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Gamma positioning and market quality ☆

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ABSTRACT

In this paper, we study the effect of the gamma positioning of dynamic hedgers on market quality through simulations. In our zero-intelligence model, the presence of dynamic hedgers enhances market liquidity under normal conditions. However, positive gamma helps sustain liquidity in stressed scenarios, while negative gamma depletes it. We find that an increase in the net gamma positioning of dynamic hedgers reduces volatility and increases market stability, whereas a negative gamma positioning increases volatility and makes the market more prone to failure. Price discovery typically worsens when dynamic hedgers become more prevalent, regardless of the sign of their positioning. Our findings imply that steering the net gamma position of dynamic hedgers can be considered a policy instrument to improve market quality, especially for instruments with low liquidity or low traded volume.

1. Introduction

In November 2014 an unexpectedly large number of sell orders caused U.S. Treasuries to drop 1.6% before rebounding fully by an equally unprecedented number of buy orders. The intraday largest Treasury move since 2009 is attributed to a large short option position amongst delta neutral traders (Levine et al., 2017). This phenomenon is called a gamma trap or a gamma squeeze: excessive price volatility induced by dynamic hedgers who involuntarily act as momentum traders due to their short gamma position and preference for an overall delta-neutral position.

Risk sensitivities of option contracts to certain parameters are denoted by 'Greeks,' which are the partial first derivatives of the option price in the Black and Scholes (1973) formula to option characteristics. The most prominent of these Greeks is delta, which measures the price sensitivity of the option with respect to price changes in the underlying security. Gamma is a second-order Greek and measures the degree by which this delta changes when the underlying security's price moves.[1] All option contracts exhibit some non-zero gamma; option contracts that are close to maturing and 'at the money' exhibit the largest gamma. Dynamic hedgers are market participants whose objective is to maintain a constant delta. Dynamic hedging trading desks are prevalent at banks, insurers,

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[1] Gamma is the sensitivity of delta with respect to changes in the underlying price. Since delta itself is the sensitivity of the option value with respect to changes in the underlying, gamma is considered a second-order Greek; it is the second derivative of the Black and Scholes (1973) option-pricing formula to the price of the underlying. Other frequently used Greeks are vega (price sensitivity with regards to changes in volatility), theta (price changes due to the evolution of time), and rho (price changes due to changes in interest rates).

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high-frequency trading firms, and hedge funds, which hedge option positions by dynamically replicating their exposure using the underlying security. The existence of gamma makes their hedging strategy dynamic, as changes in the price of the underlying alters the delta of the option portfolio and this requires a re-balancing in the hedge portfolio.

An open question is what the activity of dynamic hedgers, who by definition act indifferently to the fundamental or long-term value of securities, does to the market quality of the underlying security. The presence of arbitrageurs is generally seen as favorable for market quality (Baruch et al., 2007; Rappoport and Tuzun, 2020; Rösch, 2021). On the other hand, the presence of high-frequency traders introduces a trade-off between liquidity and efficiency (Arifovic et al., 2022). It is not at all obvious, though, that dynamic hedgers should meet similar praise as their presence is anecdotally linked to flash crashes, gamma traps, and excessive volatility.[2]

On the one hand, the theoretical option pricing literature shows that the dynamic replication of an option portfolio dampens or exacerbates volatility through the so-called 'feedback effect' of the overall net gamma position of dynamic hedgers: a long net gamma position dampens volatility whereas a short position amplifies this. Recently, this feedback effect has garnered attention from researchers, who incorporate feedback effects in theoretical option pricing models. Bank et al. (2017, March) for instance explicitly solve how optimal hedging strategies are affected by temporary price impact, whereas Bouchard et al. (2015) do so for linear impact. Similarly, Almgren (2015) and Abergel and Loeper (2017) show that optimal dynamic hedge strategies are altered when the non-endogeneity constraint is relaxed. Anderegg et al. (2022) derive a simple expression of the feedback effect under linear permanent market impact. When the total gamma exposure is of the magnitude of a hundred times the permanent market impact, the equation becomes quasi-linear. Relatedly, Jeannin et al. (2008) theoretically study the effect of hedging on stock pinning. They show that stock prices tend to move closer to the strike price of heavily traded options as maturity nears.

The empirical literature, on the other hand, contains very few studies of the effect of dynamic hedging on market quality dimensions. This is due to the fact that proprietary data on option positions of dynamic hedgers is required, but is typically not available. Anderegg et al. (2022) use a classification algorithm instead to infer the net option position among dynamic hedgers. They observe a relation between option positioning and realized volatility in the foreign exchange market for the euro-US dollar and US dollar-Japanese yen. Baltussen et al. (2021) acknowledge the importance of gamma trading on market functioning and specifically its impact on asset allocation choices. They assume, however, from the outset that dealers are short gamma. Whilst their metric succeeds in predicting intraday momentum, their implicit assumption that the dealers' clients are more prone to buy puts than calls on average limits their study to a one-sided effect of gamma trading.

In this paper, we study the effect of dynamic hedgers on different dimensions of market quality of the underlying security, such as volatility, liquidity, price discovery, and market failure through a simulation study. We expand on the literature by studying more than just the feedback effect and explore both short and long gamma positioning. We study the effects of gamma positioning on market quality via simulations. We first use a Monte Carlo simulation that allows us to isolate the gamma channel by comparing parallel markets that only differ in gamma positioning. Secondly, we add further complexity to the simulation by letting multiple stochastic processes interact under the presence or absence of a dynamic hedge heuristic. Since all data is self-generated, we have an exact description of all revealed and latent information at any moment. The presence of perfect information allows us to utilize sophisticated liquidity measures, to exactly trace the price discovery process, and measure the impact on market metrics.

We develop a simulation model in which heterogeneous agents interact in the order book of a risky asset. A set of stochastic order arrival processes representing generic end-users in combination with a stochastic news-shock process, and a continuous order removal function, generates a constantly evolving order book. Obviously, many of these order arrivals interact with the order book and result in transactions. Agents are heterogeneous on two dimensions: their demand for immediacy and their informativeness. Agents with a high (low) demand for immediacy, typically use market (limit) orders. Informed (uninformed) agents use the fundamental value (midpoint) as their price reference point. In our experimental setup, we add a dynamic hedger to this market, with varying degrees of gamma positions. As such, we are able to directly observe the effect of dynamic hedging on several metrics of market quality.

Our study reveals that net positive gamma positioning has an overall positive impact on three dimensions of market quality compared to a situation without dynamic hedging: volatility, liquidity, and market failure.[3] Negative gamma positioning, on the other hand, enhances liquidity even more, but at the cost of amplified volatility and increased susceptibility to market failure.[4,5] Our final finding is that price discovery typically worsens when dynamic hedgers become more prevalent, regardless of the sign of their positioning, although the channels differ: In positive gamma scenarios price informative signals are absorbed by dynamic hedgers, leading to slower price discovery, whereas in negative gamma scenarios, both informative and uninformative signals are amplified, leading to overshoots.

We contribute to several strands of literature. First, as indicated above, we study how intermediaries impact market quality, as in Baruch et al. (2007); Rappoport and Tuzun (2020); Rösch (2021). We are the first to focus on the market quality effects of dynamic hedgers. Secondly, we contribute to the literature on the feedback effect on option pricing as in Bouchard et al. (2015); Almgren (2015); Bank et al. (2017, March); Abergel and Loeper (2017); Anderegg et al. (2022). Finally, we contribute to the agent-based literature, especially those agent-based models focusing on market microstructure. To give a few examples, Chiarella and Iori (2002)

[2] See, e.g., https://www.risk.net/risk-management/market-risk/

[3] We define a market failure as a dry-up of liquidity, such that price, volatility, and liquidity are temporarily undefined.

[4] Our model setup can explain this counter-intuitive result. In negative gamma scenarios, absolute price shocks tend to be more pronounced, leading the simulation to frequently find itself in a deeper part of the order book with more resting orders.

[5] Our model does not incorporate higher-order behaviors. We do not assume that market makers want to be compensated for the increased market volatility in the negative gamma scenario by posting worse quotes (and thus worsening liquidity). Furthermore, we do not model a rapid deterioration of market liquidity (due to the retraction of resting quotes) after a large price shock.

and Chiarella et al. (2009) study the impact of heterogeneous investors on price formation and order flow in a limit order book market. More recently, Biondo (2019) studies the impact of market design on market quality in an agent-based setting. Similar to our work, various prior studies have utilized random order arrival processes to model markets; pioneering works such as Farmer et al. (2005a) and Farmer et al. (2005b) were among the first to explore the market impact of zero-intelligence traders in a double auction framework. Furthermore, Leal et al. (2016) study the generation of flash crashes within a framework where high-frequency traders place random orders on both sides of the market. A comprehensive analysis of financial agent-based models involving non-intelligent agents is available in Bouchaud (2018). Our contribution to the agent-based literature is the introduction of dynamic hedgers into a market populated by zero-intelligence traders, shedding light on new dimensions of market behavior in such scenarios.

Market quality is relevant for institutions that are not themselves secondary market participants, such as policymakers, central banks, regulators, and security issuers. This paper aims to elucidate the causal workings and market quality impact of the dynamic hedger's gamma position, such that it could become relevant for such institutions. In particular, we want to stress the fact that net gamma positioning is not exogenously given. Indeed, it was the conspiring of Reddit users that jointly created a net short gamma position among dynamic hedgers during the meme-stock saga of 2021.[6] We recognize that the net gamma position among dynamic hedgers is generally best left to the market. However, given the obvious market quality-enhancing potential shown in our paper, the net gamma position can be steered to lead to more welfare-enhancing outcomes.

From a policy perspective, our results imply that it would be beneficial to provide incentives to increase the net positive gamma position of dynamic hedgers. For many institutions, such as central banks, regulators, and security issuers, utilization of the gamma channel might be considered as a policy instrument to improve market quality. Temporarily or permanently altering the net gamma position of dynamic hedgers has the potential to improve market quality and is a policy avenue that is hitherto unexplored.

The remainder of this paper is organized as follows. Section 2 presents our model and simulation, and Section 3 explains our measures of market quality. Section 4 presents evidence supporting the option position effect of dynamic hedgers on market quality. Section 5 concludes.

2. Model and simulation

2.1. Dynamic hedgers

Several types of market participants are indifferent to the fundamental or long-term value of the instruments. As such, they should not contribute to the price discovery process as their actions contain no information about the fundamental value of the asset. Arbitrageurs bridge supply and demand imbalances between similar instruments and thus only care about the asset price in relation to another asset price. Dynamic hedgers are similar in their indifference, as they only wish to trade an asset in order to mimic the sensitivity of another asset. An open question is what the presence of dynamic hedgers, who are presumably indifferent, does to market quality.

Dynamic hedgers are agents who aim to maintain a constant delta. Examples of such dynamic hedgers are banks, insurers, high-frequency trading firms, and hedge funds, which hedge option positions by dynamically replicating their exposure using the underlying security. A concrete example is that these institutions can be the writers (sellers) of option contracts, and wish to hedge the delta risk of this short option position by taking a long position in the underlying asset. The relation between the prices of an option and its underlying asset nonlinear, but convex. In other words, the second derivative of the option price to the underlying price, the gamma, is not zero. By definition, the delta of the underlying asset is one, and the gamma is zero. Combined, this makes their hedging strategy dynamic, as changes in the price of the underlying alters the delta of the option portfolio; this requires continuous re-balancing in the hedge portfolio to maintain an overall delta of zero.

