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Commun.Fac.Sci.Univ.Ank.Ser. A1 Math. Stat.
Volume 73, Number 1, Pages 274–284 (2024)
DOI:10.31801/cfsuasmas.1313970
ISSN 1303-5991 E-ISSN 2618-6470

Research Article; Received: June 13, 2023; Accepted: October 31, 2023

APPLICATION OF THE GKM TO SOME NONLINEAR PARTIAL EQUATIONS

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Abstract. In this manuscript, the strain wave equation, which plays an important role in describing different types of wave propagation in microstructured solids and the (2+1) dimensional Bogoyavlensky Konopelchenko equation, is defined in fluid mechanics as the interaction of a Riemann wave propagating along the y-axis and a long wave propagating along the x-axis, were studied. The generalized Kudryashov method (GKM), which is one of the solution methods of partial differential equations, was applied to these equations for the first time. Thus, a series of solutions of these equations were obtained. These found solutions were compared with other solutions. It was seen that these solutions were not shown before and were presented for the first time in this study. The new solutions of these equations might have been useful in understanding the phenomena in which waves are governed by these equations. In addition, 2D and 3D graphs of these solutions were constructed by assigning certain values and ranges to them.

1. Introduction

Nonlinear evolution equations (NLEEs) have been utilized to make mathematical models of encountered problems in various scientific circles. A number of solution methods have been developed by various scientists to solve NLEEs, which have very important areas of use [1–10]. In this study, one of these methods, GKM, has been taken into consideration and applied to the strain wave and (2+1)-dimensional Bogoyavlensky-Konopelchenko (BK) equations.

2020 *Mathematics Subject Classification.* 35A25,35C07,35C08.

Keywords. Generalized Kudryashov method, strain wave equation, (2+1)-dimensional Bogoyavlensky-Konopelchenko equation.

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Strain wave equation is given as [11]:

$$u_{tt} - u_{xx} - \epsilon\alpha_1(u^2)_{xx} - k\alpha_2 u_{xxt} + \delta\alpha_3 u_{xxxx} \\ -(\delta\alpha_4 + k^2\alpha_7)u_{xxtt} + k\delta(\alpha_5 u_{xxxxt} + \alpha_6 u_{xxttt}) = 0, \tag{1}$$

where $u(x,t)$ is the micro-strain wave function. ϵ indicates elastic strain, δ shows the elastic stresses and the rate between the wavelength and size of the microstructure, k reflects the dissipative effect and $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7$ are arbitrary constants. Assuming $\delta = O(\epsilon)$ on Eq. (1), an equilibration takes place between dispersion and nonlinearity. If $k = 0$ is selected in this equation, the undistributed state of the micro-stress wave is obtained. In this way, the following equation for the bi-dispersion in microstructured solids is obtained [12–16]:

$$u_{tt} - u_{xx} - \epsilon(\alpha_1(u^2)_{xx} - \alpha_3 u_{xxxx} + \alpha_4 u_{xxtt}) = 0. \tag{2}$$

Recently, the solutions of strain wave equation investigated by various researchers with different methods. Seadawy et al. used the modified extended mapping method for strain wave equation [11]. Ayati et al. applied the functional variable method and Kudryashov method to strain wave equation [12]. Arshad et al. practiced the modified direct algebraic method to strain wave equation [13]. Gao et al. used the F-expansion method for strain wave equation [14]. Irshad et al. practiced the generalized Jacobi elliptic function method to strain wave equation [15]. Kumar et al. used the generalized exponential rational function method for strain wave equation [16]. Joseph implemented the new rational F-expansion method to strain wave equation [17].

