{j=1}^{m}\left(y_j^0 - \beta_j\right)^2 \tag{7}$$

For the identification of c_i as a half of the width of the carrier bearing $\widetilde{A}_i$ the minimization of the fitness function J_2 is defined in the form

$$\min J_2 = \min \sum_{j=1}^{m}\sum_{i=0}^{n}\left|c_{j,i}\right| \tag{8}$$

Genetic algorithms

To solve the minimization problem under the conditions (6), (7) we use the genetic algorithms [13]. Genetic algorithms (GA) belong to stochastic heuristic optimization methods. They find their greatest application in solving optimization problems, most often minimizing the so-called fitness function of the solved problem. The general formulation of the GA optimization problem is expressed by the relation

$$\bar{x}_{opt} = \arg opt\{f(\bar{x})|\bar{x} \in C\} \tag{9}$$

where C is the set of admissible solutions, f is the fitness function, $\bar{x}$ is the admissible solution, and $\bar{x}_{opt}$ is the optimal solution sought. What is important is the strategy of finding generate possible solutions in the set C. GA uses a strategy inspired by natural evolution.

GAs are iterative in nature. In individual iterations, one does not work with an isolated solution, but with a so-called *population*. GA works in each iteration with a set of solutions contained in the population and tries to use genetic operations on these solutions to ensure the appearance of increasingly better solutions in subsequent iterations.

Each solution in the population is represented by a so-called *chromosome.* For each chromosome, the value of the objective function J is calculated.

Generation of the population for the next iteration is done based on the existing population by genetic operations called *crossover* and *mutation.* In the crossover operation, the chromosome of the new population is obtained by combining two chromosomes of the existing population. The solutions entering this operation are determined so that the use of each member of the population in the crossing operation is proportional to the quality of its value of its fitness function J. The mutation operation is applied to the population thus obtained and the procedure continues with the next iteration.

The iterative GA run is represented by a sequence of individual solutions whose fitness function gradually decreases (minimizes), i.e. converges to zero. The number of chromozomes in population (PopulationSize) must be set. The GA run is terminated when the size of the reduction in the value of J in the next two iterations falls below the specified limit (FunctionTolerance) or the prescribed maximum number of iterations (MaxGeneration) takes place. Parameter StallGenLimit option controls the number of steps GA looks over to see whether it is making progress. The optimal solution is then the best solution contained in the last iteration step.

In the case of identifying the fuzzy regression model (1), the values of the mean value (core) α_i of fuzzy number $\widetilde{A}_i$ are sought such that the value of the fitness function $J_{1\,i}$ is minimized (5) while minimizing c_i as a half of the width of the carrier bearing $\widetilde{A}_i$ through the minimization of the fitness function J_2 (6). In the case of model identification, two genetic algorithms are used – namely *GA1* for function minimization J_1 searching α_i and *GA2* for function minimization J_2 searching c_i.

The implementation of genetic algorithms *GA1* and *GA2* was performed in the Global Optimization Toolbox of the MATLAB simulation system [20]. The parameters of new population generation, crossover and mutation procedures are predefined in the system. During the tuning of the GA, such values of the GA parameters are sought that minimize the computation time of the genetic algorithm and at the same time minimize the probability of the GA getting stuck in the local minimum of the fitness function value.

The *GA1* tuning results in the following parameter values: *PopulationSize* = 100, *MaxGenerations* = 100, *StallGenLimit* = 200, the *GA1* tuning results in *PopulationSize* = 100, *MaxGenerations* = 10, *StallGenLimit* = 200. The aspiration limit is set to $H = 0.50$.

The running of the *GA1* and *GA2* genetic algorithms is documented by the graphs of the convergence of the size of the fitness function in Figures 4a and Figure 4b.