The concept of dynamic hedging is a defining feature of the option pricing literature. The ability to dynamically replicate an option makes arbitrage-free valuation possible. The seminal literature regarding option pricing explicitly requires the absence of market impact arising from dynamically hedging the replicating portfolio (Black and Scholes, 1973). Successful attempts have been made to loosen the assumption of a perfect underlying market by incorporating the hedging impact on realized volatility. This phenomenon is called the feedback effect; see Platen and Schweizer (1998), Frey and Stremme (1999), and Wilmott and Schönbucher (2000). The central idea is that the transactions arising from market participants who dynamically replicate an option portfolio might have market impact. In particular, the gamma determines the magnitude and direction of the market impact. The overall net position of dynamic hedgers determines the feedback effect: with a positive (negative) net-gamma position, an increase in prices will lead to an increase (decrease) in the delta of dynamic hedgers, forcing them to sell (buy) the underlying, and vice-versa for an initial decrease in prices, to maintain a portfolio delta of zero. In other words, dynamic hedgers with a positive gamma have a contrarian strategy in the market of the underlying, whereas dynamic hedgers with a negative gamma have a momentum strategy in the market of the underlying. Hence, a positive net-gamma position among dynamic hedgers is expected to dampen price movements and volatility, whereas a negative gamma position amplifies these. The active trading by dynamic hedgers also directly impacts liquidity through the additional orders in the order book, which can be liquidity taking or liquidity making depending on the gamma.

Several authors have incorporated the feedback effect into option pricing models. Bank et al. (2017, March) show how optimal hedging strategies are affected by temporary price impact, whereas Bouchard et al. (2015) do so for linear impact. Similarly, Almgren and Li (2016) and Abergel and Loeper (2017) show that optimal dynamic hedge strategies are altered when the non-endogeneity

[6] See www.forbes.com/gamma.

constraint is relaxed. Anderegg et al. (2022) derive a simple expression of the feedback effect under linear permanent market impact. We set up an agent-based model in which the orders of the end-users arrive randomly, but the strategy of the dynamic arbitrageur is explicit such that we can study their causal impact on market quality.

2.2. Model

This paper illustrates the effect of dynamic hedgers on market quality by utilizing Monte Carlo simulations. A set of agent-mimicking stochastic order arrival processes in combination with a stochastic news-shock process under a continuous order removal decay function generates a constantly evolving order book. Many of these order arrivals interact with the order book and result in transactions.

The simulation allows us to draw inferences about the moments of the variables of interest. By varying a subsection of the simulation parameters but keeping the same random seeds, we create parallel simulations that isolate the mechanism we aim to study. The subsequent subsections describe the elements of the model.

2.2.1. The market

Since our simulation focuses on short time horizons, we assume without loss of generality an asset that does not yield coupons or dividends in a world with no time-value of money. The market is represented by a single order book, where both market and limit orders are structured as two-dimensional vectors $\langle x, y\rangle$ with x denoting the volume (negative for sell orders) and y denoting the price. Bids and offers are placed at discrete multiples of the minimal tick size of 0.01. All orders are considered good-till-canceled, so there is no discontinuity between trading days. Limit orders that immediately transact upon placement leave their balance. Orders of similar prices are executed based on time priority, which we only have to track for the dynamic hedger. At-market buy and sell orders are structured as $\langle x, \infty\rangle$ and $\langle -x, -\infty\rangle$ respectively. The average of the highest bid, denoted by p_b, and the lowest offer, denoted by p_o, is considered the mid-price p.

Because we have events related to the simulation happening at different intervals (order arrivals, hedge actions, news arrivals, hedge measurements, and option transactions), we index time over a variety of parameters. Each simulation consists of T trading sessions, with each trading session t being separated into H trade intervals. With each trade interval h consisting of Q sub-intervals. Within each sub-interval q a number of orders arrive following a Poisson distribution with parameter λ. Time can be uniquely mapped to an interval $[1, T + \frac{H}{T} + \frac{Q}{HT}]$ from (t, h, v) via $(t + \frac{h}{H} + \frac{q}{HQ})$.

$$\tau_{t,h,q} = t + \frac{h}{H} + \frac{q}{HQ}. \tag{1}$$

Summing over the re-indexed time τ is more convenient. Finally, we define τ^* as the index that identifies every tick:

$$\tau^*_{t,h,q,l} = t + \frac{h}{H} + \frac{q}{HQ} + \frac{l}{HQL_{t,h,q}}, \tag{2}$$

here l denotes the l'th Poisson arrival and $L_{t,h,q}$ represents the total amount of Poisson arrivals in interval (t, h, q).[7] A schematic overview of the simulation process is given in Fig. 1.

As will become clear when we introduce the agent behavior, dynamic hedger behavior, and the exogenous news process; THQ structure is chosen to let events occur at different intervals. In particular:

- The portfolio of the dynamic hedger is reset every t.
- Dynamic hedgers are evaluated on their delta position at every h. Additionally, news shocks occur at every h.
- The hedge actions of the dynamic hedger are performed at the end of every q.
- One (random) new order by a non-dynamic hedger occurs every τ^*, the number of order arrivals per q is random. Resting orders also decay at every τ^*.

2.2.2. Fundamental value

The news process is a Gaussian random walk whose shocks are independent of the order book state. News shocks cause the fundamental price to go up or down. The true price p^* at trade interval $t + h/H$ is given by:

$$p^*_{t+h/H} = p^*_0 + \Sigma_i^{t+h/H} \eta_i, \tag{3}$$

with η_i an independent stochastic process given by $\eta_i \sim N(0, \sigma_n)$. The variables p^*_0 and σ_n are simulation input parameters and will be discussed in Section 2.3 and Section 4.2 respectively.

[7] Such that every τ^* represents an individual order arrival.

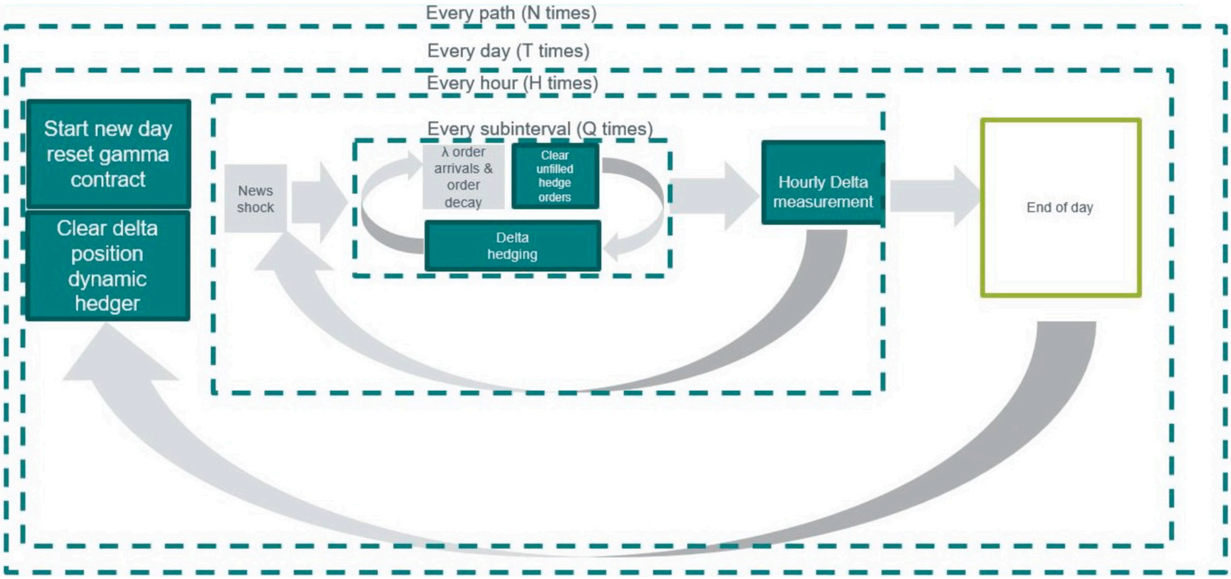

Fig. 1. Schematic overview of the simulation process.

Notes: This figure gives a graphical representation of the simulation model. Each simulation consists of T trading sessions, with each trading session t being separated in H trade intervals. With each trade interval h consisting of Q sub-intervals. Within each sub-interval q a number of orders arrive following a Poisson distribution with parameter λ. Dynamic hedgers attempt to hedge their delta by sending orders at q sub-intervals. Dynamic hedgers are evaluated on their delta position every h interval. Dynamic hedgers empty their entire portfolio every t interval, at which they receive new $(t+2)$ ATM stradles.

2.2.3. Agent representing stochastic order arrival processes

Our base set-up without a dynamic hedger contains four agent-representing order arrival processes. All of these order arrivals have the same volume distribution, given by the uniform distribution.[8] Order volumes are not scaled for endowment, i.e., if the asset trades lower, the volume metrics such as the traded volume and order-book volume are expected to be equal in nominal terms but lower in cash-equivalent terms:

$$v \sim U(v_{min}, v_{max}), \tag{4}$$

with v_{min} and v_{min} the boundaries of the uniform volume distribution. Each order arrival is taken from one of the following four types:

- Agents (end-users) with a high demand for immediacy:
  - Uninformed (noise) end-users: place at-market orders in random direction: $\langle v, \infty \rangle$ and $\langle -v, -\infty \rangle$ with equal probability.
  - Informed end-users: place at-market orders in the direction of the fundamental price: either $\langle v, \infty \rangle$ if $(p < p^*)$ or $\langle -v, -\infty \rangle$ if $(p > p^*)$.
- Agents (market makers) with a low demand for immediacy:
  - Uninformed market makers: place orders at random around a distance from the mid-price: $\langle v, \sim N(p - \mu_p, \sigma_p) \rangle$ and $\langle -v, \sim N(p + \mu_p, \sigma_p) \rangle$ with equal probability.
  - Informed market makers: Place orders at random around a distance from the fundamental price: $\langle v, \sim N(p^* - \mu_p, \sigma_p) \rangle$ and $\langle -v, \sim N(p^* + \mu_p, \sigma_p) \rangle$ with equal probability.

The parameters μ_p and σ_p describing the order placement distribution for the agents are input variables for the simulation and their values will be discussed in Section 2.3. The same goes for the input parameters (u) and (i) that describe the relative proportions of order types. In particular, orders with a high demand for immediacy have probability (u), and orders with low demand for immediacy have probability $(1 - u)$. Similarly, informed orders have probability (i) and uninformed orders have probability $(1 - i)$. These order arrival types are independent such that uninformed orders with a high demand for immediacy have probability $i(1 - u)$.

Immediacy-demanding agents necessarily consume liquidity from the order book, whereas market makers generally provide liquidity. Since their order placement is stochastic, some of their orders actually immediately execute causing them to occasionally reduce liquidity.

[8] Empirically, order volumes follow a multi-modal distribution, as different type of agents each have their preferred median order size. However, given that our simulation already requires a lot of parameters to tweak, we keep the volume generation process uniform.

While these order arrival processes are intended to mimic agent behavior, they are aggregated abstractions and lack any agent-like properties. We do not track inventories of individual agents, nor are any of their order placement heuristics strategically considering other agents' behavior.

