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A new route to fast accurate design tool for open planar circuit analysis

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SUMMARY

Spectral Domain Method (SDM) is one of the best known techniques for developing full-wave design tool at the operating frequency of interest. However full-wave analysis requires more computer power and takes time to yield results. The aim of this paper is to enhance SDM to make it as fast as the simpler methods. The SDM analysis requires the definition of the unknown current distribution on the metallisation of the circuit. The minimisation of the number of basis functions required is crucial to the efficiency of the technique, therefore, sub-gridding, the inclusion of a priori knowledge of current distribution by using precalculated current basis functions and re-mapping of precalculated basis function are employed in the present implementation.

***Key words**: spectral domain method, planar circuits, sub-gridding, current basis functions, re-mapping, priori-knowledge, precalculated current basis function.*

1. INTRODUCTION

Improvements in technology have always affected society as well as their standards of living. The last few decades have witnessed advances in communication and computer technology (referred to collectively as information technology). These developments have transformed society from the *industrial* to the so-called *information* society [1, chapter 1].

One of the key technologies in an information society is the means of information exchange. All means of modern communication rely on the modulation of electromagnetic waves which are broadcast around the world through the atmosphere or transmitted along the cable from one point to another. Therefore the increase in demand for data transmission necessitated communication systems to be moved upward to less-used frequency bands with more available bandwidth since lower frequency bands can provide only a limited amount of the spectrum and are already heavily used.

The rapid increase of higher frequency usage has created a demand for accurate design tools. A design tool which allows "right first time" prediction is invaluable to the realisation of cost-effective research and development. The accuracy of the design tool is often balanced against the computational effort required. Existing packages allow rapid design, but their accuracy is not sufficient for today's applications. Alternatively there are highly-accurate analysis tools available, but they require a large amount of computer resources.

The demands of the design engineer require a technique which is accurate, yet retains the interactive design capabilities of the simpler techniques. High speed computers do not solve the problem, because the complexity of the circuits to be designed and the computational overhead required for accurate modelling will outstretch any development in computer technology. Therefore a design tool, which reverses the trend for the requirement of ever-increasing computer resources, is required.

The Spectral Domain Method (SDM) has been chosen to fulfil these requirements and to analyse open planar microwave circuits. It is a very popular method but has been criticised for being computationally intensive, due to the need for the formation and inversion of a large matrix. Another criticism is that SDM is severely limited in the complexity of the metallisation patterns which it can handle. The results of this research overcome the limitations mentioned above and allow an interactive design tool to be developed for the computer aided design of passive open planar microwave circuits as well as planar antennas.

The choice of current basis functions for the Method of Moments solution is crucial to the efficiency of the technique. To obtain an efficient numerical solution, which converges for a small number of basis functions, the set of basis functions must represent the actual current distribution as closely as possible. If they are not chosen carefully, then a large number of basis functions will be required for convergence. However this criterion must not sacrifice the range of the metallisation patterns which can be analysed.

The basic SDM formulation can be applied to simple structures in an efficient way, but problems exist in the application of the method to complex circuits. An entire domain basis function which is valid on the entire metallisation of the circuit can be used to define the unknown current distribution of simple structures such as an infinite microstrip line [2, 3], a microstrip resonator [4] or a hairpin resonator [5]. However the implementation of an arbitrary metallisation pattern requires sub-domain basis functions which are valid on a portion of the metallisation. It has been shown and commonly used that the rooftop function [6, 7] as a sub-domain current basis function allows an irregular shaped microwave circuit's current to be defined. But, the use of the rooftop functions to define the unknown current distribution on the metallisation of the circuit results in a large number of basis functions for convergence. As the size of the impedance matrix to be calculated for each spot frequency is proportional to double of the number of basis functions, it is therefore very important to minimise the set of basis functions required.

It was proposed by Railton and Meade [5, 6] that precalculated basis functions be used to reduce the number of basis functions required. They applied the proposed technique to complex shielded planar microwave circuits showing that inclusion of a priori knowledge of the current distribution for realistically complex structures reduces the size of the impedance matrix to be calculated. The precalculated basis function is derived from the sub-domain basis functions, which are identical apart from a shift in origin, to exploit the benefit of using the FFT. In addition the number of rooftop functions plus 1 in both directions must be an integer power of 2. These conditions restrict the geometry of the circuit and the size of the rooftop function.

The present implementation applies all available enhanced current basis functions in the literature. The effectiveness of rooftop functions is improved by introducing sub-gridding in the spectral domain. The sub-gridding is used in the sense that sizes of the rooftop functions are defined as functions of their locations. In this paper the precalculated function is derived from two different precalculations. It can be either a linear combination of rooftop functions with precalculated coefficients, or a current wave with a precalculated wave number as explained in Subsection 5.4. In both cases, there is no restriction of geometry. This paper also describes the combination of sub-gridding, precalculation and re-mapping of the precalculated functions.

To measure the performance of the present implementation, a simplified two-dimensional version of SDM is used to derive the dispersion characteristics of infinite microstrip lines, for which numerical results are shown in Section 7. This allows the calculation of the surface current distribution of the microstrip line, which is going to be used in the full three-dimensional version of the technique. In addition, complex threedimensional circuits are analysed by using the proposed technique and numerical results are compared with available measurements and published data in Section 8.

2. CLASSES OF CURRENT BASIS FUNCTIONS

Generally, there are four classes of current basis functions:

- ***Entire domain***: A basis function which is valid for an entire metallisation of the circuit, for example the resonant mode of a microstrip hairpin resonator [5].
- ***Sub-domain***: This is the smallest current basis function used in this paper. The piecewise linear rooftop is preferred and defined as a product of two separable functions, a step function in the direction perpendicular to the current flow and a triangle function in the direction of the current flow.
- ***Region***: A basis function is defined for a section of the metallisation which is bigger than a subdomain but smaller than an entire domain.
- ***Transfer***: This is a special case of a region basis function and used to join neighbouring regions. Transfer functions can be either sub-domain basis functions which overlap into neighbouring regions or a precalculated basis function.

Throughout this contribution, the above classes of current basis functions are used to define the unknown current distribution of an arbitrary shaped metallisation pattern. Further details of each class of basis functions are given in the following sections.

3. ENTIRE DOMAIN BASIS FUNCTION

An entire domain current basis function is a function which is valid over the entire metallisation of the circuit. The definition of the unknown current distribution for a simple circuit can be possible by using just a single entire domain function, but the implementation of an arbitrary metallisation pattern requires sub-domain current basis functions. Therefore the entire domain basis function is only mentioned very briefly in this paper.

4. SUB-DOMAIN BASIS FUNCTION

4.1 Definition of sub-domain basis function: rooftop basis function

The definitions of the known basis functions which are non-zero only on the metallisation of the circuit of interest in the space domain and of which analytical Fourier transforms exist are required for the Method of Moments solution. The same set is also used as a set of weighting functions for metallisation. The choice of basis functions is very important for the efficiency of the technique and they must approximate the unknown current distribution as closely as possible.