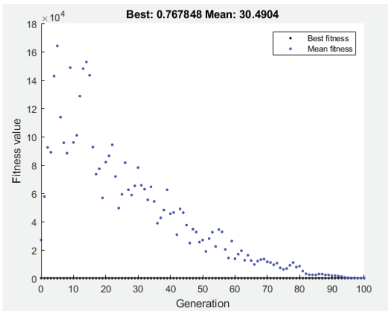


Figure 4a Course of *GA1* convergence

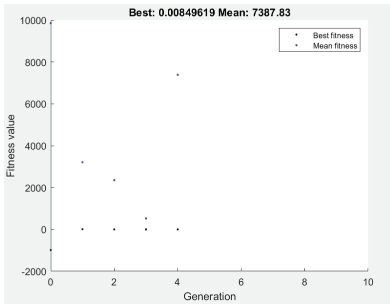


Figure 4b Course of *GA2* convergence

Fuzzy regression models of the trend and seasonal cycles of time series of economic variables bring new information and enable a deeper understanding of their behavior. In the following chapter, time series of selected variables, namely the discount rate, inflation rate and unemployment, are analyzed in this way.

3. ANALYSIS OF SELECTED ECONOMIC VARIABLES

Analysis and modeling of economic variables is a complex and very demanding task, especially in the period of two successive crises: the Covid-19 pandemic and unprecedentedly high energy prices as a result of the war and the tense world situation. This also applies to the analysis of long time series, during which revolutionary changes in economic reality can and do occur, when during the analyzed time period from 2019 to 2021 the global and Czech economy went through pre-crisis, crisis and post-crisis phases and is on the verge of another crisis. This makes the need for fuzzy regression analysis even more important, both for its scientific purpose and for use in practice.

For these reasons, the main economic entities - state, households and companies - are under continuous pressure, to which they submit their decisions in the area of demand, supply, investments, consumption and savings. New conditions force them to make decisions in situations of high uncertainty and high risk. Because of the rapid changes and also because of the new nature of crises, previously best practices and decisions are shown to be inadequate or flawed. Purely economic decisions are thus under the pressure of psychology, behavioral sciences, but also demographic development, immigration, environmental issues, etc.

The time series analyzed in this work consist of twelve monthly data for each of the monitored years 2019 to 2021. The analyzed time period was chosen due to the use of completely up-to-date data and also due to the fact that the first monitored year 2019 was the year before the outbreak of the crisis due to the Covid-19 pandemic, 2020 was the peak of this crisis and 2021 its gradual decline. The monitored period is therefore a classic cycle starting with a day, through a rise, a peak and a fall again. The economic variables were chosen with regard to their mutual connection and interconnectedness, as well as their dominant importance for public and private economy and finance.

The discount rate as an independent variable plays an important and irreplaceable role for both households and companies, as it generally determines the price of capital. Both external resources in the form of operating loans, investment loans and mortgages, as well as deposits and term deposits and financial market products. It also indirectly affects the exchange rate of the national currency through indirect foreign exchange interventions and foreign exchange interest. However, the central bank‘s decision made today will have the greatest impact on inflation in 12 to 18 months, which is why a macroeconomic forecast is important when deciding on rates, which mainly describes and estimates the future development of the economy with a focus on risks and uncertainties.

However, the central bank's measures to change interest rates will be reflected in the economy with a certain delay. The so-called time lags is the total time that elapses between the detection and recognition of economic problems and the moment after the implementation of corrective measures, when these measures begin to have a measurable effect on the economy. Recognition lag is the time that elapses from the actual occurrence of an economic disturbance to its recording and measurement by the relevant institutions. Decision lag is the period necessary to find a consensus and take relevant measures, in this case a change in the discount rate. And finally, the implementation lag is the period of time required for the measures adopted through the national economic transmission mechanism to be reflected in the economy in a desirable way.

Inflation as an analyzed dependent variable is, according to the prevailing opinion mainstream, the main and most significant economic disorder and imbalance. It is the subject of scientific research all over the world and the leading target of central banks. Inflation is very complicated state of economic imbalance, which manifests itself mainly in the growth of common price level of goods and services in the economy and, in some time, in growth of GDP and the decrease in unemployment. The price level expresses the relationship between the total amount of money and the total volume of goods and services for which it is exchanged.