Due to their directional bias, the order placement heuristic of informed traders is such that they create price pressure towards the fundamental price. There is no bias for uninformed orders, as their order placement is symmetric around the current price.

2.2.4. Order removal

The order arrival process is discrete, but as we do not track agent inventory beyond that of the dynamic hedger, our order removals cannot be attributed to agent-specific considerations. As such we rely on a continuous approximation of order removals that is standard in the differential equations utilized in the market microstructure literature[9]:

$$v_{p_i,\tau^*_{t,h,q,l+1}} = v_{p_i,\tau^*_{t,h,q,l}} e^{-\zeta(p-p_i)^2}, \tag{5}$$

with p the midprice, p_i the corresponding price at the i'th price level of the order book (and thus v_{p_i} being the resting volume at p_i) and ζ a decay parameter. The combination of linear liquidity addition via order arrivals and non-linear decay allows for an order book that is consistent with empirically observed limit order book distributions, without having to incorporate higher-order agent behavior.

The decay should not have the same periodicity as the dynamic hedger, since this would bias the result. For this reason, the decay process happens after each order arrival τ^* rather than after a fixed time interval. This implies that sub-intervals with more (less) order arrivals due to the stochastic nature of the Poisson process also contain more (less) order removals.

2.2.5. The dynamic hedger

The main object of our study is the analysis of the impact of dynamic hedgers' net gamma position on market quality. We represent the collective of dynamic hedgers via a single agent, who buys its options from a single entity. This entity, the counterparty, never interacts with the order book. We assume the dynamic hedger and the counterparty are engaged in an evergreen gamma contract, i.e. a contract that automatically renews before its term expires. An evergreen gamma contract is given as follows: when the dynamic hedger is long (short) the evergreen gamma contract, then every day the dynamic hedger buys (sells) a $t+2$ at-the-money option straddle position from the counterparty and sells (buys) back all the $t+1$ options it has previously bought (sold), along with the full residual position in the underlying asset.[10] As such the dynamic hedger with a long (short) evergreen gamma contract starts every day with no delta position and a positive (negative) gamma exposure. If the underlying market remains unchanged, the agent long (short) the evergreen contract loses (gains) the value between a $t+2$ and $t+1$ option straddle. This gain or loss of time value, or theta, is always opposite in direction to the gamma position. We assume the evergreen gamma contract is maintained for the duration of the entire simulation.[11]

The dynamic hedger's orders are not generated stochastically, but based on a predetermined heuristic. Furthermore, the dynamic hedger is the only agent whose orders and inventory are being tracked for the purpose of removing or adjusting limit orders. The dynamic hedger attempts to minimize its absolute delta, as given by the Black and Scholes sensitivity of its entire portfolio to the underlying.[12] In our experiment the dynamic hedger has no control over the options within its portfolio, thus the only way to manage its delta exposure is by buying or selling the underlying.

We assume that at the end of every interval h during the trading session the dynamic hedger is evaluated on its net delta, i.e. the sum of the option delta and the underlying delta. Thus the dynamic hedger aims to minimize its net delta at the end of interval h, while allowing the delta to deviate from zero during the interval. More specifically, its hedge heuristic is given by placing a buy (sell) order at the end of each q with a volume equal to the absolute portfolio delta with a distance to the best offer (bid) proportional to $\frac{q}{Q}$. In particular, the bid is given by[13]:

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[9] Bouchaud et al. (2002) note a humped-shape distribution of the order book, suggesting resting orders with a large distance to the current mid-price have a greater propensity to be removed.

[10] Note that a straddle is a combination of a call and a put option contract with the same strike price and time to maturity. A long (short) straddle position has a positive (negative) gamma.

[11] Instead of making the evergreen gamma contract explicit via option transactions, one could opt to stipulate that the dynamic hedgers have a constant gamma position. Doing so would eliminate the need for having the trading sessions T and one could simply simulate an uninterrupted sequence of trading intervals. However, gamma is not constant and is also reliant on the distance to the strike price. By constantly re-striking options our simulation is more realistic as dynamic hedgers would hedge less aggressively when options are far out of (or in) the money.

[12] Note that our price process is not a log-normal Brownian motion (none of the order arrivals are scaled to the underlying price), but a regular Brownian motion. As such we apply the classic Black and Scholes formula without using the lognormal Martingale correction. Failing to omit the Martingale correction would result in biased delta estimates of the option contract values.

[13] One could generalize this heuristic for $Q \neq 4$ via:

$$\langle x, y \rangle_{\text{bid}} = \left\langle -\delta, p_b + (p_o - p_b)\left(\frac{32}{3}\left(\frac{q}{Q}\right)^3 - 16\left(\frac{q}{Q}\right)^2 + \frac{28}{3}\left(\frac{q}{Q}\right) - 2\right)\right\rangle$$

and

$$\langle x,y\rangle_{\text{bid}} = \begin{cases} \langle -\delta, p_b - (p_o - p_b)\rangle & \text{if } \dfrac{q}{Q} = \dfrac{1}{4} \\ \langle -\delta, p_b\rangle & \text{if } \dfrac{q}{Q} = \dfrac{1}{2} \\ \left\langle -\delta, p_b + \frac{1}{2}(p_o - p_b)\right\rangle & \text{if } \dfrac{q}{Q} = \dfrac{3}{4} \\ \langle -\delta, p_o\rangle & \text{if } \dfrac{q}{Q} = \dfrac{4}{4}, \end{cases} \tag{6}$$

whereas the offer is given by:

$$\langle x,y\rangle_{\text{offer}} = \begin{cases} \langle -\delta, p_o + (p_o - p_b)\rangle & \text{if } \dfrac{q}{Q} = \dfrac{1}{4} \\ \langle -\delta, p_o\rangle & \text{if } \dfrac{q}{Q} = \dfrac{1}{2} \\ \left\langle -\delta, p_o - \frac{1}{2}(p_o - p_b)\right\rangle & \text{if } \dfrac{q}{Q} = \dfrac{3}{4} \\ \langle -\delta, p_b\rangle & \text{if } \dfrac{q}{Q} = \dfrac{4}{4}. \end{cases} \tag{7}$$

Since dynamic hedgers attempt to keep their delta constant by trading the underlying, the volumes are given by $-\delta$. The time-based mixed strategy where the dynamic hedger switches from passive to aggressive orders is equivalent to Ellersgaard and Tegnér (2017).

2.2.6. Transactions between counterparty and dynamic hedger

Every day the dynamic hedger and the counterparty roll over their existing option contracts. While the valuation of this option is by itself not required to demonstrate the feedback effect on market quality, the option delta is. The option delta is a component of the portfolio delta, which is an input to the hedge heuristic. For the evaluation of the Greeks, we assume a constant implied volatility term structure and assume homoskedasticity of implied volatility throughout the simulation. Since there are no news shocks outside the trading sessions and the order book does not empty overnight, there are no jumps in prices in between trading sessions. As such we can use the time, interval, and sub-interval mapping for intraday option Greek evaluation. Specifically, we denote the auxiliary variables $d_{1,\tau,t}$ and $d_{2,\tau,t}$[14]:

$$d_{1,\tau,t} = \frac{p_\tau - K}{\sigma\sqrt{(m-\tau)}}, \tag{8}$$

$$d_{2,\tau,t} = d_{1,\tau,t} - \sigma^2\sqrt{(m-\tau)}, \tag{9}$$

for an option written on time t, with implied volatility σ evaluated at time τ. K denotes the option strike, p_τ the midprice at time τ and m the corresponding maturity time.

A straddle is an option position consisting of a put and a call with the same strike. Then the valuation of the option straddle with strike K is given by:

$$NPV_{put+call,\tau} = 2\Phi(d_1)p_\tau - 2\Phi(d_2)K, \tag{10}$$

with Φ the cumulative normal distribution. The delta for the straddle is given by:

$$\delta_{put+call,\tau} = 2\Phi(d_1) - 1. \tag{11}$$

Since $\Phi(d_1) = 0.5$ when $p_\tau = K_t$, which is the case at the onset of a new contract. Hence, the straddle starts with $\delta = 0$. We do not require other Greeks for the heuristic, but we can track the gamma position to quantify the relation of gamma to market quality. The straddle-gamma is given by:

$$\gamma_{put+call,\tau} = \frac{2\phi(d_1)}{S\sigma\sqrt{m-\tau}}, \tag{12}$$

with ϕ the standardized normal probability density function. Since ϕ is maximized when $d_1 = 0$, the strike where $p_\tau = K_t$ gives maximum gamma. Furthermore, lower values of $(m-\tau)$ provide more gamma than higher values of $(m-\tau)$. Thus short-dated at-the-money straddles exhibit the most gamma whilst exhibiting no initial delta.

$$\langle x,y\rangle_{\text{offer}} = \left\langle -\delta, p_o - (p_o - p_b)\left(\frac{32}{3}\left(\frac{q}{Q}\right)^3 - 16\left(\frac{q}{Q}\right)^2 + \frac{28}{3}\left(\frac{q}{Q}\right) - 2\right)\right\rangle.$$

A 3rd degree polynomial equation that equals Eq. (6) and Eq. (7) when $Q = 4$.

[14] These auxiliary variables are used to make the subsequent formulas more compact and readable.

Table 1
Benchmark parameter setting.

Ticksize	0.01	σ_p	0.1
N	100	μ_p	-0.1
T	20	σ	0.089
H	10	v_{min}	0
Q	4	v_{max}	1
λ	40	p_0	5
u	0.3	μ_n	0
i	0.25	σ_n	0
		ζ	0.05

Notes: This table gives the benchmark parameter settings of the simulation model used in Section 4.1. The same parameters are used for Section 4.2, however there news shocks are added to the simulation such that $\sigma_n = 0.1$ rather than 0.

2.3. Simulation parameters and initialization

We logically limit our analysis to simulations with parameters that result in a stable order book evolution. A u parameter that is too high, representing too much demand for immediacy, will result in more liquidity being consumed than provided, which increases the likelihood of market failure. This also holds for ζ (order decay propensity), σ_p (standard deviation of order placement of informed market makers), and more. Since our main market quality metrics are unusable in the case of market failure, we simulate a market that is not prone to fail using the following characteristics, as displayed in Table 1.

Our asset has a tick size of 0.01 and starts at $p_0 = 5$.[15] The incoming orders are split 75/25 between market makers and market takers ($i = 0.25$). The market makers, agents with a low demand for immediacy, place bids (offers) normally distributed 10 cents below (above) their reference price with a standard deviation of 0.1 (i.e., $\mu_p = -0.1$ and $\sigma_p = 0.1$). For informed market makers, this reference price is the fundamental price p^*, whereas for the uninformed market makers, this is the mid-price p. In our simulation, we use $u = 0.3$ meaning that 30 percent of the market makers and market takers place their orders conditional on the (latent) fundamental price and $(1 - u) = 0.7$ place their orders conditional on the current mid price. Volumes are uniformly distributed between $[0, 1]$.[16]

Our $N = 100$ simulation paths span $T = 20$ trading sessions each, all consisting of $H = 10$ trading intervals, which consist of $Q = 4$ trading sub-intervals. During each of these sub-intervals, a random number of Poisson distributed λ orders arrive, excluding one additional order for the dynamic hedger, which is the first one to execute its order during each interval. We initialize an order book with 10.0 volume on the bid and offer side at $[4.94, 4.96, 4.98, 5.02, 5.04, 5.06]$. The decay parameter $\zeta = 0.05$, in conjunction with the initialized order book, ensures the order book's exponential decay matches the net liquidity production after approximately one trading session. For this reason, we omit all data before the second trading session.