(2+1)-dimensional BK equation is given as [18]:

$$u_{xt} + h_1 u_{xxxx} + h_2 u_{xxxy} + h_3 u_{xx} u_x + h_4(u_{xy} u_x + u_{xx} u_y) = 0, \tag{3}$$

where h_1, h_2, h_3 and h_4 are arbitrary constants. If $h_1 = a, h_2 = \beta, h_3 = 6a, h_4 = 4\beta$ values are selected for the h_1, h_2, h_3, h_4 constants in Eq. (3), Eq. (3) can be written as.

$$u_{xt} + \alpha u_{xxxx} + \beta u_{xxxy} + 6\alpha u_{xx} u_x + 4\beta u_{xy} u_x + 4\beta u_{xx} u_y = 0. \tag{4}$$

The resulting Eq. (4) is handled as a two-dimensional generalization of the KdV equation, and under favorable conditions, it can be converted to the KdV equation [19]. This equation provides the Calogero-Bogoyavlensky-Schiff equation for $\alpha = 0$ and is also defined as the interplay of a Riemann wave spreading along the y-axis and a long wave spreading along the x-axis in fluid mechanics [20, 21]. For Eq. (4) $u_y = v_x$ is transformed and integrated, and the following equation is found:

$$u_t + \alpha u_{xxx} + \beta v_{xxx} + 3\alpha(u_x)^2 + 4\beta u_x v_x = 0. \tag{5}$$

Accordingly, Eq. (4) can be expressed as a system as follows:

$$u_t + \alpha u_{xxx} + \beta v_{xxx} + 3\alpha(u_x)^2 + 4\beta u_x v_x = 0,$$
$$u_y = v_x. \tag{6}$$

When looking at the past works on (2+1)-dimensional BK equation. Zhou et al. gave based on its bilinear form, the N th-order breather solutions of the (2+1)-dimensional generalized BK equation [21]. Ray got infinitesimal generators of (2+1)-dimensional BK equation by using Lie group analysis method and investigated symmetry analysis and similarity reduction of (2+1)-dimensional BK equation [18,22]. Chen and Ma obtained the symbolic solutions of the (2+1)-dimensional BK equation that own a Hirota bilinear form [23].

The purpose of this article is to detect soliton solutions of strain wave equation and (2+1)-dimensional BK equation using GKM [24–27]. First of all, the features of GKM, which is the method we used in our study, are explained. Subsequently, some soliton solutions of the strain wave equation and (2+1)-dimensional BK equation were found using this method.

2. Analysis of the Method

Consider a general nonlinear partial differential equation for a function v that depends on three variables, as follows:

$$K(v, v_t, v_y, v_x, v_{xx}, ...) = 0. \tag{7}$$

Step 1: First, the traveling wave transform is discussed in the following form;

$$v(x, y, t) = v(\eta), \eta = x + y - mt. \tag{8}$$

Eq. (7) is transformed into an ordinary differential equation using the transformations in Eq. (8) as follows:

$$L(t, y, x, v, v^{'}, v^{''}, \cdots) = 0, \tag{9}$$

where superscripts demonstrate ordinary derivatives according η

Step 2: Assume that the solutions of Eq. (9) are treated as follows:

$$v(\eta) = \frac{\sum_{i=0}^{\sigma} a_i Q^i(\eta)}{\sum_{j=0}^{\rho} b_j Q^j(\eta)} = \frac{P[Q(\eta)]}{S[Q(\eta)]}, \tag{10}$$

where Q is $\frac{1}{1 \pm e^{\eta}}$. It is stated that Q is the solution of the following equation

$$Q_\eta = Q^2 - Q. \tag{11}$$

Step 3: The solution of Eq. (9) is sought according to this method as follows:

$$v(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2 + \cdots + a_\sigma Q^\sigma}{b_0 + b_1 Q + b_2 Q^2 + \cdots + b_\rho Q^\rho}. \tag{12}$$

The values of σ and ρ in Eq. (10) can be determined through the homogeneous balance principle. For this, a balance is established between the highest-order

derivative and the highest-order nonlinear term in Eq. (9).
Step 4: Eq. (10) is inserted into Eq. (9). Thus, a polynomial $R(Q)$ of Q is obtained. Thereafter all coefficients of $R(Q)$ are set equal to zero, to obtain a system of algebraic equations. Solving the resulting system determines c and the coefficients $a_0, a_1, a_2, \ldots, a_\sigma, b_0, b_1, b_2, \ldots, b_\rho$. Finally, the soliton solutions of Eq. (9) are obtained.