High inflation rates make markets inefficient and causes difficulties for businesses to make long term plans, price calculations and budgets. Inflation can cause a drop in productivity, as businesses are required to advance resources from providing goods and services to financial operations with a goal to hedge against losses. Insecurity about the future value of money reduces investments and savings. Inflation also causes tax inflation – a hidden increase in taxation when nominally higher incomes push taxpayers into higher income tax rate bands.

Unemployment is another dependent economic variable that is analyzed in the paper. It is a condition where there is an imbalance in labour market, when demand for jobs is higher than supply of jobs. A certain level of unemployment is natural, as not all able-bodied people can be employed at any given time. It is related on the one hand to their social behavior and on the other hand to the fluctuation of the real economy around the potential GDP. This simultaneously creates a certain reserve on the labor market and a reservoir of labor for companies and the state. Unemployment is also relatively seasonal, and the labor market is also characterized by a low degree of elasticity. The seasonal, inelastic and also the political and social dimension of unemployment and the behavior of trade unions make this phenomenon a difficult economic construct that is difficult to predict and influence.

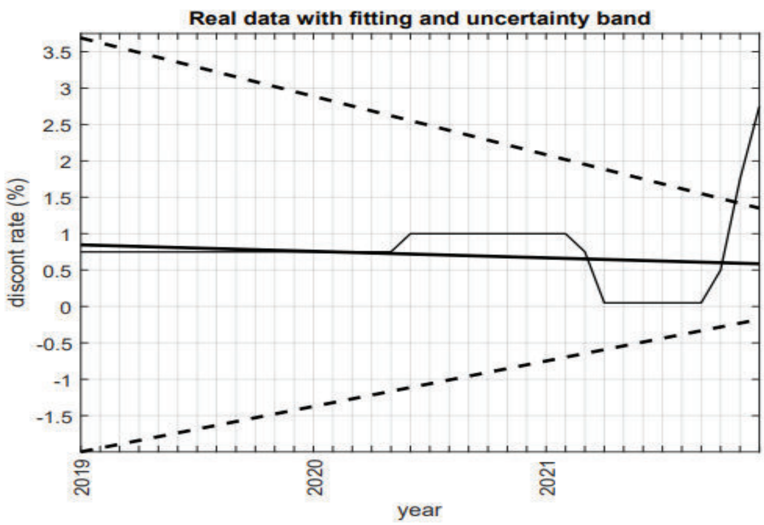
Relationship between dependent analyzed variables - inflation and unemployment - is interconnected, it is inverse. This fact is described by the so-called Phillips curve, which illustrates the inversely proportional relation among the rate of inflation and the unemployment rate. A lower unemployment rate in national economy correlates to higher inflation rate and vice versa. Modern Phillips curve models define two different models: the short run Phillips curve and the long run Phillips curve. It is due to the fact, that in the short run, there is general a fair inverse relationship between inflation and unemployment rate. In the long period of time, however, this phenomenon does not hold, and the whole economy might returns to natural rate of unemployment, regardless on the rate of inflation. Since the presented paper deals with a period of 3 years, i.e. a medium-term period, we will try to confirm the existence of the Phillips curve using the fuzzy linear regression method.

The presented fuzzy linear regression of economic variables was performed using classical algorithms in Global Optimal Toolbox of MATLAB software [20].

The results are presented as fuzzy regression models of time series for the discount rate (DIS; figures 5 and 6), the inflation rate (INF; figures 7 and 8) and unemyployment (UNE; figures 9 and 10). These figures (5 – 10) reprezent their fuzzy trends and fuzzy seasonal cycles.