Preliminary analysis showed that the simulation parameters produce a realized price standard deviation of $\sigma = 0.089$.[17,18] This standard deviation is used for the evaluation of the option contracts, most notably for the determination of the option delta.

3. Market quality metrics

As noted in the introduction, we study the effect of gamma positioning on four dimensions of market quality. We operate under the assumption that low volatility, high market liquidity, and high (/fast) price discovery are intrinsically good characteristics of markets and that market failure is undesirable in the eyes of end users.

3.1. Volatility

If we denote a price shock via:

$$\epsilon_\tau = p_\tau - p_{\tau-1}, \tag{13}$$

where p_τ is the mid market price at τ, and the mean shock by:

$$\bar{\epsilon} = \frac{1}{THQ} \sum_{\tau=1}^{THQ} \epsilon_\tau, \tag{14}$$

[15] This price is set mainly for computational reasons, as it allows us to initialize an order book from 0.01 to 10.0 with a negligible probability of orders being placed out of bounds.
[16] All input variables are chosen by the authors and are not derived from data.
[17] Not shown here, but available upon request. The effect of this parameter is negligible for options with a very short time to expiry.
[18] Not to be confused with the dispersion of market makers' order placement σ_p and the standard deviation of the random walk of the news process σ_n.

then we can sample the average standard-deviation of price shocks across the simulations[19]:

$$S_1 = \frac{1}{N}\sum_{n=1}^{n}\frac{1}{THQ}\sum_{\tau=1}^{THQ}(\bar{\epsilon_n} - \epsilon_{\tau,n})^2. \tag{15}$$

Furthermore, we track the differential between the highest price and lowest price via the high-low spread of a single simulation by:

$$S_2 = \frac{1}{N}\sum_{n=1}^{N} max(\mathbf{p_n}) - min(\mathbf{p_n}), \tag{16}$$

where $\mathbf{p_n}$ is vector of mid-prices (at every τ^*) of the n-th simulation.

We also look at more specific downside risks and analyze average percentile-based shocks. In particular, we denote $\epsilon^*_{0.95,Q}$ as the subperiod-shock that is lower than 95 percent of other subperiod-shocks. Similarly, $\epsilon^*_{0.99,Q}$ as the subperiod-shock that is lower than 99 percent of other subperiod-shocks. Finally, we denote $\epsilon^*_{0.95,T}$ as the daily shock that is lower than 95 percent of other daily shocks. Due to symmetry in the simulation parameters, studying the negative shocks suffices.

Subsequently, we denote:

$$S_3 = \frac{1}{N}\sum_{n=1}^{N}\epsilon^*_{0.95,Q}, \tag{17}$$

$$S_4 = \frac{1}{N}\sum_{n=1}^{N}\epsilon^*_{0.99,Q}, \tag{18}$$

and

$$S_5 = \frac{1}{N}\sum_{n=1}^{N}\epsilon^*_{0.95,T}, \tag{19}$$

as the sample averages of the downside risk measures.

3.2. Market liquidity

We utilize four different measures for liquidity: the average bid-ask spread, the average total posted volume, the total traded volume, and a price impact measure.

The bid-ask spread averaged over time, per simulation, is given by:

$$L_1 = \frac{1}{N}\sum_{n=1}^{N}\frac{\sum_{\tau=1}^{THQ} p_{o,\tau,n} - p_{b,\tau,n}}{THQ}, \tag{20}$$

where $p_{o,\tau,n}$ represents the best offer price, and $p_{b,\tau,n}$ the best bid price (at time τ in simulation n). The total order book volume averaged over time and per simulation is given by:

$$L_2 = \frac{1}{N}\sum_{n=1}^{N}\frac{1}{THQ}\sum_{\tau=1}^{THQ}\sum_{\rho=p_{min}}^{p_{max}}|v_{\rho,\tau,n}|, \tag{21}$$

where $|v_{\rho,\tau,n}|$ denotes the absolute order book volume at price level ρ used as an index variable, at time τ in simulation n. Summation is done over the whole range of outstanding order price levels in the order book, so between the lowest order p_{min} and the highest order p_{max}. Traded volume L_3 quantifies the total volume traded, averaged over the simulations.

We combine price and volume information in a hybrid liquidity measure by means of the limit order book slope (LOS), as defined in Buis et al. (2020), which we transform here to:

$$L_4 = \frac{1}{N}\sum_{n=1}^{N}\frac{1}{THQ}\sum_{\tau=1}^{THQ}\sum_{\rho=p_{min}}^{p_{max}}\frac{v_{\rho,\tau,n}}{p_{\tau,n} - \rho}, \tag{22}$$

where the fraction $\frac{0}{0}$ entry is put to 0, if applicable. In this measure, order book volumes are divided by price distance such that volumes closer to the mid-price have a greater weighting in the measure.[20]

[19] THC denotes the total of all sub-trading intervals Q within the trade intervals H within the trading sessions T.

[20] Note that no absolute sign is needed, as offer volumes are negative but $p_{\tau,n} - \rho$ is negative as well (offers are by definition higher than the mid-price).

3.3. Price discovery

While price discovery is a pervasive topic in the empirical literature (Hasbrouck, 1995), we have the luxury of knowing the fundamental value of the asset via the latent news process. First, we quantify price discovery by measuring the squared distance between the mid-price p to the (latent) fundamental price p^*:

$$D_1 = \frac{1}{N}\sum_{n=1}^{N}\frac{\sum_{t=1}^{THQ}(p^*_{\tau,n} - p_{\tau,n})^2}{THQ}. \tag{23}$$

Another way of quantifying price discovery is by correlating trading interval ϵ_h price shocks with the prior η_{h-1} news shocks. This quantifies the degree by which the order book responds to news shocks:

$$D_2 = \frac{1}{N}\sum_{n=1}^{N}\frac{\sum_{\tau=2}^{TH}((\bar{\eta_n} - \eta_{h-1,n})(\bar{\epsilon_n} - \epsilon_{h,n}))}{\sqrt{\sum_{\tau=1}^{TH}(\bar{\epsilon_n} - \epsilon_{h,n})^2}\sqrt{\sum_{\tau=1}^{TH}(\bar{\eta_n} - \eta_{h,n})^2}}. \tag{24}$$

Note that, as opposed to other market quality variables, D_2 is calculated at the trading interval level, similar to Equation (13) but instead using $\epsilon_h = p_h - p_{h-1}$, since news shocks only occur at the beginning of each trading interval.

Finally, we measure price discovery as the degree to which positive price shocks are followed by negative price shocks. As noted before, shocks induced by uninformed investors are not informative, and therefore transitory and should be reversed. One way to measure the speed of mean reversion is by the first-order autocorrelation of price shocks, as given by:

$$D_3 = \frac{1}{N}\sum_{n=1}^{N}\frac{\sum_{\tau=2}^{THQ}((\bar{\epsilon_n} - \epsilon_{\tau,n})(\bar{\epsilon_n} - \epsilon_{\tau-1,n}))}{\sum_{\tau=1}^{THQ}(\bar{\epsilon_n} - \epsilon_{\tau,n})^2}. \tag{25}$$

4. Results

4.1. Market without exogenous news shocks

This section analyzes a market where there is no news arrival process. By putting $\sigma_n = 0$, we ensure that the (latent) fundamental price is constant $p^* = 5$. The reason we start without the news process is to isolate the gamma channel in a very controlled simulation environment. The results for the market with news are presented in Section 4.2. We vary in the gamma dimension by letting the evergreen gamma contract range from $[-80, 80]$ option contracts (with negative numbers indicating that the dynamic hedger has a short option position), with increments of 20.

First, we present a grid of figures of one representative simulation path with -80, 0, and 80 options in Fig. 2. The figure in the top left quadrant shows the time series of the price process around the fundamental value of five. We clearly observe that the variability is substantially less in the setup with +80 contracts compared to the other two settings. The top right quadrant displays the order book volume over time. We observe that the volume is generally higher in the case of a positive gamma position, followed by a negative position, and no position. The bottom left quadrant shows the shape of the order book. Both situations including options clearly increase the depth of the book; the shape, however, depends on the sign. The variety in distributions provides clarity on how gamma positioning affects the shape of the order book: negative gamma positioning (among dynamic hedgers) seemingly bolsters liquidity close to the mid-price, whereas positive gamma positioning reinforces deep order liquidity. The bottom right quadrant, finally, shows a time series of the bid-ask spreads. These are the lowest for the negative gamma situation, followed by the positive gamma and neutral setups.

Fig. 3 shows the price distribution of prices in the three cases. In all cases, the distributions are symmetric around the fundamental value, but there are differences in the spread. Clearly, the price is closest to the fundamental value of 5.0 in the setup with positive gamma. The setup with negative gamma, on the other hand, gives a more pronounced spread around the fundamental value.

The statistical results are presented in Table 2. We report the general statistics and Greeks (panel A) and the statistical moments (panel B) as outright statistics, and the market quality metrics (panel C, D, and E) as ratios to the baseline of no dynamic hedgers (0 option contracts). Appendix A Tables A1 and A2 gives the actual numbers.

First, with respect to the Greeks of the dynamic hedger, two observations can be made. The long evergreen gamma contracts have a lower absolute delta and a higher absolute gamma on average.[21] It can thus be concluded that the hedge heuristic is more effective for long gamma positions, as it has a smaller absolute delta position at the measurement moment on average. This implies that there is less liquidity available for the short gamma hedger, right before the measurement moment compared to the dynamic hedger with a long gamma position. The Greeks demonstrate that when both long and short strategies are subject to the same delta constraints, the short strategies need to opt for a more aggressive hedge heuristic to achieve the same results. Thus our observed effect of market quality worsening under short gamma hedgers is slightly understated.

When looking at market volatility, we observe the feedback effect in the dispersion of the observed mid-prices, thereby corroborating the directional findings of Anderegg et al. (2022): the standard deviation of the price shocks decreases with the net gamma

[21] The delta and gamma, calculated by Equations (11) and (12), are averaged across all τ.