3. Application of GKM to the equations

Example 1. Initially, the following transformation is considered.

$$u(x,t) = u(\eta), \eta = x - ct. \tag{13}$$

Substituting Eq. (13) into Eq. (2) yields the following equation.

$$(c^2-1)u - \epsilon\alpha_1 u^2 + \epsilon(\alpha_3 - c^2\alpha_4)u'' = 0. \tag{14}$$

If the balance principle is applied to Eq. (14), the following equation is obtained

$$\sigma = \rho + 2$$

If $\rho = 1$, then $\sigma = 3$. Thus the following equations are found.

$$u(\eta) = \frac{a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3}{b_0 + b_1 Q}, \tag{15}$$

$$\begin{aligned} u'(\eta) &= \left(Q^2 - Q\right) \\ &\times \left[\frac{(a_1 + 2a_2 Q + 3a_3 Q^2)(b_0 + b_1 Q) - b_1\left(a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3\right)}{(b_0 + b_1 Q)^2}\right], \end{aligned}$$

$$\begin{aligned} u''(\eta) = {} & \frac{(Q^2 - Q)(2Q - 1)}{(b_0 + b_1 Q)^2} \\ & \times \left[(a_1 + 2a_2 Q + 3a_3 Q^2)(b_0 + b_1 Q) - b_1\left(a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3\right)\right] \\ & + \frac{(Q^2 - Q)^2}{(b_0 + b_1 Q)^3}\left[(2a_2 + 6a_3 Q)(b_0 + b_1 Q)^2 - 2b_1(a_1 + 2a_2 Q + 3a_3 Q^2)(b_0 + b_1 Q)\right] \\ & + \frac{(Q^2 - Q)^2}{(b_0 + b_1 Q)^3}\left[2b_1^2\left(a_0 + a_1 Q + a_2 Q^2 + a_3 Q^3\right)\right]. \end{aligned}$$

The soliton solutions of the strain wave equation are obtained in different cases as follows;

Case 1.

$$a_0 = 0, a_1 = \frac{6b_0(\alpha_3 - \alpha_4)}{\alpha_1(-1 + \epsilon\alpha_4)}, a_3 = \frac{6b_1(-\alpha_3 + \alpha_4)}{\alpha_1(-1 + \epsilon\alpha_4)},$$
$$a_2 = \frac{6(-b_0 + b_1)(\alpha_3 - \alpha_4)}{\alpha_1(-1 + \epsilon\alpha_4)}, c = -\frac{\sqrt{-1 + \epsilon\alpha_3}}{\sqrt{-1 + \epsilon\alpha_4}}.$$

By substituting the above equalities into Eq. (15), the following solution of Eq. (2) is found.

$$u_1(x,t) = \frac{3(\alpha_3 - \alpha_4)}{\left(1 + cosh\left[x + t\frac{\sqrt{-1+\epsilon\alpha_3}}{\sqrt{-1+\epsilon\alpha_4}}\right]\right)\alpha_1(-1+\epsilon\alpha_4)}. \tag{16}$$

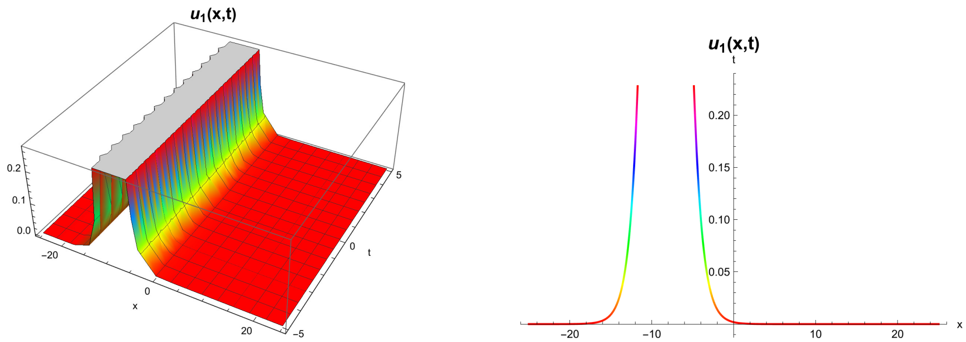

FIGURE 1. 3D and 2D plots of $u_1(x,t)$ solution.

