UDC 519.68:519.85:621.3
Original scientific paper
Received: 13.10.2003.

Computationally efficient solution for multilayer microwave circuits

Hasan H. Balik
Department of Electrical & Electronics Engineering, University of Firat, 23119 Elazig, TURKEY
e-mail: balik@firat.edu.tr

SUMMARY

The Spectral Domain Method (SDM) is widely applied to analyse two layer planar microwave circuits. Adaptation of this technique to multilayer structures requires an efficient derivation of the Green's function. This paper provides an iterative derivation for computer design tool programmers.

Key words: *Spectral Domain Method (SDM), multilayer, iterative solution.*

1. INTRODUCTION

Recently, multilayer planar microwave circuits are becoming essential components of electronic communication systems utilising the microwave region of electromagnetic spectrum because of their potential applications in the compact design of new devices. Meanwhile the rapid increase in the use of higher frequencies for communication and computation over recent years has created a demand for accurate design tools, because the simplified circuit theory which is valid at low frequencies cannot simply be employed for the analysis of the microwave circuits of interest. Existing CAD tools allow rapid design, but accuracy is not sufficient in todays applications. This situation forced the researchers to full-wave techniques.

The high-speed computer has also effected the computation of electromagnetic problems to the point that most practical computations of fields are now done numerically by computers. This is because most of the practical problems in the electromagnetics can be solved numerically but can not be done analytically. Therefore computers are necessary for numerical solutions. As a result the science of numerical computation of electromagnetics is a mixture of electromagnetic theory, mathematics and numerical analysis.

A number of numerical full-wave techniques are reported in the literature for the analysis of multilayer and multiconductor transmission line [l, 2] and microstrip antenna with multiayer substrate [3].

In Refs. [4-6] the authors presented an efficient technique, which was implemented for fast rigorous analysis of two layered open planar microwave circuits in Spectral Domain Technique (SDM).

SDM is a frequency domain technique and a convenient form of the Green's function is available in the spectral domain. But in the case of multilayer structure analysis, it is very important to find an efficient way to derive Green's function. This derivation must be suitable for computer programming as well. The Green's function can directly be derived by solving Maxwell's equation in the spectral domain with suitable boundary condition but extending this procedure for multiple dielectric layers becomes too complicated. A generalised spectral domain Green's function for multilayer dielectric substrates is derived by Das [7] in terms of suitable components of vector electric and magnetic potential. With these vector potentials, the boundary conditions were simplified into equivalent transmission lines problems. Despite this simplification, his technique is found by the author to be still complicated and not much favorable for computer programming.

The iterative derivation technique has been developed by using the immitance approach introduced by Itoh [8]. In this technique a transmission line is assumed to be terminated by another transmission line of different characteristic impedance. In the iterative derivation, the conventional transmission line theory is used to find the characteristic impedance of the corresponding layer. The Asymptotic forms of the Green's function which was proved to be effective for single layer structures [5] is also derived in this paper for the circuits with multiple dielectric layer.

2. DERIVATION OF DYADIC GREEN'S FUNCTION

Although the method may be applied to other printed circuit structures, a simple microstrip resonator shown in Figure 1 is used for the formulation.