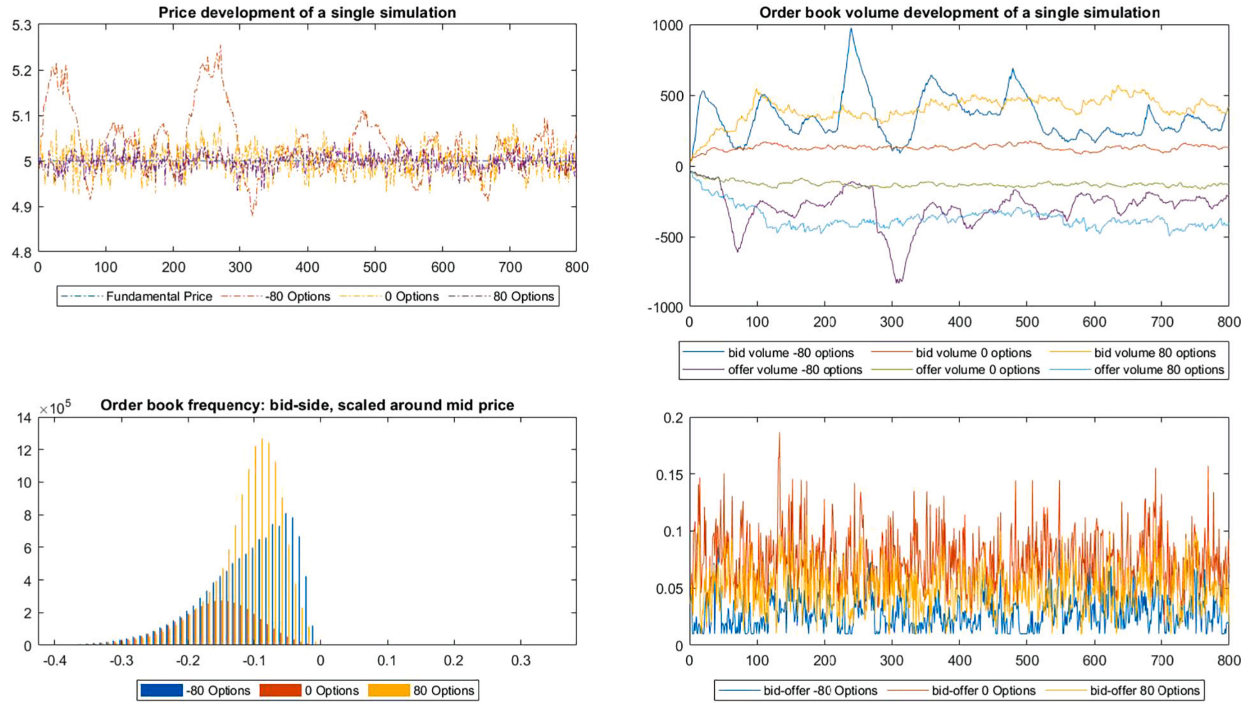

Fig. 2. Graphical results of the simulation without exogenous news shocks.

Notes: This figure shows one representative simulation path for the model without news shocks. Top left: time series plot of prices depicting the short gamma (-80 options), no dynamic hedger (0 options), and the long gamma scenario (80 options) as well as the fundamental price. Top right: Volume development, with the offer side depicted with negative volumes, for three gamma scenarios. Bottom left: An order book frequency plot of three gamma scenarios scaled around the mid-price. Bottom right: time-series plot of the bid-ask spread for three gamma scenarios. For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

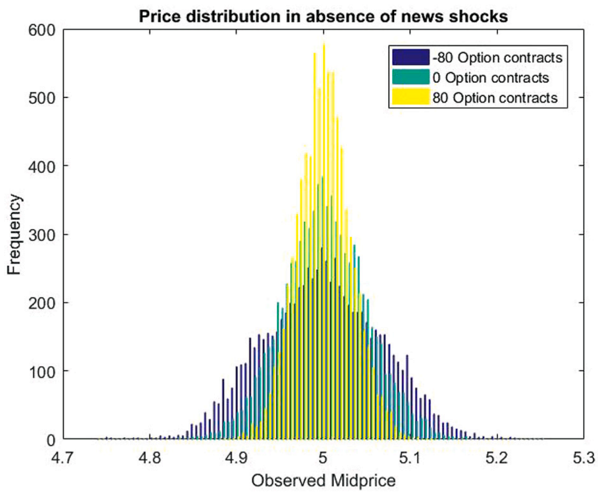

Fig. 3. Price distributions.

Notes: This figure shows the price distribution of the underlying for three gamma scenarios, given a constant fundamental value of 5.0. The price distributions clearly show that the standard deviation decreases (increases) when dynamic hedgers are long (short) option contracts. For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

position. We observe a non-linear relation between the gamma position and market volatility, as illustrated by the higher effect size variation between the most negative scenarios compared to the positive scenarios. This is also reflected in the average high-low spread between simulations. The presence of dynamic hedgers, who are short gamma and thus trade in the same direction as the price shock, severely increases the degree by which prices overshoot in either direction.

From a price discovery perspective, we observe that the autocorrelation of price shocks is strongly linked to the number and direction of evergreen gamma contracts: the inherent autocorrelation of -0.254 flipping sign to a positive 0.091, with the largest

Table 2
Simulation results without news shocks.

#options	-80	-60	-40	-20	0	20	40	60	80
	Panel A: General and Greeks								
Returns	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean absolute delta	16.390	11.161	7.593	4.253	0	4.311	7.272	9.375	11.203
	(0.279)	(0.215)	(0.144)	(0.047)	(0.000)	(0.048)	(0.060)	(0.068)	(0.061)
Average Gamma	-117.750	-89.010	-59.520	-29.810	0	29.890	59.840	89.840	119.810
	(0.205)	(0.091)	(0.042)	(0.010)	(0.000)	(0.009)	(0.014)	(0.019)	(0.022)
	Panel B: Moments								
Bid mean	-0.103	-0.115	-0.128	-0.14	-0.151	-0.133	-0.119	-0.109	-0.101
	(0.002)	(0.002)	(0.001)	0.000	0.000	0.000	0.000	(0.001)	(0.001)
Bid standard dev.	0.033	0.032	0.032	0.032	0.033	0.035	0.036	0.038	0.039
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid skewness	7.685	7.261	7.215	7.513	7.791	8.466	8.919	9.377	9.733
	(0.110)	(0.069)	(0.042)	(0.032)	(0.015)	(0.020)	(0.038)	(0.032)	(0.039)
Bid kurtosis	64.015	56.524	55.987	61.156	66.035	78.136	86.935	96.348	103.981
	(2.049)	(1.190)	(0.712)	(0.543)	(0.257)	(0.385)	(0.759)	(0.695)	(0.861)
	Panel C: Volatility								
S.1 St.dev.	1.789	1.474	1.211	1.053	1.000	0.816	0.711	0.632	0.579
	(0.162)	(0.131)	(0.101)	(0.076)		(0.068)	(0.065)	(0.042)	(0.040)
S.2 High-low spread	2.474	1.798	1.393	1.133	1.000	0.827	0.734	0.694	0.671
	(0.145)	(0.091)	(0.098)	(0.031)		(0.022)	(0.024)	(0.020)	(0.019)
	Panel D: Price Discovery								
D.1 Auto Corr.	-0.358	-0.146	-0.055	0.331	1.000	1.417	1.429	1.378	1.366
	(0.068)	(0.040)	(0.051)	(0.038)		(0.066)	(0.073)	(0.067)	(0.067)
D.2 Sqd.price err	3.703	2.617	1.712	1.001	1.000	0.746	0.586	0.535	0.153
	(0.790)	(0.134)	(0.045)	(0.020)		(0.030)	(0.075)	(0.121)	(0.220)
	Panel E: Liquidity								
L.1 Bid-Ask spread	0.403	0.472	0.583	0.750	1.000	0.875	0.792	0.722	0.681
	(0.004)	(0.003)	(0.003)	(0.004)		(0.004)	(0.004)	(0.004)	(0.003)
L.2 Traded vol.	1.439	1.320	1.220	1.119	1.000	1.063	1.106	1.138	1.169
	(0.008)	(0.006)	(0.007)	(0.004)		(0.004)	(0.004)	(0.004)	(0.005)
L.3 Order book vol.	2.367	1.814	1.463	1.223	1.000	1.526	2.055	2.577	3.054
	(0.048)	(0.045)	(0.012)	(0.008)		(0.013)	(0.016)	(0.021)	(0.036)
L.4 LOS	2.534	2.110	1.714	1.318	1.000	1.200	1.421	1.696	1.892
	(0.301)	(0.211)	(0.123)	(0.105)		(0.091)	(0.112)	(0.128)	(0.164)

Notes: This table gives the general statistics (Panel A), statistical moments of the bid-price relative to the mid-price (Panel B), volatility (Panel C), Price Discovery (Panel D), and Liquidity (Panel E) metrics for the simulation model without exogenous news. The simulation parameters are given in Table 1. All metrics are averages over the 100 simulation paths; numbers in parentheses represent standard errors over the 100 paths. Panel C, D, and E are reported as ratios to the base case scenario of 0 gamma contracts. The reported standard errors are calculated as the standard error of ratios of means.

negative gamma short position among dynamic hedgers. Long gamma positions greatly improve price discovery, with autocorrelation dropping to -0.347 for the biggest long gamma position.[22] When looking at the squared pricing error, we also observe that positive (negative) gamma positions decrease (increase) the pricing error, and thus increase (decrease) price discovery and the efficiency of the price process.

At first glance, the liquidity measures seem to reflect ambivalence with regard to the sign of the dynamic hedger's gamma position. Bid-ask spreads decrease and traded volume increases regardless of the sign of the overall net gamma position. Indeed, regardless of the sign, the hedge heuristic polynomial dictates that the dynamic hedger is a liquidity provider in 75% of the cases and in at least 50% of the cases it does so within the spread. However, further examining the traded volumes and the frequency distribution of the order book reveals directional effects: the traded volume is higher for short gamma positions compared to long gamma positions of similar magnitude, signaling that short gamma hedgers consume more liquidity relative to their long gamma counterparts. This is also reflected in the order book volumes: 80 long contracts increase the summed order book volume by 205 percent whereas 80 short contracts do so by only 137 percent. Finally, the LOS and bid-offer measures seem to favor the short gamma scenarios in terms of liquidity. To analyze this counterintuitive finding further, we add four measures to Table 2 in Panel B that capture the shape of the order book. Whereas the mean and standard deviation of the bids in the order book are quite similar for positive and negative gamma positions, the skewness and kurtosis are substantially higher for positive than for negative gamma positions. This implies that although the bid-ask spread in the simulation with positive gamma contracts is higher than with negative gamma, the order book is able to digest a larger amount of buys with limited price impact because there is a larger amount of volume posted close to the best bid.

[22] The in text numbers are the actual autocorrelations, found in Appendix A Tables A1 and A2, instead Table 2 presents them as auto correlations relative to the base scenario without gamma hedgers.