Arbitrary shaped planar microwave circuits have been modelled using rooftop functions as sub-domain basis functions by numerous authors [5-7]. The geometry of a rooftop function is illustrated in Figure 1 and is defined in space domain by:

$$J_{xn}(x,z)=\begin{cases}1-\dfrac{|x-x_n|}{l_x} & \begin{matrix} x_n-l_x \le x \le x_n+l_x \\ z_n-l_z \le z \le z_n+l_z \end{matrix} \\ 0 & \text{otherwise}\end{cases} \quad (1)$$

$$J_{zn}(x,z)=\begin{cases}1-\dfrac{|z-z_n|}{l_z} & \begin{matrix} x_n-l_x \le x \le x_n+l_x \\ z_n-l_z \le z \le z_n+l_z \end{matrix} \\ 0 & \text{otherwise}\end{cases} \quad (2)$$

where x_n and z_n are the coordinates of the centre of the n^{th} rooftop and l_x and l_z are the sizes of the grid where the rooftop function is located. The rooftop function is defined by two separable functions: a triangle function in the direction of the current flow and a step function in the direction perpendicular to the current flow. Figure 1 shows a rooftop function to model the current flow in the z-direction as two separable functions, the step function which is $2l_x$ wide and the triangle function which is $2l_z$ wide. Thus the function overlaps in both directions.

The two-dimensional Fourier transform of the rooftop function is given by:

$$\boldsymbol{J}_{xn}(k_x,k_z)=\frac{4}{k_x^2k_zl_x}\left(1-\cos(k_xl_x)\right)\sin(k_zl_z)\,e^{j(k_xx_n+k_zz_n)} \quad (3)$$

$$\boldsymbol{J}_{zn}(k_x,k_z)=\frac{4}{k_xk_z^2l_z}\sin(k_xl_x)\left(1-\cos(k_zl_z)\right)e^{j(k_xx_n+k_zz_n)} \quad (4)$$

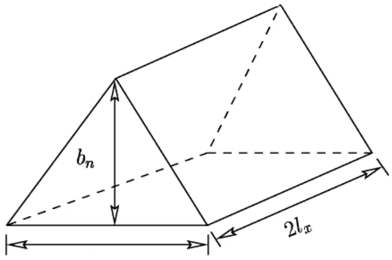


Fig. 1 Z-directed rooftop function

The rooftop functions define an area of metallisation of the circuit by forming a grid of overlapping rectangular sub-domains as illustrated in Figure 2. Each function overlaps in its neighbouring rooftops in both directions. Therefore the x and z-directed components are co-located and are defined on the same grid.

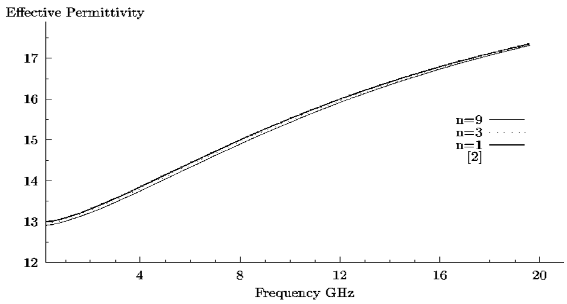


Fig. 2 Illustration of sub-domain basis function gridding

4.2 Sub-gridding

The unknown current distribution on an arbitrary shaped metallisation can be defined as a combination of the rooftop functions, but this approach results in a large number of rooftop functions for convergence. The modelling of a corner singularity shown in Figure 3 requires even more rooftop functions in order to get accurate results for the entire circuit if all rooftop functions are defined as identical, apart from a shift in origin. This is because fine rooftops are required to model the singularity and the same sized rooftops must be used to model the entire circuit. In this case, unnecessarily fine rooftops would be used in the region where no discontinuity exists. This is a major drawback of the technique and a reduction in the number of basis functions required would be an improvement.

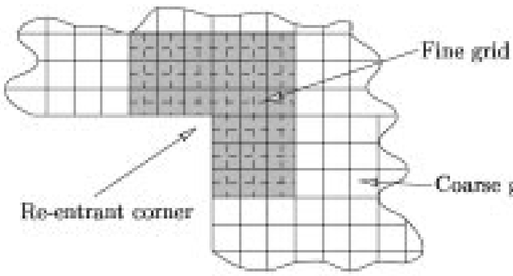


Fig. 3 Illustration of sub-gridding

In contrast to the formulation for boxed structures [5], the FFT is not employed for the calculation of the impedance matrix elements in this implementation. The restrictions for the FFT usage no longer apply and alternative calculation methods are introduced by the author in Ref. [8]. The author has defined the size of the rooftop function in Ref. [9] as a function of its location, allowing sub-gridding. Fine rooftops are used where rapid change in current distribution occurs and a coarse size where only slow changes in current distribution occur, as illustrated for the re-entrant corner discontinuity in Figure 3. The fine grid is located next to the discontinuity and a coarse grid is defined where the current distribution is less dependent on its position.

It must be emphasised that the ratio of the fine and coarse grid size, unlike sub-gridding in FDTD, can be any arbitrary number. It is not necessary to have this ratio as an integer number. The discontinuity region is treated individually and only fine rooftops are used to connect the fine grid region to adjacent coarse grid regions as illustrated in Figure 4.

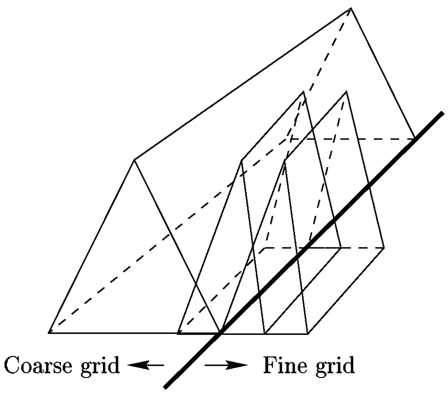


Fig. 4 Connection of coarse and fine grids

4.3 Comparison with other implementations

In accordance with this implementation, sub-domain basis functions are used to allow the definition of an arbitrary shaped metallisation of the circuit in Refs. [5-7, 10-12]. The methods generally use a rectangular gridding system to define the location of the sub-domain function. There are two forms of rectangular grid basis functions implemented, piecewise linear [5-7, 10] and piecewise sinusoid [11, 12]. Both are referred to as rooftop functions, but in this research this term is reserved for the previous piecewise linear definition. The latter piecewise sinusoidal sub-domain rooftop functions are commonly used in the space domain Integral Equation Method (IEM), because the Fourier transform is not required. The piecewise linear rooftop functions are more suited to the spectral domain formulation, because the Fourier transform of the piecewise linear rooftop function is simpler than its piecewise sinusoidal counterpart.

As mentioned in Subsection 4.1, a rooftop function is defined as two separate functions: a triangle function in the direction of the current flow and a step function in the direction perpendicular to the current flow (see Figure 1). As illustrated in Figure 2, a rooftop function in this paper is $2l_x$ wide (l_x is the grid size in x direction which is the direction perpendicular to the current flow) compared to l_x in Refs. [7, 10]. The rooftop function overlaps in both directions identical to Refs. [5, 6] and in contrast to Refs. [7, 10] in which it overlaps only in the direction of the current flow.