Case 2.

$$a_0 = \frac{b_0(\alpha_3 - \alpha_4)}{\alpha_1(1+\epsilon\alpha_4)}, a_1 = \frac{(-6b_0 + b_1)(\alpha_3 - \alpha_4)}{\alpha_1(1+\epsilon\alpha_4)},$$
$$a_2 = \frac{6(b_0 - b_1)(\alpha_3 - \alpha_4)}{\alpha_1(1+\epsilon\alpha_4)},$$
$$a_3 = \frac{6b_1(\alpha_3 - \alpha_4)}{\alpha_1(1+\epsilon\alpha_4)}, c = \frac{\sqrt{1+\epsilon\alpha_3}}{\sqrt{1+\epsilon\alpha_4}}.$$

By substituting the above equalities into Eq. (15), the following solution of Eq. (2) is found.

$$u_2(x,t) = \frac{\left(-2 + cosh\left[x - t\frac{\sqrt{1+\epsilon\alpha_3}}{\sqrt{1+\epsilon\alpha_4}}\right]\right)(\alpha_3 - \alpha_4)}{\left(1 + cosh\left[x - t\frac{\sqrt{1+\epsilon\alpha_3}}{\sqrt{1+\epsilon\alpha_4}}\right]\right)\alpha_1(1+\epsilon\alpha_4)}. \tag{17}$$

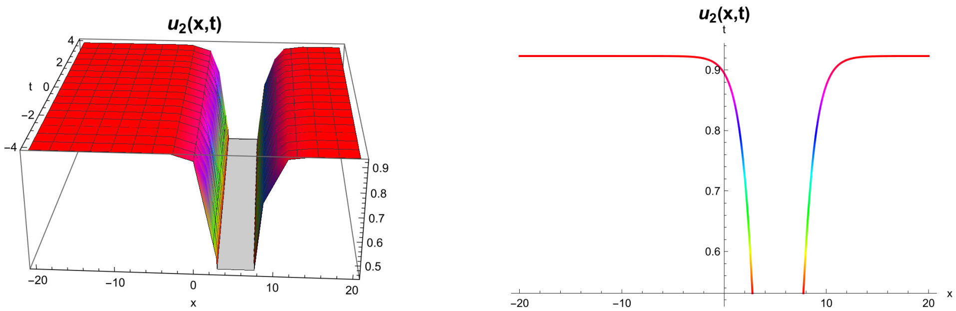

FIGURE 2. 3D and 2D plots of $u_2(x,t)$ solution.

Example 2. First, he following transformation is taken into account.

$$u(x,y,t)=u(\eta),v(x,y,t)=v(\eta),\eta=kx+my-ct. \tag{18}$$

Substituting Eq. (18) into system (6) yields the following equation.

$$-cu'+(\alpha k^3+m\beta k^2)u'''+(3\alpha k^2+4m\beta k)(u')^2=0. \tag{19}$$

The following equation is obtained by transformation $u'=g$ in Eq. (19).

$$-cg+(\alpha k^3+m\beta k^2)g''+(3\alpha k^2+4m\beta k)g^2=0. \tag{20}$$

As a result of applying (18) transformation to this system,
$v=\frac{m}{k}u$ equality is obtained. If the balance principle is applied to Eq. (20), the following equation is obtained.

$$\sigma=\rho+2$$

If $\rho=1$, then $\sigma=3$. Thus the following equations are found.