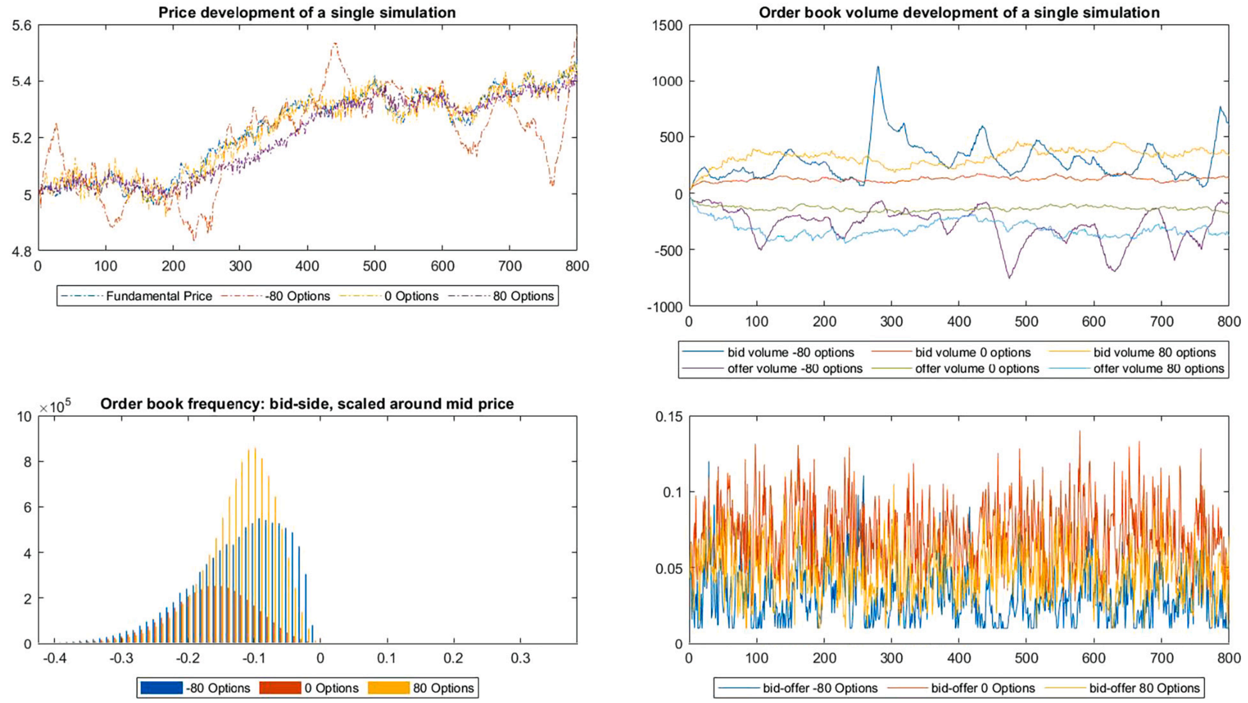

Fig. 4. Graphical results of the simulation with exogenous news shocks.

Notes: This figure shows one representative simulation path, for the model with news shocks. Top left: time series plot of prices depicting the short gamma (-80 options), no dynamic hedger (0 options) and the long gamma scenario (80 options) as well as the fundamental price. Top right: Volume development, with the offer side depicted with negative volumes, for three gamma scenarios. Bottom left: An order book frequency plot of three gamma scenarios scaled around the mid-price. Bottom right: time-series plot of the bid-ask spread for three gamma scenarios. For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

4.2. *Market quality in the presence of exogenous news shocks*

While the previous analysis provides us with insights regarding the effect of dynamic hedgers in a controlled setting, the simulation is too static with pricing fluctuation around the equilibrium value of $p = 5.0$. With no exogenous news shocks, it never occurs that vast quantities of order book liquidity are being consumed, such that the price jumps to a level far from the fundamental value. Furthermore, by having a fundamental price that is unchanged, the informed traders will sell when prices rise and buy when prices drop. Such behavior makes them act functionally similar to dynamic hedgers who are long gamma, biasing the results. If we instead introduce news shocks to the fundamental value and set $\sigma_n = 0.01$, we will not only generate more realistic price paths, and thus more realistic order book liquidity dynamics, but this will also allow us to study the effect of gamma on price discovery.

As in the previous section, we first visually inspect one representative simulation path with -80, 0 and 80 options, see Fig. 4. The top left quadrant shows again the time series of prices. The fundamental price is now moving stochastically. The setup with negative gamma again clearly shows increased volatility. The top right quadrant displays the evolution of the order book and is consistent with Fig. 2 in the sense that both positive and negative gamma positions increase the order book volume, especially for positive gamma. The order book's configuration is depicted in the bottom-left quadrant. Positive and negative gamma positions contribute to the overall volume in the book. Our findings confirm that the distributions are influenced by the dealer option positioning's sign. Although the shapes of the figures share a resemblance, it is important to note the varying scale on the y-axis when comparing Fig. 2 to Fig. 4. Positive gamma positions result in a prominent peak around -0.1, indicating a capacity to absorb substantial price shocks, albeit with visibly less liquidity around the mid-price compared to scenarios with negative gamma positions. The bottom right quadrant, finally, shows the evolution of the bid-ask spread over time. Fig. 4 confirms that both gamma setups decrease the spread, but the negative gamma setup does so even more than the positive gamma setup.

The statistical results for the setup with exogenous news over the 100 simulation paths are presented in Table 3. An immediate observation that can be made from Panel A, is that there is an asymmetry in the degree by which gamma hedgers contribute to the total order flow: the larger the shocks, the greater the delta that needs to be hedged by the dynamic hedgers.

Overall, the simulation including exogenous shocks also captures the feedback effect. Given that this is a more realistic scenario, we also include the tail-risk measures. At the highest granularity, we observe no worsening of tail risk with negative gamma positions: indeed both the positive and negative gamma inventories among dynamic hedgers reduce the amplitude of the most extreme shocks. This result can be explained by the overall increase in market liquidity that the dynamic hedgers provide. However, when we consider daily shocks, the results coincide with the diffusion metrics: The more negative the gamma position the greater the daily tail risk. This result can be extended for positive gamma positions, where large positive gamma positions greatly reduce the tail risk measured on

Table 3
Simulation results with a news process.

#options	-80	-60	-40	-20	0	20	40	60	80
	Panel A: General and Greeks								
Proportion of flow	0.524	0.401	0.295	0.192	0.000	0.181	0.271	0.331	0.366
Returns	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean absolute delta	18.448	12.174	7.682	4.056	0	3.819	6.457	8.530	10.302
	(0.196)	(0.109)	(0.044)	(0.017)	(0.000)	(0.017)	(0.028)	(0.032)	(0.035)
Average Gamma	-117.610	-88.995	-59.631	-29.905	0	29.979	60.031	90.102	120.200
	(0.429)	(0.315)	(0.213)	(0.105)	(0.000)	(0.104)	(0.204)	(0.299)	(0.384)
	Panel B: Moments								
Bid mean	-0.105	-0.115	-0.126	-0.138	-0.148	-0.132	-0.122	-0.114	-0.108
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid standard dev.	0.033	0.032	0.031	0.031	0.032	0.033	0.034	0.035	0.036
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid skewness	7.563	7.215	7.063	7.212	7.424	7.838	8.115	8.362	8.637
	(0.024)	(0.016)	(0.011)	(0.009)	(0.008)	(0.013)	(0.017)	(0.019)	(0.022)
Bid kurtosis	61.840	55.875	53.483	56.065	59.662	66.522	71.342	75.838	81.082
	(0.423)	(0.265)	(0.177)	(0.151)	(0.132)	(0.229)	(0.306)	(0.368)	(0.430)
	Panel C: Volatility								
S.1 St.dev.	1.243	1.122	1.043	1.009	1.000	0.957	0.922	0.887	0.852
	(0.056)	(0.052)	(0.050)	(0.049)		(0.048)	(0.047)	(0.046)	(0.046)
S.2 High-low sprd	1.380	1.197	1.080	1.033	1.000	0.920	0.867	0.828	0.782
	(0.051)	(0.043)	(0.039)	(0.038)		(0.036)	(0.035)	(0.035)	(0.034)
S.3 Q-shock .05%	0.637	0.851	1.151	1.176	1.000	0.902	0.828	0.750	0.500
	(0.006)	(0.009)	(0.011)	(0.011)		(0.008)	(0.008)	(0.007)	(0.003)
S.4 Q-shock .01%	0.724	0.914	1.172	1.155	1.000	0.931	0.862	0.828	0.379
	(0.012)	(0.014)	(0.017)	(0.017)		(0.013)	(0.012)	(0.013)	(0.004)
S.5 H-shock .05%	2.596	1.840	1.372	1.106	1.000	0.851	0.766	0.713	0.617
	(0.104)	(0.078)	(0.047)	(0.038)		(0.027)	(0.027)	(0.023)	(0.022)
	Panel D: Price Discovery								
D.1 Auto Corr.	-0.504	-0.242	-0.075	0.298	1.000	1.409	1.425	1.409	1.369
	(0.025)	(0.016)	(0.016)	(0.016)		(0.021)	(0.021)	(0.021)	(0.020)
D.2 Sqd.price err	7.617	3.461	1.728	0.917	1.000	0.883	1.810	2.673	3.514
	(0.547)	(0.124)	(0.040)	(0.017)		(0.009)	(0.016)	(0.022)	(0.029)
D.3 ρ(news,price)	0.330	0.429	0.516	0.703	1.000	0.912	0.714	0.604	0.495
	(0.045)	(0.036)	(0.037)	(0.040)		(0.053)	(0.041)	(0.039)	(0.037)
	Panel E: Liquidity								
L.1 Bid-Ask sprd	0.406	0.478	0.594	0.754	1.000	0.870	0.768	0.696	0.652
	0.000	0.000	0.000	0.000		0.000	0.000	0.000	0.000
L.2 Traded vol.	1.456	1.329	1.217	1.114	1.000	1.066	1.112	1.150	1.183
	(0.004)	(0.003)	(0.002)	(0.002)		(0.002)	(0.002)	(0.002)	(0.002)
L.3 Order book vol.	2.419	1.843	1.456	1.206	1.000	1.421	1.792	2.130	2.455
	(0.019)	(0.012)	(0.006)	(0.005)		(0.006)	(0.009)	(0.012)	(0.016)
L.4 LOS	2.653	1.971	1.507	1.188	1.000	1.192	1.418	1.648	1.905
	(0.104)	(0.067)	(0.043)	(0.028)		(0.028)	(0.034)	(0.040)	(0.050)

Notes: This table gives the general statistics (Panel A), statistical moments of the bid-price relative to the mid-price (Panel B), volatility (Panel C), Price Discovery (Panel D), and Liquidity (Panel E) metrics for the simulation model with exogenous news shocks. The simulation parameters are given in Table 1. All metrics are averages over the 100 simulation paths; numbers in parentheses represent standard errors over the 100 paths. Panel C, D, and E are reported as ratios to the base case scenario of 0 gamma contracts. The reported standard errors are calculated as the standard error of ratios of means.

a daily frequency. The discrepancy between tail risk measures at different timescales can be attributed to the simulation set-up and hedge heuristic.[23] Dynamic hedgers holding positive gamma positions counteract market shocks, thereby reducing autocorrelation and preventing the accumulation of large shocks at the hourly and daily levels. Conversely, negative gamma positions among dynamic hedgers compel them to trade in the direction of the market, exacerbating autocorrelation and magnifying shock magnitudes at the hourly and daily levels.

When observing the squared pricing error and the correlation of lagged news shocks with price shocks, it can be inferred that dynamic hedgers generally worsen price discovery. Indeed, by trading unconditionally on the fundamental price they act as noise traders. The precise mechanism by which gamma positioning affects price discovery will be studied in Section 4.3. The worsening of price discovery, though, is stronger for negative gamma simulations than for positive gamma simulations.

[23] We note that at the lowest time frequency (the quarter-hourly scale), the hedge heuristic effectively mitigates the occurrence of extreme values in price shocks. However, in the negative gamma scenario, the heightened auto-correlation of these quarter-hourly price shocks does result in increased extreme values at aggregated time frequencies.

Table 4
Simulation results price discovery event study.

#options	-80	-60	-40	-20	0	20	40	60	80
Time to *n*th cross									
n = 1	11	12	11	11	12	22	29	42	53
n = 2	57	45	32	18	14	23	31	43	54
n = 3	60	46	33	20	16	26	35	47	58
n = 10	109	63	55	39	34	44	52	59	77
n = 20	149	130	95	71	54	62	82	94	108
n = 100	617	509	414	315	326	369	347	329	360
Mean crossing time									
Steady state (n > 20)	5.810	4.190	3.095	4.381	4.333	2.952	2.619	2.762	2.571

Notes: This table shows the average crossing times of 100 simulations. A crossing time is the number of subintervals it takes the mid-price p to cross the fundamental price n times after the fundamental price p^* drops from 5.0 to 4.0. The bottom row gives the mean crossing time after the fundamental has dropped and the price has already crossed the fundamental price 20 times.