A novel feature of this implementation is that, the size of the rooftop function is defined as a function of its location as explained in Subsection 4.2. To the author's knowledge this method has not been used elsewhere, for example in Refs. [5, 6], the grid sizes must be identical to exploit the benefit of using the Fast Fourier transform (FFT).

5. REGION BASIS FUNCTIONS

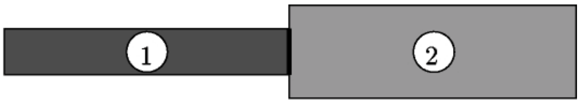
5.1 Division of a circuit into region

Following Ref. [13], the analysis of an arbitrary shaped metallisation pattern is based on dividing a complex metallisation into regions. In each region, a set of basis functions which are called region basis functions are defined . The division of the metallisation of the circuit into regions is central to the form of the set of basis functions and hence the criteria governing this are crucial to the efficiency and feasibility of the method.

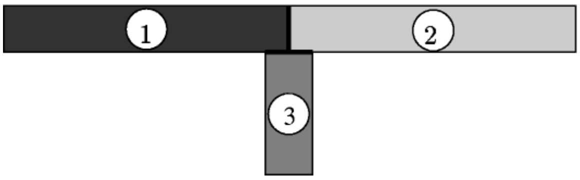
The division of a metallisation into regions is such that a region is defined between each discontinuity. This is illustrated by an example of microstrip circuits in Figure 5. The rules of division are not strict and can be adjusted to fit each specific example. As shown in Figure 5(a), the discontinuity has been sandwiched between two regions for a microstrip step. If more than two regions are neighbours of a discontinuity, a "*T*" junction is produced by the position of the stubs as shown in Figure 5(b). For a *90°* bend discontinuity, three possible sub-divisions have been illustrated in Figure 5(c). A special case of basis functions is used to join two regions and can be either rooftop functions, or precalculated current basis functions as described in detail in Section 6.

5.2 Definition of region basis function

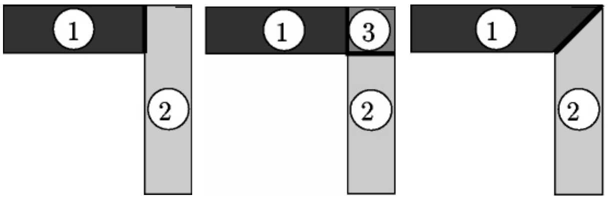
The inclusion of a priori knowledge of the surface current distribution is essential to the efficiency of the technique. A technique which allows precalculated basis functions to be used was presented by Railton in Ref. [5] for shielded microstrip structures and is applied to the open planar structures in this present implementation of SDM.



(a) Microstrip step in width



(b) Microstrip stub



(c) *90°* bend

Fig. 5 Illustration of division into regions

Railton [5] introduced the concept of precalculated basis functions for the modes of a microstrip resonator. This was extended by Meade [6] to allow a set of arbitrary basis functions to be defined as:

$$\boldsymbol{J}(r) = \sum_{m=1}^{M} a_m \psi_m(r) \tag{5}$$

where $\psi_m(r)$ is the current distribution of m^{th} region basis function. A precalculated region basis function ($\psi_m(r)$ in Eq. 5) is defined in Refs. [5, 6] as a linear combination of the rooftop functions which are identical apart from a shift in origin as:

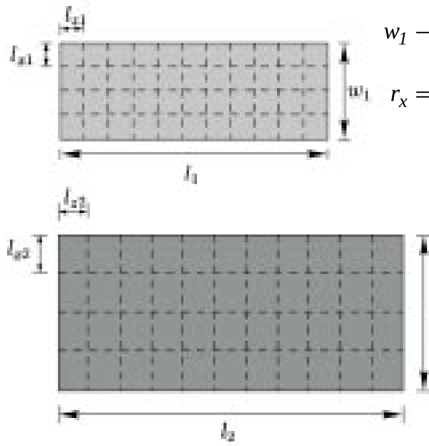
$$\psi_m(r) = \sum_{n=1}^{N} b_{mn} \boldsymbol{R}_n(r) \tag{6}$$

In addition, the number of rooftop functions (N in Eq. (6)) plus 1 must be chosen to be an integer power of 2 to allow the use of the FFT [13, page 32]. These limitations restrict the geometry of the circuit. Since the FFT is not applicable to open structures, the rooftop functions are defined in this paper as functions of their locations, allowing sub-gridding in the region where rapid changes occur in the current distribution. Some precalculated basis functions are also produced from an analytical current wave function with precalculated propagation constants. Details of this procedure are given in Subsection 5.4. The final set of current basis functions for an arbitrary shaped metallisation pattern ($\boldsymbol{J}(r)$ in Eqs. (5)) is a combination of the region and rooftop functions.

5.3 Re-mapping of precalculated basis function

The use of precalculated basis functions and the inclusion of a priori knowledge reduce the number of basis functions which are required to define the unknown current distribution on the complex metallisation of the circuit. An arbitrary shaped metallisation pattern is analysed by dividing the metallisation into regions for which basis functions are easy to determine. The region basis functions are calculated and stored in the library to be used in the full-wave analysis. In some cases, the dimensions of the region to be modelled by the precalculated basis function can be slightly different from the region for which the stored function has been previously calculated. In this case, new precalculated basis functions are required to be calculated, which is computationally intensive.

It is proposed and illustrated in Figure 6 that scaling of the function from one grid size to another one in either the x or z direction may be used to avoid the need for re-calculating new basis functions. However, the accuracy of the procedure is inversely proportional to the variation in length. The adaptation of the mapping is possible to more complicated regions, but care must be taken in its application to complex regions.



$$w_1 \rightarrow w_2 \;\; and \;\; l_1 \rightarrow l_2$$

$$r_x = \frac{w_2}{w_1} \;\; and \;\; r_z = \frac{l_2}{l_1}$$

$$l_{x2} = r_x \cdot l_{x1}$$

$$l_{z2} = r_z \cdot l_{z1}$$

Fig. 6 Illustration of re-mapping

5.4 Microstrip line region basis function

A straight microstrip line with no discontinuity is the simplest region and its basis function is the simplest region basis function. The standing current wave set up on a microstrip line can be modelled by a set of line modes [13]. In this case, the variation of the current distribution in the space domain is given by:

$$J_z(x,z) = J_z(x) \overbrace{\left(\sum_{n=0}^{N} a_n^{'} \cos\left(\frac{n\pi z}{L}\right) + \sum_{n=1}^{N} a_n^{''} \sin\left(\frac{n\pi z}{L}\right) \right)}^{J_z(z)} \tag{7}$$

$$J_x(x,z) = J_x(x)\overbrace{\left(\sum_{n=0}^{N} b_n' \sin\left(\frac{n\pi z}{L}\right) + \sum_{n=1}^{N} b_n'' \cos\left(\frac{n\pi z}{L}\right)\right)}^{J_x(z)} \tag{8}$$