$$u(\eta)=\frac{a_0+a_1Q+a_2Q^2+a_3Q^3}{b_0+b_1Q}, \tag{21}$$

$$u'(\eta)=\left(Q^2-Q\right)\times\left[\frac{(a_1+2a_2Q+3a_3Q^2)(b_0+b_1Q)-b_1\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)}{(b_0+b_1Q)^2}\right],$$

$$\begin{aligned}u''(\eta)=&\frac{(Q^2-Q)(2Q-1)}{(b_0+b_1Q)^2}\\&\times\left[(a_1+2a_2Q+3a_3Q^2)(b_0+b_1Q)-b_1\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)\right]\\&+\frac{(Q^2-Q)^2}{(b_0+b_1Q)^3}\left[(2a_2+6a_3Q)(b_0+b_1Q)^2-2b_1(a_1+2a_2Q+3a_3Q^2)(b_0+b_1Q)\right]\\&+\frac{(Q^2-Q)^2}{(b_0+b_1Q)^3}\left[2b_1^2\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)\right]\end{aligned}.$$

The soliton solutions of the (2+1)-dimensional BK equation are obtained in different cases as follows;;

Case 1.

$$a_0 = 0, a_1 = -\frac{a_2}{6}, a_3 = -a_2, b_0 = 0, c = \frac{k^2 m\beta a_2}{3a_2 - 6kb_1},$$
$$\alpha = -\frac{2m\beta(2a_2 - 3kb_1)}{3k(a_2 - 2kb_1)}.$$

Replacing the above equations in Eq. (21), the following solution of system (6) is reached.

$$u_1(x,y,t) = \frac{a_2}{2b_1}\left(tanh\left[\frac{kx}{2} + \frac{my}{2} - \frac{k^2 mt\beta a_2}{6a_2 - 12kb_1}\right] - \frac{kx}{3} - \frac{my}{3} + \frac{k^2 mt\beta a_2}{9a_2 - 18kb_1}\right). \quad (22)$$
$$v_1(x,y,t) = \frac{ma_2}{2kb_1}\left(tanh\left[\frac{kx}{2} + \frac{my}{2} - \frac{k^2 mt\beta a_2}{6a_2 - 12kb_1}\right] - \frac{kx}{3} - \frac{my}{3} + \frac{k^2 mt\beta a_2}{9a_2 - 18kb_1}\right).$$

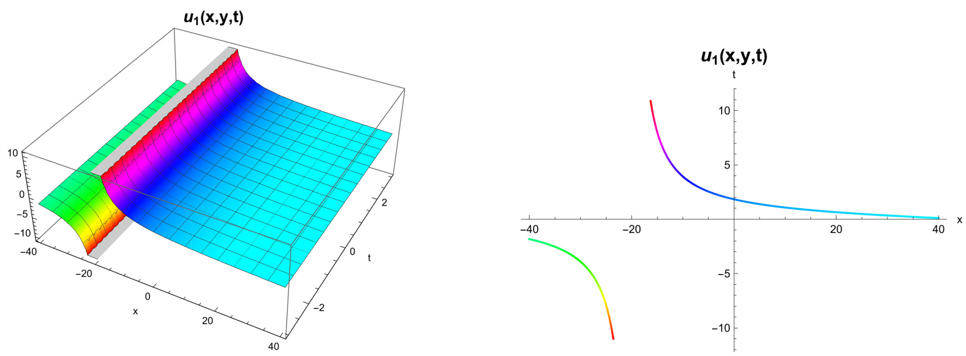

FIGURE 3. 3D and 2D plots of $u_1(x,y,t)$ solution.

Case 2.

$$a_0 = 0, a_1 = -\frac{k(k\alpha + m\beta)b_1}{3k\alpha + 4m\beta}, a_2 = \frac{6k(k\alpha + m\beta)b_1}{3k\alpha + 4m\beta},$$
$$a_3 = -\frac{6k(k\alpha + m\beta)b_1}{3k\alpha + 4m\beta}, b_0 = 0, c = -k^2(k\alpha + m\beta).$$

Replacing the above equations in Eq. (21), the following solution of system (6) is reached.

$$u_2(x,y,t) = -\frac{k(k\alpha + m\beta)\left(kx + my + k^2 t(k\alpha + m\beta) - 3tanh\left[\frac{1}{2}\left(kx + my + k^2 t(k\alpha + m\beta)\right)\right]\right)}{3k\alpha + 4m\beta}$$
$$(23)$$
$$v_2(x,y,t) = -\frac{m(k\alpha + m\beta)\left(kx + my + k^2 t(k\alpha + m\beta) - 3tanh\left[\frac{1}{2}\left(kx + my + k^2 t(k\alpha + m\beta)\right)\right]\right)}{3k\alpha + 4m\beta}$$