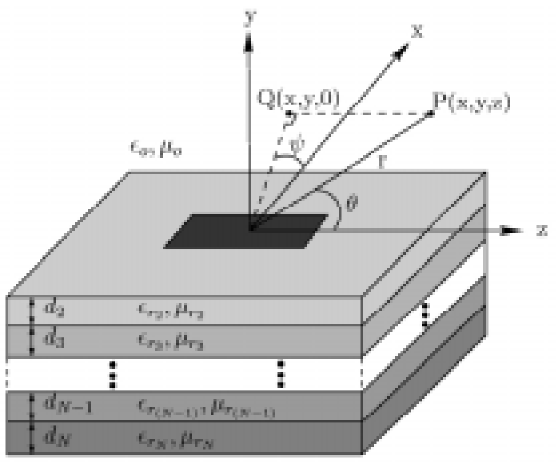

Fig. 1 Cross-section of a multilayered microstrip resonator

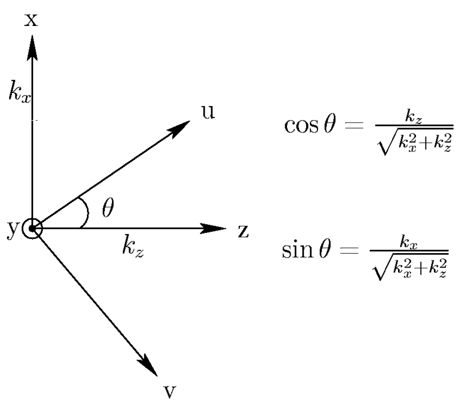

Fig. 2 Coordinate transformation

The basic concept can be understood if the inverse transform of the Fourier transform in the Eq. (1) for the field is examined:

$$\phi(x,z)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(k_x,k_z)e^{-j(k_x x+k_z z)}dk_x dk_z \quad (1)$$

From this expression, all field components are a superposition over k_x and k_z of inhomogeneous (in y) plane waves which are propagating in the direction of θ from the z-axis , where $\theta = cos^{-1}\left(\frac{k_z}{\sqrt{k_x^2+k_z^2}}\right)$. For each θ, waves may be decomposed into TM-to-y ($\boldsymbol{E}_y,\boldsymbol{E}_v,\boldsymbol{H}_u$), and TE-to-$y$ ($\boldsymbol{H}_y,\boldsymbol{H}_v,\boldsymbol{E}_u$) where the coordinates v and u are as shown in Figure 2 and related with (x,z) via:

$$u=z\, sin\theta - x\, cos\theta \quad (2)$$

$$v=z\, cos\theta + x\, sin\theta \quad (3)$$

The current $\boldsymbol{J}_v$ creates only the TM fields, because it is concerned with $\boldsymbol{H}_u$ and likewise $\boldsymbol{J}_u$ creates the TE fields. Therefore, an equivalent circuit for the TM and TE fields can be drawn, as shown in Figure 3. The characteristic impedance in each region is given by:

$$\boldsymbol{Z}_{TM_i} = \frac{\boldsymbol{E}_v}{\boldsymbol{H}_u} = \frac{\gamma_i}{j\omega\in_i} \quad (4)$$

$$\boldsymbol{Z}_{TE_i} = -\frac{\boldsymbol{E}_u}{\boldsymbol{H}_v} = \frac{j\omega\mu_i}{\gamma_i} \quad (5)$$

$$\gamma_i = \sqrt{k_x^2 + k_z^2 - k_i^2} \quad (6)$$

where $i=1...N$ and $k_i^2=\omega^2\mu_i\in_i$. The γ_i is the propagation constant in the y direction in the i^{th} region. All boundary conditions for the TM and TE waves are incorporated in the equivalent circuits. The electric fields $\boldsymbol{E}_v$ and $\boldsymbol{E}_u$ are continuous at $y=0$ and given by:

$$\boldsymbol{E}_v(k_x,k_z,0) = \boldsymbol{Z}_e(k_x,k_z,0)\ \boldsymbol{J}_v(k_x,k_z,0) \quad (7)$$

$$\boldsymbol{E}_u(k_x,k_z,0) = \boldsymbol{Z}_h(k_x,k_z,0)\ \boldsymbol{J}_u(k_x,k_z,0) \quad (8)$$