where L is the length of the microstrip line and $J_s(x)$ ($s=z,x$) is the current distribution along the line perpendicular to the current flow and precalculated by using the two-dimensional version of the technique. The Fourier transform of the current basis function in the direction perpendicular to the current flow ($\boldsymbol{J}_s(k_x)$) is a sum of the Fourier transforms of the corresponding rooftop functions with precalculated coefficients. The Fourier transforms of the line modes in the direction of the current flow as defined in Eqs. (7) and (8) are given by:

$$\boldsymbol{J}_z(k_z) = \sum_{n=0}^{N} a_n' \frac{2(-1)^n \frac{n\pi}{L}}{k_z^2 - \left(\frac{n\pi}{L}\right)^2} \cos\left(\frac{k_z L}{2}\right) + j\sum_{n=1}^{N} a_n'' \frac{2(-1)^n k_z}{k_z^2 - \left(\frac{n\pi}{L}\right)^2} \cos\left(\frac{k_z L}{2}\right) \tag{9}$$

$$\boldsymbol{J}_x(k_z) = j\sum_{n=0}^{N} b_n' \frac{2(-1)^n k_z}{k_z^2 - \left(\frac{n\pi}{L}\right)^2} \cos\left(\frac{k_z L}{2}\right) + \sum_{n=1}^{N} b_n'' \frac{2(-1)^n \frac{n\pi}{L}}{k_z^2 - \left(\frac{n\pi}{L}\right)^2} \cos\left(\frac{k_z L}{2}\right) \tag{10}$$

Each line mode can be represented by a precalculated basis function with x and z components. The number of modes required to fully describe a given length of line are restricted by the minimum guide wavelength (λ_{min}) to be modelled. Therefore the minimum line mode wavelength (λ_N) must be less than the minimum microstrip guide wavelength required:

$$l_N < l_{min} \tag{11}$$

where λ_N is the wavelength of the highest order line mode and the wavelength of the n^{th} line mode is given by:

$$\lambda_n = \frac{2L}{n} \tag{12}$$

The microstrip guide wavelength (l_{min} in Eq. (11)) is taken as the wavelength for an infinite microstrip line and calculated by using the two-dimensional version of SDM. This allows the user to specify the maximum operating frequency and, accordingly the number of modes and the related line mode current basis functions are automatically calculated.

At high frequencies, a large number of line modes are needed to accurately model the unknown current distribution of a microstrip line. For this reason, a wave function is proposed in the direction of current flow as a current basis function for the microstrip line of interest and given in the space domain by:

$$\boldsymbol{J}_z(z) = \begin{cases} e^{-jk_n z} & \frac{L}{2} \le z \le \frac{L}{2} \\ 0 & otherwise \end{cases} \tag{13}$$

$$\boldsymbol{J}_x(z) = \begin{cases} e^{-jk_n\left(z-\frac{\pi}{2}\right)} & \frac{L}{2} \le z \le \frac{L}{2} \\ 0 & otherwise \end{cases} \tag{14}$$

where k_n is the precalculated wave number of an infinite microstrip line, where the width is identical to the microstrip line region, and L is the length of the microstrip line region. The precalculation of k_n is carried out by the two-dimensional version of the technique. The same precalculated transverse basis functions ($J_z(x)$ and $J_x(x)$ in Eqs. (7) and (8)) are used in the direction perpendicular to the current flow. The Fourier transforms of Eqs. (13) and (14) are given by:

$$\boldsymbol{J}_z(k_z) = \frac{2}{(k_z - k_n)} \cos(k_z - k_n) \tag{15}$$

$$\boldsymbol{J}_x(k_z) = \frac{j2}{(k_z - k_n)} \cos(k_z - k_n) \tag{16}$$

Note that the centre of the microstrip line region is located at the origin and can be shifted by multiplying the Fourier transforms by $e^{jk_z z_{off}}$, where z_{off} is the offset from the origin.

5.5 Resonant modes

A microstrip resonator shown in Figure 7 is another special case of the region basis function which includes a priori knowledge of the end effect on the current distribution. This microstrip line is opencircuited at both ends and can be completely modelled by a set of resonant modes. The variation of the current distribution for the microstrip resonator in the space domain is given by:

$$J_z(x,z) = J_z(x)\sum_{n=1}^{N} a_n \cos\left(\frac{n\pi z}{L}\right) \tag{17}$$

$$J_x(x,z) = J_x(x)\sum_{n=1}^{N} b_n \sin\left(\frac{n\pi z}{L}\right) \tag{18}$$

where L is the length of the microstrip resonator and $J_s(x)$ ($s=z,x$) is the transverse current distribution, calculated by the two-dimensional version of the technique. The Fourier transform of the current basis function in the direction perpendicular to the current flow ($\boldsymbol{J}_s(k_x)$) is also a sum of the Fourier transforms of the corresponding rooftop functions with precalculated coefficients, as mentioned in Subsection 5.4. The Fourier transforms of the resonance modes in the direction of current flow as defined in Eqs. (17) and (18) are given by:

$$\boldsymbol{J}_z(k_z) = \sum_{n=1}^{N} \frac{2(-1)^n \frac{n\pi}{L}}{k_z^2 - \left(\frac{n\pi}{L}\right)^2} \cos\left(\frac{k_z L}{2}\right) \qquad (19)$$

$$\boldsymbol{J}_x(k_z) = \sum_{n=1}^{N} j \frac{2(-1)^n k_z}{k_z^2 - \left(\frac{n\pi}{L}\right)^2} \cos\left(\frac{k_z L}{2}\right) \qquad (20)$$

The maximum number of the resonant modes is determined by the criterion given for the line modes in Eq. (12). The origin is in the centre of the resonator and can be shifted by multiplying by $e^{jk_z z_{off}}$, where z_{off} is the offset from the origin.

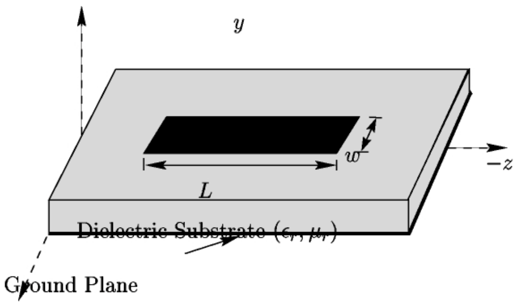


Fig. 7 Microstrip resonator

5.6 Precalculated discontinuity function

The set of line modes and resonator modes are not sufficient to model an arbitrary shaped metallisation pattern including several discontinuities. It is possible to add extra rooftop functions in the area of the discontinuity, similar to Ref. [7], but this requires a relatively high number of basis functions. Moreover, the effect of a discontinuity can be significant along a large portion of a circuit. Therefore, Meade and Railton [6] have introduced precalculated corner basis functions for shielded structures, to include a priori knowledge of the edge and corner singularities in the set of basis functions. In this section, the basic philosophy of the calculation of the precalculated discontinuity functions is given in brief. Further details can be found in Refs. [6] and [13, chapter 4].

This technique has been developed to "extract" the required function from a spot frequency solution for the current distribution of a region, by comparing this with the current distribution for a similar region which does not contain any discontinuity. It has been found by Meade [6] that a single function is required to model the discontinuity over the entire frequency band of interest.