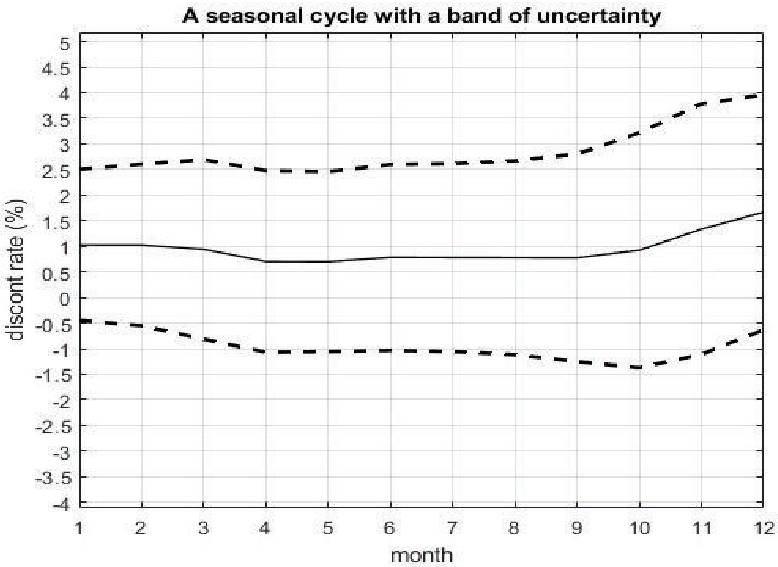
Discount Rate – Fuzzy Linear Regression Function

Figure 5



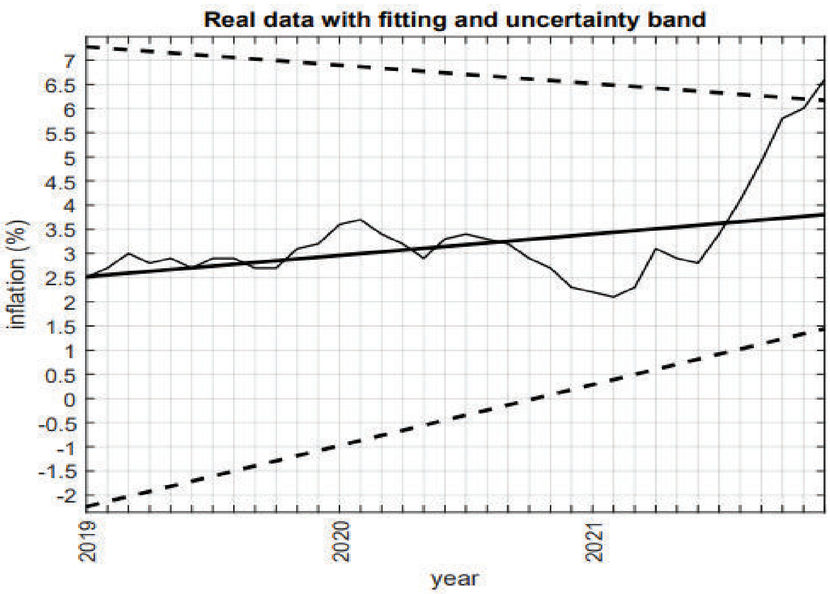
Discount Rate – Fuzzy Seasonal Cycles Function

Figure 6



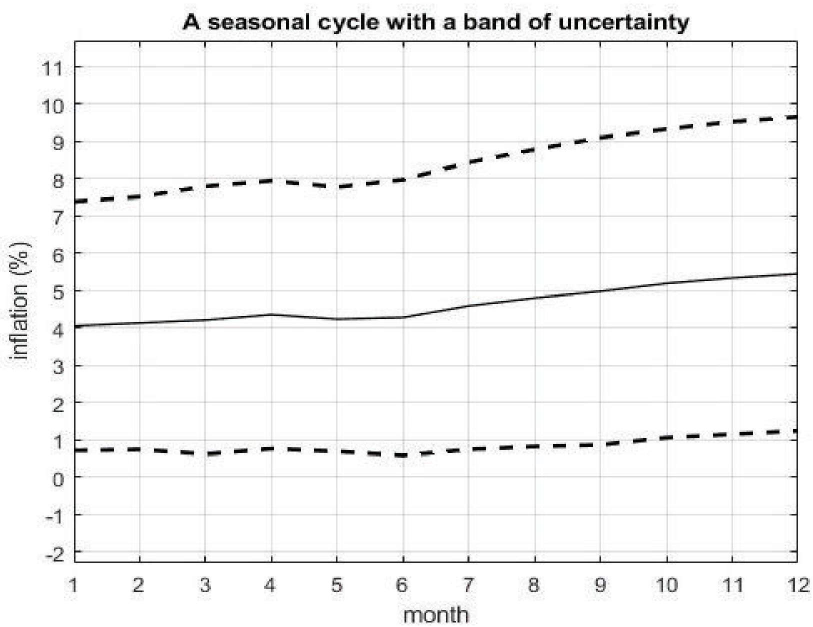
Inflation – Fuzzy Linear Regression Function