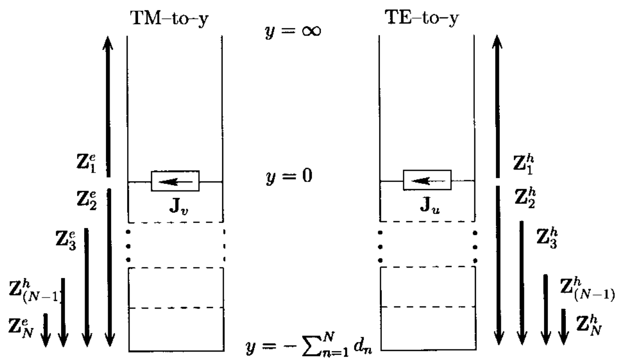

Fig. 3 Equivalent transmission lines for the TM and TE fields

$\boldsymbol{Z}_e$ and $\boldsymbol{Z}_h$ are the input impedances looking into the equivalent circuits at $y=0$ and are given by:

$$\boldsymbol{Z}_e(k_x,k_z,d)=\left(\frac{1}{\boldsymbol{Z}_1^e}+\frac{1}{\boldsymbol{Z}_2^e}\right)^{-1} \tag{9}$$

$$\boldsymbol{Z}_h(k_x,k_z,d)=\left(\frac{1}{\boldsymbol{Z}_1^h}+\frac{1}{\boldsymbol{Z}_2^h}\right)^{-1} \tag{10}$$

where $\boldsymbol{Z}_1{}^e$ and $\boldsymbol{Z}_2{}^e$ are input impedances looking into the corresponding regions at $y=0$ in the TM equivalent circuit, whereas $\boldsymbol{Z}_1{}^h$ and $\boldsymbol{Z}_2{}^h$ are those impedances in the TE circuit. The definition of $\boldsymbol{Z}_1{}^e$ and $\boldsymbol{Z}_1{}^h$ are:

$$\boldsymbol{Z}_1{}^e=\boldsymbol{Z}_{TM_1} \qquad \boldsymbol{Z}_1{}^h=\boldsymbol{Z}_{TE_1} \tag{11}$$

As seen in Figure 3, a transmission line is terminated by another transmission line of different characteristic impedance. Therefore, conventional transmission line theory can be used to find the $\boldsymbol{Z}_2{}^e$ and $\boldsymbol{Z}_2{}^h$ as:

$$\begin{aligned}
\boldsymbol{Z}_{N-1}^e &= \boldsymbol{Z}_{TM_{(N-1)}} \frac{\boldsymbol{Z}_N^e \coth\gamma_{N-1}d_{N-1}+\boldsymbol{Z}_{TM_{(N-1)}}}{\boldsymbol{Z}_{TM_{(N-1)}}\coth\gamma_{N-1}d_{N-1}+\boldsymbol{Z}_N^e}\\
\boldsymbol{Z}_{N-1}^h &= \boldsymbol{Z}_{TE_{(N-1)}} \frac{\boldsymbol{Z}_N^h \coth\gamma_{N-1}d_{N-1}+\boldsymbol{Z}_{TE_{(N-1)}}}{Z_{TE_{(N-1)}}\coth\gamma_{N-1}d_{N-1}+Z_N^h}\\
\vdots &= \vdots\\
\boldsymbol{Z}_2^e &= \boldsymbol{Z}_{TM_2}\frac{\boldsymbol{Z}_3^e\coth\gamma_2 d_2+\boldsymbol{Z}_{TM_2}}{\boldsymbol{Z}_{TM_2}\coth\gamma_2 d_2+\boldsymbol{Z}_3^e}\\
\boldsymbol{Z}_2^h &= \boldsymbol{Z}_{TE_2}\frac{\boldsymbol{Z}_3^h\coth\gamma_2 d_2+\boldsymbol{Z}_{TE_2}}{\boldsymbol{Z}_{TE_2}\coth\gamma_2 d_2+\boldsymbol{Z}_3^h}
\end{aligned} \tag{12}$$

where:

$$\boldsymbol{Z}_N^e=\frac{\boldsymbol{Z}_{TM_N}}{\coth\gamma_N d_N} \qquad \boldsymbol{Z}_N^h=\frac{\boldsymbol{Z}_{TE_N}}{\coth\gamma_N d_N}$$