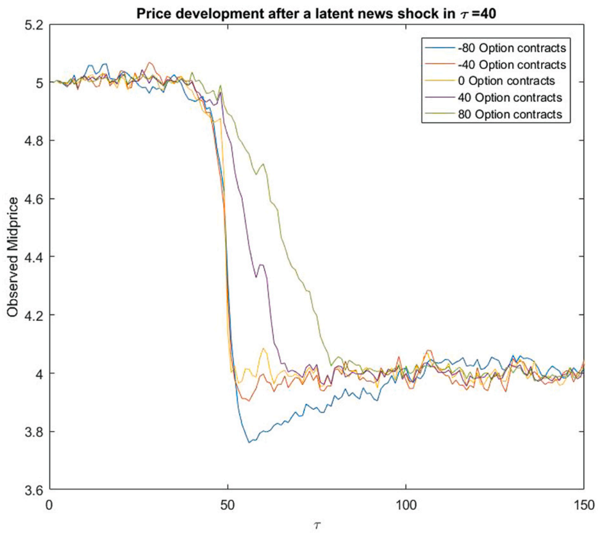

Fig. 5. Event study.

Notes: This figure depicts the average price development of 100 simulations whereby the fundamental price drops from $p = 5.0$ to $p = 4.0$ at $\tau = 40$ under different gamma positions amongst dynamic hedgers. For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.

The liquidity measures, finally, seem to suggest that dynamic hedgers improve market liquidity, and those with negative gamma improve the bid-ask spread whereas those with positive gamma improve overall volumes. This is similar to the case with the constant fundamental in Table 2.

4.3. Price discovery event study

From a price discovery perspective, dynamic hedgers act as noise traders. However, given the way these dynamic hedgers act conditional on their gamma position, it is worthwhile to investigate in more detail how they distort price discovery.

In this simulation, we analyze price discovery, price overshoots, and pinning more closely. We will employ the exact same simulation conditions as in the previous section, except we change the way our latent variable is constructed. We do so by making the fundamental price discontinuous; in particular $p^* = 5 - I_{t>1}$, where I is an indicator function which is one if $(t > 1)$ and zero otherwise. Thus the fundamental price is no longer a random walk, it starts at five in the first trading session such that the order book is properly filled and from the second trading session on it drops to four.

Subsequently, we analyze the time it takes for the mid-price to reach the new fundamental price under different gamma positions among dynamic hedgers. Furthermore, we can analyze the time it takes to cross this price twice, three times, etc., such that we can see if the price overshoots the fundamental price. Finally, we can measure the mean crossing time in equilibrium: if the market price has already crossed the fundamental price 20 times, we can measure the average time for another crossing.

Table 4 and Fig. 5 distinctly reveal the inhibiting impact of gamma on price discovery. The first-crossing analysis clearly reveals how positive gamma impedes price discovery. Notably, scenarios featuring non-positive gamma exhibit a mean time to achieve the first price between 11-12 subintervals, roughly equivalent to three trading hours. However, scenarios characterized by positive gamma extend this duration significantly, requiring at least twice the time, with figures reaching up to 53 subintervals, i.e., more than a single day. However, when we analyze subsequent crossings we see the consequences of price-overshoots with negative gamma positions: The most negative gamma scenario of 80 contracts reaches the fundamental price slightly faster than the scenario without a dynamic hedger. However, it takes more than a full trading session to reach the fundamental price again after having overshot its target. In the most positive gamma scenario, the fundamental price is reached last but breaches it again from the other side the next day and will continue to do this in rapid succession.

Finally, we measure the frequency by which the price breaches the fundamental value once in equilibrium. After roughly 20 crossings, the mean time between crossings becomes relatively constant and we measure its average. The mean crossing times show a direct relation between the gamma position and the degree by which the prices gravitate around the fundamental price.

In summary, these results reinforce the notion that both positive and negative gamma positioning hinder price discovery, yet their impact operates through distinct mechanisms. Negative gamma positioning accelerates price movements (potentially beneficial) but leads to price overshoots (detrimental for price discovery). On the other hand, positive gamma positioning restrains directional movement toward the fundamental value (detrimental for price discovery) while offering stability once the price aligns with its fundamental value (potentially beneficial).

4.4. Market failure

So far we have studied simulation set-ups that were unlikely to result in market failure. We did this with the idea that we did not want to bias our estimates and standard errors. Next, we apply variation to several coefficients of the model, and combine these with variations in the dynamic hedger's gamma position. An important motivation for this paper is the occurrence of gamma squeezes. Therefore, we also study the proportion of market failure in the simulations. When a market buy (sell) order arrives with a size larger than the total offer (bid) volume, we could argue that the market has failed in its function to provide immediacy. By tracking the market failure rates across the simulations, we can assess whether gamma positions affect the propensity for markets to fail:

$$F = \frac{1}{N}\sum_{n=1}^{N} I_{F,n}, \tag{26}$$

where I_F is an indicator function denoting that a market failure occurred during the nth simulation.[24] The results are presented in Table 5.

Increasing the proportion of market orders relative to the number of limit orders, i, increases the probability of market failure. Table 5 shows that this is indeed the case, but it can be seen that a positive gamma position prevents market failure even with immediacy as high as 0.4. A negative gamma position clearly increases the probability of market failure. Higher news variance, σ_n also has a more negative impact on market stability when dynamic hedgers are short gamma. Similar results are observed when we vary the deep-order book liquidity via increasing the decay rate ζ: having less deep-order book liquidity is especially risky when the market is net short gamma, and not very risky when it is long gamma. Greater bid-ask spreads via a greater average market-making spread, μ_p, are disruptive for short and long gamma positions, though slightly more so for short positions.

Finally, we run a number of experiments in which a fraction of 0.02 of the immediate orders, both informed and non-informed, have a much higher volume v_e rather than a uniformly distributed volume between zero and one. These extreme volume orders are thus either $\langle v_e, \infty\rangle$ or $\langle -v_e, -\infty\rangle$. This occurrence is rare enough to have an insignificant impact on dispersion metrics in the absence of dynamic hedgers. The results are presented in Panel E of Table 5. Here again, it becomes evident that long gamma positions among dynamic hedgers prevent market failure due to large shocks over neutral and negative gamma positions.

The increased propensity of market failure where there are negative gamma positions among dynamic hedgers coincides with the anecdotal evidence of gamma traps. While the presence of negative gamma within dynamic hedgers' portfolios is good for liquidity in normal circumstances, it also has a low probability of consuming all liquidity, causing non-linear price moves and extreme volatility and markets moving far from their fundamental value.

5. Conclusion

This paper studies an important and often overlooked fact about financial markets, namely that the presence and net position of dynamic hedgers can have a substantial impact on market quality. The relative homogeneity of dynamic hedgers and the predictability of their actions conditional on their gamma position make the gamma channel a potent medium for market quality improvement.

In a simulated market for a risky asset where trading orders are given by heterogeneous agents, we add a dynamic hedger with varying degrees of gamma positioning but with a preference for delta neutrality. Our findings highlight the significant impact of this dynamic hedger's transactional behavior on market quality. Specifically, we observe that long gamma positions among dynamic hedgers enhance market quality across multiple dimensions: they lead to reduced volatility, increased liquidity, and a lower rate of market failure. However, the presence of long gamma dynamic hedgers typically negatively affects price discovery by acting as an additional noise trader. Conversely, negative gamma positions amplify volatility, reduce price discovery and greatly affect the propensity of markets to fail. We conclude that, markets featuring positive gamma dynamic hedgers will function better compared to

[24] Because market failure makes all the previous market quality metrics unintelligible (infinite bid-ask spread, infinite variance, etc.), and incommensurable (as failure can happen at different points in time, leading to unequal simulation lengths), we study market failure in isolation. Unless specifically mentioned, none of the simulations used to calculate the previous metrics reached a failed state.

Table 5
Failure rate of simulated markets.

	-80	-60	-40	-20	0	20	40	60	80
	Panel A: immediacy i								
i = 0.3	0%	0%	0%	0%	0%	0%	0%	0%	0%
i = 0.325	5%	10%	0%	0%	0%	0%	0%	0%	0%
i = 0.35	50%	45%	5%	0%	0%	0%	0%	0%	0%
i = 0.375	95%	90%	45%	0%	20%	0%	0%	0%	0%
i = 0.4	100%	95%	100%	90%	85%	0%	0%	0%	0%
i = 0.45	100%	100%	100%	100%	100%	95%	65%	10%	10%
i = 0.5	100%	100%	100%	100%	100%	100%	100%	100%	95%
	Panel B: news volatility σ_n								
$\sigma_n = 0.01$	0%	0%	0%	0%	0%	0%	0%	0%	0%
$\sigma_n = 0.025$	10%	0%	0%	0%	0%	0%	0%	0%	0%
$\sigma_n = 0.05$	35%	15%	10%	5%	0%	0%	5%	10%	0%
$\sigma_n = 0.075$	75%	70%	55%	35%	20%	20%	20%	20%	40%
$\sigma_n = 0.1$	100%	100%	90%	75%	65%	55%	55%	70%	65%
$\sigma_n = 0.125$	100%	100%	90%	85%	80%	100%	95%	80%	65%
	Panel C: order decay ζ								
$\zeta = 0.05$	0%	0%	0%	0%	0%	0%	0%	0%	0%
$\zeta = 0.075$	50%	15%	0%	0%	0%	0%	0%	0%	0%
$\zeta = 0.1$	70%	40%	5%	0%	0%	0%	0%	0%	0%
$\zeta = 0.125$	80%	80%	25%	5%	0%	0%	0%	0%	0%
$\zeta = 0.15$	100%	100%	50%	15%	0%	0%	0%	0%	5%
$\zeta = 0.175$	100%	100%	85%	35%	10%	5%	0%	0%	0%
	Panel D: market maker spread μ_p								
$\mu_p = -0.1$	0%	0%	0%	0%	0%	0%	0%	0%	0%
$\mu_p = -0.15$	0%	5%	0%	0%	0%	0%	0%	0%	0%
$\mu_p = -0.2$	25%	20%	0%	0%	0%	0%	0%	0%	0%
$\mu_p = -0.25$	90%	75%	20%	0%	0%	0%	0%	0%	0%
$\mu_p = -0.3$	100%	100%	95%	20%	0%	10%	40%	30%	30%
$\mu_p = -0.35$	100%	100%	100%	90%	5%	75%	85%	75%	75%
	Panel E: volume shocks υ_e								
$\upsilon_e = 5$	0%	0%	0%	0%	0%	0%	0%	0%	0%
$\upsilon_e = 10$	50%	5%	15%	10%	20%	0%	0%	0%	0%
$\upsilon_e = 15$	80%	60%	45%	85%	100%	15%	5%	0%	0%
$\upsilon_e = 20$	90%	95%	100%	100%	100%	65%	15%	0%	0%
$\upsilon_e = 25$	100%	100%	100%	100%	100%	100%	70%	15%	0%
$\upsilon_e = 30$	100%	100%	100%	100%	100%	100%	100%	75%	25%

Notes: This table presents the failure rate of simulated markets over 100 price paths, varied over the gamma position of the dynamic hedger and mutations in the variables immediacy, standard deviation of news shocks, order decay rate, market maker mid-price bias, and the magnitude of extreme shocks. A market failure is defined as the moment a market order arrives of greater magnitude than the opposing order book liquidity, as in Equation (26). Once a market fails, that part of the simulation is halted; hence, a market cannot fail more than once in the same run.

those populated by negative gamma dynamic hedgers. And when price discovery takes a lower weight in the subjective assessment of the various dimensions of market quality, one can conclude that having positive gamma dynamic hedgers is preferred over not having dynamic hedgers at all.