Figure 7



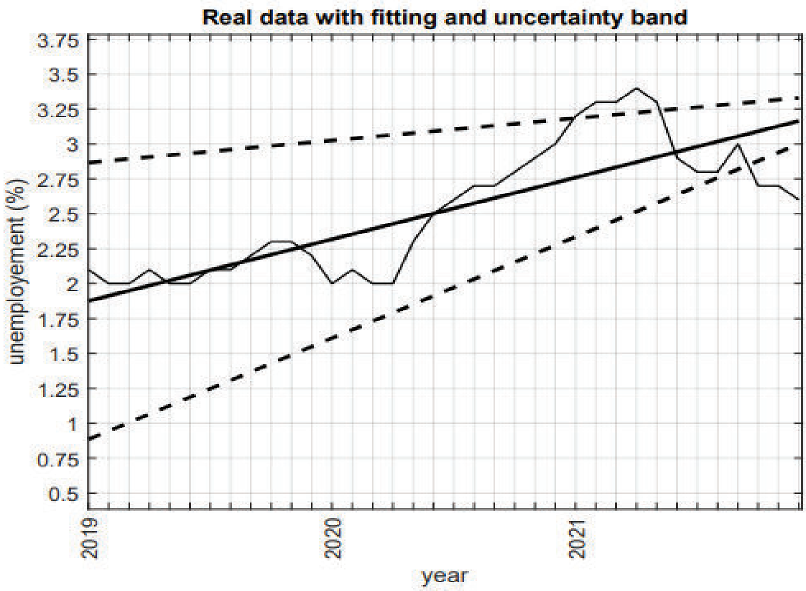
Inflation – Fuzzy Seasonal Cycles Function

Figure 8



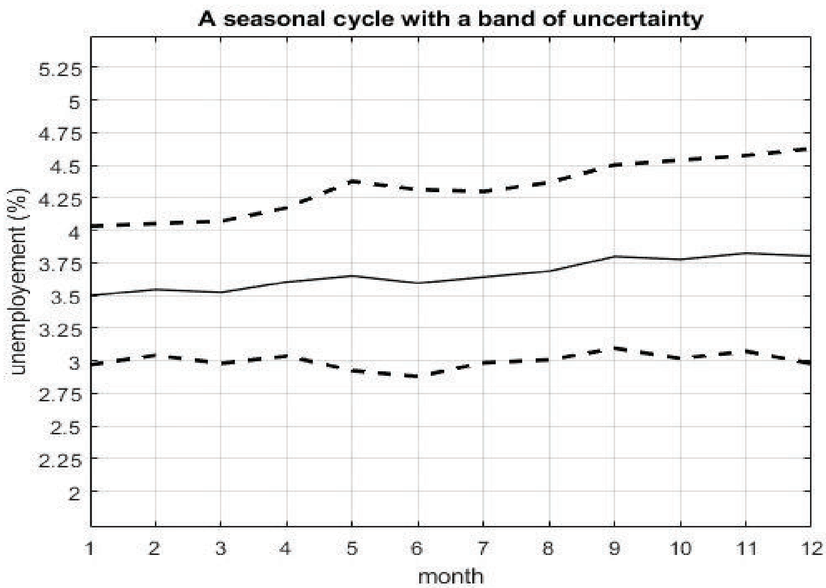
Unemployment – Fuzzy Linear Regression Function

Figure 9



Unemployment – Fuzzy Seasonal Cycles Function

Figure 10



The results of the fuzzy linear regression of the analyzed quantities (DIS, INF and UNE) showed in the monitored period of time series 2019 – 2021 the interrelatedness and dependence of the selected quantities, but in some cases they showed a demonstrable degree of vagueness and fuzziness.