The final part of the formulation is to map from the (u, v) to (x, z) coordinate system for the spectral wave corresponding to each θ given by k_x and k_z. Because of the coordinate transform in the Eqs. (2) and (3), $\boldsymbol{E}_x$ and $\boldsymbol{E}_z$ are linear combination of $\boldsymbol{E}_u$ and $\boldsymbol{E}_v$. Similarly $\boldsymbol{J}_x$ and $\boldsymbol{J}_z$ are superpositions of $\boldsymbol{J}_u$ and $\boldsymbol{J}_v$. When the above notations are applied, the Green's function elements are found to be:

$$\boldsymbol{G}_{zz}=N_z{}^2\boldsymbol{Z}_e+N_x{}^2\boldsymbol{Z}_h \tag{13}$$

$$\boldsymbol{G}_{zx}=N_xN_z(-\boldsymbol{Z}_e+\boldsymbol{Z}_h) \tag{14}$$

$$\boldsymbol{G}_{xz}=\boldsymbol{G}_{zx} \tag{15}$$

$$\boldsymbol{G}_{xx}=N_x{}^2\boldsymbol{Z}_e+N_z{}^2\boldsymbol{Z}_h \tag{16}$$

where $\boldsymbol{N}_x$ and $\boldsymbol{N}_z$ are transforming ratios given by:

$$N_x=\frac{k_x}{\sqrt{k_x^2+k_z^2}} \qquad N_z=\frac{k_z}{\sqrt{k_x^2+k_z^2}} \tag{17}$$

Note that $\boldsymbol{Z}_e$ and $\boldsymbol{Z}_h$ are functions of $k_x{}^2+k_z{}^2$ and the ratio of k_x and k_z enters through N_x and N_z.

3. ASYMPTOTIC FORM OF GREEN'S FUNCTION

The Green's function, which has been derived in Section 2, is computationally complex and this complexity increases with the number of layers. There is however a possibility to use the asymptotic form of the Green's function. For large transform variables (k_x and k_z), the original Green's function for a multilayer structure converges to the Green's function of a simple two layer structure. For this, they must satisfy the conditions which are given by:

$$\gamma_i=\sqrt{k_x^2+k_z^2-k_{max}^2}\approx\sqrt{k_x^2+k_z^2} \tag{18}$$

$$\coth\gamma_{ij}\, d_{min}\approx 1 \tag{19}$$

where k_{max}= max (k_i) and d_{min}= min (h_i).

So, for the large Fourier transform variables, all multilayer structures are equivalent to the structure containing the immediate two layers, extended to infinity, on each side of the strip. The reason is that large values of transform variables account for the reactive field of the source, which is a localised effect.

With the above argument, the asymptotic form of the Green's function of multilayered structures can be directly obtained from the asymptotic form of the Green's function for two layer structure, which is given in detail in Ref. [9, Chapter 4] as:

$$\boldsymbol{G}_{st}^{\infty}=h_1\boldsymbol{K}_{st1}^{\infty}+h_2\boldsymbol{K}_{st2}^{\infty} \tag{20}$$

where:

$$\begin{aligned}
h_1&=-\frac{j}{\omega\epsilon_0\,(1+\epsilon_{r2})} & h_2&=\frac{j\omega\mu_0\mu_r}{(1+\mu_{r2})}\\
K_{zz1}^{\infty}&=\frac{k_z^2}{\sqrt{k_x^2+k_z^2}} & K_{zz2}^{\infty}&=\frac{k_z^2}{\sqrt{\left(k_x^2+k_z^2\right)^3}}\\
K_{zx1}^{\infty}&=-\frac{k_xk_z}{\sqrt{k_x^2+k_z^2}} & K_{zx2}^{\infty}&=\frac{k_xk_z}{\sqrt{\left(k_x^2+k_z^2\right)^3}}\\
K_{xz1}&=K_{zx1} & K_{xz2}&=K_{zx2}\\
K_{xx1}^{\infty}&=\frac{k_x^2}{\sqrt{k_x^2+k_z^2}} & K_{xx2}^{\infty}&=\frac{k_x^2}{\sqrt{\left(k_x^2+k_z^2\right)^3}}
\end{aligned} \tag{21}$$

with $\omega=2\pi f$ and ϵ_{r2}, μ_{r2} the relative permittivity and permeability of the electric substrate next to the strip respectively. It must be noted that the first layer is free-space because of the open structure.