The precalculated discontinuity function depends only on the geometry of the corner itself, not on the surrounding circuitry. Moreover, the function is also independent of substrate thickness, permittivity and permeability. It is not limited to the exact circuit for which it was originally defined, only the relative dimensions of the region and the adjacent metallisation pattern of interest. This reduces the necessity to define new functions for new circuits.

6. TRANSFER FUNCTION

The transition of the current between neighbouring regions is achieved by using either rooftop functions or a set of precalculated current basis functions. The microstrip stub (shown in Figure 5(b)) has been taken as an example and rooftop functions which overlap into adjacent regions are defined for the "*T*" shaped area. This is illustrated in Figure 8, which shows the location of the transfer functions and of the area of overlap into neighbouring regions.

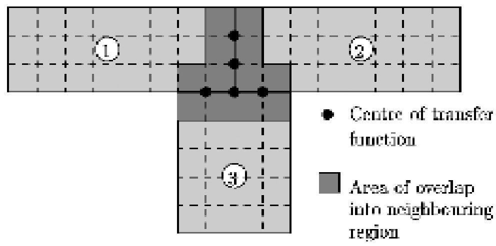


Fig. 8 Illustration of transfer functions for T-shaped junction

7. TWO-DIMENSIONAL NUMERICAL EXAMPLES

7.1 Introduction

Numerical considerations are introduced in this section using the results from the two-dimensional version of the technique. The two-dimensional version of SDM is formulated as an eigenvalue problem to calculate the propagation constant (k_z) of infinite microstrip lines of the form in Figure 9. From this solution, the effective permittivity, the current distribution in the transverse direction, the guide wavelength and the characteristic impedance can be derived. In this two-dimensional analysis, overlapping rooftop functions (given in Subsection 4.1) are used.

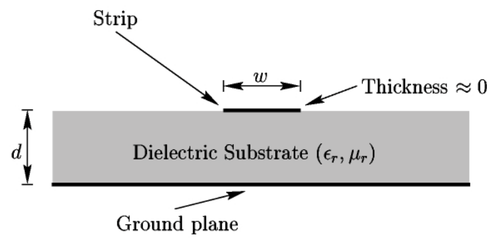


Fig. 9 Infinite microstrip line for two-dimensional version of SDM

7.2 Dispersion characteristics of infinite microstrip line

Dispersion characteristics for microstrip lines are commonly presented as the effective permittivity (ϵ_{eff}) versus frequency defined as:

$$\epsilon_{eff} = \left(\frac{k_z}{k_0} \right)^2 \tag{21}$$

Also, the concept of microstrip guide wavelength is used where:

$$\lambda_g = \frac{2\pi}{k_z} \tag{22}$$

where k_z is the propagation constant.

As an initial test of confidence, the dispersion characteristics for a simple infinite microstrip line are calculated. The tested microstrip line is completely open with a width of *1.27 mm*, on a substrate with a thickness of *1.27 mm* and relative permittivity of 20. Figure 10 is a plot of effective permittivity for a frequency range of *0.1 GHz* to *20 GHz*. The results are compared to the published results of Itoh [2]. A series of curves are also plotted for a different number of basis functions. A convergence to the calculated reference data [2] can be noted.

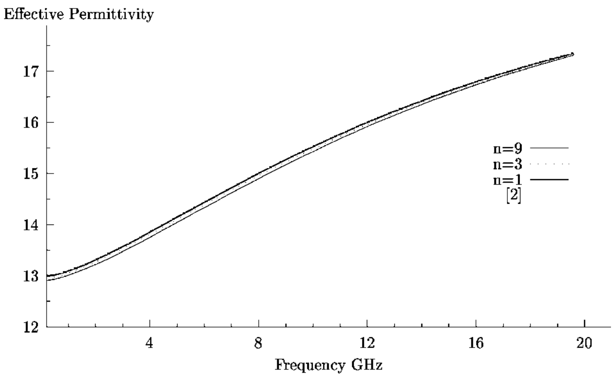


Fig. 10 Dispersion characteristics for the open infinite microstrip line

8. THREE-DIMENSIONAL NUMERICAL EXAMPLES

8.1 Introduction

In this section, three-dimensional example structures are analysed to illustrate and verify the theoretical discussions in the proceeding sections. The numerical results are presented in the forms of resonant frequencies and of scattering parameters. In the three-dimensional analysis, non-overlapping rooftop functions in the direction perpendicular to the current flow are not favoured and only overlapping rooftop functions (see Subsection 4.1) in both directions are used to analyse complex metallisation patterns.

8.2 Precalculated transverse basis function

Inclusion of a priori knowledge of the current distribution reduces the size of the impedance matrix to be calculated. The precalculated transverse function is the first step to include a priori knowledge of the current distribution. It is derived by applying the two-dimensional version of SDM, at a spot frequency and assumed to be valid over a wide frequency band. The choice of the spot frequency is arbitrary, but in practice it is in the middle of the intended operating frequency range. The spot frequency has been chosen to be *5 GHz*. The Fourier transform of the precalculated transfer function is derived from the sum of the Fourier transform of the rooftop functions with precalculated coefficients. In this section, the practical similarity of the results given by a set of rooftop functions and the precalculated transverse basis function is verified. The precalculated function is made up of the same number of rooftop functions in the direction perpendicular to the current flow.

8.2.1 Microstrip Resonator

The microstrip resonator is the first example used to test the similarity of the results given by a precomputed transverse basis function and a set of rooftop functions. This is a useful integrated circuit component at microwave and millimetre-wave frequencies for building filters, oscillators, etc. Before presenting the numerical results to compare the published results and experimental data, it should be noted that the structures used in Ref. [4] for both numerical computations and experiments are scale models of millimetre-wave integrated circuits. In practice, after a circuit has been designed and tested at a low frequency, a miniature structure for millimetre-wave integrated circuits may be obtained by reducing all dimensions of the circuit structure while keeping the ratios of all the dimensions to the wavelength constant. The value of the relative dielectric constant of the substrate is assumed to be unchanged in these two frequency ranges. Since the field distributions in both the scale model and the actual structure are the same, the field problem of millimetre-wave integrated circuit structures is being solved even though the actual operating frequency is in the UHF range.

To test the accuracy of the two approaches, the first resonant frequency of the microstrip resonator on a substrate of thickness *1.27 cm* and of relative permittivity 3.82 is calculated. The calculation of the resonant frequency is given in Refs. [4, 14]. The analysis employs a set of rooftop functions and a precalculated transverse basis function, in which the same number of rooftop functions are used for precalculation by using the two-dimensional version of the technique, in the direction perpendicular to the current flow to test the similarity in accuracy. Both models are compared with experimental and published data [4]. For the first model, a total of 48 rooftop functions as current basis functions are required, whereas the number of basis functions required are reduced to 16 by the use of the precalculated transverse basis function for the same definition ($l_x=\omega/4$ and $l_z=L/9$).