The DIS variable ranged from 0.05 % to 2.75 % in the monitored period, with considerable fluctuations over the three years monitored. Its amount followed the course of crisis caused by pandemy of Covid-19. For approximately half of the observed period - since its beginning - its amount reached 0.75 %. If we start with the assumption that the Covid-19 pandemic started in the spring of 2020, then with a time delay of 1 year, the central bank sharply reduced the discount rate to the so-called technically zero level of 0.05 %. A delay of 1 year corresponds to the cumulative sum of the recognition and decision lags mentioned above. The rate cut was supposed to support economy affected by a sharp drop in demand due to the government‘s administrative measures. In the end of analyzed time series, i.e. in the end of 2021, central bank proceeds to gradually increase the DIS again up to 2.75 %, due to the onset of accelerating inflation, which still persists in the economy. DIS is generally characterized by a significant degree of volatility, which is also evident from the regression results and which accelerates at the end of the monitored time series period. This is clearly evident from the figure showing real data with fitting and uncertainty band. From the point of view of monetary theory, it can be assessed that the behavior of the central bank in times of crisis and in times of rising inflation was standard and corresponded to the mainstream of monetary policy in developed countries.

The INF variable shows a value above the central bank‘s inflation target throughout the monitored period. It was 2 % throughout the monitored period with a tolerance band of ± 1 %, with the ideal and desirable rate of inflation being close to and below the targeted 2 %. In most of the monitored period, however, the inflation rate exceeded the tolerated 3 %, and by the end of the monitored period, it was already very significant. The mandate of the central bank was exceeded twice at the end of 2021, or even three times. The linear regression of the variables demonstrates the interconnectedness of DIS and INF in two respects. Firstly, the data of both variables develop in the same time series trend throughout the monitored period, and secondly, a time delay in the implementation of monetary policy measures was confirmed in this case as well. The regression clearly shows a lag of about half a year in the increase in DIS after INF started to rise. The seasonal monthly cycle with a band of uncertainty clearly illustrates the dependence of the INF variable on the DIS variable and safely demonstrates the time delay of DIS after INF throughout the observed period. Thus, the traditional concept of monetary policy and the use of interest discount rates as a basic and standard monetary policy tool was confirmed. From the linear regression, it can also be proven that if the INF is relatively unchanged in the long term and is within its inflation target, central bank does not make sudden changes, or even any changes in the field of interest rate policy or monetary policy at all.

Variable UNE - unemployment is calculated by the unemployment rate, what is the number of people who are unemployed as a percentage of total labour force (the total number of people employed added to those unemployed). UNE variable in seasonal cycles with a band of uncertainty graph has an increasing trend, which can be explained by the period of the Covid-19 pandemy crisis, especially since the beginning of 2020. Within the annual cycle, there is a classic, albeit slight, fluctuation of the unemployment rate depending at the time of year. Since unemployment rate is calculated from total amount of labour force, i.e. also seasonal employees in agriculture, construction, transport, etc., the UNE shows a cyclical character despite the crisis period of the time series. On the other hand, cyclicality is not significant, mainly as a result of global warming, a longer business cycle, the introduction of new technologies, etc.

When comparing real year-on-year data with fitting and uncertainty band, it is possible to demonstrate - in the crisis period from 2020 - a clearly interdependent inversely proportional relationship between the variables INF and UNE. The measured data and the result of the linear regression confirm the progress within the so-called Phillip‘s curve during the observed crisis period. The Phillip‘s curve is an empirical relationship between two quantities, namely the inflation rate and the unemployment rate, with these variables having a stable and at the same time inverse relationship. It states that with the growth of the real product over time comes inflation, which in turn leads to the creation of jobs and a decrease in the unemployment rate.

Unemployment rate did not exceed its natural level, which ranges between 4 and 5 %, throughout the monitored period of time series. In the monitored period, the value of UNE was the highest at the beginning of 2021 at 3.4 %. With a defined delay, INF growth did occur, but not above the level that would immediately be the cause of the rising inflation. Therefore, the so-called NAIRU vehicle - Non-Accelerating Inflation Rate of Unemployment - was monitored as a certain level of unemployment that occurs in the economy and that does not cause an increase in the inflation rate. Thus, if unemployment is within the limits of the NAIRU, inflation is unchanged. The NAIRU often represents a balance between the level of output and the stability of the labour market.