The functions $\boldsymbol{K}_{sti}{}^{\infty}$ ($s,t=z$ or x; $i=1,2$) are just functions of k_x and k_z, thus they are independent of frequency and metallisation pattern. The constants h_i ($i=1,2$) need only be calculated once per frequency, as they are independent of k_x and k_z.

An important point to be considered for the spectral domain integration of the impedance matrix elements is the treatment of the surface wave poles. It can be easily proved that all the poles of the Green's function lie between k_0 and $\sqrt{\in_{r\,max} * \mu_{r\,max}}\; k_0$. The same technique given in Ref. [9, Chapter 4] is used for the impedance matrix elements integration.

4. NUMERICAL EXAMPLES

To verify the accuracy of the Green's function which has been derived by the technique given in this paper, the simple two layer microstrip line *1.27 mm* wide on a substrate *1.27 mm* thick and of permittivity *8.875* is taken as a test structure. The microstrip line is analysed for the effective permittivity by the existing two dimensional version of the technique, then the substrate is divided into three, assuming that the structure is now a four layer microstrip line with identical substrate parameters. The effective permittivities are plotted in Figure 4 and compared with published data in Ref. [10].

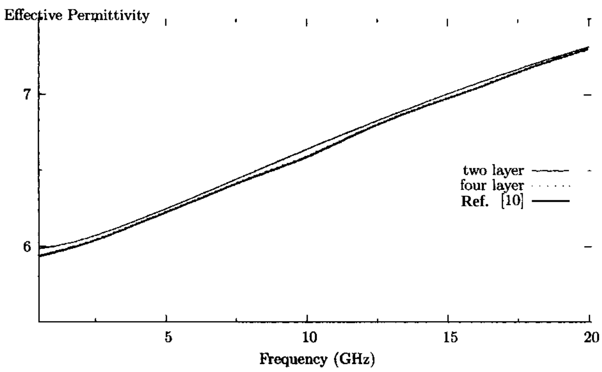

Fig. 4 Illustration of an accuracy

To further test the accuracy of the Green's function, results are compared with the simple closed-form expression for the effective dielectric constant of a microstrip on a two layer substrate [11]. The geometry of the two substrate layer microstrip used in the present test is shown in Figure 5.

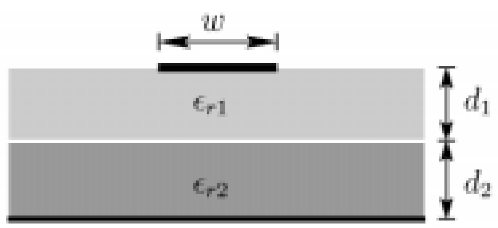

Fig. 5 Geometry of coupled microstrip with two substrate layers

Following Ref. [11], the total height of the substrate layers, $d_{tot}=d_1+d_2$, is held constant and the design formula uses the following as design parameters:

- ratio of the width to total height, ω/d_{tot},
- the height ratio, $d_r=d_2/d_{tot}$,
- the dielectric constants, $\in_{r1}$ and $\in_{r2}$,
- the effective dielectric permittivities of the single layer cases, $\in_{reff}(d_r=0)$ and $\in_{reff}(d_r=1)$

Figure 6 shows the effective dielectric permittivity as a function of the height ratio for different dielectric constants of the lower substrate. The results are compared with published data in Ref. [11]. In Figure 6, a linear normalisation is used, described by:

$$\in_{reff(norm)}(d_r)=\in_{reff}(d_r)-\in_{reff}(0)-d_r[\in_{reff}(1)-\in_{reff}(0)] \qquad (22)$$

where $\in_{reff}(0)$ and $\in_{reff}(1)$ are the effective permittivity for the single substrate cases given by:

$$\begin{aligned} &\in_{reff}(0)=\in_{reff},\ d_r=0 \\ &\in_{reff}(1)=\in_{reff},\ d_r=1 \end{aligned} \qquad (23)$$