The first resonant frequency versus the length of the resonator is plotted in Figure 11, in which the results obtained by using the precalculated transverse basis function and those obtained using a set of rooftop functions are almost identical. Other theoretical results are based on the Spectral Domain Method by using the entire domain basis function, quasi-TEM and parallel-plate transmission line models. The experiments were conducted by Itoh [4], using a strip with a thickness of *0.254 mm* which is negligibly small compared to other dimensions. As seen in Figure 11, the results for the two kinds of current basis functions are indistinguishable on the graphical figure and the agreements between experimental data and the results computed by the proposed technique are extremely good.

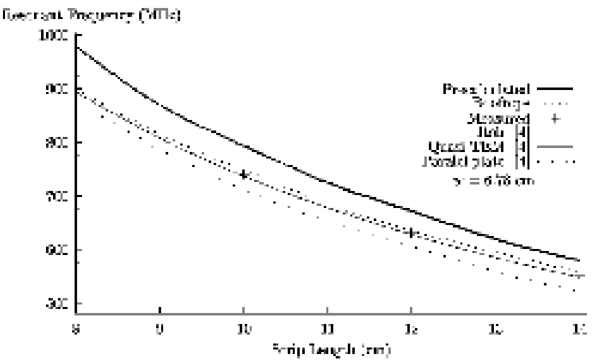


Fig. 11 Comparison of the results from the present method with those obtained from experiments and other methods

8.2.2 Microstrip Gap

In order to further prove that a precalculated transverse basis function is valid for a wide frequency band and gives similar results to a set of rooftop functions, the microstrip gap discontinuity consisting of two identical end-coupled microstrip lines, as shown in Figure 12, is modelled. The dimensions of the circuit are given in Figure 12.

72 basis functions in total (36 x and 36 z components) are required for the sub-domain analysis for the definition ($l_x=l_z=w/4=0.318$ *mm*). The transverse basis function is precalculated for the same definition ($l_x=w/4=0.318$ *mm*) and used as one basis function which is a sum of a number of rooftop functions with precalculated coefficients, as a result 24 basis functions become sufficient for the same circuit. It is emphasised that a plot of S-parameters in Figure 13 clearly shows the agreement of a set of rooftop functions to a precalculated transverse basis function on a wide frequency band over the frequency range *1 GHz* to *20 GHz*, although the precalculated basis function has been calculated at a single spot frequency.

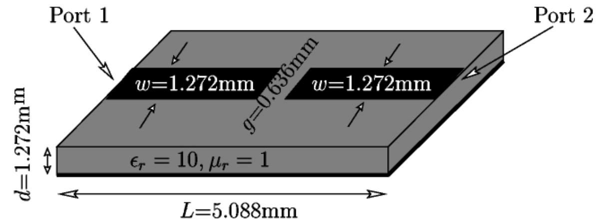


Fig. 12 Microstrip gap layout

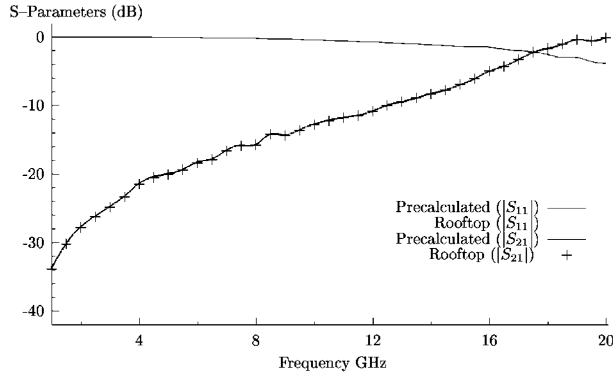


Fig. 13 Plot of S-parameters's magnitude for the microstrip gap discontinuity comparing basis function sets

8.3 Region basis function

The use of region basis functions allows further inclusion of a priori knowledge of current distribution. In this section, several precalculated region basis functions are applied to microstrip circuits and the results are compared with the results yielded by the use of rooftop functions and of precalculated transverse basis functions.

8.3.1 Microstrip line region basis function

The straight microstrip line with no discontinuity is the simplest region and a microstrip line in a finite length shown in Figure 14 is the simplest microwave circuit to test the proposed region basis function discussed in Subsection 5.4. The microstrip line in Figure 14 is of *6 mm* lenght, and of *1.272 mm* width on a substrate of *1.272 mm* thickness. The permittivity and permeability are 10 and 1 respectively.

Three sets of S-parameter results are plotted in Figure 15. These are the results calculated by using 36 rooftop functions for each current components, the results calculated by utilising precalculated transverse basis functions in which case the number of basis functions required are reduced to 6 for the same grid size (l_x=*w/4* and l_z=*L/7*) and the results calculated by using the single proposed microstrip line region basis function given in Subsection 5.4. As seen in Eqs. (13) and (14), the precalculated current wave function is an analytical function with a precalculated wave number and thus different from precalculated functions in Refs. [5, 6].

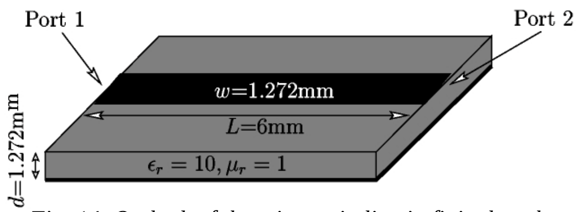


Fig. 14 Outlook of the microstrip line in finite length

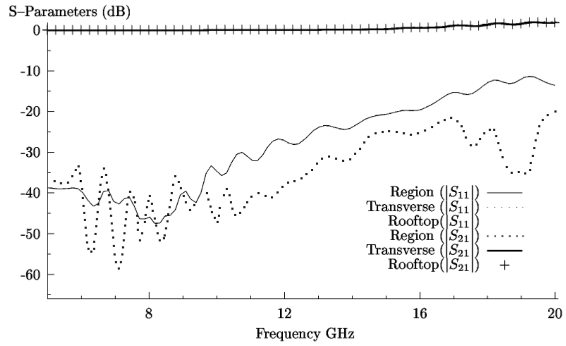


Fig. 15 S-parameters' magnitude for the microstrip line

The accuracy of the analysis is a function of the number of basis functions used, because at high frequencies a large number of rooftop functions or a large number of line modes are required to accurately model the unknown current distribution on the microstrip line. It must be emphasised that a single microstrip line region basis function may give more accurate results than coarse rooftop functions at high operating frequencies.

8.3.2 Resonant mode: edge-coupled filter

As illustrated in Figure 16, the filter consists of a central microstrip resonator and two feedlines with a length *of 12.72 mm* and a width of *1.272 mm*. The substrate has a thickness of *1.272 mm*, relative permittivity 10 and relative permeability 1. The edge-coupled filter is analysed using rooftop functions, precalculated transverse functions and resonant mode region basis functions.