The regression curves of the INF and UNE values clearly demonstrate an inverse character. While the UNE rate is highest at the beginning of 2021, the INF value is the lowest at the beginning of this year. The value of INF reached 2.1 % (the lowest) during this period, but the value of UNE reached 3.4 % (the highest) with a delay of 2 months.

Other influences that affect the labor market also play a significant role in the sometimes non-standard course of the UNE regression. This is, for example, the inelasticity of the labor market caused by the inflexibility of wages and also the legal protection of employees given by the Labor Code. Employee unions also play their role, protecting employees as the weaker party in labor and legal relations. Last but not least, it is also necessary to count the free movement of labor force within the EU, which is currently among the basic freedoms of the single market in the EU. Employees from other EU countries come to the national economy, and Czech employees also leave to work abroad. There were approximately 650,000 people in the Czech Republic during the monitored period. foreign workers, which, given the total number of the labor force of 6.8 million, constitutes over 9 %. Geopolitical influences on the labor market are therefore unavoidable.

4. CONCLUSION

Mathematical linear regression is a statistical method used to gain a prescription using which we will be capable to predict the value of one variable from knowledge of the value of another variable on condition, that there is a causal relationship between these two variables. It is a statistical tool for modeling a linear trend for data showing a linear distribution around a defined trend and for which we assume interdependence among dependent and independent variables. Prerequisite for regression and subsequent application of data is that the source data - variables are clearly defined, their collection was reliable, they are conclusive, the time series are long enough, etc. Problems can arise precisely in these cases, when the data is unreliable, ambiguous, vague, comes from from sources that are difficult to compare with each other, time series are insufficient, etc.

To increase the adequacy quality of complex real systems modelling, the new methods of artificial intelligence are used. In the presented paper, we proposed and defined vague data as a source for fuzzy sets, and using principles of fuzzy modelling we created fuzzy linear regression model. We defined uncertainty of data sets and regression models as vague parameters. Using a genetic algorithms, we identified fuzzy data coefficients of fuzzy regression model. The linear approximation of generated vague data was used, which the work presents analytically and graphically.

In the time series from 2019 to 2021, data on the development of the discount rate (DIS), inflation rate (INF) and the rate of unemployment (UNE) were modeled in this way. Fuzzy regression model considered their mutual relationship, cyclicality and seasonality. The results were presented with the assumption of time lags, which occur by default in the transmission mechanism of monetary and fiscal policy.

DIS is an independent variable, it is the monetary vehicle of the central bank, with which the bank determines the price of money, especially short-term credit sources. Its regulatory effect and effectiveness in influencing the rate of INF was observed and proven in all three years of the monitored time series. And that even taking into account the aforementioned delays as well as exogenous and foreign inflationary influences. It has also been observed that as DIS rises, the price of money rises, cooling the economy and slowing the growth rate of real GDP. The fuzzy regression analysis performed in this paper demonstrated the interdependence of these variables in the classical structure of the market economy.

Partial interdependence was also demonstrated in relation between amount of DIS and the rate of unemployment (UNE). Changes in the amount of DIS affected the behavior of households and firms in relation to consumption and investment, which had a consequence in the amount of UNE. The price of resources, i.e. money, affects the real GDP through consumption and investments and thus the UNE, which was proven in all three monitored years. However, the relationship between DIS and UNE was looser, due to the existence of market disturbances in the labor market, market distortions, inflexibility of the labor market, free movement of labor to the Czech labor market, etc. However, the dependence and connection between the amount of INF and the amount of UNE was proven, and inversely proportional. Built on time series analysis from 2019 to 2021, the presented paper confirmed the existence of a long-term Phillip‘s curve in the environment of the Czech economy. State interventions in the economy also played a special role - the level of independence of the central bank and the setting of DIS rate, the system of subsidies and interventions, the inflexibility of the labor market and the protection of employees, i.e. various non-market interventions, which may be one of causes of the entirely standard and fuzzy development of selected measured economic quantities at every moment of the monitored period.

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