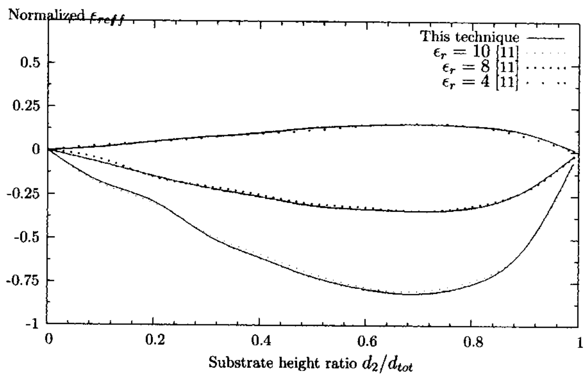

Fig. 6 $\in_{reff}$ of a microstrip on a two layer substrate normalised using the Eq. (22) as a function of the substrate height ratio for different values of $\in_{r2}$ ($\omega/d_{tot}=1$ and $\in_{r1}=6$)

As shown in Figures 4 and 6, the immitance approach is shown to be an accurate way to find the Green's function for multilayer structures.

To observe the effects of the order of dielectric substrates with different parameters, the microstrip line *1 mm* wide is modelled and a series of results are plotted in Figure 7. In the analysis, each layer thickness has been chosen to be *0.25 mm*.

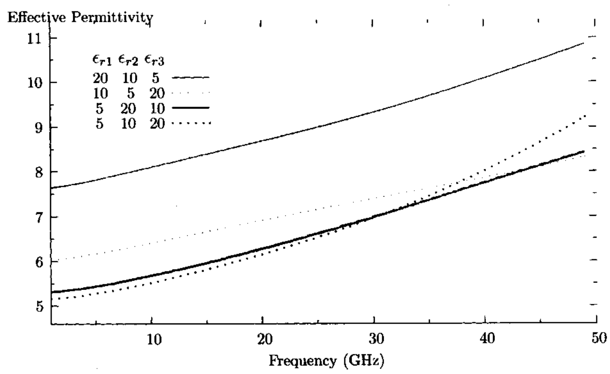

Fig. 7 Illustration of the effects of multilayering on effective permittivity

As shown in Figure 7, the order of the dielectric layers is very effective. At low frequencies the value of the effective permittivity is determined by the relative permittivity of the substrate next to the strip. It is clearly shown in Figure 7 that the parameters of substrate next to the ground plane is effective on the increment of the effective permittivity from one frequency point to another.

The following illustrates the effects of multilayering, a simple three layer microstrip resonator *1 mm* wide and *8 mm* long as shown in Figure 8 is modelled and the Table 1 has been prepared. The calculation of the resonant frequency is given in Refs. [12, 13]. As shown in Table 1, the order of the dielectric layers is dominant on the resonant frequency. The resonant frequency is inversely proportional to the value of parameters of dielectric substrate next to the conducting strip.

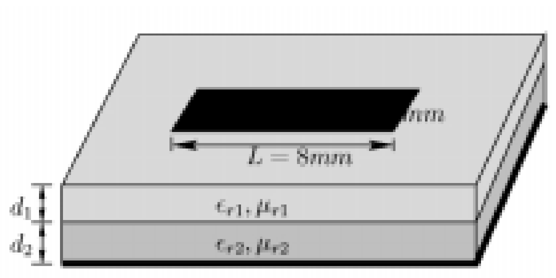

Fig. 8 Two layer microstrip resonator

Table 1 Illustration of the effects of multilayering on resonant frequency (GHz)

	$\in_{r1}$	d_1=0.636 mm		
$\in_{r2}$		20	10	5
d_1=0.636 mm	20	$3.05+j1.7010^{-3}$	$3.93+j6.7110^{-3}$	$5.13+j1.8510^{-2}$
	10	$3.38+j3.8110^{-3}$	$4.24+j8.7710^{-3}$	$5.40+j2.0610^{-2}$
	5	$3.83+j6.4410^{-3}$	$4.70+j1.2710^{-2}$	$5.82+j2.2810^{-2}$

5. CONCLUSION

This paper presents an iterative solution to derive the Green's function for the multi dielectric layer structures. This derivation can be easily applied to computer programming. The derivation of the Asymptotic Form of the Green's function was also mentioned along with two key conditions.