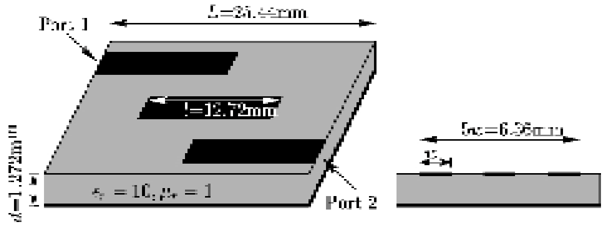


Fig. 16 Microstrip edge-coupled filter

For the resonant mode analysis:

- The transverse current distributions on the microstrip resonator and feedlines are precalculated by using the two-dimensional version of the technique.
- The longitudinal current distribution corresponding to the resonant modes has been precalculated using the three-dimensional version of the technique, but it is not necessary to have an excitation. The resonant frequency is found by seeking the operating frequency which makes the determinant of the impedance matrix identical to zero. The resonant mode current distribution is calculated as an eigenvalue solution of the resonant frequency and stored in the data library. In the 3-D analysis, the precalculated transverse current function can also be utilised. The resulting resonant mode current distributions are shown in Figure 17.

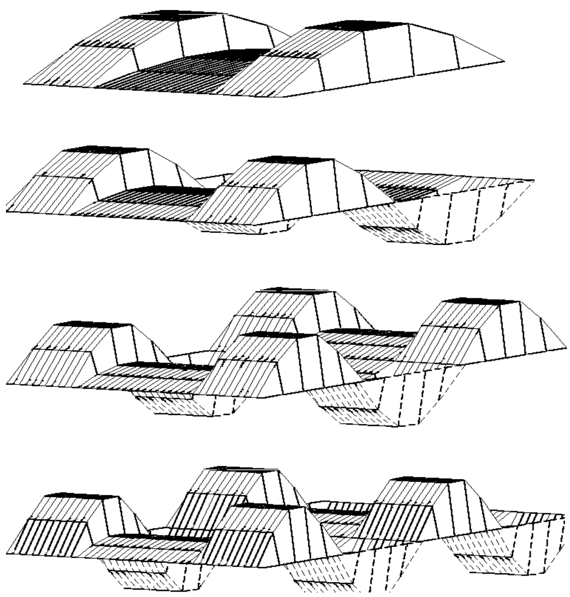


Fig. 17 Current distributions for the first four resonant modes of a strip resonator

- The complete structure is then analysed using two different models. For Model 1, the longitudinal current distributions on the feedlines are assumed to be the same as the current on the resonator as in Eqs. (17) and (18) and shown in Figure 17. However, this does not include all modes existing on the feedlines. Therefore in Model 2, the resonant modes are used only for the middle strip and the precalculated transverse basis function as well as rooftop functions are used for the feedlines in the longitudinal directions. The results from these two models are compared to those obtained from modelling the entire metallisation with rooftop functions and to those obtained using rooftop functions in the longitudinal direction and precalculated basis functions in the transverse direction.

Note that for Models 1 and 2 only one basis function is required to fully describe the strip resonator over the frequency range of interest. In total 114 basis functions (*57 x* and *57 z* components) are required for the same structure if only precalculated transverse basis functions are used. In contrast to this, 342 rooftop functions (*171 x* and *171 z* components) would be required for the basic method in which rooftop functions are used as sub-domain basis functions for equivalent accuracy for the same rooftop sizes ($l_x=0.318$ *mm* and $l_z=0.636$ *mm*). This allows a large saving in computer memory and computational time per frequency. Figure 18 shows the results for a magnitude of S-parameters plotted against operating frequency. Model 2 agrees more closely with the basic rooftop, the precalculated transverse basis function calculation and measured data [15].

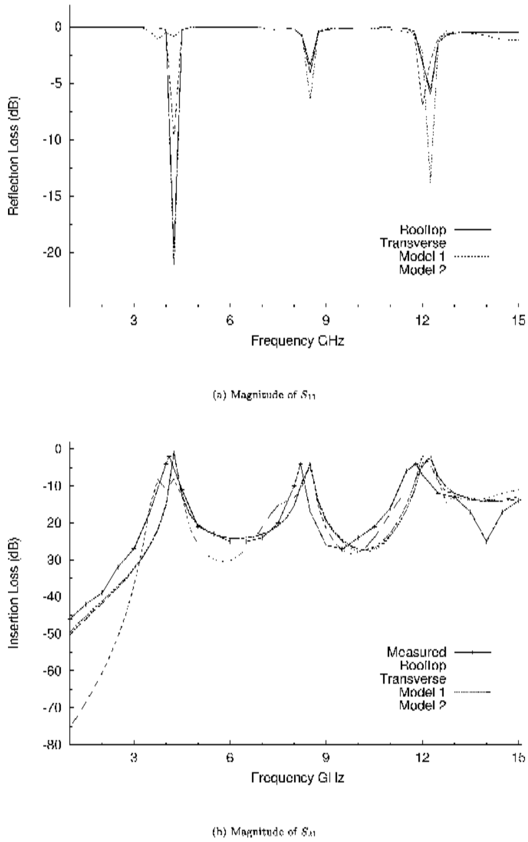


Fig. 18 Plot of S-parameters' magnitude for the edge-coupled filter, comparing basis function sets

8.4 Sub-gridding

As mentioned in Subsection 4.1, the grid sizes must be identical in order to exploit the benefit of using the Fast Fourier transform (FFT) for shielded planar microwave circuits [5]. A major difference between the boxed and the open case is that the discrete Fourier transform is employed in the former, whereas the continuous transform is required for the latter. Therefore the benefits exploited by using the FFT are not available. In this implementation the sub-gridding is employed for the analysis of complex metallisation patterns. The sub-gridding is used so that the sizes of the rooftop functions are defined as functions of their location. Fine rooftops are used next to the discontinuity. Moreover the precalculated current basis functions are used where only slow changes in the current distribution occur. As a test of confidence, the microstrip step discontinuity shown in Figure 19 is taken as an example structure. The planar circuit in Figure 19 is completely open on a substrate with a thickness of *1.272 mm* and relative permittivity 10. The other dimensions are given in Figure 19.

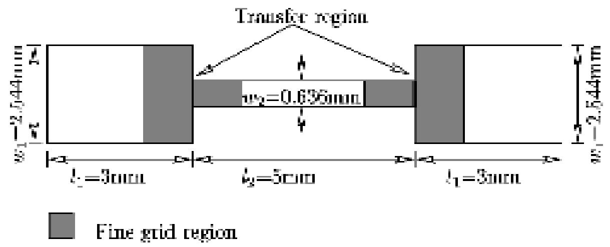


Fig. 19 Microstrip step discontinuity

Three sets of S-parameter results are plotted in Figure 20.