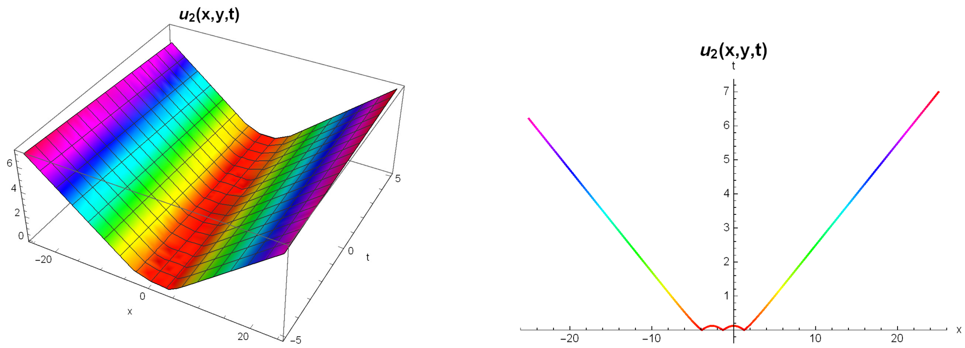

FIGURE 4. 3D and 2D plots of $u_2(x, y, t)$ solution.

4. Results and Discussion

In this study, strain wave and (2+1)-dimensional BK equations are studied. Hyperbolic solutions for the strain wave equation and dark soliton solutions for the (2+1)-dimensional BK equation are obtained. When these solutions are compared with previous studies in the literature, it is seen that the solutions are new and presented for the first time in this study. The graphical representations of the obtained solutions are made for the following values.

Figure 1, depicts singular kink soliton for 3D plot of solution (16) for $\alpha_1 = 2, \alpha_3 = 3, \alpha_4 = 0.5, \epsilon = 4, -25 \leq x \leq 25$ values with $-5 \leq t \leq 5$ range and 2D plot of solution for $t = 2.5$ with these values. Figure 2, shows singular kink soliton for 3D plot of solution (17) for $\alpha_1 = 1.5, \alpha_3 = 2, \alpha_4 = 0.2, \epsilon = 1.5, -20 \leq x \leq 20$ values with $-4 \leq t \leq 4$ range and 2D plot of solution for $t = 3$ with these values. Figure 3, represents soliton solution for 3D plot of solution (22) for $a_2 = 2, b_1 = 1, k = 0.05, m = 1, \beta = 1, y = 1, -40 \leq x \leq 40$ values with $-3 \leq t \leq 3$ range and 2D plot of solution for $t = 2$ with these values. Figure 4, depicts smooth soliton for 3D plot of solution (23) for $k = 1, m = 0.2, \alpha = 0.2, \beta = 0.5, y = 2, -25 \leq x \leq 25$ values with $-5 \leq t \leq 5$ range and 2D plot of solution for $t = 3$ with these values.

5. Conclusions

In this study, GKM was considered. GKM was applied to the strain wave equation and (2+1)-dimensional BK equations. Thus, hyperbolic soliton solutions of the strain wave equation and dark soliton solutions of the (2+1)-dimensional BK equation were obtained using this method. These solutions were different from the found solutions in other studies and were presented for the first time in this study. The accuracy of the results was confirmed by putting the obtained solutions back into the original equation. The new solutions of these equations studied could have helped to understand the phenomena in which waves are governed by these equations. In addition, some special values and intervals were given to the results obtained using Wolfram Mathematic 2D and 3D graphical representations of the solutions were made.

The considered method can also be applied to other nonlinear partial differential equations. The most important advantage of this method is that all solutions are obtained from a single algebraic equation. This means that it is sufficient to set up a single algorithm and there is no unnecessary computational overhead.

Author Contribution Statements The authors contributed equally and they read and approved the final manuscript.

Declaration of Competing Interests The authors report that they have no competing interests.

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