6. REFERENCES

[1] M.S. Alam, M. Koshiba, K. Hirayama and Y. Hayashi, Hybrid-mode analysis of multilayered and multiconductor transmission lines, *IEEE Transaction on Microwave Theory and Technique*, Vol. 45, pp. 205-211, 1997.

[2] A.R. Djordjevic, SPICE-compatible models for multiconductor transmission lines in laplace-transform domain, *IEEE Transaction on Microwave Theory and Technique*, Vol. 45, pp. 569-579, 1997.

[3] A. Gharsallah, A. Mami, R. Douma, A. Gharbi and H. Baudrand, Analysis of microstrip antenna with fractal multilayer substrate using iterative method, *International Journal of RF and Microwawe Computer-Aided Engineering*, Vol. 11, pp. 212-218, 2001.

[4] H.H. Balik and C.J. Railton, New compensation functions for efficient excitation of open planar microwave circuits in SDM, *IEEE Transaction on Microwave Theory and Technique*, Vol. 47, pp. 106-108, 1999.

[5] H.H. Balik and C.J. Railton, A new route to fast accurate design tool for open planar circuit analysis, *International Journal for Engineering Modelling*, Vol. 15, No. 1-4, pp. 31-43, 2002.

[6] H.H. Balik and C.J. Railton, Fast rigorous impedance matrix calculations in SDM for analysis of open planar structures, *International Journal for Engineering Modelling*, Vol. 15, No. 1-4, pp. 21-29, 2002.

[7] N.K. Das and D.M. Pozar, A generalised spectral-domain Green's function for multilayer dielectric substrates with application to multilayer transmission lines, *IEEE Transaction on Microwave Theory and Technique*, Vol. 35, pp. 326-335, 1987.

[8] T. Itoh, Spectral domain immitance approach for dispersion characteristics of generalized printed transmission lines, *IEEE Transaction on Microwave Theory and Technique*, Vol. 28, pp. 733-736, 1980.

[9] H.H. Balik, Passive Open Planar Circuit Analysis by Enhanced Spectral Domain Method, Ph.D. Thesis, University of Bristol, 1997.

[10] T. Itoh and R. Mittra, Spectral domain approach for calculating the dispersion characteristics of microstrip lines, *IEEE Transaction on Microwave Theory and Technique*, pp. 496-499, 1973.

[11] J.P.K. Gilb and C.A. Balanis, Closed-form expressions for the design of microstrip lines with two substrate layers, IEEE MTT-S INT. Microwave Symp., pp. 1005-1008, 1993.

[12] T. Itoh, Analysis of microstrip resonators, *IEEE Transaction on Microwave Theory and Technique*, Vol. 22, pp. 946-952, 1974.

[13] T. Itoh and W. Menzel, A full-wave analysis method for open microstrip structures, *IEEE Transactions on Antennas and Propagation*, Vol. 29, pp. 63-67, 1981.

NUMERIČKI UČINKOVITO RJEŠENJE ZA VIŠESLOJNE MIKROVALNE KRUGOVE

SAŽETAK

Metoda spektralne domene uvelike se primjenjuje u analizi dvosložnih planarnih mikrovalnih krugova. Prilagodba ove tehnike na višeslojne strukture zahtijeva efikasnu derivaciju Green-ove funkcije. Ovaj rad nudi iterativnu derivaciju programerima računalnih alata.

Ključne riječi: *metoda spektralne domene, višeslojan, iterativno rješenje.*