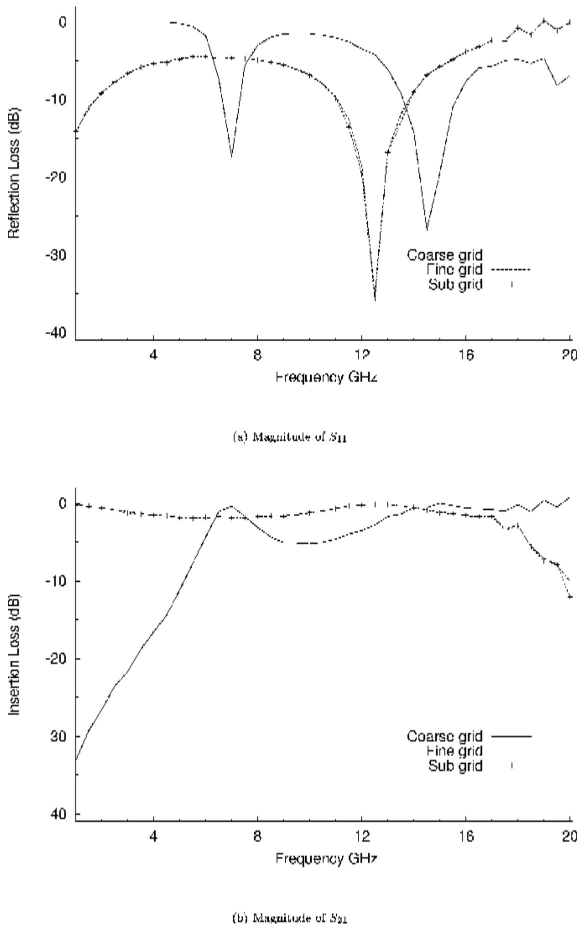


Fig. 20 Plot of S-parameters' magnitude for the step discontinuity

To illustrate the convergence pattern, the circuit in Figure 19 is analysed by using fine rooftop functions, which are 162 in total (*81 x* and *81 z* components) for the definition ($l_x=0.318$ *mm* and $l_z=0.5$ *mm*), for the entire metallisation pattern. For the coarse grid analysis, only three microstrip line region basis functions and two transfer current basis functions, which are fine rooftop functions, overlapping into neighbouring regions, are required. In the sub-grid analysis, the benefits of using precalculated region basis functions are exploited and almost the same accuracy has been achieved with only 42 current basis functions (*21 x* and *21 z* components). For this analysis, fine rooftops are used to model the current distribution in the shaded area, *1 mm* long for each region. The current basis functions required for an accurate analysis are reduced by *74%* and the results are found almost identical to the results obtained by the fine grid analysis.

9. CONCLUSION

The choice of the current basis functions to model the unknown current distribution has been discussed. The class of current basis functions used in this contribution has been given along with their drawbacks. The overlapping sub-domain rooftop functions in both directions have been described and compared to other implementations in the literature. The rooftop functions in this paper have been defined as functions of their locations to allow sub-gridding. The analysis of complex circuits performed by dividing the metallisation into "regions" has been discussed and a set of basis functions for these regions has been described. The example region basis functions such as the microstrip line region, microstrip resonator and precomputed discontinuity functions have been described. To demonstrate and verify the proposed enhancements, the numerical results for a series of two and three-dimensional example microwave circuits have been presented. It has been shown that the precalculated region basis function and sub-gridding schemes can achieve over *70%* saving in the number of basis functions required without limiting the generality and accuracy of the method.

10. REFERENCES

[1] I.J. Croddock, Enhanced numerical techniques for time domain electromagnetic analysis, Ph.D. Thesis, University of Bristol, 1995.

[2] T. Itoh and R. Mittra, Spectral-domain approach for calculating the dispersion characteristics of microstrip lines, *IEEE Transaction on Microwave Theory and Technique*, pp. 496-499, 1973.

[3] T. Itoh and R. Mittra, A technique for computing dispersion characteristics of shielded microstrip lines, *IEEE Transaction on Microwave Theory and Technique*, pp. 896-898, 1974.

[4] T. Itoh, Analysis of microstrip resonators, *IEEE Transaction on Microwave Theory and Technique*, Vol. 22, pp. 946-952, 1974.

[5] C.J. Railton and S. A. Meade, Fast rigorous analysis of shielded planar filters, *IEEE Transaction on Microwave Theory and Technique*, Vol. 40, pp. 978-985, 1992.

[6] S.A. Meade and C.J. Railton, Efficient implementation of the spectral domain method including precalculated corner basis functions, *IEEE Transaction on Microwave Theory and Technique*, Vol. 42, pp. 1678-1684, 1994.

[7] R.W. Jackson, Full-wave, finite element analysis of irregular microstrip discontinuities, *IEEE Transaction on Microwave Theory and Technique*, Vol. 37, pp. 81-89, 1989.

[8] H.H. Balik and C.J. Railton, Fast rigorous impedance matrix calculations in sdm for analysis of open planar structures, *The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Submitted for publication, 2002.

[9] H.H. Balik and C.J. Railton, Sub-gridding in the spectral domain method for the analysis of planar circuits and antennas, Proc. 3rd International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Services, pp. 592-595, 1997.

[10] J.C. Rautio and R.F. Harrington, An electromagnetic time-harmonic analysis of shielded microstrip circuits, *IEEE Transaction on Microwave Theory and Technique*, Vol. 35, pp. 726-729, 1987.

[11] R.W. Jackson and D.M. Pozar, Full-wave analysis of microstrip open-end and gap discontinuities, *IEEE Transaction on Microwave Theory and Technique*, Vol. 33, pp. 1036-1042, 1985.

[12] W.P. Harokopus and P.B. Katehi, Characterisation of microstrip discontinuities on multilayer dielectric substrates including radiation losses, *IEEE Transaction on Microwave Theory and Technique*, Vol. 37, pp. 2058-2066, 1989.

[13] S.A. Meade, The rapid and rigorous mathematical modelling of passive planar microwave circuits, Ph.D. Thesis, University of Bristol, 1994.

[14] T. Itoh and W. Menzel, A full-wave analysis method for open microstrip structures, *IEEE Transactions on Antennas and Propagation*, Vol. 29, pp. 63-67, 1981.

[15] T. Shibata, T. Hayashi, and T. Kimura, Analysis of microstrip circuits using three-dimensional fullwave electromagnetic field analysis in the time domain, *IEEE Transaction on Microwave Theory and Technique*, Vol. 36, pp. 1064-1070, 1988.

NOVI PRISTUP BRZOM I TOČNOM PROJEKTIRANJU U ANALIZI OTVORENOG RAVNINSKOG STRUJNOG KRUGA

SAŽETAK

Metoda spektralne domene (SDM) je jedna od najboljih poznatih metoda za razvijanje punovalnog alata pri željenoj radnoj frekvenciji. Međutim, punovalna analiza zahtijeva veću snagu kompjutora i potrebno je dosta vremena da bi se postigli rezultati. Cilj ovog rada je poboljšati metodu spektralne domene kako bi postala isto tako brza kao i jednostavnije metode. Analiza u kojoj se primjenjuje SDM metoda zahtijeva definiranje nepoznate distribucije struje na metalizaciji strujnog kruga. Svođenje na minimum broja potrebnih baznih funkcija bitno je za efikasnost ove metode te se stoga kod ove primjene upotrebljava: progušćavanje mreže, uključivanje 'a priori' poznavanja distribucije struje, koristeći prethodno izračunate strujne bazne funkcije i njihovo ponovno preslikavanje.

Ključne riječi: *metoda spektralne domene (SDM), ravninski strujni krugovi, progušćavanje mreže, strujne bazne funkcije, ponovno preslikavanje, 'a priori' znanje.*