Bank trading desks are functionally delta-constrained and operate in a non-linear market. Therefore, the various capital regulation frameworks for banks also impact market quality. The existing regulation already disincentivizes negative gamma positions for systemic reasons through, e.g., value-at-risk, stressed-value-at-risk, and stress tests, via penalties for negative non-linear exposure. Our results, however, provide an argument for such constraints from a market quality perspective.

A potential application of the gamma market quality relation could be the selling of short-dated at-the-money options at a discount (e.g. by a central bank or a government) in order to prevent a market liquidity crisis.[25] Should a situation arise where a lack of market maker involvement or a sudden extreme demand for immediacy causes deterioration of limit order book conditions to such an extent that market quality causes real-world effects, such as defaults, stop-outs, margin spirals, etc., then a ‘gamma infusion’ could offer a temporary remedy until the market makers can resume their activities. However, institutions should thoroughly assess the costs (selling options below market value) and potential side effects (moral hazard, disruption in the options market, etc.) of such intervention against the expected welfare loss due to the real-world implications of a liquidity crisis.

In contrast to such crisis measures, one might consider permanently increasing the natural average gamma position of an asset by selling (or providing) nontransferable options to delta-constrained market participants. These market participants, due to their

[25] To avoid directional market impact we recommend using at-the-money straddles, which is a combination of a put and call option with their strikes equal to the forward price of the underlying security.

delta constraint, will functionally act as dynamic hedgers. Given that they cannot sell the options, their impact on the existing option market of that asset is contained. This avenue is worth considering for regular issuers of securities, in particular debt securities. For such issuers, it might be economical, as the reduction of the illiquidity premium might offset the cost of the gamma remuneration. There is evidence of fee payments to market makers by debt issuers conditional on their liquidity provision, as shown by Buis et al. (2020) for the eurozone government bond market. It is even the case already in the European sovereign bond market, that for some bond issues the issuers provide payment not in the form of a cash fee but in the form of an embedded option (DSTA, 2022; NTMA, 2021). The embedded option is called a non-competitive bid: the trading desks get the option to buy more debt from the issuer at the average auction price at a predetermined time in the future. Such an embedded option makes the trading desk functionally long gamma. Our study provides a strong argument that such non-competitive bid agreements should be utilized more by issuers given the market quality-enhancing features.[26] As far as we know, the market quality effects of non-competitive bids have not been studied.

Our simulation model is highly stylized, and purpose-built to illustrate the effect of dynamic hedging on market quality. The purpose of this paper is to isolate the causal mechanism between gamma positioning and market quality, and thereby illustrate that the described mechanism is first-order. The real-life effect size, however, is hard to quantify. This particular model is not suitable for calibration to real-life data, which also makes it challenging to provide details on the effectiveness of the proposed policies. However, the model does generate testable implications. Although it is hard to find data on the net gamma position of market participants, it is possible to reason for which (type of) assets the net gamma position is higher or lower. As such, an empirical cross-sectional study would be an option.

Appendix A

Table A1
Simulation results without a news process, original values.

#options	-80	-60	-40	-20	0	20	40	60	80
	Panel A: General and Greeks								
Returns	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean absolute delta	16.390	11.161	7.593	4.253	0	4.311	7.272	9.375	11.203
	(0.279)	(0.215)	(0.144)	(0.047)	(0.000)	(0.048)	(0.060)	(0.068)	(0.061)
Average Gamma	-117.750	-89.010	-59.520	-29.810	0	29.890	59.840	89.840	119.810
	(0.205)	(0.091)	(0.042)	(0.010)	(0.000)	(0.009)	(0.014)	(0.019)	(0.022)
	Panel B: Moments								
Bid mean	-0.103	-0.115	-0.128	-0.14	-0.151	-0.133	-0.119	-0.109	-0.101
	(0.002)	(0.002)	(0.001)	0.000	0.000	0.000	0.000	(0.001)	(0.001)
Bid standard dev.	0.033	0.032	0.032	0.032	0.033	0.035	0.036	0.038	0.039
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid skewness	7.685	7.261	7.215	7.513	7.791	8.466	8.919	9.377	9.733
	(0.110)	(0.069)	(0.042)	(0.032)	(0.015)	(0.020)	(0.038)	(0.032)	(0.039)
Bid kurtosis	64.015	56.524	55.987	61.156	66.035	78.136	86.935	96.348	103.981
	(2.049)	(1.190)	(0.712)	(0.543)	(0.257)	(0.385)	(0.759)	(0.695)	(0.861)
	Panel C: Volatility								
S.1 St.dev.	0.068	0.056	0.046	0.04	0.038	0.031	0.027	0.024	0.022
	(0.005)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
S.2 High-low spread	0.428	0.311	0.241	0.196	0.173	0.143	0.127	0.12	0.116
	(0.023)	(0.014)	(0.016)	(0.003)	(0.004)	(0.002)	(0.003)	(0.002)	(0.002)
	Panel D: Price Discovery								
D.1 Auto Corr.	0.091	0.037	0.014	-0.084	-0.254	-0.36	-0.363	-0.350	-0.347
	(0.017)	(0.010)	(0.013)	(0.009)	(0.010)	(0.009)	(0.012)	(0.010)	(0.010)
D.2 Sqd.price err	0.0050	0.0036	0.0023	0.0014	0.0014	0.0010	0.0008	0.0007	0.0002
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	Panel E: Liquidity								
L.1 Bid-Ask spread	0.029	0.034	0.042	0.054	0.072	0.063	0.057	0.052	0.049
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
L.2 Traded vol.	14.508	13.311	12.306	11.287	10.085	10.724	11.159	11.479	11.786
	(0.073)	(0.048)	(0.058)	(0.030)	(0.026)	(0.025)	(0.033)	(0.034)	(0.034)
L.3 Order book vol.	636.10	487.30	393.00	328.50	268.70	410.00	552.20	692.40	820.50
	(12.482)	(11.840)	(2.427)	(1.174)	(1.537)	(2.600)	(2.744)	(4.002)	(8.583)
L.4 LOS	3661	3049	2477	1904	1445	1734	2053	2450	2734
	(391.20)	(260.40)	(121.40)	(115.00)	(75.40)	(96.20)	(121.80)	(133.70)	(189.10)

Notes: This table gives the general statistics (Panel A), moments (Panel B), volatility (Panel C), price discovery (Panel D), and liquidity (Panel E) metrics for the simulation model without exogenous news shocks. The simulation parameters are given in Table 1. All metrics are averages over the 100 simulation paths; numbers in parentheses represent standard errors over the 100 paths.

[26] As opposed to our previous solution, such an option does not need to be a straddle. The positive price impact of the hedging of call options could be a desired side effect. And issuers have a monopoly on issuing their own paper, so issuing more debt is less costly/inconvenient than being forced to buy back debt.

Table A2
Simulation results with a news process, original values.

#options	-80	-60	-40	-20	0	20	40	60	80
	Panel A: General and Greeks								
Proportion of flow	0.524	0.401	0.295	0.192	0	0.181	0.271	0.331	0.366
Returns	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean absolute delta	18.448	12.174	7.682	4.056	0	3.819	6.457	8.530	10.302
	(0.196)	(0.109)	(0.044)	(0.017)	(0.000)	(0.017)	(0.028)	(0.032)	(0.035)
Average Gamma	-117.610	-88.995	-59.631	-29.905	0	29.979	60.031	90.102	120.200
	(0.429)	(0.315)	(0.213)	(0.105)	(0.000)	(0.104)	(0.204)	(0.299)	(0.384)
	Panel B: Moments								
Bid mean	-0.105	-0.115	-0.126	-0.138	-0.148	-0.132	-0.122	-0.114	-0.108
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid standard dev.	0.033	0.032	0.031	0.031	0.032	0.033	0.034	0.035	0.036
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bid skewness	7.563	7.215	7.063	7.212	7.424	7.838	8.115	8.362	8.637
	(0.024)	(0.016)	(0.011)	(0.009)	(0.008)	(0.013)	(0.017)	(0.019)	(0.022)
Bid kurtosis	61.840	55.875	53.483	56.065	59.662	66.522	71.342	75.838	81.082
	(0.423)	(0.265)	(0.177)	(0.151)	(0.132)	(0.229)	(0.306)	(0.368)	(0.430)
	Panel C: Volatility								
S.1 St.dev.	0.143	0.129	0.120	0.116	0.115	0.110	0.106	0.102	0.098
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
S.2 High-low spread	0.672	0.583	0.526	0.503	0.487	0.448	0.422	0.403	0.381
	(0.017)	(0.014)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
S.3 Q-shock .05%	-0.026	-0.034	-0.046	-0.047	-0.040	-0.036	-0.033	-0.030	-0.020
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
S.4 Q-shock .01%	-0.042	-0.053	-0.068	-0.067	-0.058	-0.054	-0.050	-0.048	-0.022
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
S.5 H-shock .05%	-0.244	-0.173	-0.129	-0.104	-0.094	-0.080	-0.072	-0.067	-0.058
	(0.007)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
	Panel D: Price Discovery								
D.1 Auto Corr.	0.127	0.061	0.019	-0.075	-0.252	-0.355	-0.359	-0.355	-0.345
	(0.006)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
D.2 Sqd.price err	0.008	0.004	0.002	0.001	0.001	0.001	0.002	0.003	0.004
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
D.3 Corr(news,prices)	0.030	0.039	0.047	0.064	0.091	0.083	0.065	0.055	0.045
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)
	Panel E: Liquidity								
L.1 Bid-Ask spread	0.028	0.033	0.041	0.052	0.069	0.060	0.053	0.048	0.045
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
L.2 Traded vol.	14.691	13.410	12.276	11.236	10.087	10.755	11.212	11.597	11.935
	(0.037)	(0.026)	(0.016)	(0.011)	(0.011)	(0.012)	(0.012)	(0.013)	(0.016)
L.3 Order book vol.	648.580	494.160	390.400	323.340	268.120	381.100	480.410	571.130	658.180
	(4.780)	(2.853)	(1.314)	(0.886)	(0.770)	(1.139)	(1.833)	(2.774)	(3.745)
L.4 LOS	3990	2964	2267	1786	1504	1793	2133	2479	2865.200
	(140.3)	(86.0)	(51.4)	(28.6)	(26.0)	(29.6)	(35.19	(42.8)	(55.9)

Notes: This table gives the general statistics (Panel A), moments (Panel B), volatility (Panel C), price discovery (Panel D), and liquidity (Panel E) metrics for the simulation model with exogenous news. The simulation parameters are given in Table 1. All metrics are averages over the 100 simulation paths; numbers in parentheses represent standard errors over the 100 paths.

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